SUBSET INTERVAL GROUPOIDS

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PREFACE

The study of groupoids is meager and we have recently introduced the new notion of subset groupoids and have studied them. It is interesting to keep on record that interval groupoids have been studied by us in 2010.

Further when the subsets of a loop are taken they also form only a subset groupoid and not a subset loop. Thus we do not have the concept of subset interval loop they only form a subset interval groupoid.

Special elements like subset interval zero divisors, subset interval idempotents and subset interval units are studied. Concept of subset interval groupoid homomorphism is developed.

It is interesting to note several nice properties are developed in this book on subset interval groupoids nearly 12 theorems are given about these subset interval groupoids to satisfy special identities.

An interesting feature is we can have interval subset groupoids to enjoy special Smarandache properties. Matrix subset interval groupoids and polynomial interval subset groupoids are developed and described in this book. Interval matrix subset groupoids using a special type of loops is interesting for the operation on them is only natural product $\times_n$

and every singleton set is such that their product is identity matrix.

Subset matrix interval groupoids S using the loops $L_n(m)$ has no S-Cauchy elements.

Over 10 theorems about these subset interval matrix groupoids using these special types of loops and subset interval polynomial groupoids using these loops are developed and described.

On these subset interval groupoids two non associative topological spaces are built this is the first time we develop a new type of non associative topological spaces

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Chapter One

INTRODUCTION

Here we give basic concepts and introduction about subset non associative structures. We have discussed and developed about subset algebraic structures which are associative in [39]. We have also studied algebraic structures of subset polynomials and subset matrices in [41].

Here we mainly study the subset groupoids and subset loop groupoids using the subsets of groupoids and loops respectively. The operation on the subset collection is the operation of the groupoid or loop. We see these subsets of a loop is only a groupoid and not a loop. However we see the loop L is a subset in the subsets of a loop L.

We study the properties of these new algebraic structures, like conditions on them to satisfy special identities and so on. We also obtain the condition under which they are Smarandache.

We first recall the definition of a groupoid and give examples of various types of groupoids.

DEFINITION 1.1: *Let G be a non empty set with a binary operation * defined on G. That is for all a, b $\in$ G;*

*a * b $\in$ G and * in general is non associative on G. We define (G, *) to be a groupoid.*

We may have groupoids of infinite or finite order.

Example 1.1: Let G = $\{a_0, a_1, a_2, a_3, a_4\}$ be the groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4
a_0	a_0	a_4	a_3	a_2	a_1
a_1	a_1	a_0	a_4	a_3	a_2
a_2	a_2	a_1	a_0	a_4	a_3
a_3	a_3	a_2	a_1	a_0	a_4
a_4	a_4	a_3	a_2	a_1	a_0

Example 1.2: Let (G, *) be the groupoid given by the following table:

*	a_1	a_2	a_3
a_1	a_1	a_3	a_2
a_2	a_2	a_1	a_3
a_3	a_3	a_2	a_1

Example 1.3: Let (G, *) be the groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
a_0	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8
a_1	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9
a_2	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0
a_3	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1
a_4	a_4	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2
a_5	a_5	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3
a_6	a_6	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4
a_7	a_7	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5
a_8	a_8	a_0	a_2	a_4	a_6	a_8	a_0	a_2	a_4	a_6
a_9	a_9	a_1	a_3	a_5	a_7	a_9	a_1	a_3	a_5	a_7

Clearly (G, *) is a grouped of order 10.

Example 1.4: Let G = (Z, *, (3, –1)) be a groupoid. If a, b $\in$ Z; a * b = 3a + b(–1) = 3a – b; that is if 8, 0 $\in$ Z then
8 * 0 = 3 × 8 + 0 (–1) = 24.

For 5, 10 $\in$ Z, 5 * 10 = 3 × 5 – 10 * 1 = 15 – 10 = 5.

Clearly G is an infinite groupoid. That is o(G) = ∞.

Example 1.5: Let (Q, *, (7/3, 2)) = G be a groupoid. o(G) = ∞.
Let x = 3 and y = –2 $\in$ Q then
3 * (–2) = 3 × 7/3 – 2 × 2 = 7 – 4 = 3.

Example 1.6: Let G = (R, *, ($\sqrt{3}$, 0)) be a groupoid. o(G) = ∞.

Let x = 8, y = $-5\sqrt{2} \in$ R.

$$x * y = 8 * (-5\sqrt{2}) = 8 \times \sqrt{3} - 0 \times (-5\sqrt{2})$$
$$= 8\sqrt{3} \in R.$$

Clearly o(G) = ∞.

Example 1.7: Let G = {C, *, (3–i, 4 + 5i)} be a groupoid of infinite order.

$a * b = a (3 – i) + b (4 + 5i)$ for $a, b \in C$.

Take $a = –3i$ and $b = 2 + i \in C$.

$$\begin{aligned} a * b &= –3i \times (3–i) + 2 + i) (4 + 5i) \\ &= –9i – 3 + 8 + 4i + 10i – 5 \\ &= 5i \in C. \end{aligned}$$

$o(G) = \infty$ so G is of infinite order.

Finally for more about groupoids please refer [18, 20-1, 23 and 40]. For the notion of loops refer [24]. For interval matrices and interval polynomials refer [28]. For the concept of interval groupoids refer the book [31].

Finally we also give the topology on these collections in a very special way. To every subset groupoid S we have two topologies defined on S.

Thus, we have discussed and developed these new type of topologies, which are basically non associative and also non commutative in general. This study is very innovative. For to the best of our knowledge here we get examples of non associative topologies with finite or infinite number of elements in them. These topologies are unique in their nature and may find lot of applications. For more about the topologies of these types refer [35-6].

We define the notion of Smarandache topological spaces as well as those special type of Smarandache topologies which satisfy special identities; this is also analysed in this book.

Chapter Two

SUBSET INTERVAL GROUPOIDS

In this chapter we for the first time introduce the notion of subset interval groupoids of both finite and infinite order. We describe develop and define these concepts. We give the necessary and sufficient condition for a Smarandache subset interval groupoid to be idempotent.

The notion of groupoids and subset groupoids are described in [18, 20-1, 23, 31 and 40].

DEFINITION 2.1: *Let (G, *) be an interval groupoid where (G, *) = {[a, b] | [a, b] $\in$ G with a and b elements from some groupoid}. Take $S_G = \{\{[a_1, b_1], \ldots, [a_t, b_t]\} \mid [a_i, b_i] \in G, 1 \leq i \leq t\}$ to be the collection of subsets from the interval groupoid G. We define (S_G, *) to be the subset interval groupoid of the interval groupoid G.*

We will illustrate this situation by some examples.

Example 2.1: Let $S_G = \{\{[a_1, b_1], \ldots, [a_t, b_t]\} \mid [a_i, b_i] \in G = [Z_9, Z_9]$, where on Z_9; * is the defined operation using $(3, 2)\}; 1 \leq i \leq t$.

Let

$$A = \{[3, 2], [4, 0], [2, 5], [4, 5]\} \text{ and}$$
$$B = \{[0, 1], [3, 0], [7, 1]\} \in S_G.$$

A * B

$= \{[3, 2], [4, 0], [2, 5], [4, 5]\} * \{[0, 1], [3, 0], [7, 1]\}$

$= \{[3, 2] * [0, 1], [4, 0] * [0, 1], [2, 5] * [0, 1],$
$[4, 5] * [0, 1], [3, 2] * [3, 0], [4, 0] * [3, 0],$
$[2, 5] * [3, 0], [4, 5] * [3, 0], [3, 2] * [7, 1],$
$[4, 0] * [7, 1], [2, 5] * [7, 1], [4, 5] * [7, 1]\}$

$= \{[9, 6 + 4], [12, 2], \ldots, [12 + 14, 15+2]\}$

$= \{[0, 1], [3, 2], \ldots, [8,8]\}.$

This is the way '*' operation on S_G is performed.

We see * on S_G is non associative.

Let $A = \{[3, 0]\}$, $B = \{[8, 1]\}$ and $C = \{[2, 1]\} \in S_G$.

$$\begin{aligned}(A * B) * C &= (\{[3, 0]\} * \{[8, 1]\}) * C \\ &= \{[3 * 8, 0 * 1]\} * C \\ &= \{[9 + 16, 2]\} * [2, 1] \\ &= \{[7, 2]\} * \{[2, 1]\} \\ &= \{[21 + 4, 6 + 2]\} \\ &= \{[7, 8]\} \in S_G.\end{aligned}$$

Thus
$(A * B) * C = \{[7, 8]\}$ … I

$$\begin{aligned}A * (B * C) &= A * (\{[8, 1]\} * \{[2, 1]\}) \\ &= A * \{[1, 5]\} \\ &= \{[3, 0]\} * \{[1, 5]\} \\ &= \{[9 + 2, 10]\} \\ &= \{[2, 1]\} \quad \ldots \quad \text{II}\end{aligned}$$

We see clearly I and II are distinct, hence $(S_G, *)$ is non associative.

We check for the commutativity of S_G.

Consider A = {[8, 0]} and B = {[2, 5]} in S_G.

$$\begin{aligned} A * B &= \{[8, 0]\} * \{[2, 5]\} \\ &= \{[24 + 4, 10]\} \\ &= \{[1, 1]\} \qquad \ldots \qquad \text{I} \end{aligned}$$

$$\begin{aligned} B * A &= \{[2, 5]\} * \{[8, 0]\} \\ &= \{[6 + 16, 15]\} \\ &= \{[4, 6]\} \qquad \ldots \qquad \text{II} \end{aligned}$$

Clearly I and II are distinct so S_G is non commutative.

Example 2.2: Let

$S_G = \{\{[a_1, b_1], \ldots, [a_n, b_n]\} \mid [a_i, b_i] \in \{G, *, (2, 0)\}$ where $G = Z_7\}$ be subset interval groupoid.

It is easily verified that S_G is both non associative and non commutative.

Take A = {[3, 2], [0, 6]} and B = {[0, 4], [2, 1]} $\in S_G$.

$$\begin{aligned} A * B &= \{[3, 2], [0, 6]\} * \{[0, 4], [2, 1]\} \\ &= \{[3, 2] * [0, 4], [0, 6] * [0, 4], [3, 2] * [2, 1], \\ &\quad [0, 6] * [2, 1]\} \\ &= \{[6, 4], [6, 4], [0, 0], [0, 0]\} \\ &= \{[6, 4], [0, 0]\} \qquad \ldots \qquad \text{I} \end{aligned}$$

Consider

$$\begin{aligned} B * A &= \{[0, 4], [2, 1]\} * \{[3, 0], [0, 6]\} \\ &= \{[0, 4] * [3, 0], [2, 1] * [3, 0], \\ &\quad [0, 4] * [0, 6], [2, 1] * [0, 6]\} \\ &= \{[0, 1], [4, 2]\} \qquad \ldots \qquad \text{II} \end{aligned}$$

We see A * B ≠ B * A as I and II are distinct.

Example 2.3: Let S_G = {A | A is subset of the collection of intervals from the interval groupoid G = {[a, b] | a, b ∈ Z_{12}, *, (3, 4)}} be the subset interval groupoid of the interval groupoid G.

Take A = {[2, 8], [4, 6], [8, 0]}
and B = {[0, 2], [4, 0]} ∈ S_G.

We find

A * B = {[2, 8], [4, 6], [8, 0]} * {[0, 2], [4, 0]}
= {[2, 8] * [0, 2], [4, 6] * [0, 2], [8, 0] * [0, 2], [2, 8] * [4, 0], [4, 6] * [4, 0], [8, 0] * [4, 0]}
= {[6, 24 + 8], [6 + 16, 24], [12, 18 + 8], [12 + 16, 18], [24, 8], [24 + 16, 0]}
= {[6, 8], [10, 0], [0, 2], [4, 6], [0, 8], [4, 0]} … I

B * A = {[0, 2], [4, 0]} * {[2, 8], [4, 6], [8, 0]}
= {[0, 2] * [2, 8], [4, 0] * [2, 8], [0, 2] * [4, 6], [4, 0] * [4, 6], [0, 2] * [8, 0], [4, 0] * [8, 0]}
= {[8, 6+32], [0 + 16, 6 + 24], [0, 32, 6] [12 + 8, 32], [12 + 16, 24], [12 + 32, 0]}
= {[8, 2], [4, 6], [8, 6], [8, 8] , [4, 0], [8, 0]} … II

We see I and II are distinct thus S_G is a non commutative and non associative subset interval groupoid.

Example 2.4: Let S_G = {A | A is the collection of all interval subsets of the interval groupoid of the groupoid Z_6, *, (3, 3)} be the interval subset groupoid. It is easily verified SG is commutative but non associative.

Take A = {[0, 5], [5, 0]} and B = {[2, 4], [4, 3]} ∈ S,

A * B = {[0, 5] * [2, 4], [5, 0] * [2, 4], [0, 5] * [4, 3], [5, 0] * [4, 3]}
= {[6, 15+12], [15+6, 12], [12, 15+9], [15+12, 9]}
= {[0, 3], [3, 0], [0, 0], [3, 3]} … I

$$\begin{aligned} B * A &= \{[2, 4], [4, 3]\} * \{[0, 5], [5, 0]\} \\ &= \{[2, 4] * [0, 5], [4, 3] * [0, 5], [2, 4] * [5, 0], \\ &\quad [4, 3] * [5, 0]\} \\ &= \{[6, 12 + 15], [6+15, 12], [12, 9+15], \\ &\quad [12 + 15, 9]\} \\ &= \{[0, 3], [3, 0], [0, 0], [3, 3]\} \quad \ldots \quad \text{II} \end{aligned}$$

Clearly I and II are identical, thus A * B = B * A.

Now

A = {[4, 2], [3, 1]}, B = {[4, 5]} and C = {[0, 4]} $\in S_G$.

Consider

$$\begin{aligned} A * (B * C) &= A * (\{[4, 5] * [0, 4]\} \\ &= A * (\{12, 15 + 12]\}) \\ &= \{[4, 2], [3, 1]\} * \{[0, 3]\} \\ &= \{12 + 0, 6 + 9], [9, 3 + 9]\} \\ &= \{[0, 3], [3, 3]\} \quad \ldots \quad \text{I} \end{aligned}$$

We now find

$$\begin{aligned} (A * B) * C &= (\{[4, 2], [3, 1]\} * \{[4, 5]\}) * C \\ &= \{[4, 2] * [4, 5], [3, 1] * [4, 5]\} * C \\ &= \{[12 + 12, 6 + 15], [9 + 12, 3 + 15]\} * C \\ &= \{[0, 3] , [3, 0]\} * C \\ &= \{[0, 3] * [0, 4], [3, 0] * [0, 4]\} \\ &= \{0, 9+12], [9, 12]\} \\ &= \{[0, 3], [3, 0]\} \quad \ldots \quad \text{II} \end{aligned}$$

Clearly I and II are distinct and we see S_G is only a commutative subset interval groupoid but S_G is a non associative subset interval groupoid of {Z, *, (3, 3)}.

In view of all these we can get a class of commutative non associative subset interval groupoids.

It is pertinent at this juncture to keep on record that all the examples given are only of finite order. We say S_G is of finite order if the number of interval subsets in S_G has only finite number of elements.

THEOREM 2.1: *Let S_G be the interval subset groupoid where intervals are taken from $G = \{Z_n, *, (t, t)\}$ $(1 < t < n)$. Clearly S_G is commutative but non associative subset interval groupoid of finite order.*

The proof is direct and hence left as an exercise to the reader.

We now give examples of subset interval groupoids of infinite order.

Example 2.5: Let S_G = {Collection of all subsets of intervals of the interval groupoid built using the groupoid $\{Z, *, (3, 7)\}$}.

S_G is a non associative non commutative interval subset groupoid of a interval groupoid of infinite order.

Example 2.6: Let S_G = {Collection of all subsets of the interval groupoid $G = (R, *, (8, -3))$} be the subset interval groupoid of the interval groupoid G.

Clearly S_G is of infinite order which is also non commutative.

Take $A = \{[0, 4], [0, 2], [-1, 0]\}$,
$B = \{[4, 6], [0, 1]\}$ and $C = \{[5, 6]\}$ in S_G.

Consider

$$\begin{aligned}
A * (B * C) &= A * (\{[4, 6], [0, 1]\} * \{[5, 6]\}) \\
&= A * \{[4, 6] * [5, 6], [0, 1] * [5, 6]\} \\
&= A * \{[32 - 15, 48 - 18], [-15, 8-18]\} \\
&= \{[0, 4], [0, 2], [-1, 0]\} * \{[17, 30], \\
&\quad [-15, -10]\}
\end{aligned}$$

$= \{[0, 4] * [17, 30], [0, 2] * [17, 30],$
$[-1, 0] * [17, 30], [0, 2] * [-15, -10],$
$[0, 4] * [-15, -10], [-1, 0] * [-15, -10]\}$

$= \{[-51, 32-90], [-51, 16-90], [-8-51, -150],$
$[45, 16 + 30], [45, 32 + 30], [-8 + 45, 30]\}$

$= [-51, -58], [-51, -74], [-59, -150], [45, 46],$
$[45, 62], [37, 30]\}$ … I

$(A * B) * C = \{[0, 4], [0, 2], [-1, 0]\} * \{[4, 6], [0, 1]\}) * C$

$= \{[0, 4] * [4, 6], [0, 2] * [4, 6], [-1, 0] * [4, 6],$
$[0, 4] * [0, 1], [0, 2] * [0, 1], [-1, 0] * [0, 1]\}$
$* C$

$= \{[-12, 32-18], [0, 32-3], [32, 16-18],$
$[0, 16-3], [-8-12, -18], [-8, -3]\} * C$

$= \{[-12, 14], [0, 29], [32, -2], [0, 13],$
$[-20, -18], [-8, -3]\} * C$

$= \{[-12, 14] * [5, 6], [0, 29] * [5, 6],$
$[32, -2] * [5, 6], [0, 13] * [5, 6],$
$[-20, -18] * [5, 6], [-8, -3] * [5, 6]\}$

$= \{[-96-15, 112-18], [-15, 212-18],$
$[256 -15, -16-18], [-15, 104-18],$
$[-160-15, -144 -18], [-64-15, -24-18]\}$

$= \{[-111, 94], [-15, 194], [241, -34], [-15, 86],$
$[-175, -152], [-79, -42]\}$ … II

Clearly I and II are distinct hence S_G is a non associative subset interval groupoid of infinite order which is also non commutative.

Example 2.7: Let S_G = {Collection of all subsets of intervals of the interval groupoid G = {[a, b] | a, b $\in$ R, *, (7, 0)}} be the subset interval groupoid of infinite order.

We see S_G is non associative as well as non commutative.

Take A = {[3, 2], [7, 1]} and B = {[1, 1], [0, 2]} $\in$ S.

A * B = {[3, 2], [7, 1]} * {[1, 1], [0, 2]}
= {[3, 2] * [1, 1], [7, 1] * [1, 1], [3, 2] * [0, 2], [7, 1] * [0, 2]}
= {[21, 14], [49, 7], [21, 14], [49, 7]}
= {[21, 14], [49, 7]} … I

B * A = {[1, 1], [0, 2]} * {[3, 2], [7, 1]}
= {[1, 1] * [3, 2], [1, 1] * [7, 1], [0, 2] * [3, 2], [0, 2] * [7, 1]}
= {[7, 7], [0, 14]} … II

I and II are not identical so A * B $\neq$ B * A.

Example 2.8: Let S_G = {Collection of all subsets of the interval groupoid of the groupoid, G where a, b $\in$ G = {$Q^+ \cup$ {0}, *, (0, 9)}} be the subset interval groupoid.

S_G is of infinite order and S_G is an infinite non commutative subset interval groupoid.

Example 2.9: Let G = {$Z^+ \cup$ {0}, *, (4, 4)} be an infinite commutative groupoid.

S = {Collection of all intervals [a, b] of G where a, b $\in$ G}, S is an interval groupoid.

S_G = {Collection of all subsets of the interval groupoid S}. S_G is the subset interval groupoid of infinite order.

S_G is commutative.

In view of this we have the following theorem.

THEOREM 2.2: *Let S_G be the subsets of the interval groupoid $S = \{[a, b] \mid a, b \in Z$ or Q or R or $Z^+ \cup \{0\}$; $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or C; $*$, $(t, t)\}$. S_G is commutative and of infinite order.*

The proof is obvious from the very fact for if A = {[m, n]} and B = {[s, r]} $\in S_G$, then

$$\begin{aligned} A * B &= \{[m, n]\} * \{[s, r]\} \\ &= \{[m * s, n * r]\} \\ &= \{[tm + ts, tn + tn]\} \quad \ldots \quad \text{I} \end{aligned}$$

$$\begin{aligned} B * A &= \{[s, r]\} * \{[m, n]\} \\ &= \{[s * m, r * n]\} \\ &= [ts + tm, tr + tn]\} \quad \ldots \quad \text{II} \end{aligned}$$

Thus A * B = B * A. Hence SG is a commutative subset interval groupoid.

Corollary 2.1: In the above theorem if (t, t) is replaced by (1, 1), we see S_G is an associative interval subset groupoid of infinite order.

Example 2.10: Let S_G = {Collection of all interval subsets of the groupoid G = {Z_8 (g), *, (8, g) where $g^2 = 0$}} be the interval subset groupoid which is both non associative and non commutative of finite order.

Example 2.11: Let S_G = {Collection of all interval subsets of the interval groupoid of the groupoid; G = {$C(Z_{12})$, *, (6, $3i_F$)}}. We see S_G is of finite order and is a finite complex subset interval groupoid.

Next we proceed onto give examples of subset interval groupoid of dual numbers and finite complex modulo integers.

Example 2.12: Let G = {Z_8, (g, g_1), *, (3, $4g_1$) where $g_1^2 = 0$ and $g^2 = g$, $g_1g = 0 = gg_1$} be the mixed dual number groupoid. S = {[a, b] | a, b $\in$ G}, (S, *) is the interval groupoid of G.

Let S_G = {Collection of all subsets of the interval groupoid S}; S_G is a finite subset interval mixed dual number groupoid which is non commutative.

Example 2.13: Let G = $\{C(Z_{17})\ (g) \mid g^2 = -g, *, (8, g)\}$ be the finite complex modulo integer special quasi dual numbers groupoid. S = $\{\{[a, b]\} \mid a, b \in G, *, (8, g)\}$ be the finite complex modulo integer interval groupoid.

We take

S_G = {Collection of all subsets from the interval groupoid S}; S_G is a subset interval groupoid of finite order of complex modulo integer interval groupoid.

Example 2.14: Let S_G = {Collection of all subset intervals from the interval groupoid S = $\{\{[a, b] \mid a, b \in G = \{C(Z_{15})\ (g, g_1, g_2) \mid g^2 = 0,\ g_1^2 = g_1$ and $g_2^2 = -g_2$ with $i_F^2 = 14$, $gg_i = g_ig = 0 = g_ig_j = g_jg_i$; $1 \leq i, j \leq 2, *, (3, 0)\}\}$ be the subset interval groupoid of finite order which is both non associative and non commutative.

Thus having seen examples of subset interval groupoids of finite and infinite order of all types; we now proceed onto define special properties related to substructures and concept of subset zero divisors, subset idempotents and subset nilpotents in S_G.

However the definition is a matter of routine so we only illustrate these concepts by appropriate examples.

Example 2.15: Let S_G = {Collection of all interval subsets of the interval groupoid S where S = $\{[a, b] \mid a, b \in G = \{Z_{12}, *, (4, 6)\}\}$ be the interval subset groupoid of S.

We now give subset interval zero divisors and subset interval idempotents in S_G.

Take A = {[3, 2], [6, 4]} and B = {[0, 4], [3, 8]} $\in S_G$.

$$\begin{aligned} A * B &= \{[3, 2], [6, 4]\} * \{[0, 4], [3, 8]\} \\ &= \{[3, 2] * [0, 4], [6, 4] * [0, 4]\}\ \{[3, 2] * [3, 8], \\ &\quad [6, 4] * [3, 8]\} \\ &= \{[0, 0]\}. \end{aligned}$$

Thus we see A, B $\in S_G$ are subset zero divisors. In this case B * A = {[0, 0]}. However it is often natural to get in S_G subset intervals A, B $\in S_G$ with A * B = {[0, 0]} but B * A $\neq$ {[0, 0]}.

Now S_G has also non trivial subset idempotents for take A = {[4, 0]} $\in S_G$, we see A * A = {[4, 0]} = A. Thus S_G has non trivial subset idempotents.

Let S_G = {Collection of all interval subsets of the interval groupoid S = {[a, b] | a, b $\in$ G = {Z, *, (5, –5)}} be the subset interval groupoid. We take A = {[3, 3]} $\in S_G$.

$$\begin{aligned} A * A &= \{[3, 3]\} * \{[3, 3]\} \\ &= \{[3, 3]\} * \{[3, 3]\} \\ &= \{[15–15, 15–15]\} \\ &= \{[0, 0]\}. \end{aligned}$$

Thus A is a subset nilpotent element of order two. Infact S_G has infinite number of subset nilpotent elements of order two.

Let P = {[a, a] | a $\in$ Z} $\subseteq S_G$ be the collection of all subset nilpotents of order two.

We will proceed to give examples in case of building groupoids of subset intervals using Z_n.

Example 2.16: Let S = {Collection of all intervals of the form [a, b] where a, b $\in$ G = {Z_{15}, *, (8, 7)}} be the interval groupoid of finite order.

Take S_G = {Collection of all subsets of the set S}, S_G is the interval subset groupoid.

We see all subsets A = {[s, s]} where s $\in Z_{15}$ in S_G are subset nilpotents of order two.

$$\begin{aligned} A * A &= \{[s, s]\} * \{[s, s]\} \\ &= \{[8s + 7s, 8s + 7s]\} \\ &= \{[0, 0]\}, \text{ hence the claim.} \end{aligned}$$

Thus all subsets $B = \{[a, a]\} \mid a \in Z_{15}\} \subseteq S_G$ are subset nilpotent elements collection.

Infact B has a substructure only a subset interval subgroupoid of S_G.

THEOREM 2.3: *Let S_G = {Collection of all subset intervals from S where $S = \{[a, b] \mid a, b \in G = \{Z_n, *, (t, s); t, s \in Z_n \setminus \{n, 0\}\}\}$ be the interval subset groupoid.*

Let $P = \{A = \{[a, a]\} / a \in Z_n\} \subseteq S_G$. Every $x \in P$ is such that $x * x = [0, 0]$ if $t + s \equiv 0 \pmod{n}$.

The proof is left as an exercise to the reader.

However we give a hint to the proof of the theorem.

Let $A = \{[a, a]\} \in P$;

$$\begin{aligned} A * A &= \{[a, a]\} * \{[a, a]\} \\ &= \{[ta + sa, ta + sa]\} \\ &= \{[0, 0]\} \text{ only if } (t + s) \equiv 0 \pmod{n} \text{ for all } A \in P. \end{aligned}$$

Corollary 2.2: If Z_n in theorem 2.3 is replaced by Z or Q or R, (s, t) should be such that $s = -t$ that is $s + t = 0$, then the conclusion of the above theorem is true.

Now we proceed onto give examples of substructures.

Example 2.17: Let $S = \{[a, b] \mid a, b \in G = \{(Z_9, *, (3, 2)) \times (Z_{20}, *, (0,4))\}$ be the interval groupoid.

S_G = {Collection of all subset intervals from S} be the subset interval groupoid.

Take P = {Collection of all intervals from the subgroupoid $Z_9 \times \{0\}\} \subseteq S$ and P_G = {Collection of all subset intervals of P} $\subseteq S_G$.

Clearly P_G is a subset interval subgroupoid of S_G as well as the subset interval ideal of S_G.

We do have subset interval subgroupoids which are not subset interval ideals but every subset interval ideal is a subset interval subgroupoid of S_G.

Example 2.18: Let M = {Collection of all intervals of the groupoid G = $\{Z_{42}, *, (3, 9)\}\}$ be the interval groupoid of G. S_G = {Collection of all subsets of M} be the subset interval groupoid of M.

S_G has subset interval subgroupoids. S_G has subset zero divisors and subset units.

Take $A = \{[7, 14]\} \in S_G$ we see

$$\begin{aligned} A * A &= \{[7, 14]\} * \{[7, 14]\} \\ &= \{[7 * 7], [14 * 14]\} \\ &= \{[0, 0]\}. \end{aligned}$$

Hence A is a subset interval zero divisor in S_G.

Let $A = \{[5, 2]\}$ and $B = \{[3, 4]\} \in S$.

We see

$$\begin{aligned} A * B &= \{[5, 2]\} * \{[3, 4]\} \\ &= \{[5 * 3, 2 * 4]\} \\ &= \{[0, 0]\} \text{ is again a subset interval zero divisor in S.} \end{aligned}$$

Example 2.19: Let $G = \{\langle Z_{12} \cup I\rangle, *, (1, 0)\}$ be a groupoid. M = {Collection of all intervals [a, b] with a, b $\in$ G} be the interval groupoid of G.

S = {Collection of all subsets of the interval groupoid M} be the subset interval groupoid. We call S as the neutrosophic subset interval groupoid of M.

Clearly $o(S) < \infty$.

S has subset units and subset zero divisors. Infact S has subset idempotents for A = {[4I, 4]} $\in$ S is such that A * A = {[4I, 4]} = A; so is a subset idempotent of S.

A = {[I, 4]} $\in$ S is a subset idempotent as $A^2 = A$. B = {[I, 4I]} $\in$ S is also a subset interval idempotent. A = {I, I]} $\in$ S is also a subset interval idempotent of S.

B = {[1, 7]} $\in$ S is such that

B * B = {[1, 7]} * {[1, 7]}
= {[1, 7] * [1, 7]}
= {[1 * 1, 7 * 7]}
= {[1, 7]} is a subset interval idempotent of S_G.

Example 2.20: Let G = {C ($\langle Z_9 \cup I\rangle$), *, (I, 0)} be a groupoid M = {Collection of all subsets [a, b] where a, b $\in$ G} be the interval groupoid of G.

S = {Collection of all subsets of the interval groupoid M} is the subset interval groupoid of M.

Let A = {[I, 0]} $\in$ S is such that A * A = A is a subset interval idempotent of S. A = {[I, 0]} $\in$ S is such that

A * A = {[I, 0]} * {[I, 0]}
= {[I, 0]} $\in$ S is a subset interval idempotent.

Let A = {[aI, 0]} and B = {[0, bI]} $\in$ S.

We see A * B = {[aI, 0]}* {[0, bI]}
= {[aI + 0, 0]}

$= \{[aI, 0]\}$
$= A.$

Thus we get $A * B = A$ and

$B * A = \{[0, bI]\} * \{[aI, 0]\}$
$= \{[0, bI]\} = B.$

These are special pair of subset interval elements; that is $A, B \in S$ is a special pair of interval elements if $A * B = A$ and $B * A = B$.

$A = \{[aI, 0] \mid a \in Z_9\}$ and $B = \{[0, bI] \mid b \in Z_9\} \in S$ are such that $A * B = A$ and $B * A = B$.

Interested reader can work with more examples.

We see $A * A = A$ for all $A = \{[aI, 0] \mid a \in Z_9\}$.

Further $B = \{[0, bI] \mid b \in Z_9\}$ is such that $B * B = \{[0, bI] \mid b \in Z_9\} = B$.

We see A and B form a class of subset interval idempotents infact $A_C = \{A = [aI, 0] \mid a \in Z_9\}$ and $B_C = B = \{[0, bI] \mid b \in Z_9\}$ are such that A_C and B_C are subset interval subgroupoids of S.

They can also be called as subset interval idempotent subgroupoids of S.

Let S = {Collection of all subsets of the interval groupoid M of a groupoid G}. If N is a interval subgroupod of M then S_N = {Collection of all subsets of the interval subgroupoid N of M} $\subseteq$ S is also a subset interval subgroupoid of S.

Further N is a interval subgroupoid of M of the groupoid G if $P \subseteq G$ is such that P a subgroupoid of G and
N = {Collection of all intervals of the subgroupoid P of G}.

We will first give some examples of this.

Example 2.21: Let S = {Collection of all subset intervals of the interval groupoid M = {all intervals [a, b] where a, b $\in$ G = {$C(Z_{12})$, *, (4, 3)}}} be the subset interval groupoid of the interval groupoid M of the groupoid G.

Take P_1 = {Collection of all subset intervals of the interval subgroupoid; N = {Collection of all intervals of the subgroupoid H = {Z_{12}, *, (4, 3)} $\subseteq$ G} $\subseteq$ M}} $\subseteq$ S is a subset interval subgroupoid of S.

We see P_1 is such that N is a interval subgroupoid of M of the subgroupoid H of G.

Example 2.22: Let S = {Collection of all subset intervals of the interval groupoid M = {Collection of all intervals of the groupoid G = {$C(Z_9)$ (g_1, g_2) | *, (3, 0), $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$}}} be the subset interval groupoid of M and M, the interval groupoid of G.

Take S_1 = {Collection of all subset intervals of the interval subgroupoid N_1 = {Collection of all intervals of the subgroupoid H_1 = {$C(Z_9)(g_1) \subseteq C(Z_9)(g_1, g_2)$, *, (3, 0)} $\subseteq$ G}}} $\subseteq$ S be the subset interval subgroupoid of S.

Let S_2 = {Collection of all subset intervals of the interval subgroupoid N_2 = {Collection of the intervals [a, b] from the subgroupoid H_2 = {$C(Z_9)(g_2) \subseteq C(Z_9)(g_1, g_2)$, *, (3, 0)} $\subseteq$ G} $\subseteq$ S be the subset interval subgroupoid of S.

Let S_3 = {Collection of all subsets of intervals from the interval subgroupoid N_3 of H_3 = {$C(Z_9)$, *, (3, 0)} $\subseteq$ G}} be the subset interval subgroupoid of S.

S_4 = {Collection of all subsets of intervals from the interval subgroupoid N_4 of H_4 = {Z_9, *, (3, 0)} $\subseteq$ G} be the subset interval subgroupoid of S.

Thus we see there are several subset interval subgroupoids of S.

However we see all these subset interval subgroupoids of S are not subset interval ideals of S be it left or right. However it is important to keep on record that we have for this S also both right subset interval ideals and left subset interval ideals and subset interval ideals.

First before we proceed onto work with the concept of ideals we just include the most important theorem.

THEOREM 2.4: *Let S be the subset interval groupoid of the interval groupoid M of the groupoid G. If G has a subgroupoid H then*

(i) *M has atleast two interval subgroupoids M_1 and M_2 isomorphic to H.*
(ii) *S has atleast two subset interval subgroupoids S_1 and S_2 isomorphic with M_1, M_2 and H.*

We just indicate the proof of the result.

Let G be the given groupoid $H = \{h_1, \ldots, h_t\} \subseteq G$ be the proper subgroupoid of G.

Let $M_1 = \{[0, h_i] \mid h_i \in H; 1 \leq i \leq t\} \subseteq M$, where M is the interval groupoid of the groupoid G.
$M_2 = \{[h_i, 0] \mid h_i \in H, 1 \leq i \leq t\} \subseteq M$.

We see both M_1 and M_2 are interval subgroupoids of the interval groupoid M of G. Further M_1 and M_2 are isomorphic as interval groupoids.

For take $\eta : M_1 \to M_2$ a map
where $\eta([0, h_i]) = [h_i, 0]$ for all $i = 1, 2, \ldots, n$.

Clearly η is a interval groupoid isomorphism of M_1 and M_2, that is $M_1 \cong M_2$.

Further $\eta_i : M_i \to H$ is defined by
$\eta_i ([h_j, 0]) (\eta_i ([0, h_j])) = h_j$; $1 \leq j \leq t$ and $i = 1, 2$.

Thus $M_1 \cong H$ and $M_2 \cong H$ as groupoids. Hence the claim.

Now consider $S_1 = \{\{[0, h_i]\} \mid [0, h_i] \in M_1; 1 \leq i \leq t\} \subseteq S$; $S_1 = \{\{[0, h_1]\}, \{[0, h_2]\}, \{[0, h_3]\}, \ldots, \{[0, h_t]\}\} \subseteq S$ is again a subset interval subgroupoid of S.

Define map $\delta_1 : S_1 \to M$, by
$\delta_1 (\{[0, h_i]\}) = [0, h_i]$; $i = 1, 2, \ldots, t$.

It is easily verified δ_1 is a groupoid isomorphism. But we know M_1 is isomorphic to the subgroupoid H, hence $S_1 \cong M_1$ and $S_1 \cong H$ as $M_1 \cong H$ by the map $f_1: S_1 \to H$.

$$f_1 (\{[0, h_i]\}) = h_i,\ i = 1, 2, \ldots, t.$$

Similarly we can prove $S_2 \cong M_2$ and $S_2 \cong H$ as $M_2 \cong H$. Finally we see $S_1 \cong S_2$ for let $g : S_1 \to S_2$ be a map such that

$g(\{[0, h_i]\}) = \{[h_i, 0]\}$; $i = 1, 2, \ldots, t$. It is easily verified g is a subset interval subgroupoid isomorphism.

Hence $S_1 \cong S_2$.

Now we give some more illustrations before we prove more properties.

Example 2.23: Let S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_5, *, (4, 1)\}\}\}$ be the subset interval groupoid of the interval groupoid M of G. S has atleast three interval subset subgroupoids all of which are isomorphic with G.

Take $S_1 = \{\{[0, a]\} \mid [0, a] \in M, a \in G\} \subseteq S$, S_1 is a subset interval subgroupoid of S and S_1 is isomorphic with the groupoid G.

Take $S_2 = \{\{[a, 0]\} \mid [a, 0] \in M, a \in G\} \subseteq S$, S_2 is a subset interval subgroupoid of S and is isomorphic with the groupoid G.

$S_3 = \{\{[a, a]\} \mid [a, a] \in M, a \in G\} \subseteq S$, S_3 is again a subset interval subgroupoid of S and is isomorphic with the groupoid G. That is $S_3 \cong G$, hence $S_1 \cong S_2$, $S_2 \cong S_3$ and $S_3 \cong S_1$ as subset interval groupoids.

Example 2.24: Let S = {Collection of all subset intervals of the interval groupoid M = {Collection of all intervals of the form [a, b]; b, a $\in$ G = {Z, *, (10, –6)}}} be the subset interval groupoid. Take S_1 = {Collection of all subset intervals of the interval subgroupoid M_1 = {Collection of all intervals of the form [a, b], a, b $\in$ {2Z, *, (10, –6)} $\subseteq$ G}} be the subset interval subgroupoid of S.

$S_2 = \{\{[a, 0]\} \mid \{[a, 0]\} \in S, a \in Z\} \subseteq S$ is a subset interval subgroupoid of S.

$S_3 = \{\{[0, a]\} \mid \{[0, a] \in S, a \in Z\} \subseteq S$ is a subset interval subgroupoid of S.

$S_4 = \{\{[a, a]\} \mid \{[a, a] \in S, a \in Z\} \subseteq S$ is a subset interval subgroupoid of S.

