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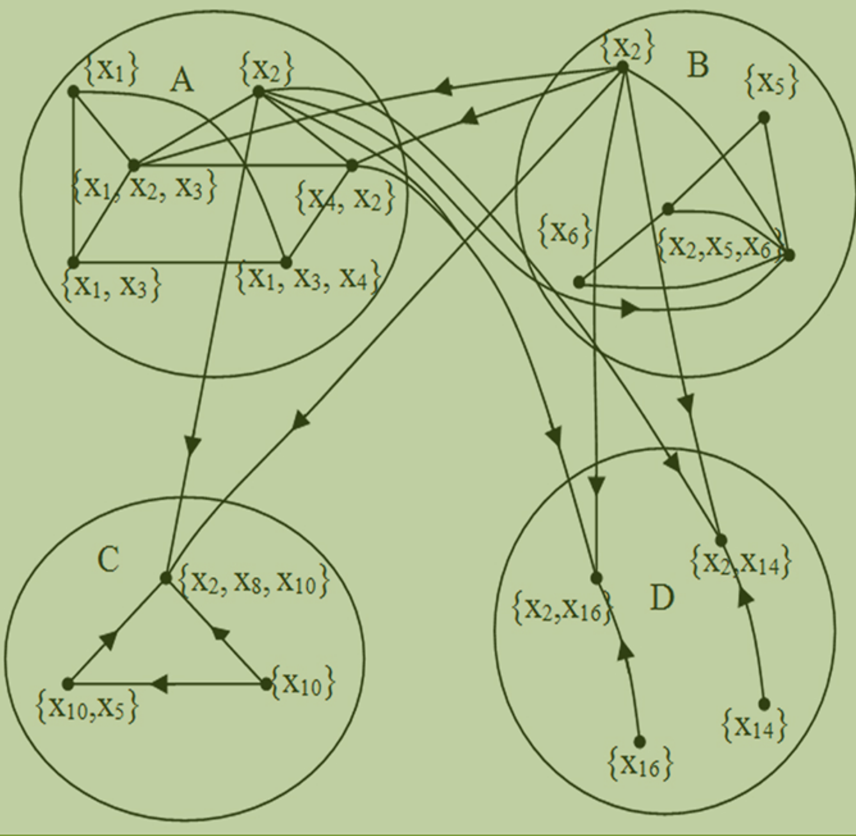

SUBSET VERTEX GRAPHS FOR SOCIAL NETWORKS

Subset Vertex Graphs for Social Networks

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PREFACE

In this book authors for the first time introduce the notion of subset vertex graph using the vertex set as the subset of the power set *P(S), S* is assumed in this book to be finite; however it can be finite or infinite. We have defined two types of subset vertex graphs, one is directed and the other one is not directed.

The most important fact which must be kept in record is that for a given set of vertices there exists one and only one subset vertex graph be it of type I or type II. Several important and innovative features of these graphs are defined, described and developed in this book.

The set *S* considered can be real or complex or indeterminate or dual numbers depending on the problem in hand. However by the definitions of the subset vertex graphs of type I and type II the edges or relation between two vertices are always real.

Now based on the property that in case of subset vertex graphs for a given set of vertices the graph is unique, these graphs are well suited for social networking. This is one of the special features enjoyed by these subset vertex graphs.

The applications of these new types of subset vertex graphs are dealt in chapter three of this book. Authors suggest several open problems for any researcher. Finally these new types of subset vertex graphs will be very useful in general to researchers in computer networks and in particular to researchers in social information networks.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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Chapter One

INTRODUCTION

Social information networks concept was introduced or perceived by researchers Emile Durkheim and Ferdinand Tonnies as social groups as early as 1890's [1, 17]. However Tonnies argued that social groups can exist as personal and direct social ties that either link individuals who share values and beliefs or impersonal, formal and instrumental social links but Durkheim gave a non individualistic explanation of social facts arguing that social phenomena arise when interacting individuals constitute a reality that can no longer be accounted for in terms of the properties of individual actors. Georg Simmel analyzed the network size on interaction and examined and likelihood of interaction in loosely knit networks rather than groups [13].

Major developments in the field can be seen in the 1930s by several groups in psychology, anthropology and mathematics working independently [47]. Jacob L. Moreno [9-11] has studied by systematic recording and analysis of social interaction in small groups, like class rooms and work groups.

He was the one who defined and developed sociometry a quantitative method for measuring social relationship. He studies the relationship between social structures and psychological wellbeing. "Sociometric explorations reveal the hidden structures that give a group its form: the alliances, the subgroups, the hidden beliefs, the forbidden agendas, the ideological agreements the 'starts of the show' [47].

One of Moreno's innovations in sociometry was the development of the sociogram, a systematic method for graphically representing individuals as points/nodes and the relationships between them as lines/arcs. Moreno who wrote extensively of his thinking, applications and findings also founded a journal sociometry.

For more about the development please refer [9-11]. Finally just for the better understanding of the reader, Facebook is a social network service and website which is largely based on the sociometry of its users.

In this chapter some basic properties of graphs are recalled so as to make this book a self-contained one. It is well known when we have G = (V, E) the graph, then V is the set of vertices $v_1, v_2, \ldots, v_n$ and $E = \{e_{ij} / 1 \leq i, j \leq n\}$ are edges of the graph G.

If e_{ii} in G exists we call them as loops or a graph with self connection of vertices.

We will just provide some examples.

Example 1.1: Let G = (V, E) be the graph given by the following figure.

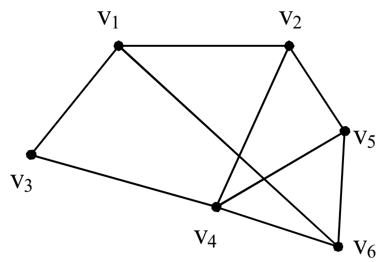


Figure 1.1

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the vertex set of the graph G and $\{e_{12}, e_{25}, e_{56}, e_{46}, e_{34}, e_{13}, e_{45}, e_{16}\} = E$ is the edge set. As the graph is not directed we have every $e_{ij} \in E$; $e_{ij} = e_{ji}$. $1 \leq i, j \leq 8$.

Study in this direction can be had from [3].

Further if there is no direction given in edges we call it as a graph and is not a directed graph. Further this graph G has no loops.

We give the matrix associated with G which is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

We see the matrix is a square matrix with main diagonal entries to be zero. Clearly M is symmetric about the diagonal.

Now we can categorically state that if M is any arbitrary $n \times n$ symmetric matrix with zero diagonal entries then with M we can always associate a graph G which is not a directed graph with n vertices. Here we can say we have a 1 - 1 (one to one) correspondence between a non directed graph G with n vertices and a set of $n \times n$ symmetric matrices with the main diagonal entries to be zero and other entries are just 0 or 1.

This observation is essential for we have a one to one map from $M \in S = \{(a_{ij}) / a_{ii} = 0, a_{ij} = a_{ji}; a_{ij} \in \{0, 1\}; i \neq j, 1 \leq i, j \leq n\}$ and $G \in N$ = {collection of all graphs with n vertices which is not directed and has no loops.

This will have to be analyzed in case of subset vertex graphs which is defined and developed in this book.

Suppose we have a 7×7 symmetric matrix with entries from {0, 1} and the main diagonal entries are zero given by

$$M = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}.$$

Now M is a symmetric square matrix with diagonal entries to be zero and the entries of M are either 0 or 1. Now the graph G associated with M with vertices $x_1, x_2, \ldots, x_7$ are as follows.

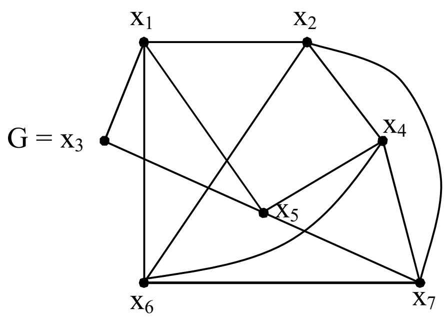


Figure 1.2

For more about these structures one can refer from [ ]. A good knowledge in this direction is essential for any one who is interested in reading this book.

However we have tried our level best to make this book as self contained as possible.

Next we know one can define subgraphs of a graph which is left for the reader to find all subgraphs of G given in Figure 1.2.

Now we can just mention about the directed graphs and describe their properties.

Example 1.2. Let G be the directed graph given by the following Figure 1.3.

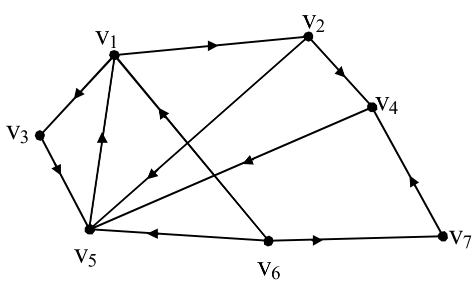


Figure 1.3

Now we give the adjacency matrix M associated with the graph G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we are forced to make the following observations which may be helpful in comparison with the nondirected graph.

We see both the nondirected graphs and directed graphs without loops will have the main diagonal entries to be zero.

In case of nondirected graphs without loops we have the corresponding adjacency matrix to be symmetric, however in case of directed graphs we see the diagonal entries are zero but is not symmetric.

Thus given any square non-symmetric n × n matrix with entries from the set {0, 1} with diagonal entries zero we can always associate a directed graph with n vertices.

This is illustrated by the following example.

$$\text{Let } M = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{array}\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

be the 6 × 6 matrix with zero diagonal entries which is not symmetric.

Now we give the directed graph G associated with this M. Clearly G has six vertices $y_1, y_2, ..., y_6$ and 15 edges which is as follows.

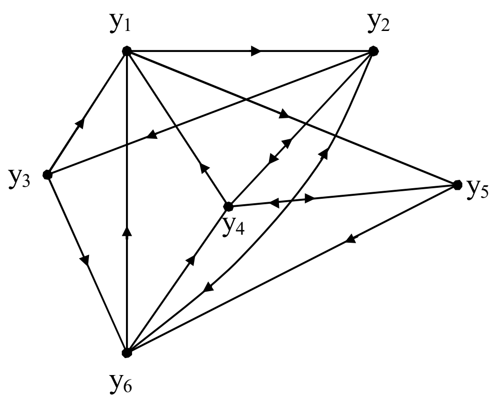


Figure 1.4

Thus we just record we can have a one to one (1 - 1) correspondence between the collection of all square non symmetric matrices with zero diagonal entries and entries from the set {0, 1} and all directed graphs without loops.

For more information [3].

Similarly we can find subgraphs of the directed graphs G and the subgraphs of G will continue to be directed graphs. This property of directional will also be transmitted in substructures also.

Directed graphs play a role in the study of social networks and Fuzzy Cognitive Maps model [5, 27].

Next we proceed onto describe the notion of bigraphs and the types of matrices associated with them by some examples.

