SUBSET GROUPOIDS

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Educational Publisher Inc.
Ohio
2013

This book can be ordered from:

Education Publisher Inc.
1313 Chesapeake Ave.
Columbus, Ohio 43212,
USA
Toll Free: 1-866-880-5373

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ISBN-13: 978-1-59973-222-0
EAN: 9781599732220

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PREFACE

In this book authors introduce the new notion of constructing non associative algebraic structures using subsets of a groupoid. Thus subset groupoids are constructed using groupoids or loops. Even if we use subsets of loops still the algebraic structure we get with it is only a groupoid. However we can get a proper subset of it to be a subset loop which will be isomorphic with the loop which was used in the construction of the subset groupoid.

To the best of the authors' knowledge this is the first time non associative algebraic structures are constructed using subsets.

We get a large class of finite subset groupoids as well as a large class of infinite subset groupoids. Here we find the conditions under which these subset groupoids satisfy special identities like Bol identity, Moufang identity, right alternative identity and so on. In fact it is a open problem to find subset groupoids to satisfy special identities even if the groupoids over which they are defined do not satisfy any of the special identities.

We define the notion of Smarandache strong Bol subset groupoids or Smarandache Bol groupoids if the respective subset groupoids are strong Bol groupoids or Bol groupoids respectively.

On similar lines we define Smarandache subset strong Moufang groupoid and Smarandache subset Moufang groupoid, Smarandache strong subset P-groupoids and Smarandache subset P-groupoids and so on. We have illustrated this by several examples. However we see we are yet to construct Smarandache strong subset Moufang groupoids (Bol groupoids or P-groupoids and so on) using Z the integers or Q the rationals or the reals R.

This book has three chapters. The first chapter is introductory in nature. In second chapter we introduce the notion of subset groupoids using groupoids. In chapter three we build subset groupoids using the loops. Several innovative results are developed and described in this book.

We thank Dr. K.Kandasamy for proof reading and being extremely supportive.

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Chapter One

INTRODUCTION

In this chapter we recall most of the important definitions and properties that are used in this book. This is mainly done to keep this book as self contained as possible.

We first recall the definition of a groupoid and give examples of various types of groupoids.

DEFINITION 1.1: *Let G be a non empty set with a binary operation * defined on G. That is for all a, b ∈ G;*

*a * b ∈ G and * in general is non associative on G. We define (G, *) to be a groupoid.*

We may have groupoids of infinite or finite order.

Example 1.1: Let G = $\{a_0, a_1, a_2, a_3, a_4\}$ be the groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4
a_0	a_0	a_4	a_3	a_2	a_1
a_1	a_1	a_0	a_4	a_3	a_2
a_2	a_2	a_1	a_0	a_4	a_3
a_3	a_3	a_2	a_1	a_0	a_4
a_4	a_4	a_3	a_2	a_1	a_0

Example 1.2: Let (G, *) be the groupoid given by the following table:

*	a_1	a_2	a_3
a_1	a_1	a_3	a_2
a_2	a_2	a_1	a_3
a_3	a_3	a_2	a_1

Example 1.3: Let (G, *) be the groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
a_0	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8
a_1	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9
a_2	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0
a_3	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1
a_4	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2
a_5	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3
a_6	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4
a_7	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5
a_8	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6
a_9	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7

Clearly (G, *) is a grouped of order 10.

Example 1.4: Let G = (Z, *, (3, –1)) be a groupoid. If a, b $\in$ Z; a * b = 3a + b(–1) = 3a – b; that is if 8, 0 $\in$ Z then
8 * 0 = 3 × 8 + 0 (–1) = 24.

For 5, 10 $\in$ Z, 5 * 10 = 3 × 5 – 10 * 1 = 15 – 10 = 5.

Clearly G is an infinite groupoid. That is o(G) = ∞.

Example 1.5: Let (Q, *, (7/3, 2)) = G be a groupoid. o(G) = ∞. Let x = 3 and y = –2 $\in$ Q then
3 * (–2) = 3 × 7/3 – 2 × 2 = 7 – 4 = 3.

Example 1.6: Let G = (R, *, ($\sqrt{3}$, 0)) be a groupoid. o(G) = ∞.

Let x = 8, y = $-5\sqrt{2} \in$ R.

$$x * y = 8 * (-5\sqrt{2}) = 8 \times \sqrt{3} - 0 \times (-5\sqrt{2})$$
$$= 8\sqrt{3} \in R.$$

Clearly o(G) = ∞.

Example 1.7: Let G = {C, *, (3–i, 4 + 5i)} be a groupoid of infinite order.

a * b = a (3 – i) + b (4 + 5i) for a, b $\in$ C.

Take a = –3i and b = 2 + i $\in$ C.
a * b = –3i × (3–i) + 2 + i) (4 + 5i)
= –9i – 3 + 8 + 4i + 10i – 5
= 5i $\in$ C.

o(G) = ∞ so G is of infinite order.

Now if (G, *) is a groupoid. Let H be a subset of G, if (H, *) is a groupoid then we define (H, *) to be a subgroupoid of G.

We will illustrate this situation by some examples.

Example 1.8: Let G be a groupoid given by the following:

*	a_0	a_1	a_2	a_3
a_0	a_0	a_2	a_0	a_2
a_1	a_3	a_1	a_3	a_1
a_2	a_2	a_0	a_2	a_0
a_3	a_1	a_3	a_1	a_3

Take $P = \{a_0, a_2\} \subseteq G$ and

$Q = \{a_1, a_3\} \subseteq G$ be subsets of G.

Both P and Q are subgroupoids.

Example 1.9: Let (G, *) be a groupoid;

*	a_0	a_1	a_2	a_3	a_4	a_5
a_0	a_0	a_4	a_2	a_0	a_4	a_2
a_1	a_2	a_0	a_4	a_2	a_0	a_4
a_2	a_4	a_2	a_0	a_4	a_2	a_0
a_3	a_0	a_4	a_2	a_0	a_4	a_2
a_4	a_2	a_0	a_4	a_2	a_2	a_4
a_5	a_4	a_2	a_0	a_4	a_2	a_0

Consider $P = \{a_0, a_2, a_4\} \subseteq G$, P is a subgroupoid.

Example 1.10: Let (G, *) be a groupoid which is as follows:

*	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
a_0	a_0	a_3	a_6	a_9	a_0	a_3	a_6	a_9	a_0	a_3	a_6	a_9
a_1	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}
a_2	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}
a_3	a_3	a_6	a_9	a_0	a_3	a_6	a_9	a_0	a_3	a_6	a_9	a_0
a_4	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1
a_5	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2
a_6	a_6	a_9	a_0	a_3	a_6	a_9	a_0	a_3	a_6	a_9	a_0	a_3
a_7	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4
a_8	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5
a_9	a_9	a_0	a_3	a_6	a_9	a_0	a_3	a_6	a_9	a_0	a_3	a_6
a_{10}	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7	a_{10}	a_1	a_4	a_7
a_{11}	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8	a_{11}	a_2	a_5	a_8

Take $K = \{a_0, a_3, a_6, a_9\} \subseteq G$ and $H = \{a_2, a_5, a_8, a_{11}\} \subseteq G$. Both K and H are subgroupoids of G.

We can define ideal in a groupoid G.

Let G be a groupoid. P a non empty proper subset of G. P is said to be a left ideal of the groupoid G if

(i) P is a subgroupoid of G.
(ii) For all $x \in G$ and $a \in P$; $xa \in P$.

On similar line we define right ideal of G. If both P is a right or a left ideal then P is an ideal of G.

We will give some examples of ideals of a groupoid.

Example 1.11: Let G be a groupoid given by the following table;

*	a_0	a_1	a_2	a_3	a_4	a_5	a_6
a_0	a_0	a_4	a_1	a_5	a_2	a_6	a_3
a_1	a_3	a_0	a_4	a_1	a_5	a_2	a_6
a_2	a_6	a_3	a_0	a_4	a_1	a_5	a_2
a_3	a_2	a_6	a_3	a_0	a_4	a_1	a_5
a_4	a_5	a_2	a_6	a_3	a_0	a_4	a_1
a_5	a_1	a_5	a_2	a_6	a_3	a_0	a_4
a_6	a_4	a_1	a_5	a_2	a_6	a_3	a_0

Example 1.12: Let (G, *) be a groupoid given below:

*	a_0	a_1	a_2	a_3
a_0	a_0	a_2	a_0	a_2
a_1	a_3	a_1	a_3	a_1
a_2	a_2	a_0	a_2	a_0
a_3	a_1	a_3	a_1	a_3

P = $\{a_0, a_3\}$ is a right ideal of G. Clearly P is not a left ideal of G. Q = $\{a_1, a_3\}$ is a right ideal of G. Clearly Q is not a left ideal of G.

Example 1.13: Let (G, *) be a groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4	a_5
a_0	a_0	a_4	a_2	a_0	a_4	a_2
a_1	a_2	a_0	a_4	a_2	a_0	a_4
a_2	a_4	a_2	a_0	a_4	a_2	a_0
a_3	a_0	a_4	a_2	a_0	a_4	a_2
a_4	a_2	a_0	a_4	a_2	a_0	a_4
a_5	a_4	a_2	a_0	a_4	a_2	a_0

Clearly P = $\{a_0, a_2, a_4\}$ is both a right ideal and a left ideal of G; infact an ideal of G.

Let G be a groupoid. A subgroupoid V of G is said to be a normal subgroupoid of G if

$$aV = Va;$$
$$(Vx)\, y = V(xy)$$
$$y(xV) = (yx)\, V$$
for all x, y, a $\in$ V.

We see P in example 1.13 is a normal subgroupoid.

We define a groupoid G to be normal if

$$aG = Ga,$$
$$G(xy) = (Gx)\, y$$
$$y(xG) = (yx)G \text{ for all } a, x, y \in G.$$

We are more interested mainly on groupoids built using Z_n or $C(Z_n)$ or $Z_n(g)$ or $Z_n(g_1, g_2)$ or Z or Z(g) or Q or Q(g) or C.

We do not wish to build abstract groupoids using some elements like $(a_1, \ldots, a_n)$ in G; G a groupoid.

The authors feel these are more non abstract structures built on known algebraic structures.

We will first illustrate this situation by examples.

Example 1.14: Let G = $\{Z_5, *, (3, 0)\}$ be a groupoid.

For 4, 2 $\in$ G we see

$$\begin{aligned} 4 * 2 &= 3 \times 4 + 2 \times 0 \pmod 5 \\ &= 2 \pmod 5. \\ 2 * 4 &= 3 \times 2 + 4 \times 0 \pmod 5 \\ &= 1 \pmod 5 \end{aligned}$$

Clearly a * b $\neq$ b * a in general.

Example 1.15: Let G = {Z_8, *, (6, 2)} be a groupoid.
Take 3, 4, 5 $\in Z_8$.

$$\begin{aligned}(3 * 4) * 5 &= (3 \times 6 + 4 \times 2) * 5 \\ &= 2 * 5 = 2 \times 6 + 2 \times 5 \\ &= 12 + 10 = 6.\end{aligned}$$

Consider 3 * (4 * 5)

$$\begin{aligned}&= 3 * [4 \times 6 + 5 \times 2] \\ &= 3 * 10 = 3 * 2 \\ &= 3 \times 6 + 2 \times 2 \\ &= 18 + 4 = 22 = 6.\end{aligned}$$

We see (a * b) * c = a * (b * c) for this a = 3, b = 4 and c = 5

Consider (1 * 3) * 7 = (6 × 1 + 3 × 2) * 7

$$\begin{aligned}&= (6 + 6) * 7 \\ &= 4 * 7 \\ &= 24 + 14 = 6 \qquad \text{I}\end{aligned}$$

$$\begin{aligned}1 * (3 * 7) &= 1 * (3 \times 6 + 7 \times 2) \\ &= 1 * (18 + 14) \\ &= 1 * 0 \\ &= 6 \qquad \text{II}\end{aligned}$$

We see (1 * 3) * 7 = 1 * (3 * 7)

Consider (2 * 1) * 4

$$\begin{aligned}&= (2 \times 6 + 1 \times 2) * 4 \\ &= (12 + 2) * 4 = 6 * 4 \\ &= 6 \times 6 + 4 \times 2 = 4 \qquad \text{I}\end{aligned}$$

$$\begin{aligned}2 * (1 * 4) &= 2 * (6 + 4 \times 2) \\ &= 2 * 6 \\ &= 6 \times 2 + 6 \times 2 = 0 \qquad \text{II}\end{aligned}$$

I and II are not equal that is (2 * 1) * 4 ≠ 2 * (1 * 4). Thus G is a groupoid.

Example 1.16: Let G = $\{Z_6, *, (3, 2)\}$ be a groupoid. Take $5, 1, 4 \in Z_6$,

$$(5 * 1) * 4 = (15 + 2) * 4 = 5 * 4$$
$$= 5 \times 3 + 4 \times 2$$
$$= 3 + 3 = 0 \qquad \text{I}$$

Consider $5 * (1 * 4) = 5 * (3 + 8) = 5 * 5$
$$= 3 \times 5 + 2 \times 5$$
$$= 15 + 10 = 1 \qquad \text{II}$$

I and II are distinct. Thus * operation on G is non associative.

Example 1.17: Let G = $\{Z_3, *, (0, 1)\}$ be a groupoid of order three.

It is interesting and important to keep on record that for a given Z_n we can have several groupoids which is impossible in case of semigroups. Take Z_3, we see $G_1 = \{Z_3, *, (1, 0)\}$, $G_2 = \{Z_3, *, (0, 1)\}$, $G_3 = \{Z_3, *, (2, 0)\}$, $G_4 = \{Z_3, *, (0, 2)\}$, $G_5 = \{Z_3, *, (1, 2)\}$ and $G_6 = \{Z_3, *, (2, 1)\}$ are six distinct groupoids.

We test what happens in case $G_7 = (Z_3, *, (2, 2))$.

Take x = 2 and y = 1
$$x * y = 2 \times 2 + 2 \times 1 = 0$$
$$(x * y) * 1 = 2 \qquad \text{I}$$

$$x * (y * 1) = 2 * (2 + 2)$$
$$= 2 * 1 = 4 * 2$$
$$= 0 \qquad \text{II}$$

I and II are distinct. Thus we have seven distinct groupoids of order three.

It is important to mention that as we increase n, that is Z_n for larger n we have more number of distinct groupoids.

Thus by defining in this way we get more number of groupoids.

Now if we take Z_n find the number of groupoids constructed in general using Z_n for a fixed n. Now we can find groupoids of dual numbers.

Example 1.18: Let G = $\{Z_5(g), *, (2, g) \mid g^2 = 0\}$ be a dual number groupoid of order 25.

Example 1.19: Let G = $\{C(Z_5), *, (i_F, 0)\}$ be the complex finite modulo integer groupoid of order 25.

Example 1.20: Let P = $\{C(Z_{90}), *, (48, 49i_F)\}$ be a finite complex modulo integer groupoid of finite order.

Example 1.21: Let G = $\{Z_9(g, g_1), *, (g, g_1) \mid g_1^2 = g_1$ and $g^2 = 0, g_1g = gg_1 = 0\}$ be a groupoid of finite order. G is a mixed dual number groupoid.

Example 1.22: Let G = $\{C(Z_{10})(g), *, (5g, 5), g^2 = 0\}$ be a finite groupoid of dual finite complex modulo integer.

Example 1.23: Let G = $\{C(Z_7), *, (3, 2i_F)\}$ be a complex finite modulo integer groupoid.

We say a groupoid G to be Smarandache groupoid if G contains a non empty proper subset H such that H under the operations of G is a semigroup.

We will give some examples of a Smarandache groupoid.

Example 1.24: Let G be a groupoid given by the table:

*	0	1	2	3	4
0	0	4	3	2	1
1	2	1	0	4	3
2	4	3	2	1	0
3	1	0	4	3	2
4	3	2	1	0	4

{1}, {0}, {2}, {3} and {4} are subsets of G which are semigroups so G is a Smarandache groupoid.

Example 1.25: Let G = $\{Z_6(g) \mid g^2 = 0, *, (2g, g)\}$ be a groupoid. G is a Smarandache groupoid.

For more about Smarandache groupoids please refer [4].

We define the concept of Moufang groupoid. If the groupoid G satisfies the identity
$(x * y) * (z * x) = (x * (y * z)) * x$ for all x, y, z in G then we define G to be a Moufang groupoid.

If in a groupoid G; $((x * y) * z) * y = x * ((y * z) * y)$ for all $x, y, z \in G$ we define G to be a Bol groupoid.

Example 1.26: The groupoid G given by the following table is a Bol groupoid:

*	a_0	a_1	a_2	a_3	a_4	a_5
a_0	a_0	a_3	a_0	a_3	a_0	a_3
a_1	a_2	a_5	a_2	a_5	a_2	a_5
a_2	a_4	a_1	a_4	a_1	a_4	a_1
a_3	a_0	a_3	a_0	a_3	a_0	a_3
a_4	a_2	a_5	a_2	a_5	a_2	a_5
a_5	a_4	a_1	a_4	a_1	a_4	a_1

A groupoid (G, *) is said to be a P-groupoid if for all $x, y \in G$, we have $(x * y) * x = x * (y * x)$.

A groupoid G is said to be a right alternative groupoid if $(x * y) * y = x * (y * y)$ for all $x, y \in G$. G is said to be a left alternative groupoid if $(x * x) * y = x * (x * y)$ for all $x, y \in G$.

A groupoid G is said to be an alternative groupoid if it is both left and right alternative simultaneously. We define a proper subset H of a groupoid G to be a subgroupoid if H is a groupoid under the operations of G.

We call a groupoid G to be an idempotent groupoid if $x * x = x$ for all $x \in g$. We can have zero divisors in a groupoid (G, *); we say $x, y \in G$ is a zero divisor of G if $x * y = 0$ and $y * x = 0$.

We can in case of a groupoid G have $x * y = 0$ and $y * x \neq 0$ then we define x to be a right zero divisor. If $y * x = 0$ we define y to be a left zero divisor.

We define centre of a groupoid C(G) of G to be $\{x \in G \mid x * a = a * x \text{ for all } a \in G\}$.

We say $a, b \in G$ to be a conjugate pair if $a = b * x$ (or $x * a = b$) and $b * y = a * y$ (or $y * a$ for some $y \in G$).

We have the concept of infinite groupoids also. Finally we can define the notion of Smarandache strong Bol groupoid [4].

If G is a groupoid we can define Smarandache Moufang groupoid and Smarandache strong Moufang groupoid.

On similar lines we have defined the notion of Smarandache P-groupoid and Smarandache strong P-groupoid [4].

Likewise we define Smarandache right alternative (left alternative) groupoid and Smarandache strong right alternative (left alternative) groupoid. Finally the notion of Smarandache alternative groupoid and Smarandache strong alternative groupoid.

We wish to state that if the reader is familier with these notions then only he/she can study these concepts in case of subset groupoids with ease.

Next we proceed onto describe the properties of loops. Mainly we study the special class of loops $L_n(m) \in L_n$; for we see abstract definition of loop makes only the concept more complicated. However the natural new class of loops happens to be nice is not that abstract and one easily sees that how under non associative operation this structure behaves.

We just call a non empty set L to be a loop if in L is defined a closed binary operation * such that we have a unique element e in L with $e * a = a * e = a$ for all $a \in L$ called the identity element of L with respect to the operation *. For every ordered pair $(a, b) \in L \times L$ there exist a unique pair $(x, y) \in L$ with $ax = b$ and $ya = b$.

We work only with finite loops and finite subset loop groupoids of these finite loops or more precisely we only work with the new class of loops in L_n.

A loop is said to be a Moufang loop if it satisfies any one of the following identities.

$$x (y (xz)) = ((xy) x) z$$
$$((xy)z) y = x (y (zy))$$
$$(xy) (zx) = (x (yz)) x \text{ for all } x, y, z \in L.$$

A loop L is called a Bruck loop if $x (yx) z = x (y(xz))$ and $(xy)^{-1} = x^{-1} y^{-1}$ for all $x, y, z \in L$.

A loop L is a Bol loop if $((xy)z)y = x((yz)y)$ for all $x, y, z \in L$.

A loop L is right alternative if $(xy) y = x (yy)$ and left alternative if $(xx)y = x (xy)$ for all $x, y \in L$ and alternative if L is both left and right alternative simultaneously. We say a loop L satisfies the weak inverse property if $(x * y) * z = e$ imply

$$x * (y * z) = e \text{ for all } x, y, z \text{ in } L.$$

A non empty subset H of a loop L is a subloop of L if H itself is a loop under the operations of L.

A subloop H of L is said to be normal if

(i) xH = Hx;
(ii) (Hx)y = H(xy)
(iii) y (xH) = (yx) H for all x, y $\in$ L.

A loop is simple if L has no normal subloops.

We say for x, y $\in$ L; the commutator (x, y) is defined by xy = (yx) (x, y).

We use the classical theorems of finite groups in case of finite loops [5].

Let (L, *) be a finite loop. For $\alpha \in$ L; define a right multiplication R_α as a permutation of the loop (L, *) as follows:

$$R_\alpha : x \mapsto x * \alpha.$$

We will call the set $\{R_\alpha \mid \alpha \in L\}$ the right regular representation of (L, *) or briefly representation of L.

For any predetermined a, b in a loop L, a principal isotope (L, o) of the loop (L, *) is defined by x o y = X * Y where X * a = x and b * Y = y. L is a G-loop if it is isomorphic to all of its principal isotopes.

Construction of a new class of loops.

Let $L_n(m) = \{e, 1, 2, \ldots, n\}$; n > 3 and n is an odd integer. m is a positive integer such that m < n and (m, n) = 1 and (m – 1, n) = 1 with m < n.

Define on $L_n(m)$ a binary operation * as follows:

(i) $e * i = i * e = i$ for all $i \in L_n(m)$
(ii) $i * i = e$ for all $i \in L_n(m)$
(iii) $i * j = t$ where $t = (m_j – (m – 1)i)$ (mod n) for all $i, j \in L_n(m)$, $i \neq j$, $i \neq e$ and $j \neq e$.

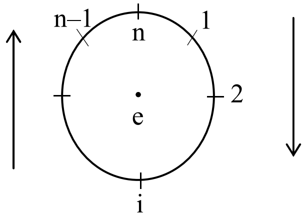
Then $L_n(m)$ is a loop. We give one example.

Consider the following the table of $L_7(4) = \{e, 1, 2, 3, 4, 5, 6, 7\}$ which is as follows:

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	5	2	6	3	7	4
2	2	5	e	6	3	7	4	1
3	3	2	6	e	7	4	1	5
4	4	6	3	7	e	1	5	2
5	5	3	7	4	1	e	2	6
6	6	7	4	1	5	2	e	3
7	7	4	1	5	2	6	3	e

is a loop of order 8. We just give the physical interpretation of the operation in the loop $L_n(m)$.

We give a physical interpretation of this class of loops which is as follows: $L_n(m) = \{e, 1, 2, \ldots, n\}$ be a loop of this new class. Suppose that the non identity elements of the loop are equidistantly placed on a circle with e as its centre. We assume the elements to move always in the clockwise direction.



Let i, j $\in L_n(m)$ ($i \neq j$, $i \neq e$, and $j \neq e$). If j is the rth element from i counting in the clockwise direction then i * j will be the ith element from j in the clockwise direction where $t = (m-1)r$.

We see in general i . j need not be equal to j * i. When i = j we define $i * i = i^2 = e$ and i.e = e.i = i for all $i \in L_n(m)$, e acts as the identity in $L_n(m)$.

Let L_n denote the class of all loops $L_n(m)$ in L_n for a fixed n and various m's satisfying the conditions m < n, (m, n) = 1 and (m – 1, n) = 1, that is
$L_n = \{L_n(m) | n > 3;\ n \text{ odd};\ m < n, (m, n) = 1 \text{ and } (m - 1, n) = 1\}$.

We will use the following results in case of subset loop groupoids of the loops $L_n(m) \in L_n$.

Result 1.1: Let L_n be the class of loops for any n > 3 if $n = p_1^{\alpha_1}, \ldots, p_k^{\alpha_k}$ ($\alpha_i \geq 1$, i = 1, 2,…, k) then $|L_n| = \prod_{i=1}^{k} (p_i - 2) p_i^{\alpha_i - 1}$ where $|L_n|$ denotes the number of loops in L_n.

Results 1.2: L_n contains one and only one commutative loop.

Result 1.3: L_n for $n = p_1^{\alpha_1}, \ldots, p_k^{\alpha_k}$ contains exactly F_n loops which are strictly non commutative where $F_n = \prod_{i=1}^{k} (p_i - 3) p_i^{\alpha_i - 1}$.

Result 1.4: If n = 3t the class of loops L_n does not contain any strictly non commutative loop.

Result 1.5: If $n = p_i$, p a prime the loop in L_n is either commutative or strictly non commutative.

Result 1.6: The class of loops L_n contains exactly one left alternative loop and one right alternative loop and does not contain any alternative loop.

Result 1.7: The right and left alternative loops in L_n are non commutative.

Result 1.8: The class of loops L_n does not contain any Moufang loops.

Result 1.9: The class of loops L_n does not contain a Bruck loop.

Result 1.10: Let $L_n(m) \in L_n$. Then $L_n(m)$ is a weak inverse property loop if and only if

$$(m^2 - m + 1) \equiv 0 \pmod{n}.$$

Result 1.11: A left or right alternative loop of L_n is not a WIP loop.

Result 1.12: The class of loops L_n does not contain any associative loop.

Result 1.13: In the class of loops L_n we have for every $x, y \in L_n(m)$, $(xy)\,x = x\,(yx)$.

Result 1.14: Let L_n be the class of loops. The number of strictly non right (left) alternative loops is F_n where

$$F_n = \prod_{i=1}^{k} (p_i - 3)p_i^{\alpha_i - 1} \text{ and } n = \prod_{i=1}^{k} p_i^{\alpha_i}$$

Result 1.15: If $n = p$, p a prime then in the class L_n a loop is either right (left) alternative or strictly non right (left) alternative.

Result 1.16: The exact number of distinct non identity pairs in $L_n(m)$; L_n which commute is $n(d-1)/2$ where $d = (n, 2m-1)$.

Result 1.17: Let $L_n(m) \in L_n$ if $(2m-1, n) = 1$ then the loop is strictly non commutative if $(2m-1, n) = n$ then the loop is commutative.

Result 1.18: For $L_n(m) \in L_n$, the number of distinct non identity pairs which satisfy the right alternative law is given by $n(d-2)/2$ where $d^2 = (m^2 - 2m, n)$.

Result 1.19: Let $L_n(m) \in L_n$. Then the number of distinct non identity pairs which satisfy the left alternative law is given by $n(d-1)/2$ where $d = (m^2 - 1, n)$.