Thus all these three interval subset subgroupoids are isomorphic with G.

This is the case of infinite subset interval subgroupoids also.

In view of all these we have the following theorem.

THEOREM 2.5: *Let S = {Collection of all subsets of the interval groupoid M of a groupoid G} be the subset interval groupoid.*

(i) S has atleast three subset interval subgroupoids.
(ii) Each of these subset interval subgroupoids are isomorphic with G as groupoids.

The proof is direct follows from the simple fact that if $S_1 = \{\{[a, a]\}\} \mid \{[a, a]\} \in S, a \in G\} \subseteq S$ be a subset interval subgroupoid of S which is isomorphic with the groupoid G by the map $\eta_1 : S_1 \to G$; $\eta_1 (\{[a, a]\}) = a$.

$S_2 = \{\{[a, 0]\} \mid \{[a, 0]\} \in S$ where $a \in G\} \subseteq S$; is such that $S_2 \cong G$ with $\eta_2 (\{[a, 0]\}) = a$.

$S_3 = \{\{[0, a]\} \mid \{[0, a]\} \in S$ where $a \in G\} \subseteq S$; is such that $S_3 \cong G$ with $\eta_3 : S_3 \to G$; $\eta_3 \{\{[0, a]\}\} = a$.

Hence the claim.

Example 2.25: Let S = {Collection of all subsets of the interval groupoid $H = \{[a, b] \mid a, b \in G = \{Z_{12}(g), *, (3, 0)\}\}\}$ be the subset interval groupoid of the groupoid G.

$S_1 = \{\{[0, a]\} \mid a \in G, \{[0, a]\} \in S\} \subseteq S$ is a subset interval subgroupoid of S.

Find all other subset interval subgroupoids of S.

$S_2 = \{\{[a, b]\}\} \mid [a, b] \in \{$intervals from $\{0, 3, 3g, 6g, 9g, 6, 9\} \subseteq Z_{12} (g)\}$ that is $a, b \in \{0, 3, 3g, 6g, 9g, 6, 9\} \subseteq Z_{12} (g)$ be the subset interval subgroupoid of S.

We can find also other subset interval subgroupoids of S.

Example 2.26: Let S = {Collection of all interval subsets of the interval subset groupoid $M = \{[a, b] \mid a, b \in G = \{Z_{21} (g), *, (7, 3), g^2 = 0\}\}\}$ be the subset interval groupoid of M.

This also has subset interval subgroupoids.

Now we see subset interval ideals of a subset interval groupoid.

Example 2.27: Let S = {Collection of all interval subsets of the interval groupoid $M = \{\{[a, b]\} \mid a, b \in G = \{Z_{20}, *, (10, 0)\}\}\}$ be the interval subset groupoid of the interval groupoid M.

Consider $S_1 = \{\{[0, 0]\}, \{[0, 10]\}, \{[10, 0]\}, \{[10, 10]\} \subseteq S$, S_1 is a subset interval ideal of S.

Let $S_2 = \{\{[0, 0], [0, 10], [10, 0], [10, 10]\}\} \subseteq S$, S_2 be a subset interval ideal of S.

$S_3 = \{\{[0, 0]\}, \{[2, 0]\}, \{[0, 2]\}, \{[2, 2]\}, \{[0, 4]\}, \{[4, 0]\}, \{[4, 4]\}, \{[6, 6]\}, \{[0, 6]\}, \{[6, 0]\}, \{[2, 4]\}, \{[4, 2]\}, \{[2, 6]\}, \{[6, 2]\}, \{[4, 6]\}, \ldots, \{[10, 10]\}\}$

= {Collection all intervals of the subgroupoid {0, 2, 4, 6, 8, 10}}.

Clearly S_3 is also a subset interval ideal of S.

We have seen examples of subset interval ideals of a subset interval groupoid. We just show all subset interval subgroupoids of a subset interval groupoid need not in general be a subset interval ideal of S.

In view of this we give some examples.

Example 2.28: Let S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_{13}, *, (1, 0)\}\}\}$ be the subset interval groupoid of the interval groupoid M.

Consider $S_2 = \{\{[a, a]\} \mid a \in Z_{13}\} \subseteq S$ is only a subset interval subgroupoid of G but is not a subset interval ideal of S. Hence we see S_2 is a subset interval subgroupoid which is not a subset interval ideal of S.

In view of this we have the following theorem.

THEOREM 2.6: *Let S = {Collection of all subset intervals of the interval groupoid M of the groupoid G} be the subset interval groupoid of G. S has subset interval subgroupoid which in general is not subset interval ideal.*

Proof is left as an exercise to the reader.

Example 2.29: Let S = {Collection of all interval subsets of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_6, *, (4, 5)} of the groupoid G}} be the interval subset groupoid of the interval groupoid M.

A = {[1, 1], [0, 0], [3, 3], [5, 5]} $\subseteq$ S is a Smarandache left subset interval ideal of S. It is easily verified that A is not a Smaradache right subset interval ideal of S.

Example 2.30: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_7, *, (3, 3)}}} be the subset interval groupoid of the interval groupoid M.

We see G has no subgroupoids.

However M has three interval subgroupoids.

M_1 = {[a, a] | a $\in$ Z_7} $\subseteq$ M,
M_2 = {[0, a] | a $\in$ Z_7} $\subseteq$ M and
M_3 = {[a, 0] | a $\in$ Z_7} $\subseteq$ M which are all interval subgroupoids of M isomorphic to G.

S has six subset interval subgroupoids given in the following.

S_1 = {{[a, a]} | [a, a] $\in$ M, a $\in$ G} $\subseteq$ S,
S_2 = {{[0, a]} | [0, a] $\in$ M, a $\in$ G} $\subseteq$ S,
S_3 = {{[a, 0]} | [a, 0] $\in$ M, a $\in$ G} $\subseteq$ S,
S_4 = {[0, 0], [0, 1], [0, 2], [0, 3], [0, 4], [0, 5], [0, 6]} $\subseteq$ S,
S_5 = {[0, 0], [1, 0], [2, 0], [3, 0], [4, 0], [5, 0], [6, 0]} $\subseteq$ S and
S_6 = {[0, 0], [1, 1], [2, 2], [3, 3], [4, 4], [5, 5], [6, 6]} $\subseteq$ S are the six subset interval subgroupoids of S.

Thus even if the basic groupoid G on which S is built has no subgroupoids yet S can have atleast six subset interval subgroupoids.

Example 2.31: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_6, (5, 5), *}}} be the subset interval groupoid of the interval groupoid M. S has more than six subset interval subgroupoids.

Now we as in case of usual subset groupoids define in case of subset interval groupoids also the concept of Smarandache subset interval Bol groupoids, Smarandache subset interval Moufang groupoid, Smarandache subset interval P-groupoid, Smarandache subset interval idempotent groupoid, Smarandache subset interval commutative groupoid and their Smarandache strong analogous.

This is considered by the authors as a matter of routine and hence is left as an exercise to the reader.

However the authors will provide examples of all these new concepts.

Example 2.32: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_6, *, (0, 2)}}} be the subset interval groupoid of the groupoid G.

S is a non commutative subset interval groupoid as G is a non commutative groupoid.

Consider P_1 = {Collection of all interval subsets of the set {0, 3} $\subseteq$ Z_6} $\subseteq$ S is such that

P_1 = {{[0, 0]}, {[0, 3]}, {[3, 0]}, {[3, 3]}, {[0, 0], [0, 3]}, {[0, 0], [3, 0]}, {[0, 0], [3, 3]}, {[0, 3], [3, 0]}, {[3, 3], [0, 3]}, {[3, 3], [3, 0]}, {[0, 0], [0, 3], [3, 0]}, {[0, 0], [3, 3], [0, 3]}, {[0, 0], [3, 0], [3, 3]}, {[3, 3], [0, 3], [3, 0]} and {[0, 0], [0, 3], [3, 0], [3, 3]}} $\subseteq$ S is a subset interval subgroupoid of S which is very unique for P_1 * P_1 = {[0, 0]}.

However P_1 is not a two sided subset interval ideal but a one sided subset interval ideal of S.

Example 2.33: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_4, *, (3, 2)}}} be the subset interval groupoid of M.

S is a Smarandache idempotent subset interval groupoid as G the groupoid over which this is built is an idempotent groupoid.

Example 2.34: Let S = {Collection of all subsets interval of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{24}, *, (9, 16)}}} be the subset interval groupoid of the interval groupoid M. S is a Smarandache interval subset idempotent groupoid of the interval groupoid M.

In view of this we have the following theorem.

THEOREM 2.7: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] / a, b $\in$ G = {Z_n, (t, u), *}}} be the subset interval groupoid of the groupoid G.*

S is a Smarandache subset interval idempotent groupoid if and only if $t + u \equiv 1 \pmod{n}$.

The proof is similar to that of usual groupoids.

Another concept of Smarandache subset inner commutative interval groupoid.

Example 2.35: Let S = {Collection of all subset intervals of the interval groupoid M = {Collection of all intervals [a, b] where a, b $\in$ G = {Z_5, *, (3, 3)}}} be the subset interval groupoid. S is Smarandache inner commutative.

Example 2.36: Let S = {Collection of all subset intervals of the interval groupoid M = [a, b] where a, b $\in$ G = {Z_{23}, *, (12, 12)}} be the subset interval groupoid of M.

Is S Smarandache inner commutative?

Example 2.37: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{12}, *, (3, 9)}}} be the subset interval groupoid of the groupoid G.

S is a Smaradache Moufang subset interval groupoid which is not a Smarandache strong Moufang subset interval groupoid.

Example 2.38: Let S = {Collection of all subsets of the interval groupoid M = {{[a, b] | a, b $\in$ G = {Z_{10}, *, (5, 6)}}} be the subset interval groupoid of the interval groupoid M.

S is not a Smarandache strong Moufang subset groupoid though G is a Smarandache strong Moufang groupoid.

Example 2.39: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{12}, *, (3, 4)}}} be the subset interval groupoid of the interval groupoid M.

We see G is a Smarandache strong Bol groupoid so S is a Smarandache strong subset interval Bol groupoid of G.

Example 2.40: Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_4, *. (2,3)}}} be the subset interval groupoid of M.

Clearly S is only a Smarandache Bol groupoid and S is not a subset interval Smarandache strong Bol groupoid.

For {x}, {y}, {z} $\in$ S.

We see $[(\{x\} * \{y\}) * \{z\})] * \{y\}$

$$\begin{aligned}
&= [(2x + 3y) * \{z\}] * \{y\} \\
&= \{4x + 6y + 3z\} * \{y\} \\
&= \{4x + 6y + 3y\} \\
&= \{3y + 2z\} \\
&= \{[3y+2z,0]\} \qquad \ldots \qquad \text{I}
\end{aligned}$$

Consider $\{x\} * [(\{y\} * \{z\}) * \{y\}]$

$$= \{x\} * [\{2y + 3z\} * \{y\}]$$

$$
\begin{aligned}
&= \{x\} * \{4y + 6z + 3y\} \\
&= \{x\} * \{2z + 3y\} \\
&= \{2x + 6z + 9y\} \\
&= \{2x + y + 2z\} \\
&= \{[2x + y + 2z\ , 0]\} \qquad \ldots \qquad \text{II}
\end{aligned}
$$

Clearly I and II are distinct hence S is not a subset interval Smarandache strong Bol groupoid as the singleton sets $\{x\} = \{[x, 0]\}$, $\{y\} = \{[y, 0]\}$, $\{z\} = \{[z, 0]\} \in S$ ($x, y, z \in G$) are such that I and II are not equal. Here we denote $\{x\} = \{[x, 0]\}$, $\{y\} = \{[y, 0]\}$ and $\{z\} = \{[z, 0]\}$ (or $\{x\} = \{[0, x]\}$, $\{y\} = \{[0, y]\}$ and $\{z\} = \{[0, z]\}$ just for brevity only).

Interested reader can construct more examples about these subset interval groupoids.

We now proceed onto describe the Smarandache subset interval P-groupoids.

Example 2.41: Let S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_6, *, (4, 3)\}\}$} be the subset interval groupoid of M.

S is a Smarandache strong subset interval P-groupoid as G is a Smarandache strong P-groupoid.

Example 2.42: Let S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_{12}, *, (5, 10)\}\}$} be the subset interval groupoid of the interval groupoid M.

S is only a Smarandache subset interval P-groupoid S is not a Smarandache strong subset interval P-groupoid of G.

For take any $\{x\}$ ($= \{[x, 0]\}$) and $\{y\}$ ($= \{[y, 0]\}$) $\in S$.

$$
\begin{aligned}
(\{x\} * \{y\}) * \{x\} &= \{5x + 10y\} * \{x\} \\
&= \{25x + 50y + 10x\} \\
&= \{11x + 2y\} \\
&= \{[11x + 2y, 0]\} \qquad \ldots \qquad \text{I}
\end{aligned}
$$

Now $\{x\} * (\{y\} * \{x\})$

$$= \{x\} * \{5y + 10x\}$$
$$= \{5x + 50y + 100x\}$$
$$= \{9x + 2y\}$$
$$= \{[9x + 2y, 0]\} \qquad \ldots \qquad \text{II}$$

(for brevity we use $\{x\}$ instead of $\{[x, 0]\}$ or $\{[0, x]\}$).

Clearly I and II are distinct so S is not a Smarandache strong subset interval P-groupoid of G.

Now we just proceed onto give one or two examples of Smarandache strong subset interval alternative groupoid and Smarandache subset interval alternative groupoid.

Example 2.43: Let $G = \{Z_{14}, *, (7,8)\}$ be a groupoid. $M = \{[a,b] \mid a, b \in Z_{14}\}$ be the interval groupoid of G. S = {Collection of all subsets from M} be the subset interval groupoid of the interval groupoid M.

We see S is a Smarandache strong alternative subset interval groupoid as G is a Smarandache strong alternative groupoid.

Clearly S is both right and left alternative S-strong subset interval groupoid of G.

Example 2.44: Let S = {Collection of all subsets intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_{12}, *, (1, 6)\}\}\}$ be the subset interval groupoid of the interval groupoid M.

It is easily verified S is only Smarandache alternative subset interval groupoid which is not a Smarandache strong alternative subset interval groupoid of M.

We see all the characterization theorems on groupoids is true in case of subset interval groupoids as we have basically defined these structures in a very different way that is every property dependent on the property of G using which it is defined. We can also define subset idempotent interval groupoids.

We first see only elements of the subsets P in S must be one for it to be an idempotent in case subsets P in S has more than one element which is an idempotent then these idempotents must be mutually orthogonal.

We just give an example or two of subset interval idempotent groupoids.

Example 2.45: Let S = {Collection of all subsets of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{10}, *, (5, 6)}}} be a subset interval groupoid of M which is a Smarandache idempotent groupoid.

Example 2.46: Let S = {Collection of all subsets of the interval from the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{12}, *, (4, 9)}}} be the subset interval groupoid of the groupoid G.

Clearly S is a Smarandache subset interval idempotent groupoid as well as Smarandache subset interval P-groupoid.

We can just state a few theorems for a subset interval groupoid to be Smarandache subset interval P-groupoid (idempotent groupoid, Bol groupoid and Moufang groupoid).

First of all we clearly state in case of a subset interval groupoid S if we need S to be Smarandache subset interval groupoid of the groupoid G enjoying a special property then necessarily S enjoys that property which is enjoyed by G.

We just give a few theorems and the proof of them can be had from with simple appropriate modifications [18].

THEOREM 2.8: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] / a, b $\in$ G = {Z_n, *, (t, u)}}} be the subset interval groupoid of G. S is a Smarandache subset interval idempotent groupoid if t + u $\equiv$ 1 (mod n).*

THEOREM 2.9: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_n, *, (t, u) with t + u $\equiv$ 1 (mod n)}}} be the subset interval groupoid of the groupoid G is a Smarandache subset interval P-groupoid if and only if $t^2 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.*

THEOREM 2.10: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_n, *, (t, u) with t + u $\equiv$ 1 (mod n)}}} be the subset interval groupoid of the groupoid G. S is a Smarandache subset interval alternative groupoid if and only if $t^2 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.*

THEOREM 2.11: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_n, *, (t, u); t + u $\equiv$ 1 (mod n)}}} be the subset interval groupoid of G. S is a Smarandache strong Bol groupoid if and only if $t^3 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.*

THEOREM 2.12: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_n, *, (t, u) with t + u $\equiv$ 1 (mod n)}}} be the subset interval groupoid of G. S is a Smarandache strong Moufang subset interval groupoid if and only if $t^2 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.*

THEOREM 2.13: *Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_p, *, $\left.\left.\left.\left(\frac{p+1}{2}, \frac{p+1}{2}\right)\right\}\right\}\right\}$ be the subset interval groupoid of G.*

(i) Then S is a Smarandache subset interval groupoid.
(ii) S is a Smarandache subset interval idempotent groupoid.

S is a Smarandache subset interval groupoid if G is a Smarandache groupoid. It is not known if G is not a Smarandache groupoid still can S be a Smarandache subset interval groupoid?

Another question is if S is a subset interval groupoid of the groupoid G and if G is simple, can S be simple?

For instance take G = $\{Z_{12}, *, (5, 7)\}$ to be a simple groupoid. Will S = {Collection of all subset intervals of the interval groupoid M = $\{[a, b] \mid a, \in G = \{Z_{12}, *, (5, 7)\}\}\}$ be the subset interval groupoid of the groupoid G. Will S be simple?

We suggest some problems.

Problems:

1. Obtain some special properties enjoyed by subset interval groupoids built using the groupoid $G = \{Z_p, *, (t, r)\}$ p a prime $(t, r) = 1$.

2. Does there exist a subset interval groupoid which has no zero divisors?

3. Can infinite subset interval groupoid have zero divisors?

4. Can infinite subset interval groupoid have subset idempotents but no subset zero divisors?

5. Characterize all those finite subset interval groupoids which has only subset zero divisors and no subset idempotents.

6. Let S_G = {Collection of all subsets from intervals of the interval groupoid $S = \{[a, b] \mid a, b \in G = \{Z_{11}, *, (5, 0)\}\}\}$ be the subset interval groupoid.

   (i) Find $o(S_G)$.
   (ii) Can S_G have subset zero divisors?
   (iii) Can S_G have subset idempotents?
   (iv) Can S_G have subset nilpotents?
   (v) Prove S_G is non commutative.

7. Characterize those infinite subset interval groupoids which have no subset idempotents.

8. Find those finite dual number subset interval groupoid which has subset zero divisors.

9. Let S = {[a, b] | a, b $\in$ G = {Z_{16} (g) | $g^2 = 0$, *, (6, 8)}} be the interval groupoid of G. S_G = {Collection of all subsets of S}. (S_G, *) be the subset interval groupoid of S.

   Study question (i) to (v) of problem 6.

10. Let G = {Z, *, (8, –4)} be the groupoid. M = {[a, b] | a, b $\in$ G} be the interval groupoid. S_G the subset interval groupoid of the interval groupoid M of infinite order.

    Study questions (i) to (v) of problem 6 for this S_G.

11. G = {($Z^+ \cup$ {0}) (g) | $g^2 = 0$, *, (4, 5g)} be the groupoid. S = {[a, b] where a, b $\in$ G = {$Z^+ \cup$ {0}) (g)}} be the interval groupoid. S_G = {Collection of all subsets of S} be the subset interval groupoid.

    Study questions (i) to (v) of problem 6 for this S_G.

12. Let M = {Collection of all intervals of the groupoid G={Z_{49},*,(7,0)}} be the interval groupoid. S = {Collection of all subsets of M}, S is a subset interval groupoid of M.

    (i) Find o(M).
    (ii) Find o(S).
    (iii) Can S have subset idempotents which are not Smarandache subset idempotents?
    (iv) Can S have subset S-units?
    (v) Find all the subset zero divisors of S.
    (vi) Find all subset interval subgroupoids of S.
    (vii) Find all subset interval ideals of S.
    (viii) Find all subset interval S-subgroupoids which are subset S-ideals of S.

13. Let S = {Collection of all subsets of the interval groupoid of the group G = $\{Z_{18}, *, (3, 0)\}$} be the subset interval groupoid.

    Study questions (i) to (viii) of problem 12 for this S.

14. Let S = {Collection of all subsets of the interval groupoid, M = {all intervals [a, b] where a, b $\in$ G = $\{Z_{19}, *, (3, 16)\}$}} be the subset interval groupoid.

    Study questions (i) to (viii) of problem 12 for this S.

15. Let S = {Collection of all subsets of the interval groupoid; M = {all intervals of the form [a, b] where a, b $\in$ G = $\{Z^{+} \cup \{0\}, *, (3,2)\}$}} be the subset interval of the interval groupoid M of G.

    Study questions (i) to (viii) of problem 12 for this S.

16. Let S = {Collection of all subset intervals from the interval groupoid M = {Collection of all [a, b] where a, b $\in$ G = $\{C(Z_7), *, (i_F, 6i_F)\}$}} be subset interval groupoid of the interval groupoid M.

    Study questions (i) to (viii) of problem 12 for this S.

17. Let S = {Collection of all subset intervals from the groupoid M = {Collection of all intervals [a, b] from the groupoid G = $\{C(Z_{12})\ (g_1, g_2) \mid g_1^2 = 0,\ g_2^2 = g^2,\ g_1g_2 = g_2g_1 = 0, *, (3, 9g_1)\}$}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (viii) of problem 12 for this S.

18. Let S = {Collection of all subset intervals from the interval groupoid, M = {Collection of all intervals of the groupoid G = $\{C(Z_{45})\ (g_1, g_2, g_3) \mid g_1^2 = 0,\ g_2^2 = g_2,\ g_3^2 = -g_3,\ g_ig_j = g_jg_i = 0,\ 1 \leq i, j \leq 3,\ i \neq j\}$}} be the subset interval

special mixed dual number groupoid of the interval groupoid M.

Study questions (i) to (viii) of problem 12 for this S.

19. Let S = {Collection of all subset intervals from the interval groupoid M = {[a, b] | a, b ∈ Z_{18} of the groupoid G where G = {Z_{18}, *, (6, 3)}}} be the subset interval groupoid of the groupoid G.

    (i) Find all subset interval subgroupoids of S.
    (ii) Find o(S).
    (iii) Find all S-subset interval subgroupoids of S.
    (iv) Find all S-subset interval ideals of S.

20. Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b ∈ G = {Z_{18}, *, (7, 11)}}} be the subset interval groupoid of the groupoid G.

    (i) Is S simple?
    (ii) Is S a Smarandache simple subset interval groupoid?

21. Let S = {Collection of all subset intervals from the interval groupoid M = {[a, b] | a, b ∈ G = {Z_{13}, *, (11, 2)}}} be the subset interval groupoid of the groupoid G.

    Study questions (i) and (ii) of problem 20 for this S.

22. Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b ∈ G = {Z_n, *, (t, u) where t and u are primes and n = t + u}}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) and (ii) of problem 20 for this S.

23. Let S = {Collection of all subset intervals from the interval groupoid M = {[a, b] | a, b ∈ G = {Z_{12}, *, (3, 9)}}} be the subset interval groupoid of G.

(i) Does S have normal subset interval subgroupoids?
(ii) Find o(S).
(iii) Find all subset interval subgroupoids of S.
(iv) Find all subset interval ideals of S.
(v) Does S contain subset interval Smarandache ideals?
(vi) Can S have S-subset idempotents?
(vii) Can S have S-subset units?
(viii) Can S have S-subset zero divisors?

24. Let S_1 = {Collection of all interval subsets of the interval groupoid M = {[a, b] | a, b ∈ G = {Z_{12}, *, (10, 8)}}} be the interval subset groupoid of S.

    Study questions (i) to (viii) of problem 23 for this S_1.
    Is $o(S) = o(S_1)$?

25. Let S_2 = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b ∈ G = {Z_8, *, (2, 8)}}} be the subset interval groupoid.

    (i) Study questions (i) to (viii) of problem 23 for this S_2.
    (ii) Can S_2 have subset interval subgroupoids which are Smarandache semiconjugate?

26. Let S = {Collection of all subset intervals of the form [a, b] where [a, b] ∈ M = {Collection of all intervals [a, b] with a, b ∈ G = {Z, *, (5, –5)}}} be the subset interval groupoid of the groupoid G.

    (i) Study questions (i) to (ii) of problem 25 for this S.
    (ii) Obtain any other special property enjoyed by this S.

27. Let S = {Collection of all subset intervals from the interval groupoid M = {[a, b] | a, b ∈ G = {C (Z_{18}), *, (6,9)}}} be the subset interval groupoid of the groupoid G.

    Study questions (i) to (viii) of problem 23 for this S.

28. Let S = {Collection of all subset intervals from the interval groupoid M = {[a, b] | a, b $\in$ G = {C (Z_{10}) (g_1, g_2, g_3) | $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = -g_3$, $g_ig_j = g_jg_i = 0$, $1 \leq i, j \leq 3$, $i \neq j$}}} be the subset interval groupoid of the groupoid G.

    Study questions (i) to (viii) of problem 23 for this S.

29. Characterize those subset interval groupoids which has no Smarandache conjugate subset interval subgroupoids.

30. Charactetize those subset interval groupoids which has Smarandache semiconjugate subgroupoids.

31. Characterize those subset interval groupoids which has no Smarandache semiconjugate subgroupoids.

32. Characterize those subset interval groupoids which has Smarandache conjugate subset interval subgroupoids.

33. Characterize those subset interval groupoids which are normal.

34. Characterize those subset interal groupoids which are not normal.

35. Characterize those subset interval groupoids which has interval subset subgroupoids which are normal.

36. Characterize those subset interval groupoids which are Smarandache strong interval subset Moufang groupoid.

37. Let S = {Collection of all subset intervals from the interval groupoid M = [a, b] | a, b $\in$ G = {Z_{10}, *, (1, 5)}}} be the subset interval groupoid of the groupoid G.

    (i) Study o(S).
    (ii) Find all subset interval subgroupoids of S.
    (iii) Find all subset interval zero divisors of S.

(iv) Prove if $A_1 = \{\{[0, a]\} \mid \{[0, a]\} \in S\}$ and $A_2 = \{\{[a, 0]\} \mid \{[a, 0]\} \in S\}$ are subset interval subgroupoids of S. Prove $A_1 * A_2 \neq \{[0, 0]\}$.

(v) Find all other special features enjoyed by S.

38. Is it true if S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_n, *, (t, u)$; with $t + u \equiv 0 \pmod n)\}\}\}$ be the subset interval groupoid of the interval groupoid M then S has a Smarandache subset interval normal subgroupoid?

39. Let S = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_{27}, *, (10, 10)\}\}\}$ be the subset interval groupoid of the groupoid G.

(i) Is S Smarandache inner commutative?
(ii) Is S Smarandache commutative?
(iii) Can we say if we replace (10, 10) by (t, t); $2 \leq t \leq 26$; will that S be Smarandache inner commutative?

40. Give an example of a Smarandache inner commutative subset interval groupoid of infinite order.

41. Give an example of a subset interval groupoid of finite order which is not Smarandache inner commutative.

42. Give an example of a subset interval groupoid of infinite order which is not Smarandache inner commutative.

43. Let S_P = {Collection of all subset intervals of the interval groupoid $M = \{[a, b] \mid a, b \in G = \{Z_n, *, (p, p)\}$ where $1 < p < n\}\}$ be the subset interval groupoid of G.

(i) Find o(S).
(ii) Is every S_p, $p \in Z_n \setminus \{0, 1\}$, a Smarandache inner commutative subset interval groupoid?
(iii) Obtain any of the special properties enjoyed by S.
(iv) Can S_p satisfy any of the special identities?

44. Give an example of a Smarandache strong subset interval Moufang groupoid of infinite order.

45. Give an example of a subset interval groupoid of infinite order which is not a Smarandache Moufang subset interval groupoid.

46. Give an example of a Smarandache subset interval Moufang groupoid of infinite order which is not a Smarandache strong subset interval Moufang groupoid.

47. Does there exists a finite subset interval Smarandache strong Moufang groupoid?

48. Give an example of a Smarandache strong Bol subset interval groupoid of finite order.

49. Give an example of a Smarandache strong Bol subset interval groupoid of infinite order.

50. Give an example of a Smarandache Bol subset interval groupoid which is not a Smarandache strong Bol subset interval groupoid.

51. Give an example of a finite interval subset groupoid which is not a Smarandache Bol subset interval groupoid.

52. Describe the special properties enjoyed by Smarandache subset interval P-groupoid of a P-groupoid.

53. Give an example of a Smarandache subset interval P-groupoid of infinite order.

54. Give an example of a Smarandache P-subset interval groupoid.

    (i) Using $G = \{Z, *, (p, q); p, q \in Z\}$.
    (ii) Using $G = \{Z_n, *, (p, q); p, q \in Z_n\}$.
    (iii) Using $G = \{C(Z_n), *, (p, q); p, q \in C(Z_n)\}$.

(iv) Using G = {C(Z_n) (g_1, g_2, g_3); *, (p, q);
p,q ∈ C(Z_n) (g_1,g_2,g_3) where $g_1^2 = 0, g_2^2 = g_2$ and
$g_3^2 = -g_3$}.

(v) Using G = {C, *, (p, q) | p, q ∈ C}.

55. Characterize those subset interval groupoids which are simple.

56. Does there exist a subset interval groupoid of infinite order which is simple?

57. Give an example of a subset interval groupoid of infinite order which is not simple.

58. Give an example of a subset interval groupoid of finite order which is not Smarandache.

59. Is it possible to have a subset interval groupoid S in which every A ∈ S is such that A * A = {0}.

60. Let S = {Collection of all subsets of the interval groupoid M = {[a, b] | a, b ∈ G = {C(Z_p), *, (r, s)}; p a prime}}} be the subset interval groupoid of G.

(i) For what values of r and s in Z_p, S will be Smarandache strong Bol subset interval groupoid?
(ii) Can we have a (r, s) so that S is simple?
(iii) Do we have a (r, s) so that S is a Smarandache strong P-subset interval groupoid?
(iv) Does their exist a pair (r, s) in C(Z_p) × C(Z_p) so that S satisfies none of the special identities?
(v) Does there exist a pair (r, s) so that S is not Smarandache?
(vi) Does there exist a (r, s) so that S has no S-subset ideals?
(vii) Does there exist a (r, s) so that S has no S-subset interval subgroupoid?

61. Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {$C(Z_{18})$ (g), *, (p, q); p, q $\in$ $C(Z_{18})$ (g) with $g^2 = 0$}}} be the subset interval groupoid of G.

    Study questions (i) to (vii) of problem (60) of this S.

62. Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{19} (g_1, g_2, g_3), *, (p, q), p, q $\in$ Z_{19} (g_1, g_2, g_3); $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$, $g_ig_j = g_jg_i = 0$ if $i \neq j$; $1 \leq i, j \leq 3$}}} be the subset interval groupoid of G.

    Study questions (i) to (vii) of problem 60 for this S.

63. S_1 = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {$\langle Z_{25} \cup I\rangle$, *, (p, q) where p, q $\in \langle Z_{25} \cup I\rangle$}}} be the subset interval groupoid of the groupoid G.

    Study questions (i) to (vii) of problem 60 for this S_1.

64. Let S_2 = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {P = $\langle C(Z_{25}) \cup I\rangle$, *, (p, q), p, q $\in$ P}}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (vii) of problem 60 for this S_2.

65. Let S_3 = {Collection of all subset intervals of interval groupoid M = {[a, b] | a, b $\in$ G = {$\langle Z_{23} \cup I\rangle$, *, (p, q), p, q $\in$ $\langle Z_{23} \cup I\rangle$}}} be the subset interval groupoid of the groupoid G.

    Study questions (i) to (vii) of problem (60) for this S_3.

66. Let S_4 = {Collection of all subset intervals of interval groupoid M = {[a, b] | a, b $\in$ G = {Z_{25} (g_1, g_2, g_3, g_4), *,

(p, q) where p, q $\in Z_{15}$ (g_1, g_2, g_3, g_4) and $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = -g_3$, $g_4^2 = g_4$ with $g_ig_j = g_jg_i = 0$; $i \neq j$; $1 \leq i, j \leq 4$}}} be the subset interval groupoid of the interval groupoid M.

Study questions (i) to (vii) of problem 60 for this S_4.

67. Let S_5 = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {$\langle C \cup I\rangle$, *, (p, q); p, q $\in$ $\langle C \cup I\rangle$}}} be the subset interval groupoid of the interval groupoid M of G.

    Study questions (i) to (vii) of problem 60 for this S_5.

68. Let S_6 = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b $\in$ G = {$C\langle Z_7 \cup I\rangle$, *, (p, q), p, q $\in$ $C(\langle Z_7 \cup I\rangle)$}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (vii) of problem 60 for this S_6.

69. Enumerate any of the special properties associated with subset interval groupoid of an interval groupoid M of infinite order.

70. Show there exists subset interval groupoid of infinite or finite order which has no S-subset idempotents and S-subset zero divisors.

71. Let S = {Collection of all interval subsets of the interval groupoid M = {[a, b] | a, b $\in$ G = {$C(\langle Z_{20} \cup I\rangle)$, *, (p, q), q, p $\in$ $\langle Z_{20} \cup I\rangle$}}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (vii) of problem 60 for this S.

72. Can we have a subset interval groupoid S which has every element to be subset nilpotent?

73. Does there exists a subset interval groupoid S which has no subset nilpotents but has subset zero divisors?

74. Does there exists a subset interval groupoid S in which every subset zero divisor is a S-subset zero divisor?

75. Does there exists a subset interval groupoid S in which every subset idempotent is a S-subset idempotent?

76. Can we have a subset interval groupoid S in which every subset interval subgroupoid is a Smarandache subset interval subgroupoid?

77. Can we have a subset interval groupoid in which no subset interval subgroupoid is a Smarandache subset interval subgroupoid?

78. Can we have subset interval groupoid in which every subset interval ideal is a Smarandache subset interval ideal?

79. Can we have a subset interval groupoid such that every subset interval ideal is a Smarandache subset interval ideal?

80. Let S = {Collection of all subset intervals of the interval groupoid M = $\{\{[a, b] \mid a, b \in G = \{P = C(\langle Z_{15} \cup I\rangle), *, (p, 0), p \in P\}\}\}$ be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (vii) of problem 60 for this S.

81. Can we have a subset interval groupoid S so that S has no Smarandache subset subgroupoids?

82. Can we built subset interval groupoid S of infinite order using $\langle R \cup I\rangle$ so that S has every subset interval subgroupoid to be Smarandache?

83. Does there exist a subset interval groupoid S of finite order which is S-simple?

84. Let S = {Collection of all subset intervals of the interval groupoid of the groupoid G = {(⟨$Z^+ \cup \{0\} \cup I$⟩), *, (p, q), p, q ∈ ⟨$Z^+ \cup \{0\} \cup I$⟩}} be the subset interval groupoid of the interval groupoid G.

    Study questions (i) to (vii) of problem 60 for this S.

85. Let S = {Collection of all subset intervals of the interval groupoid M = {[a, b] | a, b ∈ G = {⟨$R^+ \cup I \cup \{0\}$⟩, *, (p, q); p, q ∈ ⟨$R^+ \cup I \cup \{0\}$⟩}}} be the subset interval groupoid of the interval groupoid M.

    Study questions (i) to (vii) of problem 60 for this S.

Chapter Three

SUBSET INTERVAL MATRIX GROUPOIDS AND SUBSET INTERVAL POLYNOMIAL GROUPOIDS

In this chapter we for the first time define subset interval matrix groupoids and subset interval matrix polynomial groupoids, describe and develop them. We have introduced the notion of subset interval groupoids in chapter II. Just interval matrix groupoids and interval polynomial groupoids and their neutrosophic analogue are introduced in [18, 23, 39-41].

DEFINITION 3.1: *Let S = {Collection of all subsets of $1 \times n$ row interval matrices from M of a groupoid (G,*); that is M = {([a_1, b_1], ..., [a_n,b_n])| a_i,b_i $\in$ G; $1 \leq i \leq n$}}. We define S to be a subset interval row matrix groupoid under the operation of G.*

We illustrate this situation by an example or two.

Example 3.1: Let S = {Collection of all interval row matrices from the row matrix interval groupoid M = {([a_1, b_1], [a_2, b_2], [a_3, b_3]) | a_i, b_i $\in$ G = {Z_{42}, *, (6, 00}, $1 \leq i \leq 3$}}} be the subset interval row matrix groupoid.

Let A = {([0, 1], [2, 0], [0, 0]), ([6, 0], [1, 0], [5, 0])} and B = {([0, 0], [0, 0], [2, 1]), ([3, 0], [0, 0], [0, 0])} ∈ S.

A * B = {([0, 1], [2, 0], [0, 0]), ([6, 0], [1, 0], [5, 0])} * {([0, 0], [0, 0], [2, 1]), ([3, 0], [0, 0], [0, 0])}

= {([0, 1], [2, 0], [0, 0]) * ([0, 0], [0, 0], [2, 1]),
([0, 1], [2, 0], [0, 0]) * ([3, 0], [0, 0], [0, 0]),
([6, 0], [1, 0], [5, 0])* ([0, 0], [0, 0], [2, 1]),
([6, 0], [1, 0], [5, 0]) * ([3, 0], [0, 0], [0, 0])

= {([0, 1] * [0, 0], [2, 0] * [0, 0], [0, 0] * [2, 1]),
([0, 1] * [3, 0], [2, 0] * [0, 0], [0, 0] * [0, 0])
([6, 0] * [0, 0], [1, 0] * [0, 0], [5, 0] * [2, 1])

= {([0, 6], [12, 0], [0, 0]), ([0, 6], [12, 0], [0, 0]),
([36, 0], [6, 0], [30, 0]), ([36, 0], [6, 0], [30, 0])}
∈ S.

This is the way operations are performed on S.

Example 3.2: Let S = {Collection of all subsets of the row interval groupoid M = {([a_1, b_1], [a_2, b_2]) | a_i, b_i ∈ G = {Z_{10}, *, (5, 6)}, 1 ≤ i ≤ 2)}}} be the subset row interval matrix groupoid of finite order.

Let A = {([0, 5], [2, 2]), ([3, 0], [0, 9)] and B = {([1, 2], [2, 4])} ∈ S.

A * B = {([0, 5], [2, 2]), ([3, 0], [0, 9)] * {([1, 2], [2, 4])}

= {([0, 5], [2, 2]) * ([1, 2], [2, 4]),
([3, 0], [0, 9]) * ([1, 2], [2, 4])}

= {([0, 5] * [1, 2], [2, 2] * [2, 4]),
([3, 0] * [1, 2], [0, 9] * [2, 4])}

= {([6, 7], [2, 4]), ([1, 2], [2, 9])} ∈ S.

This is the way operation on S is performed.

Example 3.3: Let M = $\{([a_1, b_1], [a_2, b_2], \ldots, [a_{10}, b_{10}]) \mid a_i, b_i \in G = \{C(Z_{16}), *, (8, 4)\}; 1 \leq i \leq 10\}\}$ be the interval row matrix groupoid. S = {Collection of all subsets of the interval row matrix groupoid M} be the subset interval row matrix groupoid.