Example 1.3 Let H be the bigraph given by the following Figure 1.5.

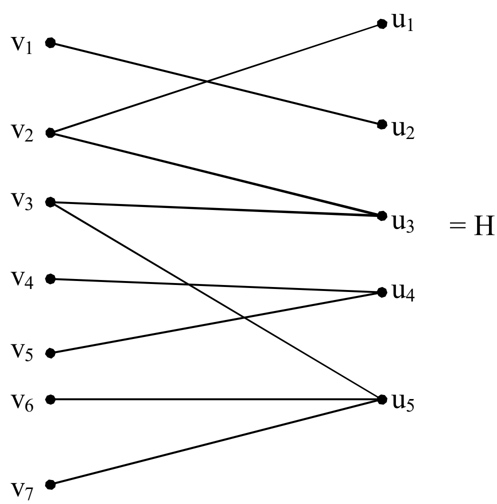


Figure 1.5

The matrix N associated with this graph H is as follows.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

This is a bigraph and we see if we taken the transpose of this matrix any N^t we will get the corresponding bigraph.

$$N^t = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

The bigraph associated with H^t is as follows

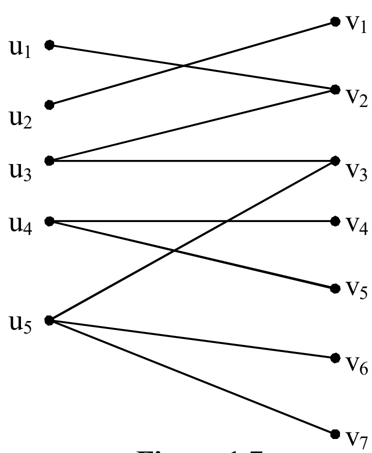


Figure 1.7

For more about bigraphs, complete graphs, complete bigraphs and other relevant concepts refer [3].

Next we proceed onto define and describe the notion of weighted graphs and weighted directed graphs by some example. For more refer [3].

Example 1.4. Let G = {V, E} be a graph with 9 vertices. If edge weights w_{ij} are associated with the edges e_{ij} and $w_{ij} \in R$ we call G the weighted graph which is described below. Here $1 \leq i,j \leq 9$ and e_{ij} denotes the edge from v_i to v_j and w_{ij} are real values from R.

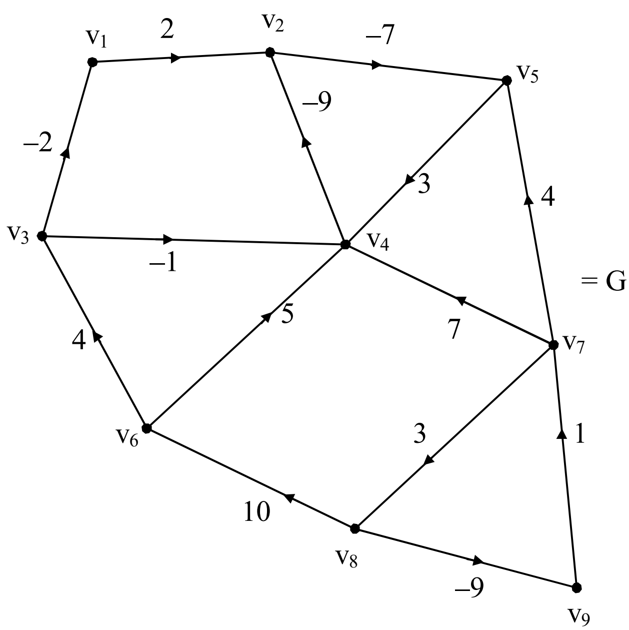


Figure 1.8

Clearly in this graph G we see $e_{ij} \neq e_{ji}$ as it is a directed weighted graph. However G has no loops. Associated with these graph is the weighted matrix M which is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & -9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

This weighted matrix of the weighted graph G is not symmetric. However as the diagonal elements are zero we see this graph has no weighted loops only weighted edges.

These type of graphs find their applications in neural networks etc.

Next we proceed onto describe weighted bigraphs by an example.

Example 1.5. Let H be a weighted bigraph with vertex set $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ connected with the vertex set $\{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8\}$ given b y the following Figure 1.9.

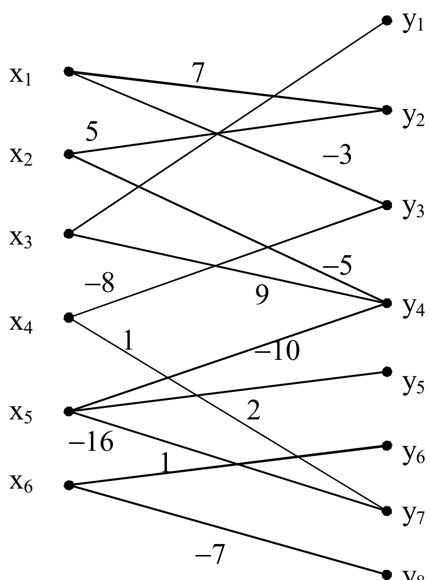


Figure 1.9

We now call the matrix W associated with the graph H as the weighted matrix which is as follows.

$$W = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 \end{array} \\ \begin{bmatrix} 0 & 7 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -10 & 2 & 0 & -16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -7 \end{bmatrix} \end{array}.$$

This sort of weighted bigraphs play a role in the study of certain types of ANN in soft computing. Thus these graphs also find their applications in Fuzzy Relational Equations, Neutrosophic Relational Equations, Neutrosophic Relational Maps and Fuzzy Relational Maps.

These graphs can be vertex complex valued graphs or vertex neutrosophic valued graphs or vertex complex-neutrosophic valued graphs. However the edges in all cases are only real it may be directed or otherwise. Other properties related with these types of graphs can be had from [28].

Several interesting properties are enjoyed by subset vertex graphs using subsets from the power set P(S) of a finite set S. The finite set S can be real or complex or neutrosophic or complex neutrosophic.

We do not describe it explicitly as the definition of the subset vertex graphs, do not distinguish them. But it is pertinent to keep on record that this concept of imaginary nodes and indeterminate nodes in social networks has not been studied so far. We have defined two types of subset vertex graphs. Type I subset vertex graphs are not directed. They are defined in chapter II of this book.

Two distinct vertex subsets v_1 and v_2 have an edge if $v_1 \cap v_2 \neq \phi$. However $v_i \cap v_i = v_i$ but we define there exists no edge that is no loop in this case.

The main advantage of using these subset vertex type I graphs are given a set of vertices we can have one and only one graph associated with that set of vertices. This makes these graphs more useful for no ambiguity is even possible in all these cases for by the above said definition they are defined uniquely. Further the subgraphs of these subsets vertex graphs of type I are also unique. So given the set of vertices $V \subseteq P(S) = 2^n$ where $S = \{x_1, x_2, \ldots, x_n\}$ we can find the total number of subset vertex subgraphs of type I.

Study in this direction is very interesting and has been carried out in a unique and an innovative way. Type I subset vertex graphs are not directed but unique once the vertex set is given. However subset vertex graphs of type II are directed and they are also unique once the vertex set is given.

They are directed for if v_1 and v_2 are subsets in P(S) the edge from v_1 to v_2

$$\underset{v_1}{\bullet}\!\!\longrightarrow\!\!\underset{v_2}{\bullet}$$

exists if and only if $v_1 \subset v_2$ and edge from v_2 to v_1 is given by

$$\underset{v_2}{\bullet}\!\!\longrightarrow\!\!\underset{v_1}{\bullet}$$

exists if and only if $v_2 \subset v_1$.

If $v_1 = v_2$ then no loop exists; if $v_1 \not\subset v_2$ or $v_2 \not\subset v_1$ no edge exist. By this unique way we have for a given set of vertex set there is a unique directed graph. Number of subgraphs of these type II graphs are also is only a fixed number.

Several interesting problems in this direction are proposed some are usual or routine exercises and some of them are open conjectures. Both the types of graphs are ideal for social networking and community networks. In chapter III of this book such situations are dealt with. It is important to record at this juncture that working with these subset vertex graphs of type I (or type II) can certainly save time and economy.

Further when these graphs are used in community networking with appropriate vertex set it can automatically fix the independence or interdependence of the different groups or community. This is the first time such automatic fixing of edges is done which can totally avoid the human bias. This situation is elaborately dealt in the last chapter of this book.

Chapter Two

SUBSET VERTEX GRAPHS OF TYPE I

In this chapter new type of graphs which are unique when vertex set is taken from P(S), where P(S) is the power set of a finite set S. This special type of graph takes the vertex set from P(S) = {collection of subsets of S}. The edges are defined in a specific way which results in a unique graph. S can be a real set or a complex set or a neutrosophic set or a complex neutrosophic set of finite order.

We will first describe this situation by some examples before we make an abstract definition of the graph.

Example 2.1. Let $S = \{x_1, x_2, x_3, x_4\}$ and P(S) be the power set of S. Let G = {V, E} be the graph where $V = \{v_1 = \{x_1, x_2, x_3\}, v_2 = \{x_1, x_2\}, v_3 = \{x_4\}, v_4 = \{x_2, x_3\}, v_5 = \{x_4, x_2\}, v_6 = \{x_3, x_4, x_1\}\}$.

The edge set $E = \{e_{12}, e_{14}, e_{15}, e_{16}, e_{24}, e_{25}, e_{26}, e_{35}, e_{36}, e_{45}, e_{46}, e_{56}\}$ is given by the following Figure 2.1.

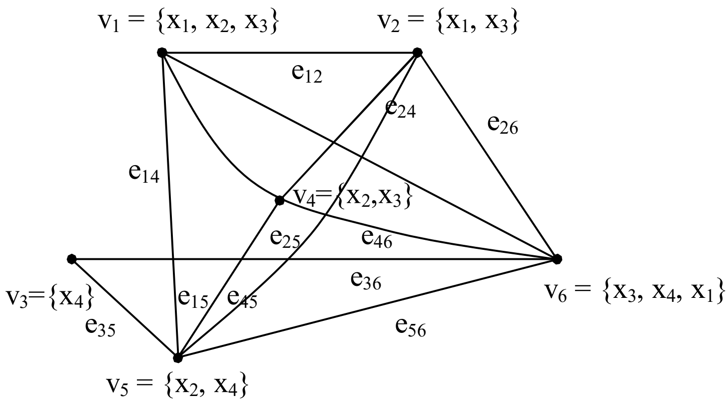


Figure 2.1

It is pertinent to make the following observations. In this graph we make or mark the edges if and only if $v_i \cap v_j \neq \phi$. If $v_i \cap v_j = \phi$, then the edges do not exist ($i \neq j$).