Result 1.20: Let $L_n(m) \in L_n$. For every t/n there exists t subloops of order k+1 where k = n/t.

Result 1.21: Let $H_i(t)$ and $H_j(t)$ be subloops in $L_n(m)$.

$$H_i(t) \cap H_j(t) = \{e\} \text{ where } i \neq j.$$

Result 1.22: Let $H_i(t)$'s be subloops $\bigcup_{i=1}^{t} H_i(t) = L_n(m)$ for every t dividing n.

Result 1.23: $H_i(t) \cong H_j(t)$ for every t dividing n.

Result 1.24: $L_n(m) \in L_n$ contains a subloop of order k + 1 if and only if k/n.

Result 1.25: Let r and s $(1 < r < s)$ such that r/s implies r+1/s+1 then $s > r^2 + r + 1$.

Result 1.26: Let $L_n(m) \in L_n$.

The Lagrange's theorem for groups is satisfied by every subloop $L_n(m)$ if and only if n is an odd prime.

Result 1.27: For any loop $L_n(m)$ of L_n there exist only 2-Sylow subloops.

Result 1.28: Let $L_n(m) \in L_n$. No $L_n(m)$ satisfies Cauchy theorem for any odd prime.

We just give an example of a right regular representation of this new class of loops.

Example 1.27: Let $L_7(4) \in L_n$ where $L_7(4) = \{e, 1, 2, \ldots, 7\}$ given by the following table:

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	5	2	6	3	7	4
2	2	5	e	6	3	7	4	1
3	3	2	6	e	7	4	1	5
4	4	6	3	7	e	1	5	2
5	5	3	7	4	1	e	2	6
6	6	7	4	1	5	2	e	3
7	7	4	1	5	2	6	3	e

I ; (e 1) (2 5 3) (4 6 7)
(e 2) (1 5 7) (3 6 4)
(e 3) (1 2 6) (4 7 5)
(e 4) (1 6 5) (2 3 7)
(e 5) (1 3 4) (2 7 6)
(e 6) (1 7 3) (2 4 5)
(e 7) (1 4 7) (3 5 6)

where I is the identity permutation on the loop $L_7(4)$.

Example 1.28: Let L = {1, 2, 3, 4, 5, 6}.

The composition table of (L, *) is as follows:

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	1	5	3	6	4
3	3	5	1	6	4	2
4	4	3	6	1	2	5
5	5	6	4	2	1	3
6	6	4	2	5	3	1

The principal isotope (L, o) of (L, *) is as follows:

o	1	2	3	4	5	6
1	6	4	5	3	2	1
2	5	3	4	6	1	2
3	4	5	2	1	6	3
4	2	1	6	5	3	4
5	3	6	1	2	4	5
6	1	2	3	4	5	6

.

Example 1.29: We now give the loop ($L_5(2)$, *) and its principal isotope with a = e and b = 3 with identity e′ = 3 have the property x o x = e′ = 3.

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

o	e	1	2	3	4	5
e	3	2	5	e	1	4
1	5	3	4	1	e	2
2	4	e	3	2	5	1
3	e	1	2	3	4	5
4	2	5	1	4	3	e
5	1	4	e	5	2	3

Now we have given most of the results used in this book. For more refer [4, 5].

Chapter Two

SUBSET GROUPOIDS

In this chapter we for the first time introduce the notion of subset non associative structure. Here we define, describe and develop them.

DEFINITION 2.1: *Let*
*S = {Collection of all subsets from a groupoid (G, *)}. For any A, B ∈ S define A*B (that is if* $A = \{a_1, \ldots, a_n\}$ *and* $B = \{b_1, \ldots, b_t\}$*;* $A * B = \{a_i * b_j \mid a_i \in A \text{ and } b_j \in B \text{ and } a_i, b_j \in G\}$ *and if A * B ∈ S; then we define S under the operation of the groupoid G to be the subset groupoid of G.*

We will first illustrate this situation by some examples.

Example 2.1: Let
S = {Collection of all subsets of the groupoid G=(Z_6, (2,5), *)}; S is a subset groupoid of G. Take A = {3, 4} and B = {5, 1, 2} ∈ S.

A * B = {3 * 5, 3 * 1, 3 * 2, 4 * 5, 4 * 1, 4 * 2}

= {(6 + 25) (mod 6), (18 + 5) (mod 6) (6 + 10) (mod 6), (8 + 25) (mod 6) (8 + 5) (mod 6), (8+10) (mod 6)}

= {1, 5, 4, 3, 0} ∈ S.

Now if C = {4} ∈ S we find (A * B) * C and A * (B * C).

Now A * B = {1, 5, 4, 3, 0} and
(A * B) * C = {1 * 4, 5 * 4, 4 * 4, 3 * 4, 0 * 4}
= {2 + 20 (mod 6), {10 + 20} (mod 6),
(8 + 20) (mod 6), {6 + 20} (mod 6)
{20} (mod 6)}
= {4, 0, 2} … I

We find B * C = {5, 1, 2} * {4}
= {5 * 4, 1 * 4, 2 * 4}
= {(10 + 20) (mod 6) (2 + 20) (mod 6) (4 + 20) (mod 6)}
= {0, 4}.

A * (B * C) = {3, 4} * {0, 4}
= {3 * 0, 3 * 4, 4 * 0, 4 * 4}
= {0, 2, 4} … II

We see (A * B) * C = A * (B * C) for this A, B, C ∈ S.

Take A = {2}, B = {3} and C = {4} ∈ S.

We now find (A * B) * C = ({2} * {3}) * C
= {2 × 2 + 3 × 5} * C
= {1} * C = {4}
= {1 * 4} = {2 + 20 (mod 6)}
= {4} … I

A * (B * C) = A * ({3} × {4})
= A * {3 × 2 + 4 × 5 (mod 6)}
= A * {2}
= {4 + 10} = {2} … II

We see (A * B) * C ≠ A * (B * C) as equations I and II are distinct.

Thus S is a subset groupoid.

Example 2.2: Let
S = {Collection of all subsets of the groupoid G = {Z_{40}, *, (5, 0)}} be the subset groupoid of G.

We do not adjoin the empty set ϕ. We never do that as we want to have a non associative operation on S.

Example 2.3: Let
S = {Collection of all subsets of the groupoid G = {Z_{19}, *, (1, 3)}} be the subset groupoid of G.

Take A = {3, 7, 5} and B = {1, 9} in S.

A * B = {3, 7, 5} * {1, 9}
= {3 * 1, 3 * 9, 7 * 1, 7 * 9, 5 * 1, 5 * 9}
= {(3 + 3) (mod 19), (3+27) (mod 19), (7 + 3) (mod 19), (7 + 27) (mod 19), (5 + 3) (mod 19), (5 + 27) (mod 19)}
= {6, 11, 10, 15, 8, 13}.

Let C = {2} ∈ S.

(A * B) * C = {6, 11, 10, 15, 8, 13} * {2}
= {6 * 2, 11 * 2, 10 * 2, 15 * 2, 8 * 2, 13 * 2}
= {12, 17, 16, 2, 14, 0} … I

Consider A * (B * C)
= A * ({1, 9} * {2})
= A * (1 * 2, 9 * 2)
= A * (7, 15)
= {3, 7, 5} * {7, 15}
= {3 * 7, 3 * 15, 7 * 7, 7 * 15, 5 * 7, 5 * 15}
= {(3 + 21) (mod 19), (3 + 45) (mod 19), (7*21) (mod 19), (7 + 45) (mod 19), (5 × 21) (mod 19), (5 + 45) (mod 19)}
= {5, 10, 9, 14, 7, 12} … II

We see equations I and II are not equal.
Thus (A * B) * C ≠ A * (B * C) for A, B, C ∈ S.
Hence the subset groupoid S is non associative.

Example 2.4: Let S = {Collection of all subsets of the groupoid G = {Z_{16} (g), *, (2, 0), $g^2 = 0$}} be the subset groupoid of G. Let A = {1 + 3g, 8g, 4g + 5} and B = {g + 3} be in S.

A * B = {(1 + 3g) * (g + 3), 8g * (g + 3), (4g + 5) * (g + 3)}
= {2 + 6g + 0, 16g + 0, 8g + 10 + 0}
= {2 + 6g, 0, 8g + 10} $\in$ S.

We see this subset groupoid is the groupoid of dual numbers.

Example 2.5: Let S = {Collection of all subsets of the groupoid C(Z_4) = {a + bi_F | a, b $\in$ Z_4, $i_F^2 = 3$}, *, (2, i_F)}} be the subset groupoid of G.

Let A = {3i_F + 1, 1+i_F, 2i_F} and B = {2 + 2i_F, 2} $\in$ S

A * B = {3i_F + 1 * 2 + 2i_F, 1 + i_F * 2 + 2i_F, 2i_F * 2 + 2i_F,
3i_F + 1 * 2, 1 + i_F * 2, 2i_F * 2}
= {6i_F + 2 + 2i_F + 3, 2i_F + 2 + 2i_F + 3, 0 + 2i_F + 3,
6i_F + 2 + 2i_F, 2 + 2i_F + 2i_F + 2i_F}
= {1, 2i_F + 3, 2, 2i_F} $\in$ S.

This subset groupoid will also be known as finite complex modulo integer groupoid.

Example 2.6: Let S = {Collection of all subsets of the special dual like number groupoid. G = {Z_6 (g); *, (3, 2) | $g^2 = g$}}; be the subset groupoid of G.

For A, B $\in$ S where A = {3g + 4, 5g, 2g + 3} and B = {2g, 3g+3}.

A * B = {3g + 4 * 2g, 5g * 2g, 2g + 3 * 2g, 3g + 4 * 3g + 3,
5g * 3g + 3, 2g + 3 * 3g + 3}
= {9g + 12 + 4g, 15g + 4g, 6g + 9 + 4g, 9g + 12 + 6g
+ 6, 15g + 6g + 6, 6g + 9 + 6g + 6}
= {g, 4g + 3, 3g, 3} $\in$ S.

S is a subset special dual like number groupoid.

Example 2.7: Let S = {Collection of all subsets of the special quasi dual number groupoid G = {Z_3 (g), *, (2, g) | $g^2 = -g$}} be the subset groupoid of G.

Let A = {2 + 2g, 2g} and B = {g + 1, g + 2} $\in$ S.

A * B = {2 + 2g * g + 1 2 + 2g * g + 2, 2g * g + 1, 2g * g + 2}
= {4 + 4g + g^2 + g, 4 + 4g + g^2 + 2g, 4g + g^2 + g, 4g + g^2 + 2g}
= {1 + g, 1 + 2g, g, 2g} $\in$ S.

Thus S is a special quasi dual like number subset groupoid.

Example 2.8: Let S = {Collection of all subsets of the groupoid G = {C(Z_5) (g), *, (4g, 0), $g^2 = 0$}} be the subset finite complex modulo integer dual number groupoid of G.

Let A = {3g + 1, 4g + 2, g + 3} and B = {3g, 2g} $\in$ S.

A * B = {12g^2 + 4g + 0, 16g^2 + 8g + 0, 4g^2 + 12g + 0}
= {4g, 3g, 2g} $\in$ S.

S is a groupoid.

We see all the eight subset groupoids given in examples 2.1 to 2.8 are all of finite order.

Now we proceed onto give examples of subset groupoids of infinite order.

Example 2.9: Let
S = {Collection of all subsets of the groupoid G = {Z, *, (2, –1)}} be the subset groupoid of G. Clearly S is a infinite order.

If A = {3, 2, 0, 5, 1} and B = {6, 2, 4} $\in$ S.

Then A * B = {3 * 6, 2 * 6, 0 * 6, 5 * 6, 1 * 6, 3 * 2, 2 * 2, 0 * 2, 5 * 2, 1 * 2, 3 * 4, 2 * 4, 0 * 4, 5 * 4, 1 * 4}

= {6 – 6, 4 – 6, –6, 10 – 6, 2–6, 6 –2, 4 – 2, 0 – 2, 10 – 2, 2 – 2, 6 – 4, 4 – 4, –4, 10 – 4, 2 –4}

= {0, –2, –6, 4, –4, 2, 8, 6}.

It is easily verified (A * B) * C = ≠ A * (B * C).

For take A = {3} B = {–5} and C = {2};

(A * B) * C = ({3} * {5}) * {2}
= {6–5} * {2}
= {1} * {2} = 2–2
= {0}. … I

Consider {A} * ({B} * {C})
= {3} * ({5} * {2})
= {3} * {10–2}
= {3} * {8}
= {6–8}
= {–2}. … II

Clearly (A * B) * C ≠ A * (B * C); since equations I and II are distinct.

Thus S is a subset groupoid of infinite order.

Example 2.10: Let S = {Collection of all subsets of the groupoid G = {Z^+ ∪ {0}, *, (5, 0)}}be the subset groupoid of G.

Take A = {6, 3, 4, 8, 1} and B = {5, 2, 1} ∈ S.

A * B = {6, 3, 4, 8, 1} × {5, 2, 1}
= {30 + 0, 15 + 0, 20+0, 40+0, 5+0}
= {30, 15, 20, 40, 5} ∈ S.

We see the subset groupoids given in examples 2.8 and 2.9 are of different infinite cardinality.

Example 2.11: Let
S = {Collection of all subsets of the groupoid G = {Q, *, (7, –1/2)}} be the subset groupoid of infinite order of G.

Example 2.12: Let S = {Collection of all subsets of the groupoid G = {$R^+ \cup \{0\}$, *, ($\sqrt{3}$, $-\sqrt{5}$)}} be the subset groupoid of infinite order.

If A = {$\sqrt{3}$, $\sqrt{5}$, 2, 4} and B = {5, $\sqrt{5}$, -1, 0} ∈ S then A * B = {$\sqrt{3}$ * 5, $\sqrt{3}$ * $\sqrt{5}$, $\sqrt{3}$ * –1, $\sqrt{3}$ * 0, $\sqrt{5}$ * 0, 2 * 5, 2 * $\sqrt{5}$, 2 * –1, 2 * 0, 4 * 5, 4 * $\sqrt{5}$, 4 * (–1), 4 * 0}

= {$3-5\sqrt{5}$, 3–5, $3-\sqrt{5}$, 3, $\sqrt{15}-5\sqrt{5}$, $2\sqrt{3}-5$, $2\sqrt{3}-\sqrt{5}$, $2\sqrt{3}$, $4\sqrt{3}-5\sqrt{5}$, $4\sqrt{3}-5$, $4\sqrt{3}+\sqrt{5}$, $4\sqrt{3}$} ∈ S.

Thus we see (S, *) is a subset groupoid of infinite order of the groupoid G.

Example 2.13: Let
S = {Collection of all subsets the groupoid G = {C, *, (i, 0)}} be the subset groupoid of complex numbers. S is of infinite order.

Example 2.14: Let S = {Collection of all subsets of the groupoid G = {C, *, (3, –2)}} be the complex number subset groupoid of G of infinite order.

Example 2.15: Let S = {Collection of all subsets of the groupoid G = {R(g), *, (3g+2, 2g), $g^2 = 0$}} be the subset groupoid of real dual numbers of G.

Example 2.16: Let S = {Collection of all subsets of the groupoid G = {($Z^+ \cup \{0\}$) (g), *, $g^2 = g$, (2, 3+g)}} be the subset groupoid of special dual like numbers of infinite order.

Example 2.17: Let S = {Collection of all subsets of the groupoid G = {($Q^+ \cup \{0\}$) (g), *, (3g, 2) where $g^2 = 0$}} be the subset groupoid of dual numbers of infinite order.

Example 2.18: Let S = {Collection of all subsets of the groupoid G = {$(Z^+ \cup \{0\})$ (g_1, g_2, g_3), *, (0, 5), $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = 0$, $g_ig_j = 0$, $i \neq j$; $1 \leq i, j \leq 3$}} be the subset mixed dual number groupoid of infinite order of G.

Example 2.19: Let S = {Collection of all subsets of the groupoid G = {$C(g_1, g_2, g_3)$, *, (0, 3i) | $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = -g_3$, $g_ig_j = 0$, $i \neq j$; $1 \leq i, j \leq 3$}} be the subset special mixed dual number groupoid of infinite order of G.

Now having seen examples of subset groupoids of finite and infinite order we now proceed onto define properties about them.

DEFINITION 2.2: *Let S = {Collection of subsets of a groupoid G} be the subset groupoid.*

If for A, B ∈ S we have A × B = {0} we say A is a subset zero divisor of the subset groupoid S where (A ≠ {0} and B ≠ {0}); (we may have right zero divisors as well as left zero divisors).

*If {A} * {A} = {A} we define {A} ∈ S to be a subset idempotent of the subset groupoid S; A ≠ {0}.*

We will first illustrate this situation by an example or two.

Example 2.20: Let S = {Collection of all subsets of the groupoid G = {Z_{16}, *, (8, 0)}} be the subset groupoid of G. Take A = {0, 2, 4, 6, 10, 12} and B = {5, 7, 8, 11, 4, 3, 1} ∈ S.

A * B = {0}.
B * A = {8, 0} ≠ {0}.
So we see A * B = {0} but B * A ≠ {0}.

Thus in groupoids we can have subset right zero divisors which are not subset left zero divisors and vice versa.

Only in very rare cases we get a zero divisor which is both a right zero divisor and a left zero divisor.

Example 2.21: Let S = {Collection of all subsets of the groupoid G = {Z, *, (2, –2)}} be the subset groupoid of G.

Take A = {5} $\in$ S we see
A * A = {5} * {5} = {5 * 5} = {10–10} = {0}.

We see infact S is rich in subset nilpotent elements of order two.

All A = {n} $\in$ S for n $\in$ Z are subset nilpotent elements of order two.

Example 2.22: Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (2, 10)}} be the subset groupoid of G.

Consider A = {10} and B = {2} $\in$ S.
A * B = {10} * {2}
= {10 * 2}
= {20 + 20} = {0}.

B * A = {2} * {10}
= {2 * 10}
= {(4 + 100)}
= {4} $\neq$ {0}.

Thus A * B = {0} but B * A $\neq$ {0}.

Example 2.23: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (3, 0)}} be the subset groupoid of G.

Take A = {2} and B = {5, 4, 3, 1} $\in$ S.

Consider
A * B = {2} * {5, 4, 3, 1}.
= {2 * 5, 2 * 4, 2 * 3, 2 * 1}
= {0}.

$$\begin{aligned}\text{But } B * A &= \{5, 4, 3, 1\} * \{2\} \\ &= \{5 * 2, 4 * 2, 3 * 2, 1 * 2\} \\ &= \{15, 12, 9, 3\} \\ &= \{3, 0\} \neq \{0\}.\end{aligned}$$

Thus A * B ≠ B * A.

Example 2.24: Let S = {Collection of all subsets of the groupoid G = {$Z^+ \cup \{0\}$, *, (3, 4)}} be the subset groupoid.

We see for A, B ∈ S.
A ≠ {0}, B ≠ {0}, A * B ≠ {0}.

Thus S has no subset zero divisors.

Example 2.25: Let S = {Collection of all subsets of the groupoid G = {Z, *, (n, –n)}} be the subset groupoid. Every A = {m} where m ∈ Z is such that A * A = {0}.

Thus S has subset zero divisors which are nilpotents of order two.

Example 2.26: Let S = {Collection of all subsets of the groupoid G = {Z_{15}, *, (5, 0)}} be the subset groupoid of G.

Take A = {3, 6, 9} and B = {1, 2, 3, 4, 5, 6, 7} ∈ S.

A * B = {0} but B * A ≠ {0}.

Infact we see for every B ∈ S;

A * B = {0} but B * A may or may not be zero.

We see this is a special property enjoyed by A ∈ S.

We see A = {3} ∈ S is also such that A * B = {0} for all B ∈ S.

However B * A may be equal to zero or may not be equal to zero. This is yet another element in S such that this sort of special property is enjoyed.

Take A = {6} ∈ S, A is also such that A * B = {0} for all B ∈ S.

Likewise A_1 = {3, 6}, A_2 = {9}, A_3 = {3, 9} and A_4 = {6, 9} in S are such that A_i * B = {0} for all B ∈ S, $1 \leq i \leq 4$.

So in view of this we make the following new definition.

DEFINITION 2.3: *Let S = {Collection of all subsets of the groupoid G} be the subset groupoid of G. If for A ∈ S we have A * B = {0} for all B ∈ S then we define A to be a right annihilator subset of the subset groupoid.*

*If A ∈ S then B * A = {0} for all B ∈ S then we define A to be the left subset annihilator of the subset groupoid.*

We have in case of subset groupoids subset right or left annihilators of a subset groupoid.

We will illustrate this by some examples.

Example 2.27: Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (0, 4)}} be the subset groupoid. Consider A = {5, 10} ∈ S is such that B * A = {0} for all B ∈ S.

Thus A is the subset left annihilator of the subset groupoid S.

A_1 = {5, 15}, A_2 = {5}, A_3 = {10}, A_4 = {15}, A_5 = {5, 10} A_6 = {10, 15}, A_7 = {0, 5}, A_8 = {0, 10}, A_9 = {0, 15}, A_{10} = {10, 35, 15} and A_{11} = {0, 5, 10, 15} are all subset left annihilators of the subset groupoid S.

Example 2.28: Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (4, 0)}} be the subset groupoid of G. Consider A = {5, 10} ∈ S, A is a right annihilator subset of the subset groupoid S.

Infact all A_i given in example 2.27 are right annihilator subset of the subset groupoid S.

Example 2.29: Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (10, 0)}} be the subset groupoid of G. A_1 = {2}, A_2 = {4}, A_3 = {6}, A_4 = {8}, A_5 = {10}, A_6 = {12}, A_7 = {14}, A_8 = {16}, A_9 = {18}, A_{10} = {0, 2}, A_{11} = {0, 2}, …, A_t = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18} are all right annihilators subsets of the subset groupoid S.

Infact A_1, A_2, …, A_t will be left annihilator subsets of the subset groupoid G′ = {Z_{20}, *, (0, 10)}.

Example 2.30: Let S = {Collection of all subsets of the groupoid G = {Z_7, *, (6, 0}} be the subset groupoid. We see S has no set A ∈ S such that S is a left or right subset annihilator of S.

Example 2.31: Let S = {Collection of all subsets of the groupoid G = {Z_{43}, *, (25, 0)}} be the subset groupoid; we see S has no element A such that A is a left or right subset annihilator in S.

Example 2.32: Let S = {Collection of all subsets of the groupoid G = {Z_{19}, *, (10, 0)}} be the subset groupoid of G; we see S does not contain any subset A ≠ {0} ∈ S such that A is the left or right annihilator subset of S.

Inview of this we have the following theorem.

THEOREM 2.1: *Let S = {Collection of all subsets of the groupoid G = {Z_n, *, (t, 0) / n-not a prime t/n}} be a subset groupoid of G. If A_1, ..., A_t are subset right annihilators of the subset groupoid S then A_1, A_2, ..., A_t are subset left annihilators* *of the subset groupoid S′ = {Collection of all subsets of the groupoid G′ = {Z_n, *, (0, t)}}.*

The proof is direct hence left as an exercise to the reader.

Example 2.33: Let S = {Collection of all subsets of the groupoid G = {Z, *, (–3, 3)}}. We see A_n = {n}; n ∈ Z are all neither right nor left annihilator subsets of G.

Only A_n * A_n = {0}.
If A_n * B ≠ {0} for B = {4, 8} in S and
A_n * B = {n * 4, n * 8} = {–3n + 12, –3n + 24}.

If A_n * B = {0} then + 3n = +12 and +3n = 24 forces n = 0. Hence the claim.

Example 2.34: Let S = {Collection of all subsets of the groupoid G = {Z, *, (–n, n)}, n ∈ Z}. We see S has no annihilator subset associated with it. For no A in S we have A * B = {0} for all B ∈ S.

A * B = {0} only when A = B = {t}; t ∈ Z.

Example 2.35: Let S = {Collection of all subsets of the groupoid G = {R, *, (t, s)} t ≠ s ∈ R \ {0}} be a subset groupoid of G. S has no subset which left annihilates S or right annihilates S.

Example 2.36: Let S = {Collection of all subsets of the groupoid G = {C, *, (a, b)} a, b ∈ C \ {0}} be a subset groupoid of G. S has no left or right annihators.

Example 2.37: Let S = {Collection of all subsets of the groupoid G = {R^+ ∪ {0}, *, (a, b)}, a, b ∈ R^+}} be the subset groupoid of G. S has no left or right annihilator subset.

Example 2.38: Let S = {Collection of all subsets of the groupoid G = {Q^+ ∪ {0}, *, (a, b)}, a, b ∈ Q^+}} be the subset groupoid of G. S has no left or right subset annihilators.

In view of all this we have the following theorems.

THEOREM 2.2: *Let S = {Collection of all subsets of the groupoid G = {$Z^+ \cup \{0\}$, *, (a, b) | a, b ∈ Z^+}} be the subset groupoid of the groupoid G; S has no annihilator subset groupoid.*

Proof is obvious. If $Z^+ \cup \{0\}$ is replaced by $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$, still the results hold good.

THEOREM 2.3: *Let S = {Collection of all subsets of the groupoid G = {Q, *, (a, b) / a, b ∈ $Q^+ \setminus \{0\}$}} be the subset groupoid. S has no subset annihilators right or left.*

This proof is also direct and hence left as an exercise to the reader. If in the above theorem Q is replaced by Z or R or C still the results hold good.

Now we proceed onto define substructures in subset groupoids.

DEFINITION 2.4: *Let S = {Collection of all subsets of the groupoid G} be the subset groupoid of G. Let P ⊆ S; if P itself is a subset groupoid under the operations of S, then we define P to be the subset subgroupoid of S.*

We will first illustrate this by some examples.

Example 2.39: Let S = {Collection of all subsets of the groupoid G = {$Z^+ \cup \{0\}$, *, (4, 8)}} be a subset groupoid. P = {Collection of all subsets of the groupoid G′ = {$2Z^+ \cup \{0\}$, *, (4, 8)}} is the subset subgroupoid of S.

Example 2.40: Let S = {Collection of all subsets of the groupoid G = {Z_{24}, *, (4, 0)}} be the subset groupoid of G. Take P = {Collection of subsets from $2Z_{24}$} ⊆ S, P is a subset subgroupoid of S.

Example 2.41: Let S = {Collection of all subsets of the groupoid G = {Z_{40}, *, (0, 10)}} be the subset groupoid. This has subset subgroupoids.

In view of this we have the following theorem.

THEOREM 2.4: *Let S = {Collection of all subsets of a groupoid G} be a subset groupoid of G.*

If G has a subgroupoid P, then T = {Collection of all subsets of the subgroupoid P of the groupoid G} ⊆ S is a subset subgroupoid of S.

The proof is direct hence left as an exercise to the reader.

