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Soft Neutrosophic Groupoids and Their Generalization

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic groupoid and their generalization with the discuissionf of some of their characteristics. We also introduced a new type of soft neutrophic groupoid, the so called soft strong neutrosophic goupoid which is of pure neutrosophic character. This notion also found in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

1 Introduction

Florentine Smarandache for the first time introduced the concept of neutrosophy in $1995,$ which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W.B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali et al. introduced nuetrosophic LA-semigroups.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in $[2,9,10]$. Some properties and algebra may be found in $[1]$. Feng et al. introduced soft semirings in $[5]$. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in $[6]$. Some other concepts together with fuzzy set and rough set were shown in $[7,8]$.

This paper is about to introduced soft nuetrosophic groupoid, soft neutrosophic bigroupoid, and soft neutrosophic N-groupoid and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic groupoid, soft neutrosophic strong groupoid, and some of their properties are discussed. In the next section, soft neutrosophic bigroupoid are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N-groupoid and their corresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

2.1 Neutrosophic Groupoid

Definition 2.1.1. Let G be a groupoid, the groupoid generated by G and I i.e. $G \cup I$ is denoted

by $\langle G \cup I \rangle$ is defined to be a neutrosophic groupoid where I is the indeterminacy element and termed as neutrosophic element.

Definition 2.1.2. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. A proper subset P of $\langle G \cup I \rangle$ is said to be a neutrosophic subgroupoid, if P is a neutrosophic groupoid under the operations of $\langle G \cup I \rangle$. A neutrosophic groupoid $\langle G \cup I \rangle$ is said to have a subgroupoid if $\langle G \cup I \rangle$ has a proper subset which is a groupoid under the operations of $\langle G \cup I \rangle$.

Theorem 2.1.3. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. Suppose P_1 and P_2 be any two neutrosophic subgroupoids of $\langle G \cup I \rangle$, then $P_1 \cup P_2$, the union of two neutrosophic subgroupoids in general need not be a neutrosophic subgroupoid.

Definition 2.1.4. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid under a binary operation $*$. P be a proper subset of $\langle G \cup I \rangle$. P is said to be a neutrosophic ideal of $\langle G \cup I \rangle$ if the following conditions are satisfied.

1. P is a neutrosophic groupoid.
2. For all $p \in P$ and for all $s \in \langle G \cup I \rangle$ we have $p * s$ and $s * p$ are in P.

2.2 Neutrosophic Bigroupoid

Definition 2.2.1. Let $(BN(\mathrm{G}), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{G}), *, \circ)$ is said to be a neutrosophic bigroupoid if $BN(\mathrm{G}) = P_1 \cup P_2$ where atleast one of $(P_1, *)$ or $(P_2, \circ)$ is a neutrosophic groupoid and other is just a groupoid. P_1 and P_2 are proper subsets of $BN(\mathrm{G})$.

If both $(P_1, *)$ and $(P_2, \circ)$ in the above definition are neutrosophic groupoids then we call $(BN(\mathrm{G}), *, \circ)$ a strong neutrosophic bigroupoid. All strong neutrosophic bigroupoids are trivially neutrosophic bigroupoids.

Definition 2.2.2. Let $(BN(\mathrm{G}) = P_1 \cup P; : *, \circ)$ be a neutrosophic bigroupoid. A proper subset $(T, \circ, *)$ is said to be a neutrosophic subbigroupoid of $BN(\mathrm{G})$ if

1) $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and

$T_2 = P_2 \cap T$ and

2) At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic groupoid.

Definition 2.2.3. Let $(BN(\mathrm{G}) = P_1 \cup P, *, \circ)$ be a neutrosophic strong bigroupoid. A proper subset T of $BN(S)$ is called the strong neutrosophic subbigroupoid if $T = T_1 \cup T_2$ with $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and if both $(T_1, *)$ and $(T_2, \circ)$ are neutrosophic subgroupoids of $(P_1, *)$ and $(P_2, \circ)$ respectively. We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbigroupoid, if atleast one of $(T_1, *)$ or $(T_2, \circ)$ is a groupoid then $T = T_1 \cup T_2$ is only a neutrosophic subgroupoid.

Definition 2.2.4. Let $(BN(\mathrm{G}) = P_1 \cup P_2, *, \circ)$ be any neutrosophic bigroupoid. Let J be a proper subset of $BN(\mathrm{J})$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(\mathrm{G})$.

Definition 2.2.5. Let $(BN(\mathrm{G}), *, \circ)$ be a strong neutrosophic bigroupoid where $BN(S) = P_1 \cup P_2$ with $(P_1, *)$ and $(P_2, \circ)$ be any two neutrosophic groupoids. Let J be a proper subset of $BN(\mathrm{G})$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic groupoids P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(\mathrm{G})$.

Union of any two neutrosophic biideals in general is not a neutrosophic biideal. This is true of neutrosophic strong biideals.

2.3 Neutrosophic N-groupoid

Definition 2.3.1. Let $\{N(\mathrm{G}), *_1, ..., *_2\}$ be a non-empty set with N-binary operations defined on it. We call $N(G)$ a neutrosophic N-groupoid (N a positive integer) if the following conditions are satisfied.

1) $N(\mathrm{G}) = G_1 \cup ... \cup G_N$ where each G_i is a proper subset of $N(G)$ i.e. $G_i \subset G_j$ or $G_j \subset G_i$ if $i \neq j$.

2) $(\mathrm{G}_i, *_i)$ is either a neutrosophic groupoid or a groupoid for $i = 1, 2, 3, ..., N$.

If all the N-groupoids $(\mathrm{G}_i, *_i)$ are neutrosophic groupoids (i.e. for $i = 1, 2, 3, ..., N$) then we call $N(G)$ to be a neutrosophic strong N-groupoid.

Definition 2.3.2. Let
$$\mathrm{N(G)} = \{\mathrm{G}_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$$ be a neutrosophic N-groupoid. A proper subset
$$P = \{\mathrm{P}_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$$ of $N(G)$ is said to be a neutrosophic N-subgroupoid if
$$P_i = P \cap G_i, i = 1, 2, ..., N$$ are subgroupiids of G_i in which atleast some of the subgroupoids are neutrosophic subgroupoids.