If in the definition 3.1 if we replace the subset row interval matrices by subset column interval matrices then we call S to be a subset interval column matrix groupoid.

We will give examples of them.

Example 3.4: Let S = {Collection of all subsets of 4 × 1 column interval matrices

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in \{Z_{29}, *, (3, 2)\}; 1 \leq i \leq 4\}\right\}$$

be the subset column interval matrix groupoid of finite order.

We see if $A = \left\{ \begin{bmatrix} [0,1] \\ [2,0] \\ [1,2] \\ [0,4] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [2,0] \\ [1,0] \\ [0,2] \end{bmatrix}, \begin{bmatrix} [1,0] \\ [1,1] \\ [3,0] \\ [4,1] \end{bmatrix} \right\}$ and

$$B = \left\{ \begin{bmatrix} [1,2] \\ [2,1] \\ [0,4] \\ [4,0] \end{bmatrix} \right\} \in S.$$

We now find

$$A * B = \left\{\begin{bmatrix}[0,1]\\[2,0]\\[1,2]\\[0,4]\end{bmatrix}, \begin{bmatrix}[0,0]\\[2,0]\\[1,0]\\[0,2]\end{bmatrix}, \begin{bmatrix}[1,0]\\[1,1]\\[3,0]\\[4,1]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[1,2]\\[2,1]\\[0,4]\\[4,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,1]\\[2,0]\\[1,2]\\[0,4]\end{bmatrix} * \begin{bmatrix}[1,2]\\[2,1]\\[0,4]\\[4,0]\end{bmatrix}, \begin{bmatrix}[0,0]\\[2,0]\\[1,0]\\[0,2]\end{bmatrix} * \begin{bmatrix}[1,2]\\[2,1]\\[0,4]\\[4,0]\end{bmatrix}, \begin{bmatrix}[1,0]\\[1,1]\\[3,0]\\[4,1]\end{bmatrix} * \begin{bmatrix}[1,2]\\[2,1]\\[0,4]\\[4,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,1]*[1,2]\\[2,0]*[2,1]\\[1,2]*[0,4]\\[0,4]*[4,0]\end{bmatrix}, \begin{bmatrix}[0,0]*[1,2]\\[2,0]*[2,1]\\[1,0]*[0,4]\\[0,2]*[4,0]\end{bmatrix}, \begin{bmatrix}[1,0]*[1,2]\\[1,1]*[2,1]\\[3,0]*[0,4]\\[4,1]*[4,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[2,7]\\[10,2]\\[3,14]\\[8,12]\end{bmatrix}, \begin{bmatrix}[2,4]\\[10,2]\\[3,8]\\[8,4]\end{bmatrix}, \begin{bmatrix}[5,4]\\[7,5]\\[9,8]\\[1,3]\end{bmatrix}\right\}$$

is in S. This is the way '*' the product is performed on elements of S.

Example 3.5: Let S = {Collection of all subsets of the interval column matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{40}, *, (2, 20)\}; 1 \leq i \leq 12 \right\}\}$$

be the subset column interval matrix groupoid of finite order. Clearly S is non commutative.

S has zero divisors and subset column interval matrix subgroupoids.

Example 3.6: Let S = {Collection of all subsets interval column matrices of the interval column matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z, *, (3, -2)\}; 1 \leq i \leq 3 \right\}\}$$

be the subset interval column matrix groupoid of M.

We see S is of infinite order.

Let

$$A = \left\{ \begin{bmatrix} [9,2] \\ [0,3] \\ [1,5] \end{bmatrix}, \begin{bmatrix} [0,1] \\ [-1,2] \\ [0,-1] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [0,-1] \\ [1,0] \end{bmatrix} \right\},$$

$$B = \left\{ \begin{bmatrix} [1,0] \\ [0,0] \\ [2,1] \end{bmatrix}, \begin{bmatrix} [0,5] \\ [1,-1] \\ [0,0] \end{bmatrix} \right\} \in S.$$

We find

$$A * B = \left\{\begin{bmatrix}[9,2]\\ [0,3]\\ [1,5]\end{bmatrix}, \begin{bmatrix}[0,1]\\ [-1,2]\\ [0,-1]\end{bmatrix}, \begin{bmatrix}[0,0]\\ [0,-1]\\ [1,0]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[1,0]\\ [0,0]\\ [2,1]\end{bmatrix}, \begin{bmatrix}[0,5]\\ [1,-1]\\ [0,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[9,2]\\ [0,3]\\ [1,5]\end{bmatrix} * \begin{bmatrix}[1,0]\\ [0,0]\\ [2,1]\end{bmatrix}, \begin{bmatrix}[0,1]\\ [-1,2]\\ [0,-1]\end{bmatrix} * \begin{bmatrix}[1,0]\\ [0,0]\\ [2,1]\end{bmatrix}, \begin{bmatrix}[0,0]\\ [0,-1]\\ [1,0]\end{bmatrix} * \begin{bmatrix}[1,0]\\ [0,0]\\ [2,1]\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}[9,2]\\ [0,3]\\ [1,5]\end{bmatrix} * \begin{bmatrix}[0,5]\\ [1,-1]\\ [0,0]\end{bmatrix}, \begin{bmatrix}[0,1]\\ [-1,2]\\ [0,-1]\end{bmatrix} * \begin{bmatrix}[0,5]\\ [1,-1]\\ [0,0]\end{bmatrix}, \begin{bmatrix}[0,0]\\ [0,-1]\\ [1,0]\end{bmatrix} * \begin{bmatrix}[0,5]\\ [1,-1]\\ [0,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[25,6]\\ [0,9]\\ [1,13]\end{bmatrix}, \begin{bmatrix}[-2,3]\\ [-3,6]\\ [-4,-5]\end{bmatrix}, \begin{bmatrix}[-2,0]\\ [0,-3]\\ [-1,2]\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}[27,-4]\\ [-2,11]\\ [3,15]\end{bmatrix}, \begin{bmatrix}[0,-7]\\ [-5,8]\\ [0,-3]\end{bmatrix}, \begin{bmatrix}[0,-10]\\ [-2,-1]\\ [3,0]\end{bmatrix}\right\} \in S.$$

This is the way * operation is performed on S.

S is of infinite order. S is non commutative for take

$$A = \left\{\begin{bmatrix}[7,2]\\ [0,5]\\ [4,0]\end{bmatrix}\right\} \text{ and } B = \left\{\begin{bmatrix}[0,7]\\ [2,0]\\ [3,0]\end{bmatrix}\right\} \text{ in S.}$$

$$\text{Consider } A * B = \left\{\begin{bmatrix}[7,2]\\ [0,5]\\ [4,0]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[0,7]\\ [2,0]\\ [3,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[7,2]*[0,7]\\ [0,5]*[2,0]\\ [4,0]*[3,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[21,-8]\\ [-4,15]\\ [6,0]\end{bmatrix}\right\} \qquad \ldots \qquad \text{I}$$

$$\text{Now } B * A = \left\{\begin{bmatrix}[0,7]\\ [2,0]\\ [3,0]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[7,2]\\ [0,5]\\ [4,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,7]*[7,2]\\ [2,0]*[0,5]\\ [3,0]*[4,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[-14,17]\\ [6,-10]\\ [1,0]\end{bmatrix}\right\} \qquad \ldots \qquad \text{II}$$

Clearly I and II are distinct hence S is not a commutative subset interval column matrix groupoid of infinite order.

Example 3.7: Let S = {Collection of all subsets of the interval column matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z, *, (5, -5)\};\ 1 \le i \le 5 \right\}\}$$

be the subset interval column matrix groupoid.

S has infinite number of subset nilpotent elements of order two.

$$\text{For take } A = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \right\} \in S.$$

$$\text{Consider } A * A = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [a_1, b_1]*[a_1, b_1] \\ [a_2, b_2]*[a_2, b_2] \\ [a_3, b_3]*[a_3, b_3] \\ [a_4, b_4]*[a_4, b_4] \\ [a_5, b_5]*[a_5, b_5] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\} \in S.$$

A is subset nilpotent of order two.

Example 3.8: Let S = {Collection of all subsets of interval row matrix M = $\{([a_1, b_1], \ldots, [a_7, b_7]) \mid a_i, b_i \in G = \{Z_8, *, (4, 4)\};$ $1 \leq i \leq 7\}\}$ be the subset interval row matrix groupoid of finite order. S has nilpotent elements of order two.

Consider

$A = \{([a_1, a_1], [a_2, a_2], \ldots, [a_7, a_7]) \mid a_i \in G; 1 \leq i \leq 7\}$.

We see

$A * A = \{([a_1, a_1], [a_2, a_2], \ldots, [a_7, a_7])\} * \{([a_1, a_1], [a_2, a_2], \ldots, [a_7, a_7])\}$

$= \{([a_1, a_1] * [a_1, a_1], \ldots, [a_7, a_7] * [a_7, a_7])\}$

$= \{([0, 0], [0, 0], \ldots, [0, 0])\} \in S$.

Hence the claim.

In view of all these we have the following theorems.

THEOREM 3.1: *Let S = {Collection of all subsets of interval $1 \times n$ row matrix groupoid M =$\{([a_1, b_1], [a_2, b_2], \ldots, [a_n, b_n]) /$ $a_i, b_i \in G = \{R, *, (t, -t)); \text{where } t \in R, b_i, a_i \in R, 1 \leq i \leq n\}\}\}$ be the subset row matrix interval groupoid. S has infinite number of subset nilpotent elements of order two.*

We see M = $\{([a_1, a_1], [a_2, a_2], \ldots, [a_n, a_n]) \mid a_i \in G; a_i \in R, 1 \leq i \leq n\}\} \subseteq S$ is such that for all A $\in$ M.

$$A * A = \{([0, 0], [0, 0], \ldots, [0, 0])\}.$$

However $A * B \neq \{([0, 0], [0, 0], \ldots, [0, 0])\}$ in general if $A \neq B$ and A, B $\in$ S.

Example 3.9: Let S = {Collection of all subsets from column matrix interval groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{15}, b_{15}] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{27}, *, (20, 7)\}; 1 \leq i \leq 15 \right\}\}$$

be the subset interval column matrix groupoid of finite order.

Clearly S is of finite order.

If

$$P = \left\{ \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{15}, b_{15}] \end{bmatrix} \right\} \in S; a_i \in Z_{27}, 1 \leq i \leq 15 \right\} \subseteq S,$$

P is such that for every A $\in$ P we have $A * A = \left\{ \begin{bmatrix} [0, 0] \\ [0, 0] \\ \vdots \\ [0, 0] \end{bmatrix} \right\}$.

However

$$A * B \neq \left\{\begin{bmatrix}[0,0]\\ [0,0]\\ \vdots\\ [0,0]\end{bmatrix}\right\}$$ in general if A ≠ B and A, B ∈ P.

In view of this we have the following theorem.

THEOREM 3.2: *Let S = {Collection of all subsets from the interval column matrix groupoid*

$$M = \left\{\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ \vdots\\ [a_n,b_n]\end{bmatrix} \right| a_i,\ b_i \in G = \{Z \text{ (or } Z_n),*, (t, -t) \text{ for}$$

$$t \in Z \text{ or (or } Z_n),\ 1 \leq i \leq n\}\}\}$$

be the subset interval column matrix groupoid of M.

$$P = \left\{\left\{\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ \vdots\\ [a_n,b_n]\end{bmatrix}\right\} \in S,\ a_i \in Z \text{ (or } Z_n),\ 1 \leq i \leq n\right\} \subseteq S$$

is such that every element in P is subset nilpotent of order two.

Proof is direct and hence is left as an exercise to the reader.

Example 3.10: Let S = {Collection of all subsets from of the interval row matrix groupoid M ={([a_1, b_1], …, [a_{10}, b_{10}]) | a_i, b_i ∈ G = {Z_{25}, *, (5, 0)}; 1 ≤ i ≤ 10}}} be the subset interval row matrix groupoid of M. S has subset zero divisors.

For take T = {Collection of all subsets of the row interval matrices ($[a_1, b_1], \ldots, [a_{10}, b_{10}]$) where $a_i, b_i \in \{0, 5, 10, 15, 20\} \subseteq Z_{25}$; $1 \leq i \leq 25\} \subseteq M\} \subseteq S$.

Every element in T is a subset zero divisor. Further T is also a subset interval row matrix subgroupoid of S of finite order.

Example 3.11: Let S = {Collection of all subsets of the column interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{10}, *, (0, 5)\}; 1 \leq i \leq 10\} \right\}$$

be the subset interval column matrix groupoid of M.

S has a subset interval column matrix subgroupoid P_1 such that in P_1 every pair is a subset zero divisor where

$$P = \left\{ \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \right\} \middle| \; a_i, b_i \in \right.$$

$$\{2Z_{10} = \{0, 2, 4, 6, 8\} \subseteq Z_{10}\} \subseteq G; 1 \leq i \leq 10\} \subseteq S$$

is such that P_1 = {Collection of all subsets from the interval matrix groupoid P of M} $\subseteq$ S.

Thus we see we can have subset interval column matrix subgroupoid of S such that every element is a subset zero divisor.

Example 3.12: Let S = {Collection of all subsets from the interval column matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{14}, *, (7, 0)\}; 1 \leq i \leq 10\} \right\}$$

be the subset column interval matrix groupoid of finite order.

Let P = {Collection of all subsets from the column interval matrix groupoid

$$T = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in \{0,2,4,6,8,10,12\} \subseteq Z_{14}, 1 \leq i \leq 10\} \right\}$$

be the subset interval column matrix subgroupoid of S.

We see in P every element is subset zero divisor and further P is a subset interval column interval matrix subgroupoid of finite order.

Now we proceed onto give examples of subset interval matrix groupoids.

Example 3.13: Let S = {Collection of all subsets of the interval 3 × 2 matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{bmatrix} \middle| \; a_i, b_i \in G = Z_{40}, *, (10, 4)\}; 1 \leq i \leq 6\} \right\}$$

be the subset interval matrix groupoid of M.

$$\text{Let A } = \left\{\begin{bmatrix}[0,0] & [1,0] \\ [2,0] & [4,0] \\ [0,0] & [0,1]\end{bmatrix}, \begin{bmatrix}[1,6] & [0,0] \\ [10,0] & [0,4] \\ [1,0] & [0,0]\end{bmatrix}\right\} \text{ and}$$

$$B = \left\{\begin{bmatrix}[5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5]\end{bmatrix}\right\} \in S.$$

$$A * B = \left\{\begin{bmatrix}[0,0] & [1,0] \\ [2,0] & [4,0] \\ [0,0] & [0,1]\end{bmatrix}, \begin{bmatrix}[1,6] & [0,0] \\ [10,0] & [0,4] \\ [1,0] & [0,0]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,0] & [1,0] \\ [2,0] & [4,0] \\ [0,0] & [0,1]\end{bmatrix} * \begin{bmatrix}[5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5]\end{bmatrix}, \right.$$

$$\left. \begin{bmatrix}[1,6] & [0,0] \\ [10,0] & [0,4] \\ [1,0] & [0,0]\end{bmatrix} * \begin{bmatrix}[5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,0]*[5,1] & [1,0]*[0,2] \\ [2,0]*[0,6] & [4,0]*[0,0] \\ [0,0]*[0,0] & [0,1]*[1,5]\end{bmatrix}, \right.$$

$$\left. \begin{bmatrix}[1,6]*[5,1] & [0,0]*[0,2] \\ [10,0]*[0,6] & [0,4]*[0,0] \\ [1,0]*[0,0] & [0,0]*[1,5]\end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} [20,4] & [10,8] \\ [20,24] & [0,0] \\ [0,0] & [4,30] \end{bmatrix}, \begin{bmatrix} [30,24] & [0,8] \\ [20,24] & [0,0] \\ [10,0] & [4,20] \end{bmatrix} \right\} \text{ is in S.}$$

Consider

$$B * A = \left\{ \begin{bmatrix} [5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [0,0] & [1,0] \\ [2,0] & [4,0] \\ [0,0] & [0,1] \end{bmatrix}, \begin{bmatrix} [1,6] & [0,0] \\ [10,0] & [0,4] \\ [1,0] & [0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5] \end{bmatrix} * \begin{bmatrix} [0,0] & [1,0] \\ [2,0] & [4,0] \\ [0,0] & [0,1] \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} [5,1] & [0,2] \\ [0,6] & [0,0] \\ [0,0] & [1,5] \end{bmatrix} * \begin{bmatrix} [1,6] & [0,0] \\ [10,0] & [0,4] \\ [1,0] & [0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [5,1]*[0,0] & [0,2]*[1,0] \\ [0,6]*[2,0] & [0,0]*[4,0] \\ [0,0]*[0,0] & [1,5]*[0,1] \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} [5,1]*[1,6] & [0,2]*[0,0] \\ [0,6]*[10,0] & [0,0]*[0,4] \\ [0,0]*[1,0] & [1,5]*[0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [10,4] & [4,20] \\ [8,20] & [16,0] \\ [0,0] & [10,14] \end{bmatrix}, \begin{bmatrix} [14,34] & [0,20] \\ [0,20] & [0,16] \\ [4,0] & [10,10] \end{bmatrix} \right\} \in \text{S.}$$

We see clearly A * B ≠ B * A.

Thus S is a subset 3 × 2 matrix interval groupoid of finite order which is non commutative. S has subset zero divisors.

Example 3.14: Let S = {Collection of all subsets of the interval 2 × 5 matrix groupoid

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] & [a_5,b_5] \\ [a_6,b_6] & [a_7,b_7] & [a_8,b_8] & [a_9,b_9] & [a_{10},b_{10}] \end{bmatrix}\right.$$

where $a_i, b_i \in G = \{Z, *, (5, -5)\}; 1 \leq i \leq 10\}\}$

be the subset interval matrix groupoid of infinite order. Clearly S is non commutative. S has infinite number of nilpotent elements. Let P = {Collection of all subsets from the interval matrix subgroupoid

$$B = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] & [a_5,b_5] \\ [a_6,b_6] & [a_7,b_7] & [a_8,b_8] & [a_9,b_9] & [a_{10},b_{10}] \end{bmatrix}\right| a_i, b_i \in$$

$$G = \{Z, *, (5, -5)\}; 1 \leq i \leq 10\} \subseteq S\}\}$$

be the subset interval matrix subgroupoid of S. We see in P, for every A ∈ P

we have $A * A = \left\{\begin{pmatrix} [0,0] & \ldots & [0,0] \\ [0,0] & \ldots & [0,0] \end{pmatrix}\right\}$.

However $A * B \neq \left\{\begin{pmatrix} [0,0] & [0,0] & [0,0] & [0,0] & [0,0] \\ [0,0] & [0,0] & [0,0] & [0,0] & [0,0] \end{pmatrix}\right\}$

in general if A ≠ B.

Example 3.15: Let S = {Collection of all subsets of the 4 × 2 matrix interval groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \end{bmatrix} \middle| \; a_i, b_i \in G = \{C(Z_{20}), *, (i_F, 19i_F)\} \; 1 \leq i \leq 8 \} \right\}$$

be the subset interval matrix groupoid of finite complex modulo integers.

Example 3.16: Let S = {Collection of all subsets from the 8×8 interval matrix groupoid M = $\{([a_i, b_i]) \mid a_i, b_i \in G = \{C, *, (10, -i)\}; 1 \leq i \leq 64\}\}$ be the complex subset matrix interval groupoid of infinite order.

Example 3.17: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{11}, b_{11}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{\langle Z_{21} \cup I \rangle, *, (I, 0)\}; 1 \leq i \leq 11\} \right\}$$

be the neutrosophic subset interval matrix groupoid of finite order; S is non commutative and has subset zero divisors.

Example 3.18: Let S = {Collection of all subsets from the interval matrix groupoid M = {all 3×8 interval matrices with entries from the groupoid G = $\{Z_{10}(g_1, g_2), *, (5, 5g_1) \mid g_1^2 = 0, g_2^2 = g_2, g_1g_2 = g_2g_1 = 0\}\}\}$ be the subset interval matrix groupoid of mixed dual numbers.

We see S is non commutative and is of finite order.

Example 3.19: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \\ [a_9, b_9] & [a_{10}, b_{10}] \end{array}\right] \middle| \; a_i, b_i \in G = \{\langle C \cup I\rangle, *, (10, -10)\}; \right.$$

$$\left. 1 \leq i \leq 10 \right\}$$

be the subset interval matrix groupoid of complex neutrosophic numbers of infinite order.

Example 3.20: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left[\begin{array}{cccc} [a_1, b_1] & [a_2, b_2] & \ldots & [a_7, b_7] \\ [a_8, b_8] & [a_9, b_9] & \ldots & [a_{14}, b_{14}] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{36}, b_{36}] & [a_{37}, b_{37}] & \ldots & [a_{42}, b_{42}] \end{array}\right] \middle| \; a_i, b_i \in \right.$$

$$G = \{C(Z_{20}) (g_1, g_2, g_3); *, (10, 2) \mid g_1^2 = 0, \; g_2^2 = g_2,$$
$$\left. g_3^2 = -g_3, g_ig_j = g_jg_i = 0, i \neq j, 1 \leq i, j \leq 3\}; 1 \leq i \leq 42 \right\}\}$$

be the subset interval matrix groupoid of complex modulo mixed dual numbers of finite order.

Example 3.21: Let S = {Collection of all subsets from the interval 7×7 matrix groupoid with entries from G ={$C\langle Z_7 \cup I\rangle$, *, (3, 0)}} be the subset matrix interval groupoid of finite order of finite neutrosophic complex modulo integer groupoid.

Example 3.22: Let S = {Collection of all subsets from the 10×7 interval matrix groupoid G, where G = {$C\langle Z_{40} \cup I\rangle$ (g_1, g_2, g_3), *, (10, 2), $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = -g_3$, $g_ig_j = g_jg_i = 0$, $i \neq j$, $1 \leq i$, $j \leq 3$}} be the subset finite neutrosophic complex modulo integer mixed dual number groupoid of finite order.

Now having seen examples of all types of subset matrix interval groupoids we can study substructures in them which is considered as a matter of routine.

Now we proceed onto discuss other properties about these subset matrix interval groupoids.

In the first place as in case of usual groupoids we say a subset interval matrix groupoid S is Smarandache if S has a proper subset P $\subseteq$ S such that P is a subset matrix interval semigroup under the operations of S.

One of the ways to get Smarandache subset matrix interval groupoids S is that if the matrix interval groupoid M, from which subsets are taken for S is Smarandache then S will be Smarandache.

However it is not known if G or M is not a Smarandache groupoid will S be a Smarandache groupoid.

This task is solved for if H is a subset interval groupoid isomorphic with G as a singleton subset in S then that G as a whole is a semigroup for G * G = G.

We will illustrate this situation by some simple examples.

Example 3.23: Let S = {Collection of all subsets from the interval 1 × 3 matrix groupoid G = {$([a_1, b_1], [a_2, b_2], [a_3, b_3])$ | $a_i, b_i \in Z_3$, *, (2, 2), $1 \leq i \leq 3$}} be the subset row matrix interval groupoid of finite order.

Let P = {([0, 0], [0, 0], [0, 0]), ([1, 0], [0, 0], [0, 0]), ([2, 0], [0, 0], [0, 0])} $\subseteq$ S is a subset interval row matrix semigroup in S. For P * P = P. Thus S is a Smarandache subset interval row matrix groupoid.

Example 3.24: Let S = {Collection of all subsets from the matrix interval groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{12}, *, (5, 5)\}; 1 \leq i \leq 4\} \right\}$$

be the subset interval matrix groupoid.

$$P = \left\{ \begin{bmatrix} [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [1,1] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [2,2] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [3,3] \\ [0,0] \\ [0,0] \end{bmatrix}, \right.$$

$$\begin{bmatrix} [0,0] \\ [4,4] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [5,5] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [6,6] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [7,7] \\ [0,0] \\ [0,0] \end{bmatrix},$$

$$\left. \begin{bmatrix} [0,0] \\ [8,8] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [9,9] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [10,10] \\ [0,0] \\ [0,0] \end{bmatrix}, \begin{bmatrix} [0,0] \\ [11,11] \\ [0,0] \\ [0,0] \end{bmatrix} \right\} \subseteq S,$$

is such that P * P = P and P $\in$ S is a subset matrix interval subsemigroup of S.

Hence S is a Smarandache subset matrix interval groupoid.

Example 3.25: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{\left.\begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_5, b_5] \end{bmatrix}\right| a_i, b_i \in G = \{Z^+ \cup \{0\}, *, (2, 4)\}; 1 \leq i \leq 5\}\right\}$$

be the subset interval matrix groupoid.

Take P = {Collection of all subsets of the interval matrix subgroupoid N of M where

$$N = \left\{\left.\begin{bmatrix} [a_1, b_1] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}\right| a_1, b_1 \in G\right\} \subseteq M\};$$

P is a subset interval matrix subgroupoid of S.

Clearly if A, B $\in$ P then A * B $\neq$ B * A in general.

For take $A = \left\{\begin{bmatrix} [5,2] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}\right\}$ and $B = \left\{\begin{bmatrix} [8,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}\right\} \in P$.

$$A * B = \left\{\begin{bmatrix} [5,2] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}\right\} * \left\{\begin{bmatrix} [8,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} [5,2]*[8,0] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [42,4] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\} \qquad \ldots \qquad \text{I}$$

Consider

$$B * A = \left\{ \begin{bmatrix} [8,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [5,2] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [8,0]*[5,2] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \\ [0,0]*[0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [36,8] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\} \qquad \ldots \qquad \text{II}$$

Clearly I and II are distinct, hence P is a non commutative infinite subset interval matrix subgroupoid. Infact S has many such subset interval subgroupoids.

Example 3.26: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \\ [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \middle| \; a_i, b_i \in \right.$$

$$G = \{C(Z_{16}) *, (2, 0)\} \; 1 \leq i \leq 12\}\}$$

be the subset interval matrix complex modulo integer groupoid of finite order.

Take P = {Collection of all subsets of the interval matrix subgroupoid

$$N = \left\{ \begin{bmatrix} [0,0] & [0,0] & [0,0] \\ [0,0] & [a_1, b_1] & [0,0] \\ [0,0] & [0,0] & [0,0] \\ [0,0] & [a_2, b_2] & [0,0] \end{bmatrix} \middle| \; a_i, b_i \in \right.$$

$$G = \{C(Z_{16}) *, (2, 0)\} \; 1 \leq i \leq 2\} \subseteq M\}.$$

P is a subset interval matrix subgroupoid of finite order.

Clearly P is also a non commutative finite complex modulo integer subset interval matrix subgroupoid of S.

Example 3:27: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] & [a_9,b_9] \end{bmatrix} \right| a_i, b_i \in$$

$$G = \{\langle R^+ \cup I\rangle, *, (I, 0), 1 \leq i \leq 9\}\}$$

be the subset real positive neutrosophic interval matrix groupoid of infinite order.

S is a S-subset interval matrix groupoid and has several subset interval matrix subgroupoids.

Example 3.28: Let S = {Collection of all subsets of the matrix interval groupoid

$$M=\left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{15}, *, (7, 9)\}, 1 \leq i \leq 4\}\}$$

be the subset interval matrix groupoid of finite order.

We see S is a Smarandache subset interval matrix idempotent groupoid as G is an idempotent groupoid.

So S has subset matrix idempotent subsets.

For take $A = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \right\} \in S$, we see

$$A * A = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [a_1,b_1]*[a_1,b_1] & [a_2,b_2]*[a_2,b_2] \\ [a_3,b_3]*[a_3,b_3] & [a_4,b_4]*[a_4,b_4] \end{bmatrix} \right\}$$

$$= \left\{\begin{bmatrix} [7a_1+9a_1, 7b_1+9b_1] & [7a_2+9a_2, 7b_2+9b_2] \\ [7a_3+9a_3, 7b_3+9b_3] & [7a_4+9a_4, 7b_4+9b_4] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix}\right\} = A.$$

However if A is not a singleton subset (say);

$$A = \left\{\begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix}, \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix}\right\}, \in S$$

then A * A $\neq$ A in general.

$$\text{For } A * A = \left\{\begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix}, \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix}\right\} *$$

$$\left\{\begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix}, \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix} * \begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix},\right.$$

$$\begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix} * \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix},$$

$$\begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix} * \begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix},$$

$$\left.\begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix} * \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} [a_1, b_1] & [0,0] \\ [a_2, b_2] & [0,0] \end{bmatrix}, \begin{bmatrix} [a_3, b_3] & [0,0] \\ [0,0] & [a_4, b_4] \end{bmatrix}, \right.$$

$$\begin{bmatrix} [7a_1 + 9a_3, 7b_1 + 9b_3] & [0,0] \\ [7a_2, 7b_2] & [9a_4, 9b_4] \end{bmatrix},$$

$$\left. \begin{bmatrix} [7a_3 + 9a_1, 7b_3 + 9a_1] & [0,0] \\ [9a_2, 9b_2] & [7a_4, 9b_4] \end{bmatrix} \right\} \in S.$$

Clearly A * A ≠ A however we observe A ⊆ A * A.

Now in this context we define a new notion about subset matrix interval groupoids.

Let S be any subset matrix interval groupoid if for any A ∈ S, A * A is such that A ⊆ A * A then we call A to be the idempotent inclusive subset of S. If every A ∈ S is such that A ⊆ A * A then we define S to be subset interval matrix idempotent inclusive matrix groupoid.

Clearly all subset interval matrix idempotent inclusive groupoid is a Smarandache subset interval matrix idempotent groupoid.

Example 3.29: Let S = {Collection of all subsets from the matrix interval groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \\ [a_6, b_6] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{17}, *, (10, 8)\}, 1 \leq i \leq 6\} \right\}$$

be the subset interval matrix groupoid of finite order. Clearly S is a subset interval matrix idempotent inclusive groupoid. Also S is a Smarandache subset interval matrix idempotent groupoid.

$$\text{For take A} = \left\{\begin{bmatrix}[3,2]\\[0,5]\\[7,3]\\[5,1]\\[0,0]\\[1,0]\end{bmatrix}\right\} \in \text{S}.$$

$$\text{Consider A * A} = \left\{\begin{bmatrix}[3,2]\\[0,5]\\[7,3]\\[5,1]\\[0,0]\\[1,0]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[3,2]\\[0,5]\\[7,3]\\[5,1]\\[0,0]\\[1,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[3,2]*[3,2]\\[0,5]*[0,5]\\[7,3]*[7,3]\\[5,1]*[5,1]\\[0,0]*[0,0]\\[1,0]*[1,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[30+24\ (\text{mod}\ 17), 20+16\ (\text{mod}\ 17)]\\ [0, (50+40)\ (\text{mod}\ 17)]\\ [70+56\ (\text{mod}\ 17), 30+21\ (\text{mod}\ 17)]\\ [50+40\ (\text{mod}\ 17), 10+8\ (\text{mod}\ 17), 1]*[5,1]\\ [0,0]\\ [10+8\ (\text{mod}\ 17), 0]\end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} [3,2] \\ [0,5] \\ [7,3] \\ [5,1] \\ [0,0] \\ [1,0] \end{bmatrix} \right\} = A \in S.$$

$$\text{Let } B = \left\{ \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix}, \begin{bmatrix} [2,1] \\ [3,0] \\ [0,5] \\ [4,1] \\ [0,0] \\ [5,4] \end{bmatrix} \right\} \in S.$$

$$B * B = \left\{ \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix}, \begin{bmatrix} [2,1] \\ [3,0] \\ [0,5] \\ [4,1] \\ [0,0] \\ [5,4] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix}, \begin{bmatrix} [2,1] \\ [3,0] \\ [0,5] \\ [4,1] \\ [0,0] \\ [5,4] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix} * \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix}, \begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix} * \begin{bmatrix} [2,1] \\ [3,0] \\ [0,5] \\ [4,1] \\ [0,0] \\ [5,4] \end{bmatrix}, \right.$$

$$\left.\begin{bmatrix}[2,1]\\[3,0]\\[0,5]\\[4,1]\\[0,0]\\[5,4]\end{bmatrix}*\begin{bmatrix}[2,1]\\[3,0]\\[0,5]\\[4,1]\\[0,0]\\[5,4]\end{bmatrix},\begin{bmatrix}[2,1]\\[3,0]\\[0,5]\\[4,1]\\[0,0]\\[5,4]\end{bmatrix}*\begin{bmatrix}[1,2]\\[0,3]\\[5,0]\\[1,4]\\[0,0]\\[4,5]\end{bmatrix}\right\}$$

$$=\left\{\begin{bmatrix}[1,2]*[1,2]\\[0,3]*[0,3]\\[5,0]*[5,0]\\[1,4]*[1,4]\\[0,0]*[0,0]\\[4,5]*[4,5]\end{bmatrix},\begin{bmatrix}[1,2]*[2,1]\\[0,3]*[3,0]\\[5,0]*[0,5]\\[1,4]*[4,1]\\[0,0]*[0,0]\\[4,5]*[5,4]\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}[2,1]*[2,1]\\[3,0]*[3,0]\\[0,5]*[0,5]\\[4,1]*[4,1]\\[0,0]*[0,0]\\[5,4]*[5,4]\end{bmatrix},\begin{bmatrix}[2,1]*[1,2]\\[3,0]*[0,3]\\[0,5]*[5,0]\\[4,1]*[1,4]\\[0,0]*[0,0]\\[5,4]*[4,5]\end{bmatrix}\right\}$$

$$=\left\{\begin{bmatrix}[1,2]\\[0,3]\\[5,0]\\[1,4]\\[0,0]\\[4,5]\end{bmatrix},\begin{bmatrix}[2,1]\\[3,0]\\[0,5]\\[4,1]\\[0,0]\\[5,4]\end{bmatrix},\right.$$

$$\left.\begin{bmatrix} [10+16, 20+8] \\ [24, 30] \\ [50, 40] \\ [10+32, 40+8] \\ [0, 0] \\ [40+40, 50+32] \end{bmatrix}, \begin{bmatrix} [20+8, 10+16] \\ [30, 24] \\ [40, 50] \\ [40+8, 10+32] \\ [0, 0] \\ [50+32, 40+40] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [1,2] \\ [0,3] \\ [5,0] \\ [1,4] \\ [0,0] \\ [4,5] \end{bmatrix}, \begin{bmatrix} [2,1] \\ [3,0] \\ [0,5] \\ [4,1] \\ [0,0] \\ [5,4] \end{bmatrix}, \begin{bmatrix} [9,11] \\ [7,13] \\ [16,13] \\ [8,14] \\ [0,0] \\ [12,14] \end{bmatrix}, \begin{bmatrix} [11,9] \\ [13,7] \\ [16,13] \\ [14,8] \\ [0,0] \\ [14,12] \end{bmatrix}\right\} \in S.$$

Clearly $A \subseteq A * A$. Thus we see S is a subset interval matrix inclusive idempotent groupoid as well as Smarandache subset interval idempotent groupoid.

Example 3.30: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{\begin{bmatrix} [a_1, b_1] & \dots & [a_6, b_6] \\ [a_7, b_7] & \dots & [a_{12}, b_{12}] \\ \vdots & \vdots & \vdots \\ [a_{31}, b_{31}] & \dots & [a_{36}, b_{36}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{41}, *, (20, 22)\}, \; 1 \leq i \leq 36\right\}\}$$

be the subset interval matrix groupoid.

S is both a Smarandache subset interval matrix idempotent groupoid and subset interval matrix inclusive idempotent groupoid.

Example 3.31: Let S = {Collection of all subsets from the interval matrix groupoid M; where M = {([a_1, b_1], [a_2, b_2], …, [a_9, b_9]) | a_i, b_i ∈ G = {Z_{23}, *, (10, 14)}, 1 ≤ i ≤ 9}} be the subset interval row matrix groupoid which is both subset interval row matrix idempotent inclusive groupoid as well as Smarandache subset interval row matrix idempotent groupoid.

Example 3.32: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_9, b_9] \\ [a_7, b_7] & \ldots & [a_{18}, b_{18}] \\ [a_{19}, b_{19}] & \ldots & [a_{27}, b_{27}] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{27}, *, (13, 15)\}, \right.$$

$$1 \leq i \leq 27\}\}$$

be the subset interval matrix groupoid.

Clearly S is both Smarandache subset interval matrix idempotent groupoid as well as subset matrix interval idempotent inclusive groupoid.

Example 3.33: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ \vdots & \vdots & \vdots \\ [a_{34}, b_{34}] & [a_{35}, b_{35}] & [a_{36}, b_{36}] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{18}, *, \right.$$

$$(11, 8)\}\ 1 \leq i \leq 36\}\}$$

be the subset interval matrix groupoid.

Clearly S is a Smarandache subset interval matrix idempotent groupoid as well as subset interval matrix idempotent inclusive groupoid.

In view of all these examples we give a class of subset interval matrix groupoid as well as subset interval matrix idempotent inclusive groupoid.

THEOREM 3.3: *Let S = {Collection of all subsets from interval matrix groupoid M = {Collection of t × s interval matrix from the groupoid G = {Z_n, *, (p, r) where p + r = 1 (mod n)}}} be the subset interval matrix groupoid. S is a subset matrix interval inclusive idempotent groupoid and S-subset interval matrix idempotent groupoid.*

Proof follows from simple number theoretic techniques by using p + r ≡ 1 (mod n).

Example 3.34: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & \ldots & [a_{10},b_{10}] \\ [a_{11},b_{11}] & [a_{12},b_{12}] & \ldots & [a_{20},b_{20}] \\ [a_{21},b_{21}] & [a_{22},b_{22}] & \ldots & [a_{30},b_{30}] \end{bmatrix} \right| a_i, b_i \in G = \right.$$

$$\{Z_{12}, *, (0, 4)\}\ 1 \leq i \leq 30\}\}$$

be the subset interval matrix groupoid. S is a Smarandache subset matrix interval groupoid.

Example 3.35: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] \\ [a_2,b_2] \\ \vdots \\ [a_9,b_9] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{24}, *, (0, 9)\};\ 1 \leq i \leq 9\}\right\}$$

be the subset interval matrix groupoid which is a Smarandache subset matrix interval P-groupoid of finite order.

Example 3.36: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \\ [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in G = \{(Z_{15}) *, (0, 10)\}\ 1 \leq i \leq 12\}\}$$

be the subset interval matrix groupoid. S is a Smarandache subset matrix interval P-groupoid of finite order.

Example 3.37: Let S = {Collection of all subsets of the interval matrix groupoid M = {($[a_1, b_1], \ldots, [a_9, b_9]$) | $a_i, b_i \in G = \{Z_{10}, *, (0, 6)\}; 1 \leq i \leq 9\}\}$ be the subset interval matrix groupoid.

Clearly S is a Smarandache subset interval matrix P-groupoid of finite order.

In view of all these we give a class of Smarandache subset interval matrix P-groupoids.