Example 2.42: Let S = {Collection of all subsets of a groupoid G = {Z, *, (3, –1)}} be a subset groupoid. Take P = {{a} | a ∈ Z} ⊆ S, P is a subset subgroupoid of S.

Example 2.43: Let S = {Collection of all subsets of the groupoid G = {Z_{40}, *, (8, 4)}} be a subset groupoid of G.

Take P = {{a} | a ∈ Z_{40}} ⊆ S; P is a subset subgroupoid of S. Infact P ≅ G.

Example 2.44: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (8, 0)}} be a subset groupoid.

P = {{a} | a ∈ G} ⊆ S, is a subset subgroupoid of S. Infact P ≅ G as groupoid by the map {a} ↦ a for {a} ∈ P and a ∈ G.

Example 2.45: Let S = {Collection of all subsets of the groupoid G = {Z_5, *, (3, 1)}} be a subset groupoid of G.

P = {{0}, {1}, {2}, {3}, {4}} ⊆ S is a subset subgroupoid of S and P ≅ G.

In view of this we give the following theorem.

THEOREM 2.5: *Let S = {Collection of all subsets of the groupoid G} be a subset groupoid of G. Take P = {{a} | a ∈ G} ⊆ S; P is a subset subgroupoid of G and P ≅ G.*

The proof is direct and hence left as an exercise to the reader.

Example 2.46: Let S = {Collection of all subsets of the groupoid G = {Z_{10}, *, (5, 0)}} be a subset groupoid of G. P = {{a} | {a} ∈ G} is a subset subgroupoid of G and P ≅ G.

Now as in case of subset semigroups we can in case of subset groupoids define the notion of left subset ideals, right subset ideals and subset ideals of a subset groupoid.

This can be done as a matter of routine so left as an exercise to the reader.

Example 2.47: Let S = {Collection of all subsets of the groupoid G = {Z, *, (4, 0)}} be the subset groupoid of G. Take P = {Collection of all subsets from 2Z} ⊆ S, P is a subset ideal of S and P is both a subset left ideal as well as subset right ideal of S.

Take p ∈ P and s ∈ S, where p = {2, 4, 0, 8} ∈ P and s = {1, 2, 3} ∈ S; now

p * s = {2 * 1, 2 * 2, 2 * 3, 4 * 1, 4 * 2, 4 * 3, 0 * 1, 0 * 2,
0 * 3, 8 * 1, 8 * 2, 8 * 3}
= {8, 4, 16, 0, 32} ∈ P.

Consider
s * p = {1, 2, 3} * {2, 4, 0, 8}
= {1 * 2, 1 * 4, 1 * 0, 1 * 8, 2 * 2, 2 * 4, 2 * 0, 2 * 8,
3 * 2, 3 * 4, 3 * 0, 3 * 8}
= {4, 8, 12, 0} ∈ P.

Thus P is a subset ideal of the subset groupoid S.

Example 2.48: Let S = {Collection of all subsets of the groupoid G = $\{Z^+ \cup \{0\}, *, (0, 8)\}\}$ be a subset groupoid. P = {all subsets of the set $4Z^+ \cup \{0\}\} \subseteq$ S is a subset ideal of S.

Infact S has infinite number of subset ideals.

Example 2.49: Let S = {Collection of all subsets of the groupoid G = $\{Z^+ \cup \{0\}, *, (1, 0)\}\}$ be a subset groupoid of G.

Take P = {all subsets of the set $2Z^+ \cup \{0\}\} \subseteq$ S, P is a subset subgroupoid of S.

Clearly P is not a subset ideal of S. For take p = {2, 4, 8, 0} $\in$ P and s = {1, 3, 5} $\in$ S. We see

$$p * s = \{2 * 1, 2 * 3, 2 * 5, 4 * 1, 4 * 3, 4 * 5, 8 * 1, 8 * 3, 0 * 1, 0 * 3, 0 * 5\}$$

$$= \{2, 4, 8, 0\} \in P.$$

Consider

$$s * p = \{1, 3, 5\} * \{0, 2, 4, 8\}$$
$$= \{1 * 0, 1 * 2, 1 * 4, 1 * 8, 3 * 0, 3 * 2, 3 * 4, 3 * 8, 5 * 0, 5 * 2, 5 * 4, 5 * 8\}$$
$$= \{1, 3, 5\} \notin P.$$

Thus P is only a right subset ideal of S and is not a left subset ideal of S.

Hence we have in subset groupoids which are subset right ideals and are not subset left ideals and vice versa.

Example 2.50: Let S = {Collection of all subsets of the groupoid G = $\{Z^+ \cup \{0\}, *, (0, 1)\}\}$ be the subset groupoid of G.

Take M = {Collection of all subsets of $3Z^+ \cup \{0\}$} ⊆ S; M is a subset subgroupoid of S.

We find for s ∈ S and m ∈ M; s * m and m * s.

Let
s = {2, 4, 6, 10, 8, 0} and m = {12, 3, 24, 6, 9, 27, 0, 15} ∈ M.

We find

s * m = {2, 4, 6, 8, 10, 0} * {0, 3, 6, 9, 12, 15, 24, 27}

= {2 * 0, 2 * 3, 2 * 6, …, 2 * 27, 4 * 0, 4 * 3, 4 * 6, …, 4 * 27, 6 * 0, 6 * 3, 6 * 6, …, 6 * 27, 8 * 0, 8 * 3, 8 * 6, …, 8 * 27, 10 * 0, 10 * 3, 10 * 6, …, 10 * 27, 0 * 0, 0 * 3, 0 * 6, …, 0 * 27}
= {0, 3, 6, 9, 12, 15, 24, 27} ∈ M.

Consider

m * s = {0, 3, 6, 9, 12, 15, 24, 27} * {2, 4, 6, 8, 10, 0}

= {0 * 2, 0 * 4, …, 0 * 0, 3 * 2, 3 * 4, …, 3 * 0, 6 * 2, 6 * 4, …, 6 * 6, 9 * 2, 9 * 4, …, 9 * 0, 12 * 2, 12 * 4, …, 12 * 0, 15 * 2, 15 * 4,…,15 * 0, 24 * 2, 24 * 4, …, 24 * 0, 27 * 2, 27 * 4, …, 27 * 0}

= {2, 4, 6, 8, 10, 0} ∉ M.

Thus M is only a left subset ideal of S which is not a right subset ideal of S.

Example 2.51: Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (1, 0)}} be the subset groupoid. If M = {collection of all subsets from {2, 4, 6, 8, 10, 12, 14, 16, 18, 0} ⊆ G} ⊆ S be the subset subgroupoid of S. Then M is not a subset ideal of S only a right subset ideal of S and is not a left subset ideal of S.

Similarly if G in example 2.51 is replaced by $G' = \{Z_{20}, *, (0, 1)\}$ then S′ the subset groupoid of G′; we see M′ = {Collection of all subsets from the set {0, 2, 4, 6, 8, 10, 12, 14, 16, 18} ⊆ G′} ⊆ S′ is only a left subset ideal of S′ and is not a right subset ideal of S′.

In view of the above examples we have the following theorem.

THEOREM 2.6: *Let S = {Collection of subsets of the groupoid G = {Z (or $Z^+ \cup \{0\}$ or Z_n (n a composite number)), *, (1, 0)} or G′ = {Z (or $Z^+ \cup \{0\}$ or Z_n (n a composite number), *, (0, 1)}} be a subset groupoid. S has subset right ideals which are not subset left ideals and vice versa.*

The proof is direct and hence left as an exercise to the reader.

Example 2.52: Let S = {Collection of all subsets of the groupoid G = {Z, *, (4, 2)}} be the subset group. S has subset ideals which are both left and right.

Now we make the following statement.

THEOREM 2.7: *If S is the subset groupoid of a groupoid G and if*

(i) If G has a ideal then S has a subset ideal.
(ii) If G has a left ideal then S has a subset left ideal.
(iii) If G has a right ideal then S has a subset right ideal.

The proof is direct hence left as an exercise to the reader. Now we proceed onto define other structures over these subset groupoids.

DEFINITION 2.5: *Let S = {Collection of all subsets of a groupoid G} be the subset groupoid. If S contains a non empty subset H ⊆ S such that H is* *a subset semigroup of S then we define S to be a Smarandache subset groupoid.*

We will first illustrate this situation by some examples.

Example 2.53: Let
S = {Collection of all subset the groupoid G = {Z_6, *, (4, 5)}} be the subset groupoid. S is a Smarandache subset groupoid for H = {{0}, {3}, {0, 3}} ⊆ S is a subset semigroup.

Example 2.54: Let
S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (3, 9)}} be a subset groupoid of G.

S is a Smarandache subset groupoid as
H = {{0}, {6}, {0, 6}} ⊆ S is a subset semigroup.

Example 2.55: Let S = {Collection of all subsets of the groupoid G = {Z_{10}, *, (5, 6)}} be a subset groupoid of G.

H = {{0}, {2}, {0, 2}} ⊆ S is a subset semigroup. S is a Smarandache subset groupoid.

Example 2.56: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (3, 4)}} be a subset groupoid of G. S is a Smarandache subset groupoid for H_1 = {{0}, {2}} and H_2 = {{0}, {4}, {0, 4}} are subset semigroups of S.

In view of all these we have the following theorem.

THEOREM 2.8: *Let*
S = {Collection of all subsets of a groupoid G} be a subset groupoid of the groupoid G. If G is a Smarandache groupoid then S is a Smarandache subset groupoid.

Proof: If G is a Smarandache groupoid then G contains a non empty subset H ⊆ G such that H is a semigroup under the operations of G. Take P = {Collection of all subsets of H} ⊆ S, P is also a subset semigroup, hence S is a Smarandache subset groupoid.

It is left as an open question to test if S is a Smaradache subset groupoid over G should G be a Smarandache groupoid?

Now we discuss some more properties of subset groupoids.

DEFINITION 2.6: *Let S be a subset groupoid of a groupoid G. Suppose P $\subseteq$ S be a subset subgroupoid of G. If these exists a M $\subseteq$ P such that M is a subset semigroup then we define P to be a subset Smarandache subgroupoid of S.*

We will first illustrate this situation by an example or two.

Example 2.57: Let S = {Collection of all subsets of the groupoid G = {Z_8, *, (2, 6)}} be a subset groupoid of G. Let P = {Collection of all subsets of the set; {0, 2, 4, 6} $\subseteq$ Z_8} $\subseteq$ S. P is a subset Smarandache subgroupoid of G.

For H = {{0}, {4}, {0, 4}} $\subseteq$ S is a subset semigroup of P.

Example 2.58: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (4, 5)}} be a subset groupoid of G. P = {Collection of all subsets of the set {1, 3, 5} $\subseteq$ Z_6} $\subseteq$ S, P is a Smarandache subset subgroupoid of S as H = {{3}} is a subset semigroup of S.

Now we proceed onto define Smarandache commutative subset groupoid, Smarandache subset right ideal, Smarandache subset left ideal, Smarandache subset seminormal groupoid, Smarandache subset normal groupoid, Smarandache semiconjugate subset subgroupoid and Smarandache subset conjugate subgroupoid and describe them with examples [4].

DEFINITION 2.7: *Let S be a subset groupoid of the groupoid G. If S contains a subset semigroup H such that H is commutative then we define S to be a Smarandache commutative subset groupoid.*

DEFINITION 2.8: *Let S be a subset groupoid of the groupoid G. Let P be a subset Smarandache subgroupoid of S. If for s ∈ S and p ∈ P; sp ∈ P then we define P to be a Smarandache subset left ideal of S. If for s ∈ S and p ∈ P; ps ∈ P we define P to be a Smarandache subset right ideal of S. If P is both a Smarandache subset left ideal as well as Smarandache subset right ideal we define P to be a Smarandache subset ideal of S.*

DEFINITION 2.9: *Let S be a subset groupoid of a groupoid G. V be a Smarandache subset subgroupoid of S. We say V is a Smarandache seminormal subset groupoid if*

(i) $aV = X$ *for all* $a \in S$
(ii) $Va = Y$ *for all* $a \in S$

where either X or Y is a Smarandache subset subgroupod of G but X and Y are both subset subgroupoids.

We say V is a Smarandache normal subset groupoid if Va = Y and aV = X for all s ∈ S where both X and Y are Smarandache subset groupoids.

Now we proceed onto define Smarandache subset semiconjugate subgroupoid and Smarandache subset conjugate subgroupoid.

DEFINITION 2.10: *Let S be a subset Smarandache groupoid of a groupoid G. We say two subset subgroupoids H and P of S are said Smarandache semiconjugate subset subgroupoids of S if*

(i) *H and P are Smarandache subset subgroupoids of S.*
(ii) $H = xP$ *or* Px *or*
(iii) $P = xH$ *or* Hx *for some* $x \in S$.

DEFINITION 2.11: *Let S be a Smarandache subset groupoid. H and P be two subset subgroupoids of S. We say H and P are Smarandache conjugate subset subgroupoids of S if*

(i) *H and P are Smarandache subset subgroupoids of S.*
(ii) *H = xP or Px and*
(iii) *P = xH or Hx for some x $\in$ S.*

We will first illustrate these definitions by some examples and describe a few properties associated with them.

Example 2.59: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (4, 5)}} be a subset groupoid of a groupoid G. Clearly S is a Smarandache commutative groupoid and is not a commutative groupoid.

Example 2.60: Let
S = {Collection of all subsets of a groupoid G = {Z_{12}, *, (1, 3)}} be a subset groupoid. S is a Smarandache commutative subset groupoid of G.

In view of these examples we give the following theorem.

THEOREM 2.9: *Let S be a commutative subset groupoid; if S is a Smarandache subset groupoid then S is a Smarandache commutative subset groupoid. Conversely if S is a Smarandache subset commutative groupoid then S need not in general be a commutative subset groupoid.*

The proof is direct and hence left as an exercise to the reader.

Example 2.61: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (4, 5)}} be the subset groupoid. Take I = {Collection of all subsets of the set {1, 3, 5}} $\subseteq$ S, I is a Smarandache subset left ideal of S and is not a Smarandache subset left ideal of S.

Example 2.62: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (2, 4)}} be a subset groupoid of the groupoid G.

P = {Collection of all subsets from {0, 2, 4}} ⊆ S; P is a subset ideal of S however P is not a Smarandache subset ideal. That is P is not even a Smarandache subset subgroupoid of S.

In view of all these we have the following theorem.

THEOREM 2.10: *Let S be a subset groupoid of a groupoid G. If P ⊆ S is a Smarandache subset ideal of S then P is a subset ideal of S however if P is a subset ideal of S; P need not in general be a Smarandache subset ideal of S.*

THEOREM 2.11: *Let S be a subset groupoid of G. If G has a Smarandache ideal then S has a Smarandache subset ideal.*

Example 2.63: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (4, 5)}} be the subset groupoid of G. P = {Collection of all subsets of the set {1, 3, 5}} ⊆ S be a subset Smarandache subgroupoid of S. P is a Smarandache subset seminormal groupoid of S. Clearly P is not a Smarandache normal subset groupoid.

Example 2.64: Let S = {Collection of all subsets of the groupoid G = {Z_8, *, (2, 6)}} be the subset groupoid of G. P = {Collection of all subsets {0, 2, 4, 6}} ⊆ S. P is a Smarandache subset normal groupoid of S. Further P is a Smarandache subset seminormal groupoid of S.

In view of all these we have the following result.

THEOREM 2.12: *Let S be a subset groupoid. Every Smarandache normal subset groupoid is a Smarandache seminormal subset groupoid and not conversely.*

The proof is direct and hence left as an exercise to the reader.

Next we proceed onto give examples of Smarandache semiconjugate subset subgroupoids.

Example 2.65: Let S = {Collection of all subsets of the groupoid G = {Z_8, *, (2, 6)}} be the subset groupoid.

Let
P = {Collection of all subsets of the set {0, 3, 2, 4, 6} $\subseteq Z_8$} be the Smarandache subset subgroupoid of S.

Let Q = {Collection of all subsets of the set {0, 2, 4, 6} $\subseteq Z_8$} be the Smarandache subset subgroupoid of S. Now {7}P = Q. Hence P and Q are Smarandache semiconjugate subset groupoids.

Example 2.66: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (1, 3)}} be the subset groupoid of G.

Let P_1 = {Collection of all subsets of {0, 3, 6, 9} $\subseteq Z_{12}$} be Smarandache subset subgroupoid of S and
P_2 = {Collection of all subsets of {2, 5, 8, 11} $\subseteq Z_{12}$} be another Smarandache subset subgroupoid of S. We see {3}P_2 = P_1 and {2}P_1 = P_2.

Hence P_1 and P_2 are Smarandache subset conjugate subgroupoids of G.

We just give some theorem the proofs of which are direct and hence left as an exercise to the reader.

THEOREM 2.13: *Let S be a Smarandache subset groupoid of a groupoid G. If P_1 and P_2 are two Smarandache subset subgroupoids of S which are Smarandache conjugate then they are Smarandache semiconjugate. But Smarandache semiconjugate subset subgroupoids need not in general be Smarandache subset conjugate subgroupoids.*

THEOREM 2.14: *Let S be a subset groupoid of a groupoid G. If G has two Smarandache subgroupoids A_1 and A_2 such that A_1 and A_2 are Smarandache semiconjugates then S has two Smarandache subset subgroupoids which are semiconjugate.*

However we are not aware of the fact about the following question.

Problem 2.1: Let G be a groupoid which has no Smarandache semiconjugate subgroupoids.

S = {Collection of all subsets of the groupoid G} be the subset groupoiid of G. Can S have Smarandache semiconjugate subset subgroupoids?

THEOREM 2.15: *Let G be a groupoid such that G has two Smarandache subgroupoids P_1 and P_2 such that P_1 and P_2 are Smarandache conjugate subgroupoids.*

Let S = {Collection of all subsets of the groupoid G} be the subset groupoid of G, then S has two Smarandache subset subgroupoids Q_1 and Q_2 such that Q_1 and Q_2 are Smarandache conjugate subset subgroupoids.

However we have the following problem.

Problem 2.2: Let G be a groupoid which has no Smarandache conjugate subgroupoids.

Let S = {Collection of all subsets of G} be the subset groupoid of G. Can S contain Smarandache subset subgroupoids T_1 and T_2 such that T_1 and T_2 are Smarandache subset conjugate subgroupoids of G?

We just say a subset groupoid S of a groupoid G to be inner commutative if for every proper subset P of S which is a subset semigroup is commutative.

Our results in case of subset inner commutative groupoid is little difficult. We will first study one example.

Ex*ample 2.67:* Let
S = {Collection of all subsets of the groupoid G = {0, 1, 2, 3}} under the operation * given by the table:

*	0	1	2	3
0	0	3	2	1
1	2	1	0	3
2	0	3	2	1
3	2	1	0	3

be the subset groupoid of G. A_1 = {1} and {2} = A_2 are subsets of S which are subset semigroups of S under the binary operation '*'. P_1 = {{0}, {2}, {0, 2}}, and P_2 = {{1}, {3}, {1,3}} $\subseteq$ S are subset subgroupoids which are infact Smarandache subset subgroupoids of G. Clearly P_1 and P_2 are not commutative subset Smarandache subgroupoids of S.

Now consider all the subsets of G.

S = {{0}, {1}, {2}, {3}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {0, 1, 2}, {0, 1, 3}, {0, 3, 2}, {1, 2, 3}, {0, 1, 2, 3}} is the subset groupoid of order 15 which is also a Smarandache subset groupoid which is not commutative but is Smarandache commutative.

Is S Smarandache inner commutative?

So we just propose the following problem.

Problem 2.3: Let
S = {Collection of all subsets of a groupoid G} be the subset groupoid of G.

(i) If G is a Smarandache inner commutative groupoid, will S be a Smarandache inner commutative subset groupoid?
(ii) If G is not a Smarandache inner commutative groupoid; can S be a Smarandache inner commutative subset groupoid?

However the following result is direct hence left as an exercise to the reader.

THEOREM 2.16: *Let S be a Smarandache inner commutative subset groupoid then S is a Smarandache commutative subset groupoid but a Smarandache commutative subset groupoid in general is not a Smarandache inner commutative subset groupoid.*

Next we proceed onto define identities in Smarandache subset groupoids.

DEFINITION 2.12: *Let S be a subset groupoid of a groupoid G. We say S is a Smarandache subset Moufang groupoid if there exists a subset Smarandache subgroupoid H such that*

*(A * B) * (C * A) = (A * ((B * C)) * A for all A, B, C in H* $\subseteq$ *S.*

If S is a Smarandache subset groupoid such that every Smarandache subset subgroupoid H of S satisfies the Moufang identity for all A, B, C in H then we define S to be Smarandache strong subset Moufang groupoid.

First we will illustrate this situation by some examples.

Example 2.68: Let S = {Collection of all subsets of the groupoid G = {Z_{10}, *, (5, 6)}} be the subset groupoid of G. Clearly S is a Smarandache subset groupoid of G as {2} $\in$ S is a subset semigroup.

Take A = {4, 2, 1}, B = {3, 0}, and C = {5, 6} in S.

Consider (A * B) * (C * A)

= {4 * 3, 4 * 0, 2 * 3, 2 * 0, 1 * 3, 1 * 0} * {5 * 4, 5 * 2, 5 * 1, 6 * 4, 6 * 2, 6 * 1}
= {20 + 18, 20, 10 + 18, 10, 5 + 18, 5} * {25 + 24, 25 + 12, 25 + 6, 30 + 24, 30 + 12, 30 + 6}
= {8, 0, 3, 5} * {9, 7, 1, 4, 2, 6}

= {8 * 9, 8 * 7, 8 * 1, 8 * 4, 8 * 2, 8 * 6, 0 * 9, 0 * 7, 0 * 1, 0 * 4, 0 * 2, 0 * 6, 3 * 9, 3 * 7, 3 * 1, 3 * 4, 3 * 2, 3 * 6, 5 * 9, 5 * 7, 5 * 1, 5 * 4, 5 * 2, 5 * 6}

= {40 + 54, 40 + 42, 40 + 6, 40 + 24, 40 + 12, 40 + 36, 54, 42, 6, 24, 12, 36, 15 + 54, 15 + 42, 15 + 6, 15 + 24, 15 + 12, 15+36, 25 + 54, 25 + 42, 25 + 6, 25 + 24, 25 + 12, 25 + 36}

= {4, 2, 6, 9, 7, 1}. … I

Consider [A * (B * C)] * A

= ({4, 2, 1} * ({3, 0} * {5, 6}) * {4, 2, 1}
= ({4, 2, 1} * {3 * 5, 3 * 6, 0 * 5, 0 * 6}) * {4, 2, 1}
= ({4, 2, 1} * {15+30, 15+36, 30, 36}) * {4, 2, 1}
= ({4, 2, 1}) * {5, 1, 6}) * {4, 2, 1}
= {4 * 5, 4 * 1, 4 * 6, 2 * 5, 2 * 1, 2 * 6, 1 * 5, 1 * 1, 1 * 6} * {4, 2, 1}
= {20 + 30, 20 + 6, 20 + 36, 10 + 30, 10 + 6, 10 + 36, 5 + 30, 5 + 6, 5 + 36} * {4, 2, 1}
= {0, 6} * {4, 2, 1}
= {0 * 4, 0 * 2, 0 * 1, 6 * 4, 6 * 2, 6 * 1}
= {24, 12, 6, 30+24, 30+12, 30+6}
= {4, 2, 6}. … II

We see S is not a Smarandache strong subset Moufang groupoid though G is a Smarandache strong Moufang groupoid for I and II are distinct.

But S is a Smarandache subset Moufang groupoid for P = {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}} ⊆ S is a Smarandache strong Moufang subgroupoid of S.

Example 2.69: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, (3, 9), *}} be the subset groupoid of a groupoid G. S is not a Smarandache strong Moufang groupoid. However S is a Smarandache Moufang subset groupoid.

In view of all these we can just have the following theorem.

THEOREM 2.17: *Every Smarandache strong Moufang subset groupoid is a Smarandache Moufang subset groupoid and not conversely.*

We on similar lines define a subset groupoid S to be a Smarandache Bol subset groupoid if it satisfies atleast for a Smarandache subset subgroupoid H $\subseteq$ S the Bol identity.

$((\{A\} * \{B\}) * \{C\}) * \{B\} = \{A\} * ((\{B\} * \{C\}) * \{B\})$ … I

for all {A}, {B}, {C} $\in$ H.

If the Bol identity I is satisfied by every triple of subset {A}, {B}, {C} $\in$ S then we define S to be a Smarandache strong subset Bol groupoid.

We will illustrate these situations by some examples.

Example 2.70: Let S = {Collection of all subsets of the groupoid G = {Z_4, *, (2, 3)}} be the subset groupoid of the groupoid G.

We see S is just a Smarandache subset Bol groupoid as P = {{0}, {2}, {0, 2}} $\subseteq$ S is such that (P, *) is a Smarandache subset subgroupoid given by the following table:

*	{0}	{2}	{0,2}
{0}	{0}	{2}	{0,2}
{2}	{0}	{2}	{0,2}
{0,2}	{0}	{2,0}	{0,2}

for all {2} $\subseteq$ P is a subset semigroup in S. We see P satisfies Bol identity so P is a Smarandache subset Bol groupoid.

But S is not a Smarandache strong subset Bol groupoid for take A = {1}, B = {2} and C = {2, 3} $\in$ S.

((A * B) * C) * B

= ({(1 * 2)} * C) * B
= ({0} * {2, 3}) * B
= {0 * 2, 0 * 3} * B
= {2, 1} * {2}
= {2, 0}. … I

Now consider A * [(B * C) * B]

= A* [({2} * {2, 3}) × {2}]
= A * [{2 * 2, 2 * 3} × {2}]
= A * ({2} * {2})
= A * {2 * 2}
= A * {2} = {1} * {2}
= {1 * 2}
= {2 + 2} = {0} … II

Clearly I and II are different so S is not a Smarandache strong subset Bol groupoid.

Example 2.71: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (3, 4)}} be the subset groupoid of the groupoid G.

P = {{1}, {2}, …, {11}, {0}} ⊆ S is a Smarandache strong Bol subset subgroupoid of S.

So S is clearly a Smarandache subset Bol groupoid.