Definition 2.3.3. Let
$$\mathrm{N(G)} = \{\mathrm{G}_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$$ be a neutrosophic strong N-groupoid. A proper subset
$$T = \{\mathrm{T}_1 \cup T_2 \cup .... \cup \mathrm{T}_N, *_1, *_2, ..., *_N\}$$ of $N(G)$ is said to be a neutrosophic strong sub N-groupoid if each $(T_i, *_i)$ is a neutrosophic subgroupoid of $(\mathrm{G}_i, *_i)$ for $i = 1, 2, ..., N$ where $T_i = G_i \cap T$.

If only a few of the $(T_i, *_i)$ in T are just subgroupoids of $(\mathrm{G}_i, *_i)$, (i.e. $(T_i, *_i)$ are not neutrosophic subgroupoids then we call T to be a sub N-groupoid of $N(G)$.

Definition 2.3.4. Let
$$\mathrm{N(G)} = \{\mathrm{G}_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$$ be a neutrosophic N-groupoid. A proper subset
$$P = \{\mathrm{P}_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N\}$$ of $N(G)$ is said to be a neutrosophic N-subgroupoid, if the following conditions are true,

1. P is a neutrosophic sub N-groupoid of $N(G)$.

2. Each $P_i = G \cap P_i, i = 1, 2, ..., N$ is an ideal of G_i.

Then P is called or defined as the neutrosophic N-ideal of the neutrosophic N-groupoid $N(G)$.

Definition 2.3.5. *Let*
$$\mathrm{N(G)} = \{\mathrm{G}_1 \cup G_2 \cup .... G_N, *_1, *_2, ..., *_N\}$$ *be a neutrosophic strong N-groupoid. A proper subset*
$$J = \{\mathrm{J}_1 \cup J_2 \cup .... \mathrm{J}_N, *_1, *_2, ..., *_N\}$$ *where*
$$J_t = J \cap G_t \text{ for } t = 1, 2, ..., N \text{ is said to be a neutrosophic strong } N\text{-ideal of } N(G) \text{ if the following conditions are satisfied.}$$

1) Each it is a neutrosophic subgroupoid of
    $G_t, t = 1, 2, ..., N$ i.e. It is a neutrosophic strong N-

subgroupoid of $N(G)$.

2) Each it is a two sided ideal of G_t for $t = 1, 2, ..., N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $N(G)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

2.4 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 2.4.1. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(\mathrm{a})$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 2.4.2. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$ and

2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 2.4.3. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.4.4. Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in C = A \cap B$.

Definition 2.4.5. The extended intersection of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cap_\varepsilon (K,B) = (H,C)$.

Definition 2.4.6. The restricted union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as

$(F,A) \cup_R (K,B) = (H,C)$.

Definition 2.4.7. The extended union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cup_\varepsilon (K,B) = (H,C)$.

3 Soft Neutrosophic Groupoid and Their Properties

3.1 Soft Neutrosophic Groupoid

Definition 3.1.1. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groupoid and (F,A) be a soft set over $\{\langle G \cup I \rangle, *\}$. Then (F,A) is called soft neutrosophic groupoid if and only if $F(a)$ is neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Example 3.1.2. Let

$$\langle Z_{10} \cup I \rangle = \begin{cases} 0, 1, 2, 3, \ldots, 9, I, 2I, \ldots, 9I, \\ 1 + I, 2 + I, \ldots, 9 + 9I \end{cases}$$

be a neutrosophic groupoid where $*$ is defined on $\langle Z_{10} \cup I \rangle$ by $a * b = 3a + 2b \pmod{10}$ for all $a, b \in \langle Z10 \cup I \rangle$. Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic groupoid over $\{\langle Z10 \cup I \rangle, *\}$, where

$$F(a_1) = \{0, 5, 5I, 5 + 5I\},$$
$$F(a_2) = (Z_{10}, *).$$

Theorem 3.1.3. A soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ always contain a soft groupoid over $(G, *)$.

Proof. The proof of this theorem is straightforward.

Theorem 3.1.4. Let (F,A) and (H,A) be two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then their intersection $(F,A) \cap (H,A)$ is again a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Proof. The proof is straightforward.

Theorem 3.1.5. Let (F,A) and (H,B) be two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. If $A \cap B = \phi$, then $(F,A) \cup (H,B)$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Remark 3.1.6. The extended union of two soft neutrosophic groupoids (F,A) and (K,B) over a neutrosophic groupoid $\{\langle G \cup I \rangle, *\}$ is not a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Proposition 3.1.7. The extended intersection of two soft neutrosophic groupoids over a neutrosophic groupoid $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Remark 3.1.8. The restricted union of two soft neutrosophic groupoids (F,A) and (K,B) over $\{\langle G \cup I \rangle, *\}$ is not a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Proposition 3.1.9. The restricted intersection of two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Proposition 3.1.10. The *AND* operation of two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Remark 3.1.11. The *OR* operation of two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$ is not a soft nuetrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

Definition 3.1.12. Let (F,A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F,A) is called an absolute-soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ if $F(a) = \{\langle G \cup I \rangle, *\}$, for all $a \in A$.

Theorem 3.1.13. Every absolute-soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ always contain absolute soft

groupoid over $\{G, *\}$.

Definition 3.1.14. Let (F, A) and (H, B) be two soft neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then (H, B) is a soft neutrosophic subgroupoid of (F, A), if

1. $B \subseteq A$.
2. $H(a)$ is neutrosophic subgroupoid of $F(a)$, for all $a \in B$.