In the first place can we assume the vertex set as the empty set ϕ. If in our working we assume empty set as non existence of anything we cannot take ϕ as a vertex set. However as in case of lattices (algebraic) assume $\phi = \{0\}$ and $s = \{1\}$ then we assume the vertex set $\{0\} = \phi$ can be taken as a vertex.

The assumption and working can be followed as conceptual and it is not a difficult job to ascertain it.

There are 3 types of subset vertex graphs.

i) If S is a set P(S) the power set of S. Let $v_i, v_j \in P(S)$ a edge $e_{ij} = e_{ij}$ exist if $v_i \cap v_j \neq \phi$ if $v_i \cap v_j = \phi$ then no edge exists between the vertices v_i and v_j.

ii) e_{ij} exist if $v_i \subset v_j$ and $e_{ij} = e_{ji}$. In this case the graph becomes a directed graph if we define $v_i \longrightarrow v_j$ if $v_i \subset v_j$ which is dealt in chapter III of this book.

iii) e_{ij} exist if the researcher feels he/she can connect v_i and v_j even if $v_i \cap v_j = \phi$. This is not dealt in this book.

We define those subset vertex graphs whose edges $e_{ij} = e_{ji}$ if $v_i \cap v_j \neq \phi$ as type I vertex subset graphs or subset vertex graphs.

Clearly these graphs are not directed graphs.

We will give some more examples of them before we make the routine abstract definition.

Example 2.2. Let $S = \{x_1, x_2, x_3, x_4, x_5\}$ be the given set. $P(S)$ = power set of S = {collection of all subsets of the set S including S and ϕ}. We see how vertex subset graphs or subset vertex graphs using $P(S)$ can be defined.

We only follow the rule if $v_i, v_j \in P(S) \setminus \{\phi\}$; e_{ij} exist if and only if $v_i \cap v_j \neq \phi$ and in that case $e_{ij} = e_{ji}$. However if ϕ is taken as one of v_i or v_j then we have a different set up.

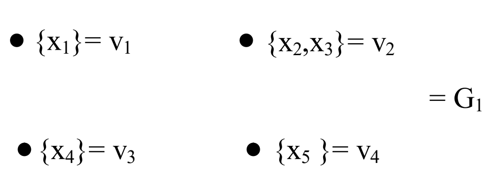


Figure 2.2

Thus $G_1 = \{V, E\}$ is a point set subset vertex graph and cardinality of V is four and edge set E is empty.

Let $G_2 = \{V, E\}$ be the vertex subset graph given by the following Figure 2.3.

● $v_1 = \{x_1\}$ ● $v_2 = \{x_2\}$

$= G_2$

● $v_3 = \{x_3\}$ ● $v_4 = \{x_4\}$ ● $v_5 = \{x_5\}$

Figure 2.3

Clearly G_2 is again a vertex subset graph of type I and it is only a point set graph or empty graph with edge set E to be empty. Consider $G_3 = \{V, E\}$ where $V = \{v_1 = \{x_1\}, v_2 = \{x_3\}, v_3 = \{x_2, x_4, x_5\}\}$ and clearly $E = \phi$.

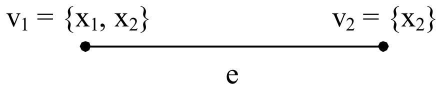
G_3 also contributes only to a point set, subset vertex graph or vertex subset graph of type I and $o(V) = 3$.

Consider $G_4 = \{V, E\}$ where $V = \{\{x_1, x_5, x_3\} = v_1, v_2 = \{x_2, x_4\}\}$; clearly $v_i \cap v_j = \phi$ so G is again a point set vertex subset graph of type Iand $E = \phi$.

Given S the set and P(S) the power set of S how many point set vertex subset graphs exists? ($S = \{1, 2, \ldots, n\}$; $2 \leq n < \infty$).

Next we see $H_1 = \{V, E\}$ where $V = \{v_1 = \{x_1, x_2\}, v_2 = \{x_2\}\}$ then $E = \{e\}$ is a subset vertex graph given by the following Figure 2.4.

H_1 =



$H_2 = \{\{x_2, x_3, x_4\} = v_1, \{x_2, x_4\} = v_2\}, E = \{e\}$

Figure 2.4

The subset vertex graph H_2 is given by the following Figure 2.5.

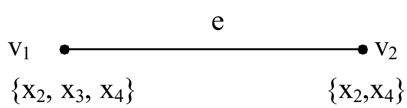


Figure 2.5

Let $H_3 = \{\{\{x_1, x_2, x_3, x_5\} = v_1, v_2 = \{x_5\}\} = V, E = \{e\}\}$ be the subset vertex graph given by the following Figure 2.6.

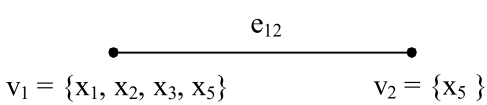


Figure 2.6

We can get several line graphs with two vertices and one edge.

Another simple exercise is given $S = \{x_1, x_2, \ldots, x_n\}$ and P(S) its power set.

How many line graphs that is vertex subset graphs with two vertices and one edge exist?

In case if $S = \{x_1, x_2\}$ we can have five subset vertex graphs of type I which are line graphs given by

$H_1 = \{v_1 = \{x_1, x_2\}, v_2 = \{x_3\}\}$ and $H_2 = \{\{x_2, x_2\} = v_1, \{x_1\} = v_2\}$ which is given by the following diagrams;

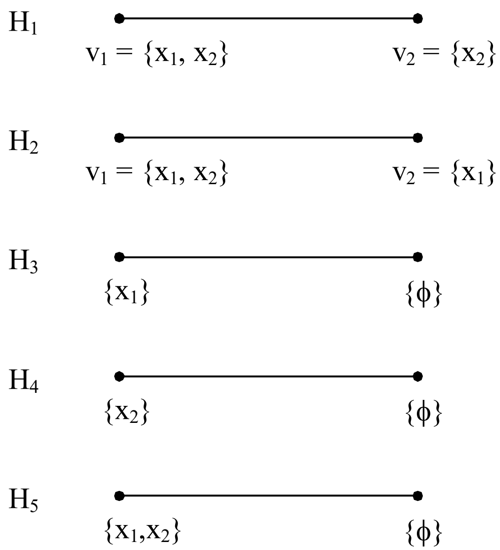


Figure 2.7

We have only one point set subset vertex graph given by $K = \{\{x_1\} = v_1, \{x_2\} = v_2\}$ which is described in the following.

Figure 2.8

The other subset vertex graphs got from P(S) are as follows.

$M = P\{\{x_1, x_2\}, \{x_1\}, \phi\}$

and the figure associated with M is as follows.

M =

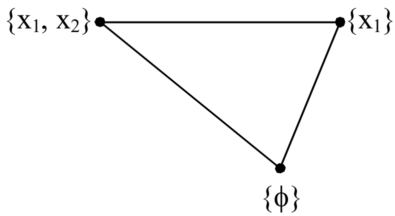


Figure 2.9

$L = \{\{x_1\} = v_, \{x_2\} = v_2, \{\phi\} = v_3\}$ is given by the following figure.

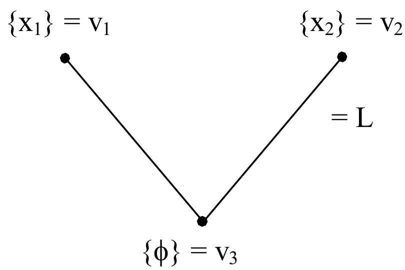


Figure 2.10

Consider $R = \{\{x_1, x_2\} = v_1, \{x_2\} = v_{2,} \{x_3\}= v_3, \{\phi\} = v_4\}$ be the vertex subset graph whose figure is given in the following;

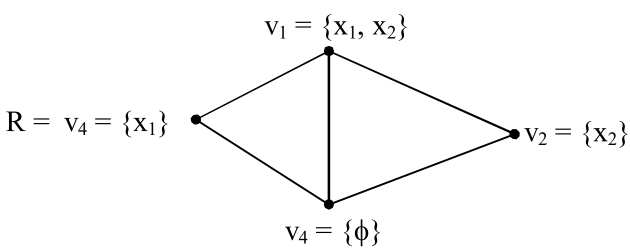


Figure 2.11

Interested reader is left with the task of finding all vertex subset graphs of type I which are just point set graphs, one edge graphs, two edge graphs and no four edge graphs and only one five edge graph which is given by R.

So at this juncture we are forced to make the following statement.

If B is a Boolean algebra associated with the power set P(S) associated with S the set on which B is built then B is not a subset vertex graph (or vertex subset graph of P(S)) of type I.

Thus using $S = \{x_1, x_2\}$ the subset vertex graphs of type I built using P(S) are listed below.

Point set vertex subset graphs are type I or empty subset vertex graph is

$P_1 = \{\{x_1\}, \{x_2\}\}$.

We do not include just one point set as point set subset vertex graphs of type I.

There is only one point set subset graph given by P_1.

Next we give all subset vertex graphs with one edge and two vertices;

$P_2 = \{\{x_1\}, \phi\}$ which enjoys the following figure.

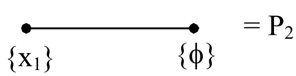


Figure 2.12

$P_3 = \{\{x_2\}, \phi\}$ whose figure is as follows.

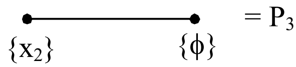


Figure 2.13

$P_4 = \{\{x_1, x_2\}, \phi\}$ whose figure is as follows.

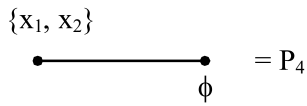


Figure 2.14

$P_5 = \{\{x_1, x_2\}, \{x_1\}\}$ which is described by the following figure

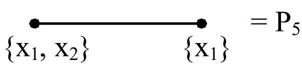


Figure 2.15

$P_6 = \{\{x_1, x_2\}, \{x_2\}\}$ whose graph is described below.

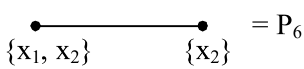


Figure 2.16

We see if $o(P(S)) = 2^2$ we have 5 subset vertex graphs with two vertices and one edge.

Now we describe those subset vertex graphs which have three vertices and only two edges.

$P_7 = \{\{x_1, x_2\}, \{x_1\}, \{x_2\}\}$

The figure associated with P_7 is as follows.

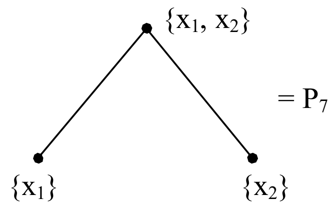


Figure 2.17

Let $P_8 = \{\{x_1\}, \{x_2\}, \phi\}$. The subset vertex graph figure is described below.

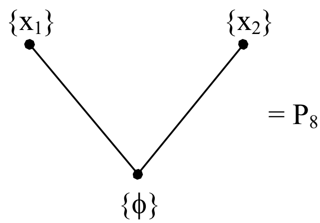


Figure 2.18

Now we see there are only two subset vertex graphs with two edges and three vertices.

