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Some GIS Topological Concepts via Neutrosophic Crisp Set Theory

Abstract

In this paper we introduce and study the neutrosophic crisp pre-open, semi-open, β- open set, neutrosophic crisp continuity and neutrosophic crisp compact spaces are introduced. Furthermore, we investigate some of their properties and characterizations. Possible application to GIS topology rules are touched upon.

Keywords

Neutrosophic crisp topological spaces, neutrosophic crisp sets, neutrosophic crisp continuity, neutrosophic crisp compact space.

1. Introduction

Smarandache [26, 27] introduced the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set [28]. In Zadah's sense, there is no precise definition for the set. Later on, Atanassov presented the idea of the intuitionistic fuzzy set [1], where he goes beyond the degree of membership introducing the degree of non-membership of some element in the set. The new presented concepts attracted several authors to develop the classical mathematics. For instance, Chang [2] and Lowen [6] started the discipline known as "Fuzzy Topology", where they forwarded the concepts from fuzzy sets to the classical topological spaces. Furthermore, Salama et al. [14, 17, 20] established several notations for what they called, "Neutrosophic topological spaces"].

In this paper, we study in more details some weaker and stronger structures constructed from the neutrosophic crisp topology introduced in [7], as well as the concepts neutrosophic crisp interior and the neutrosophic closure.

The remaining of this paper is structured as follows: in **§2**, some basic definitions are presented, while the new concepts of neutrosophic crisp nearly open sets are introduced in **§3**, in addition to providing a study of some of its properties. The neutrosophic crisp continuous function and neutrosophic crisp compact spaces are presented in **§4** and **§5**, respectively.

2. Terminologies

We recollect some relevant basic preliminaries, in particular, the work introduced by We recollect some relevant basic preliminaries, in particular, the work introduced by Smarandache and Salama [7], Salama et al. [8] and Smarandache [25,26,27]. The neutrosophic components T, I, F: X⟶$]0^{-}, 1^{+}[$to represent the membership, indeterminacy, and non-membership values of some universe X, respectively, where $]0^{-}, 1^{+}[$is the non-standard unit Interval.

Definition 2.1 [7]

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) A is an object having the form $A = (A_1, A_2, A_3)$ where A_1, A_2 and A_3 are subsets of X. Where A_1 contains all those members of the space X that accept the event A and A_3 contains all those members of the space X that rejected the event A, while A_2 contains those who stand in a distance from accepting or rejecting A.

Definition 2.2

Salama [7] defined the object having the form $A = (A_1, A_2, A_3)$ to be

1) ***(Neutrosophic Crisp Set with Type 1)***,if satisfying $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_2 \cap A_3 = \emptyset$. (NCS -Type 1).
2) ***(Neutrosophic Crisp Set with Type 2)***, if satisfying $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_2 \cap A_3 = \emptyset$ and $A_1 \cup A_2 \cup A_3 = X$ (NCS -Type 2).
3) ***(Neutrosophic Crisp Set with Type 3)*** if satisfying $A_1 \cap A_2 \cap A_3 = \emptyset$ and $A_1 \cup A_2 \cup A_3 = X$. ($NCS$ -Type3 for short) .

Every neutrosophic crisp set A of a non-empty set X is obviously aNCS having the form $A = (A_1, A_2, A_3)$.

Definition 2.3 [7]

Let $A = (A_1, A_2, A_3)$ a NCS on X, then the complement of the set A , (A^c for short) was presented in [7], to have one of the following forms:

(C_1) $A^c = (A_1^c, A_2^c, A_3^c)$ or
(C_2) $A^c = (A_3, A_2, A_1)$ or
(C_3) $A^c = (A_3, A_2^c, A_1)$.

Several relations and operations between NCS were defined in [7], which we are introducing in the following:

Definition 2.4 [7]

Let X be a non-empty set, and $NCSA$ and B in the form $A = (A_1, A_2, A_3)$, $B = (B_1, B_2, B_3)$, then we may consider two possible definitions for subsets ($A \subseteq B$).