Example 3.1.15. Let

$$\langle Z_4 \cup I \rangle = \left\{ \begin{matrix} 0, 1, 2, 3, I, 2I, 3I, 1+I, 1+2I, 1+3I \\ 2+I, 2+2I, 2+3I, 3+I, 3+2I, 3+3I \end{matrix} \right\}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b \pmod 4$ for all $a, b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2, 2I, 2+2I\},$$

$$F(a_2) = \{0, 2, 2+2I\},$$

$$F(a_3) = \{0, 2+2I\}.$$

Let $B = \{a_1, a_2\} \subseteq A$. Then (H, B) is a soft neutrosophic subgroupoid of (F, A), where

$$H(a_1) = \{0, 2+2I\},$$
$$H(a_2) = \{0, 2+2I\}.$$

Definition 3.1.16. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groupoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft Lagrange neutrosophic groupoid if and only if $F(a)$ is a Lagrange neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Example 3.1.17. Let

$$\langle Z_4 \cup I \rangle = \left\{ \begin{matrix} 0, 1, 2, 3, I, 2I, 3I, 1+I, 1+2I, 1+3I \\ 2+I, 2+2I, 2+3I, 3+I, 3+2I, 3+3I \end{matrix} \right\}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b \pmod 4$ for all $a, b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a

soft Lagrange neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2, 2I, 2+2I\},$$
$$F(a_2) = \{0, 2+2I\}.$$

Theorem 3.1.18. Every soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true. We can easily show the converse by the help of example.

Theorem 3.1.19. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange neutrosophic groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is a soft Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.1.20. Let (F, A) and (K, C) be two soft Lagrange neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F, A) \wedge (K, C)$ may not be a soft Lagrange neutrosophic groipoid over $\{\langle G \cup I \rangle, *\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

6. Their OR operation $(F, A) \vee (K, C)$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.1.21. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groipoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft weak Lagrange neutrosophic groupoid if atleast one $F(a)$ is not a La-

grange neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for some $a \in A$.

Example 3.1.22. Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as

$a * b = 2a + b(\mathrm{mod}\,4)$ for all $a,b \in \langle Z_4 \cup I \rangle$. Let

$A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft weak Lagrange neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2, 2I, 2+2I\},$$

$$F(a_2) = \{0, 2, 2+2\mathrm{I}\},$$

$$F(a_3) = \{0, 2+2I\}.$$

Theorem 3.1.23. Every soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.1.24. If $\{\langle G \cup I \rangle, *\}$ is weak Lagrange neutrosophic groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also soft weak Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.1.25. Let (F, A) and (K, C) be two soft weak Lagrange neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft weak Lagrnage neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

6. Their *OR* operation $(F, A) \vee (K, C)$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.126. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groupoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft Lagrange free neutrosophic groupoid if $F(a)$ is not a lagrange neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Example 3.1.27. Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as

$a * b = 2a + b(\mathrm{mod}\,4)$ for all $a,b \in \langle Z_4 \cup I \rangle$. Let

$A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft Lagrange free neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2I, 2+2I\},$$
$$F(a_2) = \{0, 2, 2+2\mathrm{I}\}.$$

Theorem 3.1.28. Every soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.1.29. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange free neutrosophic groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also a soft Lagrange free neutrosophic groupoid but the converse is not true.

Remark 3.1.30. Let (F, A) and (K, C) be two soft Lagrange free neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft Lagrange free neutrosophic groupoid

over $\{\langle G \cup I \rangle, * \}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, * \}$.

3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, * \}$.

4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft Lagrnage free neutrosophic groupoid over $\{\langle G \cup I \rangle, * \}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, * \}$.

6. Their OR operation $(F, A) \vee (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, * \}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.1.31. (F, A) is called soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$ if $F(a)$ is a neutrosophic ideal of $\{\langle G \cup I \rangle, * \}$, for all $a \in A$.

Theorem 3.1.32. Every soft neutrosophic ideal (F, A) over $\{\langle G \cup I \rangle, * \}$ is trivially a soft neutrosophic subgroupid but the converse may not be true.

Proposition 3.1.33. Let (F, A) and (K, B) be two soft neutrosophic ideals over $\{\langle G \cup I \rangle, * \}$. Then

1) Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

2) Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

3) Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

Remark 3.1.34. Let (F, A) and (K, B) be two soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$. Then

1) Their extended union $(F, A) \cup_E (K, B)$ is not soft

neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

2) Their restricted union $(F, A) \cup_R (K, B)$ is not soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

3) Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$.

One can easily proved $(1), (2),$ and (3) by the help of examples.

Theorem 3.1.35. Let (F, A) be a soft neutrosophic ideal over $\{\langle G \cup I \rangle, * \}$ and $\{(H_i, B_i) : i \in J \}$ is a non-empty family of soft neutrosophic ideals of (F, A). Then

1. $\underset{i \in J}{\cap}(H_i, B_i)$ is a soft neutrosophic ideal of (F, A).

2. $\underset{i \in J}{\wedge}(H_i, B_i)$ is a soft neutrosophic ideal of $\underset{i \in J}{\wedge}(F, A)$.

3.2 Soft Neutrosophic Strong Groupoid

Definition 3.2.1. Let $\{\langle G \cup I \rangle, * \}$ be a neutrosophic groupoid and (F, A) be a soft set over $\{\langle G \cup I \rangle, * \}$. Then (F, A) is called soft neutrosophic strong groupoid if and only if $F(a)$ is a neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, * \}$ for all $a \in A$.

Example 3.2.2. Let

$$\langle Z_4 \cup I \rangle = \begin{cases} 0, 1, 2, 3, I, 2I, 3I, 1+I, 1+2I, 1+3I \\ 2+I, 2+2I, 2+3I, 3+I, 3+2I, 3+3I \end{cases}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b (\mod 4)$ for all $a, b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2, a_3 \}$ be a set of parameters. Then (F, A) is a soft neutrosophic strong groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2I, 2+2I \},$$
$$F(a_2) = \{0, 2+2I \}.$$

Proposition 3.2.3. Let (F, A) and (K, C) be two soft neutrosophic strong groupoids over $\{\langle G \cup I \rangle, * \}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, * \}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F, A) \wedge (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

Remark 3.2.4. Let (F, A) and (K, C) be two soft neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted union $(F, A) \cup_R (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their OR operation $(F, A) \vee (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

Definition 3.2.5. Let (F, A) and (H, C) be two soft neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then (H, C) is called soft neutrosophic strong sublgroupoid of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic strong subgroupoid of $F(a)$ for all $a \in A$.