Next we consider subset vertex graphs with three edges and three vertices which are given in the following.

$P_9 = \{\{x_2\}, \{x_1, x_2\}, \phi\}$

be the subset vertex graph.

In following we give the figure associated with P_9

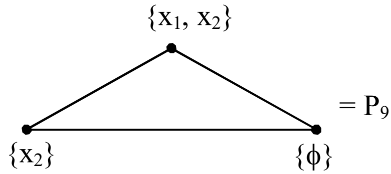


Figure 2.19

$P_{10} = \{\phi, \{x_1\}, \{x_1, x_2\}\}$ be the subset vertex graph of type I. The diagram associated with P_{10} is as follows.

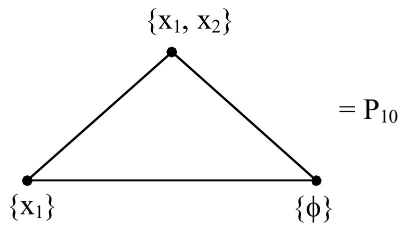


Figure 2.20

We see there are only 2 subset-vertex graph of type I with three edges and 3 vertices.

Finally we have only one subset vertex graph associated with four vertices given by the set $P(S) = \{\phi, \{x_1\}\}, \{x_2\}, \{x_1, x_2\}\}$ which has five edges.

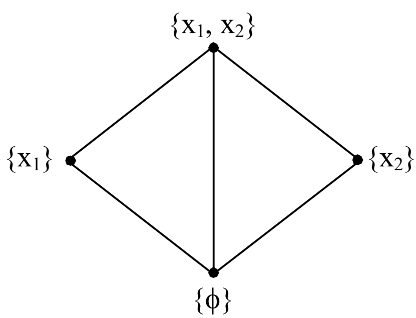


Figure 2.21

Now we see there are no subset vertex graph with four vertices and four edges.

Now we for the sake of understanding this new notion proceed onto describe the subset vertex graphs that can be constructed using the power set of S = $\{x_1, x_2, x_3\}$.

$P(S) = \{\phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$.

We can have only point set subset vertex graphs given by

$P_1 = \{x_1\}, \{x_2\}, \{x_3\}$

● ● ●

$((\{x_1\} \ \ \{x_2\}),$

● ●

$(\{\{x_2\}, \{x_3\}), \{\{x_1\}, \{x_3\}))$.

● ● ● ●

$P_2 = \{x_1\}, \quad \{x_2 \ x_3\}$

● ●

$P_3 = \{x_2\}, \quad \{x_1, x_3\}$ and

● ●

$P_4 = \{x_3\}, \quad \{x_2, x_1\}$.

● ●

Figure 2.22

Apart from the above figure 2.22 subset vertex graphs we cannot have any more.

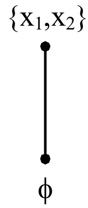
Now we proceed onto give all subset vertex graphs which has two vertices and one edge which are listed below.

P_5 =

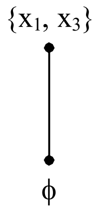
P_6 =

P_7 =

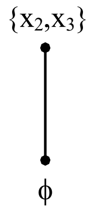
P_8 =



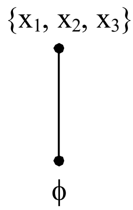
P_9 =



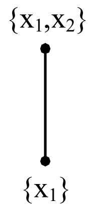
P_{10} =



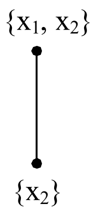
P_{11} =



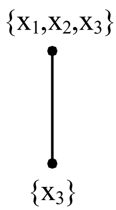
P_{12} =



P_{13} =



P_{14} =



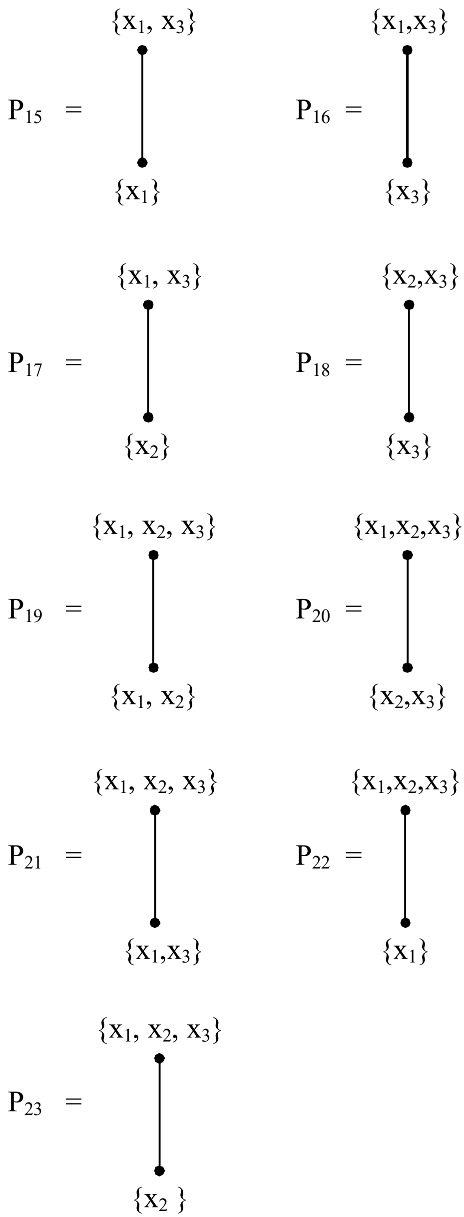
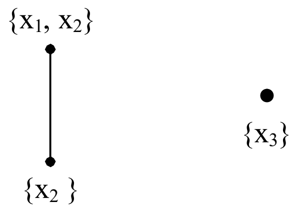

P_{15} =
{x1, x3}
{x1}
P_{16} =
{x1,x3}
{x3}
P_{17} =
{x1, x3}
{x2}
P_{18} =
{x2,x3}
{x3}
P_{19} =
{x1, x2, x3}
{x1, x2}
P_{20} =
{x1,x2,x3}
{x2,x3}
P_{21} =
{x1, x2, x3}
{x1,x3}
P_{22} =
{x1,x2,x3}
{x1}
P_{23} =
{x1, x2, x3}
{x2 }

Figure 2.23

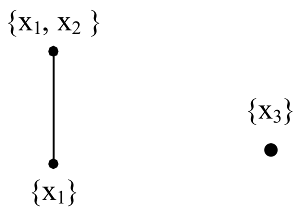
We see the above figure 2.23 gives vertex subset graphs with two vertices from P(S) and one edge.

Now we proceed onto give all subset vertex graphs with three vertices and one edge or two edges or three edges by the following figure 2.24.

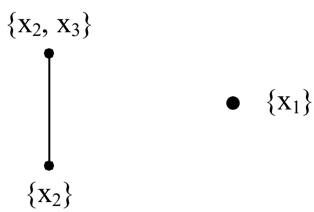
P_{25} =



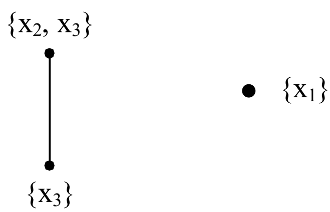
P_{26} =



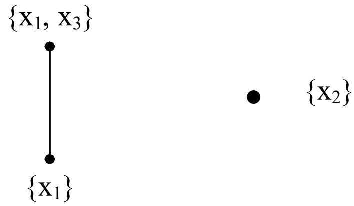
P_{27} =



P_{28} =



P_{29} =



P_{30} =

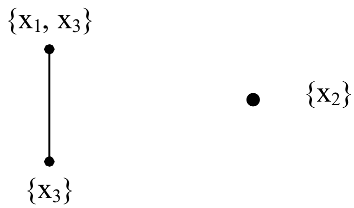
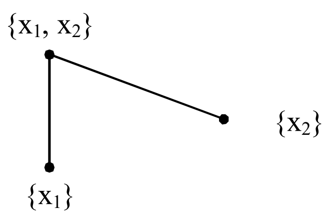


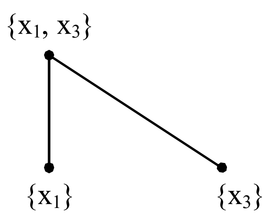
Figure 2.24

Thus there are only six subset vertex graphs with three vertices and only one edge. Now we give three vertices and two edges by the following figures.

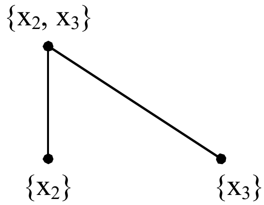
P_{31} =



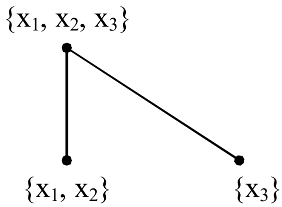
P_{32} =



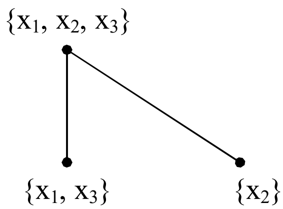
P_{33} =



P_{34} =



P_{35} =



P_{36} =

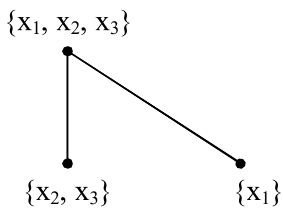
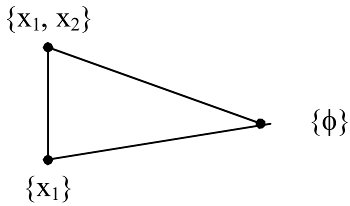


Figure 2.25

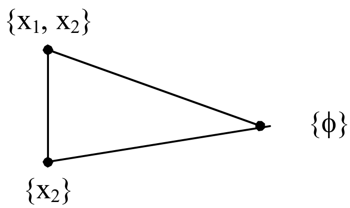
There are several subset graphs with three vertices and two edges.

Now we proceed onto give all vertex subset graphs with three vertices and three edges.