The concept of ($A \subseteq B$) may be defined as two types:

Type 1. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or

Type 2. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \supseteq B_2$ and $A_3 \supseteq B_3$

Proposition 2.5[7]

For any neutrosophic crisp set A the following are hold

$\phi_N \subseteq A$, $\phi_N \subseteq \phi_N$

$A \subseteq X_N$, $X_N \subseteq X_N$

Definition 2.6[7]

Let Xbe a non-empty set, and the two $NCSsA$ and Bgiven in the form $A = (A_1, A_2, A_3)$, $B = (B_1, B_2, B_3)$, then :

1) $A \cap B$ may be defined as two types:

i)Type 1. $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

ii) Type 2. $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2) $A \cup B$ may be defined as two types:

i) Type 1. $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$

ii) Type 2. $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

Definition 2.7[7]

A neutrosophic crisp topology (NCT) on a non-empty set X is a family $\Gamma$of neutrosophic crisp subsets of$X$ satisfying the following axioms:

i) $\emptyset_N, X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$, $\forall A_1, A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma$, $\forall \{A_j : j \in J\} \subseteq \Gamma$.

In this case, the pair (X, Γ) is called a neutrosophic crisp topological space ($NCTS$) in X. The elements of $\Gamma$are called neutrosophic crisp open sets ($NCOSs$) in X. A neutrosophic crisp set F is closed if and only if its complement F^c is an open neutrosophic crisp set.

Definition 2.8[7]

Let (X, Γ) be $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCcl(A)$) and neutrosophic interior crisp ($NCint(A)$) of A are defined by

$NCcl(A)=\cap\{K:K$ is an $NCCS$ in X and $A \subseteq K\}$

$NCint(A) = \cup\{G:G$ is an $NCOS$ in X and $G \subseteq A\}$,

Where NCS is a neutrosophic crisp set and $NCOS$ is a neutrosophic crisp open set. It can be also shown that $NCcl(A)$ is a $NCCS$ (neutrosophic crisp closed set) and $NCint(A)$ is a $NCOS$ (neutrosophic crisp open set) in X .

3. Neutrosophic Crisp Nearly Open Sets

Definition 3.1

Let (X, Γ) be a $NCTS$ and $A =\langle A_1, A_2, A_3 \rangle$ be a NCS in X, then A is called: Neutrosophic crisp α-open set iff$A \subseteq NCint(NCcl(NCint(A)))$. [24]

i) Neutrosophic crisp pre-open set iff$A \subseteq NCint(NCcl(A))$.

ii) Neutrosophic crisp semi-open set iff$A \subseteq NCcl(NCint(A))$.

iii) Neutrosophic crisp β- open set iff$A \subseteq (NCcl(NCint(NCcl(A)))$.

We shall denote the class of all neutrosophic crisp α- open sets as$NC\Gamma^{\alpha}$, and the class of all neutrosophic crisp pre-open sets as $NC\Gamma^{p}$, and the class of all neutrosophic crisp semi-open sets as $NC\Gamma^{S}$, and the class of all neutrosophic crisp β- open sets as $NC\Gamma^{\beta}$.

Definition 3.2

Let (X, Γ) be a $NCTS$ and $B = \langle B_1, B_2, B_3 \rangle$ be a NCS in X, then B is called:

i) Neutrosophic crisp α-closed set iff $(NCcl(NCint(NCcl(B)) \subseteq B$.

ii) Neutrosophic crisp pre- closed set iff $NCcl(NCint(B)) \subseteq B$.

iii) Neutrosophic crisp semi- closed set iff $NCint(NCcl(B)) \subseteq B$.

iv) Neutrosophic crisp β- closed set iff $NCint(NCcl(NCint(B)) \subseteq B$.

One can easily show that, the complement of a neutrosophic crisp (α, pre, semi, β)- open set is a neutrosophic crisp (α, pre, semi, β)- closed set,respectively.

Remark 3.3

For the class consisting of exactly all a $NC\alpha$- structure and $NC\beta$- structure, evidently, $NC\Gamma \subseteq NC\Gamma^{\alpha} \subseteq NC\Gamma^{\beta}$.

We notice that every non-empty $NC\beta$- open has $NC\alpha$-open non-empty interior.

If all neutrosophic crisp sets the family $\{B_i\}_{i\in I}$, are $NC\ \beta$- open sets, then

Proposition 3.4

Consider, $\{\cup B_i\}_{i\in I}$, is a family of $NC\beta$- open sets, then

$\{\cup B_i\}_{i\in I} \subset NCcl(NCint(B_i)) \subset NCcl(NCint(B_i))$, that is A $NC\beta$- structure is a neutrosophic closed with respect to arbitrary neutrosophic crisp unions .

We shall now characterize $NC\Gamma^{\alpha}$ in terms of$NC\Gamma^{\beta}$.