Definition 3.2.6. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic strong groupoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft Lagrange neutrosophic strong groupoid if and only if $F(a)$ is a Lagrange neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Theorem 3.2.7. Every soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.2.8. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange neutrosophic strong groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is a soft Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.2.9. Let (F, A) and (K, C) be two soft Lagrange neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ may not be a soft Lagrange strong neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F, A) \wedge (K, C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

6. Their OR operation $(F, A) \vee (K, C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.2.10. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic strong groupoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft weak Lagrange neutrosophic strong groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, *\}$ for some $a \in A$.

Theorem 3.2.11. Every soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.2.12. If $\{\langle G \cup I \rangle, *\}$ is weak Lagrange neutrosophic strong groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also soft weak Lagrange neutrosophic strong groupoid but the converse is not true.

Remark 3.2.13. Let (F, A) and (K, C) be two soft

weak Lagrange neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft weak Lagrnage neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

6. Their OR operation $(F,A) \vee (K,C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.2.14. Let $\langle L \cup I \rangle$ be a neutrosophic strong groupoid and (F,A) be a soft neutrosophic groupoid over $\langle L \cup I \rangle$. Then (F,A) is called soft Lagrange free neutrosophic strong groupoid if $F(a)$ is not a Lagrange neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Theorem 3.2.14. Every soft Lagrange free neutrosophic strong groupoid over $\langle L \cup I \rangle$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.2.15. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange free neutrosophic strong groupoid, then (F,A) over $\{\langle G \cup I \rangle, *\}$ is also a soft Lagrange free neutrosophic strong groupoid but the converse is not true.

Remark 3.2.16. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong groupoids over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

6. Their OR operation $(F,A) \vee (K,C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 3.2.17. (F,A) is called soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$ if $F(a)$ is a neutrosophic strong ideal of $\{\langle G \cup I \rangle, *\}$, for all $a \in A$.

Theorem 3.2.18. Every soft neutrosophic strong ideal (F,A) over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic strong groupoid.

Theorem 3.2.19. Every soft neutrosophic strong ideal (F,A) over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic ideal.

Proposition 3.2.20. Let (F,A) and (K,B) be two soft neutrosophic strong ideals over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F,A) \cap_E (K,B)$ is soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F,A) \cap_R (K,B)$ is soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$.

3. Their AND operation $(F,A) \wedge (K,B)$ is soft

neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$.

Remark 3.2.21. Let (F, A) and (K, B) be two soft neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not soft neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$.

2. Their restricted union $(F, A) \cup_R (K, B)$ is not soft neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$.

3. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$.

One can easily proved $(1), (2),$ and (3) by the help of examples.

Theorem 3.2.22. Let (F, A) be a soft neutrosophic strong ideal over $\left\{\left\langle G \cup I\right\rangle, *\right\}$ and $\{(H_i, B_i) : i \in J\}$ is a non-empty family of soft neutrosophic strong ideals of (F, A). Then

1. $\underset{i \in J}{\cap}(H_i, B_i)$ is a soft neutrosophic strong ideal of (F, A).

2. $\underset{i \in J}{\wedge}(H_i, B_i)$ is a soft neutrosophic strong ideal of $\underset{i \in J}{\wedge}(F, A)$.

4 Soft Neutrosophic Bigroupoid and Their Properties

4.1 Soft Neutrosophic Bigroupoid

Definition 4.1.1. Let $\{B_N(G), *, \circ\}$ be a neutrosophic bigroupoid and (F, A) be a soft set over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft neutrosophic bigroupoid if and only if $F(a)$ is neutrosophic sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Example 4.1.2. Let $\{B_N(G), *, \circ\}$ be a neutrosophic groupoid with $B_N(G) = G_1 \cup G_2$, where $G_1 = \{\left\langle Z_{10} \cup I\right\rangle \mid a * b = 2a + 3b(\mod 10); a, b \in \left\langle Z_{10} \cup I\right\rangle\}$ and $G_2 = \{\left\langle Z_4 \cup I\right\rangle \mid a \circ b = 2a + b(\mod 4); a, b \in \left\langle Z_4 \cup I\right\rangle\}$. Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$, where
$$F(a_1) = \{0, 5, 5I, 5 + 5I\} \cup \{0, 2, 2I, 2 + 2I\},$$
$$F(a_2) = (Z_{10}, *) \cup \{0, 2 + 2I\}.$$
Theorem 4.1.3. Let (F, A) and (H, A) be two soft neu-

trosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic groupoid over $\{B_N(G), *, \circ\}$.
Proof. The proof is staightforward.
Theorem 4.1.4. Let (F, A) and (H, B) be two soft neutrosophic groupoids over $\{\left\langle G \cup I\right\rangle, *\}$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic groupoid over $\{\left\langle G \cup I\right\rangle, *\}$.

Proposition 4.1.5. Let (F, A) and (K, C) be two soft neutrosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their AND operation $(F, A) \wedge (K, C)$ is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

Remark 4.1.6. Let (F, A) and (K, C) be two soft neutrosophic biloops over $\{B_N(G), *, \circ\}$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their OR operation $(F, A) \vee (K, C)$ is not a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1), (2),$ and (3) by the help of examples.
Definition 4.1.7. Let (F, A) be a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$. Then (F, A) is called an absolute soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ if $F(a) = \{B_N(G), *, \circ\}$ for all $a \in A$.
Definition 4.1.8. Let (F, A) and (H, C) be two soft neutrosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then (H, C) is called soft neutrosophic sub bigroupoid of

(F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a \in A$.