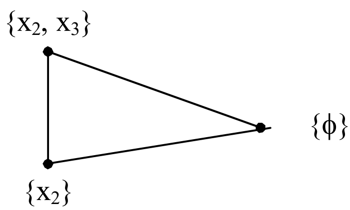
P_{37} =



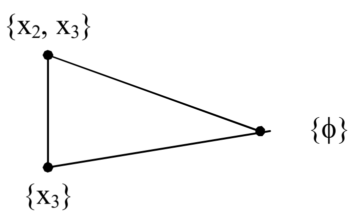
P_{38} =



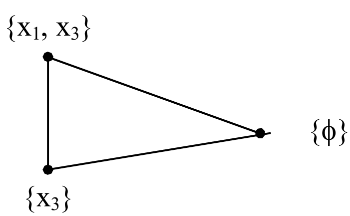
P_{39} =



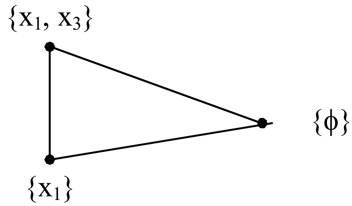
P_{40} =



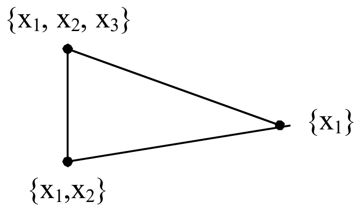
P_{41} =



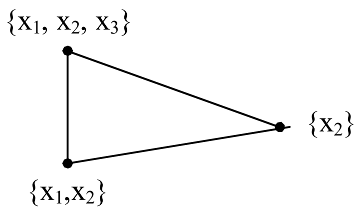
P_{42} =



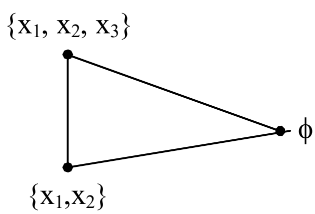
P_{43} =



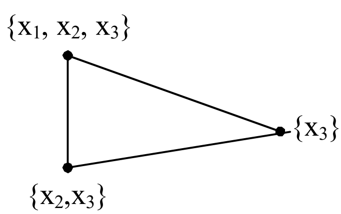
P_{44} =



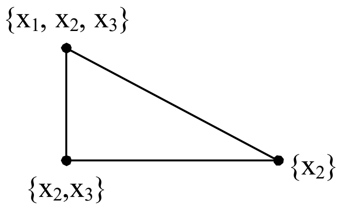
P_{45} =



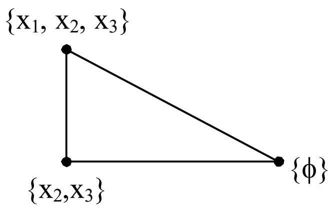
P_{46} =



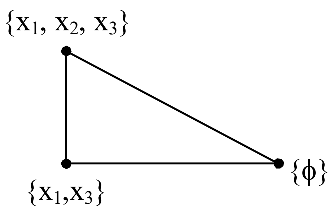
P_{47} =



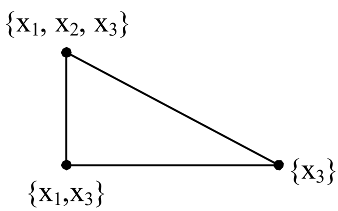
P_{48} =



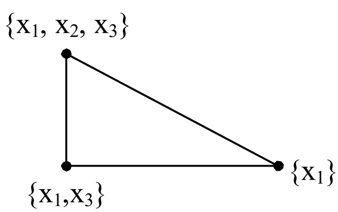
P_{49} =



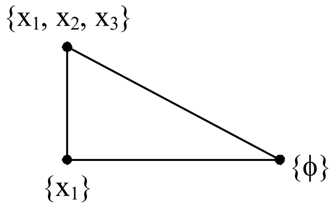
P_{50} =



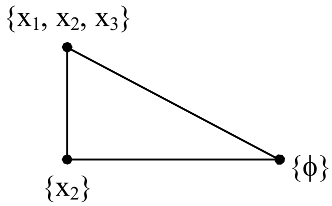
P_{51} =



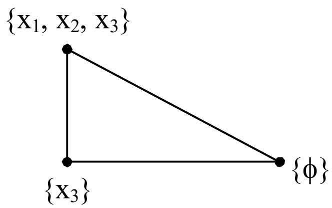
P_{52} =



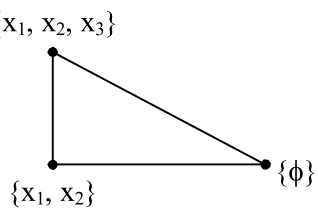
P_{53} =



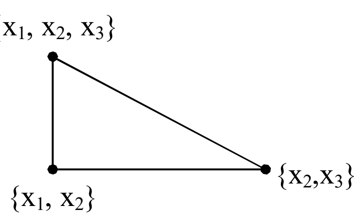
P_{54} =



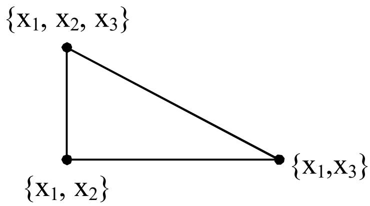
P_{55} =



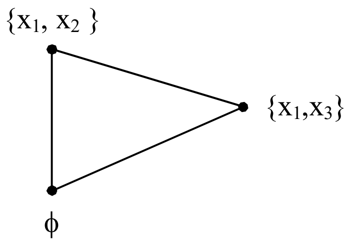
P_{56} =



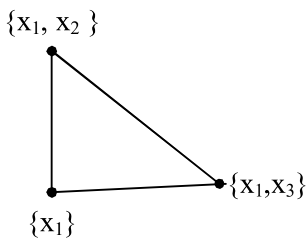
P_{57} =



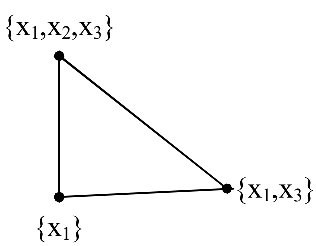
P_{58} =



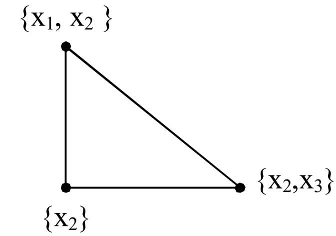
P_{59} =



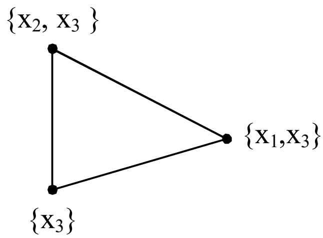
P_{60} =



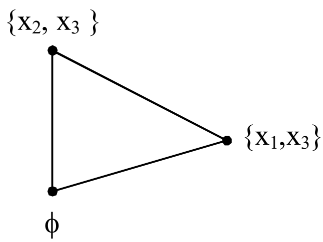
P_{61} =



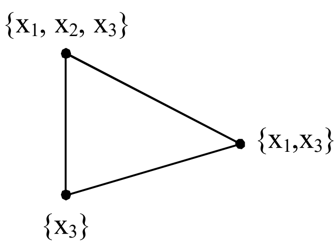
P_{62} =



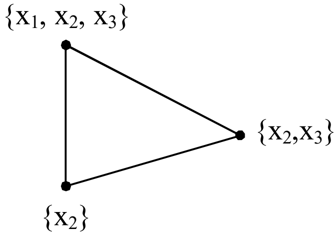
P_{63} =



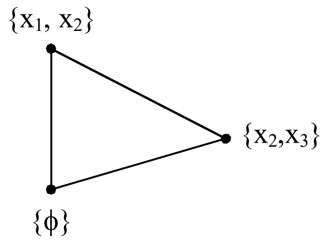
P_{64} =



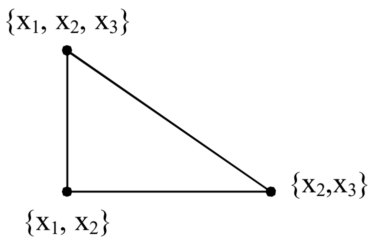
P_{65} =



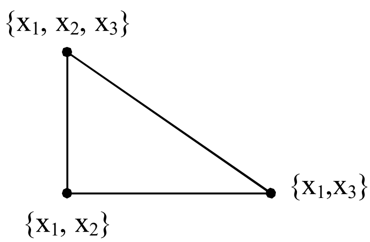
P_{66} =



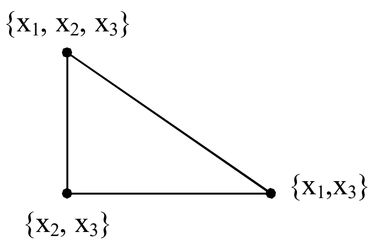
P_{67} =



P_{68} =



P_{69} =



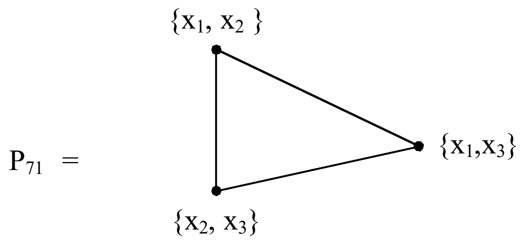


Figure 2.26

Next we proceed onto check whether there exist four vertices with one edge, four vertices with two edges, four vertex three edges four vertices with four edges and four vertices with five edges in the following.

We see we cannot have point set vertex graph with four vertices that is why we have not mentioned it.

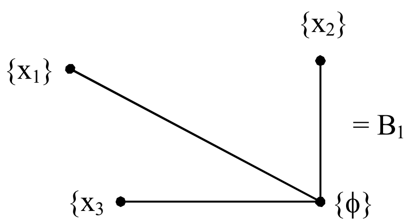


Figure 2.27

We get a vertex subset graph with 3 edges.

This set $B_1 = \{\{x_1\}, \{x_2\}, \{x_3\}, \phi\}$ is the set where the subset vertex graph has 3 edges.

Consider $B_2 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2, x_3\}\}$.

The subset vertex graph of type I associated with the set B_2 is as follows.

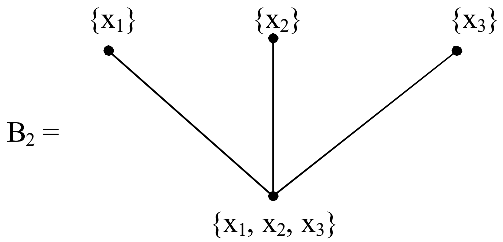


Figure 2.28

Consider the set $B_3 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}\}$. The subset vertex graph of type I associated with B_3 is

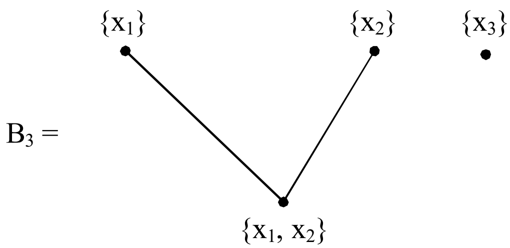


Figure 2.29

This has two edges.

Consider $B_4 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1,x_3\}$. The subset vertex graph of type I associated with B_4 is

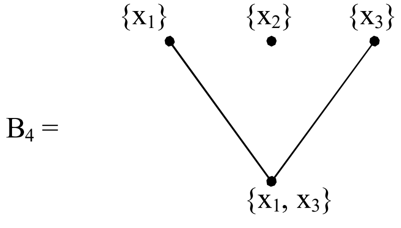


Figure 2.30

This subset vertex graph also has only two edges with four vertices.