Definition 3.5

Let (X, Γ) be a $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NCS in X, then:

$NCcl\alpha(A) = \cap\{$ G:G$\supseteq A$ and G is $NC\alpha$-closed$\}$

$NCint\alpha\ (A) = \cup\{$G:G$\subseteq A$ and G is $NC\alpha$- open$\}$

$NCcl$ pre $(A) = \cap\{$ G:G$\supseteq A$ and G is NCpre-closed$\}$

$NCint$ pre $(A) = \cup\{$G:G$\subseteq A$ and G is NCpre- open$\}$

Definition 3.6

$NCcl$ semi $(A) = \cap\{$ G:G$\supseteq A$ and G is NCsemi-closed$\}$

$NCint$ semi $(A) = \cup\{$G:G$\subseteq A$ and G is NCsemi- open$\}$

$NCcl\beta(A) = \cap\{$ G:G$\supseteq A$ and G is $NC\beta$-closed$\}$

$NCint\beta(A) = \cup\{$G:G$\subseteq A$ and G is $NC\beta$- open$\}$

Theorem 3.7

Let (X, Γ) be a $NCTS$. $NC\Gamma^{\alpha}$ Consists of exactly those $NCSA$ for which $A \cap B \in NC\Gamma^{\beta}$ for $B \in NC\Gamma^{\beta}$.

Proof

Let $A \in NC\Gamma^{\alpha}$, $B \in NC\Gamma^{\beta}$, $P \in A \cap B$ and U be a neutrosophic crisp neighborhood (for short $NCnbd$) of p.

Clearly $U \cap NCint(NCcl(NCint(A))$, too is a neutrosophic crisp open neighborhood of P, so V=$(U\cap\ NCint(NCcl(NCint(A)))) \cap NCint(B)$ is non-empty . Since $V \subset NCcl(NCint(A))$ this implies

$(U \cap NCint(A) \cap NCint(B) = V \cap NCint(A) = \emptyset_N$.

It follows that

Conversely, $A \cap B \subset NCcl(NCint(A) \cap NCint(B)) = NCcl(NCint(A \cap B))$ i.e. $A \cap B \in NC\Gamma^{\beta}$. Let $A \cap B \in NC\Gamma^{\beta}$for all $B \in NC\Gamma^{\beta}$. then in particular $A \in NC\Gamma^{\beta}$. Assume that

$P \in A \cap (NCint(NCcl(A) \cap (NCint(A)))^c$. Then $P \in NCcl(B)$, where $(NCcl(NCint(A)))^c$ Clearly $\{P\} \cup B \in NC\Gamma^{\beta}$ and consequently $A \cap \{\{P\} \cup B\} \in NC\Gamma^{\beta}$. But $A \cap \{\{P\} \cup B\} = \{P\}$. Hence $\{P\}$ is a neutrosophic crisp open. $P \in (NCcl(NCint(A)))$ implies $P \in)NCint(NCcl(NCint(A)())$, contrary to assumption. Thus $P \in A$ implies $P \in (NCcl(NCint(A))$ and $A \in NC\Gamma^{\alpha}$. Thus we have found that $NC\Gamma^{\alpha}$ is complete determined by $NC\Gamma^{\beta}$ i.e. all neutrosophic crisp topologies with the same $NC\beta$- structure also determined the same $NC\alpha$-structure, explicitly given Theorem 3.1.

We shall prove that conversely all neutrosophic crisp topologies with the same $NC\alpha$-structure, so that $NC\Gamma^{\beta}$, is completely determined by $NC\Gamma^{\alpha}$

Theorem 3.8

Every $NC\alpha$-structure is a $NC\Gamma$.

Proof

$NC\Gamma^{\beta}$ Contains the neutrosophic crisp empty set and is closed with respect to arbitrary unions. A standard result gives the class of those neutrosophic crisp sets A for which $A \cap B \in NC\Gamma^{\beta}$ for all $B \in NC\Gamma^{\beta}$ constitutes a neutrosophic crisp topology, hence the theorem.

We may now characterize $NC\Gamma^{\beta}$, in terms of $NC\Gamma^{\alpha}$ in the following way.

Proposition 3.9

Let (X, Γ) be a $NCTS$. Then $NC\Gamma^{\beta} = NC\Gamma^{\alpha\beta}$ and hence $NC\alpha$ -equivalent topologies determine the same $NC\beta$ -structure.

Proof

Let $NC\alpha\text{–}cl$ andα–int denote neutrosophic closure and Neutrosophic crisp interior with respect to $NC\Gamma^{\alpha}$. If $P \in B \in NC\Gamma^{\beta}$ and $P \in B \in NC\Gamma^{\alpha}$, then

$(NCint(NCcl(NCint(A))) \cap NCint(B)) \neq \emptyset_N$.