Now we take three subsets A = {1, 2, 0}, B = {6, 3, 2} and C = {1, 4, 8} in S. Now we find

((A * B) * C) * B

= (({1,2, 0} * {6, 3, 2}) * C) * B
= [({1 * 6, 1 * 3, 1 * 2, 2 * 6, 2 * 3, 2 * 2, 3 * 6, 3 * 3, 3 * 2}) * C] * B

= [{3 + 24, 3 + 12, 3 + 6, 6 + 24, 6 + 12, 6 + 6, 9 + 24, 9 + 12, 9 + 6} * C] * B

= ({3, 6, 9} * {1, 4, 8}) * B

= {3 * 1, 3 * 4, 3 *8, 6 * 1, 6 * 4, 6 * 8, 9 * 1, 9 * 4, 9 * 8} * B

= {9 + 4, 9 + 16, 9 + 32, 18 + 4, 18 + 16, 18 + 32, 27 + 4, 27 + 16, 27+32} * B

= {1, 5, 10, 7, 11} * {6, 3, 2}

= {1 * 6, 1 * 3, 1 * 2, 5 * 6, 5 * 3, 5 * 2, 10 * 6, 10 * 3, 10 * 2, 7 * 6, 7 * 3, 7 * 2, 11 * 6, 11 * 3, 11 * 2}

= {3 + 24, 3 + 12, 3 + 8, 15 + 24, 15 + 12, 15 + 8, 30 + 24, 30 + 12, 30+8, 21 + 24, 21 + 12, 21 + 8, 33 + 24, 33 + 12, 33 + 8} = {3, 11, 6, 2, 9, 5} … I

A * [[B * C] * B]

= A * [{{6, 3, 2} * {1, 48}} * B]

= A * ({6 * 1, 6 * 4, 6 * 8, 3 * 1, 3 * 4, 3 * 8, 2 * 1, 2 * 4, 2 * 8) * B)

= A * ({18 + 4, 18 + 16, 18 + 32, 9 + 4, 9 + 16, 9 + 32, 6 + 4, 6 + 16, 6 + 32} * B)

= A * ({10, 2, 1, 5} * {6, 3, 2})

= A * {10 * 6, 10 * 3, 10 * 2, 2 * 6, 2 * 3, 2 * 2, 1 * 6, 1 * 3, 1 * 2, 5 * 6, 5 * 3, 5 * 2}

$= \{0, 1, 2\} * \{30 + 24, 30 + 12, 30 + 8, 6 + 24, 6 + 12, 6 + 8, 3 + 24, 3 + 8, 3 + 12, 15 + 24, 15 + 8, 15 + 12\}$

$= \{0, 1, 2\} * \{6, 2, 3, 11\}$

$= \{0 * 6, 0 * 2, 0 * 3, 0 * 11, 1 * 6, 1 * 2, 1 * 3, 1 * 11, 2 * 6, 2 * 2, 2 * 3, 2 * 11\}$

$= \{24, 8, 12, 44, 3 + 24, 3 + 8, 3 + 12, 3 + 44, 6 + 24, 6 + 8, 6 + 12, 6 + 44\}$

$= \{0, 8, 3, 11, 6, 2\}$ … II

Clearly I and II are not equal so S is not a Smarandache strong subset Bol groupoid.

We have the following interesting theorem.

THEOREM 2.18: *Let S = {Collection of all subsets of a groupoid G} be the subset groupoid of the groupoid G.*

(i) If G is a Smarandache strong Bol groupoid then the subset groupoid S has a subset subgroupoid H ⊆ S which is a Smarandache strong subset Bol subgroupoid and H and G are isomorphic as groupoids.

(ii) S the subset groupoid in general is not a Smarandache strong subset Bol groupoid.

(iii) S is a Smarandache subset Bol groupoid.

The proof is direct and can be proved with appropriate modifications hence left as an exercise to the reader.

Next we proceed onto define Smarandache subset P-groupoid and Smarandache strong subset P-groupoid.

DEFINITION 2.13: *Let S be the subset groupoid. Let P ⊆ S (P a proper subset) be a Smarandache subset subgroupoid of S and satisfies the identity;*

*(A * B) * A = A * (B * A) for all A, B ∈ P; then we define S to be a Smarandache P-subset groupoid. If S is such that for every A, B ∈ S the identity, (A * B) * A = A * (B * A) is satisfied then we define S to be a Smarandache strong P-subset groupoid.*

We will illustrate this situation by some examples.

Example 2.72: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (4, 3)}} be the subset groupoid of the groupoid G.

Take M = {{0}, {1}, {2}, {3}, {4}, {5}} ⊆ S; M is a Smarandache strong P-subset subgroupoid of S so S is a Smarandache P subset groupoid.

Now we find whether for any A, B ∈ S we have
(A * B) * A = A * (B * A).

Let A = {1, 2, 0, 3} and B = {1, 4} be in S.

To find (A * B) * A

= ({1, 2, 0, 3} * {1, 4}) * A

= {1 * 1, 1 * 4, 2 * 1, 2 * 4, 0 * 1, 0 * 4, 3 * 1, 3 * 4} * A

= {4 + 7, 4 + 12, 8 + 3, 8 + 12, 3, 12, 12 + 3, 12 + 12} * A

= {5, 4, 2, 3} * {1, 2, 0, 3}

= {5 * 1, 5 * 2, 5 * 0, 5 * 3, 4 * 1, 4 * 2, 4 * 0, 4 * 3, 3 * 1, 3 * 2, 3 * 0, 3 * 3}

= {20 + 3, 20 + 6, 20, 20 + 9, 16 + 3, 16 + 6, 16, 16 + 9, 12 + 3, 12 + 6, 12, 12+9}

= {5, 2, 1, 4, 3, 0} … I

Consider A * (B * A)

= A * ({1, 4} * {1, 2, 0, 3})
= A * {1 * 1, 1 * 2, 1 * 0, 1 * 3, 4 * 1, 4 * 2, 4 * 0, 4 * 3}
= A * {4 + 3, 4 + 6, 4, 4 + 9, 16 + 3, 16 + 6, 16, 16 + 9}
= {1, 2, 0, 3} * {1, 4}
= {1 * 1, 1 * 4, 2 * 1, 2 * 4, 0 * 1, 0 * 4, 3 * 0, 3 * 4}
= {4 + 3, 4 + 12, 8 + 3, 8 + 12, 3, 12, 12 + 3, 12 + 12}
= {1, 4, 3} … II

Clearly I and II are different so S is not a Smarandache strong subset P-groupoid.

Example 2.73: Let S = {Collection of all subsets of the groupoid G = {Z_4, *, (2, 3)}} be a subset groupoid. We see S is a Smarandache P-subset groupoid as
M = {{0}, {1}, {2}, {3}} ⊆ S is a Smarandache subset subgroupoid which satisfies the identity
(A * B) * A = A * (B * A) for all A, B ∈ S.

Consider A = {0, 1} and B = {2, 3} ∈ S.

We find (A * B) * A
= ({0,1} * {2, 3}) * A
= {0 * 2, 0 * 3, 1 * 2, 1 * 3} * A
= {6, 9, 2+6, 2 + 9} * A
= {2, 1, 0, 3} * {0, 1}
= {2 * 0, 2 * 1, 1 * 0, 1 * 1, 0 * 0, 0 * 1, 3 * 0, 3 * 1}
= {0, 3, 2, 1} … I

Consider A * (B * A)

= A * {{2, 3} * {0, 1}}
= A * {2 * 0, 2 * 1, 3 * 0, 3 * 1}

= A * {4, 4 + 3, 6, 6 + 3}
= {0, 1} * {0, 3, 2, 1}
= {0 * 0, 0 * 3, 0 * 2, 0 * 1, 1 * 0, 1 * 1, 1 * 2, 1 * 3}
= {0, 9, 6, 3, 2, 2 + 3, 2 + 6, 2 + 9}
= {0, 1, 2, 3} … II

We see I and II are equal for this A and B in S.

Take A = {0, 1, 2} and B = {3} in S. To find (A * B) * A and A * (B * A)

Consider (A * B) * A

= ({0, 1, 2} * {3}) * A
= {0 * 3, 1 * 3, 2 * 3} * A
= {9, 2 + 9, 4 + 9} * A
= {1, 2} * {0, 1, 2}
= {1 * 0, 1 * 1, 1 * 2, 2 * 0, 2 * 1, 2 * 2}
= {2, 2 + 3, 2 + 6, 4, 4 + 2, 4 + 6}
= {2, 0, 1} … I

A * (B * A)

= A* ({3} * {0, 1, 2})
= A * {3 * 0, 3 * 1, 3 * 2}
= A * {6, 6 + 3, 6 + 6}
= {0, 1, 2} * {2, 1}
= {0 * 1, 0 * 2, 1 * 2, 1 * 1, 2 * 2, 2 * 1}
= {3, 6, 2 + 6, 2 + 3, 4 + 6, 4 + 3}
= {3, 2, 0, 1} … II

I and II are not the same for this A and B in S. Thus S is not a Smarandache subset strong P-groupoid.

Example 2.74: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (3, 5)}} be the subset groupoid of the groupoid G. Take B_1 = {{0}, {3}, {0, 3}} ⊆ S; B_1 is a Smarandache subset groupoid.

A = {3}, B = {0} $\in B_1$

(A * B) * A

$$
\begin{aligned}
&= (\{3\} * \{0\}) * A \\
&= \{9\} * A \\
&= \{3\} * \{3\} \\
&= \{9 + 15\} \\
&= \{0\} \qquad \ldots \qquad \text{I}
\end{aligned}
$$

$$
\begin{aligned}
A * (B * A) &= A * (\{0\} * \{3\}) \\
&= A * \{(0 * 3)\} \\
&= A * \{15\} \\
&= \{3\} * \{3\} \\
&= \{0\} \qquad \ldots \qquad \text{II}
\end{aligned}
$$

I and II are equal for this A, B in B_1.

Take A = {3} and B = {0, 3} in B_1. To find (A * B) * A.

Now (A * B) * A

$$
\begin{aligned}
&= (\{3\} * \{0, 3\}) * A \\
&= \{3 * 0, 3 * 3\} * A \\
&= \{0, 3\} * A \\
&= \{0, 3\} * \{3\} \\
&= \{0 * 3, 3 * 3\} \\
&= \{3, 0\} \qquad \ldots \qquad \text{I}
\end{aligned}
$$

Consider A * (B * A)

$$
\begin{aligned}
&= A * \{0, 3\} * \{3\} \\
&= A * \{0, 3\} \\
&= \{3\} * \{0, 3\} \\
&= \{3 * 0, 3 * 3\} \\
&= \{9, 9 + 15\} \\
&= \{3, 0\} \qquad \ldots \qquad \text{II}
\end{aligned}
$$

I and II are equal.

Thus B_1 is a Smarandache subset subgroupoid of S which satisfies the identity (A * B) * A = A * (B * A) for all A, B ∈ B. Hence S is a Smarandache subset P-groupoid.

We have the following theorem the proof of which is left as an exercise to the reader.

THEOREM 2.19: *Let S = {Collection of all subsets of a groupoid G} be a subset groupoid of the groupoid G.*

(i) *If G is a Smarandache strong P-groupoid then the subset groupoid S contains a subset subgroupoid H (⊆ S) such that H is a Smarandache strong P-subgroupoid of S.*

(ii) *S is a Smarandache P-groupoid if G is a Smarandache P-groupoid.*

(iii) *If G is a Smarandache strong P-groupoid then in general the subset groupoid S need not be a Smarandache strong subset P-groupoid.*

Next we proceed onto define the notion of Smarandache right alternative subset groupoid.

Let S be a subset groupoid of G. If H is a proper subset of S such that H is a Smarandache subset subgroupoid of S and satisfies the right alternative identity A* (B*B) = (A*B)*B for all A, B ∈ H then we define S to be a Smarandache right alternative subset groupoid.

On similar lines we can define Smarandache left alternative subset groupoid if (A*A)*B = A*(A*B) for all A, B ∈ H. If H satisfies both the left and right alternative identities then we define S to be a Smarandache subset alternative groupoid or Smarandache alternative subset groupoid.

We define S to be a Smarandache strong right alternative subset groupoid if for all A, B ∈ S; we have
(A * B) * B = A * (B * B) and Smarandache groupoid if for all A, B ∈ S. We have (A * A) * B = A * (A * B). If S is both a Smarandache strong right alternative subset groupoid and Smarandache strong left alternative subset groupoid then we define S to be a Smarandache strong alternative subset groupoid.

We will illustrate this situation by some examples.

Example 2.75: Let S = {Collection of all subsets of the groupoid G = {Z_{14}, *, (7, 8)}} be a subset groupoid. It is easily verified S is a Smarandache alternative subset groupoid as P = {{0}, {1}, {2}, …, {12}, {13}} ⊆ S is a Smarandache strong alternative subset subgroupoid of S.

To find whether S is a Smarandache strong alternative subset groupoid.

Let A = {0, 7, 1, 6} and B = {2, 4, 11, 1} ∈ S.

We first find (A * B) * A
= ({0, 7, 1, 5} * {1, 2, 4, 11}) * A

= {0 * 1, 0 * 2, 0 * 4, 0 * 11, 7 * 1, 7 * 2, 7 * 4, 7 * 11, 1 * 1, 1 * 2, 1 * 4, 1 * 11, 5 * 1, 5 * 2, 5 * 4, 5 * 11} * A

= {8, 16, 32, 88, 49 + 8, 49 + 16, 49 + 32, 49 + 88, 7 + 8, 7 + 16, 7 + 32, 7 + 88, 35 + 8, 35 + 16, 35 + 32, 35+88} * A

= {8, 2, 4, 1, 11, 9} * {0, 7, 1, 6}

= {8 * 0, 8 * 7, 8 * 1, 8 * 6, 2 * 0, 2 * 7, 2 * 1, 2 * 6, 4 * 0, 4 * 7, 4 * 1, 4 * 6, 1 * 0, 1 * 7, 1 * 1, 1 * 6, 11 * 0, 11 * 7, 11 * 1, 11 * 6, 9 * 0, 9 * 7, 9 * 11, 9 * 6}

$$= \{56, 56 + 56, 56 + 8, 56 + 48, 14, 14 + 56, 14 + 8, 14 + 48, 28, 28 + 56, 28 + 8, 28 + 48, 7 + 0, 7 + 56, 7 + 8, 7 + 48, 77 + 0, 77 + 56, 77 + 8, 77 + 48, 63 + 0, 63 + 56, 63 + 8, 63 + 48\}$$

$$= \{0, 8, 6, 7, 1, 13\} \qquad \ldots \qquad \text{I}$$

The reader is expected to find A* (B * A) and conclude.

We finally give the following theorem.

THEOREM 2.20: *Let S be the collection of all subsets of a groupoid G. S is a subset groupoid of G.*

(i) If G is a Smarandache strong alternative (left or right) groupoid then S has a subset subgroupoid such that

(a) H ≅ G.
(b) H is a Smarandache strong subset alternative (right or left) subgroupoid of S.

(ii) S is a Smarandache subset alternative (right or left) groupoid if G is a Smarandache strong alternative or left or right groupoid (or G is just a Smarandache alternative or left or right groupoid).

(iii) Even if G is a Smarandache strong alternative (or left or right) groupoid, S need not in general be a Smarandache strong alternative (or left or right) subset groupoid of G.

Now we will give some more types of groupoids built using Z_n.

Example 2.76: Let S = {Collection of all subsets of the groupoid G = {Z_7, *, (5, 3)}} be a subset groupoid of the groupoid G. S has atleast six subset semigroups.

Example 2.77: Let S = {Collection of all subsets of the groupoid G = {Z_5, * (1, 3)}} be a subset groupoid of a groupoid G. P = {0, 1, 2, 3, 4, 5} ∈ S is a subset semigroup of S. So S is a Smarandache subset groupoid.

However it is important to observe G is not a Smarandache groupoid.

Example 2.78: Let G = {Z_5, *, (1, 2)} be a groupoid; G is not a Smarandache groupoid.

However S = {Collection of all subsets of G} is the subset groupoid and is a Smarandache subset groupoid.

For P = {0, 1, 2, 3, 4} ∈ S is a subset semigroup of S.

In view of these examples we have the following theorem.

THEOREM 2.21: *Let*

S = {Collection of all subsets of a groupoid G} be a subset groupoid of a groupoid G. Even if G is not a Smarandache groupoid we may have S to be a Smarandache subset groupoid.

Proof follows from the fact if G * G = G then certainly S is a Smarandache subset groupoid of G even if G is not a S-groupoid.

We give the following problems.

Problem 2.4: Let S be a subset groupoid; can S be a Smarandache subset groupoid?

Problem 2.5: Can we have subset idempotent groupoid?

Problem 2.6: Can we have a Smarandache strong subset P-groupoid of infinite order?

Problem 2.7: Does there exist a Smarandache strong subset Bol groupoid of infinite order?

Problem 2.8: Does there exist a Smarandache subset groupoid of infinite order?

Problem 2.9: Does there exist a Smarandache strong subset Moufang groupoid of infinite order?

Problem 2.10: Can S = {Collection of all subsets of the groupoid G = {Z, *, (p, q)}} be a Smarandache strong alternative subset groupoid for any suitable values of p and q?

Problem 2.11: Let S = {Collection of all subsets of the groupoid G = {Q, *, (p, q)}} be a subset groupoid.

Does there exist a (p, q) such that S is a Smarandache Moufang subset groupoid?

Problem 2.12: Let S = {Collection of all subsets of the groupoid G = {C(Z_n), *, (p, q)}} (n a composite number)} be a subset groupoid of the groupoid G.

Is it possible for any p and q in G; so that;

(i) S is a Smarandache strong Moufang subset groupoid.
(ii) S is a Smarandache strong right alternative subset groupoid but not a Smarandache strong left alternative subset groupoid.
(iii) S is a Smarandache strong alternative subset groupoid.
(iv) S is a Smarandache strong Bol subset groupoid.

Problem 2.13: Let S = {Collection of subsets of the groupoid G = {Z (g), *, (p, q) | $g^2 = 0$}} be a subset groupoid.

(i) Does there exists p, q $\in$ Z(g) such that S is a Smarandache strong Moufang subset groupoid?

(ii) Will p, q ∈ Z(g) vary; if S is to be a S Bol subset groupoid?
(iii) Will in general p, q ∈ Z(g); vary depending on the identity which it has to satisfy?

Problem 2.14: Is it possible for the subset groupoid S to be a Smarandache strong subset P-groupoid but not a subset Moufang groupoid?

Problem 2.15: Is it possible to find for a subset groupoid S to be a Smarandache strong subset Bol groupoid?

Let S = {Collection of all subsets of the groupoid G = {Z_{11} (g_1, g_2) | $g_1^2 = g_2^2 = 0 = g_1g_2 = g_2g_1$}, *, (p, q)}} be the subset groupoid.

Can S be for any p, q ∈ G, Smarandache strong Moufang subset groupoid?

We suggest the following problems for interested reader.

Problems:

1. Find some special properties associated with subset groupoids.

2. Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (6, 2)}} be the subset groupoid of G.

   (i) Can S have subset zero divisors?
   (ii) Can S have subset idempotents?
   (iii) Find o(S).

3. Let S = {Collection of all subsets of the groupoid G = {Z_7, *, (3, 1)}} be the subset groupoid of G.
   Study questions (i) to (iii) of problem 2 for this S.

4. Let S = {Collection of all subsets of the groupoid G = {Z_{18}, *, (6, 3)}} be a subset groupoid.

(i) Can S have subset zero divisors?
(ii) Can S have subset idempotents?
(iii) Can S contain right subset ideals which are not left subset ideals?
(iv) Is S a S-subset groupoid?
(v) Can S have subset Smarandache subgroupoids which are not subset Smarandache ideals?
(vi) Can S have subset right ideals which are not Smarandache subset left ideals?
(vii) Can S have subset subgroupoid which are not Smarandache subset subgroupoids?

5. Let S = {Collection of all subsets of the groupoid G = {Z, *, (3, –5)}} be the subset groupoid of the groupoid G. Study questions (i) to (vii) of problem 4 for this S.

6. Let S = {Collection of all subsets of the groupoid G = {$(Z^+ \cup \{0\})$ g, *, (0, g)}} be the subset groupoid of G. Study questions (i) to (vii) of problem (4) for this S.

7. Let S = {Collection of all subsets of the groupoid G = {$C(Z_{10})$, *, $(2i_F, 0)$}} be the subset groupoid. Study questions (i) to (vii) of problem (4) for this S.

8. Let S = {Collection of all subsets of the groupoid G = {$C(Z_{19})(g)$; *, $(i_F, 2g)$ | $g^2 = 0$, $i_F^2 = 18$}} be the subset groupoid of G.
Study questions (i) to (vii) of problem (4) for this S.

9. Let S = {Collection of all subsets of the groupoid G = {$C(Z_{24})$ (g_1, g_2), *, $(6g_1, 8g_2)$ where $g_1^2 = 0 = g_2^2 = g_2$, $g_1g_2 = 0 = g_2g_1$}} be the subset groupoid.
Study questions (i) to (vii) of problem (4) for this S.

10. Let S = {Collection of all subsets of the groupoid G = {Z_{40} (g), *, (10, 0), $g^2 = –g$}} be the subset groupoid of G. Study questions (i) to (vii) of problem 4 for this S.

11. Let S = {Collection of all subsets of the groupoid $G = \{R(g), (-1, 1), g^2 = 0\}$} be the subset groupoid of G. Study questions (i) to (vii) of problem (4) for this S.

12. Obtain some special properties of Smarandache subset groupoids?

13. Is every subset groupoid a Smarandache subset groupoid?

14. Does there exist a subset groupoid S such that every subset subgroupoid of S is a Smarandache subset subgroupoid?

15. Does there exist a subset Smarandache groupoid S such that every right subset ideal of S is Smarandache?

16. Does there exist a subset Smarandache groupoid such that only all its subset left ideals are Smarandache and none of its subset right ideals are Smarandache?

17. Does there exists a Smarandache subset groupoid such that

    (i) None of its subset subgroupoid is Smarandache.
    (ii) None of its subset ideals (right or left) are Smarandache.

18. Give some special properties enjoyed by Smarandache seminormal subset groupoid.

19. Give some special properties enjoyed by Smarandache normal subset groupoid.

20. Let S = {Collection of all subsets of the groupoid $G = \{Z_{43}, *, 3, 2\}$} be the subset groupoid.

    (i) Is S a Smarandache normal subset groupoid?
    (ii) Is S a Smarandache seminormal subset groupoid?

21. Let S = {Collection of all subsets of the groupoid G = {Z, *, (0, 3)}} be a subset groupoid.
Can S be a Smarandache strong normal subset groupoid?

22. S = {Collection of all subsets of the groupoid G = {$Z^+ \cup \{0\}$, *, (p, q)}} be a subset groupoid.
For what values of (p, q) $\in Z^+ \cup \{0\}$ will S be a Smarandache seminormal subset groupoid?

23. Find conditions on the subset groupoid to be a Smarandache conjugate subset groupoid.

24. Let S = {Collection of all subsets of the groupoid G = {Z_{16}, *, (4, 0)}} be a subset groupoid.
Can S be a Smarandache conjugate subset groupoid?

25. Can S in problem 24 be a Smarandache strong subset Moufang groupoid?

26. Let S = {Collection of all subsets of the groupoid G = {C(Z_{12}), *, (4, 3)}} be a subset groupoid.

    (i) Can S be a Smarandache seminormal subset groupoid?
    (ii) Can S be a Smarandache normal subset groupoid?
    (iii) Can S be a Smarandache strong Moufang subset groupoid?
    (iv) Will S be a Smarandache subset groupoid?
    (v) Can S be a Smarandache strong alternative subset groupoid?

27. Let S = {Collection of all subset of the groupoid G = {$Z^+ \cup \{0\}$, *, (p, q)}} be a subset groupoid of G.
Study questions (i) to (v) of problem (26) for this S.

28. Let S = {Collection of all subsets of the groupoid G = {C(Z_{15}), (g_1, g_2), *, (3g, $3i_F$) | $g_1^2 = g_1$ and $g_2^2 = 0$, $g_1g_2 = g_2g_1 = 0$}} be a subset groupoid of G.
Study problems (i) to (v) of problem 26 for this S.

29. Does there exist a subset groupoid S which is a Smarandache strong Moufang subset groupoid?

30. Let G be a groupoid which is not a S-Moufang strong groupoid or a Smarandache Moufang groupoid.
Let S = {Collection of all subsets of G} be the subset groupoid of the groupoid G.
Prove S is not a S-Moufang strong subset groupoid.

31. Obtain some special properties enjoyed by Smarandache Moufang subset groupoids.

32. Can we have a S-Moufang subset groupoid to have its associated groupoid to be not even a Smarandache subset Moufang groupoid? Justify.

33. Find whether the subset groupoid
S = {Collection of all subsets of the groupoid G = $\{Z_{24}, *, (8, 3)\}$} is a Smarandache subset Moufang groupoid.

34. Let S = {Collection of all subsets of the groupoid $G = \{Z_8 (g), *, (4, 2) \mid g^2 = 0\}$} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

35. Is S in problem (34) a Smarandache strong P-subset groupoid?

36. Is S in problem (34) a Smarandache strong Bol subset groupoid?

37. Let S = {Collection of all subsets of the groupoid $G = \{C(Z_{16}), *, (15, 1)\}$} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

38. Let S_1 = {Collection of all subsets of the groupoid $G = \{C(Z_{16}), *, (8, 0)\}$} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

39. Let S_2 = {Collection of all subsets of the groupoid G = {C(Z_{16}), *, (14, 2)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

40. Let S = {Collection of all subsets of the groupoid G = {C(Z_{16}), *, ($8i_F$, 0)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

41. Let S = {Collection of all subsets of the groupoid G = {C(Z_{16}), *, ($10i_F$, 6, $6i_F$+10)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

42. Let S = {Collection of all subsets of the groupoid G = {C(Z_{16}, *, (8, $8i_F$)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

43. Let S = {Collection of all subsets of the groupoid G = {Z_{23}, *, (1, 0)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

44. Let S = {Collection of all subsets of the groupoid G = {Z_{23}, *, (0, 1)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.
Compare S in problems (43) and (44).

45. Let S = {Collection of all subsets of the groupoid G = {Z_{23}, *, (20, 3)}} be a subset groupoid.
Study questions (i) to (v) of problem 26 for this S.

46. Let S = {Collection of all subsets of the groupoid G = {Z_{23}, *, (3, 20)}} be a subset groupoid.
Study questions (i) to (v) of problems (26) for this S.
Compare S in problem (45) and (46).

47. Does there exists a subset groupoid of infinite order which is a Smarandache normal subset groupoid?

48. Does there exists a subset groupoid of infinite order which is a Smarandache seminormal subset groupoid?

49. Let S be a subset groupoid of infinite order.
Can S be a Smarandache strong Bol subset groupoid?

50. Let S = {Collection of all subsets of the groupoid G = {C, *, (p, q)}} be a subset groupoid.
Does we have (p, q) so that S is a Smarandache subset strong Moufang groupoid?

51. Is it possible for any subset groupoid to satisfy more than one identity?

52. Can a subset groupoid S be both Smarandache strong Bol subset groupoid as well as Smarandache strong P-subset groupoid?