THEOREM 3.4: *S = {Collection of all subsets of the interval matrix groupoid M = {Collection of all m × n interval matrices with entries from the groupoid G = {Z_t, *, (0, r)}}} is a Smarandache subset interval matrix P-groupoid if and only if $r^2 = r \pmod t$.*

Follows from the simple argument M is a S-matrix interval P-groupoid if and only if $r^2 = r$ (mod t) and G is a P-groupoid if and only if $r^2 = r$ (mod t).

It is pertinent to recall on record; we say a subset structure S enjoys a property if the basic structure over which it is built enjoys that property. We say the subset structure enjoys it as a property.

That is why we say in this theorem S is a Smarandache subset interval matrix P-groupoid for G is a P-groupoid if $r^2 = r$ (mod t).

Example 3.38: Let S = {Collection of all subsets from the interval matrix groupoid M = $\{([a_1, b_1], [a_2, b_2], [a_3, b_3], [a_4, b_4]) \mid a_i, b_i \in G = \{Z, *, (10, -10)\}, 1 \leq i \leq 4\}\}$ be the subset interval matrix groupoid of infinite order.

S has subset matrix interval subgroupoids. S has subset matrix interval ideals. S has infinite number of subset nilpotent elements of order two.

All properties studied and defined for interval matrix groupoids are true in case of subset interval matrix groupoids with appropriate modifications. The concept of S-subset interval matrix subgroupoids, subset interval matrix S-ideals can also be defined, described and developed in an appropriate way with out any difficulty.

However it is important to keep on record that the exact behaviour of subset interval matrices as subset may enjoy more properties than the interval matrix groupoid or the groupoid itself.

This is an interesting and open problem for the interested reader to address.

Likewise we see the concept of S-subset interval matrix zero divisors, S-subset idempotents can be defined and developed as a matter of routine.

Now this task is also left as an exercise to the reader. Finally we can define and find conditions for the subset interval matrix groupoids to satisfy the special identities as Moufang identity, Bruck identity, Bol identity and so on.

This task is also left as an exercise to the reader. However we suggest some problems to this end.

Now throughout in case of subset row matrix interval groupoids or subset column matrix interval groupoids or subset rectangular matrix interval groupoids we have used only natural product $\times_n$ and however the natural product, as it is inherited from the groupoid invariably the subset matrix interval groupoids are non commutative.

Also only in case of subset interval square matrix groupoids we have defined only the natural product for the other product involves also addition so we have not used it.

Now we proceed onto define the notion of subset interval polynomial groupoids.

DEFINITION 3.2: *Let S = {Collection of all subsets from the interval polynomial groupoid M =* $\left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in \{G, *;\right.$ *a groupoid of finite or infinite order}}}. S under the operations of G is defined as the subset interval polynomial groupoid of the interval polynomial groupoid M.*

We will give examples of this new structures.

Example 3.39: Let S = {Collection of all subsets of the interval polynomial groupoid M = $\left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{Z_{12}, *, (3, 0)\}\right\}\}$ be the subset interval polynomial groupoid of infinite order.

A = $\{[9, 2]x^3 + [0, 1]x + [2, 0], [3, 0]x^2 + [7, 2]\}$ and

B = $\{[7, 3]x + [0, 5]\} \in$ S.

We now show how the operations * is defined on S.

$$A * B = \{[9, 2]x^3 + [0, 1]x + [2, 0], [3, 0]x^2 + [7, 2]\} * \{[7, 3]x + [0, 5]\}$$

$$= \{([9, 2] * [7, 3]x^4 + ([0, 1] * [7, 3]x^2 + ([2, 0] * [7, 3]x + ([9, 2] * [0, 5]x^3 + ([0, 1] * [0, 5]x + [2, 0] * [0, 5], ([3, 0] * [7, 3]x^3 + ([7, 2] * [7, 3]x + ([3, 0] * [0, 5]x^2 + ([7, 2] * [0, 5]\}$$

$$= \{[3, 6]\, x^4 + [0, 3]\, x^2 + [6, 0]\, x + [3, 6]\, x^3 + [0, 3]x + [6, 0], [9, 0]x^3 + [9, 6]x + [9, 0]x^2 + [9, 6]\}$$

$$= \{[3, 6]x^4 + [0, 3]x^2 + [3, 6]x^3 + [6, 3]\, x + [6, 0], [9, 0]x^3 + [9, 0]x^2 + [9, 6]x + [9, 6]\} \in S.$$

This is the way * operation is performed on the subset polynomial interval groupoid. Clearly S is non commutative.

For take $A = \{[4, 2]\, x^3 + [3, 1]\}$ and
$B = \{[7, 3]\, x + [2, 0]\} \in S$.

Consider

$$\begin{aligned} A * B &= \{[4, 2]\, x^3 + [3, 1]\} * \{[7, 3]\, x + [2, 0]\} \\ &= \{([4, 2] * [7, 3])\, x^4 + ([3, 1] * [7, 3])x + ([4, 2] * [2, 0])x^3 + [3,1] * [2, 0]\} \\ &= \{[0, 6]\, x^4 + [9, 3]\, x + [0, 6]x^3 + [9, 3]\} \qquad \ldots \text{ I} \end{aligned}$$

Now we find

$$\begin{aligned} B * A &= \{[7, 3]\, x + [2, 0]\} * \{[4, 2]\, x^3 + [3, 1]\} \\ &= \{([7, 3] * [4, 2])\, x^4 + ([2, 0] * [4, 2])\, x^3 + ([7, 3] * [3, 1])\, x + [2, 0] * [3, 1]\} \\ &= \{[9, 9]x^4 + [6, 0]x^3 + [9, 9]x + [6, 0]\} \qquad \ldots \text{ II} \end{aligned}$$

Clearly I and II are not equal. Thus $A * B \neq B * A$ in general for $A, B \in S$.

Hence S is a non commutative subset interval polynomial groupoid.

Example 3.40: Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z_7, *, (0, 2)\}\right\}\}$$

be the subset polynomial interval groupoid.

We see S is of infinite order. S is non commutative and non associative subset interval polynomial groupoid.

Example 3.41: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z, *, (7, -7)\}\right\}\}$$

be the subset interval polynomial groupoid.

Let $A = \{[3, 3]x^7 + [13, 13]\}$ and
$B = \{[3, 3]x + [13, 13]\} \in S$.

$$\begin{aligned} A * B &= \{[3, 3]x^7 + [13, 13]\} * \{[3, 3]x + [13, 13]\} \\ &= \{([3, 3] * [3, 3])x^8 + ([13, 13] * [3, 3])x + \\ &\quad ([3, 3] * [13, 13])x^7 + [13, 13] * [13, 13]\} \\ &= \{[0, 0]x^8 + [70, 70]x + [70, -70]x^7 + [0, 0]\} \in S. \end{aligned}$$

This is the way operation is defined on S.

Clearly $A * B \neq B * A$ in general for $A, B \in S$.

Example 3.42: Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z_{10}, *, (5, 5)\}\right\}\}$$

be the subset interval polynomial groupoid of M.

Clearly S is a commutative subset polynomial interval groupoid. However S is not associative.

For take

$A = \{[2, 4]x^2 + [3, 1]x + [0, 1]\}$
$B = \{[7, 2]x\}$ and
$C = \{[1,0]\} \in S.$

We find

$$\begin{aligned}(A * B) * C &= \{([2, 4]x^2 + [3, 1]\, x + [0, 1]\} * \{[7, 2]x^3\} * C \\ &= \{([2, 4] * [7, 2])\, x^3 + ([3, 1] * [7, 2])x^2 + \\ &\quad ([0, 1]*[7,2])x\} * C \\ &= \{[5, 0]x^3 + [0, 5]x^2 + [5, 5]x\}\ \{[1, 0]\} \\ &= \{[5, 0] * [1, 0])x^3 + ([0, 5] * [1, 0])\, x^2 + \\ &\quad ([5, 5] * [1, 0]x\} \\ &= \{[0, 0]\, x^3 + [5, 5]\, x^2 + [0, 5]x\} \qquad \ldots\ \text{I}\end{aligned}$$

Consider A * (B * C)

$$\begin{aligned}&= A * (\{[7, 2]x\} * \{[1, 0]\}) \\ &= A * \{([7, 2] * [1, 0])x\} \\ &= A * \{[0, 0]\, x\} \\ &= \{[2, 4]x^2 + [3, 1]x + [0, 1]\} * \{[0, 0]x\} \\ &= \{([2, 4] * [0, 0])x^3 + ([3, 1] * [0, 0])x^2 + \\ &\quad ([0, 1]* [0, 0]x) \\ &= \{[0, 0]x^3 + [5, 5]x^2 + [0, 5]x\} \qquad \ldots\ \text{II}\end{aligned}$$

We see (A * B) * C = A * (B * C) for this A, B, C $\in$ S.

It is left as an exercise for the reader to test the associativity of S.

Example 3.43: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z_{17}, *, (8, 0)\}\right\}\}$$

be the subset interval polynomial groupoid.

Clearly S is both non commutative as well as non associative.

Let $A = \{[3, 0]x^8 + [6, 2]\}$,
$B = \{[7, 5]\, x + [2, 1]\}$ and $C = \{[3, 5]\} \in S$.

We find

$$\begin{aligned}
A * B &= \{[3, 0]x^8 + [6, 2]\} * \{[7, 5]\, x + [2, 1]\} \\
&= \{([3, 0] * [7, 5]\, x^9 + ([6, 2] * [7, 5])\, x + \\
&\quad ([3, 0] * [2, 1])x^8 + [6, 2] * [2, 1]\} \\
&= \{[7, 0]\, x^9 + [14, 16]x + [7, 0]x^8 + [14, 16]\} \qquad \text{(a)}
\end{aligned}$$

Consider

$$\begin{aligned}
B * A &= \{[7, 5]\, x + [2, 1]\} * \{[3, 0]x^8 + [6, 2]\} \\
&= \{([7, 5] * [3, 0]x^9 + ([2, 1] * [3, 0])x^8 + \\
&\quad ([2, 1] * [6, 2]) + ([7, 5] * [6, 2])x\} \\
&= \{[5, 6]x^9 + [16, 8]x^8 + [16, 8] + [5, 6]x\} \qquad \text{(b)}
\end{aligned}$$

We see (a) and (b) are distinct; hence $A * B \neq B * A$.

Consider

$$\begin{aligned}
(A * B) * C &= \{[7, 0]x^9 + [14, 16]x + [7, 0]x^8 + \\
&\quad [14, 16]\} * \{[3, 5]\}
\end{aligned}$$

(Using (a) the value of $A * B$).

$$\begin{aligned}
&= \{([7, 0] * [3, 5])\, x^9 + ([14, 16] * [3, 5])x + \\
&\quad ([7, 0] * [3, 5]x^8 + [14, 16] * [3, 5]\} \\
&= \{[5, 0]x^9 + [10, 11]\, x + [5, 0]x^8 + [10, 11] \qquad \ldots \quad \text{I}
\end{aligned}$$

We find $A * (B * C)$

$$\begin{aligned}
&= A* (\{[7, 5]x + [2, 1]\} * \{[3, 5]\}) \\
&= A* (\{[7, 5] * [3, 5])x + ([2, 1] * [3, 5])\} \\
&= A * \{[5, 6]\, x + [14, 7]\} \\
&= \{[3, 0]x^8 + [6, 2]\} + \{[5, 6]x + [14, 7]\} \\
&= \{([3, 0] * [5, 6]x^9 + ([6, 2] * [5,6]x + \\
&\quad ([3, 0] * [14, 7]x^8 + ([6, 2] * [14, 77]\} \\
&= \{[7, 0]x^9 + [14, 16]x + [7, 0]x^8 + [14, 16]\} \qquad \ldots \quad \text{II}
\end{aligned}$$

I and II are distinct; so $(A * B) * C \neq A * (B * C)$ for all A, B, $C \in S$.

Example 3.44: Let S = {Collection of all subsets from the interval polynomial subset groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{C(Z_{12}),\ *,\ (i_F, 0)\}\right\}\}$$

be the subset polynomial interval groupoid of finite complex modulo integers.

Example 3.45: Let S = {Collection of all subsets from the interval polynomial subset groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{\langle Z_{15} \cup I\rangle,\ *,\ (I, 3I)\}\right\}\}$$

be the subset finite neutrosophic interval polynomial groupoid.

Example 3.46: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{Z_{16}(g_1, g_2),\ g_1^2 = 0,\right.$$
$$\left. g_2^2 = g_2, g_1g_2 = g_2g_1 = 0, *, (4, 0)\}\right\}\}$$

be the subset polynomial interval groupoid of mixed dual numbers.

Example 3.47: Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{\langle C\cup I\rangle, *, (0, 8)\}\right\}\}$$

be the subset interval complex polynomial groupoid of M.

Example 3.48: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{\langle Z^+ \cup \{0\} \cup I\rangle, *, (7, 4)\}\right\}\}$$

be the subset interval neutrosophic polynomial groupoid.

Example 3.49: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{C\langle Z_5 \cup I\rangle (g_1, g_2), *, (2, 3) \text{ with } g_1^2 = 0,\ g_2^2 = g_2,\ g_1g_2 = g_2g_1 = 0\}\right\}\}$$

be the subset interval polynomial groupoid of finite complex neutrosophic mixed dual groupoid.

Example 3.50: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{Z_{17} (g_1, g_2, g_3), *, (10, 8),\ g_1^2 = 0,\ g_2^2 = g_2,\ g_3^2 = -g_3,\ g_ig_j = g_jg_i;\ i \neq j,\ 1 \leq i, j \leq 3\}\right\}\}$$

be the special mixed dual number interval polynomial groupoid.

Here it is pertinent to keep on record that the subset polynomial interval groupoids in general are non commutative has zero divisors.

We can as a matter of routine define the notion of subset of subset polynomial interval subgroupoids, subset polynomial interval ideals and their Smarandache analogue.

We will only provide an example or two.

Example 3.51: Let S = {Collection of all subsets from the subset polynomial interval groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z_{12}, *, (6, 0)\}\right\}\}$$

be the subset interval polynomial groupoid.

Let

$$P = \left\{\sum_{i=0}^{\infty}[a_i, 0]x^i \;\middle|\; a_i \in \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}\right\} \subseteq M$$

be the polynomial interval subgroupoid. B = {Collection of all subsets of the interval polynomial subgroupoid P of M} $\subseteq$ S is the subset interval polynomial subgroupoid of S.

Infact B is also a subset interval polynomial ideal of S.

Example 3.52: Let S = {Collection of all subsets from the subset polynomial interval groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z, *, (2, 0)\}\right\}\}$$

be the subset interval polynomial groupoid.

S has subset interval polynomial subgroupoid as well as subset interval polynomial ideals.

Example 3.53: Let S = {Collection of all subsets from the subset polynomial interval groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{R, *, (2, 0)\}\right\}\}$$

be the subset interval polynomial groupoid.

S has zero divisors.

S has subset polynomial interval subgroupoid as well as subset polynomial interval ideals.

Consider P = {Collection of all subsets from the subset interval polynomial subgroupoid

$$B = \left\{ \sum_{i=0}^{\infty} [0, a_i] x^i \,\middle|\, a_i \in G = \{R, *, (2, 0)\}\right\} \subseteq M\}$$

contained in S is a subset interval polynomial subgroupoid of S.

Now interested reader can study the notion of subset interval polynomial groupoids.

We can define for these subset polynomial interval groupoids the notion of Smarandache groupoids to be Smarandache alternative, Smarandache Bol, Smarandache Moufang and so on.

All these are considered as a matter of routine and hence left as an exercise to the reader.

However we supply some examples of these concepts.

Example 3.54: Let S = {Collection of all subsets from the subset polynomial interval groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{Z_{11}, *, (5, 5)\}\right\}\}.$$

It is easily verified S is a Smarandache polynomial P-groupoid.

However S is not a Smarandache subset interval polynomial alternative groupoid.

Example 3.55: Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{Z_{19}, *, (11, 11)\}\right\}\}$$

be the subset interval polynomial groupoid which is a Smarandache interval polynomial P-groupoid.

Example 3.56: Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [0, a_i] x^i \,\middle|\, a_i \in G = \{Z_{24}, *, (9, 9)\}\right\}\}$$

be the subset interval polynomial groupoid.

Clearly S is a Smarandache subset interval polynomial P-groupoid.

In view of these examples we have the following interesting theorem.

THEOREM 3.5: *Let S = {Collection of all subsets from the interval polynomial groupoid*

$$M = \left\{ \sum_{i=0}^{\infty} [0, a_i] x^i \,\middle|\, a_i \in G = \{Z_n, *, (t, t); t, a_i \in Z_n\}\right\}\}$$

be the subset polynomial interval groupoid. Clearly S is a Smarandache subset interval polynomial P-groupoid.

Proof is direct and hence left as an exercise to the reader.

Prove; the S in Theorem 3.5 is not a Smarandache subset interval polynomial alternative groupoid.

Almost all properties enjoyed by interval polynomial groupoids described in the book [int. group] can be derived in case of subset interval polynomial groupoids with appropriate modifications.

This work is left as an exercise to the reader.

We suggest the following problems:

Problems

1. Obtain some special features enjoyed by subset interval row matrix groupoids of finite order.

2. Let S = {Collection of all subsets of the interval 1×5 row matrix groupoid M = {([a_1, b_1], [a_2, b_2], …, [a_5, b_5]) | a_i, b_i $\in$ G = {Z_{12}, *, (4, 0)}, $1 \leq i \leq 5$}} be the subset row matrix interval groupoid of M.

   (i) Find o(S).
   (ii) Prove S has subset zero divisors.
   (iii) Find subset idempotents if any in S.
   (iv) Can S have subset S-zero divisors?
   (v) Is it possible to have subset right zero divisors which are not subset left zero divisors and vice versa?
   (vi) Can S have subset units?
   (vii) Can S have subset right units which are not subset left units?

3. Let S = {Collection of all subsets of the interval matrix groupoid M = {([a_1, b_1], …, [a_{10}, b_{10}]) | a_i, b_i $\in$ G = {Z_{40}, *, (2, 28)} where $1 \leq i \leq 10$}} be the subset interval matrix groupoid of the interval matrix groupoid M.

   (i) Find o(S).
   (ii) Find all subset interval matrix subgroupoids of S.
   (iii) Find all subset interval matrix S-ideals of S.
   (iv) Can S have subset S-zero divisors?
   (v) Can S have subset zero divisors which are not subset S-zero divisors?
   (vi) Can S have subset ideals which are not subset S-ideals?
   (vii) Can S have subset interval matrix subgroupoids which are not subset S-subgroupoids?

4. Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{29}, *, (22, 8)\}; 1 \leq i \leq 10 \right\}\}.$$

(a) Study questions (i) to (vii) of problem 3 for this S.
(b) Study questions (i) to (vii) of problem 2 for this S.

5. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & [a_{12}, b_{12}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \ldots & [a_{30}, b_{30}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z, *, (10, -10)\}; 1 \leq i \leq 30 \right\}\}$$

be the subset interval matrix groupoid of M.

(a) Study questions (i) to (vii) of problem (3) for this S.
(b) Study questions (i) to (vii) of problem (2) for this S.

6. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{43}, *, (7, 0)\}, 1 \leq i \leq 4 \right\}\}$$

be the subset matrix interval groupoid of M.

(a) Study questions (i) to (vii) of problem (2) for this S.
(b) Study questions (i) to (vii) of problem (3) for this S.

7. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ \vdots & \vdots & \vdots \\ [a_{22}, b_{22}] & [a_{23}, b_{23}] & [a_{24}, b_{24}] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{18}, *, (6, 3)\}, 1 \leq i \leq 24\}\}$$

be the subset interval matrix groupoid of M.

(a) Study questions (i) to (vii) of problem (2) for this S.
(b) Study questions (i) to (vii) of problem (3) for this S.

8. Let S = {Collection of all subsets of the matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{14}, *, (10, 5)\}, 1 \leq i \leq 9\}\}$$

be the subset interval matrix groupoid of M.

(i) Is S a Smarandache subset interval matrix P-groupoid?
(ii) Find o(S).
(iii) Is S a subset interval matrix idempotent inclusive groupoid?
(iv) Does S satisfy any other special identity?
(v) Can S have S-subset zero divisors?

9. Give an example of a subset interval matrix groupoid of infinite order which is a Smarandache Moufang groupoid.

10. Does there exist a subset interval matrix groupoid which does not satisfy any of the special identities?

11. Does there exist a finite subset interval matrix groupoid which has no subset zero divisors?

12. Give an example of a finite subset interval matrix groupoid which has subset zero divisors?

13. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{\left.\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ \vdots \\ [a_9,b_9]\end{bmatrix}\right| a_i, b_i \in G = \{Z_{12}, *, (4, 9)\}; 1 \le i \le 9\}\}$$

be the subset interval matrix groupoid of M.

(i) Is S a Smarandache subset matrix interval P-groupoid?

(ii) Is S a Smarandache subset matrix interval alternative groupoid?

(iii) Can S be a S-subset matrix interval idempotent groupoid?

(iv) Can S be a subset matrix interval idempotent inclusive groupoid?

14. Let S = {Collection of all subsets from the matrix interval groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{15}, b_{15}] & [a_1, b_{16}] \end{array} \right] \right| a_i, b_i \in G = \{Z_{42}, *, (7, 0)\};$$

$$1 \leq i \leq 16\}\}$$

be the subset interval matrix groupoid of M.

Study questions (i) to (iv) of problem 13 for this S.

15. How many subset matrix interval groupoids can constructed using M = {[a, b] | a, b $\in$ G = {Z_{42}, *, (t, s); t, s $\in$ Z_{42}}}?

16. Does there exist a formula to find the number of elements in S if the matrix order is given and order of G is given?

17. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{array} \right] \right| a_i, b_i \in G = \{Z_{53}, *, (10, 0)\},$$

$$1 \leq i \leq 6\}\}$$

be the subset interval matrix groupoid.

(i) Study questions (i) to (vii) of problem 2 for this S.
(ii) Study questions (i) to (vii) of problem 3 for this S.

18. Let S_1 = {Collection of all subsets of the interval matrix groupoid M = {([a_1, b_1], …, [a_5, b_5]) | a_i, b_i $\in$ G = {Z_{53}, *, (10, 0)}, 1 $\leq$ i $\leq$ 5}} be the subset interval groupoid.

(i) Compare S_1 with S of problem 17.
(ii) Is o (S_1) > o(S) ?

(iii) Study questions (i) to (vii) of problem (2) for this S_1.
(iv) Study questions (i) to (vii) of problem (3) for this S_1.

19. Let S_2 = {collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_7, b_7] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{53}, *, (10, 0)\}, 1 \leq i \leq 7 \right\}\Bigg\}$$

be the subset interval matrix groupoid.

(i) Compare S_2 with S_1 of problem 18 and S of problem (17).
(ii) Is $o(S_2) > o(S_1) > o(S)$?
(iii) Can one say with increase in the order of matrix there will be an increase in the order of S?
(iv) Study questions (i) to (vii) of problem (2) for this S_2.
(v) Study questions (i) to (vii) of problem (3) for this S_2.

20. Let S_3 = {Collection of all subsets of the inter matrix groupoid M = $\{([a_1, b_1], [a_2, b_2]) \mid a_i, b_i \in G = \{Z_{53}, *, (10, 0)\}, 1 \leq i \leq 2\}\}$ be the subset interval matrix groupoid of M.

(i) Find $o(S_3)$.
(ii) Is $o(S_2) > o(S_1) > o(S) > o(S_3)$?
(iii) Study questions (i) to (vii) of problem (2) for this S_3.
(iv) Study questions (i) to (vii) of problem (3) for this S_3.

21. Let S_4 = {Collection of all subsets of the interval matrix groupoid M = $\{([a_1, b_1], [a_2, b_2]) \mid a_i, b_i \in G = \{Z_{100}, *, (10, 10)\}, 1 \leq i \leq 2\}\}$ be the subset matrix interval groupoid.

(i) Is $o(S_4) > o(S_3)$; S_3 given in problem 20.

(ii) Prove with increase in the order of the groupoid G for the fixed order of the matrix the cardinality of the subset matrix interval groupoid will increase.
(iii) Prove with increase in the order of the matrix with fixed order of the groupoid the cardinality of the subset matrix interval groupoid will increase.

22. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{15}, b_{15}] & [a_{16}, b_{16}] \end{bmatrix} \right| a_i, b_i \in G = \{\langle Z \cup I\rangle, *, \right.$$

$$(10, -10I)\}; 1 \leq i \leq 16\}\}$$

be the subset matrix interval neutrosophic groupoid of infinite order.

(i) Can S have S-subset zero divisors?
(ii) Does S satisfy any of the special identities?
(iii) Can S have subset S-units?
(iv) Can S have subset idempotents which are not S-subset idempotents?
(v) Can S have subset nilpotents of order two?

23. Let S_1 = {Collection of all subsets of the interval matrix groupoids

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \dots & [a_5, b_5] \\ [a_6, b_6] & \dots & [a_{10}, b_{10}] \\ \vdots & \vdots & \vdots \\ [a_{21}, b_{21}] & \dots & [a_{25}, b_{25}] \end{bmatrix} \right| a_i, b_i \in G = \{Z (g_1, g_2), *, \right.$$

$$(10g_2 - 10g1)\}; 1 \leq i \leq 25\}\}$$

be the subset matrix interval groupoid of M.

(i) Study questions (i) to (v) of problem 22 for this S.

24. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \dots & [a_5, b_5] \\ [a_6, b_6] & \dots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \dots & [a_{15}, b_{15}] \end{bmatrix} \middle| \; a_i, b_i \in \right.$$

$$\left. G = \{\langle R^+ \cup \{0\} \cup I\rangle, *, (10, 0)\}; 1 \leq i \leq 15\} \right\}$$

be the subset matrix interval groupoid.

Study questions (i) to (v) of problem 22 for this S.

25. Give any of the special and stricking features enjoyed by subset interval matrix groupoids of finite order.

26. Can these new algebraic structures be applied to mathematical models?

27. Let S_1 = {Collection of all subsets form the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in G = \{C(Z_{12}), \; *, (9, 0)\}, \right.$$

$$\left. 1 \leq i \leq 9\} \right\}$$

be the finite subset interval matrix complex modulo integer groupoid.

(a) Study questions (i) to (vii) of problem (2) for this S_1.
(b) Study questions (i) to (vii) of problem (3) for this S_1.

28. Let S_2 = {Collection of all subsets form the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in \right.$$

$$G = \{Z_{12}\,(g_1, g_2), \; *, (9, 0)\}, 1 \le i \le 9 \text{ with}$$

$$g_1^2 = 0, \; g_2^2 = g_2, g_1g_2 = g_2g_1 = 0\}\}$$

be the subset interval matrix groupoid.

(a) Study questions (i) to (vii) of problem (2) for this S_2.
(b) Study questions (i) to (vii) of problem (3) for this S_2.

29. Let S_3 = {Collection of all subsets form the interval matrix groupoid $M = \{([a_1, b_1], \ldots, [a_{12}, b_{12}]) \mid a_i, b_i \in G = \{C(Z_{12})\,(g), \; *, (9, 0)\}, g^2 = 0; 1 \le i \le 12\}\}$be the subset interval matrix groupoid of M.

(a) Study questions (i) to (vii) of problem (2) for this S_3.
(b) Study questions (i) to (vii) of problem (3) for this S_3.

30. Compare S_1 of problem 27, S_2 of problem 28 and S_3 of problem 29 in terms of order, subset zero divisors, S-subset idempotents and S-subset ideals.

31. Let S = {Collection of all subsets of the interval matrix groupoid M = {all 10 × 5 interval matrices with entries from the groupoid $G = \{C(Z_4), *, (2, 2)\}\}\}$ be the subset matrix interval groupoid of M.

(a) Study questions (i) to (vii) of problem (2) for this S.
(b) Study questions (i) to (vii) of problem (3) for this S.

32. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_2, b_2] & [a_3, b_3] \\ \vdots & \vdots \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] \end{array} \right] \right| a_i, b_i \in$$

$$G = \{C(Z_{15})\ (g_1, g_2),\ *, (10, 5)\}, 1 \leq i \leq 22\}\}$$

be the subset interval matrix groupoid of finite complex modulo integer dual numbers of finite order.

(a) Study questions (i) to (vii) of problem (2) for this S.

(b) Study questions (i) to (vii) of problem (3) for this S.

33. Characterize those subset interval matrix groupoids which have no subset interval S-ideals but has subset interval matrix S-subgroupoids.

34. Give an example of a subset interval matrix subgroupoid which has subset S-zero divisors.

35. Does there exist a subset interval matrix groupoid which has subset S-idempotents?

36. Give an example of a subset interval matrix groupoid which has subset zero divisors but has no subset S-zero divisors.

37. Obtain some special features enjoyed by subset polynomials interval groupoids of polynomials interval groupoids where the groupoid G is of finite order.

38. Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] & [a_7, b_7] & [a_8, b_8] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{13}, b_{13}] & [a_{14}, b_{14}] & [a_{15}, b_{15}] & [a_{16}, b_{16}] \end{bmatrix} \right| a_i, b_i \in \right.$$

$$G = \{C\langle Z_{18} \cup I\rangle\ (g_1, g_2),\ g_1^2 = g_1,$$

$$g_2^2 = -g_2;\ g_1g_2 = g_2g_1 = 0, *, (9, 0)\}\}\}$$

be the subset interval matrix groupoid.

(i) Find o(S); does it depend on o(G) and order the matrix?
(ii) Study questions (i) to (vii) of problem (2) for this S.
(iii) Study questions (i) to (vii) of problem (3) for this S.

39. Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \left. \sum_{i=0}^{\infty} [a_i, b_i] x^i \right| a_i, b_i \in G = \{Z_{12}, *, (7, 7)\}\right\}$$

be the subset interval polynomial groupoid.

(i) Can S have subset zero divisors?
(ii) Can S have subset interval polynomial subgroupoids?
(iii) Is S commutative?
(iv) Can S have subset interval polynomial ideals?
(v) Is S a Smarandache subset interval polynomial groupoid?

40. Let S_1 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{Z_{48}, *, (12, 12)\}\right\}\}$$

be the subset interval polynomial groupoid.

Study questions (i) to (v) of problem 39 for this S_1.

41. Let S_2 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[0, a_i]x^i \,\middle|\, a_i \in G = \{Z_{15}, *, (3, 3)\}\right\}\}$$

be the subset interval polynomial groupoid.

Study questions (i) to (v) of problem 39 for this S_2.

42. Does there exist subset interval polynomial groupoid which has no subset interval polynomial ideals?

43. Does there exists subset interval polynomial groupoid which are not Smarandache?

44. Does there exist a subset interval polynomial groupoid which has no subset interval polynomial S-ideals?

45. Does there exist a subset interval polynomial groupoid which is a Smarandache subset interval polynomial alternative groupoid?

46. Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{C\langle Z_7 \cup I\rangle, *, (3, 4)\}\right\}\}$$

be the subset interval polynomial groupoid.

(i) Find S-subset ideals of S.
(ii) Find S-subset interval polynomial subgroupoids which are not S-subset ideals.
(iii) Find S-subset zero divisors if any in S.

47. Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{C\langle Z \cup I\rangle, \ *, (1, -1)\}\right\}\}$$

be the subset polynomial interval groupoid.

Study questions (i) to (iii) of problem 46 for this S.

48. Let S = {Collection of all subsets of the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{C\langle Z_{19} \cup I\rangle, \ *, (2, 0)\}\right\}\}$$

be the subset polynomial interval groupoid.

Study questions (i) to (iii) of problem 46 for this S.

49. Does there exist a subset interval polynomial groupoid S which has no subset S-ideals?

50. Let S = {Collection of all subsets from the subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \;\middle|\; a_i, b_i \in G = \{Z_6, \ *, (3, 4)\}\right\}\}$$

be the subset polynomial interval groupoid.

(i) Prove S satisfies some of the special identities as Smarandache subset interval polynomial groupoid.
(ii) Can P be a S-subset interval polynomial Bol groupoid?

51. Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in G = \{R, *, (3, 0)\} \right\}\}$$

be the subset polynomial interval groupoid.

(i) Define degree of subset interval polynomial in S.
(ii) Derive the usual properties enjoyed by S.
(iii) Show S has subset interval polynomials which can be subset zero divisors?
(iv) Can S ever have subset idempotents? Justify.

52. Can we say for S in problem (51) if R is replaced by C in the groupoid G all interval subset polynomials are solvable?

53. S_1 = Collection of all subsets form the interval subset polynomial

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in G = \{C(Z_p), *, (0, t); \text{p a prime}, t \in Z_p\} \right\}\}$$

be the subset interval polynomial groupoid.

Can we say all subset interval polynomial equations are solvable?

54. Study problem 53 if $C(Z_p)$ is replaced by $\{C\langle Z_{24} \cup I\rangle\}$.

55. Study problem 53 if $C(Z_p)$ is replaced by $C(Z_6)$.

Chapter Four

SUBSET INTERVAL MATRIX LOOP GROUPOIDS

In this chapter we use instead of groupoids, loops to build subset interval matrix groupoids and subset interval polynomial groupoids. We basically use loops instead of groupoids. Though we use loops in place of groupoids still the structure remains to be only a subset groupoid and not a subset loop.

But to signify the use of loops we define them as subset interval matrix loop groupoid or subset interval polynomial loop groupoid.

DEFINITION 4.1: *Let S = {Collection of all subsets from the interval loop matrix groupoid M = {Collection of all interval m $\times$ n matrices with entries from the loop L}}. S under the operations of L is a groupoid and S is defined as the subset interval matrix loop groupoid.*

We will illustrate this situation by some examples.

Example 4.1: Let S = {Collection of all subsets of the interval matrix loop groupoid M = $\{([a_1, b_1], [a_2, b_2], \ldots, [a_9, b_9]) \mid a_i, b_i$

$\in L_{11}(7);\ 1 \leq i \leq 9\}\}$. S is a subset interval row matrix loop groupoid of finite order.

Example 4.2: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_7, b_7] \end{bmatrix} \right| a_i, b_i \in L_5(2);\ 1 \leq i \leq 7 \right\}\}$$

be the subset interval matrix loop groupoid of finite order.

Example 4.3: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \right| a_i, b_i \in L_7(3);\ 1 \leq i \leq 4 \right\}$$

where the table of $L_7(3)$ is as follows:

	e	g_1	g_2	g_3	g_4	g_5	g_6	g_7
e	e	g_1	g_2	g_3	g_4	g_5	g_6	g_7
g_1	g_1	e	g_4	g_7	g_3	g_6	g_2	g_5
g_2	g_2	g_6	e	g_5	g_1	g_4	g_7	g_3
g_3	g_3	g_4	g_7	e	g_6	g_2	g_5	g_1
g_4	g_4	g_2	g_5	g_1	e	g_7	g_3	g_6
g_5	g_5	g_7	g_3	g_6	g_2	e	g_1	g_4
g_6	g_6	g_5	g_1	g_4	g_7	g_3	e	g_2
g_7	g_7	g_3	g_6	g_2	g_5	g_1	g_4	e

be the subset interval matrix loop groupoid.

Clearly S is finite non commutative and non associative subset interval matrix loop groupoid.

Let A = $\left\{\begin{bmatrix}[g_1,g_3]\\ [e,g_2]\\ [e,g_1]\\ [g_4,e]\end{bmatrix}, \begin{bmatrix}[e,g_1]\\ [g_1,g_2]\\ [g_3,e]\\ [e,g_4]\end{bmatrix}\right\}$ and B = $\left\{\begin{bmatrix}[g_7,e]\\ [e,g_6]\\ [g_2,g_5]\\ [g_1,g_3]\end{bmatrix}\right\} \in S$.

We now find

$$A * B = \left\{\begin{bmatrix}[g_1,g_3]\\ [e,g_2]\\ [e,g_1]\\ [g_4,e]\end{bmatrix}, \begin{bmatrix}[e,g_1]\\ [g_1,g_2]\\ [g_3,e]\\ [e,g_4]\end{bmatrix}\right\} * \left\{\begin{bmatrix}[g_7,e]\\ [e,g_6]\\ [g_2,g_5]\\ [g_1,g_3]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[g_1,g_3]\\ [e,g_2]\\ [e,g_1]\\ [g_4,e]\end{bmatrix} * \begin{bmatrix}[g_7,e]\\ [e,g_6]\\ [g_2,g_5]\\ [g_1,g_3]\end{bmatrix}, \begin{bmatrix}[e,g_1]\\ [g_1,g_2]\\ [g_3,e]\\ [e,g_4]\end{bmatrix} * \begin{bmatrix}[g_7,e]\\ [e,g_6]\\ [g_2,g_5]\\ [g_1,g_3]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[g_1,g_3]*[g_7,e]\\ [e,g_2]*[e,g_6]\\ [e,g_1]*[g_2,g_5]\\ [g_4,e]*[g_1,g_3]\end{bmatrix}, \begin{bmatrix}[e,g_1]*[g_7,e]\\ [g_1,g_2]*[e,g_6]\\ [g_3,e]*[g_2,g_5]\\ [e,g_4]*[g_1,g_3]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[g_5,g_3]\\ [e,g_7]\\ [g_2,g_6]\\ [g_2,g_3]\end{bmatrix}, \begin{bmatrix}[g_1,g_1]\\ [g_1,g_7]\\ [g_7,g_5]\\ [g_1,g_1]\end{bmatrix}\right\} \in S.$$

Consider $B * A = \left\{ \begin{bmatrix} [g_7, e] \\ [e, g_6] \\ [g_2, g_5] \\ [g_1, g_3] \end{bmatrix} \right\} * \left\{ \begin{bmatrix} [g_1, g_3] \\ [e, g_2] \\ [e, g_1] \\ [g_4, e] \end{bmatrix}, \begin{bmatrix} [e, g_1] \\ [g_1, g_2] \\ [g_3, e] \\ [e, g_4] \end{bmatrix} \right\}$

$$= \left\{ \begin{bmatrix} [g_7, e] \\ [e, g_6] \\ [g_2, g_5] \\ [g_1, g_3] \end{bmatrix} * \begin{bmatrix} [g_1, g_3] \\ [e, g_2] \\ [e, g_1] \\ [g_4, e] \end{bmatrix}, \begin{bmatrix} [g_7, e] \\ [e, g_6] \\ [g_2, g_5] \\ [g_1, g_3] \end{bmatrix} * \begin{bmatrix} [e, g_1] \\ [g_1, g_2] \\ [g_3, e] \\ [e, g_4] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [g_7, e]*[g_1, g_3] \\ [e, g_6]*[e, g_2] \\ [g_2, g_5]*[e, g_1] \\ [g_1, g_3]*[g_4, e] \end{bmatrix}, \begin{bmatrix} [g_7, e]*[e, g_1] \\ [e, g_6]*[g_1, g_2] \\ [g_2, g_5]*[g_3, e] \\ [g_1, g_3]*[e, g_4] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [g_3, g_3] \\ [e, g_1] \\ [g_2, g_7] \\ [g_3, g_3] \end{bmatrix}, \begin{bmatrix} [g_7, g_1] \\ [g_1, g_1] \\ [g_5, g_5] \\ [g_1, g_6] \end{bmatrix} \right\} \in S.$$

Clearly $A * B \neq B * A$.