Since $(NCint(NCcl(NCint(A)))$ is a crisp neutrosophic neighbor-hood of point p, so certainly $NCint(B)$ meets $NCcl(NCint(A))$ and therefore (big neutrosophic open) meets $NCint(A)$, proving $A \cap NCint(B) \neq \emptyset_N$. This means $B \subset NC\alpha cl(NCint(B))$ .i.e. $B \in NC\Gamma^{\alpha\beta}$ on the other hand let $A \in NC\Gamma^{\alpha\beta}$, $P \in A$. and $P \in V \in NC\Gamma$. As $V \in NC\Gamma^{\alpha}$, and $P \in NCcl(NCint(A))$, we have $V \cap NCint(A) \neq \emptyset_N$ and there exist a neutrosophic trip set $W \in \Gamma$ such that $W \subset V \cap NC\alpha int(A) \subset A$.

In other words $V \cap (NCint(A)) \neq \emptyset_N$ and $P \in NCcl(NCint(A))$. Thus we have verified $NC\Gamma^{\alpha\beta} \subset NC\Gamma^{\alpha}$, and the proof is complete combining Theorem 3.1 and Proposition 3.1. and we get $NC\Gamma^{\alpha\alpha} = NC\Gamma^{\alpha}$.

Corollary 3.10

A neutrosophic crisp topology $NC\Gamma$is a $NC\alpha$ - topology iff $NC\Gamma = NC\Gamma^{\alpha}$. Evidently $NC\Gamma^{\beta}$ is a neutrosophic crisp topology iff$NC\Gamma^{\alpha} = NC\Gamma^{\beta}$. In this case $NC\Gamma^{\beta\beta} = NC\Gamma^{\alpha\beta} = NC\Gamma^{\beta}$.

Corollary 3.11

$NC\beta$-Structure B is a neutrosophic crisp topology, then $B = B\alpha = B\beta$.

We proceed to give some results on the neutrosophic structure of neutrosophic crisp $NC\alpha$–topology

Proposition 3.12

The $NC\alpha$-open with respect to a given neutrosophic crisp topology are exactly those sets which may be written as a difference between a neutrosophic crisp open set and a neutrosophic crisp nowhere dense set. If $A \in NC\Gamma^{\alpha}$ we have $A = NCint(NCcl(NCint(A))) \cap (NCint(NCcl(NCint(A)) \cap A^c)^c$, where $(NCint(NCcl(NCint(A)) \cap A^c)$ clearly is neutrosophic crisp nowhere dense set, we easily see that

$B \subset NCcl(NCint(A))$ and consequently

$A \subset B \subset NCint(NCcl(NCint(A))$ so the proof is complete.

Corollary 3.13

A neutrosophic crisp topology is a $NC\alpha$- topology iff all neutrosophic crisp nowhere dense sets are neutrosophic crisp closed. For a neutrosophic crisp $NC\alpha$-topology may be characterized as neutrosophic crisp topology where the difference between neutrosophic crisp open and neutrosophic crisp nowhere dense set is again a neutrosophic crisp open, and this evidently is equivalent to the condition stated.

Proposition 3.14

Neutrosophic crisp topologies which are $NC\alpha$- equivalent, determine the same class of neutrosophic crisp nowhere dense sets.

Proposition 3.16

If a $NC\alpha$ -Structure B, is a neutrosophic crisp topology, then all neutrosophic crisp topologies Γ for which $\Gamma^{\beta} = B$ are neutrosophic crisp extremely disconnected.

In particular: Either all or none of the neutrosophic crisp topologies of a $NC\alpha$ – class are extremely disconnected.

Proof

Let $\Gamma^{\beta} = B$, and suppose there is $A \in \Gamma$ such that $NCcl(A) \notin \Gamma$. Let $P \in NCcl(A) \cap NCint(NCcl(A))^{c}$ with $B = \{P\} \cup NCint(NCcl(A))$, $M = NCint(NCcl(A))^{c}$

We have $\{P\} \subset M = (NCint(NCcl(A))^{c} = NCcl(NCint(M))$,

$\{P\} \subset NCcl(A) = NCcl(NCint(NCcl(A)) \subset NCcl(NCint(B))$. Hence both B and M are in Γ^{β}. The intersection $B \cap M = \{P\}$ is not neutrosophic crisp open, since $P \in NCcl(A) \cap M^{c}$ hence not $NC\beta$- open. So, $\Gamma^{\beta}=B$ is not a neutrosophic crisp topology. Now suppose B is not a topology, and $\Gamma^{\beta}=B$ There is a $B \in \Gamma^{\beta}$ such that $B \notin \Gamma^{\alpha}$. Assume that $NCcl(NCint(B)) \in \Gamma$. Then