Consider $B_5 = \{\{x_1\}, \{x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$. The subset vertex graph of type I associated with B_5 is

$B_5 =$

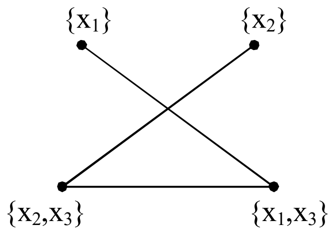


Figure 2.31

This has three edges and four vertices which is not a complete graph. Consider $B_6 = \{\{x_1\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$. The vertex subset graph of type I associated with B_6 is

$B_6 =$

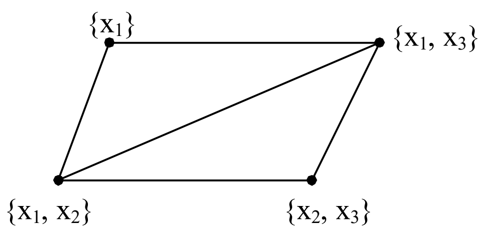


Figure 2.32

This vertex subset graph has four vertices and five edges.

Consider the subset vertex graph of type I given by the set $B_7 = \{\{\phi\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$. The graph related with B_7 is as follows.

$B_7 =$

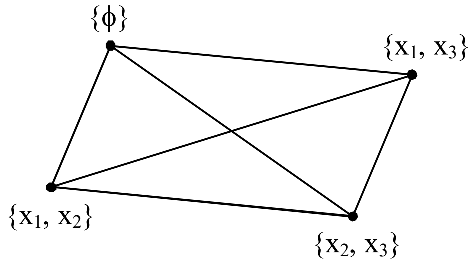


Figure 2.33

This subset vertex graph of type I is a graph with four vertices and six edges infact a complete vertex subset graph.

Consider the set $B_8 = \{\{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$.

The subset vertex graph of type I associated with B_8 is as follows.

$B_8 =$

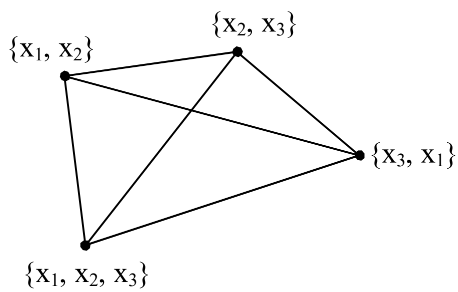


Figure 2.34

This is also complete vertex subset graph of type I.

Consider the set $B_9 = \{\{x_1\}, \{x_2\}, \{x_3, x_2\}, \{x_1, x_2, x_3\}\}$. The subset vertex graph of type I associated with B_9 is as follows.

B_9 =

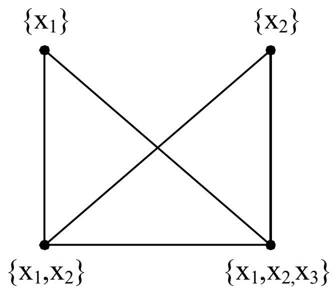


Figure 2.35

This is also a subset vertex graph of type I with five edges and four vertices.

We see the graph related with this B_9 is different from the graph got using the subset B_6 which also has five edges and four vertices.

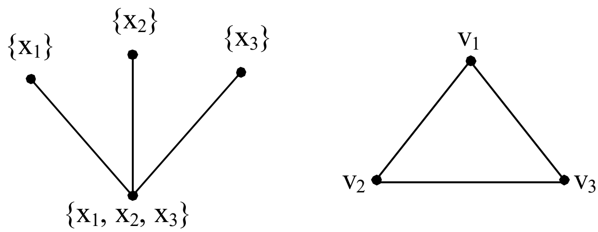
We wish to state leave it as an exercise to find all subset vertex graphs of type I with four vertices.

However when we take four vertices the following observations are pertinent.

i) There is no point set or empty graph with four vertices.

ii) When four vertices are used we can have two edges which are connected using three vertices and one point left out in case of type vertex subset graphs.

iii) In case of four vertices there are subset vertex graphs with three edges and one vertex free given by the following figure and we do not have a vertex subset graph with four vertices and three edges given by the following figure.

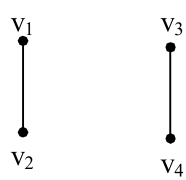


$v_1, v_2, v_3 \in P(S)$

Figure 2.36

There are subset vertex graphs of type I with four vertices and five edges and six edges only.

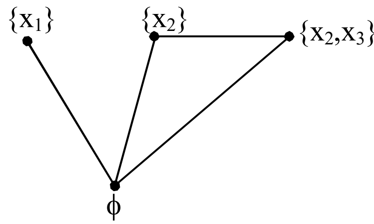
We are not in a position to find subset vertex graphs of type I with just two edges out of these subset vertex graphs of type I which can be got using four vertices and it forms a disjoint graph of the form.



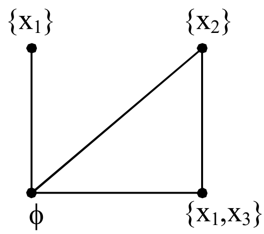
$v_1, v_2, v_3, v_4 \in P(S)$

Figure 2.37

Now we give some examples of four edge vertex subset graphs of type I using four vertices from P(S).



or



or

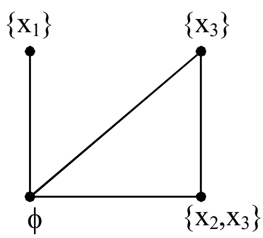


Figure 2.38

and so on.

However can we have subset vertex graph of type I of the form given below with four edges and four vertices?

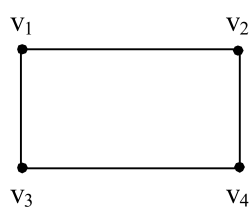


Figure 2.39

We have complete subset vertex graphs of type I with four edges and four vertices.

Now we list some of the subset vertex graphs of type I with five vertices.

We in the first place wish to keep in record that three exist atleast 56 subset vertex subgraphs of type I with five vertices given by the following figures.

D_1

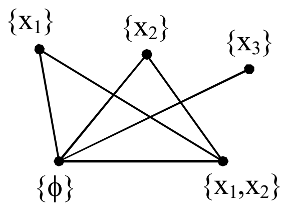
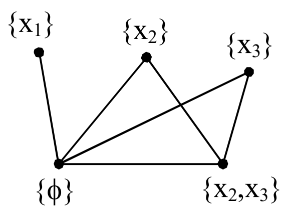


Figure 2.40

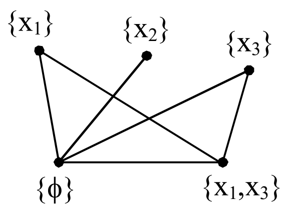
The has six edges.

We have three graph of this form in total

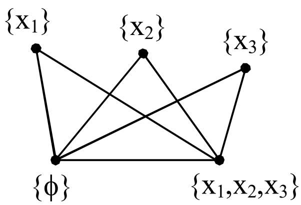
$D_2 =$



$D_3 =$

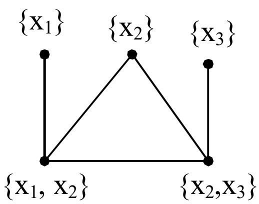


$D_4 =$

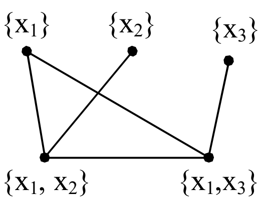


D_4 has six edges.