53. Can a subset groupoid S be both Smarandache subset Bol as well as alternative subset groupoid?

54. Give example of a subset Moufang groupoid.
Does there exist one such?

55. Does there exist a subset Bol groupoid which is not a Smarandache strong Bol subset groupoid?

56. Does there exist a subset right alternative groupoid which is not a Smarandache subset right alternative groupoid?

57. Give some special properties enjoyed by Smarandache strong subset P-groupoid?

58. Is it possible to have a subset groupoid which satisfies more than 3 identities?

59. Let S = {Collection of all subsets of the groupoid G = {C(Z_{41}), *, ($3i_F$, 0)}} be a subset groupoid.
Does S satisfy any of the special identities?

60. Let S = {Collection of all subsets of the groupoid $G = \{C, *, (i_F, -1)\}$} be a subset groupoid.
Can S be Smarandache strong Bol subset groupoid?

61. Does S in problem 60 satisfy any of the four identities Moufang identity or Bol identity or P-groupoid identity or alternative identity?

62. Does there exist a subset groupoid using the complex field C which satisfies atleast two identities?

63. Let S = {Collection of all subsets of the groupoid $S = \{R(g), *, (g, -g) \mid g^2 = 0\}$} be a subset groupoid.
Can S satisfy any one of the identities?

64. Let S = {Collection of all subsets of the groupoid $G = \{R(g), *, (g, -g), g^2 = -g\}$} be the subset groupoid of the groupoid G.

    (i) Can S be Smarandache strong Bol subset groupoid?
    (ii) Can S be a Smarandache Bol subset groupoid?
    (iii) Can S be a Smarandache strong Moufang subset groupoid?
    (iv) Can S be a Smarandache left alternative subset groupoid?

65. Let S = {Collection of all subsets of the groupoid $G = \{(R^+ \cup \{0\})\, g \mid *, (g, 0), g^2 = 0\}$} be a subset groupoid of the groupoid G.

    (i) Is it possible for S to satisfy any of the identities?
    (ii) Can S be a seminormal subset groupoid?
    (iii) Is S a normal subset groupoid?

66. Obtain some special features enjoyed by subset seminormal groupoid of a groupoid G.

67. Does there exist a subset normal groupoid of infinite order?

68. Does there exists a subset groupoid of infinite cardinality which satisfies the alternative identity?

69. Give an example of an infinite subset groupoid which does not satisfy any of the identities.

70. Does there exist an infinite subset groupoid which satisfies right alternative identity but does not satisfy the left alternative identity?

71. Let S = {Collection of all subsets of the groupoid G = {C(Z_{36}) (g),*, (6, 6g) with g^2 = 0}} be the subset groupoid of the groupoid G.

    (i) Does S satisfy any of the identities?
    (ii) Can S be a Smarandache normal subset groupoid?

72. If in problem (71); (6, 6g) in G is replaced by (0, 6g) study questions (i) to (ii) of problem (71) for this S.

73. Let S = {Collection of all subsets of the groupoid G = {Z_{12} (g), *, (10, 2g)}} be the subset groupoid of the groupoid G.

    (i) Can S be a Smarandache seminormal subset groupoid?
    (ii) Can S be a Smarandache Moufang subset groupoid?
    (iii) Can S be a Smarandache Bol subset groupoid?
    (iv) Is it possible for S to be subset right alternative?

74. Let S = {Collection of all subsets of the groupoid G = {Z_n, (m, 0), * with m * m = m (mod n)}} be the subset groupoid of the groupoid G.

    (i) Prove S is a Smarandache strong subset Bol groupoid.
    (ii) Prove S is a Smarandache strong subset Moufang groupoid.

(iii) Prove S is a Smarandache subset strong P-groupoid.
(iv) Prove S is a Smarandache subset strong alternative groupoid.

75. Let S = {Collection of all subsets of the groupoid G = {Z_{20}, *, (5, 0)}} be a subset groupoid of the groupoid G.
Study questions (i) to (iv) of problem 74 for this S.

76. Give an example of a subset groupoid which is Smarandache subset idempotent.

77. Is S = {Collection of all subsets of the groupoid G = {Z_6, *, (3, 4)}}, a Smarandache strong subset Bol groupoid?

78. Is S in problem (77) a Smarandache strong Moufang subset groupoid?

79. Is S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (4, 9)} be a Smarandache subset P-groupoid?

80. Can S in problem (79) be Smarandache alternative subset groupoid?

81. Let S = {Collection of all subsets of the groupoid G = {Z_5, *, (1, 3)}} be a subset groupoid of the groupoid G.
Can S be a Smarandache subset groupoid?

82. Let S = {Collection of all subsets of the groupoid G = {Z_9, (5, 3), *}} be a subset groupoid.
Can S be a Smarandache subset groupoid?

83. Define the notion of Smarandache subset groupoid isomorphism. Show this by an example.

84. Study the special properties enjoyed by the subset groupoid S of G where S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (5, 10)}}.

85. S = {Collection of all subsets of the groupoid G = {Z_6, *, (3, 5)}} be the subset groupoid of the groupoid G.
Is S a P-subset groupoid?

86. Let S = {Collection of all subsets of the groupoid G = {Z_{12}, *, (3, 9)}} be the subset groupoid.
Is S a Smarandache normal subset groupoid?

87. Find normal subset subgroupoid of S in problem (86).

88. Does there exist subset groupoid which is simple?

89. Characterize those groupoids which have their subset groupoids to be simple.

90. Give an example of a simple subset groupoid.

91. Can S = {Collection of all subsets of the groupoid G = {C(Z_{24}), *, (13,11)}} be the subset simple groupoid?

92. Let S = {Collection of all subsets of the groupoids G = {Z_{16}, *, (11, 5)}} be a subset groupoid.
Can S be subset simple?

93. Let S = {Collection of all subsets of the groupoid G = {C(Z_{21}), *, (19, 2)}} be the subset groupoid.
Can S be subset simple?

94. Let S = {Collection of all subsets of the groupoid G = {Z_5, *, (3, 2)}} be the subset groupoid.

(i) Can S be subset simple?
(ii) Find o (S).

95. Can S = {Collection of all subsets of the groupoid G = {C(Z_{17}), *, (5, 2)}} be a simple subset groupoid?

96. Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (3, 3)}} be a subset groupoid.

(i) Can S be simple?
(ii) Find subset zero divisors if any in S.
(iii) Is S subset seminormal?
(iv) Can S be a normal subset groupoid?
(v) Find o(S).
(vi) Is S subset commutative?
(vii) Can S satisfy any of the special identities?
(viii) Is S an idempotent subset groupoid?

97. Is it possible for the subset groupoid S = {Collection of all subsets of the groupoid G = $\{Z_{11}, *, (3, 8)\}$} to be Smarandache strong Bol subset groupoid as well as Smarandache strong Moufang subset groupoid?

98. If (3, 8) in G in the above problem is replaced by (0, 1) study question (i) to (viii) for S in problem 97.

99. For S in problem 97 study questions (i) to (viii) of the problem 96.

100. Characterize those subset groupoids which are S-right subset alternative and not S- subset left alternative?

101. Give an example of a subset groupoid which is commutative.

102. Give an example of a subset groupoid which is Smarandache subset commutative.

103. Give an example of a subset groupoid which is Smarandache subset inner commutative.

Chapter Three

SUBSET LOOP GROUPOIDS

In this chapter authors for the first time define the notion of subset loop groupoids and study their properties.

DEFINITION 3.1: *Let (L, *) be a loop S = {Collection of all subsets of L}. {S, *} is a subset groupoid and (S, *) is defined as the subset loop groupoid of the loop L.*

It is important to mention here that S will not be a subset loop, but only a subset groupoid.

We will first illustrate this situation by some examples.

Example 3.1: Let S = {Collection of all subsets of the loop (L, *) = {e, a_1, a_2, a_3, a_4, a_5} given by the following table;

*	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	a_4	a_5
a_1	a_1	e	a_3	a_5	a_2	a_4
a_2	a_2	a_5	e	a_4	a_1	a_3
a_3	a_3	a_4	a_1	e	a_5	a_2
a_4	a_4	a_3	a_5	a_2	e	a_1
a_5	a_5	a_2	a_4	a_1	a_3	e

{S, *} is the subset loop groupoid of the loop L.

For A = {e, a_2, a_3} and B = {a_4, a_5, a_1} in S we find
A * B = {e * a_4, e * a_5, e * a_1, a_2 * a_4, a_2 * a_5, a_2 * a_1,
a_3 * a_4, a_3 * a_5, a_3 * a_1}
= {a_4, a_5, a_1, a_3, a_2} $\in$ S.

We now find B * A
= {a_4, a_5, a_1} * {e, a_3, a_2}
= {a_4 * e, a_5 * e, a_1 * e, a_4 * a_3, a_5 * a_3, a_1 * a_3,
a_4 * a_2, a_5 * a_2, a_1 * a_2}
= {a_4, a_5, a_1, a_2, a_3} $\in$ S.

We see A * B = B * A
But in general A * B $\neq$ B * A for take

A = {a_1} and B = {a_3}.

A * B = {a_1 * a_3} = {a_5} and
B * A = {a_3 * a_1} = {a_4}.

Thus A * B $\neq$ B * A in general for A, B $\in$ S.

Example 3.2: Let S = {Collection of all subsets of the loop (L, *) where L = {e, a_1, a_2, a_3, a_4, a_5, a_6, a_7} given by the following table:

*	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
e	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	a_1	e	a_5	a_2	a_6	a_3	a_7	a_4
a_2	a_2	a_5	e	a_6	a_3	a_7	a_4	a_1
a_3	a_3	a_2	a_6	e	a_7	a_4	a_1	a_5
a_4	a_4	a_6	a_3	a_7	e	a_1	a_5	a_2
a_5	a_5	a_3	a_7	a_4	a_1	e	a_2	a_6
a_6	a_6	a_7	a_4	a_1	a_5	a_2	e	a_3
a_7	a_7	a_4	a_1	a_5	a_2	a_6	a_3	e

be the subset loop groupoid.

We see for every A, B $\in$ S we have A * B = B * A. This is so because L is a commutative loop. Thus S is a commutative subset loop groupoid of L.

Take A = $\{a_1, a_3, a_5, a_6\}$ and B = $\{a_1, a_5, a_7\} \in$ S.

$$A * B = \{a_1 * a_1, a_1 * a_5, a_1 * a_7, a_3 * a_1, a_3 * a_5, a_3 * a_7, a_5 * a_1, a_5 * a_5, a_5 * a_7, a_6 * a_1, a_6 * a_5, a_6 * a_7\}$$
$$= \{e, a_3, a_4, a_2, a_5, a_6, a_7\} \in S.$$

$$B * A = \{a_1, a_5, a_7\} * \{a_1, a_3, a_5, a_6\}$$
$$= \{a_1 * a_1, a_1 * a_3, a_1 * a_5, a_1 * a_6, a_5 * a_1, a_5 * a_3, a_5 * a_5, a_5 * a_6, a_7 * a_1, a_7 * a_3, a_7 * a_5, a_7 * a_6\}$$
$$= \{e, a_2, a_3, a_7, a_4, a_5, a_6\} \in S.$$

Clearly A * B = B * A. This is the way subset operations are performed on S.

We see S cannot be a loop. It is a subset groupoid and not a subset semigroup as the operation * is non associative.

For take A = $\{a_1, a_2\}$, B = $\{a_3\}$ and C = $\{a_7, a_6\} \in$ S.

Consider (A * B) * C

$$= (\{a_1, a_2\} * \{a_3\}) * C$$
$$= \{a_1 * a_3, a_2 * a_3\} * C$$
$$= \{a_2, a_6\} * \{a_7, a_6\}$$
$$= \{a_2 * a_7, a_2 * a_6, a_6 * a_7, a_6 * a_6\}$$
$$= \{a_1, a_4, a_3, e\} \quad \ldots \quad I$$

$$A * (B * C) = A * (\{a_3\} * \{a_6, a_7\})$$

$$= A * (\{a_3 * a_6, a_3 * a_7\})$$
$$= \{a_1, a_2\} * \{a_1, a_5\}$$
$$= \{a_1 * a_1, a_1 * a_2, a_2 * a_1, a_2 * a_5\}$$
$$= \{e, a_5, a_7\} \quad \ldots \quad II$$

Clearly (A * B) * C ≠ A *(B * C) as I and II are distinct.

Example 3.3: Let S = {Collection of all subsets of the loop L given by the following table; L = {e, a_1, a_2, a_3, a_4, a_5}

*	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	a_4	a_5
a_1	a_1	e	a_5	a_4	a_3	a_2
a_2	a_2	a_3	e	a_1	a_5	a_4
a_3	a_3	a_5	a_4	e	a_2	a_1
a_4	a_4	a_2	a_1	a_5	e	a_3
a_5	a_5	a_4	a_3	a_2	a_1	e

be the subset loop groupoid.

S is a non commutative subset loop groupoid of the loop L. Take A = {a_1} and B = {a_5} ∈ S,

we see A * B = {a_1} * {a_5} = {$a_1 * a_5$} = {a_2} … I

B * A = {a_5} * {a_1} = {$a_5 * a_1$} = {a_4} … II

Clearly I and II are not the same so S is a non commutative subset groupoid.

In view of these we give the following theorem the proof of which is left as an exercise to the reader.

THEOREM 3.1: *Let*
*S = {Collection of all subsets of the loop (L, *)} be the subset loop groupoid of L. S is a commutative subset loop groupoid if and only if L is a commutative loop.*

Proof: Suppose L is a commutative loop. Let A = {a_1, …, a_n} and B = {b_1, …, b_m} ∈ S where a_i, b_j ∈ L, $1 \leq i \leq n$ and $1 \leq j \leq m$.

We see

$A * B = \{a_i * b_j \mid a_i \in A \text{ and } b_j \in B, 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \in S$.

Consider $B * A = \{b_j * a_i \mid b_j \in B, a_i \in a; 1 \leq i \leq n$ and $1 \leq j \leq m\} \in S$. We know $a_i * b_j = b_j * a_i$ for all $1 \leq i \leq n$ and $1 \leq j \leq m$; since we are given L is a commutative loop.

So $A * B = B * A$ for all $A, B \in S$.

Thus S is a subset loop groupoid which is commutative.

Now if we assume S to be commutative we see if $A, B \in S$ we have $A * B = B * A$ have to prove L is a commutative loop.

Take $A = \{a\}$ and $B = \{b\}$ in S where $a, b \in L$.

We see $A * B = B * A$ so $A * B = a * b = B * A = b * a$ for all $a, b \in L$ as S is given to be commutative, hence L is commutative.

Hence the theorem.

Example 3.4: Let S = {Collection of all subsets of the loop $L_5(2)$ given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	4	5	2
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

}

be the subset loop groupoid. Clearly S is a non commutative subset loop groupoid of $L_5(2)$.

$$o(S) = 2^6 - 1.$$

Example 3.5: Let S = {Collection of all subsets of the loop $L_5(3)$ where the table of $L_5(3)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	4	2	5	3
2	2	4	e	5	3	1
3	3	2	5	e	1	4
4	4	5	3	1	e	2
5	5	3	1	4	2	e

be the subset loop groupoid of $L_5(3)$. $o(S) = 2^6 - 1$.

Clearly S is a commutative subset loop groupoid.

Now consider the subset groupoid of order $2^6 - 1$ given by $G = \{Z_6, (3, 0), *\}$.

Example 3.6: Let S_1 = {Collection of all subsets of the groupoid $G = \{Z_6, *, (3, 0)\}$} be the subset groupoid. Clearly S_1 is non commutative and $o(S_1) = 2^6 - 1$.

Compare S and S_1. Does S_1 contain a subset which is a loop?

It is to keep on record that if S is a subset loop groupoid of the loop L, then S has a subset collection which is isomorphic to the loop L.

However it is interesting to study if S_1 is any subset groupoid can S_1 contain a substructure which is a loop?

We know S_1 has substructure which is a groupoid.

Example 3.7: Let
S = {Collection of all subsets of the loop L = $L_{45}(8)$} be a subset loop groupoid of L. P = {{e}, {1}, {2}, …, {45}} ⊆ S is a subset loop subgroupoid of L. We see P ≅ L as loops by the map; {a} ↦ a for all {a} ∈ P and a ∈ L = $L_{45}(8)$.

Now we define substructures of a subset loop groupoid.

DEFINITION 3.2: *Let S = {Collection of all subsets of a loop L} be the subset loop groupoid of L. Let P ⊆ S if P is a subset groupoid and not a loop we define P to be a subset loop subgroupoid of S.*

If W ⊆ S is such that W is a subset semigroup then we define W to be a Smarandache subset loop semigroup of S. If V ⊆ S such that V is a subset loop we call S to be a super special subset loop-loop groupoid of S. Suppose M ⊆ S such that M is a subset group we call M to be a subset Smarandache loop-group of S.

We will illustrate all these situations by an example or two.

Example 3.8: Let S = {Collection of all subsets of the loop L = $L_7(4)$ given by the following table;

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	5	2	6	3	7	4
2	2	5	e	6	3	7	4	1
3	3	2	6	e	7	4	1	5
4	4	6	3	7	e	1	5	2
5	5	3	7	4	1	e	2	6
6	6	7	4	1	5	2	e	3
7	7	4	1	5	2	6	3	e

}

be a subset loop groupoid of L. Consider

V = {{e}, {1}, {2}, {3}, {4}, {5}, {6}, {7}} ⊆ S, clearly V is a loop so V is a super special subset loop-loop groupoid of S.

Consider W = {{e}, {5}, {e, 5}} ⊆ S. The table of W is as follows:

*	{e}	{5}	{e,5}
{e}	{e}	{5}	{e,5}
{5}	{5}	{e}	{e,5}
{e,5}	{e,5}	{e,5}	{e,5}

We see W is a subset semigroup so W is a Smaradache subset loop semigroup of S.

Now consider T = {{e}, {6}} ⊆ S; the table of T is as follows:

*	{e}	{6}
{e}	{e}	{6}
{6}	{6}	{e}

Clearly T is a subset group of S; so T is defined as subset S; Smarandache subset loop-group of S.

Take M_1 = {{e}, {6}} ⊆ S or M_2 = {{e}, {7}} ⊆ S or M_3 = {{e}, {2}} ⊆ S or M_4 = {{e}, {3}} ⊆ S or M_5 = {{e}, {4}} ⊆ S or M_6 = {{e}, {5}} ⊆ S are all subset groups which are also subset subloops.

Example 3.9: Let
S = {Collection of all subsets of the loop $L_{45}(8)$} be the subset loop groupoid of $L_{45}(8)$.

Take V = {{e}, {1}, {6}, {11}, {16}, {21}, {26}, {31}, {36}, {41}} ⊆ S is a super special subset loop of S.

Now if M = {Collection of all subsets of T = {e, 1, 6, 11, 16, 21, 26, 31, 36, 41} ⊆ L} ⊆ S, M is a subset loop subgroupoid of S.

We define a subset loop groupoid S to be super Smarandache loop groupoid if S has a subset which is a subset loop.

On similar lines we define a subset loop subgroupoid P to be a super Smarandache loop subgroupoid if S has a subset loop subgroupoid P which is a super Smarandache subset loop subgroupoid.

A subset loop groupoid S is said have a normal subset loop subgroupoid, H if H ⊆ S and the following conditions are satisfied:

(i) H is a subset loop subgroupoid of S.
(ii) xH = Hx
(iii) (Hx) y = H(xy)
(iv) y(xH) =(yx) H for all x, y ∈ S.

A subset loop groupoid S is said to be simple if S has no normal subset loop subgroupoid.

Example 3.10: Let
S = {Collection of all subsets of the loop $L_5(3)$} be the subset loop groupoid. S is a simple subset loop groupoid.

For take H = {$L_5(3)$} = {e, 1, 2, 3, 4, 5} ⊆ S.
We see

(i) xH = Hx for all x ∈ S
(ii) (Hx) y = H(xy)
(iii) y(xH) =(yx)H for all x, y ∈ S.

Hence S is not simple in a trivial way. However it is important to mention that H in S is a singleton. It is not just like e ∈ $L_5(3)$ and ex = xe = x;

e(xy) =(ex) y and y(xe) =(yx) e for all x, y ∈ $L_5(3)$.

Hence we can say for the subset loop groupoid S over $L_5(3)$ = {e, 1, 2, 3, 4, 5}. H = {e, 1, 2, 3, 4, 5} = $L_5(3)$ ∈ S acts as the special pseudo identity as AH = HA = H.

For take A = {2, 4, 1} ∈ S.

A * H = {2, 4, 1} * {e, 1, 2, 3, 4, 5}

= {2 * e, 2 * 1, 2 * 2, 2 * 3, 2 * 4, 2 * 5, 4 * e, 4 * 1, 4 * 2, 4 * 3, 4 * 4, 4 * 5, 1 * e, 1 * 1, 1 * 2, 1 * 3, 1 * 4, 1 * 5}

= {2, 4, 5, 3, 1} = H.

H * A = {e, 1, 2, 3, 4, 5} * {2, 4, 1}

= {e * 2, e * 4, e * 1, 1 * 2, 1 * 5, 1 * 1, 2 * 2, 2 * 4, 2 * 1, 3 * 2, 3 * 4, 3 * 1, 4 * 2, 4 * 4, 4 * 1, 5 * 1, 5 * 2, 5 * 4}

= {2, 4, 1, 2, 5, e} = H.

Thus A* H = H*A = H, hence we can call H only as a pseudo special identity which we define as the normal element of S.

In view this we give the following theorem, the proof of which is direct.

THEOREM 3.2: *Let S = {Collection of all subsets of the loop L} be the subset loop groupoid. H = L = {all elements in L} ∈ S acts as the pseudo identity that is the normal element of S.*

The very natural question is can we have more normal elements in S.

The answer is no for if H is the normal subset element and if A ∈ S is another normal subset element A ⊆ H but A*H = H*A = H and it is not A. Hence the claim.

However if we take any subset loop subgroupoid P of S we may have an element say M ∈ P with M*T = T*M = M for all T ∈ P. If such a M exist we call M as the subnormal subset element. A subset loop groupoid may have more than one subset subnormal element and in some case no subset subnormal element.

We will illustrate this situation by some examples.

Example 3.11: Let S = {Collection of all subsets of the loop $L_5(4)$ which is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	5	4	3	2
2	2	3	e	1	5	4
3	3	5	4	e	2	1
4	4	2	1	5	e	3
5	5	4	3	2	1	e

}.

We see $L_5(4)$ has no subloops only subgroups of the form {e, 1}, {e, 2}, {3, e}, {e, 4} and {e, 5}.

So S has only H = {e, 1, 2, 3, 4, 5} ∈ S such that A*H = H*A = H for all A ∈ S is the normal element of S.

S has no subnormal elements.

Example 3.12: Let
S = {Collection of all subsets of the loop $L_7(4)$} be the subset loop groupoid of the loop $L_7(4)$ which is as follows:

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	5	2	6	3	7	4
2	2	5	e	6	3	7	4	1
3	3	2	6	e	7	4	1	5
4	4	6	3	7	e	1	5	2
5	5	3	7	4	1	e	2	6
6	6	7	4	1	5	2	e	3
7	7	4	1	5	2	6	3	e

Clearly the loop $L_7(4)$ has no subloops the only subgroups of $L_7(4)$ are {e, 1}, {e, 2}, {e, 3}, {e, 4}, {e, 5}, {e, 6} and {e, 7}.

Let H = {e, 1, 2, 3, 4, 5, 6, 7} $\in$ S; H is the unique normal element of S and we see H*A = A*H = H for all H $\in$ S.

Example 3.13: Let S = {Collection of all subsets of the loop $L_{45}(8)$} be the subset loop groupoid of $L_{45}(8)$.

$H_1(15)$ = {e, 1, 16, 31} is a subloop of $L_{45}(8)$.
$H_1(5)$ = {e, 1, 6, 11, 16, 21, 26, 31, 36, 41} is also a subloop of $L_{45}(8)$.

Let P = {Collection of all subsets of the subloop $H_1(15)$ of $L_{45}(8)$} $\subset$ S. P is a subset subloop subgroupoid of S.

We see T = {e, 1, 16, 31} $\in$ P is the subnormal subset element of P for each A $\in$ P we see T*A = A*T = T.

However H = $L_{45}(8)$ is the normal subset element of S. Take M = {Collection of all subsets of the subloop $H_1(5)$ = {e, 1, 6, 11, 16, 21, 26, 31, 36, 41} $\subseteq L_{45}(8)$}; M is a subset subloop subgroupoid of S.

We see V = {e, 1, 6, 11, 16, 21, 26, 31, 36, 41} ∈ M acts as a subnormal subset element of S.

Thus this S has subnormal subset elements. Infact the number of subnormal subset elements of the subset loop groupoid of the loop L equals to the number of subloops of L which are not subgroups. In case of simple loops we have no subnormal subset element one and only one normal subset element.

We will denote the subset loop groupoid of this loop $L_n(m) \in L_n$ by $S_n(m)$ in S_n; that is

S_n = {Collection of all subset loop groupoids of the loop $L_n(m) \in L_n$}

= {$S_n(m)$ | $S_n(m)$ is the collection of all subsets of the loop $L_n(m) \in L_n$}.

We see in case of S_5 we have only 3 subset loop groupoids and all the three subset loop groupoids are simple S_5 = {$S_5(3)$, $S_5(4)$, $S_5(2)$}. Similarly S_7 = {$S_7(2)$, $S_7(3)$, $S_7(4)$, $S_7(5)$ and $S_7(6)$} and all the 5 subset loop groupoids are simple.

$|S_n| = |L_n| = \prod_{i=1}^{k} (p_i - 2)p_i^{\alpha_i - 1}$ and n is an odd prime; we see number of subset loop groupoids of L_n is n–2.

So for a given odd prime p, p > 3 we can have p–2 number of subset loop groupoids each of $S_n(m)$ is of order $2^{n+1} - 1$.

For proof of these refer [5].

Now we will study some of the properties associated with the subset loop groupoids S_n of $L_n(m) \in L_n$.

Let $S_n(m) \in S_n$(n > 3 n an odd number).

THEOREM 3.3: *Let*
S_n = {Collection of all subsets of the loop $L_n(m) \in L_n$}

= {$S_n(m)$ |(n, m) = 1 =(n, m–1), n > 3, n odd} has one and only one commutative subset loop groupoid. (We see each $S_n(m)$ is a subset loop groupoid of order $2^{n+1} - 1$).

Proof: We know L_n the class of loops of order n + 1 contains a loop given by $L_n(m)$ where m =(n+1)/2.

So S_n contains one and only one commutative subset loop groupoid given by $S_n\left(\frac{n+1}{2}\right)$.

For working refer [5]. We call a loop L to be strictly non commutative if xy ≠ yx for any x, y ∈ L; x ≠ y.