Example 4.1.9. Let $\{B_N(G), *, \circ\}$ be a neutrosophic groupoid with $B_N(G) = G_1 \cup G_2$, where
$G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b (\mod 10); a, b \in \langle Z_{10} \cup I \rangle\}$
and
$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b (\mod 4); a, b \in \langle Z_4 \cup I \rangle\}$.
Let $A = \{a_1, a_2\}$ be a set of parameters. Let (F, A) is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$, where
$$F(a_1) = \{0, 5, 5I, 5 + 5I\} \cup \{0, 2, 2I, 2 + 2I\},$$
$$F(a_2) = (Z_{10}, *) \cup \{0, 2 + 2I\}.$$
Let $B = \{a_1\} \subseteq A$. Then (H, B) is a soft neutrosophic sub bigroupoid of (F, A), where
$$H(a_1) = \{0, 5\} \cup \{0, 2 + 2I\}.$$

Definition 4.1.10. Let $\{B_N(G), *, \circ\}$ be a neutrosophic strong bigroupoid and (F, A) be a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft Lagrange neutrosophic bigroupoid if and only if $F(a)$ is a Lagrange neutrosophic sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Theorem 4.1.11. Every soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

One can easily see the converse by the help of examples.

Theorem 4.1.12. If $\{B_N(G), *, \circ\}$ is a Lagrange neutrosophic bigroupoid, then (F, A) over $\{B_N(G), *, \circ\}$ is a soft Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 4.1.13. Let (F, A) and (K, C) be two soft Lagrange neutrosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid

over $\{B_N(G), *, \circ\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

6. Their *OR* operation $(F, A) \vee (K, C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 4.1.14. Let $\{B_N(G), *, \circ\}$ be a neutrosophic bigroupoid and (F, A) be a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft weak Lagrange neutrosophic bigroupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub bigroupoid of $\{B_N(G), *, \circ\}$ for some $a \in A$.

Theorem 4.1.15. Every soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic groupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

Theorem 4.1.16. If $\{B_N(G), *, \circ\}$ is weak Lagrange neutrosophic bigroupoid, then (F, A) over $\{B_N(G), *, \circ\}$ is also soft weak Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 4.1.17. Let (F, A) and (K, C) be two soft weak Lagrange neutrosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft weak Lagrnage neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

6. Their *OR* operation $(F, A) \vee (K, C)$ is not a

soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 4.1.18. Let $\{B_N(G), *, \circ\}$ be a neutrosophic bigroupoid and (F, A) be a soft neutrosophic groupoid over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft Lagrange free neutrosophic bigroupoid if $F(a)$ is not a Lagrange neutrosophic sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Theorem 4.1.19. Every soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

Theorem 4.1.20. If $\{B_N(G), *, \circ\}$ is a Lagrange free neutrosophic bigroupoid, then (F, A) over $\{B_N(G), *, \circ\}$ is also a soft Lagrange free neutrosophic bigroupoid but the converse is not true.

Remark 4.1.21. Let (F, A) and (K, C) be two soft Lagrange free neutrosophic bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

6. Their OR operation $(F, A) \vee (K, C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 4.1.22. (F, A) is called soft neutrosophic biideal over $\{B_N(G), *, \circ\}$ if $F(a)$ is a neutrosophic biideal of $\{B_N(G), *, \circ\}$, for all $a \in A$.

Theorem 4.1.23. Every soft neutrosophic biideal (F, A) over $\{B_N(G), *, \circ\}$ is a soft neutrosophic bigroupoid.

Proposition 4.1.24. Let (F, A) and (K, B) be two soft neutrosophic biideals over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic biideal over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic biideal over $\{B_N(G), *, \circ\}$.

3. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic biideal over $\{B_N(G), *, \circ\}$.

Remark 4.1.25. Let (F, A) and (K, B) be two soft neutrosophic biideals over $\{B_N(G), *, \circ\}$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not soft neutrosophic biideals over $\{B_N(G), *, \circ\}$.

2. Their restricted union $(F, A) \cup_R (K, B)$ is not soft neutrosophic biidleals over $\{B_N(G), *, \circ\}$.

3. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic biideals over $\{B_N(G), *, \circ\}$.

One can easily proved $(1), (2)$, and (3) by the help of examples

Theorem 4.1.26. Let (F, A) be a soft neutrosophic biideal over $\{B_N(G), *, \circ\}$ and $\{(H_i, B_i) : i \in J\}$ is a non-empty family of soft neutrosophic biideals of (F, A). Then

1. $\bigcap_{i \in J}(H_i, B_i)$ is a soft neutrosophic biideal of (F, A).

2. $\bigwedge_{i \in J}(H_i, B_i)$ is a soft neutrosophic biideal of $\bigwedge_{i \in J}(F, A)$.

4.2 Soft Neutrosophic Strong Bigroupoid

Definition 4.2.1. Let $\{B_N(G), *, \circ\}$ be a neutrosophic bigroupoid and (F, A) be a soft set over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft neutrosophic strong bigroupoid if and only if $F(a)$ is neutrosophic strong sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Example 4.2.2. Let $\{B_N(G), *, \circ\}$ be a neutrosophic groupoid with $B_N(G) = G_1 \cup G_2$, where
$G_1 = \left\{ \langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b (\mathrm{mod}\,10); a, b \in \langle Z_{10} \cup I \rangle \right\}$
and
$G_2 = \left\{ \langle Z_4 \cup I \rangle \mid a \circ b = 2a + b (\mathrm{mod}\,4); a, b \in \langle Z_4 \cup I \rangle \right\}$.
Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$, where

$$F(a_1) = \{0, 5 + 5I\} \cup \{0, 2 + 2I\},$$
$$F(a_2) = \{0, 5I\} \cup \{0, 2 + 2I\}.$$

Theorem 4.2.3. Let (F, A) and (H, A) be two soft neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

Proof. The proof is staightforward.

Theorem 4.2.4. Let (F, A) and (H, B) be two soft neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

Proposition 4.2.5. Let (F, A) and (K, C) be two soft neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

Remark 4.2.6. Let (F, A) and (K, C) be two soft neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their *OR* operation $(F, A) \vee (K, C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1), (2)$, and (3) by the help of ex-amples.

Definition 4.2.7. Let (F, A) and (H, C) be two soft neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then (H, C) is called soft neutrosophic strong sub bigroupoid of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic strong sub bigroupoid of $F(a)$ for all $a \in A$.

Definition 4.2.8. Let $\{B_N(G), *, \circ\}$ be a neutrosophic strong bigroupoid and (F, A) be a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$. Then (F, A) is called soft Lagrange neutrosophic strong bigroupoid if and only if $F(a)$ is a Lagrange neutrosophic strong sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Theorem 4.2.9. Every soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

One can easily see the converse by the help of examples.