Thus S is a non commutative subset interval matrix loop groupoid of finite order.

Example 4.4: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in L_7(4); 1 \leq i \leq 4 \right\}\}$$

be the subset interval matrix loop groupoid of finite order. The table of the loop $L_7(4)$ is as follows:

*	e	g_1	g_2	g_3	g_4	g_5	g_6	g_7
e	e	g_1	g_2	g_3	g_4	g_5	g_6	g_7
g_1	g_1	e	g_5	g_2	g_6	g_3	g_7	g_4
g_2	g_2	g_5	e	g_6	g_3	g_7	g_4	g_1
g_3	g_3	g_2	g_6	e	g_7	g_4	g_1	g_5
g_4	g_4	g_6	g_3	g_7	e	g_1	g_5	g_2
g_5	g_5	g_3	g_7	g_4	g_1	e	g_2	g_6
g_6	g_6	g_7	g_4	g_1	g_5	g_2	e	g_3
g_7	g_7	g_4	g_1	g_5	g_2	g_6	g_3	e

We see clearly S the subset interval matrix loop groupoid of finite order which is commutative but non associative.

We only use the natural product and in this natural product $\times_n$ by *.

We will just illustrate how the product operation * on S is performed.

Let $A = \left\{ \begin{bmatrix} [g_1, e] & [g_7, g_1] \\ [e, g_2] & [g_3, g_2] \end{bmatrix}, \begin{bmatrix} [e, g_6] & [g_6, g_2] \\ [g_2, g_4] & [g_1, e] \end{bmatrix} \right\}$ and

$$B = \left\{ \begin{bmatrix} [e, g_4] & [g_1, e] \\ [g_1, g_5] & [g_3, g_7] \end{bmatrix} \right\} \in S.$$

We now find

$$A * B = \left\{ \begin{bmatrix} [g_1, e] & [g_7, g_1] \\ [e, g_2] & [g_3, g_2] \end{bmatrix}, \begin{bmatrix} [e, g_6] & [g_6, g_2] \\ [g_2, g_4] & [g_1, e] \end{bmatrix} \right\} *$$

$$\left\{ \begin{bmatrix} [e, g_4] & [g_1, e] \\ [g_1, g_5] & [g_3, g_7] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [g_1, e] & [g_7, g_1] \\ [e, g_2] & [g_3, g_2] \end{bmatrix} * \begin{bmatrix} [e, g_4] & [g_1, e] \\ [g_1, g_5] & [g_3, g_7] \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} [e, g_6] & [g_6, g_2] \\ [g_2, g_4] & [g_1, e] \end{bmatrix} * \begin{bmatrix} [e, g_4] & [g_1, e] \\ [g_1, g_5] & [g_3, g_7] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [g_1, e]*[e, g_4] & [g_7, g_1]*[g_1, e] \\ [e, g_2]*[g_1, g_5] & [g_3, g_2]*[g_3, g_7] \end{bmatrix} \right.$$

$$\left. \begin{bmatrix} [e, g_6]*[e, g_4] & [g_6, g_2]*[g_1, e] \\ [g_2, g_4]*[g_1, g_5] & [g_1, e]*[g_3, g_7] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [g_1, g_4] & [g_4, g_1] \\ [g_1, g_7] & [e, g_1] \end{bmatrix}, \begin{bmatrix} [e, g_5] & [g_7, g_2] \\ [g_5, g_1] & [g_2, g_7] \end{bmatrix} \right\} \in S.$$

This is the way product is defined on S.

We see it is easy to verify S is commutative but non associative as a subset interval matrix groupoid.

Example 4.5: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & \ldots & [a_{12}, b_{12}] \end{bmatrix} \middle| \; a_i, b_i \in L_9(5); \; 1 \leq i \leq 12 \right\} \}$$

be the subset interval matrix loop groupoid of finite order.

We see S is also a commutative subset matrix interval loop groupoid of finite order.

Example 4.6: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{\begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in L_{45}(8);\ 1 \leq i \leq 10\}\right\}$$

be the subset interval matrix groupoid of finite order.

Clearly S has subset interval matrix subgroupoids.

S is also non commutative and of finite order.

Let P_1 = {Collection of all subsets from the interval matrix loop subgroupoid

$$M_1 = \left\{\begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in H_1(15) \subseteq L_{45}(8);\right.$$

$$\left. 1 \leq i \leq 10\right\}\} \subseteq S;$$

be a subset interval matrix loop subgroupoid of S.

P_2 = {Collection of all subsets of the interval matrix loop subgroupoid

$$M_2 = \left\{\begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in H_2(15) \subseteq L_{45}(8);\right.$$

$$\left. 1 \leq i \leq 10\right\}\} \subseteq S;$$

is a subset interval matrix loop subgroupoid of S.

Take

N_1 = {Collection of all subsets of the interval matrix loop subgroupoid}.

B_1 = {Collection of all subsets of the interval matrix loop subgroupoid

$$M_3 = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \\ \vdots & \vdots \\ [a_9,b_9] & [a_{10},b_{10}] \end{bmatrix} \right| a_i, b_i \in H_1(9) \subseteq L_{45}(8); \right.$$

$$1 \leq i \leq 10\} \subseteq M\}$$

be the subset interval matrix loop subgroupoid of S.

Example 4.7: Let S = {Collection of all subsets from the matrix of the interval loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] & [a_9,b_9] \\ [a_{10},b_{10}] & [a_{11},b_{11}] & [a_{12},b_{12}] \end{bmatrix} \right| a_i, b_i \in L_{43}(8); \right.$$

$$1 \leq i \leq 12\}\}$$

be the subset interval matrix loop groupoid of M.

$L_{43}(8)$ has no subloops other than groups of order two say $H_i = \{e, i\}$ where $i \in L_{43}(8)$.

We see P_i = {Collection of all subsets from the subset interval loop subgroupoid

$$M_i = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \\ [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in H(e, i) \right.$$

with $i \in L_{43}(8) \setminus \{e\}; 1 \leq i \leq 12\}\} \subseteq M\} \subseteq S$

is a subset interval matrix loop subgroupoid of S.

Clearly we have 43 such subset interval matrix loop subgroupoids all of them are associative, that is these 43 subset interval matrix loop subgroupoids are subset interval matrix loop semigroups of S.

Example 4.8: Let S = {Collection of all subsets from the subset interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{bmatrix} \right| a_i, b_i \in L_{29}(9); 1 \leq i \leq 20\} \right\}$$

be the subset interval matrix loop groupoid of M.

S has subset interval matrix loop subgroupoids which are not associative.

Example 4.9: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & \ldots & [a_{30}, b_{30}] \\ [a_{31}, b_{31}] & \ldots & [a_{40}, b_{40}] \end{bmatrix} \right| a_i, b_i \in L_{53}(4); 1 \leq i \leq 40\} \right\}$$

be the subset interval matrix loop groupoid.

S has atleast 53 subset interval matrix loop subgroupoids which are associative and S has atleast 53 subset interval matrix loop semigroups of finite order.

Example 4.10: Let S = {Collection of all subsets from the interval matrix loop groupid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] & [a_7, b_7] & [a_8, b_8] \\ [a_9, b_9] & [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in L_{165}(8); \; 1 \leq i \leq 12 \right\}\}$$

be the subset interval matrix loop groupoid. Clearly S has 165 subset interval matrix loop semigroups and 124 subset interval matrix loop subgroupoids which are not associative.

In view of all these we have the following theorems.

THEOREM 4.1: *Let S = {Collection of all subsets from the interval matrix loop groupoid M = {m × n interval matrices with entries from the loop $L_p(m)$; p a prime}} be the subset interval matrix loop groupoid. S has p number of subset interval matrix loop semigroups.*

The proof is direct and hence left as an exercise to the reader.

However it is left as an open problem for the reader to find existence of a subset interval matrix loop subgroupoid which is not a semigroup.

THEOREM 4.2: *Let S = {Collection of all subsets from the interval matrix groupoid M = {All s × t matrices with entries from the loop $L_n(m)$; $n = p_1 p_2 p_3$ (each of the p_i's are distinct primes)}} be the subset interval matrix loop groupoid. S has* *atleast $p_1 + p_2 + p_3 + p_1p_2 + p_2p_3 + p_1p_3$ number of subset interval matrix loop subgroupoids.*

The proof is left as an exercise to the reader.

We will give examples of them.

Infact for any n = $p_1^{\alpha_1} \ldots p_t^{\alpha_t}$; $\alpha_i \geq 1$, $1 \leq i \leq t$ we can find the number of subset interval matrix loop subgroupoids.

Example 4.11: Let S = {Collection of all subsets from the subset interval groupoid

$$M = \left\{ \left[\begin{matrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_9, b_9] \\ [a_{10}, b_{10}] & [a_{11}, b_{11}] & \ldots & [a_{18}, b_{18}] \end{matrix} \right] \middle| \, a_i, b_i \in L_{315}(23); \right.$$

$$1 \leq i \leq 18\}\}$$

be the subset interval matrix loop groupoid.

The subloops of $L_{315}(23)$ are

$H_1(3)$ = {e, 4, 7, 10, 13, …, 313}
$H_2(3)$ = {e, 5, 8, 11, 14, …, 314}
and $H_3(3)$ = {e, 6, 9, 12, 15, …, 315}.

$H_i(9)$ = {e, 10, 19, …, 307} and so on
$H_9(9)$ = {e, 9, 18, …, 306, 315}.

$H_1(5)$ = {e, 6, 11, …, 310}
$H_2(5)$ = {e, 7, 12, …, 311} and so on.
$H_5(5)$ = {e, 5, 10, …, 315}.
$H_1(15)$ = {e, 16, …, 301}
and so on.

$H_{15}(15)$ = {e, 15, …, 300, 315}.
$H_1(7)$ = {e, 8, 15, …, 309}
$H_2(7)$ = {e, 9, 16, …, 310}
and so on.

$H_7(7) = \{e, 7, 14, \ldots, 308, 315\}$.
$H_i(21)$, $H_j(35)$, $i = 1, 2, \ldots, 21$ and $j = 1, 2, \ldots, 35$.

Thus we have atleast
$3 + 9 + 5 + 7 + 15 + 21 + 35 + 45 + 63 = 203$ number of subloops.

Corresponding to each of these 203 subloops we can have 203 subset interval matrices subloop subgroupoids.

Hence the claim.

Example 4.12: Let S = {Collection of all subsets from the interval matrix groupoid $M = \{([a_1, b_1] [a_2, b_2], [a_3, b_3]) \mid a_i, b_i \in L_7(5), 1 \leq i \leq 3\}\}$ be the subset interval matrix loop groupoid.

Let $T = \{([e, g_1], [e, e], [e, e]), ([e, g_2], [e, e], [e, e]), ([e, g_3], [e, e], [e, e]), ([e, g_4], [e, e], [e, e]), ([e, g_5], [e, e], [e, e]), ([e, g_6], [e, e], [e, e]), ([e, g_7], [e, e], [e, e]), ([e, e], [e, e], [e, e])\} \subseteq S$ be a subset interval matrix loop subgroupoid which is such that $T * T = T$.

If $P = \{A_1 = \{([e, e], [e, e], [e, e])\}, A_2 = \{([g1, g1], [e, e], [e, e])\}, A_3 = \{([g_2, g_2], [e, e], [e, e])\}, A_4 = \{([g_3, g_3], [e, e], [e, e])\}, A_5 = \{([g_4, g_4], [e, e], [e, e])\}, A_6 = \{([g_5, g_5], [e, e], [e, e])\}, A_7 = \{([g_6, g_6], [e, e], [e, e])\}$ and $A_8 = \{([g_7, g_7], [e, e], [e, e])\}\} \subseteq S$.

We see P is a subset interval matrix loop subgroupoid of S and $P \cong L_7(5)$ by mapping $\eta (\{([e, e], [e, e], [e, e])\}) = e$.

$\eta (\{([g_i, g_i], [e, e], [e, e])\}) = g_i$; $i = 1, 2, \ldots, 7$ and $\eta : P \to L_7(5)$ where P is a subset interval matrix loop groupoid which is a subset interval matrix loop and η is a loop isomorphic from P into the loop $L_7(5)$.

Thus the subset interval matrix loop groupoid has subset interval matrix loop which are isomorphic with the loop $L_7(5)$.

Example 4.13: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & \ldots & [a_5,b_5] \\ [a_6,b_6] & [a_7,b_7] & \ldots & [a_{10},b_{10}] \\ \vdots & \vdots & & \vdots \\ [a_{21},b_{21}] & [a_{22},b_{22}] & \ldots & [a_{25},b_{25}] \end{bmatrix} \right| a_i, b_i \in L_{15}(8); \right.$$

$$1 \leq i \leq 25\}\}$$

be the subset interval matrix loop groupoid.

We see S has over 15 subset interval matrix loop subgroupoids which are subset interval matrix loops and are isomorphic with the loop $L_{15}(8)$.

Further S has atleast 8 subset interval matrix loop subgroupoids.

Example 4.14: Let S = {Collection of all subsets from the interval matrix loop groupoid of

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ \vdots & \vdots \\ [a_{15},b_{15}] & [a_{16},b_{16}] \end{bmatrix} \right| a_i, b_i \in L_{11}(7); 1 \leq i \leq 16\}\} \right.$$

be the subset interval matrix loop groupoid.

We see
P_1 = {Collection of all subset interval matrices

$$\left\{ \left\{ \begin{bmatrix} [e,e] & [e,e] \\ \vdots & \vdots \\ [e,e] & [e,e] \end{bmatrix} \right\}, \left\{ \begin{bmatrix} [g_1,g_1] & [e,e] \\ [e,e] & [e,e] \\ \vdots & \vdots \\ [e,e] & [e,e] \end{bmatrix} \right\}, \right.$$

$$\left\{\begin{bmatrix} [g_2, g_2] & [e,e] \\ [e,e] & [e,e] \\ \vdots & \vdots \\ [e,e] & [e,e] \end{bmatrix}, \ldots, \begin{bmatrix} [g_{11}, g_{11}] & [e,e] \\ [e,e] & [e,e] \\ \vdots & \vdots \\ [e,e] & [e,e] \end{bmatrix}\right\} \subseteq S$$

is a subset interval matrix loop groupoid of S.

Clearly P is a subset interval matrix loop of S and $\eta : P \to L_{11}(7)$ is a loop isomorphism given by

$$\eta\left\{\begin{bmatrix} [g_i, g_i] & [e,e] \\ [e,e] & [e,e] \\ \vdots & \vdots \\ [e,e] & [e,e] \end{bmatrix}\right\} = g_i,\ i = 1, 2, \ldots, 11.$$

We have several subset interval matrix loop subgroupoids which are subset interval matrix loop and are isomorphic with $L_{11}(7)$.

The authors leave it as an open problem about the existence of subset interval matrix loop which are different from $L_{11}(7)$ and its subloops.

That is does there exist a subset interval matrix loop not isomorphic to $L_{11}(7)$ or any of its subloops?

Example 4.15: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{\begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{16}, b_{16}] \end{bmatrix} \middle|\ a_i, b_i \in L_{27}(11);\ 1 \leq i \leq 16\right\}\}$$

be the subset interval matrix groupoid.

S has subset interval matrix subgroupoids which are subset interval matrix loops and are isomorphic with $L_{27}(11)$ or $H_i(3)$ or $H_i(9)$.

Thus having seen some of the substructure of subset interval matrix loop groupoids; now we proceed onto describe those subset interval matrix loop groupoids which satisfy any of the special identities.

We will only illustrate these situations by some examples.

Example 4.16: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_9, b_9] \\ [a_{10}, b_{10}] & \ldots & [a_{18}, b_{18}] \end{bmatrix} \right| a_i, b_i \in L_{19}(10); 1 \leq i \leq 18 \right\} \}$$

be the subset interval matrix loop groupoid which is commutative and of finite order.

Example 4.17: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \end{bmatrix} \right| a_i, b_i \in L_{43}(7); 1 \leq i \leq 8 \right\} \}$$

be the subset interval matrix loop.

Clearly S is non commutative.

However S is a Smarandache strongly commutative subset matrix loop groupoid.

Example 4.18: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{27}, b_{27}] \end{bmatrix} \middle| \ a_i, b_i \in L_{29}(8); 1 \leq i \leq 27\} \right\}$$

be the subset interval matrix loop.

Clearly S is a Smarandache strongly commutative subset interval matrix loop groupoid.

In view of this we have the following theorem.

THEOREM 4.3: *Let S = {Collection of all subsets from the interval matrix loop groupoid M, where the interval elements are from $L_p(m)$, p an odd prime} be the subset interval matrix loop groupoid. S is a Smarandache strongly commutative subset interval matrix loop groupoid.*

Proof follows from the fact $L_p(m)$ where p is a prime is a Smarandache strongly commutative loop.

We say a subset interval matrix loop groupoid S is Smarandache strongly cyclic if every subgroup in S is cyclic.

We will first give examples of them before we proceed onto prove some properties.

Example 4.19: Let S = {Collection of all subsets from the interval matrix groupoid;

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{bmatrix} \middle| \ a_i, b_i \in L_{29}(3); 1 \leq i \leq 6\} \right\}$$

be the subset interval matrix loop groupoid.

S is clearly a Smarandache strongly cyclic subset interval matrix groupoid.

Example 4.20: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \\ [a_6, b_6] \end{bmatrix} \middle| \; a_i, b_i \in Z_{105}(17); 1 \leq i \leq 6 \right\}\}$$

be the subset interval matrix loop groupoid and S is also a Smarandache cyclic subset interval matrix loop groupoid.

Example 4.21: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & \ldots & [a_{12}, b_{12}] \\ \vdots & \vdots & & \vdots \\ [a_{55}, b_{55}] & [a_{56}, b_{56}] & \ldots & [a_{60}, b_{60}] \end{bmatrix} \middle| \; a_i, b_i \in Z_{57}(29); \right.$$

$$\left. 1 \leq i \leq 60 \right\}\}$$

be a subset interval matrix loop groupoid and S is a Smarandache commutative subset interval matrix loop groupoid.

THEOREM 4.4: *Let S = {Collection of all subsets from the interval matrix loop groupoid M = {set of all n × m interval matrices with entries from $L_p(t)$; p a prime}} be the subset interval matrix loop groupoid. S is a Smarandache cyclic subset interval matrix loop groupoid.*

Proof follows from the fact that $L_p(t)$; $1 < t < p$; p a prime is a Smarandache strongly cyclic loop.

We can define the notion of S-Lagrange subset interval matrix loop groupoid and S-weakly Lagrange subset interval matrix loop groupoid.

The definition for subsets is just defined as an inherited Smarandache property.

For if the subset interval matrix loop groupoid uses a loop L which a S-Lagrange loop then we define S to be a S-Lagrange subset matrix interval loop groupoid provided the subloops which give way to subset interval matrix loop groupoids H are such that o(H) | o(S).

But this study is very difficult for the order of S is very large and finding the divisibility happens to be a challenging problem.

Example 4.22: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{array} \right] \middle| \; a_i, b_i \in L_9(8); 1 \leq i \leq 9 \right\}\}$$

be the subset interval matrix loop groupoid.

Clearly S is a Smarandache subset interval matrix loop groupoid.

Example 4.23: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ \vdots & \vdots & \vdots \\ [a_{28}, b_{28}] & [a_{29}, b_{29}] & [a_{30}, b_{30}] \end{bmatrix} \middle| \; a_i, b_i \in L_{29}(9); \right.$$

$$\left. 1 \le i \le 30 \right\}\}$$

be the subset interval matrix loop groupoid. Clearly S is a Smarandache subset interval matrix loop groupoid.

In view of these examples we can say that a subset interval matrix loop groupoid S is a Smarandache subset interval matrix loop groupoid if the loop L over which S is built is a Smarandache loop.

THEOREM 4.5: *Let S = {Collection of all subsets from the interval matrix loop groupoid M = {all m × n interval matrices with elements from $L_s(t)$; s > 3, s odd and 1 < t < s with (t – 1, s) = (t, s) = 1}} be the subset interval matrix loop groupoid. S is a Smarandache subset interval matrix loop groupoid.*

Follows from the fact that $L_s(t)$ is a S-loop.

We define a subset interval matrix loop groupoid S of a loop L to be Smarandache simple subset interval matrix loop groupoid if L is a Smarandache simple loop.

Example 4.24: Let S = {Collection of all subsets from the interval matrix groupoid M = {($[a_1, b_1], [a_2, b_2], \ldots, [a_9, b_9]$) | $a_i, b_i \in L_{29}(7)$; $1 \le i \le 9$}} be the subset interval matrix loop groupoid.

Clearly S is a Smarandache subset interval matrix loop groupoid.

Example 4.25: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_{15}, b_{15}] & [a_{16}, b_{16}] \end{array} \right] \middle| \; a_i, b_i \in L_{55}(13); 1 \leq i \leq 16 \right\}\}$$

be the subset interval matrix loop groupoid.

Clearly S is a Smarandache simple subset interval matrix loop groupoid as $L_{55}(13)$ is S-simple.

In view of these examples we have the following theorem the proof is left as an exercise to the reader.

THEOREM 4.6: *Let S = {Collection of all subsets from interval matrix loop groupoid M = {t × s interval matrices with entries from the loop $L_n(m)$}} be the subset interval matrix loop groupoid of the loop $L_n(m)$. S is a Smarandache simple subset interval matrix loop groupoid.*

Example 4.26: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} [a_1, b_1] & \ldots & [a_9, b_9] \\ [a_{10}, b_{10}] & \ldots & [a_{18}, b_{18}] \\ [a_{19}, b_{19}] & \ldots & [a_{27}, b_{27}] \end{array} \right] \middle| \; a_i, b_i \in L_{23}(7); 1 \leq i \leq 27 \right\}\}$$

be the subset interval matrix loop groupoid.

We see {x} = {[x, e]} or {x} = {[x,x]} or {x} = {[e,x]} ∈ S is such that $\{x\}^2 = \{e\}$ but $2 \nmid o(S)$ as $o(S) = 2^{|M|}-1$.

We see S has no Cauchy element but in $L_{23}(7)$ every element is S-Cauchy.

So we cannot define in S the notion of Smarandache Cauchy element.

$$B = \left\{\begin{bmatrix} [a_1,b_1] & [e,e] & \ldots & [e,e] \\ [a_2,b_2] & [e,e] & \ldots & [e,e] \\ \vdots & \vdots & & \vdots \\ [e,e] & [e,e] & \ldots & [e,e] \end{bmatrix}\right\} \in S \text{ is such that}$$

$$B^2 = \left\{\begin{bmatrix} [e,e] & [e,e] & \ldots & [e,e] \\ [e,e] & [e,e] & \ldots & [e,e] \\ \vdots & \vdots & & \vdots \\ [e,e] & [e,e] & \ldots & [e,e] \end{bmatrix}\right\}.$$

Example 4.27: Let S = {Collection of all subsets from the interval matrix loop groupoid M = {($[a_1, b_1]$, $[a_2, b_2]$, $[a_3, b_3]$, $[a_4, b_4]$) | $a_i, b_i \in L_{13}(7)$; $1 \leq i \leq 4$}} be the subset interval matrix loop groupoid.

Take X_1 = {($[a_1, b_1]$, $[a_2, b_2]$, $[a_3, b_3]$, $[a_4, b_4]$)} in S. Clearly X_1^2 = {([e, e], [e, e], [e, e], [e, e])}.

Every singleton Y in S is such that Y^2 = {([e, e], [e, e], [e, e], [e, e])}.

Example 4.28: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] & [a_9,b_9] \\ [a_{10},b_{10}] & [a_{11},b_{11}] & [a_{12},b_{12}] \\ [a_{13},b_{13}] & [a_{14},b_{14}] & [a_{15},b_{15}] \end{bmatrix} \middle| \; a_i, b_i \in L_{105}(14); \; 1 \leq i \leq 15\right\}\}$$

be the subset interval matrix loop groupoid. If A ∈ S and A is such that A contains only one element then

$$A^2 = \begin{bmatrix} [e,e] & [e,e] & [e,e] \\ \vdots & \vdots & \vdots \\ [e,e] & [e,e] & [e,e] \end{bmatrix} \in S.$$

We see every singleton subset X in S is such that

$$X^2 = \left\{ \begin{bmatrix} [e,e] & [e,e] & [e,e] \\ \vdots & \vdots & \vdots \\ [e,e] & [e,e] & [e,e] \end{bmatrix} \right\} \text{ the subset identity element of S.}$$

In view of this we have the following theorem.

THEOREM 4.7: *Let S = {Collection of all subsets from the interval matrix loop groupoid M = {All s × t interval matrices with elements from the loop $L_n(m)$}} be the subset interval matrix loop groupoid.*

$$\textit{Every singleton set } X = \left\{ \begin{bmatrix} [a_1,b_1] & \ldots & [a_t,b_t] \\ [a_{t+1},b_{t+1}] & \ldots & [a_{2t},b_{2t}] \\ \vdots & \ldots & \vdots \\ [a_{(s+1)},b_{(s+1)}] & \ldots & [a_{st},b_{st}] \end{bmatrix} \right\} \in S$$

is such that

$$X^2 = \left\{ \begin{bmatrix} [e,e] & \ldots & [e,e] \\ \vdots & \ldots & \vdots \\ [e,e] & \ldots & [e,e] \end{bmatrix} \right\} \text{ where } a_i, b_i \in L_n(m);\ 1 \leq i \leq st.$$

Proof follows from the fact for every a $\in L_n(m)$ we have a * a = e.

Hence the claim.

Example 4.29: Let S = {Collection of all subsets from the interval matrix loop groupoid,

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \middle| \; a_i, b_i \in L_7(3); 1 \le i \le 5\} \right\}$$

be the subset interval matrix loop groupoid where

$o(S) = 2^{|M|} - 1$; which is odd.

Every singleton element subset X in S is such that

$$X^2 = \left\{ \begin{bmatrix} [e, e] \\ \vdots \\ [e, e] \end{bmatrix} \right\}.$$

But $2 \not| \; o(S)$, so no element in S is a S-Cauchy element of S.

THEOREM 4.8: *Let S = {Collection of all subsets from the interval matrix loop groupoid M = {Collection of all s × t interval matrices with entries from $L_n(m)$}} be the subset interval matrix loop groupoid. S has no S-Cauchy elements.*

Proof is direct and hence left as an exercise to the reader.

We can define also subset interval matrix principal isotope loop groupoid of a subset interval matrix loop groupoid.

Study of these concepts is interesting and innovative.

We will first illustrate this situation by an example.

Example 4.30: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_7, b_7] \end{bmatrix} \middle| \; a_i, b_i \in L_5(2); 1 \leq i \leq 7; \right.$$

where the loop $L_5(2)$ is as follows:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

be the subset interval matrix loop groupoid.

The principal isotope subset interval matrix loop groupoid S_{PI} of the principal isotope of the loop $L_5(2)$ denoted by ($L_5(2)$, o) is as follows:

o	e	1	2	3	4	5
e	3	2	5	e	1	4
1	5	3	4	1	e	2
2	4	e	3	2	5	1
3	e	1	2	3	4	5
4	2	5	1	4	3	e
5	1	4	e	5	2	3

S_{PI} = {Collection of all subsets from the interval matrix principal isotope loop of $L_5(2)$ groupoid;

$$M_{PI} = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_7, b_7] \end{bmatrix} \middle| \; a_i, b_i \in (L_5(2), o); 1 \leq i \leq 7 \} \right\}$$

be the principal isotope subset interval matrix loop groupoid of the loop $(L_5(2), o)$ of $(L_5(2), *)$.

Thus for every given subset interval matrix loop groupoid S of the loop (L, *) we have an associated principal isotope subset interval matrix loop groupoid S_{PI} of the loop (L, o); the principal isotope of the loop L.

Thus for every given subset interval matrix loop groupoid S we have an associated subset interval matrix principal isotope loop groupoid S_{PI}.

Interested reader can study them and compare S with S_{PI}.

Suppose S is a subset interval matrix loop groupoid of the loop $L_n(m)$, one can find the collection of all subset interval matrix loop subgroupoids of S and study the lattice of the subset interval matrix loop subgroupoid of S.

Similar study can be made for S-subset interval matrix loop subgroupoids of S; also for subset interval matrix loop ideals of S as well as S-ideals of S.

Study in this direction is both interesting and innovative; hence left as an exercise to the reader.

Cosets in subset interval matrix loop groupoids of the associated loop L can be done in an appropriate way.

Let S be a Smarandache subset interval matrix loop groupoid of the associated loop L.

Let $A \subseteq L$; $A_m = \{am \mid a \in A\}$; $m \in L$ where A is a subgroup of L. A_m is the S-right coset of A in L.

We define S-right coset of S the subset interval matrix loop groupoid in the following way;

Let $L_5(2)$ be the loop given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

Let $A = \{e, 1\} \subseteq L_5(2)$ be the subgroup of $L_5(2)$.
$A * 1 = \{e, 1\}$, $A * 2 = \{2, 3\}$,
$A * 3 = \{3, 5\}$, $A * 4 = \{4, 2\}$ and
$A * 5 = \{5, 4\}$ are the S-right cosets of A in $L_5(2)$.

The S-left cosets of A in $L_5(2)$ are
$1 * A = \{e, 1\}$, $2 * A = \{5, 2\}$,
$3 * A = \{3, 4\}$, $4 * A = \{4, 3\}$ and
$5 * A = \{5, 2\}$.

Now let $M_{A*3} = \{([a_1, b_1]\ [a_2, b_2], [a_3, b_3]);\ a_i, b_i \in \{3, 5\}\}$ be the interval matrix S-right coset of A*3 subgroupoid.

S_{A*3} = {Collection of all subsets from the M_{A*3} interval matrix S-right coset $A * 3$, subgroupoid of M} is defined as the S-right coset subset interval matrix subgroupoid of M_{A*3}.

This is the way S-right coset subset interval matrix subgroupoid M_{A*i} ($i \in L_n(m) = L_5(2)$ in this case) is defined.

We can define on similar lines the notion of subset interval matrix S - left coset subgroupoid of M_{i*A}; $i \in L_5(2)$; $A \subseteq L_5(2)$ is a subgroup of $L_5(2)$.

One can study this concept of S-subset interval matrix S-right (left) coset subgroupoids in case of S-subset interval matrix loop groupoids associated with a loop L provided L is a Smarandache loop.

Example 4.31: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in L_7(4); 1 \leq i \leq 4 \right\}\}$$

be the subset interval matrix loop groupoid.

Take

$$X = \left\{ \begin{bmatrix} [e,e] & [e,e] \\ [e,e] & [e,e] \end{bmatrix}, \begin{bmatrix} [3,e] & [e,e] \\ [e,e] & [e,e] \end{bmatrix}, \begin{bmatrix} [e,3] & [e,e] \\ [e,e] & [e,e] \end{bmatrix}, \begin{bmatrix} [3,3] & [e,e] \\ [e,e] & [e,e] \end{bmatrix} \right\} \subseteq S$$

is a subset interval matrix subgroup of S.

We can have subset interval matrix S-right (left) coset of X subgroup in S.

Let us take any

$$A = \left\{ \begin{bmatrix} [2,1] & [e,e] \\ [e,1] & [e,e] \end{bmatrix}, \begin{bmatrix} [1,e] & [e,e] \\ [3,2] & [5,1] \end{bmatrix}, \begin{bmatrix} [4,4] & [1,5] \\ [5,e] & [e,4] \end{bmatrix} \right\} \in S.$$

We can find XA and AX which will be the S-subset interval matrix right coset of X and S-subset interval matrix left coset of X respectively. It is easily verified

$$XA \neq AX, \text{ however } o(XA) = o\,(AX) = 6.$$

These S-subset interval matrix right (left) cosets; we define as genuvine S-subset interval matrix right (left) cosets of X in S.

This is an interesting aspect of S subset interval matrix loop groupoids.

We see the subsets (elements) in S are of different cardinality. Even if $A \in S$ and number of elements in A is n then we see for the above X we have XA to have 4n elements.

So the usual coset property is violated in S. This is the marked difference between usual S left (right) cosets and subset S-left (right) cosets of S.

Study of cosets in subset interval matrix loop groupoids is an interesting feature.

Now we can define the notion of subset interval matrix group-loop (loop-group) groupoid.

We will first describe it by some examples.

Example 4.32: Let S = {Collection of all subsets from the interval subset matrix loop-group groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_7, b_7] & [a_8, b_8] \end{bmatrix} \right| a_i, b_i \in L_7(6) \times S_4; 1 \leq i \leq 8\}\}$$

be the subset from interval matrix loop-group groupoid.

Clearly S has several subset interval matrix S-left (right) cosets of interval matrix subgroups A of S.

Interested reader can find them.

Example 4.33: Let S = {Collection of all subsets from the interval subset matrix loop-group groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \dots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & [a_{12}, b_{12}] & \dots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \dots & [a_{30}, b_{30}] \end{bmatrix} \middle| \; a_i, b_i \in D_{2.7} \times L_{15}(8); \right.$$

$$1 \leq i \leq 30\}\}$$

be the subset interval matrix group-loop groupoid.

S has several Smarandache subset interval matrix S-right (left) cosets related to several subset interval matrix subgroups.

Example 4.34: Let S = {Collection of all subsets from $M = \{([a_1, b_1], [a_2, b_2], [a_3, b_3]) \mid a_i, b_i \in G \times L_5(2);\ 1 \leq i \leq 3\}\}$ be the subset interval matrix group loop groupoid, where $G = \{g \mid g^4 = 1\}$.

The subset interval matrix subgroups of S are;

$T_1 = \{\langle ([(g, e), (g, e)], [(1, e), (1, e)], [(1, e), (1, e)])\rangle \mid g, 1 \in G, e \in L_5(2))\}$ is a subset interval matrix subgroup of order 4.

We have right cosets of T_1 of cardinality (order) greater than or equal to four.

We can have several such subset interval matrix subgroups and related with them. We have S-subset interval matrix right (left) cosets related to the subset interval matrix subgroups.

Next we proceed onto define subset interval matrix polynomial loop groupoid.

DEFINITION 4.2: *Let S = {Collection of all subsets from the interval polynomial loop groupoid;*

$$M = \left\{\sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L; 0 \le i \le \infty\right\}\}$$

be the subset interval polynomial groupoid associated with the loop L.

We will give examples of them.

Example 4.35: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{43}(8)\right\}\}$$

be the subset interval polynomial loop groupoid. Clearly S is non commutative.

Example 4.36: Let S = {Collection of all subsets from the interval polynomial loop groupoid;

$$M = \left\{\sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{13}(7)\right\}\}$$

be the subset interval polynomial loop groupoid.

Example 4.37: Let S = {Collection of all subsets from the interval polynomial loop groupoid;

$$M = \left\{\sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{45}(8)\right\}\}$$

be the subset interval polynomial loop groupoid.

This has subset interval polynomial semigroups of course no subset interval polynomial groups.

Let us take S_1 = {Collection of all subsets from the interval polynomial (loop or subgroupoid) semigroup

$$M_1 = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in \{e, 19\} \right\}\}$$

be the subset interval polynomial semigroup.

Infact S has at least 45 number of subset interval polynomial semigroups.

Example 4.38: Let S = {Collection of all subsets from the interval polynomial loop groupoid;

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{15}(8) \right\}\}$$

be the subset interval polynomial loop groupoid.

Take P_1 = {Collection of all subsets from the interval polynomial loop groupoid

$$M_1 = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in H_i(5) \right\} = \{e, 6, 11\} \subseteq L_{15}(8)\}\}$$

to be the subset interval polynomial loop subgroupoid of S.

We have atleast 11 such subset interval polynomial loop subgroupoids.

Now we proceed onto give more examples and their substructures.

Example 4.39: Let S = {Collection of all subsets from the interval polynomial loop groupoid;

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{29}(8) \right\}\}$$

be the subset interval polynomial loop groupoid.

P_i = {Collection of all subsets from the interval polynomial loop subgroupoid;

$$M_i = \left\{ \sum_{j=0}^{\infty} [a_j, b_j]x^j \,\middle|\, a_j, b_j \in \{e, g_i\};\ g_i = i \in L_{29}(8), \right.$$
$$\left. i = 1, 2, \ldots, 29 \right\}\}$$

be the subset interval polynomial loop subgroupoid which is associative.

Thus all these are subset interval polynomial semigroups of S.

We see S has no subset interval polynomial semigroups.

Example 4.40: Let S = {Collection of all subsets from the interval polynomial loop groupoids

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{43}(9) \right\}\}$$

be the subset interval polynomial loop groupoid.

S has subset interval polynomial loop subgroupoids.

S_1 = {Collection of all subsets from the interval polynomial loop subgroupoid

$$M_1 = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{43}(9) \right\}\}$$

be the subset interval polynomial loop subgroupoid of S.

S_2 = {Collection of all subsets from the interval polynomial subgroupoid

$$M_2 = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{43}(9)\right\}\}$$

be the subset interval polynomial loop subgroupoid of S.

We see S_1 and S_2 are isomorphic as subset interval polynomial loop subgroupoids by the isomorphism $\eta : S_1 \to S_2$ defined by $\eta\left(\left\{\sum_{i=0}^{\infty}[a_i, 0]x^i\right\}\right) = \left\{\sum_{i=0}^{\infty}[0, a_i]x^i\right\}$ is a one to one map from S_1 to S_2.

Example 4.41: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{49}(9)\right\}\}$$

be the subset interval polynomial loop groupoid.

S has subset interval polynomial loop subgroupoids.

Atleast S has 16 subset interval polynomial loop subgroupoids all of which are of infinite order apart from 49 subset interval polynomial semigroups.

Example 4.42: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{19}(2)\right\}\}$$

be the subset interval polynomial loop groupoid.

S_1 = {Collection of all subsets from the interval polynomial loop groupoid;

$$M_1 = \left\{ \sum_{i=0}^{\infty} [0, a_i] x^i \,\middle|\, a_i \in L_{19}(2) \right\}\} \subseteq M$$

is the subset interval polynomial loop subgroupoid of S.

S_2 = {Collection of all subsets from the interval polynomial loop groupoid

$$M_2 = \left\{ \sum_{i=0}^{\infty} [a_i, 0] x^i \,\middle|\, a_i \in L_{19}(2) \right\}\} \subseteq M\}$$

is the subset interval polynomial loop subgroupoid of S.

S_3 = {Collection of all subsets from the interval polynomial loop subgroupoid

$$M_3 = \left\{ \sum_{i=0}^{\infty} [a_i, a_i] x^i \,\middle|\, a_i \in L_{19}(2) \right\}\} \subseteq M\}$$

is the subset polynomial interval loop subgroupoid of S.

However apart from these three subgroupoids, we have 19 subset interval polynomial semigroups in S.

In view of this example we have the following theorem.

THEOREM 4.9: *Let S = {Collection of all subsets from the interval polynomial loop groupoid*

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_p(m); p \text{ a prime} \right\}\}$$

be the subset interval polynomial loop groupoid.