We will define the notion of strictly non commutative subset loop groupoids. We have to define a subset loop groupoid S which is strictly non commutative as A*B ≠ B*A for all distinct A and B in S(A ≠ B; A ≠ {e}, B ≠ {e}).

But we see in case of S we cannot have the notion of strictly non commutative subset loop groupoids for loops $L_n(m) \in L$.

For we see if
$S_n(m)$ = {Collection of all subsets of the loop $L_n(m) \in L_n$} be the subset loop groupoid. Let H = {$L_n(m)$} ∈ $S_n(m)$. We see for every A ∈ $S_n(m)$. A*H = H*A so the concept of strictly non commutative subset loop groupoid cannot be defined.

However we are going define specially strictly commutative subset loop groupoid as follows:

DEFINITION 3.3: *Let*
S_n = {$S_n(m)$ = Collection of all subsets of the loop $L_n(m) \in L_n$} be the collection of all subset loop groupoids of the loop $L_n(m)$ in L_n}. We say a subset loop groupoid $L_n(m) \in L_n$ is strictly non

commutative subset loop groupoid if and only if the loop $L_n(m) \in L_n$ is a strictly non commutative loop.

Example 3.14: Let S_{19} = {$S_{19}(9)$ | 19 is an odd prime > 3} = {Collection of all subset loop groupoids of the loop $L_{19}(9)$ where $L_{19}(m) \in L_{19}$}.

We see every subset loop groupoid $L_{19}(m)$ where (m,19) = 1 =(m–1, 19) is strictly non commutative except for m = 10 and $S_{19}(10)$ is the only commutative subset loop groupoid.

In view of this we have the following theorem.

THEOREM 3.4: *Let*
S_n = {$S_n(m)$ | n odd prime n ≥ 5 and(n, m) =(n, m–1) = 1} = {Collection of all subset loop groupoids of the loop $L_n(m) \in L_n$}.

All subset loop groupoids $S_n(m)$ with the exception of $S_n\left(\frac{n+1}{2}\right)$ are all strictly non commutative.

Proof: Follows from the simple fact that every loop in $L_n(m) \in L_n$; n an odd prime n ≥ 5 is strictly non commutative for all m such that(m–1, n) =(n, m) = 1 and $m \neq \frac{n+1}{2}$ [5].

In case of $m = \frac{n+1}{2}$ we see $S_n\left(\frac{n+1}{2}\right)$ is a commutative subset loop groupoid of the commutative loop $L_n\left(\frac{n+1}{2}\right)$.

Hence the claim.

THEOREM 3.5: *Let*
S_n = {$S_n(m)$ | n = 3t, n odd n > 3(m, n) =(m–1, n) = 1} = {Collection of all subset loop groupoid of the loop $L_n(m)$; (m–

1, n) =(n, m) = 1, n = 3t, n odd n > 3}. S_n does not contain any strictly non commutative subset loop groupoid.

Proof: Follows from the simple fact L_n when n = 3t does not contain any strictly non commutative loop so S_n also does not contain any strictly non commutative subset loop groupoid.

We are now interested in finding the number of strictly non commutative subset loop groupoids in S_n = {Collection of all subset loop groupoids of the loops $L_n(m)$ in L_n}.

THEOREM 3.6: *Let S_n be the class of subset loop groupoids from L_n the class of loops. If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ then S_n contains exactly F_n subset loop groupoids which are strictly non commutative where $F_n = \prod_{i=1}^{k} (p_i - 3) p_i^{\alpha_i - 1}$.*

The proof is similar to that of loops for more refer [5].

Example 3.15: Let
S = {Collection of all subsets of the loop $L_{19}(7)$} be the subset loop groupoid. S is strictly non commutative.

Example 3.16: Let
S = {Collection of all subsets of the loop $L_{23}(12)$} be the subset loop groupoid. S is a commutative subset loop groupoid.

Example 3.17: Let
S = {Collection of all subsets of the loop $L_{29}(11)$} be the subset loop groupoid. S is a strictly non commutative subset loop groupoid.

Example 3.18: Let
S = {Collection of all subsets of the loop $L_{25}(13)$} be the subset loop groupoid. S is commutative.

Now having seen examples of strictly non commutative subset loop groupoids and commutative subset loop groupoids we now proceed onto give some examples of strictly non commutative subset loop groupoids which are not strictly non commutative.

Example 3.19: Let
S = {Collection of all subsets of the loop $L_{27}(8)$} be the subset loop groupoid. S is not a strictly non commutative subset loop groupoid.

Example 3.20: Let
S = {Collection of all subsets of the loop $L_{33}(17)$} be the subset loop groupoid of $L_{33}(17)$. S is a commutative subset loop groupoid.

Example 3.21: Let
S = {Collection of all subsets of the loop $L_{33}(5)$} be the subset loop groupoid of $L_{33}(5)$. S is not a strictly non commutative subset loop groupoid.

Example 3.22: Let
S = {Collection of all subsets of the loop $L_{33}(14)$} be the subset loop groupoid of $L_{33}(14)$.

S is a strictly non commutative subset loop groupoid.

Now as in case of the subset groupoids we can in case of subset loop groupoids also define the notion of left alternative subset loop groupoids, right alternative subset loop groupoids, alternative subset loop groupoids, Moufang subset loop groupoid, Bol subset loop groupoid. Bruck subset loop groupoids and so on as in case of subset groupoids.

As the identity they satisfy be it a subset groupoid or a subset loop groupoid it is going to be the same we do not dwell into the definition. We just give some examples.

We are more interested in working only with loops from the class of loops L_n defined in chapter I of this book.

Example 3.23: Let
S = {Collection of all subsets of the loop $L_{19}(18)$} be the subset loop groupoid of the loop $L_{19}(18)$. S is a left alternative subset loop groupoid.

Example 3.24: Let
S = {Collection of all subsets of the loop $L_{21}(20)$} be the subset loop groupoid. It is easily verified S is a left alternative subset loop groupoid of $L_{21}(20)$.

It is pertinent to keep on record we call the subset loop groupoid S to be a left alternative subset loop groupoid if the basic loop on which we build S is a left alternative loop. We do not demand subsets of S to satisfy it.

Example 3.25: Let S = {Collection of all subsets of the loop L = $L_5(4)$ given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	5	4	3	2
2	2	3	e	1	5	4
3	3	5	4	e	2	1
4	4	2	1	5	e	3
5	5	4	3	2	1	e

}

be the subset loop groupoid. Clearly S is a left alternative subset groupoid of $L_5(4)$.

Example 3.26: Let
S = {Collection of all subsets of the loop $L_{27}(26)$} be the subset loop groupoid. S is a left alternative subset loop groupoid of the loop $L_{27}(26)$.

Now we see how far the left alternative identity $(x*x)*y = x*(x*y)$ is true in case of S in example 3.24.

Consider A = {1, 2, 3}, B = {e, 2, 4} be S.
To find (A * A) * B

= ({1, 2, 3} * {1, 2, 3}) * B
= {1 * 1, 1 * 2, 1 * 3, 2 * 1, 2 * 2, 2 * 3, 3 * 1, 3 * 2, 3 * 3} * B
= {e, 5, 4, 1, 3} * {e, 2, 4}
= {e * e, e * 2, e * 4, 5 * e, 5 * 2, 5 * 4, 4 * e, 4 * 2, 4 * 4, 1 * e, 1 * 2, 1 * 4, 3 * e, 3 * 2, 3 * 4}
= {e, 2, 4, 5, 3, 1} … I

Consider A *(A * B)

= A* {1, 2, 3} * {e, 2, 4}
= A * {1 * e, 2 * e, 3 * e, 1 * 2, 2 * 2, 3 * 2, 1 * 4, 2 * 4, 3 * 4}
= {1, 2, 3, 5, 4} … II

We see I and II are equal and (A * A) * B = A *(A * B) for A, B ∈ S.

In view of all these we just mention the following theorem.

THEOREM 3.7: *The class of subset loop groupoids S_n = {Collection of all subsets loop groupoids of the loop $L_n(m)$ ∈ L_n for $L_n(m)$ varying in L_n} contains exactly one left alternative subset loop groupoid.*

Proof: Follows from the fact S_n contains a subset loop groupoid of the loop $L_n(n–1)$ this subset loop groupoid is left alternative as the loop $L_n(n–1)$ is left alternative [5].

Now we give examples of right alternative subset loop groupoids.

Example 3.27: Let
S = {Collection of all subsets of the loop $L_7(2)$} be the subset loop groupoid. S is right a right alternative subset loop groupoid as $L_7(2)$ is a right alternative loop.

Example 3.28: Let
M = {Collection of all subsets of the loop $L_{19}(2)$} be a subset loop groupoid of $L_{19}(2)$. As $L_{19}(2)$ is a right alternative loop so M is also a right alternative subset loop groupoid of $L_{19}(2)$.

Example 3.29: Let
M = {Collection of all subsets of the loop $L_{25}(2)$} be the subset loop groupoid of $L_{25}(2)$. M is a right alternative subset loop groupoid as $L_{25}(2)$ is a right alternative loop.

Example 3.30: Let
S = {Collection of all subsets of the loop $L_{45}(2)$} be the subset loop groupoid of the loop $L_{45}(2)$. S is a right alternative subset loop groupoid as $L_{45}(2)$ is a right alternative loop.

We will just give without proof the following theorem.

THEOREM 3.8: *Let S_n = {Collection of all subset loop groupoids of the loops $L_n(m)$ from L_n}. S_n has only one right alternative subset loop groupoid given by*
S = {Collection of all subsets of the loop $L_n(2)$} $\in S_n$.

Proof: Follows from the fact the collection of all loops L_n contains exactly only one right alternative loop $L_n(2)$. Hence the claim of the theorem.

Example 3.31: Let S = {Collection of all subsets of the loop $L_5(2)$ where the table of $L_5(2)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

}

be the subset loop groupoid of the loop $L_5(2)$. Clearly S is a right alternative subset loop groupoid of the right alternative loop $L_5(2)$.

Let A = {e, 1, 3} and B = {4, 5} ∈ S

(A * B) * B

= ({e, 1, 3} * {4, 5}) * B
= {e * 4, e * 5, 1 * 4, 1 * 5, 3 * 4, 3 * 5} * B
= {4, 5, 2} * {4, 5}
= {4 * 4, 4 * 5, 5 * 4, 5 * 5, 2 * 4, 2 * 5}
= {e, 1, 3} … I

Consider A *(B * B)

= A *({4, 5} * {4, 5})
= A * {4 * 4, 4 * 5, 5 * 4, 5 * 5}
= A * {e, 1, 3}
= {e, 1, 3} * {e, 1, 3}
= {e * e, e * 1, e * 3, 1 * 3, 1 * 1, 3 * e, 3 * 1, 3 * 3}
= {e, 1, 3, 5, 4} … II

Clearly equations I and II are distinct so for subset A, B ∈ S in general(A * A) * B ≠ A *(A * B).

But by the definition of right alternative subset loop groupoid of the loop L we only need the loop L to be a right alternative. So we see even for subset(A * A) * B ≠ A *(A * B) still since the loop $L_5(2)$ is a right alternative loop so is the subset loop groupoid of the loop $L_5(2)$.

Hence we have seen the properties.

Example 3.32: Let S = {Collection of all subsets of the loop $L_7(5)$} be the subset loop groupoid of the loop $L_7(5)$. S is neither right alternative nor left alternative. So S is not an alternative subset loop groupoid of the loop $L_7(5)$.

Example 3.33: Let
S = {Collection of all subsets of the loop $L_{27}(8)$} be the subset loop groupoid of the loop $L_{27}(8)$. S is not an alternative subset loop groupoid as $L_{27}(8)$ is not an alternative loop.

Example 3.34: Let
S = {Collection of all subsets of the loop $L_{49}(11)$} be the subset loop groupoid of the loop $L_{49}(11)$. S is not a right alternative subset loop groupoid as $L_{49}(11)$ is not a right alternative loop. S is not a left alternative subset loop groupoid as $L_{49}(11)$ is not a left alternative loop.

Hence S is not an alternative subset loop groupoid as $L_{49}(11)$ is not an alternative loop.

In view of all these we have the following theorem.

THEOREM 3.9: *Let S_n = {Collection of all subset loop groupoids of the loops from the class of loops L_n}. S_n has no subset loop groupoid which is alternative.*

Proof: A subset loop groupoid of a loop $L_n(m)$ is alternative if and only if $L_n(m)$ is an alternative loop in L_n.

But we know the class of loops L_n has no alternative loop so S_n has no subset loop groupoid which is alternative.

We define a subset loop groupoid

S = {Collection of all subsets of the loop L} to be a weak inverse property subset loop groupoid if and only if L is a weak inverse property loop; that is for every x, y, Z $\in$ L we have if (x * y) * z = e then x *(y * z) = e.

We will first illustrate this situation by some examples.

Example 3.35: Let S = {Collection of all subsets of the loop $L_7(3)$ given by the following table:

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	4	7	3	6	2	5
2	2	6	e	5	1	4	7	3
3	3	4	7	e	6	2	5	1
4	4	2	5	1	e	7	3	6
5	5	7	3	6	2	e	1	4
6	6	5	1	4	7	3	e	2
7	7	3	6	2	5	1	4	e

be the subset loop groupoid of the loop $L_7(3)$.

Since $L_7(3)$ satisfies the weak inverse property i.e., as $L_7(3)$ is a weak inverse property loop we say S is a weak inverse property subset loop groupoid of the loop $L_7(3)$.

Example 3.36: Let
S = {Collection of all subsets of the loop $L_{31}(6)$} be the subset loop groupoid of the loop $L_{31}(6)$. $L_{31}(6)$ is a weak inverse property loop so S is a weak inverse property subset loop groupoid of the loop $L_{31}(6)$.

Example 3.37: Let
S = {Collection of all subsets of the loop $L_{57}(8)$} be the subset loop groupoid of the loop $L_{57}(8)$. S is a weak inverse property subset loop groupoid as $L_{57}(8)$ is a weak inverse property loop.

Example 3.38: Let
S = {Collection of all subsets of the loop $L_{47}(7)$} be the subset loop groupoid of the loop $L_{47}(7)$. Since $L_{47}(7)$ is a weak inverse property loop we see S is also a weak inverse property subset loop groupoid.

In view of all these examples we propose the following theorem.

THEOREM 3.10: *Let S_n = {Collection of all subset loop groupoids of the loops $L_n(m)$ from L_n}. A subset loop groupoid S of the loop $L_n(m)$ in S_n is a weak inverse property subset loop groupoid if and only if $(m^2 - m + 1) \equiv 0 (mod\ n)$.*

Proof: We know a loop $L_n(m) \in L_n$ is a weak inverse property loop if and only if $(m^2 - m + 1) \equiv 0 \pmod{n}$.

So S is a weak inverse property subset loop groupoid if and only if $L_n(m)$ is a weak inverse property loop and $L_n(m)$ is a weak inverse property loop if and only if

$$(m^2 - m + 1) \equiv 0 \pmod{n}.$$

Hence the claim.

Refer [5] for more results we just see if a subset loop groupoid is a right alternative subset loop groupoid of a loop $L_n(m)$ then S is not a weak inverse property loop.

For we know if S is to be subset right alternative then loop groupoid m = 2 so that $(m^2 - m + 1) = 4 - 2 + 1 = 3 \neq 0 \pmod{n}$ for any n as in $L_n(m)$ $n > 3$ and n a prime.

If S be a subset loop groupoid which is subset left alternative then also S is not a weak inverse property subset loop groupoid. For a subset loop groupoid of the loop $L_n(m)$ to be a left alternative subset loop groupoid we need $m = n - 1$. But for S to be a subset loop groupoid which has weak inverse property we need $(m^2 - m + 1) \equiv 0 \pmod{n}$ now if $m = n - 1$ we get $m^2 - m + 1 = (n-1)^2 - (n-1) + 1 = 1 - (n-1) + 1 \neq 0 \pmod{n}$ for any n.

Hence the claim.

We define a subset loop groupoid S of a loop $L_n(m)$ to be a strictly non right alternative if $(x*y)*y \neq x*(y*y)$ for any pair x, $y \in L_n(m)$ it may so happen that we may have subset $A, B \in S$ with $A*(B*B) =(A*B)*B$ still if $L_n(m)$ is strictly non right alternative we define S to be a strictly non right associative.

We will give examples of this situation.

Example 3.39: Let S = {Collection of all subsets of the loop $L_5(2)$ given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

} be the subset loop groupoid of $L_5(2)$.

Take A = {2, 1, e} and B = {3, 4} $\in$ S. We find

A *(B * B)

= A *({3, 4} * {3, 4})
= A * {3 * 3, 3 * 4, 4 * 3, 4 * 4}
= {1, 2, e} * {e, 5, 2}
= {1 * e, 1 * 5, 1 * 2, 2 * e, 2 * 5, 2 * 2, e * e, e * 5, e * 2}
= {e, 5, 2, 4} … I

Consider

(A * B) * B

= {1, 2, e} * {3, 4}
= {1 * 3, 1 * 4, 2 * 3, 2 * 4, e * 3, e * 4} * B
= {4, 3, 5} * B
= {4, 3, 5} * {3, 4}
= {4 * 3, 4 * 4, 3 * 3, 3 * 4, 5 * 4, 5 * 3}
= {e, 5, 2, 1} … II

I and II are distinct we do not know in general about subsets of S.

Whatever be the properties of the subsets of S we see since $L_5(4)$ is strictly not right alternative loop we say S is a strictly non right alternative subset loop groupoid.

Example 3.40: Let S = {Collection of all subsets of the loop $L_5(2)$; where the table of $L_5(2)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

be the subset loop groupoid of the loop $L_5(2)$.

Take A = {e, 1, 2, 3, 4, 5} and B = {5, 2} ∈ S
We now find(A * A) * B and A *(A * B).

Clearly S is a strictly non left alternative subset loop groupoid as $L_5(2)$ is a strictly non left alternative loop in L_5. Consider (A * A) * B = A * B as (A * A = A).

Now A * B
= {e * 5, 1 * 5, 2 * 5, 3 * 5, 4 * 5, 5 * 5, e * 2, 1 * 2, 2 * 2, 3 * 2, 4 * 2, 5 * 2}
= {5, 2, e, 4, 3} = A … I

Consider
A *(A * B)
= A * A(as A * B = A)
= A … II

We see I and II are identical still S is only as per definition strictly non left alternative loop.

Likewise if A = {1, 2, 3, 4, e, 5} is in S in example 3.37 we will have

A *(B * B) = (A * B)*B still, S in Example 3.37 is only a strictly non right alternative subset loop groupoid of the loop $L_5(4)$.

It is pertinent to keep on record if for any of the operations on S; if A = $\{L_n(m)\}$ is taken as one of the elements in any identity(that is the special normal element of S) we see the identity will be true what ever be the other subsets in S.

This is the special and a unique property enjoyed by the subset {e, 1, 2, …, n} = A $\in$ S.

Now we can find the number of strictly non right(left) alternative subset loop groupoids in S_n where

S_n = {Collection of all subset loop groupoids of the loops in L_n}.

Let n = $p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ then S_n contains $F_n = \prod_{i=1}^{k} (p_i - 3) p_i^{\alpha_i - 1}$ number of subset loop groupoids which are strictly non right(left) alternative in S_n.

This result follows from the fact
L_n = {Collection of the loops $L_n(m)$}

= {class of loops $L_n(m)$ with (n, m–1) =(n, m) = 1} is such that L_n contains only $F_n = \prod_{i=1}^{k} (p_i - 3) p_i^{\alpha_i - 1}$ number of strictly non right(left) alternative loops where n = $p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ where p_i's are distinct primes $1 \le i \le k$ [5].

Now it is to be observed that if n = p, p a prime then in the class of subset loop groupoids S_n either a subset loop groupoid is right(left) alternative or strictly non right(left) alternative.

Example 3.41: Let S_7 = {Collection of all subset loop groupoids of the loop $L_7(m) \in L_7$}.

The subset loop groupoid of the loop $L_7(2)$ is strictly non left alternative, $L_7(3)$ is strictly non left alternative and $L_7(6)$ is strictly non right alternative subset loop groupoid of $L_7(6)$ and so on.

We will on similar lines define a subset loop groupoid over a loop L is a Bol subset loop groupoid if and only if L is a Bol loop.

Example 3.42: Let
S = {Collection of all subsets of the loop L = $L_{11}(7)$} be the subset loop groupoid. S is not a subset Bol loop groupoid or Bol subset loop groupoid as $L_{11}(7)$ is not a Bol loop.

Example 3.43: Let
S = {Collection of all subsets of the loop $L_{13}(9)$} be the subset loop groupoid of $L_{13}(9)$. S is not a Bol subset loop groupoid as $L_{13}(9)$ is not a Bol loop.

Example 3.44: Let
S = {Collection of all subsets of the loop $L_{15}(8)$} be the subset loop groupoid. S is not a Bol subset loop groupoid as $L_{15}(8)$ is not a Bol loop.

In view of all these we prove the following theorem for the class of subset loop groupoid S_n.

THEOREM 3.11: *Let*
S_n = {Collection of all subset loop groupoids of the loop $L_n(m) \in L_n$}. S_n does not contain any Bol subset loop groupoids.

Proof: Proof follows from the fact L_n the class of loops does not contain any Bol loop. So if S in S_n is to be a Bol subset loop groupoid of a loop $L_n(m)$ in L_n we must have $L_n(m)$ to be a Bol loop. But no $L_n(m)$ in L_n is a Bol loop so no S is S_n can be a Bol subset loop groupoid of the loop $L_n(m)$.

Now we study about Bruck subset loop groupoid.

Example 3.45: Let
S = {Collection of all subsets of a loop $L_{19}(8)$} be a subset loop groupoid of the loop $L_{19}(8)$. S is not a Bruck subset loop groupoid as the loop $L_{33}(8)$ is not a Bruck loop.

We show S_n the class of subset loop groupoids from the class of loops L_n has no Bruck subset loop groupoid.

This follows from the fact L_n the class of loops does not contain a Bruck loop so S_n = {Class of subset loop groupoids of loops from the class of loops L_n} does not contain any subset loop groupoid which is a Bruck subset loop groupoid of the loop $L_n(m)$.

Example 3.46: Let
S = {Collection of all subsets of the loop $L_{19}(8)$} be the subset loop groupoid of the loop $L_{19}(8)$. S is not a Moufang subset loop groupoid as $L_{19}(8)$ is not a Moufang loop.

Example 3.47: Let
S = {Collection of all subsets of the loop $L_{83}(17)$} be the subset loop groupoid of the loop $L_{83}(17)$. S is not a Moufang subset loop groupoid of $L_{83}(17)$ as $L_{83}(17)$ is not a Moufang loop.

Example 3.48: Let
S = {Collection of all subsets of the loop $L_{11}(7)$} be the subset loop groupoid of the loop $L_{11}(7)$ is not a Moufang subset loop groupoid as $L_{11}(7)$ is not a Moufang loop.

In view of all these we prove the following theorem.

THEOREM 3.12: *Let S_n = {Collection of all subset loop groupoids of the loops from L_n}. S_n does not contain any Moufang subset loop groupoid of a loop $L_n(m) \in L_n$.*

Proof: Follows from the fact that if $S_n(m) \in S_n$ is to be a Moufang subset loop groupoid of a loop $L_n(m)$ we need $L_n(m)$ to be a Moufang loop but we know L_n does not contain any Moufang loop so S_n cannot contain a Moufang subset loop groupoid [4, 5].

Example 3.49: Let
S = {Collection of all subsets of the loop $L_9(8)$} be the subset loop groupoid of the loop $L_9(8)$. S is not an associative subset loop groupoid.

Example 3.50: Let
S = {Collection of all subsets of the loop $L_{19}(3)$} be the subset loop groupoid of the loop $L_{19}(3)$. S is not an associative subset loop groupoid of the loop $L_{19}(3)$.

Example 3.51: Let
S = {Collection of all subsets of the loop $L_{21}(11)$} be the subset loop groupoid of the loop $L_{21}(11)$. S is not an associative subset loop groupoid of the loop $L_{21}(11)$.

We see a subset loop groupoids of the loop L is associative if and only if the loop L is associative.

THEOREM 3.13: *Let S_n = {Collection of all subset loop groupoids of the loop $L_n(m) \in L_n$}. No subset loop groupoid $S = S_n(m)$ in S_n of the loop $L_n(m)$ is associative.*

Proof: We see any $S_n(m) \in S_n$ is a subset loop groupoid of a loop $L_n(m) \in L_n$. It is proved [5] no loop in L_n is associative so; no subset loop groupoid in S_n will be associative.

We can also define subset loop subgroupoids of a subset loop groupoids.

Recall for any loop $L_n(m)$ if t/n there exists t subloops of order k+1 where k = n/t.

$H_i(t) = \{e, i, i+t, i+2t, \ldots, i+(k-1)t\}$ is a subloop of $L_n(m)$.

We see $H_i(t)$ and $H_j(t)$ are subloops of $L_n(m)$ then
$H_i(t) \cap H_j(t) = \{e\}$; $i \neq j$ and

$$\bigcup_{i=1}^{t} H_i(t) = L_n(m).$$

Example 3.52: Let
S = {Collection of all subsets of the loop $L_{35}(2)$} be the subset loop groupoid of the loop $L_{35}(12)$. Clearly 7/35 and 5/35. Let $H_i(S)$ = {Collection of all subsets of the subloop $H_i(7)$; where

$H_1(7) = \{e, 1, 8, 15, 22, 29\}$ or
$H_2(7) = \{e, 2, 9, 16, 23, 30\}$ or
$H_3(7) = \{e, 3, 10, 17, 24, 31\}$ or
$H_4(7) = \{e, 4, 11, 18, 25, 32\}$ or
$H_5(7) = \{e, 5, 12, 19, 26, 33\}$ or
$H_6(7) = \{e, 6, 13, 20, 27, 34\}$ or
$H_7(7) = \{e, 7, 14, 21, 28, 35\}$}
'or' used in the mutually exclusive sense.

We see $\cup H_i(S) \underset{\neq}{\subset} S$.

Clearly equality can never be attained.

However each $H_i(5)$ is a subset subloop subgroupoid of S.

If we consider
$H_1(5) = \{e, 1, 6, 11, 16, 21, 26, 31\}$
$H_2(5) = \{e, 2, 7, 12, 17, 22, 27, 32\}$ and so on.

We see $H_1(5) \cap H_2(5) = \{e\}$. It is obvious from the subloops of the loop $L_n(m)$.

So we have subset subloop subgroupoids of a subset loop groupoid.

Example 3.53: Let
S = {Collection of all subsets of the loop $L_{15}(8)$} be the subset loop groupoid of $L_{15}(8)$.