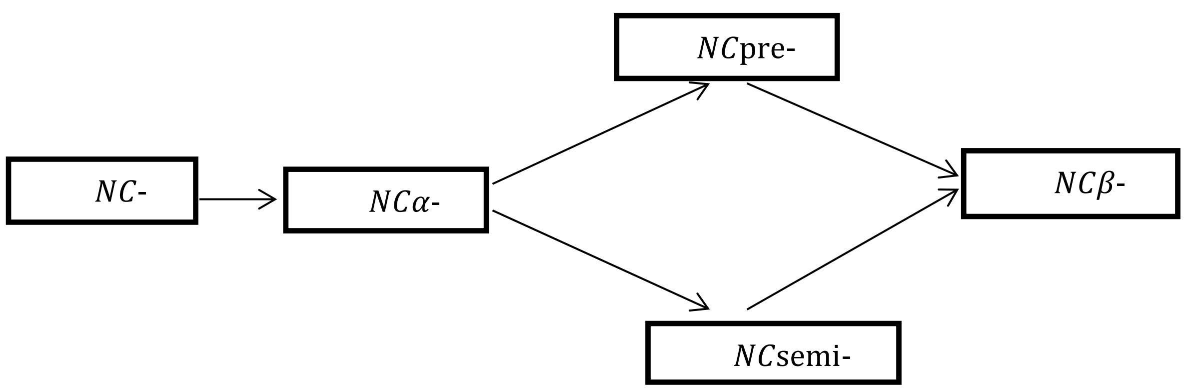
$B \subset NCcl(NCint(B)) = NCint(NCcl(NCint(B))$ i.e. $B \in \Gamma^{\alpha}$, contrary to assumption. Thus we have produced an open set whose closure is not open, which completes the proof.

Corollary 3.17

A neutrosophic crisp topology Γ is a neutrosophic crisp extremally disconnected if and only if Γ^{β} is a neutrosophic crisp topology.

Remark 3.18

The following diagram represents the relation between neutrosophic crisp nearly open sets:



4. Neutrosophic Crisp Continuity

We, introduce and study of neutrosophic crisp continuous function and we obtain some characterizations of neutrosophic continuity. Here come the basic definitions first:

Definition 4.1

Let (X, Γ) be a $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NCS in X, and $f: X \longrightarrow X$ then:

1) If $f NC\alpha$-continuous $\Longrightarrow$ inverse image of $NC\alpha$ open set is $NC\alpha$- open set
2) If f NCpre-continuous $\Longrightarrow$ inverse image of NCpre-open set is NCpre- open set
3) If f NCsemi-continuous $\Longrightarrow$ inverse image of NCsemi-open set is NCsemi- open set
4) If f $NC\beta$-continuous $\Longrightarrow$ inverse image of $NC\beta$-open set is $NC\beta$- open set

Definition 4.2

The following was given in [24]

(a) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the neutrosophic crisp image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A)=\langle f(A_1), f(A_2), f(A_3)\rangle$.

(b) If f is a bijective map then f^{-1}: $Y \longrightarrow X$ is a map defined such that:
for any NCS $B=\langle B_1, B_2, B_3\rangle$ in Y, the neutrosophic crisp preimage of B, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B)=\left\langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3)\right\rangle$.

Definition 4.3

Let (X, Γ_1) , and (Y, Γ_2) be two $NCTSs$, and let f: $X \longrightarrow Y$ be a function. Then f is said to be continuous if f the preimage of each NCS in Γ_2 is a NCS in Γ_1.

Definition 4.4

Let (X, Γ_1) , and (Y, Γ_2) be two $NCTSs$ and let f: $X \longrightarrow Y$ be a function. Then f is said to be open iff the image of each NCS in Γ_1, is a NCS in Γ_2.

Proposition4.5

Let $(X, \Gamma_o)$and(Y, ψ_o) be two $NCTSs$.

If $f: X \rightarrow Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of Definition 4.3 too.