(i) S has atleast three subset interval polynomial loop subgroupoids which are isomorphic with each other.

(ii) S has atleast p number of subset interval polynomial semigroups and all of them are isomorphic with each other.

The proof is direct and hence left as an exercise to the reader.

Example 4.43: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{35}(9) \right\}\}$$

be the subset interval polynomial loop groupoid.

S has atleast 15 non trivial subset interval polynomial loop subgroupoids and has 35 subset interval polynomial semigroups.

Example 4.44: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{55}(13) \right\}\}$$

be the subset interval polynomial loop groupoid.

S has atleast 19 number of distinct subset interval polynomial loop subgroupoids and 55 number of subset interval polynomial semigroups.

In view of this we have the following theorem.

THEOREM 4.10: *Let S = {Collection of all subsets from the interval polynomial groupoid*

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_n(m), \right.$$

n a composite number and $L_n(m)$ has t number of subloops and the t subloops are different from the n number of subgroups}} be the subset interval polynomial loop groupoid.

(1) S has atleast (t + 3) number of subset interval polynomial loop subgroupoids.
(2) S has atleast n number of subset interval polynomial semigroups.

Proof is a matter of routine and hence left as an exercise to the reader.

Example 4.45: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{85}(22) \right\}\}$$

be the subset interval polynomial loop groupoid.

Thus subset interval polynomial loop groupoid has atleast 25 subset interval polynomial loop subgroupoids and 85 subset interval polynomial subsemigroups related with the 85 subgroups of $L_{85}(22)$.

Example 4.46: Let S = {Collection of all subsets from the interval polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_7(4) \right\}\}$$

be the subset interval polynomial loop groupoid which is commutative.

S has atleast three subset interval polynomial loop subgroupoids and seven subset interval polynomial semigroups all of which are commutative.

Now we can define subset interval polynomial loop subgroupoids in this way also.

We will first illustrate this situation by some examples.

Example 4.47: Let S = {Collection of all subsets from the interval polynomial loop groupoid;

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{27}(8) \right\} \}$$

be the subset interval polynomial loop groupoid.

Consider P_2 = {Collection of all subsets from interval polynomial loop subgroupoid

$$N_2 = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^{2i} \,\middle|\, a_i, b_i \in L_{27}(8) \right\} \subseteq M \} \subseteq S;$$

P_2 is a subset interval polynomial loop subgroupoid.

P_3 = {Collection of all subsets from the interval polynomial loop subgroupoid

$$N_3 = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^{3i} \,\middle|\, a_i, b_i \in L_{27}(8) \right\} \subseteq M \} \} \subseteq S$$

be the subset interval polynomial loop subgroupoid of S.

We can have P_n = {Collection of all subsets from the interval polynomial loop subgroupoid;

$$N_n = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^{ni} \middle| a_i, b_i \in L_{27}(8)\} \subseteq M\} \right\} \subseteq S$$

be the subset interval polynomial loop subgroupoid.

Thus we have an infinite number of subset interval polynomial loop subgroupoids and none of them are ideals.

THEOREM 4.11: *Let S = {Collection of all subsets from the interval polynomial loop groupoid*

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \middle| a_i, b_i \in L_n(m)\} \right\}$$

be the subset interval polynomial loop groupoid. S has infinite number of subset interval polynomial loop subgroupoids none of which are subset interval polynomial groupoid ideals.

Proof: Follows from the fact P_n = {Collection of all subsets from the polynomial interval loop groupoid

$$M_n = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^{ni} \middle| \text{n a positie integer in } Z^+ \text{ and } a_i, b_i \in L_m(t)\} \right\}$$

$\subseteq M\} \subseteq S$ is the infinite collection of subset interval polynomial loop subgroupoids of S for varying n in Z^+ and none of them is an ideal of S. Hence the claim.

Thus we have proved in general in a subset polynomial interval loop groupoid S every subset interval polynomial loop groupoid ideal is a subset interval polynomial loop subgroupoid but however a subset polynomial interval loop subgroupoid is not in general a subset polynomial interval loop ideal of S; this is substantiated well by the above theorem.

It is important to note that if we have p(x) = [a, b] x and q(x) = [a, b] x^2; we have only p(x) * q(x) defined and we do not have addition that is suppose we get [a, b] x + [a, b] x it remains so and is not changed even a little we cannot add and put it as

2[a, b] x and so on for (S, *) is the only operation no addition. That is why (S, *) is a groupoid and does not have any other structure.

Suppose we have A = $\{[3, e]x^2 + [1, e]\}$ and B = $\{[2, e]x^2 + [1, e]\} \in S$ and 3, e, 2, 1 $\in L_5(2)$.

Now

$$\begin{aligned} A * B &= \{[3, e]x^2 + [1, e]\} * \{[2, e]x^2 + [1, e]\} \\ &= \{([3, e] * [2, e])x^4 + [1, e] * [2, e]x^2 + \\ &\quad [1, e] * [1 * e] + ([3, e] * [1, e]x^2\} \\ &= \{[1, e]x^4 + [3, e]x^2 + [e, e] + [4, e]x^2\}. \end{aligned}$$

We cannot group the x^2 coefficients even if they are the same.

We have seen substructures in subset interval polynomial loop groupoids.

We do not have any subset zero divisors in S. If at all S has subset units they are from the constant interval polynomials. S has no subset idempotents or subset nilpotents. S will also be known as groupoids free from subset zero divisors or subset units, infact the question of S subset zero divisors or S-subset units or S-subset idempotents never arises.

However S has S-subset interval polynomial loop subgroupoids and S-subset interval polynomial ideals.

Example 4.48: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{19}(3)\right\}\}$$

be the subset interval polynomial loop groupoid. S has interval subset polynomial ideals as well as S has subset interval polynomial loop subgroupoids which are not ideals.

We see subset interval polynomial loop groupoids S will be Smarandache provided the basic loop over which S is defined is Smarandache.

Now we proceed onto just give hints about interval polynomial loop groupoids which satisfy special identities.

We define those structures as Smarandache subset interval polynomial loop Moufang groupoid if L is a S-Moufang loop (Likewise in case of S-Bol loop, S-Bruck loop and so on).

We say S is a Smarandache strong interval polynomial loop Moufang groupoid if L over which S is built is a Moufang groupoid. The same results holds good in case of other identities.

All the studies carried out in case of subset matrix interval loop groupoids can be done for subset polynomial interval loop groupoids with some appropriate modifications. However all subset polynomial interval loop groupoids are of infinite order.

We suggest the following problems.

Problems

1. Let S be the subset interval matrix loop groupoid built using $L_{23}(8)$.

   What are the special and stricking features enjoyed by S?

2. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{bmatrix} \middle| \; a_i, b_i \in L_9(8); 1 \leq i \leq 6 \right\}\}$$

be the subset matrix interval loop groupoid.

(i) Find o(S).
(ii) Find all subset matrix interval loop subgroupoids of S.
(iii) Can S have Smarandache subset matrix interval loop subgroupoid?
(iv) Is S a Smarandache subset matrix interval loop groupoid?
(v) Can S have subset matrix interval loop semigroups?
(vi) Is S a normal subset interval matrix loop groupoid?
(vii) Is S a semi normal subset interval matrix loop groupoid?

3. Let S_1 = {Collection of all subsets of the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \right| a_i, b_i \in L_{27}(8);\ 1 \leq i \leq 10 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (vii) of problem 2 for this S_1.

4. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid $M = \{([a_1, b_1], [a_2, b_2], \ldots, [a_{12}, b_{12}]) \mid a_i, b_i \in L_{19}(8);\ 1 \leq i \leq 12\}$} be the subset interval matrix loop groupoid.

(i) Study questions (i) to (vii) of problem 2 for this S_2.
(ii) Compare S_1 of problem 3 with S_2 of this problem.

5. Let S_3 = {Collection of all subsets from the interval matrix loop groupoid M = {5 × 5 interval matrices with entries from the loop $L_{21}(11)$}} be the subset interval matrix loop groupoid.

Study questions (i) to (vii) of problem two for this S_3.

6. Let S_4 = {Collection of all subsets from interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{bmatrix} \right| a_i, b_i \in L_{43}(7); 1 \le i \le 20 \right\} \}$$

be the subset interval matrix loop groupoid.

(i) Can S have subset units or Smarandache subset units?
(ii) Can S have subset idempotents?
(iii) Is S simple?
(iv) Can S have subset interval matrix loop subgroupoids which are not subset interval matrix loop semigroups?
(v) Can S have S-subset interval loop semigroups?

7. Let S_1 = {Collection of all subsets from the matrix interval loop groupoid $M = \{([a_1, b_1], [a_2, b_2], \ldots, [a_{10}, b_{10}]) \mid a_i, b_i \in L_{101}(8); 1 \le i \le 10\}$} be the subset interval matrix loop groupoid.

Study questions (i) to (v) of problem 6 for this S_1.

8. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid;

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{23}, b_{23}] & [a_{24}, b_{24}] \end{bmatrix} \right| a_i, b_i \in L_{237}(11); 1 \le i \le 24 \right\} \}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (v) of problems 6 for this S_2.

9. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{13},b_{13}] & [a_{14},b_{14}] & [a_{15},b_{15}] & [a_{16},b_{16}] \end{bmatrix} \right| a_i, b_i \in L_{33}(8); 1 \leq i \leq 16 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (v) of problem (6) for this S.

10. Give an example of an infinite interval subset matrix loop groupoid built using a loop of infinite order.

11. Can an infinite subset interval matrix loop groupoid be a Smarandache strong subset interval matrix Bol loop groupoid?

12. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \\ [a_5,b_5] & [a_6,b_6] \end{bmatrix} \right| a_i, b_i \in L_{128}(8); 1 \leq i \leq 6 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (v) of problem 6 for this S.

13. Can we have infinite order subset interval matrix loop groupoid which is a Smarandache strong alternative subset interval matrix loop groupoid?

14. Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in L_5(2); 1 \leq i \leq 4 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Find o(S).
(ii) How may subset matrix interval loops exist in S which are isomorphic with $L_5(2)$?
(iii) Does S contain atleast 45 subset matrix interval loops isomorphic with $L_5(2)$?
(iv) How many subset interval matrix loop subgroupoids of S exists?

15. Let S = {Collection of all subset from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in L_{29}(3); 1 \leq i \leq 9 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Find o(S).
(ii) Find all subset interval matrix loop subgroupoids which are loops isomorphic to $L_{29}(3)$.
(iii) Find the number of proper subset interval matrix loop subgroupoids of S.
(iv) Does S contain subset units?
(v) Is S a Smarandache subset interval matrix loop groupoid?

16. Let S_1 = {collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in L_{29}(8); \; 1 \le i \le 9 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Study questions (i) to (v) of problem (15) for this S_1.
(ii) Is S_1 isomorphic with S in problem 15.

17. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_{10}, b_{10}] \\ \vdots & & \vdots \\ [a_{41}, b_{41}] & \ldots & [a_{50}, b_{50}] \end{bmatrix} \middle| \; a_i, b_i \in L_{29}(20); \; 1 \le i \le 50 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Study questions (i) to (v) of problem 15 for this S_2.
(ii) Compare S of problem 15 and S_1 of problem 16 with S_2 of problem 17.

18. Let S_3 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{18}, b_{18}] \end{bmatrix} \middle| \; a_i, b_i \in L_{55}(13); \; 1 \le i \le 18 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (v) of problem 15 for this S_3.

Study S_3 if $L_{55}(13)$ is replaced by $L_{55}(27)$.

Is S_3 and that S_1 isomorphic as subset interval matrix loop groupoids?

19. Give an example of a Smarandache strong Bruck subset interval matrix loop groupoid of infinite order.

20. Give an example of a Smarandache strong normal subset interval loop groupoid of infinite order.

21. Does there exist a subset interval matrix loop groupoid S which is both a P-groupoid as well as a Bol groupoid?

22. Does there exist a subset interval matrix loop groupoid of a loop L of finite order which is not a S-subset interval matrix loop groupoid?

23. Does there exist a subset interval matrix loop groupoid S of loop L which is a Smarandache subset matrix interval loop groupoid still the loop L is a Smarandache loop?

24. If L is any finite loop other than $L_n(m)$.
Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \right| a_i, b_i \in L; 1 \leq i \leq 9 \right\}\}$$

be the subset interval matrix loop groupoid. $o(S) = 2^{|M|} - 1$ is odd.

Can S contain X ∈ S such that $\{X\}^t = \left\{\begin{bmatrix}[e,e]\\ \vdots \\ [e,e]\end{bmatrix}\right\}$ and

t/o(S)?

25. Does there exist a subset interval matrix loop groupoid which is Smarandache Cauchy?

26. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{\begin{bmatrix}[a_1,b_1] & [a_2,b_2]\\ [a_3,b_3] & [a_4,b_4]\\ [a_5,b_5] & [a_6,b_6]\\ [a_7,b_7] & [a_8,b_8]\end{bmatrix} \middle| \; a_i, b_i \in L_{19}(3);\ 1 \le i \le 8\}\right\}$$

be the subset interval matrix loop groupoid.

(i) Find all principal isotopes of $L_{19}(3)$.
(ii) Corresponding to each of the isotopes of $L_{19}(3)$ find subset interval matrix principal isotope loop groupoid.
(iii) How many of the S_{PI}'s are commutative?
(iv) How many S_{PI}'s are isomorphic as subset interval matrix loop groupoids?

27. Let S_1 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{\begin{bmatrix}[a_1,b_1] & [a_2,b_2] & \ldots & [a_9,b_9]\\ [a_{10},b_{10}] & [a_{11},b_{11}] & \ldots & [a_{18},b_{18}]\\ [a_{19},b_{19}] & [a_{20},b_6] & \ldots & [a_{27},b_{27}]\end{bmatrix} \middle| \; a_i, b_i \in L_{27}(8);\ 1 \le i \le 27\}\right\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (iv) of problem 26 for this S_1.

28. Suppose $L_{13}(7)$ and $L_{13}(6)$ are two loops.

    Will the collection of principal isotopes related with them be different or same?

29. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix} \middle| a_i, b_i \in L_{105}(17); 1 \leq i \leq 4 \right\}\}$$

    be the subset interval matrix loop groupoid.

    Study questions (i) to (iv) of problem 26 for this S_2.

30. Does there exist any difference between the principal isotopes of the loops $L_{19}(7)$ and $L_{285}(17)$?

31. Can S-Sylow criteria be implemented on subset interval matrix loop groupoid of any loop $L_n(m)$ which satisfies S-Sylow criteria?

32. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{11}, b_{11}] \\ [a_{12}, b_{12}] & [a_{13}, b_{13}] & \ldots & [a_{22}, b_{22}] \\ [a_{23}, b_{23}] & [a_{24}, b_{24}] & \ldots & [a_{33}, b_{33}] \\ [a_{34}, b_{34}] & [a_{35}, b_{35}] & \ldots & [a_{44}, b_{44}] \end{bmatrix} \middle| a_i, b_i \in L_{15}(8); 1 \leq i \leq 44 \right\}\}$$

    be the subset interval matrix loop groupoid.

    Does S satisfy Smarandache Sylow criteria?

33. Does there exist a subset interval matrix loop groupoid which does not satisfy the S-Sylow criteria?

34. Give an example of a subset interval matrix groupoid which satisfies the S-Sylow criteria.

35. Let S = {Collection of all subsets from the interval matrix gorupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \dots & [a_5, b_5] \\ [a_6, b_6] & \dots & [a_{10}, b_{10}] \\ \vdots & \dots & \vdots \\ [a_{21}, b_{21}] & \dots & [a_{25}, b_{25}] \end{bmatrix} \right| a_i, b_i \in L_{23}(8); 1 \leq i \leq 25 \right\}\}$$

be the subset interval matrix loop groupoid.

Find all S_{PI} and compare them with S.

36. Give an example of a subset interval matrix loop groupoid which is a Smarandache weak Lagrange subset interval matrix loop groupoid.

37. Does there exist a Smarandache Lagrange subset matrix interval groupoid? Justify your claim.

38. Let S = {Collection of all subsets from the matrix interval loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{18}, b_{18}] \end{bmatrix} \right| a_i, b_i \in L_7(8); 1 \leq i \leq 18 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Find the lattice L_1 associated with the subset matrix interval loop subgroupoid of S.
(ii) Find the lattice L_2 associated with Smarandache subset interval matrix loop subgroupoid of S.
(iii) Compare the lattices L_1 and L_2. Is L_2 a sublattice of L_1?
(iv) Find the lattice L_3 of subset interval matrix loop ideals of S.
(v) Find the lattice L_4 of the subset interval matrix Smarandache ideals of S.
(vi) Is L_4 a sublattice of L_3?
(vii) Is L_3 a sublattice of L_1?
(viii) Is L_4 a sublattice of L_2?

39. Let S_1 = {Collection of all subsets of the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & \ldots & [a_{12}, b_{12}] \\ [a_{13}, b_{13}] & [a_{14}, b_{14}] & \ldots & [a_{18}, b_{18}] \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] & \ldots & [a_{24}, b_{24}] \end{bmatrix} \right| a_i, b_i \in L_{27}(8); \; 1 \leq i \leq 24 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (viii) of problem 38 for this S_1.

40. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid $M = \{([a_1, b_1], [a_2, b_2], \ldots, [a_9, b_9]) \mid a_i, b_i \in L_{105}(17); 1 \leq i \leq 9\}\}$ be the subset interval matrix loop groupoid.

Study questions (i) to (viii) of problem 38 for this S_2.

41. Let S_3 = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left[\begin{array}{cccc} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] & [a_7, b_7] & [a_8, b_8] \\ [a_9, b_9] & [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \\ [a_{13}, b_{13}] & [a_{14}, b_{14}] & [a_{15}, b_{15}] & [a_{16}, b_{16}] \end{array} \right] \right| a_i, b_i \in$$

$$L_{53}(8);\ 1 \leq i \leq 16\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (viii) of problem 38 for this S_3.

42. Does there exist a subset interval matrix loop groupoid S in which all the four lattices associated with S are modular?

43. Is it possible for S to have all the four lattices to be non modular? Justify your answer!

44. Let S_1 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{array} \right] \right| a_i, b_i \in L_{27}(8);\ 1 \leq i \leq 12\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (viii) of problem 38 for this S_1.

45. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_4, b_4] \\ [a_5, b_5] & \ldots & [a_8, b_8] \\ [a_9, b_9] & \ldots & [a_{12}, b_{12}] \\ [a_{13}, b_{13}] & \ldots & [a_{16}, b_{16}] \end{bmatrix} \right| a_i, b_i \in L_{45}(8); 1 \leq i \leq 16 \right\} \}$$

be the subset interval matrix loop groupoid.

(i) Find all S-subset interval matrix S-left coset subgroupoids of S.
(ii) Find all S-subset interval matrix S-right coset subgroupoids of S.
(iii) Can S have a S-subset interval S-coset subgroupoid?

46. Let S_1 = {Collection of all subsets from the interval matrix loop groupoid M = $\{([a_1, b_1], \ldots, [a_{10}, b_{10}]) \mid a_i, b_i \in L_9(5); 1 \leq i \leq 10\}\}$ be the subset interval matrix loop groupoid.

Study questions (i) to (iii) of problem 45 for this S_1.

47. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \end{bmatrix} \right| a_i, b_i \in L_{19}(8); 1 \leq i \leq 4 \right\} \}$$

be the subset interval matrix loop groupoid.

(i) Study questions (i) to (iii) of problem 45 for this S_2.

(ii) Can S_2 have other subset interval matrix subgroups which can give way to S-right (left) coset of S.

48. Let S_1 = {Collection of all subsets from

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in L_{11}(4); 1 \le i \le 10 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (iii) of problem 45 for this S_1.

49. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \end{bmatrix} \middle| \; a_i, b_i \in L_{13}(4); 1 \le i \le 3 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (iii) of problem 45 for this S.

50. Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in L_{23}(2); 1 \le i \le 10 \right\}\}$$

be the subset interval matrix loop groupoid.

(i) Find all genuvine S-subset interval matrix left(right) cosets of subset matrix interval subgroups of S.

(ii) Find also S-subset interval left (right) cosets of S.

51. Let S_1 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{18}, b_{18}] \end{bmatrix} \right| a_i, b_i \in L_{315}(23); 1 \leq i \leq 18 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (ii) of problem 50 for this S_1.

52. Let S_2 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & [a_{12}, b_{12}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \ldots & [a_{30}, b_{30}] \end{bmatrix} \right| a_i, b_i \in L_{147}(11); 1 \leq i \leq 30 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (ii) of problem 50 for this S_2.

53. Obtain or study some special properties enjoyed by S-subset matrix interval geneuvine right (left) cosets in S.

54. Distinguish between the S-genuvine subset interval matrix right (left) cosets and the S-coset interval matrix right (left) cosets of S.

55. Let S_3 = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in L_{1155}(17); \; 1 \leq i \leq 9 \right\}\}$$

be the subset interval matrix loop groupoid.

Study questions (i) to (ii) of problem 50 for this S_3.

56. Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_7, b_7] \\ \vdots & & \vdots \\ [a_{43}, b_{43}] & \ldots & [a_{49}, b_{49}] \end{bmatrix} \middle| \; a_i, b_i \in L_{123}(50); \; 1 \leq i \leq 49 \right\}\}$$

be the subst interval matrix loop groupoid.

Study questions (i) to (ii) of problem 50 for this S.

57. Let S = {Collection of all subset from the interval matrix group-loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in S_3 \times L_{19}(6); \; 1 \leq i \leq 10 \right\}\}$$

be the subset interval matrix group-loop groupoid.

(i) Find all subset interval matrix subgroups of S.
(ii) Find all genuvine S-subset interval matrix right (left) cosets of group-loop groupoids.
(iii) Find all S-left (right) cosets of $S_3 \times L_{19}(6)$.

(iv) Find the corresponding S-subset interval matrix left (right) cosets of subgroups in $S_3 \times L_{19}(6)$.

58. Let S_1 = {Collection of all subsets from the interval matrix group-loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ \vdots & \vdots & \vdots \\ [a_{28}, b_{28}] & [a_{29}, b_{29}] & [a_{30}, b_{30}] \end{array}\right] \middle| \; a_i, b_i \in D_{2\times 7} \times L_{29}(2); \; 1 \leq i \leq 30\right\}\}$$

be the subset interval matrix group-loop groupoid.

Study questions (i) to (iv) of problem 57 for this S_1.

59. Let S_2 = {Collection of all subsets from the interval matrix loop-group groupoid M = {([a_1, b_1] … [a_6, b_6]) | $a_i, b_i \in S_5 \times L_{105}(23)$; $1 \leq i \leq 6$} be the subset interval matrix loop-group groupoid.

(i) Find $o(S_2)$.
(ii) Study questions (i) to (iv) of problem 57 for this S_2.

60. Let S_3 = {Collection of all subsets from the interval matrix loop-group groupoid

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{15}, b_{15}] & [a_{16}, b_{16}] \end{array}\right] \middle| \; a_i, b_i \in L_{45}(8); \; 1 \leq i \leq 16 \right.$$

with G = {g | $g^{16} = 1$}}} be the subset interval matrix loop-group groupoid.

(i) Find $o(S_3)$.
(ii) Study questions (i) to (iv) of problem 57 for this S_3.

61. Let S_4 = {Collection of all subsets from the interval matrix group-loop groupoid

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \middle| a_i, b_i \in A_4 \times L_{35}(9); 1 \leq i \leq 4\right\}\}$$

be the subset interval matrix group-loop groupoid.

(i) Find $o(S_4)$.
(ii) Study questions (i) to (iv) of problem 57 for this S_4.

62. Obtain some special properties enjoyed by subset interval polynomial loop groupoid S.

63. Show S in (62) in general are non associative and non commutative.

64. Can there be a subset interval polynomial loop groupoids which are not Smarandache?

65. Does there exist a subset interval polynomial loop groupoid which is a Bol groupoid?

66. Give an example of a subset interval polynomial loop groupoid which is Bruck.

67. Give an example of a subset interval polynomial loop groupoid which is a Moufang groupoid.

68. Does there exist a subset interval polynomial loop groupoid which satisfies two different special identities?

69. Let S = {Collection of all subsets from interval polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L_{29}(3) \right\}\}$$

be the subset interval polynomial loop groupoid.

(i) Does there exist S-subset interval polynomial loop subgroupoid which is not an ideal?
(ii) How many subset interval polynomial loop subgroupoids of S exist?
(iii) Can S have subset interval polynomial loop subgroupoid which is not a S-subset interval polynomial loop groupoid ideal?

70. Give an example of subset interval polynomial loop groupoid of infinite order which is non commutative.

Chapter Five

SPECIAL TYPES OF TOPOLOGICAL SUBSET GROUPOID SPACES

In this chapter we for the first time introduce two types of topologies on subset groupoid spaces.

We will describe, define and develop these concepts.

Let S = {Collection of all subsets from a groupoid (G, *)} be the subset groupoid of G. Now on S we define usual $\cup$ and $\cap$ after adjoining the empty set we get the usual topological space on the set $S' = S \cup \{\phi\}$. But this topology has been in existence and such study is in literature for many years.

Now we wish to define topologies different from these. We take $S' = S \cup \{\phi\}$ and define for every $A \in S$ and $\{\phi\}$ in S; $A * \phi = \phi = \phi * A$.

$\{S', \cup\}$ is a closed operation on S. $\{S' \cap_*\}$ is also a closed operation where $\cap_* = *$, the usual operation on G induced on S.

We define $(S', \cup, \cap_*)$ to be a special type of topological subset groupoid space.

We will first illustrate this situation by some examples.

Example 5.1: Let S = {Collection of all subsets from the groupoid $G = \{Z_{12}, *, (4, 0)\}$} be the subset groupoid.

Here $S' = \langle S \cup \{\phi\}\rangle$; $\{S', \cup, \cap_*\}$ is a special type of topological subset groupoid space.

Let $A = \{4, 6, 8, 0\}$ and $B = \{5, 7, 11\} \in S' = S \cup \{\phi\}$.

$A \cup B = \{4, 5, 6, 8, 0, 5, 7, 11\}$.

$$A \cap_* B = \{4, 6, 8, 0\} * \{5, 7, 11\}$$

$$= \{4 * 5, 4 * 7, 4 * 11, 6 * 5, 6 * 7, 6 * 11, 8 * 5, 8 * 7, 8 * 11, 0 * 5, 0 * 7, 0 * 11, 0 * 11\}$$

$$= \{4, 0, 8\} \in S.$$

Thus $\{S', \cup, \cap_*\}$ can be given a topology and $\{S', \cup, \cap_*\}$ is defined as the special type of subset groupoid topological space of S.

Now suppose we give another operation on S′ say $\{S', \cap, \cap_*\}$ then we see for some A, B ∈ S. $A \cap B = \phi$.

$A \cap_* B = \{4, 0, 8\}$. Thus $\{S', \cap, \cap_*\}$ is another topological space; that is a special type of subset groupoid topological space.

Hence given a subset groupoid of a groupoid G we can have two special type of topological subset groupoid spaces which are distinct and different.

Example 5.2: Let S = {Collection of all subsets from the groupoid G = {Z_{17}, *, (0, 1)}} be the subset groupoid of G.

{S′, ∪, $\cap_*$} and {S′, ∩, $\cap_*$} are the two special types of topological subset spaces.

Example 5.3: Let S = {Collection of all subsets of the groupoid G = {Z_{10}, *, (7, 3)}} be the subset groupoid.

{S′, ∪, $\cap_*$} and {S′, ∩, $\cap_*$} are the two special types of topological subset groupoid spaces of S.

We first make the following observations.

We see {S′, ∩, $\cap_*$} is non associative with respect to $\cap_*$ and also non commutative with respect to $\cap_*$. So this is a very special type of topological space where with respect to one of the operations ∪ or ∩ it is commutative and associative but with respect to other operation $\cap_*$ the space is both non associative as well as non commutative.

We analyse this in case of example 5.3, take

A = {2, 3, 4, 5} and B = {7, 9} ∈ S′.

A ∩ B = {ϕ}.

A ∪ B = {2, 3, 4, 5, 7, 9}.

$A \cap_* B$ = {2, 3, 4, 5} $\cap_*$ {7, 9}

= {2 * 7, 2 * 9, 3 * 7, 3 * 9, 4 * 7, 4 * 9, 5 * 7, 5 * 9}

= {14 + 21, 14 + 27, 21 + 21, 21 + 27, 28 + 21, 28 + 27, 35 + 21, 35 + 27}

= {5, 1, 2, 8, 9, 6} ∈ S … I

Now take

$B \cap_* A = \{7, 9\} \times \{2, 3, 4, 5\}$

$= \{7 * 2, 7 * 3, 7 * 4, 7 * 5, 9 * 2, 9 * 3, 9 * 4, 9 * 5\}$

$= \{49 + 6, 49 + 9, 49 + 12, 49 + 15, 63 + 6, 63 + 9, 63+ 12, 63 + 15\}$

$= \{5, 8, 1, 4, 9, 2\}$ … II

Clearly I and II are distinct.

Thus $A \cap_* B \neq B \cap_* A$ in general. Let $C = \{0, 2\} \in S$.

We find $(A \cap_* B) \cap_* C$

$= \{5, 1, 2, 8, 9, 6\} * \{0, 2\}$
(using the resultant of equation I of A * B)

$= \{5 * 0, 1 * 0, 2 * 0, 8 * 0, 6 * 0, 9 * 0, 5 * 2, 1 * 2, 2 * 2, 8 * 2, 9 * 2, 6 * 2\}$

$= 35, 7, 14, 56, 42, 63, 35+6, 7+6, 14+6, 56+6, 42+6, 63+6\}$

$= \{5, 7, 4, 6, 2, 3, 1, 8, 9\} \in S$ (a)

Consider A * (B * C)

$= A * (\{7, 9\} * \{0, 2\})$
$= A * \{7 * 0, 9 * 0, 7 * 2, 9 * 2\}$

$= A * \{49, 63, 49+6, 63+6\}$

$= \{2, 3,. 4, 5\} * \{9, 3, 5\}$

$= \{2*9, 2 * 3, 2 * 5, 3 * 9, 3 * 3, 3 * 5, 4 * 9,$
$4 * 3, 4 * 5, 5 * 9, 5 * 3, 5 * 4, 5 * 5\}$

$= \{14 + 27, 14 + 9, 14 + 15, 21 + 27, 21 + 9,$
$21 + 15, 28 + 27, 28 + 15, 28 + 21,$
$35 + 27, 35 + 9, 35 + 12, 35 + 15\}$

$= \{1, 3, 9, 8, 0, 6, 5, 2, 4, 7\}$ (b)

Clearly (a) and (b) are distinct so
$(A \cap_* B) \cap_* C \neq A \cap_* (B \cap_* C)$ for A, B, C $\in$ S′.

Thus $\{S', \cap_*, \cup\}$ and $\{S', \cap, \cap_*\}$ in general are non associative and non commutative.

All the examples we have given are special type of topological subset groupoid spaces of finite order.

This is the first time we have succeeded in constructing special type of topological subset groupoid spaces.

Example 5.4: Let S = {Collection of all subsets from the groupoid G = $\{Z_{45}, *, (9, 9)\}$} be the subset groupoid of G.

$\{S', \cap_*, \cup\}$ and $\{S', \cap, \cap_*\}$ are two distinct special type of topological interval groupoid spaces of finite order. But they are commutative under $\cap_*$ but non associative under $\cap_*$.

Let A = {3, 4, 2, 0}, B = {4, 9} and C = {1, 10} $\in$ S.

$(A \cap_* B) \cap_* C$
$= (\{3, 4, 2, 0\} \cap_* \{4, 9\}) \cap_* C$
$= \{3 * 4, 9 * 4, 2 * 4, 0 * 4, 3 * 9, 4 * 9,$
$2 * 9, 0 * 9\} \cap_* C$
$= \{27 + 36, 36 + 36, 18 + 36, 0 + 36,$
$27 + 81, 36 + 81, 18 + 81, 81\} \cap^* C$
$= \{18, 27, 9, 36, 81, 27\} \cap_* C$
$= \{18, 27, 9, 36, 81\} * \{1, 10\}$

$= \{162 + 9, 243 + 9, 81 + 9, 324 + 9, 729 + 9,$
$162 + 90, 243 + 90 + 81 + 90, 324 +$
$90 + 729 + 90\}$
$= \{36 + 27, 0, 18, 9\}$ … I

Consider $A \cap_* (B \cap_* C)$

$= A \cap_* (\{4, 9\} \cap_* \{1, 10\})$
$= A \cap_* (4 * 1, 9 * 1, 4 * 10, 9 * 10)$
$= A \cap_* \{36 + 9, 81 + 9, 36 + 90, 81 + 90\}$
$= A \cap^* \{0, 36\}$
$= \{3, 4, 2, 0\} \cap_* \{0, 36\}$
$= \{3 * 0, 4 * 0, 2 * 0, 0 * 0, 3 * 36, 4 * 36,$
$2 * 36, 0 * 36\}$
$= \{27, 36, 18, 0, 27 + 324, 36 + 324,$
$18 + 324, 0 + 324\}$
$= \{0, 18, 27, 36, 9\}$ … II

I and II are identical.

Consider $A = \{2\}$, $B = \{3\}$ and $C = \{0\}$ in S.

$(A \cap_* B) \cap_* C = (\{2\} * \{3\}) \cap_* \{0\}$
$= \{18 + 27\} * \{0\}$
$= \{0\}$ I

Consider $A \cap_* (B \cap_* C)$
$= A \cap_* (\{3\} \cap_* \{0\})$
$= A \cap_* \{3 * 0\}$
$= A \cap_* \{27\}$
$= \{2\} \cap_* \{27\}$
$= \{2 * 27\}$
$= \{18 + 243\}$
$= \{261 \pmod{45}\}$
$= \{36\}$ … II

I and II are distinct so in general
$A \cap_* (B \cap_* C) \neq (A \cap_* B) \cap_* C$; for $A, B, C \in S$.

Example 5.5: Let S = {Collection of all subsets of the groupoid G = {R, *, (3, 0)} be the subset groupoid. Let the special type of topological spaces $T_\cup$ and $T_\cap$ be the subset groupoids of S.

We see both $T_\cup$ and $T_\cap$ has infinite number of special type topological subspaces of the subset groupoid S.

Example 5.6: Let S = {Collection of all subsets of the groupoid G = {Z_6, *, (2, 5)}} be the subset groupoid.

Let A = {3, 4, 5} and B = {1, 2, 0} $\in$ S.

Consider A $\cap_*$ A = {3, 4, 5} $\cap_*$ {3, 4, 5}
= {3 * 3, 3 * 4, 3 * 5, 4 * 3, 4 * 4, 4 * 5, 5 * 3, 5 * 4, 5 * 5}
= {3, 2, 1, 4, 5, 0} $\neq$ A.

Now B $\cap_*$ B = {1, 2, 0} * {1, 2, 0}
= {1 * 1, 1 * 2, 1 * 0, 2 * 1, 2 * 2, 2 * 0, 0 * 1, 0 * 2, 0 * 0}
= {1, 0, 2, 3, 4, 5} $\neq$ B.

Thus $T_\cap$ = {S′, $\cap_*$, $\cap$} and $T_\cup$ = {S′, $\cup$, $\cap_*$} are such that they are not like usual subset topological spaces though A $\cap$ A $\neq$ A $\cap_*$ A.

A $\cap$ B = ϕ but
A $\cap_*$ B = {3, 4, 5} $\cap_*$ {0, 1, 2}
= {3 * 0, 3 * 1, 3 * 2, 4 * 0, 4 * 1, 4 * 2, 5 * 0, 5 * 1, 5 * 2}
= {0, 5, 4, 2, 1} $\neq$ ϕ.

Thus A $\cap$ B $\neq$ A $\cap_*$ B so $T_\cap$ = {S′, $\cap$, $\cap_*$} is again a special type topological subset groupoid space and A $\cap$ A $\neq$ A $\cap_*$ A.

Example 5.7: Let S = {Collection of all subsets of the groupoid G = {Z_{12}, (4, 9), *}} be a subset groupoid.

$T_\cup$ and $T_\cap$ are the two different special types topological subset groupoid spaces.

Let A = {3, 2, 1} and B = {5, 4, 0} $\in$ S.

$A \cap B = \phi$.

$A \cup B = \{0, 1, 2, 3, 4, 5\}$.

$$\begin{aligned} A \cap_* B &= \{3, 2, 1\} \cap_* \{5, 4, 0\} \\ &= \{3 * 5, 2 * 5, 1 * 5, 3 * 4, 2 * 4, 1 * 4, \\ &\quad 3 * 0, 2 * 0, 1 * 0\} \\ &= \{12 + 45, 8 + 45, 4 + 45, 12 + 36, 8 + 36, \\ &\quad 4 + 36, 12 + 0, 8 + 0, 4 + 0\} \\ &= \{9, 5, 1, 0, 8, 4\} \qquad \ldots \qquad \text{I} \end{aligned}$$

$$\begin{aligned} B \cap_* A &= \{5, 4, 0\} \cap_* \{3, 2, 1\} \\ &= \{5 * 3, 5 * 2, 5 * 1, 4 * 3, 4 * 2, 4 * 1, \\ &\quad 0 * 3, 0 * 2, 0 * 1\} \\ &= \{20 + 27, 20 + 18, 20 + 9, 16 + 27, 16 + 18, \\ &\quad 16 + 9, 27, 18, 9\} \\ &= \{11, 2, 5, 7, 10, 1, 9, 6, 3\} \qquad \ldots \qquad \text{II} \end{aligned}$$

I and II are different so $A \cap_* B \neq B \cap_* A$ for A, B $\in$ S.

Now we have seen several examples of finite special type topological subset groupoid spaces $T_\cap$ and $T_\cup$.

Now we proceed onto give examples of infinite special type topological subset groupoid spaces.

Example 5.8: Let S = {Collection of all subsets of the groupoid G = $\{R^+ \cup \{0\}, *, (7, \sqrt{7})\}$} be the subset groupoid.

$T_\cup$ and $T_\cap$ are two special type of topological subset groupoid spaces of S.

H = $\{Q^+(\sqrt{7}) \cup \{0\}, *, (7, \sqrt{7}) \subseteq G$ pave way for a subset subgroupoid which is also special type of topological subset groupoid subspaces.

Example 5.9: Let S = {Collection of all subsets from the neutrosophic groupoid G = {$\langle Z \cup I\rangle$, *, (0, I)}} be the subset neutrosophic groupoid.

$T_\cup$ and $T_\cap$ are two special type of topological subset neutrosophic groupoid spaces.

Thus S has infinite number of special type of topological subset neutrosophic groupoid subspaces.

Example 5.10: Let S = {Collection of all subsets from the groupoid G = {$\langle Z_8 \cup I\rangle$, *, (7, I)}} be the subset groupoid of G.

$T_\cup$ and $T_\cap$ are two special types of topological subset neutrosophic groupoid spaces of S.