Take P_1 = {Collection of all subsets of the subloop

$H_2(3)$ = {e, 2, 5, 8, 11, 18} $\subseteq L_{15}(8)$}; is a subset subloop subgroupoid of S. Take P_1 = {Collection of all subsets of the subloop $H_3(5)$ = {e, 3, 8, 13} $\subseteq L_{15}(8)$}} $\subseteq$ S is also a subset subloop subgroupoid of S.

Now P_2 = {Collection of all subsets of the subloop $H_3(3)$ = {e, 3, 6, 9, 12, 15} $\subseteq L_{15}(8)$} be the subset subloop subgroupoid of S. We see $P_1 \cap P_2$ = {e}.

B_2 = {Collection of all subsets of the subloop $H_2(5)$ = {e, 2, 7, 12} $\subseteq L_{15}(8)$} $\subseteq$ S} is a subset subloop subgroupoid of S. We see $B_1 \cap B_2$ = {e}.

Thus we can get eight such subset subloop subgroupoids of S.

Example 3.54: Let
S = {Collection of all subsets of the loop $L_{21}(11)$} be the subset loop groupoid of the loop $L_{21}(11)$. Let H_1 = {Collection of all subsets of the subloop $H_1(3)$ = {e, 1, 4, 7, 10, 13, 16, 19} $\subseteq L_{21}(11)$} $\subseteq$ S be the subset subloop subgroupoid of S.

H_2 = {Collection of all subsets of the subloop $H_2(3)$ = {e, 2, 5, 8, 11, 14, 17, 20} $\subseteq L_{21}(11)$} $\subseteq$ S be the subset subloop subgroupoid of S.

H_3 = {Collection of all subsets of the subloop $H_3(3)$ = {e, 3, 6, 9, 12, 15, 18, 21} $\subseteq L_{21}(8)$} $\subseteq$ S be the subset subloop subgroupoid of S.

We see $H_i \cap H_j = \{e\}$ if $i \neq j$; $1 \leq i, j \leq 3$.

Consider P_1 = {Collection of all subsets of the subloop $H_1(7) = \{e, 1, 8, 15\} \subseteq L_{21}(11)\} \subseteq S$ be the subset subloop subgroupoid of S.

P_2 = {Collection of all subsets of the subloop $H_2(7) = \{e, 2, 9, 16\} \subseteq L_{21}(11)\} \subseteq S$ be the subset subloop subgroupoid of S. P_3 = {Collection of all subsets of the subloop $H_3(7) = \{e, 3, 10, 17\} \subseteq L_{21}(11)\} \subseteq S$ be the subset subloop subgroupoid of the subset loop groupoid S of the loop $L_{21}(11)$ and so on.

Let P_7 = {Collection of all subsets of the subloop $H_7(7) = \{e, 7, 14, 21\} \subseteq L_{21}(11)\} \subseteq S$ be the subset subloop subgroupoid of S. We see $P_i \cap P_j = \{e\}$ if $i \neq j$. $1 \leq i, j \leq 7$.

We see S has atleast ten subset subloop subgroupoids.

Example 3.55: Let
S = {Collection of all subsets of the loop $L_{55}(8)$} be a subset loop groupoid of the loop $L_{55}(8)$. Let P_1 = {Collection of all subsets of the subloop $H_1(5) = \{e, 1, 6, 11, 16, \ldots, 51\}$} be the subset subloop subgroupoid of S.

P_2 = {Collection of all subsets of the subloop; $H_2(5) = \{\{e, 2, 7, 12, 17, 22, \ldots, 52\}\}$ be the subset subloop subgroupoid of S. P_3 = {Collection of all subsets of the subloop. $H_3(5) = \{e, 3, 8, 13, 18, 23, \ldots, 53\} \subseteq L_{55}(8)$} be the subset subloop subgroupoid of S.

P_4 = {Collection of all subsets of the subloop
$H_4(5) = \{\{e, 4, 9, 14, 19, 24, \ldots, 54\} \subseteq L_{55}(8)\} \subseteq S$ be the subset subloop subgroupoid of S.

P_5 = {Collection of all subsets of the subloop $H_5(5) = \{\{e, 5, 10, 15, 20, \ldots, 55\} \subseteq L_{55}(8)\} \subseteq S$ be the subset subloop subgroupoid of S.

We see $P_i \cap P_j = \{e\}$, $i \neq j$, $1 \leq i, j \leq 5$.

Let B_1 = {Collection of all subsets of the subloop $H_1(11)$ = {{e, 1, 12, 23, 34, 45} $\subseteq L_{55}(8)$} $\subseteq$ S be the subset subloop subgroupoid of S.

B_2 = {Collection of all subsets of the subloop $H_2(11)$ = {{e, 2, 13, 24, 35, 46} $\subseteq L_{55}(8)$} $\subseteq$ S be the subset subloop subgroupoid of S.

B_3 = {Collection of all subsets of the subloop $H_3(11)$ = {{e, 3, 14, 25, 36, …, 47} $\subseteq L_{55}(8)$} $\subseteq$ S be the subset subloop subgroupoid of S and so on.

B_{11} = {Collection of all subsets of the subloop $H_{11}(11)$ = {e, 11, 22, 33, 55} $\subseteq L_{55}(8)$} $\subseteq$ S be the subset subloop subgroupoid of S.

We see $B_i \cap B_j = \{e\}$ if $i \neq j$. $1 \leq i, j \leq 11$.

Thus we are in a position to find atleast 16 subset subloop subgroupoids of the subset loop groupoid S of the loop $L_{55}(8)$.

In view of this we have the following theorem.

THEOREM 3.14: *Let S = {Collection of all subsets of the loop $L_{pq}(t)$ where p and q two distinct odd primes} be the subset loop groupoid of the loop $L_{pq}(t)$. S has atleast p+q subset subloop subgroupoids.*

Proof: We see p/pq and q/pq. We have $H_i(p)$ which gives p number of subloops; i = 1, 2, …, p so related with each of these subloops we have a subset subloop subgroupoids say $P_1, P_2, \ldots, P_p$ such that $P_i \cap P_j = \{e\}$ if $i \neq j$; $1 \leq i, j \leq p$.

Thus S has p + q number of distinct subset subloop subgroupoid where P_i = {Collection of all subsets of the subloop $H_i(p)$ = {e, I, i+p, i+2p, …, i+(q–1)p} $\subseteq L_{pq}(t)$} $\subseteq$ S be the subset subloop subgroupoid of S. Now M_j = {collection of all subset of the subloop $H_j(p)$ = {e, j, j+q, j+2q, …, j+(p–1)q} $\subseteq L_{pq}(t)$} $\subseteq$ S be the subset subloop subgroupoid of S, $1 \leq j \leq q$.

It is easily verified $P_i \cap P_j = \{e\}$ if $i \neq j$; $1 \leq i, j \leq p$ and $M_i \cap M_j = \{e\}$ if $i \neq j$. $1 \leq i, j \leq q$.

Hence the theorem.

Suppose we see p and q not relatively prime say (p, q) = d, d > 1 then what are the types of subset subloop subgroupoid we have for $L_n(m)$, n = pq (p, q) ≠ 1.

To this end we first illustrate some examples.

Example 3.56: Let
S = {Collection of all subsets of the loop $L_{45}(14)$} be the subset loop groupoid of the loop $L_{45}(14)$.

Consider $H_1(9)$ = {e, 1, 10, 19, 28, 37}, $H_2(9)$ = {e, 2, 11, 20, 29, 38}, $H_3(9)$ = {e, 3, 12, 21, 30, 39}, $H_4(9)$ = {e, 4, 13, 22, 31, 40} and so on $H_9(9)$ = {e, 9, 18, 27, 36, 45} all of them are subloops of $L_{45}(14)$.

We see P_i = {Collection of all subsets of the subloop $H_i(9)$ $\subseteq L_{45}(14)$} $\subseteq$ S is a subset subloop subgroupoid of S; $1 \leq i \leq 9$.

Consider $H_1(15)$ = {e, 1, 16, 31}, $H_2(15)$ = {e, 2, 17, 32}, $H_3(15)$ = {e, 3, 18, 33}, $H_4(15)$ = {e, 4, 19, 34}, $H_5(15)$ = {e, 5, 20, 35} and so on. $H_{15}(15)$ = {e, 15, 30, 45}; all of them are subloops of $L_{45}(14)$.

Let M_j = {collection of all subsets of the subloop $H_j(15) \subseteq L_{45}(14)$} $\subseteq$ S; M_j is a subset subloop subgroupoid of S, $1 \leq j \leq 15$.

We now find

P_1 = {Collection of all subsets of the subloop $H_1(9)$ = {e, 1, 10, 19, 28, 37} $\subseteq L_{45}(14)$} $\subseteq$ S the subset subloop subgroupoid of S.

M_3 = {Collection of all subsets of the subloop $H_3(15)$ = {e, 3, 18, 33} $\subseteq L_{45}(14)$} be the subset subloop subgroupoid of S.

We see $H_1(9) \cap H_3(15)$ = {e}.

For {e, 1, 10, 19, 28, 37} $\cap$ {e, 3, 18, 33} = {e}.

We find now $H_i(3)$ for 3/45.

$H_2(3)$ = {e, 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 31, 35, 38, 41, 44}.

We now find $H_2(3) \cap H_3(15)$ = {e} and $H_2(3) \cap H_1(9)$ = {e}.

Now we find $H_1(3)$ = {e, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43} $\subseteq L_{45}(14)$ is a subloop.

Clearly $H_1(3) \cap H_1(9)$ = {e, 1, 10, 19, 28, 37} = $H_1(9)$.

We now find $H_1(15)$ = {e, 1, 16, 31} $\subseteq L_{45}(14)$ subloop of $L_{45}(4)$.

Consider $H_1(15) \cap H_1(3)$

= {e, 1, 16, 31} = $H_1(15)$.

We now find

$H_1(9) \cap H_1(15)$ = {e, 1}.

Several conclusions can be made from this study.

Now let

N_t = {Collection of all subsets of the subloop $H_t(3) \subseteq L_{45}(45)$} $\subseteq$ S be the subset subloop subgroupoid of S.

We make the following observations

$$P_i \cap M_j = \{e\} \text{ if } i \neq j$$
$$P_i \cap N_k = \{e\} \text{ if } i \neq k$$
$$N_k \cap M_j = \{e\} \text{ if } j \neq k$$
$$1 \leq i \leq 9,\ 1 \leq j \leq 15 \text{ and } 1 \leq k \leq t.$$

Now we just study another example.

Example 3.57: Let
S = {Collection of all subsets of the loop $L_{75}(14)$} be the subset loop groupoid of the loop $L_{75}(4)$. We see $H_i(5)$, $H_j(15)$, $H_i(3)$ and $H_j(25)$ gives four classes of subloops. We have 48 subloops of $L_{75}(14)$.

Associated with these 48 subloops we can get 48 subset subloop subgroupoids of S. Some of the subgroupoids have {e} to be common and some of them are subset subloop subgroupoids of these subset subloop subgroupoids to be common if they are built on subgroupoids of subgroupoids i, i.e., $H_i(5)$ and $H_i(15)$ are such that $H_i(5) \supseteq H_i(15)$ likewise $H_i(3) \supseteq H_i(15)$

$H_i(5) \supseteq H_i(25)$

We just given only a few illustrations from them

$H_1(15) = \{e, 1, 16, 31, 46, 61\} \subseteq L_{75}(14)$ is a subloop of the loop $L_{75}(14)$.

Consider $H_1(25) = \{e, 1, 26, 51\} \subseteq L_{75}(14)$ is again a subloop of $L_{75}(14)$. Clearly $H_1(15) \cap H_1(25) = \{e, 1\}$.

But if we take $H_1(5) = \{e, 1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71\} \subseteq L_{75}(14)$ is a subloop of the loop $L_{75}(14)$.

We see $H_1(5) \cap H_1(25)$
$= \{e, 1, 26, 51\} = H_1(25)$.

Similarly $H_1(5) \cap H_1(15)$
$= \{e, 1, 16, 31, 46, 61\} = H_1(15)$.

Thus we see if in $H_1(t)$ and $H_i(m)$ if t/m the $H_i(t) \cap H_i(m) = H_i(t)$ provide i is taken to be the same index.

Consider $H_2(5) \cap H_6(15)$

$= \{e, 2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72\} \cap \{e, 6, 21, 36, 51, 66\} = \{e\}$.

We see if $i \neq j$ even if $H_i(5) \supseteq H_j(15)$. We see $H_i(5) \cap H_j(15) = \{e\}$ for $i \neq j$. Only for same value of i we have the containment relation to be true.

Example 3.58: Let

S = {Collection of all subsets of the loop $L_{105}(23)$} be the subset loop groupoid of the loop $L_{105}(23)$.

Consider $H_i(3) = \{e, I, i+3, \ldots, i+(105–i+1)\} \subseteq L_{105}(23)$ is a subloop of S.

$H_1(5) = \{e, 1, 6, 11, 16, 21, 26, 31, \ldots, 101\}$
$H_1(3) = \{e, 1, 4, 7, 10, 14, 17, 20, 23, \ldots, 103\}$
$H_1(7) = \{e, 1, 7, 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, \ldots, 99\}$
$H_1(15) = \{e, 1, 16, 31, 46, 61, 76, 91\} \subseteq L_{105}(23)$.
$H_1(21) = \{e, 1, 22, 43, 64, 85\} \subseteq L_{105}(23)$ and
$H_1(35) = \{e, 1, 36, 71\} \subseteq L_{105}(23)$ are all subloops of the loop $L_{105}(23)$.

We see $H_1(35) = H_1(7)$

$H_1(21) \subseteq H_1(7)$ and
$H_1(15) \subseteq H_1(5)$.

Further the subset subloop subgroupoid of the subloops are also contained as per above containment.

P_1 = {Collection of all subsets of the subloop $H_1(35) \subseteq L_{105}(23)$} $\subseteq$ S be the subset subloop subgroupoid of S.

M_1 = {Collection of all subsets of the subloop $H_1(7) \subseteq L_{105}(23)$} $\subseteq$ S be the subset subloop subgroupoid of S.

N_1 = {Collection of all subsets of the subloop $H_1(5) \subseteq L_{105}(23)$} $\subseteq$ S be the subset subloop subgroupoid of S.

T_1 = {Collection of all subsets of the subloop $H_1(21) \subseteq L_{105}$ (23)} $\subseteq$ S be the subset subloop subgroupoid of S.

B_1 = {Collection of all subsets of the subloop $H_1(15) \subseteq L_{105}(23)$} $\subseteq$ S be the subset subloop subgroupoid of S and

R_1 = {Collection of all subsets of the subloop $H_1(3) \subseteq L_{105}(23)$} $\subseteq$ S be the subset subloop subgroupoid of S.

We see $T_1 \subseteq R_1$, $B_1 \subseteq R_1$, $P_1 \subseteq N_1$, $P_1 \subseteq M_1$, $B_1 \subseteq N_1$. We see this is true for any appropriate i.

Example 3.59: Let
S = {Collection of all subsets of the loop $L_{63}(11)$} be the subset loop groupoid of the loop $L_{63}(11)$.

Consider $H_1(3)$ = {e, 1, 4, 7, 10, 13, 16, 19, 22, …, 61},
$H_2(3)$ = {e, 2, 5, 8, 11, 14, 17, 20, 23, …, 62} and
$H_3(3)$ = {e, 3, 6, 9, 12, 15, …, 63} subloops.

$H_1(9)$ = {e, 1, 10, 19, 28, 37, 46, 55, 64} $\subseteq L_{63}(11)$,
$H_2(9)$ = {e, 2, 11, 20, 29, 38, 47, 56, 65} $\subseteq L_{63}(11)$,
$H_3(9)$ = {e, 3, 12, 21, 30, 39, 48, 57, 66} $\subseteq L_{63}(11)$ and
$H_9(9)$ = {e, 9, 18, 27, 36, 45, 54, 63} $\subseteq L_{63}(11)$.

$H_1(7)$ = {e, 1, 8, 15, 22, 29, 36, 43, 50, 57} $\subseteq L_{63}(11)$
$H_2(7)$ = {e, 1, 2, 9, 16, 23, 30, 37, 44, 51, 58} $\subseteq L_{63}(11)$
and so on;
$H_7(7)$ = {e, 7, 14, 21, 28, 35, 42, 49, 56, 63} $\subseteq L_{63}(11)$

$H_1(21) = \{e, 1, 22, 43\} \subseteq L_{63}(11)$
$H_2(21) = \{e, 2, 23, 44\} \subseteq L_{63}(11)$
$H_3(21) = \{e, 3, 24, 45\} \subseteq L_{63}(11)$
$H_4(21) = \{e, 4, 25, 46\} \subseteq L_{63}(11)$
and so on

$H_{21}(21) = \{e, 21, 42, 63\} \subseteq L_{63}(11)$ are the subloops of $L_{63}(11)$ we have 40 subloops. Corresponding to each of the subloop we have subset subloop subgroupoid of S.

Thus we have 40 distinct subset subloop subgroupoids of S.

In view of all these we have the following results.

THEOREM 3.15: *Let S = {Collection of all subsets of the loop $L_n(m) \in L_n$} be the subset loop groupoid of the loop $L_n(m)$. If r/n and s/n then the subset subloop subgroupoids P_i and B_i of the subloops satisfy the following.*

(i) $o(P_i \cap B_j) = 1$ *if* $(r, s) = d$, $d > 1$ *and* $i \neq j \pmod d$
(ii) $o(P_i \cap B_j) = 2^{1+(n/rs)} - 1$ *if* $(r, s) = 1$
(iii) $o(P_i \cap B_j) = 2^{(1+n/lcm(r,s))} - 1$ *if* $(r, s) = d$ $(d > 1)$ *and* $i \equiv j \pmod d$

The proof is similar to that of subloops [5].

Now we can define for subset loop groupoid of the loop the analogous of Lagrange's theorem for finite groups.

We know the Lagrange's theorem for groups is true in case of loop $L_n(m)$ when n is an odd prime.

For any subloops in $L_n(m)$ are $H_i = \{e, i\} \subseteq L_n(m)$, $i \in L_n(m)$; and $o(H_i) = 2$, and 2/n+1.

However if n = 9 say then $L_9(8)$ is a loop of order 9. $H_1(3) = \{e, 1, 4, 8\}$ is a subloop of $L_9(8)$.

$o(H_1(3))$ = 4 and $4 \nmid 10$ so this subloop does not divide order of $L_9(8)$.

$o(S) = 2^{10} - 1$ and
$o(P_1)$ = o ({Collection of all subsets of the subloop $H_1(3)$}) $= 2^4 - 1 = 15$.

$15 \nmid 2^{10}-1$. So Lagranges theorem for group is not satisfied by the subset subloop subgroupoid.

Consider S the subset loop groupoid of the loop $L_{15}(8)$.
$o(L_{15}(8)) = 2^{16} - 1$.

Let P_1 = {Collection of all subsets of the subloop $H_1(5) = \{e, 1, 6, 11\} \subseteq L_{15}(8)\} \subseteq S$ be the subset subloop subgroupoid of S.

$o(P_1) = 2^4 - 1$ but $o(H_1(5)) = 4$ and 4/16 also $2^4-1 / 2^{16}-1$. We see $H_1(5)$ satisfies Lagrange theorem for finite loops and also the subset subloop subgroupoid also satisfies the Lagrange theorem for subloops.

We see if we take $H_i(3) = \{e, 1, 4, 7, 10, 13\} \subseteq L_{15}(8)$;

$o(H_i(3)) \nmid o(L_{15}(8))$. $o(B_1)= 2^6-1$.

We see $o(B_1) \nmid o(S)$ as $2^6-1 \nmid 2^{16}-1$. We see some of the subset subloop subgroupoid satisfy the Lagranges theorem for finite groups.

Example 3.60: Let S = {Collection of all subsets of the loop $L_{21}(11)$} be the subset loop groupoid of the loop $L_{21}(11)$. $H_1(7) = \{e, 1, 8, 15\}$, $H_2(7) = \{e, 2, 9, 16\}$, $H_3(7) = \{e, 3, 10, 17\}$, …, $H_7(7) = \{e, 7, 14, 21\}$.

$o(S) = 2^{21} - 1$ and

$o(P_1) = o(P_1) =$ {Collection of all subsets of the subloop $H_1(7)$}) $= 2^4 - 1$.

Clearly $2^4 - 1 \nmid 2^{21} - 1$.

$H_1(3) = \{e, 1, 4, 7, 10, 13, 16, 19\} \subseteq L_{21}(11)$. $B_1 =$ {Collection of all subsets of the subloop $H_1(3) \subseteq L_{21}(11)$} $\subseteq S$, be the subset subloop subgroupoid of the subloop $o(B_1) = 2^8 - 1$.

Clearly $2^8 - 1 \nmid 2^{21} - 1$.

Example 3.61: Let
S = {Collection of all subsets of the loop $L_{11}(8)$} be the subset loop groupoid of the loop $L_{11}(8)$.

We know $o(S) = 2^{12} - 1$. Now the only subloops of $L_{11}(8)$ are $\{e, i\} \subseteq L_n(m)$, $i \in L_{11}(8)$. Clearly the subset subloop subgroupoids are of order 3. $3/2^{12}-1$. So order of all the subset subloop subgroupoids of S divides the order of S.

Example 3.62: Let
S = {Collection of all subsets of the loop $L_{23}(7)$} be the subset loop groupoid of the loop $L_{23}(7)$. Clearly the only subset subloop subgroupoid of S are {i, e} where $i \in L_{23}(7)$. Let $P_i =$ {Collection of all subsets of {i, e}} $\subseteq S$ $(i \neq e)$ is a subset subloop subgroupoid of S of order 3. We see $o(S) = 2^{24} - 1$ and $o(P_i) = 2^2 - 1$ and $o(P_i) / o(S)$

Now having seen examples of subset loop groupoids of the loop $L_n(m)$; n a prime we formulate the following results.

THEOREM 3.16: *Let S_n = {Collection of all subset loop groupoids of the class of loops L_n}.*

S = {Collection of all subsets of the loop $L_n(m)$} be the subset loop groupoid $L_n(m)$. The Lagrange theorem for finite groups is satisfied by every subset subloop subgroupoids of S if and only if n is an odd prime.

Proof: Follows from the simple fact that $L_n(m) \in L_n$ satisfies the Lagrange theorem if and only if n is an odd prime [5].

Now if S is the subset loop groupoid of the loop $L_n(m)$ then $o(S) = 2^{n+1} - 1$.

Take all the subset subloop groupoids of S; they are $H_i = \{e, i\} \subseteq L_n(m)$ such that $i \in L_n(m)$ and $M = \{e, 1, 2, \ldots, p\} \in S$.

If P_i = {Collection of all subset subloop subgroupoids of H_i} the subset subloop subgroupoid of S then $o(P_i) = 2^2 - 1$. o(M) = 1. Thus $o(M) / 2^{p+1} - 1$ and $o(P_i) = 2^2 - 1 / 2^{p+1} - 1$, hence the claim.

We will call a subset subloop subgroupoid H of a subset loop groupoid S to be a p-Sylow subset subloop subgroupoid if $o(H) = p^k$ then $p^k / o(S)$ but $p^{k+1} \nmid o(S)$.

We first illustrate the situation of the existence / non existence of p-Sylow subset subloops of groupoids.

Example 3.63: Let S = {Collection of all subsets of the loop $L_{27}(8)$} be the subset loop groupoid of the loop $L_{27}(8)$.

The subset subloop subgroupoids of the subset loop groupoid, $L_{27}(8)$ are as follows:

$H_1(3) = \{e, 1, 4, 7, 10, 13, 16, 19, 22, 25\} \subseteq S$,
$H_2(3) = \{e, 2, 5, 8, 11, 14, 17, 20, 23, 26\} \subseteq S$ and
$H_3(3) = \{e, 3, 6, 9, 12, 15, 18, 21, 24, 27\} \subseteq S$ are subset subloop subgroupoids of S.

$H_1(9) = \{e, 1, 10, 19\} \subseteq L_{27}(8)$
$H_2(9) = \{e, 2, 11, 20\} \subseteq L_{27}(8)$
$H_3(9) = \{e, 3, 12, 21\} \subseteq L_{27}(8)$ and so on

$H_9(9) = \{e, 9, 18, 27\} \subseteq L_{27}(8)$ are all the subloops of $L_{27}(8)$.

All the associated subset subloop subgroupoids of S are P_i = {Collection of all subsets of the subloop $H_i(9)$} be the subset subloop subgroupoid of S.

$o(P_i) = 2^4 - 1 = 7$
and $o(S) = 2^{28} - 1$ and $2^4 - 1 / 2^{28} - 1$.
Also $o(H_i(9)) / o(L_{27}(8))$.

In view of this we propose the following problem that if $o(H_i) / o(L_n(m))$ then will $2^{o(H_i)} - 1 / 2^{o(L_n(m))} - 1$?

We see in most of the examples this is true.

Example 3.64: Let
S = {Collection of all subsets of the loop $L_{45}(8)$} be the subset loop groupoid of the loop $L_{45}(8)$.

$H_1(5) = \{e, 1, 6, 11, 16, 21, 26, 31, 36, 41\} \subseteq L_{45}(8)$
$H_2(5) = \{e, 2, 7, 12, 17, 22, 27, 32, 37, 42\} \subseteq L_{45}(8)$
$H_3(5) = \{e, 3, 8, 13, 18, 23, 28, 33, 38, 43\} \subseteq L_{45}(8)$
$H_4(5) = \{e, 4, 9, 14, 19, 24, 29, 34, 39, 44\} \subseteq L_{45}(8)$ and
$H_5(5) = \{e, 5, 10, 15, 20, 25, 30, 35, 40, 45\} \subseteq L_{45}(8)$ be the subloops of the loop $L_{45}(8)$.

Consider $H_1(9) = \{e, 1, 10, 19, 28, 37\} \subseteq L_{45}(8)$
$H_2(9) = \{e, 2, 11, 20, 29, 38\} \subseteq L_{45}(8)$, …,
$H_9(9) = \{e, 9, 18, 27, …, 45\} \subseteq L_{45}(8)$ be the subloops of the loop $L_{45}(8)$.

$H_1(3) = \{e, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43\} \subseteq L_{45}(8)$

$H_2(3) = \{e, 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44\} \subseteq L_{45}(8)$ and

$H_3(3) = \{e, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45\} \subseteq L_{45}(8)$ are all subloop of $L_{45}(8)$.

$o(H_i)\ (5)) = 10$, $o(H_i(3)) = 16$ and $o(H_i(9)) = 6$; clearly $o(H_i(5))$ does not divider $o(L_{45}(8))$ i.e., $10 \nmid 46$. $o(H_i(3))$ / $o(L_{45}(8))$ that is $16 \nmid 46$ and $o(H_i(9)) \nmid o(L_{45}(8))$ that is $6 \nmid 46$.