Proof

Here we consider the $NCTSs$on X and Y, respectively, as follows: $\Gamma_1 = \left\{\left\langle G, \phi, G^c\right\rangle : G \in \Gamma_o\right\}$ and $\Gamma_2 = \left\{\left\langle H, \phi, H^c\right\rangle : H \in \Psi_o\right\}$,

In this case we have, for each $\left\langle H, \phi, H^c\right\rangle \in \Gamma_2$, $H \in \Psi_o$,

$f^{-1}\left\langle H, \phi, H^c\right\rangle = \left\langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c)\right\rangle = \left\langle f^{-1}H, \phi, (f^{-1}(H))^c\right\rangle \in \Gamma_1$.

Now we obtain some characterizations of neutrosophic continuity.

Proposition 4.6

Let f: $(X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$. Then f is neutrosophic continuous iff the preimage of each neutrosophic crisp closed set **(**$NCCS$**)** in Γ_2 is a $NCCS$ in Γ_1.

Proposition 4.7

The following are equivalent to each other:

(a) f: $(X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$ is neutrosophic continuous.

(b) $f^{-1}(NCint(B) \subseteq NCint(f^{-1}(B)))$ for each $NCSB$in Y.

(c) $(NCcl\, f^{-1}(B)) \subseteq f^{-1}(NCcl(B))$. for each $NCSB$ in Y.

Corollary 4.8

Consider $(X, \Gamma_1)$and$(Y, \Gamma_2)$to be two $NCTSs$, and let $f: X \rightarrow Y$ be a function.

if $\Gamma_1 = \left\{f^{-1}(H) : H \in \Gamma_2\right\}$. Then Γ_1 will be the coarsest *NCT* on X which makes the function $f: X \rightarrow Y$ continuous. One may call it the initial neutrosophic crisp topology with respect to f.

5. Neutrosophic Crisp Compact Space

First we present the basic concepts:

Definition5.1

Let(X, Γ)be an *NCTS*.

(a) If a family $\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\}$ of *NCOSs* in X satisfies the condition $\cup\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\} = X_N$, then it is called an neutrosophic open cover of X.

(b) A finite subfamily of an open cover $\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\}$ on X, which is also a neutrosophic open cover of X , is called a neutrosophic crisp finite open subcover.

Definition5.2

A neutrosophic crisp set $A=\langle A_1,A_2,A_3\rangle$ in a NCTS (X,Γ) is called neutrosophic crisp compact iff every neutrosophic crisp open cover of A has a finite neutrosophic crisp open subcover.

Definition5.3

A family $\{\langle K_{i1},K_{i2},K_{i3}\rangle : i\in J\}$ of neutrosophic crisp compact sets in X satisfies the finite intersection property (FIP) iff every finite subfamily $\{\langle K_{i_1},K_{i_2},K_{i_3}\rangle : i=1,2,...,n\}$ of the family satisfies the condition $\cap\{\langle K_{i_1},K_{i_2},K_{i_3}\rangle : i=1,2,...,n\}\neq\phi_N$.

Definition5.4

A NCTS (X,Γ) is called neutrosophic crisp compact iff each neutrosophic crisp open cover of X has a finite open subcover.

Corollary5.5

A *NCTS* (X,Γ) is a neutrosophic crisp compact iff every family $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle : i\in J\}$ of neutrosophic crisp compact sets in X having the finite intersection properties has nonempty intersection.

Corollary5.6

Let (X,Γ_1) , (Y,Γ_2) be NCTSs and $f:X\rightarrow Y$ be a continuous surjection. If (X,Γ_1) is a neutrosophic crisp compact, then so is (Y,Γ_2).

Definition5.7

If a family $\{\langle G_{i1},G_{i2},G_{i3}\rangle : i\in J\}$ of neutrosophic crisp compact sets in X satisfies the condition $A\subseteq\cup\{\langle G_{i1},G_{i2},G_{i3}\rangle : i\in J\}$, then it is called a neutrosophic crisp open cover of A.

Let's consider a finite subfamily of a neutrosophic crisp open subcover of $\{\langle G_{i1},G_{i2},G_{i3}\rangle : i\in J\}$.

Corollary5.8

Let (X,Γ_1), (Y,Γ_2) be *NCTSs* and $f:X\rightarrow Y$ be a continuous surjection. If A is a neutrosophic crisp compact in (X,Γ_1), then so is $f(A)$ in (Y,Γ_2).

6. Conclusion

In this paper, we presented a generalization of the neutrosophic topological space. The basic definitions of the neutrosophic crisp topological space and the neutrosophic crisp compact space with some of their characterizations were deduced. Furthermore, we constructed a neutrosophic crisp continuous function, with a study of a number its properties.

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