Theorem 4.2.10. If $\{B_N(G), *, \circ\}$ is a Lagrange neutrosophic strong bigroupoid, then (F, A) over $\{B_N(G), *, \circ\}$ is a soft Lagrange neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.11. Let (F, A) and (K, C) be two soft Lagrange neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their restricted union $(F, A) \cup_R (K, C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

5. Their OR operation $(F,A) \vee (K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.12. Let $\{B_N(G), *, \circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$. Then (F,A) is called soft weak Lagrange neutrosophic strong bigroupoid if atleast one $F(a)$ is not a Lagrange neutrosophic strong sub bigroupoid of $\{B_N(G), *, \circ\}$ for some $a \in A$.

Theorem 4.2.13. Every soft weak Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

Theorem 4.2.14. If $\{B_N(G), *, \circ\}$ is weak Lagrange neutrosophic strong bigroupoid, then (F,A) over $\{B_N(G), *, \circ\}$ is also soft weak Lagrange neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.15. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft weak Lagrnage neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

6. Their OR operation $(F,A) \vee (K,C)$ is not a soft weak Lagrange neutrosophi strong bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.16. Let $\{B_N(G), *, \circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$. Then (F,A) is called soft Lagrange free neutrosophic strong bigroupoid if $F(a)$ is not a Lagrange neutrosophic strong sub bigroupoid of $\{B_N(G), *, \circ\}$ for all $a \in A$.

Theorem 4.2.17. Every soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ is a soft neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$ but the converse is not true.

Theorem 4.2.18. If $\{B_N(G), *, \circ\}$ is a Lagrange free neutrosophic strong bigroupoid, then (F,A) over $\{B_N(G), *, \circ\}$ is also a soft Lagrange free neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.19. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong bigroupoids over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

6. Their OR operation $(F,A) \vee (K,C)$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{B_N(G), *, \circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.20. (F,A) is called soft neutrosophic strong biideal over $\{B_N(G), *, \circ\}$ if $F(a)$ is a neutro-

sophic strong biideal of $\{B_N(G), *, \circ\}$, for all $a \in A$.

Theorem 4.2.21. Every soft neutrosophic strong biideal (F, A) over $\{B_N(G), *, \circ\}$ is a soft neutrosophic strong bigroupoid.

Proposition 4.2.22. Let (F, A) and (K, B) be two soft neutrosophic strong biideals over $\{B_N(G), *, \circ\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic strong biideal over $\{B_N(G), *, \circ\}$.

2. Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic strong biideal over $\{B_N(G), *, \circ\}$.

3. Their *AND* operation $(F, A) \wedge (K, B)$ is soft neutrosophic strong biideal over $\{B_N(G), *, \circ\}$.

Remark 4.2.23. Let (F, A) and (K, B) be two soft neutrosophic strong biideals over $\{B_N(G), *, \circ\}$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not soft neutrosophic strong biideals over $\{B_N(G), *, \circ\}$.

2. Their restricted union $(F, A) \cup_R (K, B)$ is not soft neutrosophic strong biidleals over $\{B_N(G), *, \circ\}$.

3. Their *OR* operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong biideals over $\{B_N(G), *, \circ\}$.

One can easily proved $(1), (2)$, and (3) by the help of examples

Theorem 4.2.24. Let (F, A) be a soft neutrosophic strong biideal over $\{B_N(G), *, \circ\}$ and $\{(H_i, B_i) : i \in J\}$ is a non-empty family of soft neutrosophic strong biideals of (F, A). Then

1. $\underset{i \in J}{\cap}(H_i, B_i)$ is a soft neutrosophic strong biideal of (F, A).

2. $\underset{i \in J}{\wedge}(H_i, B_i)$ is a soft neutrosophic strong biideal of $\underset{i \in J}{\wedge}(F, A)$.

5 Soft Neutrosophic N-groupoid and Their Properties

5.1 Soft Neutrosophic N-groupoid

Definition 5.1.1. Let $N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-groupoid and (F, A) be a soft set over $N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$. Then (F, A) is called soft neutrosophic N-groupoid if and only if $F(a)$ is neutrosophic sub N-groupoid of $N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$ for all $a \in A$.

Example 5.1.2. Let $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$ be a neutrosophic 3-groupoid, where
$G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b \pmod{10}; a, b \in \langle Z_{10} \cup I \rangle\}$,
$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b \pmod{4}; a, b \in \langle Z_4 \cup I \rangle\}$
and $G_3 = \{\langle Z_{12} \cup I \rangle \mid a * b = 8a + 4b \pmod{12}; a, b \in \langle Z_{12} \cup I \rangle\}$.

Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic N-groupoid over $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$, where
$F(a_1) = \{0, 5, 5I, 5 + 5I\} \cup \{0, 2, 2I, 2 + 2I\} \cup \{0, 2\}$,

$$F(a_2) = (Z_{10}, *) \cup \{0, 2 + 2I\} \cup \{0, 2I\}.$$

Theorem 5.1.3. Let (F, A) and (H, A) be two soft neutrosophic N-groupoids over $N(G)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic N-groupoid over $N(G)$.

Theorem 5.1.4. Let (F, A) and (H, B) be two soft neutrosophic N-groupoids over $N(G)$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic N-groupoid over $N(G)$.

Proposition 5.1.5. Let (F, A) and (K, C) be two soft neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic N-groupoid over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic N-groupoid over $N(G)$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ is a soft neutrosophic N-groupoid over $N(G)$.

Remark 5.1.4. Let (F, A) and (K, C) be two soft neutrosophic N-groupoids over $N(G)$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic N-groupoid over $N(G)$.

2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic N-groupoid over $N(G)$.

3. Their *OR* operation $(F, A) \vee (K, C)$ is not a soft neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1), (2)$, and (3) by the help of examples.

Definition 5.1.5. Let (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called an absolute soft neutrosophic N-groupoid over $N(G)$ if $F(a) = N(G)$ for all $a \in A$.