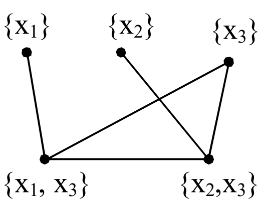
$D_5 =$



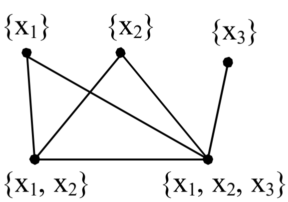
$D_6 =$



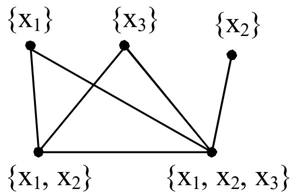
$D_7 =$



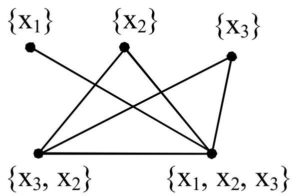
$D_8 =$



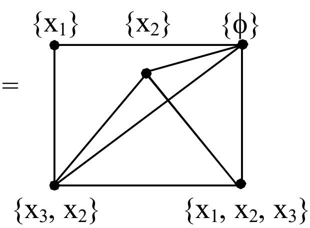
$D_9 =$



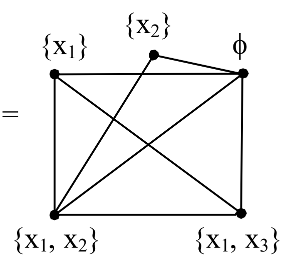
$D_{10} =$



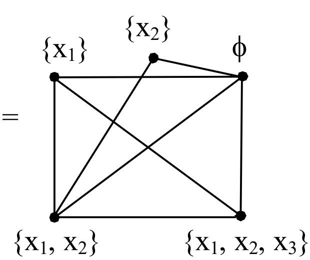
$D_{11} =$



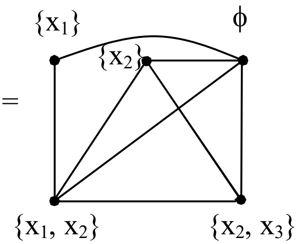
$D_{12} =$



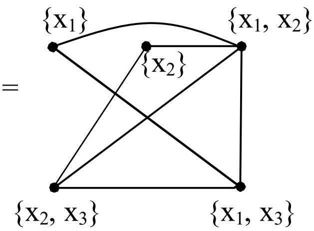
$D_{13} =$



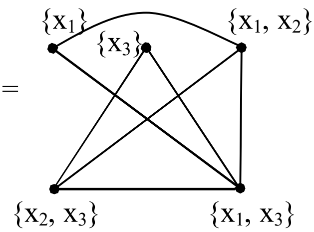
$D_{14} =$



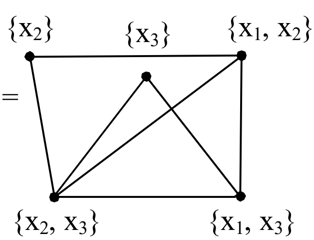
$D_{15} =$



$D_{16} =$



$D_{17} =$



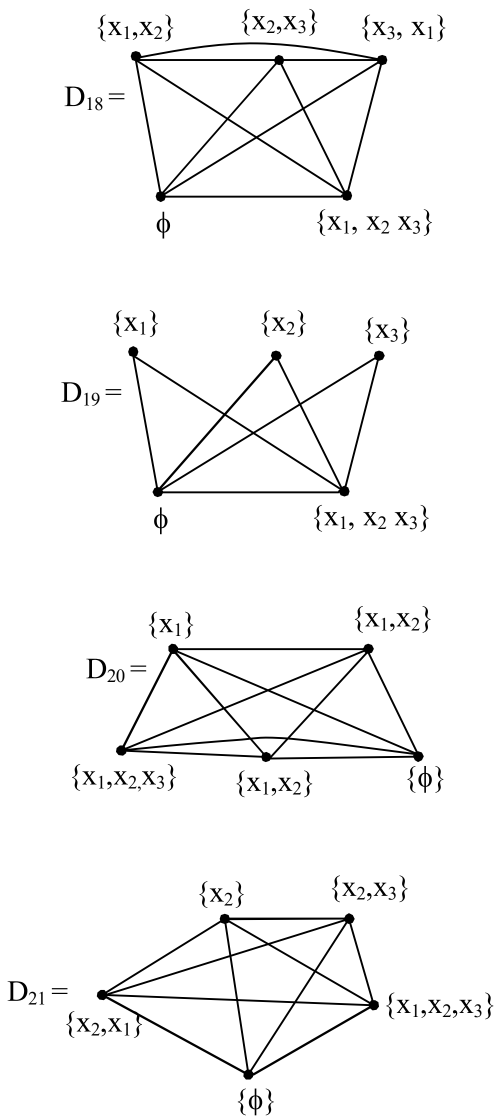

{x1,x2}
{x2,x3}
{x3, x1}
D18 =
ϕ
{x1, x2 x3}
{x1}
{x2}
{x3}
D19 =
ϕ
{x1, x2 x3}
{x1}
{x1,x2}
D20 =
{x1,x2,x3}
{x1,x2}
{ϕ}
{x2}
{x2,x3}
D21 =
{x2,x1}
{x1,x2,x3}
{ϕ}

Figure 2.41

We see D_{21} and D_{20} are complete vertex subset graphs of type I

$D_{22} =$

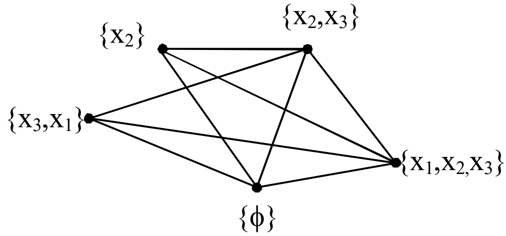


Figure 2.42

Clearly D_{22} is a not a complete subset vertex graph of type I isomorphic to D_{213} or D_{20}.

Now it is left as an exercise for the reader to find all subset vertex graphs of type I with 5 vertices and 10 edges, 9 edges, 8 edges and so on.

Can we say there are no subset vertex graphs with two edges and three edges 2?

Do we have a subset vertex graph of type I which has four edges?

We have subset vertex graphs of type I with 5 vertices has minimum of only 5 edges.

There are no subset vertex graphs of type I with five vertices which has four edges or three edges or two edges or one edge.

The minimum number of edges of a vertex subset graph of type I is only five and maximum being 10 a complete subset vertex graph of type I.

Now we proceed onto describe all subset vertex graphs of type I with six vertices. 28 subset vertex graphs with six vertices which are described below.

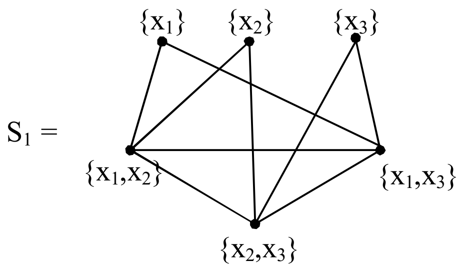


Figure 2.43

We see this has 9 edges.

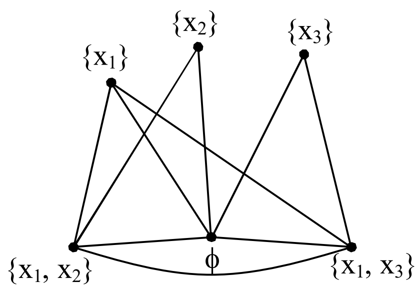


Figure 2.44

This has 10 edges.

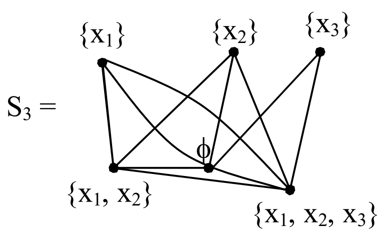


Figure 2.45

This has 11 edges

We have

$S_4 =$

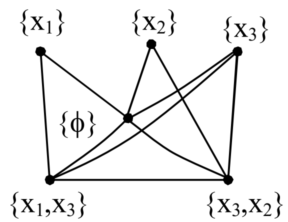


Figure 2.46

This also as 10 edges like S_2

$S_5 =$

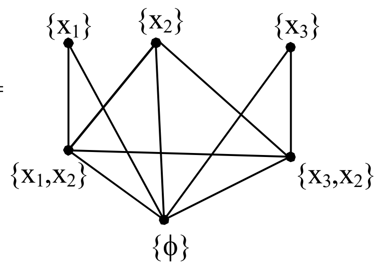


Figure 2.47

So has 10 edges S_1 and S_4

$S_6 =$

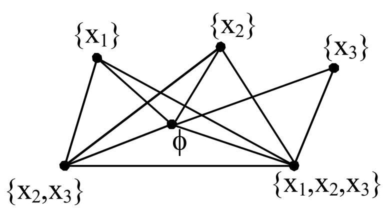


Figure 2.48

S_6 has 11 edges like S_3

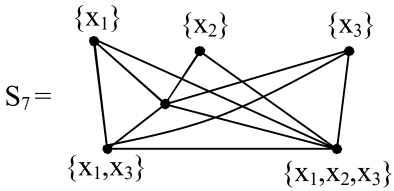


Figure 2.49

S_7 also has 11 edges like S_3 and S_6.

Consider the subset vertex graph of type I.

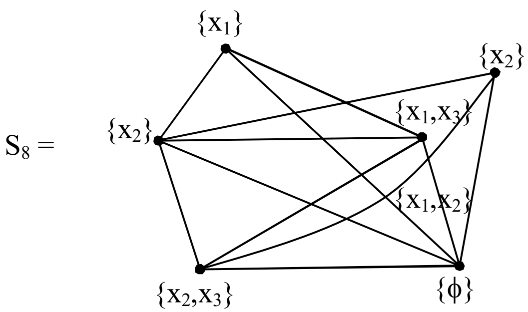


Figure 2.50

This has 12 edges.

Next we give another subset vertex graph with 12 edges in the following:

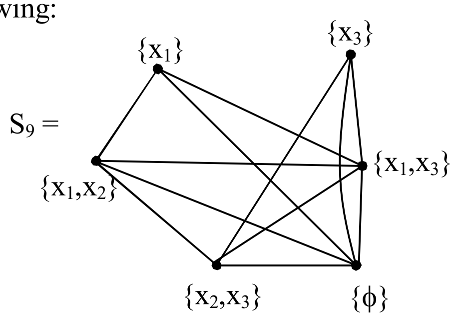


Figure 2.51

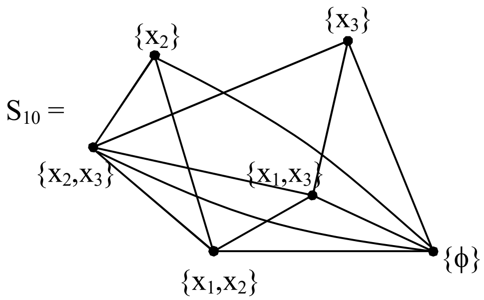


Figure 2.52

S_{10} also has 12 edges.

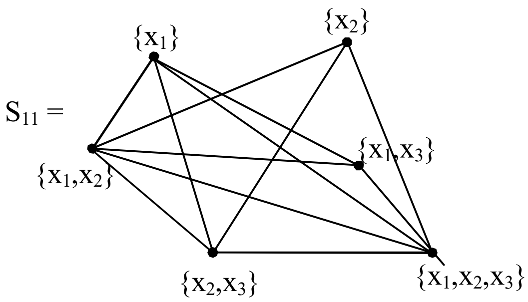


Figure 2.53

This has 12 edges.

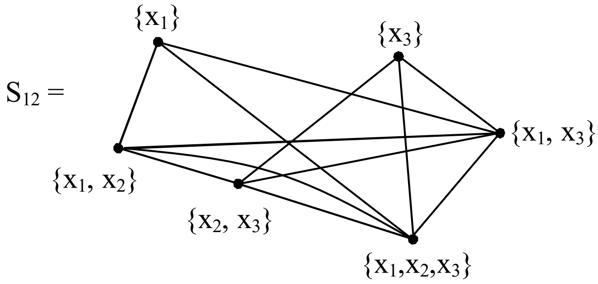


Figure 2.54

This subset vertex graph of type I with six vertices also has only 12 edges.

Similarly

$S_{13} =$

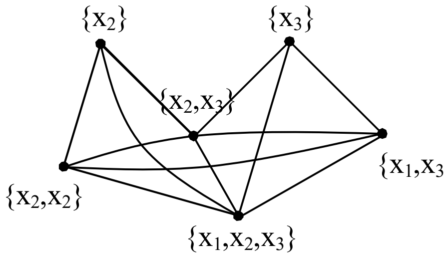


Figure 2.55

S_{13} is also a 12 edge subset vertex graph of type I. Now we give yet another example of 6 vertex subset graphs of type I.

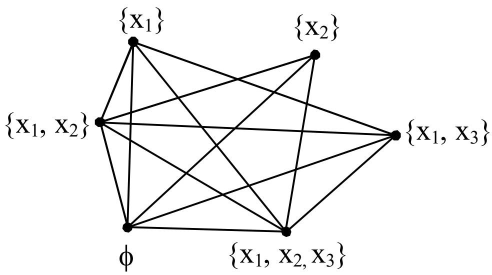
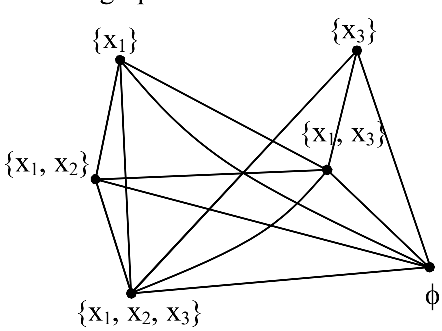


Figure 2.56

This too has only 13 edges we see we can have some more subset vertex graphs with six vertices and 13 edges.



$= S_{15}$

Figure 2.57

S_{15} has six vertices and 13 edges.