Example 5.11: Let S = {Collection of all subsets from the groupoid G = {$C\langle Z_{15} \cup I\rangle$, *, (3, 8)}} be the subset groupoid of G.

We see we have the two special types of topological subset groupoid spaces $T_\cup$ and $T_\cap$ of S.

Example 5.12: Let S = {Collection of all subsets from the groupoid G = {$C\langle Z_{19} \cup I\rangle$, *, ($i_F$, I)}} be the subset groupoid.

We see if in G; $C(\langle Z_{19} \cup I\rangle)$ is replaced by $\langle Z_{19} \cup I\rangle$ then G is not a groupoid. Also if $C\langle Z_{19} \cup I\rangle$ is replaced by $C(Z_{19})$ still G is not a groupoid.

If $C(\langle Z_{19} \cup I\rangle)$ is replaced by Z_{19} still G is not a groupoid.

Can S have subset subgroupoids?

Find special type of topological subset groupoid subspaces of S.

We can define using these subset groupoids the notion of Smarandache special type of topological Moufang subset groupoid spaces if the associated groupoid is a Smarandache Moufang groupoid. Similarly if the associated groupoid is a Smarandache Bol groupoid we call $T_\cup$ and $T_\cap$ as the Smarandache special type topological subset Bol groupoid spaces.

Thus if the associated groupoid is a S-P-groupoid we call $T_\cup$ and $T_\cap$ as the Smarandache special type topological P-subset groupoid spaces and so on.

Thus depending of the Smarandache property enjoyed by the groupoid; $T_\cup$ and $T_\cap$ will enjoy similar properties.

If G is a Bol groupoid then we call $T_\cup$ and $T_\cap$ as the Smarandache strong special type topological subset Bol groupoid space.

This is the way Smarandache strong special type topological subset Bol (P or alternative etc) groupoid spaces are defined.

We will give examples of them.

Example 5.13: Let S = {Collection of all subsets from the Smarandache groupoid G = $\{Z_6, *, (4, 5)\}$} be the S-subset groupoid.

We define both $T_\cup$ and $T_\cap$ as the Smarandache special type topological subset groupoid space.

Example 5.14: Let S = {Collection of all subsets from the Smarandache groupoid G = $\{Z_8, *, (2, 6)\}$} be the Smarandache subset groupoid.

$T_\cup$ and $T_\cap$ are the Smarandache special type topological subset groupoid spaces of S.

Example 5.15: Let S = {Collection of all subsets from the Smarandache subset groupoid G = $\{Z_{12}, *, (1, 3)\}$} be the Smarandache subset groupoid.

$T_\cup$ and $T_\cap$ are the Smarandache special type topological subset groupoid spaces of S.

We will give examples of Smarandache special type of topological subset groupoid spaces which satisfy special identities.

Example 5.16: Let S = {Collection of all subsets of the Smarandache strong Bol groupoid G = $\{Z_{12}, *, (3, 4)\}$} be the Smarandache strong subset Bol groupoid.

Both $T_\cup$ and $T_\cap$ are Smarandache strong special type topological subset Bol groupoid spaces of S.

Example 5.17: Let S = {Collection of all subsets of the Smarandache Bol groupoid G = $\{Z_4, *, (2, 3)\}$ be the Smarandache subset Bol groupoid.

$T_\cup$ and $T_\cap$ are Smarandache special type topological Bol subset groupoid spaces of S.

We have seen examples of both Smarandache strong special type topological Bol subset groupoid spaces and Smarandache special type topological Bol subset groupoid spaces of S.

Example 5.18: Let S = {Collection of subsets from the Smarandache strong P-groupoid G = $\{Z_6, *, (4, 3)\}$} be the Smarandache strong subset P-groupoid.

Let $T_\cup$ and $T_\cap$ be the Smarandache strong special type topological strong subset P groupoid spaces of S.

Example 5.19: Let S = {Collection of all subsets from the Smarandache P-groupoid G = $\{Z_6, *, (3, 5)\}$} be the Smarandache subset P-groupoid.

$T_\cup$ and $T_\cap$ are both Smarandache special type topological subset P-groupoid spaces.

Example 5.20: Let S = {Collection of all subsets from the Smarandache strong P-groupoid G = $\{Z_4, *, (2, 3)\}$} be the Smarandache strong P-subst groupoid.

$T_\cup$ and $T_\cap$ are Smarandache strong special type topological subset P-groupoid spaces of S.

Example 5.21: Let S = {Collection of all subsets from the Smarandache strong alternative groupoid G = $\{Z_{14}, *, (7, 8)\}$} be the Smarandache strong alternative subset groupoid.

$T_\cup$ and $T_\cap$ are the Smarandache strong special type topological subset groupoid spaces of S.

Example 5.22: Let S = {Collection of all subsets from the Smarandache inner commutative groupoid $(Z_5, *, (3, 3))$} be the Smarandache subset inner commutative groupoid.

$T_\cup$ and $T_\cap$ are Smarandache special topological subset inner commutative groupoid spaces of S.

We can give examples of $T_\cup$ and $T_\cap$ which satisfy special Smarandache identities.

Now we discuss the special type topological subset loop groupoid spaces $T_\cup$ and $T_\cap$ where $T_\cup$ and $T_\cap$ inherit the operations from the loop L.

We will illustrate this situation by some examples.

Example 5.23: Let S = {Collection of all subsets of the loop $L_7(3)$} be the subset loop groupoid. $T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ where $\cap_*$ is the * operation of L are a special type topological subset loop groupoid spaces of finite order.

Example 5.24: Let S = {Collection of all subsets from the loop $L_{19}(10)$} be the subset loop groupoid. Let $T_\cup$ and $T_\cap$ be the special type topological subset loop groupoid spaces of S.

Both the spaces are commutative but non associative.

Example 5.25: Let S = {Collection of all subsets from the loop $L_5(2)$ given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

be the subset loop groupoid.

Let A = {1, 2, 3} and B = {4, 5, e, 2} $\in$ S′.

We see

$$\begin{aligned} A \cap_* B &= \{1, 2, 3\} * \{4, 5, e, 2\} \\ &= \{1 * 4, 2 * 4, 3 * 4, 1 * 5, 2 * 5, 3 * 5, \\ &\quad 1 * e, 2 * e, 3 * e, 1 * 2, 2 * 2, 3 * 2\} \\ &= \{2, 1, 5, 4, 3, 2, 1, 2, 3, 3, e, 1\} \\ &= \{2, 1, 5, 3, 4, e\} \in S'. \end{aligned}$$

Consider

$$\begin{aligned} B \cap_* A &= \{4, 5, e, 2\} \cap_* \{1, 2, 3\} \\ &= \{4 * 1, 5 * 1, e * 1, 2 * 1, 4 * 2, 5 * 2, \\ &\quad e * 2, 2 * 2, 4 * 3, 5 * 3, e * 3, 2 * 3\} \\ &= \{e, 3, 1, 2, 5, 4\} \in S. \end{aligned}$$

We see A $\cap_*$ B = B $\cap_*$ A for this particular A, B $\in$ S′.

For take A = {2} and B = {3} in S; we see

$$
\begin{aligned}
A \cap_* B &= \{2\} \cap_* \{3\} \\
&= \{2 * 3\} \\
&= \{4\} & \ldots \quad \text{I}
\end{aligned}
$$

$$
\begin{aligned}
B * A &= \{3\} \cap_* \{2\} \\
&= \{3 * 2\} \\
&= \{1\} & \ldots \quad \text{II}
\end{aligned}
$$

I and II are distinct so $\cap_*$ is non commutative.

Take $C = \{4, 5\} \in S$.

$$
\begin{aligned}
\text{We find } A \cap_* (B \cap_* C) &= A \cap_* (\{3\} \cap_* \{4, 5\}) \\
&= A \cap_* \{3 * 4, 3 * 5\} \\
&= A \cap_* \{5, 2\} \\
&= \{2\} \cap_* \{5, 2\} \\
&= \{2 * 5, 2 * 2\} \\
&= \{e, 1\} & \ldots \quad \text{I}
\end{aligned}
$$

Now we find $(A \cap_* B) \cap_* C$

$$
\begin{aligned}
&= \{4\} \cap_* \{4, 5\} \quad (\text{Using I of } A \cap_* B) \\
&= \{4 * 4, 4 * 5\} \\
&= \{e, 1\} & \ldots \quad \text{II}
\end{aligned}
$$

We see
$(A \cap_* B) \cap_* C = A \cap_* (B \cap_* C)$ as I and II are identical.

However take $A = \{2\}$, $B = \{3\}$ and $C = \{5\}$ in S'

$$
\begin{aligned}
(A \cap_* B) \cap_* C &= (\{2\} * \{3\}) \cap_* C \\
&= \{4\} \cap_* \{5\} \\
&= \{4 * 5\} \\
&= \{1\} & \ldots \quad \text{I}
\end{aligned}
$$

$$
\begin{aligned}
A \cap_* (B \cap_* C) &= A \cap_* (\{3\} * \{5\}) \\
&= A \cap_* \{3 * 5\}
\end{aligned}
$$

$= A \cap_* \{2\}$
$= \{2\} \cap_* \{2\}$
$= \{2 * 2\}$
$= \{e\}$ … II

I and II are distinct hence in general
$A \cap_* (B \cap_* C) \neq (A \cap_* B) \cap_* C$ for $A, B, C \in S$.

Example 5.26: Let S = {Collection of all subsets from the loop $L_{19}(3)$} be the subset loop groupoid.

$T_\cup$ and $T_\cap$ are Smarandache special type topological subset loop groupoid spaces as the loop $L_{19}(3)$ is S-loop.

In view of this we have the following theorem.

THEOREM 5.1: *Let*
S = {Collection of all subsets from the loop $L_n(m)$} be the subset loop groupoid. Both the topological spaces $T_\cup$ and $T_\cap$ are S-special type topological subset loop groupoid spaces of S.

Follows from the fact all loops $L_n(m)$ in L_n are Smarandache loops.

All types of Smarandache strong special type topological subset loop groupoid spaces and Smarandache special type topological subset loop groupoid spaces satisfying special identities.

We now proceed onto study special type topological subset loop groupoid spaces of subset loop matrix groupoids and subset loop polynomial groupoids apart from subset matrix groupoids and subset polynomial groupoids.

Example 5.27: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{array} \right] \middle| \; a_i \in G = \{Z_{47}, *, (3, 4)\}, 1 \leq i \leq 9\} \right\}$$

be the subset matrix groupoid.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ be the special type topological subset matrix groupoid spaces of finite order.

Example 5.28: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{array} \right] \middle| \; a_i \in G = \{Z_{12}, *, (4, 9)\}, 1 \leq i \leq 12\} \right\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid spaces of S.

Example 5.29: Let S = {Collection of all subsets from the matrix groupoid M = $\{(a_1, \ldots, a_7) \mid a_i \in G = \{Z_{16}, *, (8, 0)\}; 1 \leq i \leq 7\}$ be the subset matrix groupoid of G.

$T_\cup$ and $T_\cap$ are special type topological subset matrix subset groupoid spaces of S.

Example 5.30: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix} \middle| \; a_i \in G = \{Z_6, *, (4, 0)\}, 1 \le i \le 6\} \right\}$$

be the subset matrix groupoid.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ are special type topological subset matrix groupoid spaces of S.

$$\text{Let } A = \left\{ \begin{bmatrix} 2 & 0 \\ 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 3 \end{bmatrix} \right\} \text{ and } B = \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{bmatrix} \right\} \in S.$$

$$A \cap_* B = \left\{ \begin{bmatrix} 2 & 0 \\ 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 3 \end{bmatrix} \right\} \cap_* \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 2 & 0 \\ 1 & 2 \\ 0 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 2*1 & 0*2 \\ 1*3 & 2*0 \\ 0*0 & 4*1 \end{bmatrix}, \begin{bmatrix} 3*1 & 0*2 \\ 0*3 & 3*0 \\ 3*0 & 3*1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 2 & 0 \\ 4 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} \in S'.$$

We see $\cap_*$ is both non associative and non commutative on $T_\cup$ and $T_\cap$.

We give more examples of them.

Example 5.31: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_8 \\ a_9 & a_{10} & \dots & a_{16} \\ a_{17} & a_{18} & \dots & a_{24} \end{bmatrix} \middle| \; a_i \in G = \{Z_{87}, *, (0, 3)\}, 1 \leq i \leq 24 \right\}\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid spaces of S.

Example 5.32: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ a_{31} & a_{32} & \dots & a_{40} \end{bmatrix} \middle| \; a_i \in G = \{Z, *, (3, -3)\}, 1 \leq i \leq 40 \right\}\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid spaces of infinite order.

Example 5.33: Let S = {Collection of all subsets from the matrix groupoid

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in G = \{\langle Z \cup I \rangle, *, (0, I)\}, 1 \leq i \leq 12 \right\}\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type neutrosophic topological subset matrix groupoid spaces of infinite order.

Example 5.34: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in G = \{C(\langle Z_7 \cup I \rangle), *, (I, i_F)\}, \; 1 \leq i \leq 16 \right\} \Bigg\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type finite modulo complex number neutrosophic topological subset matrix groupoid spaces of S.

Example 5.35: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| \; a_i \in \{C(Z_{42}), *, (10, 3)\}, \; 1 \leq i \leq 25 \right\} \Bigg\}$$

be the subset matrix groupoid. $T_\cup$ and $T_\cap$ are special type of topological subset matrix groupoid spaces of S.

Example 5.36: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & \dots & a_8 \\ a_9 & \dots & a_{16} \end{bmatrix} \middle| \; a_i \in \{C(\langle Z_{11} \cup I \rangle), *, (7, 5)\}, \; 1 \leq i \leq 16 \right\} \Bigg\}$$

be the subset matrix groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid space of S′.

Example 5.37: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in G = \{\langle C \cup I \rangle, *, (2, -2I)\}, 1 \leq i \leq 12 \right\}\}$$

be the subset matrix groupoid. $T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid spaces of S of infinite order.

Infact $T_\cup$ and $T_\cap$ are also complex neutrosophic special type topological subset matrix groupoid spaces of S.

Now having seen examples of special type topological subset matrix groupoid spaces we now proceed onto give substructures in them by some examples.

Example 5.38: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \end{bmatrix} \middle| \; a_i \in G = \{\langle R \cup I \rangle, *, (10, -10)\}, 1 \leq i \leq 24 \right\}\}$$

be the subset matrix groupoid of M.

Take P = {Collection of all subsets from the matrix subgroupoid

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{24} \end{bmatrix} \middle| \; a_i \in H = \{ R, *, (10, -10)\}, \right.$$

$$\left. 1 \leq i \leq 24\} \right\}$$

be the subset matrix subgroupoid of S.

$T_\cup$ and $T_\cap$ are special type topological subset matrix groupoid spaces of S.

We see if $L_\cup = \{P', \cup, \cap_*\}$ and $L_\cap = \{P', \cap, \cap_*\}$ are special type topological subset matrix subgroupoid subspace of $T_\cup$ and $T_\cap$ respectively.

Infact we have infinite number of topological subspaces for this S.

Example 5.39: Let S = {Collection of all subsets from the matrix groupoid $M = \{[a_1, a_2, a_3, a_4, a_5] \mid a_i \in G = \{Z_{12}, *, (4, 0)\}; 1 \leq i \leq 5\}\}$ be the subset matrix groupoid of M.

Let $T_\cup$ and $T_\cap$ be the special type topological subset matrix groupoid spaces of S.

Now let P = {Collection of all subsets from matrix subgroupoid $N = \{[a_1, a_2, \dots, a_5] \mid a_i \in H = \{2Z_{12}, *, (4, 0)\} \subseteq G; 1 \leq i \leq 5\}\}$ be the subset matrix subgroupoid of S.

Let $L_\cup = \{P', \cup, \cap_*\}$ and $L_\cap = \{P', \cap, \cap_*\}$ be special type topological subset matrix subgroupoid subspaces of $P \subseteq S$.

Example 5.40: Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in \{C(\langle Z_{15} \cup I \rangle), *, (5, 10)\}, 1 \le i \le 30\} \right\}$$

be the subset matrix groupoid of M.

$T_\cup$ and $T_\cap$ are both special type topological subset matrix groupoid spaces of S.

Let P = {Collection of all subsets from the matrix subgroupoid

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in H = \{(\langle Z_{15} \cup I \rangle), *, (5, 10)\}, 1 \le i \le 30\} \right\}$$

be the subset matrix subgroupoid of S.

Let $L_\cup$ and $L_\cap$ be $\{P', \cup, \cap_*\}$ and $\{P', \cap, \cap_*\}$ respectively be the special type topological subset matrix subgroupoid subspaces of $T_\cup$ and $T_\cap$ respectively.

Infact $T_\cup$ and $T_\cap$ has more number of topological subspaces associated with it.

Now we proceed onto give examples of special type topological subset polynomial groupoid spaces.

Example 5.41: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in G = \{Z_{42}, *, (6, 12)\}\right\}$$

be the subset polynomial groupoid of M.

Let $T_\cup$ and $T_\cap$ be the special type topological subset polynomial groupoid spaces of infinite order.

Example 5.42: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in G = \{C(Z_{19}), *, (2, i_F)\}\right\}$$

be the subset polynomial groupoid.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ be special type topological subset polynomial groupoid spaces of S′.

Example 5.43: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \{Z, *, (10, -10)\}\right\}$$

be the subset polynomial groupoid.

$T_\cup$ and $T_\cap$ be the special type topological subset polynomial groupoid spaces of S. $T_\cup$ and $T_\cap$ have topological subspaces.

Example 5.44: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in G = \{C(\langle Z_{15} \cup I \rangle), *, (3, 13)\}\right\}$$

be the subset polynomial groupoid.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ be special type subset polynomial topological groupoid subspaces.

Example 5.45: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{(\langle Z_{18} \cup I \rangle), *, (I, 0)\}\right\}\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset polynomial groupoid spaces of S.

We can as in case of other topological spaces define in this case also subspaces.

We will illustrate this situation by some examples.

Example 5.46: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i , b_i \in G = \{(\langle Z_{12} \cup I \rangle), *, (4, 0)\}\right\}\}$$

be the subset polynomial groupoid of M.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ be the special type topological subset polynomial groupoid spaces of S′.

Let P = {Collection of all subsets from the polynomial subgroupoid;

$$N = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in H = \{C(Z_{12}), *, (4, 0)\}\right\} \subseteq G\}$$

be the subset interval polynomial subgroupoid of S.

$L_{\cup} = \{P', \cup, \cap_*\}$ and $L_{\cap} = \{P', \cap, \cap_*\}$ are subsets of $T_{\cup}$ and $T_{\cap}$ respectively.

Further $L_{\cup}$ and $L_{\cap}$ be the special type topological subset interval polynomial groupoid subspaces of $T_{\cup}$ and $T_{\cap}$ respectively.

Infact $T_{\cup}$ and $T_{\cap}$ have more such topological subspaces.

Example 5.47: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{\left.\begin{bmatrix} a_1 \\ a_4 \\ \vdots \\ a_{10} \end{bmatrix}\right| a_i \in L_{27}(5), 1 \leq i \leq 10\right\}$$

be the subset matrix loop groupoid.

$T_{\cup} = \{S', \cup, \cap_*\}$ and $T_{\cap} = \{S', \cap, \cap_*\}$ are the special type topological subset matrix loop groupoid spaces of S.

Example 5.48: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right| a_i \in L_{23}(9), 1 \leq i \leq 12\right\}\}$$

be the subset matrix loop groupoid.

$T_{\cup}$ and $T_{\cap}$ are special type topological subset matrix loop groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.49: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \begin{bmatrix} a_1 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \; a_i \in L_{45}(8), 1 \leq i \leq 19 \right\}\}$$

be the subset matrix loop groupoid.

$T_\cup$ and $T_\cap$ are both special type topological subset matrix loop groupoid spaces of finite order.

Example 5.50: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \cdots & a_{10} \\ a_{11} & a_{12} & \cdots & a_{20} \\ \vdots & \vdots & & \vdots \\ a_{91} & a_{92} & \cdots & a_{100} \end{bmatrix} \middle| \; a_i \in L_{105}(17), 1 \leq i \leq 100 \right\}\}$$

be the subset matrix loop groupoid.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ be the special type topological subset matrix loop groupoid spaces of $S' = \{S \cup \phi\}$.

Example 5.51: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \cdots & a_{10} \\ a_{11} & a_{12} & \cdots & a_{20} \end{bmatrix} \middle| \; a_i \in L_{109}, 1 \leq i \leq 20 \right\}\}$$

be the subset matrix loop groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset matrix loop groupoid spaces of $S' = \{S \cup \phi\}$.

Example 5.52: Let S = {Collection of all subsets from the matrix loop groupoid $M = \{(a_1, a_2, a_3, a_4) \mid a_i \in L_{15}(8), 1 \leq i \leq 4\}\}$ be the subset matrix loop groupoid.

$T_\cup$ and $T_\cap$ are the two special type topological subset matrix loop groupoid spaces of $S' = \{S \cup \phi\}$.

Now we will give examples of Smarandache special type topological subset matrix loop groupoid spaces and subspaces of $T_\cup$ and $T_\cap$.

Example 5.53: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{\left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_9 \\ a_{11} & a_{12} & \ldots & a_{18} \end{array}\right] \middle| a_i \in L_7(3), 1 \leq i \leq 18\}\right\}$$

be the subset matrix loop groupoid of M.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ are special type topological subset matrix loop groupoid spaces.

Let

$$P = \left\{\left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_9 \\ a_{11} & a_{12} & \ldots & a_{18} \end{array}\right] \middle| a_i \in \{e, 8\} \subseteq L_7(3), 1 \leq i \leq 18\right\} \subseteq M.$$

W = {Collection of all subsets from P} ⊆ S be the subset matrix loop subgroupoid. It is easily verified W is a subset matrix semigroup of M, hence S is a Smarandache subset matrix loop groupoid.

So $T_\cup$ and $T_\cap$ the special type topological subset matrix loop groupoid spaces are Smarandache topological spaces.

Example 5.54: Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \middle| a_i \in L_{27}(8), 1 \leq i \leq 24\right\}\}$$

be the subset matrix loop groupoid.

S is a Smarandache subset matrix loop groupoid so $T_\cup$ and $T_\cap$ are Smarandache special type topological subset matrix loop groupoid spaces of S′.

In view of this we give the following theorem.

THEOREM 5.2: *Let S = {Collection of all subsets from the matrix loop groupoid M = {$m_1 \times n_1$ matrix with elements from $L_n(m)$}} be the subset matrix loop groupoid.*

(i) *S is a Smarandache subset matrix loop groupoid.*
(ii) *$T_\cup$ and $T_\cap$ are both Smarandache special type topological subset matrix loop groupoid spaces.*

The proof is direct and hence left as an exercise to the reader.

Now we proceed onto give examples of subset polynomial groupoids and their related special type topological subset polynomial loop groupoid spaces.

Example 5.55: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \middle| a_i \in L_{93}(8)\right\}\}$$

be the subset polynomial loop groupoid of infinite order.

$T_\cup$ and $T_\cap$ are subset topological polynomial loop groupoid spaces of S.

Example 5.56: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_7(4) \right\}$$

be the subset polynomial loop groupoid of infinite order.

$T_\cup = \{S' = S \cup \{\phi\}, \cup, \cap_\phi\}$ and
$T_\cap = \{S' = \{S \cup \{\phi\}\}, \cap, \cap_\phi\}$ be special type topological subset polynomial loop groupoid spaces of infinite order.

Example 5.57: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{245}(12) \right\}\}$$

be the subset polynomial loop groupoid of subset polynomial loop groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset polynomial loop groupoid spaces of $S' = S \cup \{\phi\}$; both $T_\cup$ and $T_\cap$ are of infinite order.

Example 5.58: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{43}(7) \right\}\}$$

be the subset polynomial loop groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset polynomial loop groupoid spaces $S' = S \cup \{\phi\}$.

Example 5.59: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{45}(14)\}\right\}$$

be the subset polynomial loop groupoid.

$T_\cup$ and $T_\cap$ are both special type topological subset polynomial loop groupoid spaces of $S' = S \cup \{\phi\}$.

Now we will give examples of substructures and the Smarandache analogue.

Example 5.60: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_9(8)\}\right\}$$

be the subset polynomial loop groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset polynomial loop groupoid spaces of $S' = S \cup \{\phi\}$.

$T_\cup$ and $T_\cap$ are Smarandache special type topological polynomial loop groupoid spaces of S' as $L_9(8)$ is a Smarandache loop.

Example 5.61: Let S = {Collection of all subsets from the polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in G = \{(\langle Z \cup I \rangle), *, (5, 0)\}\}\right\}$$

be the subset polynomial groupoid.

$T_\cup$ and $T_\cap$ has special type topological subset polynomial groupoid subspaces.

Interested reader can get more examples.

Now we just give an example or two of special type topological subset loop groupoids (subset matrix loop groupoids and subset polynomial loop groupoids).

Example 5.62: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{11}(4) \right\}\}$$

be the subset polynomial loop groupoid. $L_{11}(4)$ is a Smarandache loop so $T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ are both Smarandache special type topological subset polynomial loop groupoid spaces of S'.

Example 5.63: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{165}(23) \right\}\}$$

be the subset polynomial loop groupoid.

$L_{165}(23)$ is Smarandache loop; so both special type topological subset polynomial loop groupoid spaces are Smarandache.

We will give some subspaces of $T_\cup$ and $T_\cap$.

Let P = {Collection of all subsets from the polynomial loop subgroupoid

$$N = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in H_1(33) \subseteq L_{165}(23)\} \subseteq M \right\}$$

be the subset polynomial loop subgroupoid of S.

Let $L_\cup = \{P' = P \cup \{\phi\}, \cup, \cap_*\}$
and $L_\cap = \{P' = P \cup \{\phi\}, \cap, \cap_*\}$ be the special type topological subset loop subgroupoid subspaces of $T_\cup$ and $T_\cap$ respectively.

Example 5.64: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{29}(8) \right\}\}$$

be the subset polynomial loop groupoid.

$T_\cup$ and $T_\cap$ be special type topological subset polynomial loop groupoid spaces.

P = {Collection of all subsets from the polynomial loop subgroupoid

$$N = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{29}(8) \right\}\} \subseteq S$$

be the subset polynomial loop subgroupoid of S.

Let $L_\cup = \{P', \cup, \cap_*\}$ and $L_\cap = \{P', \cap, \cap_*\}$ be special type topological subset polynomial loop subgroupoid subspaces of $T_\cup$ and $T_\cap$ respectively.

Example 5.65: Let S = {Collection of all subsets from the polynomial loop groupoid

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{127}(12)\}\right\}$$

be the subset polynomial loop groupoid of M.

$T_\cup$ and $T_\cap$ are special type subset topological polynomial loop groupoid spaces of S.

Let

$$P = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{127}(12)\right\}$$

be the polynomial loop subgroupoid of M.

Let R = {Collection of all subsets from P} be the subset polynomial loop subgroupoid of S.

$L_\cup = \{R', \cup, \cap_*\}$ and $L_\cap = \{R', \cap, \cap_*\}$ are special type topological subset polynomial loop subgroupoid subspaces of $T_\cup$ and $T_\cap$ respectively.

Interested reader can find substructures of S, $T_\cup$ and $T_\cap$ of the subset polynomial loop groupoids.

Now we proceed onto study subset interval groupoid, subset interval loop groupoid, subset interval matrix groupoid, subset interval matrix loop groupoid, subset interval polynomial groupoid and subset interval polynomial loop groupoid.

All these cases can be analysed by appropriate changes.

However we give a few examples of these for the reader to follow.

Example 5.66: Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b $\in$ G = {Z_5, *, (2, 3)}}} be the subset interval groupoid.

$T_\cup$ = {S′ = S $\cup$ {ϕ}, $\cup$, $\cap_*$}
and $T_\cap$ = {S′ = S $\cup$ {ϕ}, $\cap$, $\cap_*$} are special type topological subset interval groupoid spaces of S′.

Example 5.67: Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b $\in$ G = {C($\langle Z_{25} \cup I\rangle$, *, (7, 0)}}} be the subset interval groupoid.

$T_\cup$ and $T_\cap$ are special type topological subset interval groupoid spaces.

Both the examples give only finite order $T_\cap$ and $T_\cup$.

Example 5.68: Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b $\in$ G = {$\langle C \cup I\rangle$, *, (3, 4)}} be the subset interval groupoid.

Both the special type topological subset interval groupoid spaces $T_\cup$ and $T_\cap$ are of infinite order.

Example 5.69: Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b $\in$ G = {$\langle Z^+ \cup \{0\} \cup I\rangle$ (g_1, g_2), *, (8, 0); $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$}}} be the subset interval groupoid.

$T_\cup$ and $T_\cap$ are special types of topological subset interval groupoid spaces of infinite order. These $T_\cup$ and $T_\cap$ have number of subspaces.

Example 5.70: Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b $\in$ $L_7(2)$}} be the subset interval loop groupoid of finite order.

$T_\cup$ and $T_\cap$ are special type topological subset interval loop groupoid spaces of $S' = S \cup \{\phi\}$ of finite order.

These topological spaces are Smarandache as $L_7(2)$ is a Smarandache loop.

Example 5.71: Let S = {Collection of all subsets from the interval groupoid $M = \{[a, b] \mid a, b \in L_{45}(8)\}\}$ be the subset interval loop groupoid.

$T_\cup$ and $T_\cap$ be special type topological subset interval loop groupoid spaces of finite order.

These spaces $T_\cup$ and $T_\cap$ has several subspaces.

For all proper subloops of $L_{45}(8)$ will pave way to subspaces. Further $T_\cup$ and $T_\cap$ are also Smarandache spaces.

Example 5.72: Let S = {Collection of all subsets from the interval groupoid $M = \{[a, b] \mid a, b \in L_{23}(8)\}\}$ be the subset interval loop groupoid.

$T_\cup$ and $T_\cap$ has atleast 25 special type topological subset interval loop subgroupoid subspaces.

Now we proceed onto describe by examples special type topological subset interval matrix loop groupoid spaces of interval matrix groupoid.

Recall a row interval matrix groupoid $M = \{([a_1, b_1], \ldots, [a_n, b_n]) \mid a_i, b_i \in G = \{Z_m, *, (t, s)\}, 1 \leq i \leq n\}\}$; (M, *) is a row interval groupoid.

G can be any groupoid not necessarily the one given in M; it can be of finite or infinite order.

Likewise N = $\left\{\left.\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ \vdots \\ [a_t,b_t]\end{bmatrix}\right| a_i, b_i \in (G, *); 1 \leq i \leq t\right\}$.

G any groupoid is an interval column matrix groupoid under the operation *.

T = {P = ($[a_{ij}, b_{kl}]$) | a_{ij}, b_{kl} ∈ (G, *), $1 \leq i, k \leq m$, $1 \leq j, l \leq n$} is the collection of all m × n interval matrices. T is a m × n matrix interval groupoid under the operation *.

If m = n we get a square matrix interval groupoid.

We will first illustrate these situations before we proceed onto define subset interval matrix groupoids of an interval matrix groupoid.

Example 5.73: Let M = {($[a_1, b_1]$, $[a_2, b_2]$), where a_i, b_i ∈ G = {Z_{14}, *, (3, 0)} $1 \leq i \leq 2$}} be an interval row matrix groupoid of finite order.

For more about interval groupoid refer [31].

Example 5.74: Let T = {($[a_1, b_1]$, $[a_2, b_2]$, …, $[a_7, b_7]$) | a_i, b_i ∈ (Z, *, (20, –1)), $1 \leq i \leq 7$} be an interval row matrix groupoid of infinite order.

Example 5.75: Let S = {($[a_1, b_1]$, $[a_2, b_2]$, …, $[a_{10}, b_{10}]$) | a_i, b_i ∈ G = {C(Z_{20}), *, (0, $2i_F$), $1 \leq i \leq 10$}} be the complex modulo integer inveral row matrix groupoid of finite order.

Example 5.76: Let S = {($[a_1, b_1]$, $[a_2, b_2]$, …, $[a_{12}, b_{12}]$) | a_i, b_i ∈ G = {⟨Z_{20} ∪ I⟩, *, (10I, 0), $1 \leq i \leq 12$}} be the neutrosophic modulo integer interval row matrix groupoid of finite order.

Example 5.77: Let S = {([a_1, b_1], [a_2, b_2], …, [a_{11}, b_{11}]) | a_i, b_i ∈ G = {Z_{10} (g_1, g_2), *, (9, 0), $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$}, 1 ≤ i ≤ 11}} be the interval row matrix groupoid of finite order.

Example 5.78: Let S = {([a_1, b_1], [a_2, b_2], …, [a_7, b_7]) | a_i, b_i ∈ G = {Z (g_1, g_2), *, (9, 2g_1)} 1 ≤ i ≤ 7}} be the interval row matrix groupoid of dual numbers of infinite order.

Example 5.79: Let S = {([a_1, b_1], [a_2, b_2], …, [a_{14}, b_{14}]) | a_i, b_i ∈ G = {($Z^+ \cup \{0\}$), *, (8, 0)} 1 ≤ i ≤ 14}} be the interval row matrix groupoid of infinite order.

Example 5.80: Let S = {([a_1, a_2]) | a_i ∈ G = {⟨C ∪ I⟩, *, (2i, 4I), 1 ≤ i ≤ 2}} be the interval matrix groupoid of infinite order.

Clearly M is a complex neutrosophic interval groupoid.

Example 5.81: Let

$$S = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_{40}, *, (10, 30), 1 \le i \le 9\} \right\}$$

be the interval column matrix groupoid of finite order.

Example 5.82: Let

$$W = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{11}, b_{11}] \end{bmatrix} \middle| \; a_i, b_i \in G = \{C(Z_{19}), *, (i_F, 9), 1 \le i \le 11\} \right\}$$

be the interval column matrix groupoid of finite order.

Example 5.83: Let

$$S = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{20}, b_{20}] \end{array}\right] \middle| \; a_i, b_i \in G = \{\langle Z_{11} \cup I\rangle, *, (I, 10I), 1 \leq i \leq 20\}\right\}$$

be the interval column matrix groupoid of finite order.

Example 5.84: Let

$$S = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{array}\right] \middle| \; a_i, b_i \in G = \{\langle Z \cup I\rangle, *, (I, 0)\}, 1 \leq i \leq 10\right\}$$

be a interval column matrix neutrosophic groupoid of infinite order.

Example 5.85: Let

$$S = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_7, b_7] \end{array}\right] \middle| \; a_i, b_i \in G = \{\langle C \cup I\rangle (g_1, g_2), *, (I, g_1); \right.$$
$$\left. g_1^2 = 0, g_1g_2 = g_2g_1 = 0, \; g_2^2 = g_2\}, 1 \leq i \leq 7\}\right\}$$

be the interval column matrix complex neutrosophic dual number groupoid.

Example 5.86: Let

$$S = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{15}, b_{15}] \end{bmatrix} \right| a_i, b_i \in G = \{\langle Z_{12} \cup I\rangle (g_1, g_2, g_3), *,$$

$$(2g_1, g_2) \text{ where } g_1^2 = 0,\ g_2^2 = g_2,\ g_3^2 = -g_3,$$
$$g_ig_j = g_jg_i = 0, i \neq j, 1 \leq i, j \leq 3\}; 1 \leq i \leq 15\}\}$$

be the neutrosophic mixed dual number of interval column matrix groupoid of finite order.

Now we proceed onto give examples of square and rectangular interval matrix groupoids.

Example 5.87: Let

$$S = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \end{bmatrix} \right| a_i, b_i \in G = \{\langle Z_9 \cup I\rangle, *,$$

$$(3I, 6)\}; 1 \leq i \leq 8\}$$

be the interval rectangular matrix groupoid of finite order.

$$\text{Let } A = \begin{bmatrix} [0,1] & [2,0] \\ [3,0] & [1,1] \\ [2,I] & [I,0] \\ [0,2I] & [0,0] \end{bmatrix} \text{ and } B = \begin{bmatrix} [8,0] & [0,0] \\ [0,1] & [I,0] \\ [I,I] & [0,3I] \\ [0,0] & [1,2] \end{bmatrix} \in P.$$

$$A * B = \begin{bmatrix} [0,1] & [2,0] \\ [3,0] & [1,1] \\ [2,I] & [I,0] \\ [0,2I] & [0,0] \end{bmatrix} * \begin{bmatrix} [8,0] & [0,0] \\ [0,1] & [I,0] \\ [I,I] & [0,3I] \\ [0,0] & [1,2] \end{bmatrix}$$

$$= \begin{bmatrix} [0,1]*[8,0] & [2,0]*[0,0] \\ [3,0]*[0,1] & [1,1]*[I,0] \\ [2,I]*[I,I] & [I,0]*[0,3I] \\ [0,2I]*[0,0] & [0,0]*[1,2] \end{bmatrix}$$

$$= \begin{bmatrix} [48,3I] & [6I,0] \\ [18I,6] & [3I+6I,3I] \\ [6I+6I,3I+6I] & [3I,18I] \\ [0,12I] & [6,12] \end{bmatrix}$$

$$= \begin{bmatrix} [3,3I] & [6I,0] \\ [0,6] & [0,3I] \\ [3I,0] & [3I,0] \\ [0,3I] & [6,3] \end{bmatrix} \in S.$$

This is the way * operation is performed on interval rectangular matrix groupoids.

It is easily verified * is a non commutative and non associative operation.

For such operation on interval matrix groupoids please refer [ ].

Example 5.88: Let

$$S = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in G = \{C(Z_5), *, (3, 0)\}; 1 \leq i \leq 4 \right\}$$

be the interval column matrix groupoid of finite order.

Let $A = \begin{bmatrix} [1,2] \\ [i_F, 0] \\ [0, 4i_F] \\ [1, 3i_F] \end{bmatrix}$ and $B = \begin{bmatrix} [2i_F, 0] \\ [0, 4] \\ [2, 4i_F] \\ [1, i_F] \end{bmatrix}$ be in S.

$$A * B = \begin{bmatrix} [1,2] \\ [i_F, 0] \\ [0, 4i_F] \\ [1, 3i_F] \end{bmatrix} * \begin{bmatrix} [2i_F, 0] \\ [0, 4] \\ [2, 4i_F] \\ [1, i_F] \end{bmatrix}$$

$$= \begin{bmatrix} [1,2]*[2i_F, 0] \\ [i_F, 0]*[0, 4] \\ [0, 4i_F]*[2, 4i_F] \\ [1, 3i_F]*[1, i_F] \end{bmatrix}$$

$$= \begin{bmatrix} [1*2i_F, 2*0] \\ [i_F*0, 0*4] \\ [0*2, 4i_F*4i_F] \\ [1*1, 3i_F*i_F] \end{bmatrix}$$

$$= \begin{bmatrix} [3,6] \\ [3i_F, 0] \\ [0, 12i_F] \\ [3, 9i_F] \end{bmatrix}$$

$$= \begin{bmatrix} [3,1] \\ [3i_F, 0] \\ [0, 2i_F] \\ [3, 4i_F] \end{bmatrix} \in S.$$

Similarly operation on interval row matrix groupoids are performed.