Definition 5.1.6. Let (F,A) and (H,C) be two soft neutrosophic N-groupoids over $N(G)$. Then (H,C) is called soft neutrosophic sub N-groupoid of (F,A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a \in A$.

Example 5.1.7. Let $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$ be a neutrosophic 3-groupoid, where
$G_1 = \{\langle Z_{10} \cup I\rangle \mid a*b = 2a + 3b \pmod{10}; a,b \in \langle Z_{10} \cup I\rangle\}$,
$G_2 = \{\langle Z_4 \cup I\rangle \mid a \circ b = 2a + b \pmod 4; a,b \in \langle Z_4 \cup I\rangle\}$
and
$G_3 = \{\langle Z_{12} \cup I\rangle \mid a*b = 8a + 4b \pmod{12}; a,b \in \langle Z_{12} \cup I\rangle\}$.

Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic N-groupoid over $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$, where

$$F(a_1) = \{0,5,5I,5+5I\} \cup \{0,2,2I,2+2I\} \cup \{0,2\},$$

$$F(a_2) = (Z_{10},*) \cup \{0,2+2I\} \cup \{0,2I\}.$$

Let $B = \{a_1\} \subseteq A$. Then (H,B) is a soft neutrosophic sub N-groupoid of (F,A), where

$$H(a_1) = \{0,5\} \cup \{0,2+2I\} \cup \{0,2\}.$$

Definition 5.1.8. Let $N(G)$ be a neutrosophic N-groupoid and (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called soft Lagrange neutrosophic N-groupoid if and only if $F(a)$ is a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a \in A$.

Theorem 5.1.9. Every soft Lagrange neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

One can easily see the converse by the help of examples.

Theorem 5.1.10. If $N(G)$ is a Lagrange neutrosophic N-groupoid, then (F,A) over $N(G)$ is a soft Lagrange neutrosophic N-groupoid but the converse is not true.

Remark 5.1.11. Let (F,A) and (K,C) be two soft Lagrange neutrosophic N-groupoids over $N(G)$. Then

1.   Their extended intersection $(F,A) \cap_E (K,C)$ may not be a soft Lagrange neutrosophic N-

groupoid over $N(G)$.

2.   Their restricted intersection $(F,A) \cap_R (K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

3.   Their AND operation $(F,A) \wedge (K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

4.   Their extended union $(F,A) \cup_E (K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

7.   Their restricted union $(F,A) \cup_R (K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

8.   Their OR operation $(F,A) \vee (K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.1.12. Let $N(G)$ be a neutrosophic N-groupoid and (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called soft weak Lagrange neutrosophic N-groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for some $a \in A$.

Theorem 5.1.13. Every soft weak Lagrange neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.1.14. If $N(G)$ is weak Lagrange neutrosophic N-groupoid, then (F,A) over $N(G)$ is also a soft weak Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 5.1.15. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic N-groupoids over $N(G)$. Then

1.   Their extended intersection $(F,A) \cap_E (K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

2.   Their restricted intersection $(F,A) \cap_R (K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

3.   Their AND operation $(F,A) \wedge (K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

4. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft weak Lagrnage neutrosophic N-groupoid over $N(G)$.

5. Their restricted union $(F, A) \cup_R (K, C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

6. Their OR operation $(F, A) \vee (K, C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 5.1.16. Let $N(G)$ be a neutrosophic N-groupoid and (F, A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F, A) is called soft Lagrange free neutrosophic N-groupoid if $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a \in A$.

Theorem 5.1.17. Every soft Lagrange free neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.1.18. If $N(G)$ is a Lagrange free neutrosophic N-groupoid, then (F, A) over $N(G)$ is also a soft Lagrange free neutrosophic N-groupoid but the converse is not true.

Remark 5.1.19. Let (F, A) and (K, C) be two soft Lagrange free neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

6. Their OR operation $(F, A) \vee (K, C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 5.1.20. (F, A) is called soft neutrosophic N-ideal over $N(G)$ if and only if $F(a)$ is a neutrosophic N-ideal of $N(G)$, for all $a \in A$.

Theorem 5.1.21. Every soft neutrosophic N-ideal (F, A) over $N(G)$ is a soft neutrosophic N-groupoid.

Proposition 5.1.22. Let (F, A) and (K, B) be two soft neutrosophic N-ideals over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic N-ideal over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic N-ideal over $N(G)$.

3. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic N-ideal over $N(G)$.

Remark 5.1.23. Let (F, A) and (K, B) be two soft neutrosophic N-ideals over $N(G)$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not a soft neutrosophic N-ideal over $N(G)$.

2. Their restricted union $(F, A) \cup_R (K, B)$ is not a soft neutrosophic N-idleal over $N(G)$.

3. Their OR operation $(F, A) \vee (K, B)$ is not a soft neutrosophic N-ideal over $N(G)$.

One can easily proved $(1), (2),$ and (3) by the help of examples

Theorem 5.1.24. Let (F, A) be a soft neutrosophic N-ideal over $N(G)$ and $\{(H_i, B_i) : i \in J\}$ be a non-empty family of soft neutrosophic N-ideals of (F, A). Then

1. $\underset{i \in J}{\cap}(H_i, B_i)$ is a soft neutrosophic N-ideal of (F, A).

2. $\underset{i \in J}{\wedge}(H_i, B_i)$ is a soft neutrosophic N-ideal of $\underset{i \in J}{\wedge}(F, A)$.

5.2 Soft Neutrosophic Strong N-groupoid

Definition 5.2.1. Let
$N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-groupoid and (F, A) be a soft set over $N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$. Then (F, A) is called soft neutrosophic strong N-groupoid if and only if $F(a)$ is neutrosophic strong sub N-groupoid of $N(G) = \{G_1 \cup G_2 \cup ... \cup G_N, *_1, *_2, ..., *_N\}$ for all $a \in A$.

Example 5.2.2. Let $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$ be a neutrosophic 3-groupoid, where
$G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b \pmod{10}; a, b \in \langle Z_{10} \cup I \rangle\}$
,
$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b \pmod{4}; a, b \in \langle Z_4 \cup I \rangle\}$
and
$G_3 = \{\langle Z_{12} \cup I \rangle \mid a * b = 8a + 4b \pmod{12}; a, b \in \langle Z_{12} \cup I \rangle\}$
.

Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic N-groupoid over
$N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$, where

$$F(a_1) = \{0, 5I\} \cup \{0, 2I\} \cup \{0, 2I\},$$

$$F(a_2) = \{0, 5 + 5I\} \cup \{0, 2 + 2I\} \cup \{0, 2 + 2I\}.$$

Theorem 5.2.3. Let (F, A) and (H, A) be two soft neutrosophic strong N-groupoids over $N(G)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic strong N-groupoid over $N(G)$.

Theorem 5.2.4. Let (F, A) and (H, B) be two soft neutrosophic strong N-groupoids over $N(G)$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic strong N-groupoid over $N(G)$.

Theorem 5.2.5. If $N(G)$ is a neutrosophic strong N-groupoid, then (F, A) over $N(G)$ is also a soft neutrosophic strong N-groupoid.

Proposition 5.2.6. Let (F, A) and (K, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic strong N-groupoid over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic strong N-groupoid over $N(G)$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ is a soft neutrosophic strong N-groupoid over $N(G)$.

Remark 5.2.7. Let (F, A) and (K, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic strong N-groupoid over $N(G)$.

2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic strong N-groupoid over $N(G)$.

3. Their *OR* operation $(F, A) \vee (K, C)$ is not a soft neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1), (2),$ and (3) by the help of examples.

Definition 5.2.8. Let (F, A) and (H, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then (H, C) is called soft neutrosophic strong sub N-groupoid of (F, A), if

1. $C \subseteq A$.

2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a \in A$.

Definition 5.2.9. Let $N(G)$ be a neutrosophic strong N-groupoid and (F, A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F, A) is called soft Lagrange neutrosophic strong N-groupoid if and only if $F(a)$ is a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a \in A$.

Theorem 5.2.10. Every soft Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

One can easily see the converse by the help of examples.

Theorem 5.2.11. Every soft Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft Lagrange neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

Theorem 5.2.12. If $N(G)$ is a Lagrange neutrosophic strong N-groupoid, then (F, A) over $N(G)$ is a soft Lagrange neutrosophic strong N-groupoid but the converse is not true.

Remark 5.2.13. Let (F, A) and (K, C) be two soft Lagrange neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

3. Their *AND* operation $(F, A) \wedge (K, C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

4. Their extended union $(F, A) \cup_E (K, C)$ may not be a soft Lagrange neutrosophic strong N-

groupoid over $N(G)$.

5. Their restricted union $(F,A) \cup_R (K,C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

6. Their OR operation $(F,A) \vee (K,C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.2.14. Let $N(G)$ be a neutrosophic strong N-groupoid and (F,A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F,A) is called soft weak Lagrange neutrosophic strong N-groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for some $a \in A$.

Theorem 5.2.15. Every soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.16. Every soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft weak Lagrange neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Remark 5.2.17. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

3. Their AND operation $(F,A) \wedge (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

4. Their extended union $(F,A) \cup_E (K,C)$ may not be a soft weak Lagrnage neutrosophic strong N-groupoid over $N(G)$.

5. Their restricted union $(F,A) \cup_R (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

6. Their OR operation $(F,A) \vee (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by

the help of examples.

Definition 5.2.18. Let $N(G)$ be a neutrosophic strong N-groupoid and (F,A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F,A) is called soft Lagrange free neutrosophic strong N-groupoid if $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a \in A$.

Theorem 5.2.19. Every soft Lagrange free neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.20. Every soft Lagrange free neutrosophic strong N-groupoid over $N(G)$ is a soft Lagrange neutrosophic strong N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.21. If $N(G)$ is a Lagrange free neutrosophic strong N-groupoid, then (F,A) over $N(G)$ is also a soft Lagrange free neutrosophic strong N-groupoid but the converse is not true.

Remark 5.2.22. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.

2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.

3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.

4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.

5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.

6. Their OR operation $(F,A) \vee (K,C)$ is not a soft Lagrange free neutrosophic stong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.2.23. (F,A) is called soft neutrosophic strong N-ideal over $N(G)$ if and only if $F(a)$ is a neutrosophic strong N-ideal of $N(G)$, for all $a \in A$.

Theorem 5.2.24. Every soft neutrosophic strong N-ideal

(F, A) over $N(G)$ is a soft neutrosophic N-groupoid.

Theorem 5.2.25. Every soft neutrosophic strong N-ideal (F, A) over $N(G)$ is a soft neutrosophic N-ideal but the converse is not true.

Proposition 15. Let (F, A) and (K, B) be two soft neutrosophic strong N-ideals over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic strong N-ideal over $N(G)$.

2. Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic strong N-ideal over $N(G)$.

3. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophicstrong N-ideal over $N(G)$.

Remark 5.2.26. Let (F, A) and (K, B) be two soft neutrosophic strong N-ideals over $N(G)$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not a soft neutrosophic strong N-ideal over $N(G)$.

2. Their restricted union $(F, A) \cup_R (K, B)$ is not a soft neutrosophic strong N-idleal over $N(G)$.

3. Their OR operation $(F, A) \vee (K, B)$ is not a soft neutrosophic strong N-ideal over $N(G)$.

One can easily proved $(1), (2),$ and (3) by the help of examples

Theorem 5.2.27. Let (F, A) be a soft neutrosophic strong N-ideal over $N(G)$ and $\{(H_i, B_i) : i \in J\}$ be a non-empty family of soft neutrosophic strong N-ideals of (F, A). Then

1. $\bigcap_{i \in J} (H_i, B_i)$ is a soft neutrosophic strong N-ideal of (F, A).

2. $\bigwedge_{i \in J} (H_i, B_i)$ is a soft neutrosophic strong N-ideal of $\bigwedge_{i \in J} (F, A)$.

Conclusion

This paper is an extension of neutrosphic groupoids to soft neutrosophic groupoids. We also extend neutrosophic bigroupoid, neutrosophic N-groupoid to soft neutrosophic bigroupoid, and soft neutrosophic N-groupoid. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic groupoids.

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