Clearly none of the subloops of the loop $L_{45}(8)$ is a Lagrange subloop of $L_{45}(8)$.

We see none of the order of the subset subloop subgroupoids of S divides order of S.

That is $2^6 - 1 \nmid 2^{46} - 1$, $2^{10} - 1 \nmid 2^{46} - 1$ and $2^{16} - 1 \nmid 2^{46} - 1$.

Hence the Lagrange theorem for subset subloop subgroupoid is not true in case of this S.

Example 3.65: Let
S = {Collection of all subsets of the loop $L_{39}(11)$} be the subset loop groupoid of the loop $L_{39}(11)$. The subloops of $L_{39}(11)$ are $H_i(3)$ and $H_j(3)$, i = 1, 2, …, 13 and j = 1, 2, 3.

$H_1(13) = \{e, 1, 14, 27\}$,

$H_2(13) = \{e, 2, 15, 28\}$ and so on $H_{13}(13) = \{e, 13, 26, 39\}$ are subloops of $L_{39}(11)$ which are Lagrange subloops as $o(H_i(13)) = 4$ and 4/40.

$H_1(13) = \{e, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37\}$
$H_2(13) = \{e, 2, 5, 8, 11, 14, 17, \ldots, 35, 38\}$ and
$H_3(3) = \{e, 1, 6, 9, 12, 15, 18, 21, 24, 27, 30, 36, 33, 39\} \subseteq L_{39}(11)$.

Clearly $o(H_j(3)) \nmid o\ (L_{39}(11))$ as $o(H_j(3)) = 14$ and that of $o(L_{39}(11)) = 40$ and $14 \nmid 40$.

Thus some subloop of $L_{39}(11)$ satisfy Lagrange theorem and some of them do not satisfy the Lagrange theorem for finite groups.

If P_i = {Collection of all subsets of the subloop $H_i(13) \subseteq L_{39}(11)$} be the subset subloop subgroupoid then $o(P_i) / o(S)$ where $o(P_i) = 2^4 - 1 / 2^{40} - 1$ and if

B_i = {Collection of all subsets of the subloop $H_i(3) \subseteq L_{39}(11)$} be the subset subloop subgroupoid then $o(B_i) = 2^{14} - 1$ does not divide $2^{40} - 1$.

Thus we have loops L such that in that L some subloops are Lagrange and some subloops are not Lagrange.

We also see some of the subloops of the loop $L_n(m) \in L_n$ are Sylow but most of them are not Sylow.

Example 3.66: Let
S = {Collection of all subsets of the loop $L_7(3)$} be the subset loop groupoid of the loop $L_7(3)$. We see the only subloops of $L_7(3)$ are $H_i = \{e, i\} \subseteq L_7(3)$; where $i \in \{1, 2, 3, \ldots, 7\}$, $1 \leq i \leq 7$.

Now
P_i = {Collection of all subsets of H_i} = $\{\{e\}, \{i\}, \{e, i\}\} \subseteq S$ is a subset subloop subgroupoid of order 3.

$o(S) = 2^8 - 1$ and $o(P_i) = 2^2 - 1$ and $o(P_i) / o\ (2^8 - 1)$.

Thus S has 3 p-Sylow subset subloop subgroupoid.

Example 3.67: Let
S = {Collection of all subsets of the loop $L_{15}(8)$} be the subset loop groupoid of the loop $L_{15}(8)$. $o(S) = 2^{16} - 1$ and $o(L_{15}(8)) = 16$.

Clearly S has 3-Sylow subset subloop subgroupoids given by the subloop (associated with the subloop), $H_i = \{e, i\} \subseteq L_{15}(8)$; $i \in \{1, 2, \ldots, 15\}$; $1 \leq i \leq 15$. Thus all subset subloop subgroupoids associated with H_i are of order $3 = 2^2 - 1$ and $2^2 - 1 / o(S) = 2^{16} - 1$. Thus S has 3-Sylow subloop subgroupoid.

Consider $H_i(5) = \{e, 1, 6, 11\} \subseteq L_{15}(8)$ be a subloop of $L_{15}(8)$. Let P_1 = {Collection of all subsets of the subloop $H_i(5)$} be the subset subloop subgroupoid of S. $o(P_i) = 2^4 - 1$ and $2^4 - 1 / 2^{16} - 1$ hence P_i's are Lagrange. As $o(P_i) = 2^4 - 1 = 15$ we cannot define any notion like p-Sylow for these subset subloop subgroupoids of S and $15/2^{16} - 1$.

Example 3.68: Let
S = {Collection of all subsets of the loop $L_{29}(7)$} be the subset loop groupoid of the loop $L_{29}(7)$. $L_{29}(7)$ has only subgroups (subloops) of order two. We see $H_i = \{e, i\} \subseteq L_{29}(7)$ where $i \in \{1, 2, \ldots, 29\}$, $1 \leq i \leq 29$ are the 29 subloops. Related with each of these subloops we get subset subloop subgroupoids of S of order $2^4 - 1$. We see order of $S = 2^{30} - 1$.

Further $2^4 - 1 / 2^{30} - 1$.

Thus all subset subloop subgroupoids of S are Lagrange and 3-Sylow.

However we see the fact that number of p-Sylow subset subloop subgroupoids of S for a given prime is of the form 1+kp where (1+kp) / o(S) for some positive integer k is general is not true.

Example 3.69: Let
S = {Collection of all subsets of the loops $L_{23}(7)$} be the subset loop groupoid of the loop $L_{23}(7)$. $o(S) = 2^{24} - 1$.

The subset subloop subgroupoids of S are P_i = {Collection of all subset of subloops of the loop $H_i = \{e, i\}$} $\subseteq$ S be the subset subloop subgroupoids of S. $o(P_i) = 2^2 - 1$. $o(P_i) / o(S)$.

All subset subloop subgroupoids are 3-Sylow subset subloop subgroupoids.

We see S has 23, 3-Sylow subset subloop subgroupoids. p = 2 and $2^{24} - 1$ S has 23 number of 3-Sylow subset subloop subgroupoids and we have 1+3k = 23 and we see these does not exist any integer k such that 1 + 3k = 23. Hence the third p-Sylow theorem is not true in case of S which is associated with the loop $L_p(m) \in L_p$; p a prime.

Example 3.70: Let
S = {Collection of all subsets of the loop $L_{27}(8)$} be the subset loop groupoid of the loop $L_{27}(8)$. The subset subloop subgroupoid of S are

$H_1(3) = \{e, 1, 4, 7, 10, 13, 16, 19, 22, 25\} \subseteq L_{27}(8)$.

P_1 = {collection of all subsets of $H_1(3)$} be the subset subloop subgroupoid of S. $o(P_1) = 2^{10} - 1$ and $o(S) = 2^{28} - 1$. We see P_1 is not a p-Sylow subset subloop subgroupoid of S.

Example 3.71: Let
S = {Collection of all subsets of the loop $L_{51}(8)$} be the subset loop groupoid of the loop $L_{51}(8)$.

Let 3/51 and 17/51 so we see we have two subset subloop subgroupoids associated with the subloops $H_i(3)$ and $H_i(17)$ where

$H_1(3) = \{e, 1, 4, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49\} \subseteq L_{51}(8)$,

$H_2(3) = \{e, 2, 5, 8, 11, 14, 17, 20, 23, 26, \ldots, 47, 50\} \subseteq L_{51}(8)$ and

$H_3(3) = \{e, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51\} \subseteq L_{51}(8)$.

$H_1(17) = \{e, 1, 18, 35\} \subseteq L_{51}(8)$,
$H_2(17) = \{e, 2, 19, 36\} \subseteq L_{51}(8)$,

$H_3(17) = \{e, 3, 20, 27\} \subseteq L_{51}(8)$, …, and
$H_{17}(17) = \{e,17, 34, 51\} \subseteq L_{51}(8)$ are the subloops of $L_{51}(8)$.

We have 20 subloops for the loop $L_{51}(8)$.

P_i = {collection of all subsets of the subloop $H_i(3)$} $\subseteq$ S, $1 \leq i \leq 3$ be the subset subloop of the subgroupoid of the subloop of $H_i(3) \subseteq L_{51}(8)$.

M_i = {collection of all subsets of the subloop $H_i(17)$} $\subseteq$ S be the subset subloop subgroupoid of the subloop $H_i(17)$; $1 \leq i \leq 17$.

$o(P_i) = 24 – 1$ as $o(H_i(3)) = 4$.

Now $o(M_i) = 2^{18} – 1$ as $o(H_i(17) = 18$; $o(S) = 2^{52} – 1$ clearly $o(P_i) / o(S)$ but $o(M_i) \nmid o(S)$ so we see P_i is not a Sylow subset subloop subgroupoid as $o(P_i) = 15$, $1 \leq i \leq 3$.

Thus S has no p-sylow subset subloop subgroupoids.

Thus $o(H_i(3)) / o(L_{51}(8))$; 4/52; $8 \nmid 52$ So $H_i(3)$ is a 2-Sylow subloop of $L_{51}(8)$.

However the subset subloop subgroupoid of the subloop $H_i(3)$ is not a p-Sylow subset subloop subgroupoid of S.

Example 3.72: Let
S = {Collection of all subsets of the loop $L_{33}(5)$} be the subset loop groupoid of the loop $L_{33}(5)$.

3/33 and 11/33 so we have 14 subloops for the loop $L_{33}(5)$ given by

$H_1(3) = \{e, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31\}$,
$H_2(3) = \{e, 2, 5, 8, 12, 14, 17, 20, 23, 26, 29, 32\}$ and
$H_3(3) = \{e, 3, 6, 9, 13, 15, 18, 21, 24, 27, 30, 33\} \subseteq L_{33}(5)$ are of order 12.

Thus $H_i(3)$ are not p-Sylow subloop of $L_{33}(5)$.

$H_1(11) = \{e, 1, 12, 23\}$
$H_2(11) = \{e, 2, 13, 24\}$
$H_3(11) = \{e, 3, 14, 25\}$, …, and
$H_{11}(11) = \{e, 11, 22, 33\}$
$o(L_{33}(5)) = 34$.

$o(H_i(11)) = 4$; $4 \nmid 34$ so $L_{33}(5)$ does not contain any p-Sylow subloops.

$o(P_i) = 24 – 1 = 15$ where
P_i = {Collection of all subsets of the subloop $H_i(3)$} $\subseteq$ S.

$o(M_i) = 2^{12} – 1$ where
M_i = {Collection of all subsets of the subloop $H_i(11)$} $\subseteq$ S.

We see $L_{33}(5)$ has no p-Sylow subloops as well as S has no p-Sylow subset subloop subgrouoids.

Now we proceed onto study the concept of principal isotope of a subset loop groupoid of a loop $L_n(m) \in L_n$.

Suppose $L_n(m) \in L_n$ is a loop and $(L_n(m), o)$ be the principal isotope with respect $a, b \in (L_n(m), o)$ then if

S = {Collection of all subsets of the loop $L_n(m)$} be the subset loop groupoid of $L_n(m)$ then we denote by

$S_{PI}(a, b)$ = {collection of all subsets of $(L_n(m), o)$ the principal isotope of $L_n(m)$}.

We define $S_{PI}(a, b)$ to be the principal isotope subset loop groupoid of the principal isotope of the loop $L_n(m)$.

We will first illustrate this situation by some examples.

Example 3.73: Let
S = {Collection of all subsets of the loop $L_5(2)$} be the subset loop groupoid of the loop $L_5(2)$.

The loop of $L_5(2)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

Now we get the principal isotope of ($L_5(2)$, *) for a = e and b = 3 we get the ($L_5(2)$, o) which is as follows:

o	e	1	2	3	4	5
e	3	2	5	e	1	4
1	5	3	4	1	e	2
2	4	e	3	2	5	1
3	e	1	2	3	4	5
4	2	5	1	4	3	e
5	1	4	e	5	2	3

($L_5(2)$, o) is the principal isotope of ($L_5(2)$, *) with e′ = 3 as the identity and x * x = e′ = 3.

S_{IP} = {Collection of all subsets of the principal isotope ($L_5(2)$, o)} be the principal isotope subset loop groupoid of ($L_5(2)$, o).

Clearly we see S_{IP} is not a commutative principal isotope subset loop groupoid of ($L_5(2)$, o).

Example 3.74: Let
S = {Collection of all subsets of the loop {$L_5(4)$, *}} be the subset loop groupoid.

The following loop of $L_5(4)$ is

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	5	4	3	2
2	2	3	e	1	5	4
3	3	5	4	e	2	1
4	4	2	1	5	e	3
5	5	4	3	2	1	e

Take a = 1, b = e with e′ = 1. Let ($L_5(4)$, o) be principal isotope of ($L_5(4)$, o).

The table of ($L_5(4)$, o).

o	e	1	2	3	4	5
e	1	e	5	4	3	2
1	e	1	2	3	4	5
2	4	2	1	5	e	3
3	2	3	e	1	5	4
4	5	4	3	2	1	e
5	3	5	4	e	2	1

$S_{IP}(1,e)$ = {Collection of all subsets of the principal isotope loop ($L_5(4)$, o)} be the subset loop subgroupoid of the principal isotope.

We see $S_{IP}(1, e)$ is non commutative.

Example 3.75: Let
S = {Collection of all subsets of the loop ($L_5(3)$, *)} be the subset loop groupoid.

The table for $L_5(3)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	4	2	5	3
2	2	4	e	5	3	1
3	3	2	5	e	1	4
4	4	5	3	1	e	2
5	5	3	1	4	2	e

The principal isotope loop of ($L_5(3)$, *) be ($L_5(3)$, o) a = b = 5 with e′ = e identity.

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	4	5	2
2	2	3	e	5	1	4
3	3	4	5	e	2	1
4	4	5	1	2	e	3
5	5	2	4	1	3	e

Let S_{IP} (a, b / a = b = 5) = {Collection of all subsets of the principal isotope loop ($L_5(3)$, o)} be the principal isotope subset loop groupoid of ($L_5(3)$, o).

We see S_{IP} is also a commutative subset loop groupoid.

In view of this we have the theorem.

THEOREM 3.17: *Let $L_n(m) \in L_n$. Let the principal isotope of $(L_n(m), *)$ with respect to a, b $\in L_n(m)$ be $(L_n(m), o)$.*

Let S = {Collection of all subsets of the loop $(L_n(m)\}$ be the subset loop groupoid of $(L_n(m), *)$.

Let S_{IP} = {Collection of all subsets of the loop $(L_n(m), o)\}$ be the principal isotope subset loop groupoid of $(L_n(m), o)$. S_{IP} is a non commutative subset loop grouped in the following cases

(i) a = e and b ≠ e
(ii) a ≠ e and b = e.

The proof follows from the fact if in $(L_n(m), *)$; $(L_n(m), o)$ is non commutative if

(i) a = e and b ≠ e
(ii) a ≠ e and b = e.

We give yet another example.

Example 3.76: Consider the loop $L_5(3)$ given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	4	2	5	3
2	2	4	e	5	3	1
3	3	2	5	e	1	4
4	4	5	3	1	e	2
5	5	3	1	4	2	e

Take a = 4, b = e the principal isotope $(L_5(3), o)$ composition table is as follows:

o	e	1	2	3	4	5
e	4	5	3	1	e	2
1	3	2	5	e	1	4
2	5	3	1	4	2	e
3	2	4	e	5	3	1
4	e	1	2	3	4	5
5	1	e	4	2	5	3

We see ($L_5(3)$, o) is strictly commutative. However ($L_5(3)$, *) is a commutative loop.

So S = {Collection of all subsets of the loop ($L_5(3)$, *)} is a subset loop groupoid which is commutative. But $S_{IP}(4, e)$ = {Collection of all subsets of the principal isotope ($L_5(3)$, o)} the principal isotope subset loop groupoid is strictly non commutative.

However we show by an example even the loop $L_n(m)$ is strictly non commutative yet ($L_n(m)$, o) need not be strictly non commutative.

Example 3.77: Let ($L_5(2)$, *) be the loop whose composition table is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

($L_5(2)$, *) is a strictly non commutative loop. Take a = 2 and b = 3.

The principal isotope ($L_5(2)$, o)'s composition table is as follows:

o	e	1	2	3	4	5
e	4	e	3	2	5	1
1	e	1	2	3	4	5
2	3	2	5	e	1	4
3	5	3	4	1	e	2
4	1	4	e	5	2	3
5	2	5	1	4	3	e

Clearly ($L_5(2)$, o) is not strictly non commutative as
$4 \text{ o } 5 \neq 5 \text{ o } 4$.

Thus we see
S = {Collection of all subsets of the loop ($L_5(2)$, *)} the subset loop groupoid of ($L_5(2)$, *) and

$S_{PI}(2, 3)$ = {Collection of all subsets of the loop ($L_5(2)$, o)} the subset loop groupoid of ($L_5(2)$, o) is not strictly non commutative where as S is strictly non commutative.

Here we suggest problems for this chapter.

Problems:

1. Give some special features enjoyed by subset loop groupoids.

2. Does there exist a subset loop groupoid of infinite order?

3. Let S = {Collection of all subsets of the loop $L_{17}(5)$} be the subset loop groupoid.

   (i) Find o(S).
   (ii) Can S have Smarandache subset loop subgroupoids?

4. Does there exist a commutative subset loop groupoid of infinite order?

5. Does there exist a subset loop groupoid of infinite order which is inner commutative?

6. Does there exist a subset loop groupoid of infinite order which is strictly non commutative?

7. Does there exist a subset loop groupoid of infinite order which is not commutative but not strictly non commutative?

8. Does there exists a Bol subset loop groupoid of infinite order?

9. Does there exist a Burck subset loop groupoid of infinite order?

10. Give an example of a Bol subset loop groupoid.

11. Give an example of a Burck subset loop groupoid.

12. Give an example of a strictly non left alternative subset loop groupoid of infinite order.

13. Does there exist a Moufang loop of infinite order?

14. Give some examples of Moufang loops of infinite order.

15. Does there exist a Moufang subset loop groupoid of infinite order?

16. Give an example of a Moufang subset loop groupoid of finite order.

17. Give some special features enjoyed by Moufang subset loop groupoids.

18. Does there exist a subset loop groupoid S of the loop L which is Moufang?
(That is every A, B, C $\in$ S satisfies the Moufang).

19. Does there exist a subset loop groupoid, S of the loop L which is Bol?

20. Does there exist a subset loop groupoid S of a loop which is subset right alternative but not subset left alternative?

21. Let S be a subset loop groupoid of a loop L. Can S be subset left alternative but not subset right alternative?

22. Can S be a subset loop groupoid of a loop L such that S is a weak inverse property subset loop groupoid?

23. Does there exist a subset loop groupoid S of the loop L so that S is a P-subset loop groupoid?

24. Let S_n = {Collection of all subsets of loop $L_n(m) \in L_n$} be a subset loop subgroupoid of the loop $L_n(m)$.
Find the subset loop groupoid which is commutative.

25. Obtain some special features enjoyed by the S_{PI} of any loop $(L_n(m), *)$.

26. Compare S and S_{PI} for the loop $L_{43}(8)$, e = a and b = 42.

27. Compare S and S_{PI} for the loop $L_{29}(3)$, a = 10 and b = 17.

28. Compare S and S_{PI} for the loop $L_{31}(8)$ for a = e and b = 8.

29. Compare S and S_{PI} for the loop $L_{27}(8)$ for a = b = 8.

30. Characterize those S_{PI} loops which are commutative.

31. Characterize those S_{PI} loops which are strictly non commutative.

32. Characetrize those S_{PI} loops which satisfy Lagrange theorem for finite groups.

33. When will both S and S_{PI} be commutative?

34. When will both S and S_{PI} be strictly non commutative?

35. Characterize those loops $L_n(m) \in L_n$ which are satisfy Lagrange theorem for finite groups.

36. Characterize those loops $L_n(m) \in L_m$ such that none of the subloops of $L_n(m)$ divide the order of $L_n(m)$.

37. Characterize those loop $L_n(m) \in L_n$ which have some subloops $H_i(t)$ such that $o(H_i(t)) \not| \, o(L_n(m))$ and some subloops $H_j(s)$ such that $o(H_j(s)) \,/\, o(L_n(m))$.

38. Study the above three questions in case of subset loop groupoids of these three types of loops.

    (i) $L_n(m)$ left alternative
    (ii) $L_n(m)$ right alternative
    (iii) $L_n(m)$ alternative.

39. Does there exist a loop L which is alternative so that every pair of elements in
S = {Collection of all subsets of L} the subset loop groupoid is alternative?

40. Let S = {Collection of all subsets of the loop $L_{49}(3)$} be the subset loop groupoid of the loop $L_{49}(3)$.

    (i) Find all subset subloop subgroupoids of S.
    (ii) How many of these subset subloop subgroupoids satisfy the Lagrange theorem for finite groups?
    (iii) Can S have p-Sylow subset subloop subgroupoids?

(iv) Find $H_i(7)$ of $L_{49}(3)$.
(v) Let P_i = {Collection of all subsets of $H_i(7) \subseteq L_{49}(3)$}, $1 \leq i \leq 7$. Study the special properties enjoyed by P_i.
(vi) Does P_i satisfy any of the special identities as subset subloop subgroupoid of S, $1 \leq i \leq 7$?

41. Let S = {Collection of all subsets of the loop $L_{65}(12)$} be the subset loop groupoid of the loop $L_{65}(12)$. Let $H_i(13)$ and $H_j(5)$; $1 \leq i \leq 13$ and $1 \leq j \leq 4$ be subloops.

Study the special associated properties of the subset subloop subgroupoids of the subloops $H_i(13)$ and $H_j(5)$, $1 \leq i \leq 13$ and $1 \leq j \leq 5$.

Can S have more than 24 subset subloop subgroupoids. Justify your claim.

42. Let S_n = {Collection of all subsets loop groupoids of the loops $L_{105}(m) \in L_{105}$} be the subset loop groupoids.

(i) Find the number of subset loop groupoids in S_n.
(ii) How many of S in S_n satisfy Lagrange theorem for finite groups?
(iii) How many of S in S_n have p-Sylow subset subloop subgroupoids?
(iv) Find any other special properties enjoyed by S in S_n.

43. Let S_{43} = {collection of all subset loop groupoids of the loops $L_{43}(m) \in L_{43}$} be the subset loop groupoids.
Study questions (i) to (iv) of problem 42 for this S_{43}.

44. Let S_{19} = {Collection of all subset loop groupoids of $L_{19}(m) \in L_{19}$}.

(i) Compare S_{43} and S_{19}
(ii) Study all questions (i) to (iv) of problem 42 for this S_{19}.
(iii) Compare S_n in problem 42 and this S_{19}.

45. Let
S_{27} = {Collection of all subset loop groupoids of the loop $L_{27}(m) \in L_{27}$}.

(i) Study questions (i) to (iv) of problem 42 for this S_{27}.

(ii) Compare S_{27} with S_{19} of problem 44 and S_{43} of problem 43.

46. Let S_{57} = {Class of subset loop groupoids of $L_{57}(m) \in L_{57}$}.

(i) Compare S_{57} with S_{27} of problem 45.

(ii) Do they enjoy any common features?

(iii) Study questions (i) to (iv) of problem 42 for this S_{57}.

47. Let S_{147} = {collection of all subset loop groupoids of the loop $L_{147}(m) \in L_{147}$}.

(i) Study questions (i) to (iv) of problem (42).
(ii) Compare S_{147} with S_7, S_{29}, S_9 and S_{21}.

48. Let S_{99} = {Collection of all subset loop groupoids of $L_{99}(m) \in L_{99}$}.

(i) What are the special identities satisfied by the subset loop groupoids $S \in S_{99}$?
(ii) Compare S_{99} with S_{11}, S_9, S_{13} and S_3.

49. Let S_{1479} = {collection of all subset loop groupoids of $L_{1479}(m) \in L_{1479}$}.
(i) How many subset loop groupoid in S_{1479} satisfy Lagrange theorem for finite groupoid?
(ii) Find $o(S_{1479})$.
(iii) Study questions (i) to (iv) of problem 42 for S_{1479}.
(iv) Compare S_{1479} with S_7, S_{15}, S_9 and S_{147}.

50. Find any special features about S_n.
    (i) n prime.
    (ii) n odd.

51. Can S be of infinite order with the associated loop to be of infinite order satisfy any of the special identities?

FURTHER READING

1. Birkhoff. G, *Lattice theory*, 2nd Edition, Amer-Math Soc. Providence RI 1948.

2. Smarandache. F. (editor), *Proceedings of the First International Conference on Neutrosophy*, Neutrosophic Probability and Statistics, Univ. of New Mexico-Gallup, 2001.

3. Vasantha Kandasamy, W.B., *Smarandache Semigroups*, American Research Press, Rehoboth, 2002.

4. Vasantha Kandasamy, W.B., *Groupoids and Smarandache Groupoids*, American Research Press, Rehoboth, 2002.

5. Vasantha Kandasamy, W.B., *Smarandache Loops*, American Research Press, Rehoboth, 2002.

6. Vasantha Kandasamy, W.B., *Smarandache Rings*, American Research Press, Rehoboth, 2002.

7. Vasantha Kandasamy, W.B., *Smarandache Semirings, Semifields and Semivector spaces*, American Research Press, Rehoboth, 2002.

8. Vasantha Kandasamy, W.B., *Linear Algebra and Smarandache Linear Algebra*, Bookman Publishing, US, 2003.

9. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Neutrosophic Rings*, Hexis, Arizona, 2006.

10. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Set Linear Algebra and Set Fuzzy Linear Algebra*, InfoLearnQuest, Phoenix, 2008.

11. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Finite Neutrosophic Complex Numbers*, Zip Publishing, Ohio, 2011.

12. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Dual Numbers*, Zip Publishing, Ohio, 2012.

13. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Special dual like numbers and lattices*, Zip Publishing, Ohio, 2012.

14. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Special quasi dual numbers and groupoids*, Zip Publishing, Ohio, 2012.

15. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Natural Product* $\times_n$ *on matrices*, Zip Publishing, Ohio, 2012.

16. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Set Ideal Topological Spaces*, Zip Publishing, Ohio, 2012.

17. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Quasi Set Topological Vector Subspaces*, Educational Publisher Inc, Ohio, 2012.

18. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Algebraic Structures using Subsets*, Educational Publisher Inc, Ohio, 2013.

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Dr.W.B.Vasantha Kandasamy is an Associate Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 646 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 83rd book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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The authors in this book construct a large class of finite subset groupoids as well as a large class of infinite subset groupoids. Here the conditions under which these subset groupoids satisfy special identities like Bol identity, Moufang identity, right alternative identity and so on are found. In fact it is an open problem to find subset groupoids to satisfy special identities even if the groupoid over which they are defined do not satisfy any of the special identities.

ISBN 9781599732220

90000 >

9 781599 732220

US $35.00

Educational Publisher Inc.