Example 5.89: Let

$$S = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \end{bmatrix} \middle| \; a_i, b_i \in G = \{Z_6, *, (2, 4)\};\ 1 \le i \le 6 \right\}$$

be the interval matrix groupoid.

$$\text{Let } A = \begin{bmatrix} [3,0] & [0,2] & [1,3] \\ [1,2] & [4,0] & [5,0] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [1,2] & [0,1] & [0,4] \\ [2,1] & [2,2] & [4,1] \end{bmatrix} \text{ be in S.}$$

$$A * B = \begin{bmatrix} [3,0] & [0,2] & [1,3] \\ [1,2] & [4,0] & [5,0] \end{bmatrix} * \begin{bmatrix} [1,2] & [0,1] & [0,4] \\ [2,1] & [2,2] & [4,1] \end{bmatrix}$$

$$= \begin{bmatrix} [3,0]*[1,2] & [0,2]*[0,1] & [1,3]*[0,4] \\ [1,2]*[2,1] & [4,0]*[2,2] & [5,0]*[4,1] \end{bmatrix}$$

$$= \begin{bmatrix} [3*1,0*2] & [0*0,2*1] & [1*0,3*4] \\ [1*2,2*1] & [4*2,0*2] & [5*4,0*1] \end{bmatrix}$$

$$= \begin{bmatrix} [6+4,0+8] & [0+0,4+4] & [2+0,6+16] \\ [2+8,4+4] & [8+8,0+8] & [10+16,4] \end{bmatrix}$$

$$= \begin{bmatrix} [4,2] & [0,2] & [2,4] \\ [4,2] & [4,2] & [2,4] \end{bmatrix} \in S.$$

The product is the natural product $\times_n$ on matrices. Infact $A * B \neq B * A$; in general even under natural product be it a interval row matrix groupoid or a interval column matrix groupoid or interval rectangular matrix groupoid or a interval square matrix groupoid.

Consider A = [3, 4] and B = [5, 2] interval matrix of singletons from G = $\{Z_7, *, (3, 0)\}$

$$\begin{aligned} A * B &= [3, 4] * [5, 2] \\ &= [3 * 5, 4 * 2] \\ &= [9, 12] = [2, 5] \qquad \ldots \qquad \text{I} \end{aligned}$$

$$\begin{aligned} B * A &= [5, 2] * [3, 4] \\ &= [5 * 3, 2 * 4] \\ &= [15, 6] \\ &= [1, 6] \qquad \ldots \qquad \text{II} \end{aligned}$$

We see $A * B \neq B * A$ as I and II are different hence our claim the product on interval matrix groupoids in general are non commutative.

Consider A = ([3, 5], [1, 2], [0, 7]) and
B = ([0, 2], [5, 0], [1, 3]) $\in$ S

M = {Collection of all 1×3 interval row matrices with entries from G = {Z_{10}, *, (2, 0)}}

$$\begin{aligned} A * B &= ([3, 5], [1, 2], [0, 7]) * ([0, 2], [5, 0], [1, 3]) \\ &= ([3, 5]* [0, 2], [1, 2]* [5, 0], [0, 7]* [1, 3]) \\ &= ([3 * 0, 5 * 2], [1 * 5, 2 * 0], [0 * 1, 7 * 3]) \\ &= ([6,0], [2, 4], [0, 4]) \qquad \ldots \qquad \mathrm{I} \end{aligned}$$

Consider

$$\begin{aligned} B * A &= ([0, 2], [5, 0], [1, 3]) * ([3, 5], [1, 2], [0, 7]) \\ &= ([0, 2] * [3, 5], [5, 0] * [1, 2], [1, 3]* [0, 7]) \\ &= ([0*3, 2*5], [5*1, 0*2], [1*0, 3*7]) \\ &= ([0, 4], [0, 0], [2, 6]) \qquad \ldots \qquad \mathrm{II} \end{aligned}$$

Clearly $A * B \neq B * A$ as I and II are distinct.

We see in case of interval row matrix groupoids the product '*' is not commutative. As * is non associative.

Clearly * on these interval matrix groupoids will be non associative.

Example 5.90: Let S = {Collection of all intervals matrices of the form

$$G = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \right| a_i, b_i \in G = \{Z_{15}, *, (10, 5)\}; \; 1 \leq i \leq 9 \right\}$$

be the interval square matrix groupoid of finite order.

Clearly S is a non commutative and a non associative groupoid of finite order.

We just indicate how we can have two operation on S; one the usual matrix product and the other natural matrix product.

$$\text{Let } A = \begin{bmatrix} [3,0] & [2,1] & [0,1] \\ [1,2] & [0,5] & [0,0] \\ [0,0] & [1,0] & [2,1] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0,0] & [1,2] & [5,0] \\ [0,2] & [0,0] & [0,1] \\ [1,5] & [0,3] & [0,0] \end{bmatrix} \in S.$$

We find A * B using natural product $\times_n$.

$$A * B = \begin{bmatrix} [3,0] & [2,1] & [0,1] \\ [1,2] & [0,5] & [0,0] \\ [0,0] & [1,0] & [2,1] \end{bmatrix} * \begin{bmatrix} [0,0] & [1,2] & [5,0] \\ [0,2] & [0,0] & [0,1] \\ [1,5] & [0,3] & [0,0] \end{bmatrix}$$

$$= \begin{bmatrix} [3,0]*[0,0] & [2,1]*[1,2] & [0,1]*[5,0] \\ [1,2]*[0,2] & [0,5]*[0,0] & [0,0]*[0,1] \\ [0,0]*[1,5] & [1,0]*[0,3] & [2,1]*[0,0] \end{bmatrix}$$

$$= \begin{bmatrix} [3*0,0*0] & [2*1,1*2] & [0*5,1*0] \\ [1*0,2*2] & [0*0,5*0] & [0*0,0*1] \\ [0*1,0*5] & [1*0,0*3] & [2*0,1*0] \end{bmatrix}$$

$$= \begin{bmatrix} [30,0] & [25,20] & [25,10] \\ [30,30] & [0,50] & [0,5] \\ [5,25] & [10,15] & [20,10] \end{bmatrix}$$

$$= \begin{bmatrix} [0,0] & [10,5] & [10,10] \\ [0,0] & [0,5] & [0,5] \\ [5,10] & [10,0] & [5,10] \end{bmatrix} \qquad \ldots \quad \mathrm{I}$$

One can consider B * A under natural product and show A * B ≠ B * A.

We will describe them by examples.

Example 5.91: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1,b_1] \\ [a_2,b_2] \\ \vdots \\ [a_{12},b_{12}] \end{bmatrix} \middle| \, a_i, b_i \in G = \{Z_{42}, *, (6, 7)\}; 1 \leq i \leq 12 \right\}\}$$

be the subset interval matrix groupoid of M.

$T_\cup = \{S', \cup, \cap_*\}$ and $T_\cap = \{S', \cap, \cap_*\}$ are both special type topological subset, interval matrix groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.92: Let S = {collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \middle| \, a_i, b_i \in G = \{Z_{18}, *, (3, 0)\}; 1 \leq i \leq 4 \right\}\}$$

be the subset interval matrix groupoid of M.

$T_\cup$ and $T_\cap$ are special type topological subset interval matrix groupoid spaces of M.

Let A = $\left\{\begin{bmatrix}[0,3] & [2,1]\\ [5,1] & [2,0]\end{bmatrix}, \begin{bmatrix}[1,1] & [4,0]\\ [0,7] & [2,5]\end{bmatrix}\right\}$

and B = $\left\{\begin{bmatrix}[0,0] & [1,1]\\ [2,0] & [0,6]\end{bmatrix}, \begin{bmatrix}[2,0] & [9,1]\\ [0,0] & [0,1]\end{bmatrix}\right\} \in S$

$$A \cap_* B = \left\{\begin{bmatrix}[0,3] & [2,1]\\ [5,1] & [2,0]\end{bmatrix}, \begin{bmatrix}[1,1] & [4,0]\\ [0,7] & [2,5]\end{bmatrix}\right\} \cap_* \left\{\begin{bmatrix}[0,0] & [1,1]\\ [2,0] & [0,6]\end{bmatrix}, \begin{bmatrix}[2,0] & [9,1]\\ [0,0] & [0,1]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,3] & [2,1]\\ [5,1] & [2,0]\end{bmatrix} * \begin{bmatrix}[0,0] & [1,1]\\ [2,0] & [0,6]\end{bmatrix}, \begin{bmatrix}[0,3] & [2,1]\\ [5,1] & [2,0]\end{bmatrix} * \begin{bmatrix}[2,0] & [9,1]\\ [0,0] & [0,1]\end{bmatrix}\right.$$

$$\left.\begin{bmatrix}[1,1] & [4,0]\\ [0,7] & [2,5]\end{bmatrix} * \begin{bmatrix}[0,0] & [1,1]\\ [2,0] & [0,6]\end{bmatrix}, \begin{bmatrix}[1,1] & [4,0]\\ [0,7] & [2,5]\end{bmatrix} * \begin{bmatrix}[2,0] & [9,1]\\ [0,0] & [0,1]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,3]*[0,0] & [2,1]*[1,1]\\ [5,1]*[2,0] & [2,0]*[0,6]\end{bmatrix}, \begin{bmatrix}[0,3]*[2,0] & [2,1]*[9,1]\\ [5,1]*[0,0] & [2,0]*[0,1]\end{bmatrix}, \right.$$

$$\left.\begin{bmatrix}[1,1]*[0,0] & [4,0]*[1,1]\\ [0,7]*[2,0] & [2,5]*[0,6]\end{bmatrix}, \begin{bmatrix}[1,1]*[2,0] & [4,0]*[9,1]\\ [0,7]*[0,0] & [2,5]*[0,1]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,9] & [6,3]\\ [15,3] & [6,0]\end{bmatrix}, \begin{bmatrix}[0,9] & [6,3]\\ [15,3] & [6,0]\end{bmatrix}, \right.$$

$$\left.\begin{bmatrix}[3,3] & [12,0]\\ [0,3] & [6,15]\end{bmatrix}, \begin{bmatrix}[3,3] & [12,0]\\ [0,3] & [6,15]\end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} [0,9] & [6,3] \\ [15,3] & [6,0] \end{bmatrix}, \begin{bmatrix} [3,3] & [12,0] \\ [0,3] & [6,15] \end{bmatrix} \right\}$$

is in $T_{\cup}$ and $T_{\cap}$.

$$\text{Now } A \cup B = \left\{ \begin{bmatrix} [0,3] & [2,1] \\ [5,1] & [2,0] \end{bmatrix}, \begin{bmatrix} [1,1] & [4,0] \\ [0,7] & [2,5] \end{bmatrix} \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,0] & [1,1] \\ [2,0] & [0,6] \end{bmatrix}, \begin{bmatrix} [2,0] & [9,1] \\ [0,0] & [0,1] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0,3] & [2,1] \\ [5,1] & [2,0] \end{bmatrix}, \begin{bmatrix} [1,1] & [4,0] \\ [0,7] & [2,5] \end{bmatrix}, \begin{bmatrix} [0,0] & [1,1] \\ [2,0] & [0,6] \end{bmatrix}, \begin{bmatrix} [2,0] & [9,1] \\ [0,0] & [0,1] \end{bmatrix} \right\}$$
$$\in T_{\cup}.$$

Now

$$A \cap B = \left\{ \begin{bmatrix} [0,3] & [2,1] \\ [5,1] & [2,0] \end{bmatrix}, \begin{bmatrix} [1,1] & [4,0] \\ [0,7] & [2,5] \end{bmatrix} \right\} \cap$$

$$\left\{ \begin{bmatrix} [0,0] & [1,1] \\ [2,0] & [0,6] \end{bmatrix}, \begin{bmatrix} [2,0] & [9,1] \\ [0,0] & [0,1] \end{bmatrix} \right\}$$

$$= \{\phi\} \in T_{\cap}.$$

This is the way operations on special type topological subset interval matrix groupoid spaces are performed.

Example 5.93: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ \vdots & \vdots & \vdots \\ [a_{10}, b_{10}] & [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in \right.$$

$$G = \{\langle Z \cup I \rangle, *, (9, -9)\}; 1 \le i \le 12\}\}$$

be the special type topological subset interval matrix groupoid spaces of $S' = S \cup \{\phi\}$.

Both $T_\cup$ and $T_\cap$ are of infinite spaces.

Example 5.94: Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{bmatrix} \right| a_i, b_i \in G = \{\langle C \cup I \rangle, *, \right.$$

$$(10, 1)\}; 1 \le i \le 20\}\}$$

be the subset interval matrix groupoid.

Both $T_\cup$ and $T_\cap$ are special type topological subset interval matrix groupoid spaces of $S' = S \cup \{\phi\}$.

Now we can have Smarandache substructures and subspaces which is considered as a matter of routine and left as an exercise to the reader.

We give examples of subset interval matrix loop groupoids and their Smarandache structures and substructures.

Example 5.95: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{16}, b_{16}] \end{bmatrix} \right| a_i, b_i \in L_{27}(8); 1 \leq i \leq 16 \right\}\}$$

be the subset interval matrix loop groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset interval matrix loop groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.96: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_5, b_5] \\ [a_6, b_6] & \ldots & [a_{10}, b_{10}] \\ \vdots & \ldots & \vdots \\ [a_{21}, b_{21}] & \ldots & [a_{25}, b_{25}] \end{bmatrix} \right| a_i, b_i \in L_{105}(23); 1 \leq i \leq 25 \right\}\}$$

be the subset matrix interval loop groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset matrix interval loop groupoid spaces of S.

Take P = {Collection of all subsets from the matrix interval subgroupoid

$$N = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_5, b_5] \\ [a_6, b_6] & \ldots & [a_{10}, b_{10}] \\ \vdots & \ldots & \vdots \\ [a_{21}, b_{21}] & \ldots & [a_{25}, b_{25}] \end{bmatrix} \right\} a_i, b_i \in H_1(21) \underset{\neq}{\subset} L_{105}(23);$$

$$1 \leq i \leq 25\}\} \subseteq S$$

be the subset interval matrix loop subgroupoid of S.

$L_{\cup} = \{P' = P \cup \{\phi\}, \cup, \cap_*\}$
and $L_{\cap} = \{P' = P \cup \{\phi\}, \cap, \cap_*\}$ are special type topological interval matrix loop subgroupoid subspaces of $T_{\cup}$ and $T_{\cap}$ respectively.

Infact $T_{\cup}$ and $T_{\cap}$ have several other subspaces.

Further as $L_{45}(8)$ is a Smarandache loop $T_{\cup}$ and $T_{\cap}$ are also Smarandache topological spaces.

Example 5.97: Let S = {Collection of all subsets of the interval matrix groupoid

$$M = \left\{\left.\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ \vdots & \vdots \\ [a_{19},b_{19}] & [a_{20},b_{20}] \end{bmatrix}\right| a_i, b_i \in L_{23}(12), 1 \leq i \leq 20\right\}\}$$

be the subset interval matrix loop groupoid of M.

$T_{\cup}$ and $T_{\cap}$ are both special type topological subset matrix interval loop groupoid spaces. $T_{\cup}$ and $T_{\cap}$ are both Smarandache topological spaces of $S' = S \cup \{\phi\}$.

Example 5.98: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{\left.\begin{bmatrix} [a_1,b_1] & \ldots & [a_{12},b_{12}] \\ [a_{13},b_{13}] & \ldots & [a_{24},b_{24}] \end{bmatrix}\right| a_i, b_i \in L_{123}(23); 1 \leq i \leq 24\right\}\}$$

be the subset interval matrix loop groupoid.

$T_{\cup}$ and $T_{\cap}$ are special type topological subset interval matrix loop groupoid spaces of $S' = S \cup \{\phi\}$.

This has subspaces and $T_{\cup}$ and $T_{\cap}$ are both Smarandache.

Example 5.99: Let S = {Collection of all subsets from the interval matrix loop groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{25}, b_{25}] \end{bmatrix} \middle| \; a_i, b_i \in L_{237}(5); 1 \leq i \leq 25 \right\}\}$$

be the subset interval matrix loop groupoid of finite order.

$T_\cup$ and $T_\cap$ are both special type topological subset interval matrix loop groupoid spaces.

$T_\cup$ and $T_\cap$ have special type topological subset interval matrix loop groupoid subspaces.

We now give examples of special type topological subset interval polynomial groupoids and (loop groupoid) spaces.

Example 5.100: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \middle| \; a_i, b_i \in G = \{Z_{47}, *, (3, 0)\} \right\}\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are both special type topological subset interval polynomial groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.101: Let S = {collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \middle| \; a_i, b_i \in G = \{C(Z_{42}), *, (6, 0)\}, 0 \leq i \leq \infty \right\}\}$$

be the subset interval polynomial groupoid of M.

$T_\cup$ and $T_\cap$ are special type topological subset interval polynomial groupoid spaces. Both $T_\cup$ and $T_\cap$ have subspaces.

Example 5.102: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in G = \{\langle Z^+ \cup I \cup \{0\}\rangle, *, (3,0)\}\right\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset interval polynomial groupoid spaces of $S' = S \cup \{\phi\}$. $T_\cup$ and $T_\cap$ has infinite number of subspaces.

Example 5.103: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{37}(8)\right\}\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset interval polynomial groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.104: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{109}(12)\right\}\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset interval polynomial groupoid spaces of $S' = S \cup \{\phi\}$.

Example 5.105: Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{45}(8)\}\right\}$$

be the subset interval polynomial groupoid.

$T_\cup$ and $T_\cap$ are the special type topological subset interval polynomial groupoid spaces.

Let P = {Collection of all subsets from the interval polynomial loop subgroupoid

$$N = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in H_2(15) \subseteq L_{45}(8)\}\right\}$$

be the subset interval polynomial loop subgroupoid.

$L_\cup = \{P' = P \cup \{\phi\}, \cup, \cap_*\} \subseteq T_\cup$
and $L_\cap = \{P' = P \cup \{\phi\}, \cap, \cap_*\} \subseteq T_\cap$ are special type topological subset polynomial interval loop groupoid subspaces of $P' = P \cup \{\phi\}$.

Infact $T_\cup$ and $T_\cap$ has atleast 80 non trivial special type topological subset interval polynomial loop groupoid subspaces.

Thus we can always state that in case of subset polynomial interval loop groupoid associated with the loop $L_n(m)$ has always special type topological subspaces.

If n is a prime then both $T_\cup$ and $T_\cap$ has atleast p + 3 distinct special type topological subset polynomial interval loop groupoid subspaces.

If n is a composite number and if the associated loop $L_n(m)$ has t number of subloops which are not subgroups then each of $T_\cup$ and $T_\cap$ has (t + 3 + n) number of special type topological subset interval polynomial loop subgroupoid subspaces of which n of them will be associative subspaces of $T_\cup$ (and $T_\cap$).

Thus we see in general subset interval polynomial subset loop groupoid of the loop $L_n(m)$ has both special type topological subset interval polynomial loop subgroupoid subspaces which are associative (n in number) and 3 special type topological subset interval polynomial loop subgroupoid subspaces and if n; composite number as many subspaces as that of the proper subloops of $L_n(m)$ which are not subgroupoids.

We suggest the following problems for this chapter.

Problems:

1. Let S = {Collection of all subsets from the groupoid G = {Z, *, (3, –3)} be the subset groupoid.

   (i) Compare both the basis of $T_\cup$ and $T_\cap$, the special type of subset groupoid topological spaces.
   (ii) Do both of the spaces $T_\cup$ and $T_\cap$ have same number of elements in the basic set?

2. Find some special properties enjoyed by special type topological subset groupoid spaces of finite order.

3. Do we have apart from these special type of topological subset groupoid spaces $T_\cup$ and $T_\cap$ any other non associative topological spaces?

4. Find some nice applications of these new structures.

5. Let S = {Collection of all subsets from the groupoid G = {Z_{14}, *, (2, 7)}} be the subset groupoid of G.

(i) Find o(S).
(ii) Find $T_{\cap} = \{S', \cap, \cap_*\}$ and $T_{\cup} = \{S', \cup, \cap_*\}$ of S′.
(iii) Can $T_{\cup}$ and $T_{\cap}$ have special types of subset groupoid topological subspaces?
(iv) Find a basic set for both $T_{\cup}$ and $T_{\cap}$.

6. Let S_1 = {Collection of all subsets of the groupoid $G = \{Z_{28}, *, (21, 7)\}$}be the subst groupoid of G.

   Study questions (i) to (iv) of problem 5 for this S_1.

7. Let S_2 = {Collection of all subsets of the groupoid $G = \{Z_{17}, *, (9, 8)\}$} be the subset groupoid of G.

   Study questions (i) to (iv) of problem 5 for this S_2.

8. Let S_3 = {Collection of all subsets from the groupoid $G = \{Z_7, *, (5, 3)\}$} be the subset groupoid of G.

   Study questions (i) to (iv) of problem 5 for this S_3.

9. Let S_4 = {Collection of all subsets from the groupoid $G = \{Z_{71}, *, (2, 5)\}$ be the subset groupoid of G.

   (i) Study questions (i) to (iv) of problem (5) for this S_4.
   (ii) Compare S_3 of problem 8 with this S_4.

10. Let S_5 = {Collection of all subsets from the groupoid $G = \{C(Z_{10}), *, (3, 0)\}$} be the subset groupoid.

    (i) Study questions (i) to (iv) of problem (5) for this S_5.
    (ii) Compare S_6 = {Collection of all subsets from the groupoid $G = \{Z_{10}, *, (3, 0)\}$} the subset groupoid with S_5.

11. Let S_7 = {Collection of all subsets from the groupoid $G = \{\langle Z_{10} \cup I\rangle, *, (3, 0)\}$} be the subset groupoid of G.

    (i) Study questions (i) to (iv) of problem 5 for this S_7.

(ii) Compare S_7 with S_5 and S_6 of problem 10.
(iii) Is $S_7 \cong S_6$?

12. Let S_1 = {Collection of all subsets of the groupoid $G = \{C(\langle Z_{17} \cup I\rangle), *, (10, 8)\}$} be the subset groupoid.

    (i) Study questions (i) to (iv) of problem 5 for this S_1.
    (ii) Study S_1 if $C(\langle Z_{17} \cup I\rangle)$ is replaced by $C(Z_{17})$.
    (iii) Study S_1 if $C(\langle Z_{17} \cup I\rangle)$ is replaced by $\langle Z_{17} \cup I\rangle$
    (iv) Study S_1 if $C(\langle Z_{17} \cup I\rangle)$ is replaced by Z_{17}.

13. Let S_2 = {Collection of all subsets from the groupoid $G = \{C(\langle Z_{16} \cup I\rangle), *, (5, 0)\}$} be the subset groupoid.

    Study questions (i) to (iv) of problem 12 for this S_2.

14. Let S_3 = {Collection of all subsets from the groupoid $G = \{\langle C \cup I\rangle, *, (3, 0)\}$} be the subset groupoid.

    Study questions (i) to (iv) of problem 12 for this S_3.

15. Let S_4 = {Collection of all subsets from the groupoid $G = \{C(\langle Z_{18} \cup I\rangle), *, (0, I)\}$} be the subset groupoid of G.

    Can questions (i) to (iv) of problem 12 be adopted for this S_4? Justify your answer.

16. Let S = {Collection of all subsets from the groupoid $G = \{Z_{12}, *, (4, 9)\}$} be the subset groupoid.

    (i) Does S satisfy any special identities?
    (ii) Is $T_\cup$ and $T_\cap$ S-special type of topological subset groupoid spaces?

17. Give an example of a S-strong special type of topological Moufang groupoid spaces $T_\cup$ and $T_\cap$ of S.

18. Give an example of a S-strong special type of topological subset Bol groupoid spaces $T_\cup$ and $T_\cap$ of S.

19. Does there exist a S-strong special type of topological subset Moufang groupoid spaces of infinite order?

20. Does there exist a Smarandache strong special type topological subset right alternative groupoid spaces which are not a left alternative subset groupoid spaces?

21. Give an example of a Smarandache strong special type topological subset right (left) alternative groupoid spaces of infinite order.

22. Does there exist a subset groupoid S whose associated special type topological spaces $T_\cup$ and $T_\cap$ are of infinite order and satisfy any two of the special identities?

23. Does there exist a special type of topological subset loop groupoid spaces of infinite order?

24. Does there exist a special type of topological subset loop groupoid spaces of infinite order which is Smarandache left alternative?

25. Does there exist a special type of topological subset loop groupoid spaces of infinite order which is Smarandache Bol?

26. Give an example of a special type of topological subset loop groupoid spaces $T_\cup$ and $T_\cap$ of infinite order which is Smarandache alternative.

27. Let S = {Collection of all subsets from the loop groupoid $L_{25}(2)$} be the subset loop groupoid.

    Does the special type topological subset loop groupoid spaces of S which is Smarandache satisfy any of the special identities?

28. Give examples of special types topological subset loop groupoid spaces $T_\cup$ and $T_\cap$ which are

    (i) Smarandache alternative.
    (ii) Smarandache P subset groupoid.
    (iii) Smarandache Bol.
    (iv) Smarandache Moufang.

29. Let S = {Collection of all subsets from the loop $L_{19}(10)$} be a subset loop groupoid.

    Prove both the $T_\cup$ and $T_\cap$ are Smarandache special type topological subset pseudo commutative loop groupoid spaces!

30. Give the special features enjoyed by special type of topological subset matrix groupoid spaces of infinite (and finite) order.

31. Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_{13} \\ a_3 & a_{14} \\ \vdots & \vdots \\ a_{12} & a_{24} \end{array} \right] \middle| \; a_i \in G = \{Z_6, *, (3, 0)\}, 1 \leq i \leq 24 \right\}\}$$

    be the subset matrix groupoid.

    (i) Find o(S).
    (ii) Find $T_\cap = \{S', \cap, \cap_*\}$ and $T_\cup = \{S', \cup, \cap_*\}$ of S.
    (iii) Find all topological subspaces of $T_\cup$ and $T_\cap$.

32. Do we have a special type topological subset spaces of $T_\cup$ and $T_\cap$ of S so that both $T_\cup$ and $T_\cap$ have no topological subspaces?

33. Let S_1 = {Collection of all subset from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & \ldots & a_{10} \\ a_{11} & \ldots & a_{20} \\ a_{21} & \ldots & a_{30} \end{bmatrix} \middle| \; a_i \in G = \{Z_{10}(g), *, (5, g)\}, \right.$$

$$\left. g^2 = 0,\ 1 \leq i \leq 30 \right\}\}$$

be the subset matrix groupoid of dual numbers.

Study questions (i) to (iii) of problem 31 for this S_1.

34. Let S_2 = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in G = \{\langle Z_7 \cup I\rangle, *, (3, 4)\},\ 1 \leq i \leq 4 \right\}\}$$

be the subset matrix groupoid.

Study questions (i) to (iii) of problem 31 for this S_2.

35. Let S_3 = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \; a_i \in G = \{\langle Z^+ \cup I\rangle, *, (3, 4)\},\ 1 \leq i \leq 8 \right\}\}$$

be the subset matrix groupoid.

Study questions (i) to (iii) of problem 31 for this S_3.

36. Let S_5 = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \left[\begin{matrix} a_1 & a_2 & \cdots & a_6 \\ a_7 & a_8 & \cdots & a_{12} \\ \vdots & \vdots & & \vdots \\ a_{31} & a_{32} & \cdots & a_{36} \end{matrix} \right] \right| a_i \in G = \{\langle Z_8 \cup I \rangle (g_1, g_2),$$

$$*, (4, 0),\ g_1^2 = 0,\ g_2^2 = g_2, g_1g_2 = g_2g_1 = 0\}, 1 \leq i \leq 36\}\}$$

be the subset matrix groupoid.

Study questions (i) to (iii) of problem 31 for this S_5.

37. Let S = {Collection of all subsets from the matrix groupoid

$$M = \left\{ \left[\begin{matrix} a_1 & a_2 & a_3 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{matrix} \right] \right| a_i \in G = \{C(Z_{16}), *, (4, 0)\},$$

$$1 \leq i \leq 40\}$$

be the subset matrix groupoid.

Study questions (i) to (iii) of problem 31 for this S.

38. Does there exist a special type topological subset matrix loop groupoid spaces of infinite order?

39. Does there exist a special type topological subset loop groupoid spaces of infinite order which are Smarandache?

40. Let S = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{19} & a_{20} \end{array} \right] \middle| \; a_i \in L_{19}(3), 1 \leq i \leq 38\}\right\}$$

be the subset matrix loop groupoid.

(i) Find o(S).
(ii) Find $T_{\cup}$ and $T_{\cap}$.
(iii) Find all special type of topological subset loop groupoid subspaces of $T_{\cup}$ and $T_{\cap}$.
(iv) Is $T_{\cup}$ and $T_{\cap}$ Smarandache topological spaces?
(v) Does $T_{\cup}$ and $T_{\cap}$ satisfy any Smarandache identities?

41. Let S_1 ={Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} a_1 & \ldots & a_{12} \\ a_{13} & \ldots & a_{24} \\ a_{25} & \ldots & a_{36} \end{array} \right] \middle| \; a_i \in L_{29}(8), 1 \leq i \leq 36\right\}$$

be the subset matrix loop groupoid of M.

Study questions (i) to (v) of problem 40 for this S_1.

42. Let S_2 = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_4 \\ a_7 & a_8 & a_9 \end{array} \right] \middle| \; a_i \in L_{145}(2), 1 \leq i \leq 9\}\right\}$$

be the subset matrix loop groupoid.

Study questions (i) to (v) of problem 40 for this S_2.

43. Let S_3 = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_4 \\ \vdots & \vdots & \vdots \\ a_{34} & a_{35} & a_{36} \end{array} \right] \middle| \, a_i \in L_{49}(9),\ 1 \leq i \leq 36 \right\}$$

be the subset matrix loop groupoid.

Study questions (i) to (v) of problem 40 for this S_3.

44. Let S_4 = {Collection of all subsets from the matrix loop groupoid

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \vdots & \vdots \\ a_{39} & a_{40} \end{array} \right] \middle| \, a_i \in L_{105}(23),\ 1 \leq i \leq 40 \right\} \}$$

be the subset matrix loop groupoid.

Study questions (i) to (v) of problem 40 for this S_4.

45. Does there exist a special type topological subset matrix loop groupoid space $T_\cup$ and $T_\cap$ such that they satisfy any of the special identities and is of infinite order.

46. Give an example of a special type topological subset matrix loop groupoid spaces $T_\cup$ and $T_\cap$ such that the topological spaces are subset Smarandache Bol topological spaces of infinite order.

47. Give some special features enjoyed by Smarandache special type subset topological Bruck matrix loop groupoid spaces of infinite order.

48. Find some special features enjoyed by special subset topological type matrix loop groupoid spaces.

49. Study the special features of subset polynomial loop groupoid spaces of subset polynomial loop groupoid S.

50. Let S = {Collection of all subsets from the polynomial loop groupoid M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{27}(11)\right\}\}$ be the subset polynomial loop groupoid.

(i) Find $T_\cup$ and $T_\cap$.
(ii) Show both $T_\cup$ and $T_\cap$ have infinite number of special type topological subset polynomial loop subgroupoid spaces.
(iii) Prove both $T_\cup$ and $T_\cap$ are Smarandache.

51. Let S_1 = {Collection of all subsets from the polynomial loop groupoid M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{13}(8)\right\}\}$ be the subset polynomial loop groupoid.

Study questions (i) to (iii) of problem 50 for this S_1.

52. Let S_2 = {Collection of all subsets from the polynomial loop groupoid M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{123}(5)\right\}\}$ be the subset polynomial loop groupoid.

Study questions (i) to (iii) of problem 50 for this S_2.

53. Let S_3 = {Collection of all subsets from the polynomial loop groupoid M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{315}(17)\right\}\}$ be the subset polynomial loop groupoid.

Study questions (i) to (iii) of problem 50 for this S_3.

54. Let S_4 ={Collection of all subsets from the polynomial loop groupoid M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_{43}(22)\right\}$} be the subset polynomial loop groupoid of M.

    (i) Prove $T_\cup$ and $T_\cap$ are both commutative subset special type topological spaces.
    (ii) Study questions (i) to (iii) of problem 50 for this S_4.

55. Does there exist special type topological subset polynomial loop groupoid spaces $T_\cup$ and $T_\cap$ which has no subspaces?

56. Give some special features enjoyed by special type topological subset interval groupoid spaces.

57. Let S = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b ∈ G = {Z, *, (10, –3)}}} be the subset interval groupoid.

    (i) Study $T_\cup$ and $T_\cap$.
    (ii) Show both $T_\cup$ and $T_\cap$ are non commutative and non associative subset interval groupoid topological spaces.
    (iii) Find all subspaces of $T_\cup$ and $T_\cap$.

58. Let S_1 = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b ∈ G = {$\langle Z_{15} \cup I\rangle$ *, (3, 6)}}} be the subset interval groupoid of finite order.

    (i) Study questions (i) to (iii) of problem 57 for this S_1.
    (ii) Find $o(S_1) = o(T_\cup) - 1 = o(T_\cap) - 1$.

59. Let S_2 = {Collection of all subsets from the interval groupoid M = {[a, b] | a, b ∈ G = {$C(Z_{24})$, *, (10, 2)}}} be the subset interval groupoid.

(i) Find o(S).
(ii) Study questions (i) to (iii) of problem 57 for this S_2.

60. Let S_3 = {Collection of all subsets from the interval groupoid M = $\{[a, b] \mid a, b \in G = \{\langle Q \cup I\rangle *, (10, -10)\}\}\}$ be the subset interval groupoid.

    Study questions (i) to (iii) of problem 57 for this S_3.

61. Let S_4 = {Collection of all subsets from the interval groupoid M = $\{[a, b] \mid a, b \in G = \{Z_{45}, *, (10, 0)\}\}\}$ be the subset interval groupoid.

    (i) Study questions (i) to (iii) of problem 57 for this S_4.
    (ii) Find o(S).

62. Let S = {Collection of all subsets from the interval loop groupoid M = $\{[a, b] \mid a, b \in L_{21}(17)\}\}$ be the subset interval loop groupoid.

    (i) Study questions (i) to (iii) of problem 57 for this S.
    (ii) Find o(S).
    (iii) Prove $T_\cup$ and $T_\cap$ are subset interval loop groupoid Smarandache topological spaces.

63. Let S_1 = {Collection of all subsets from the interval loop groupoid M = $\{[a, b] \mid a, b \in L_{107}(12)\}\}$ be the subset interval loop groupoid.

    (i) Find $o(T_\cup)$.
    (ii) Study questions (i) to (iii) of problem 57 for this S.
    (iii) Prove $T_\cup$ and $T_\cap$ are subset interval loop groupoid Smarandache topological spaces.

64. Let S = {Collection of all subsets from the interval loop groupoid M = $\{[a, b] \mid a, b \in L_{29}(3)\}\}$ be the subset interval loop groupoid.

(i) Find $o(T_{\cup})$.
(ii) Study questions (i) to (iii) of problem 57 for this S.
(iii) Prove $T_{\cup}$ and $T_{\cap}$ are subset interval loop groupoid topological Smarandache spaces.

65. Does there exist a special type topological subset interval loop groupoid spaces of infinite order?

Can it be Smarandache?

66. Can these special type topological subset interval loop groupoid spaces of infinite order satisfy any of the special identities?

67. Give an example of a $T_{\cup}$ and $T_{\cap}$ of subset interval loop groupoid spaces of infinite order which is S-Moufang.

68. Give an example of a special type topological subset interval matrix loop groupoid spaces of infinite order and its associated topological spaces.

69. Let S = {Collection of all subsets from the interval groupoid

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & \ldots & [a_{30}, b_{30}] \end{bmatrix} \right| a_i, b_i \in G = \{C\langle Z_{15} \cup I\rangle, \right.$$

$$*, (5, 10)\}, 1 \leq i \leq 30\}\}$$

be the subset interval matrix groupoid.

(i) Find o(S).
(ii) Find the number of subset interval matrix subgroupoids of S.
(iii) Find $T_{\cup}$ and $T_{\cap}$.
(iv) Find all subspaces of $T_{\cup}$ and $T_{\cap}$.
(v) Is $T_{\cup}$ and $T_{\cap}$ Smarandache?

70. Let S = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & \ldots & [a_{30}, b_{30}] \end{bmatrix} \middle| \; a_i, b_i \in L_{251}(8); \right.$$

$$1 \leq i \leq 30\}\}$$

be the subset matrix loop groupoid.

Study questions (i) to (v) of problem 69 for this S.

71. Let S_1 = {Collection of all subsets from the interval matrix groupoid

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in L_{77}(10); 1 \leq i \leq 10\} \right\}$$

be the subset matrix loop groupoid.

Study questions (i) to (v) of problem 69 for this S_1.

72. Obtain some special features enjoyed by special type topological subset interval polynomial groupoid spaces $T_\cup$ and $T_\cap$.

73. Let S = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \middle| \; a_i, b_i \in G = \{\langle Z_7 \cup I\rangle, *, (3, 0)\}\} \right\}$$

be the subset interval polynomial groupoid.

(i) Find $T_\cup$ and $T_\cap$.
(ii) Find all special type subset interval polynomial groupoid subspaces of $T_\cup$ and $T_\cap$.
(iii) Are $T_\cup$ and $T_\cap$ Smarandache subset interval polynomial groupoid topological spaces?

74. Let S_1 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{\langle R \cup I\rangle, *, (3, -3)\}\right\}\}$$

be the subset polynomial interval groupoid of M.

Study questions (i) to (iii) of problem 73 for this S_1.

75. Let S_2 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{C(Z_{15}), *, (10, 6)\}\right\}\}$$

be the subset polynomial interval groupoid.

Study questions (i) to (iii) of problem 73 for this S_2.

76. Let S_3 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in G = \{C(\langle Z_{12} \cup I\rangle)\,(g_1, g_2), *, (4, 3);\ g_1^2 = 0,\ g_2^2 = g_2,\ g_1g_2 = g_2g_1 = 0\}\right\}\}$$

be the subset interval polynomial groupoid of M.

Study questions (i) to (iii) of problem 73 for this S_2.

77. Let S_4 = {Collection of all subsets from the interval polynomial groupoid

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L_{21}(11) \right\}\}$$

be the subset polynomial interval groupoid of M.

Study questions (i) to (iii) of problem 73 for this S_4.

78. Does there exist commutative special type topological subset interval polynomial loop groupoid spaces which are commutative with respect to operation $\cap_*$ on $T_\cup$ and $T_\cap$?

79. Characterize those commutative topological spaces $T_\cup$ and $T_\cap$ of a subset interval polynomial loop groupoid of the loop $L_n(m)$.

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.
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When the subsets of a loop are taken they also form only a subset groupoid and not a subset loop. Thus the concept of subset interval loop is not there, and they only form a subset interval groupoid. Subset matrix interval groupoid S using the loops $L_n(m)$ has no S-Cauchy elements.

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