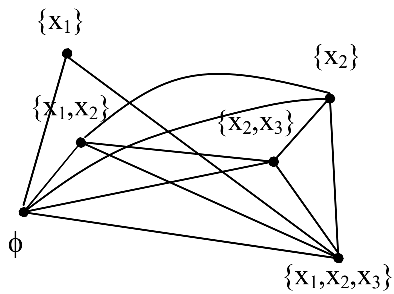


Figure 2.58

This subset vertex graph of type I has six vertices and 13 edges.

Consider the following subset vertex graphs with 6 vertices.

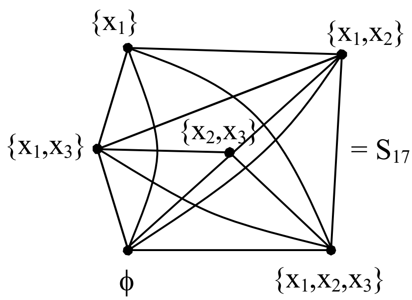


Figure 2.59

This has 14 edges and six vertices.

We leave it as an exercise for the reader to prove we have a maximum of only 14 edges for six vertices.

We further leave it as the task for the reader to prove there is no subset vertex graph of type I for any six vertices there are only maximum of 14 edges that is there does not exist a complete subset vertex graph of type I with 15 edges.

Now we proceed onto describe those subset vertex graphs of type I with seven vertices.

We will find the maximum number of edges these subset vertex graphs of type I with seven vertices can have R_1 is a subset vertex graph of type I with seven vertices and 15 edges.

$R_1 =$

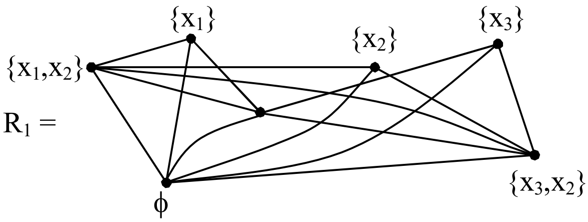


Figure 2.60

Let

$R_2 =$

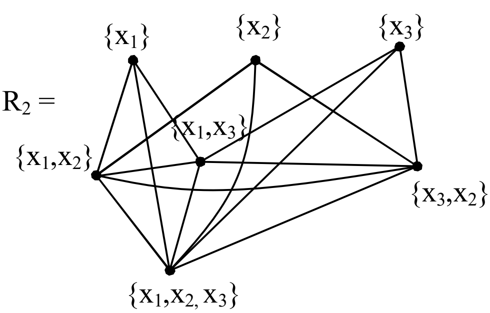


Figure 2.61

This is also a vertex subset graph with seven vertices and 15 edges.

Now we give another vertex subset graph with seven vertices and find the number of edges associated with them.

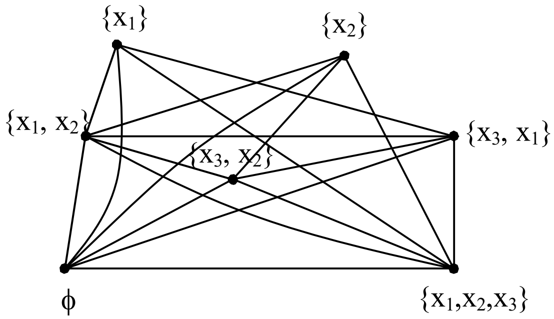


Figure 2.62

This has 20 edges.

Infact we have given here three subset vertex graphs of type I with seven vertices and 20 edges.

Infact we do not have any subset vertex graphs with seven vertices which is complete, that it has 21 edges.

The maximum number of edges which a vertex subset graph with seven vertices can have is only 20.

Next we give the unique subset vertex graph of type I with 8 vertices in the following.

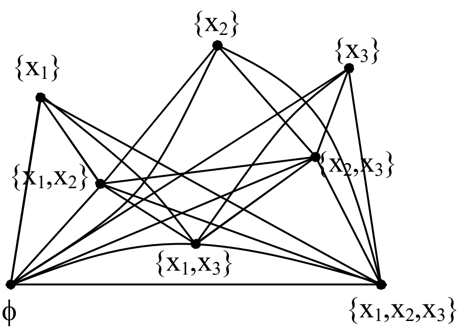


Figure 2.63

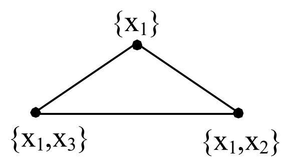
This graph has only 22 edges. Thus there is no complete subset vertex graph of type I with 8 vertices. The maximum number of edges this P can have is only 22.

In case of P(S) where S = $\{x_1, x_2, x_3\}$ we can have only complete subset vertex graph of type I with five vertices.

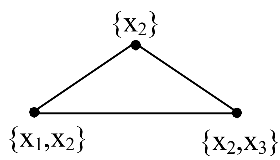
However we have 5 subset vertex graph of type I with four vertices which are complete.

Now we enlist all triads which are complete subset graphs with vertices from P(S) in the following.

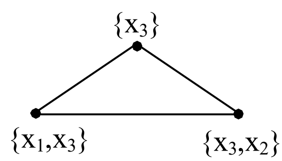
$W_1 =$



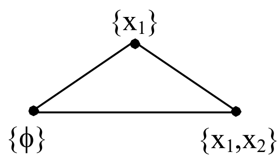
$W_2 =$



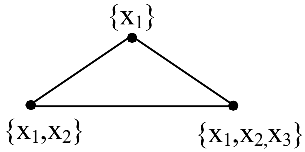
W_3



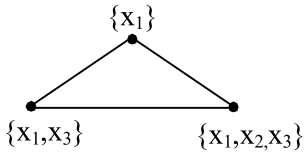
W_4



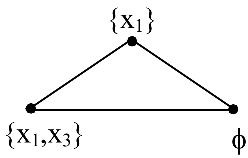
$W_5 =$



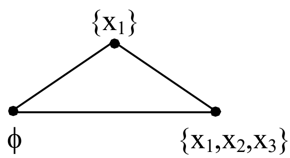
$W_6 =$



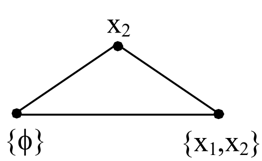
$W_7 =$



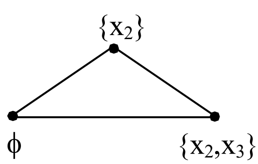
$W_8 =$



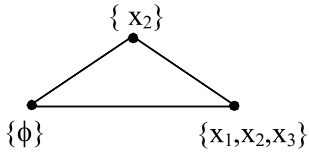
$W_9 =$



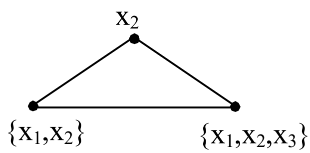
$W_{10} =$



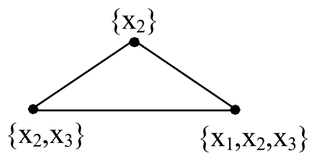
$W_{11} =$



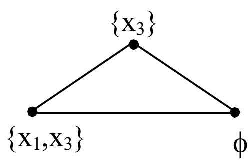
$W_{12} =$



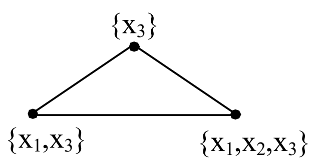
$W_{13} =$



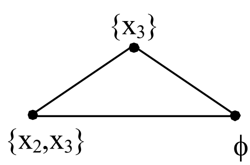
$W_{14} =$



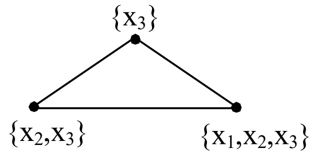
$W_{15} =$



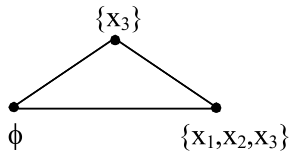
$W_{16} =$



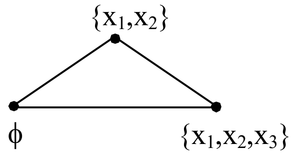
$W_{17} =$



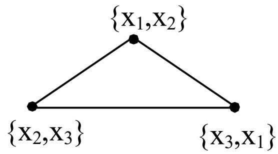
$W_{18} =$



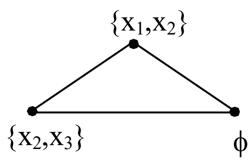
$W_{18} =$



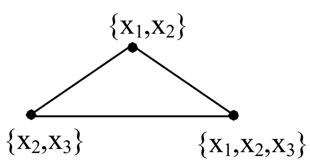
$W_{19} =$



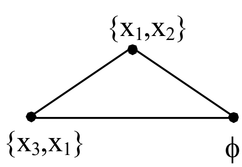
$W_{20} =$



$W_{21} =$



$W_{22} =$



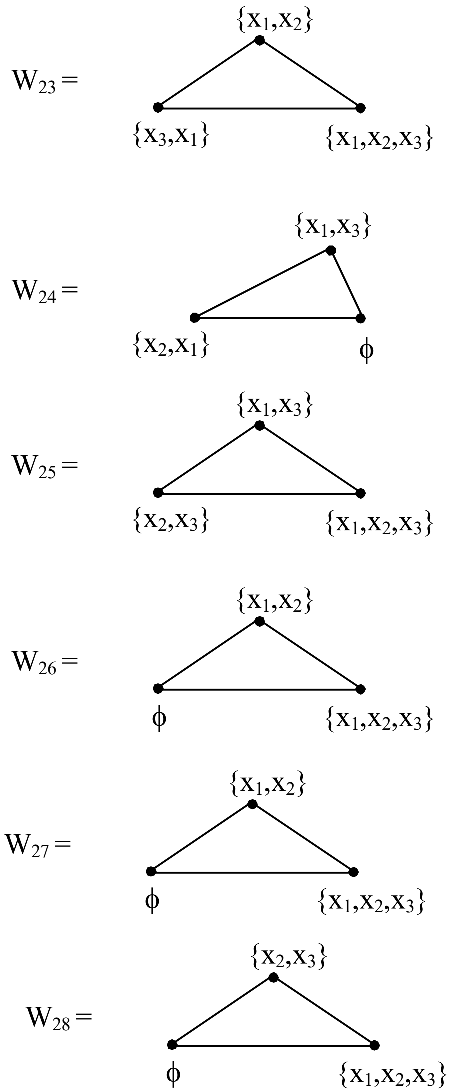
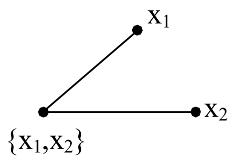


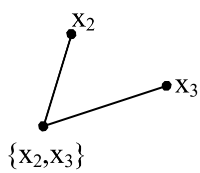
Figure 2.64

Now we enlist some of the forbidden triads of P(S).

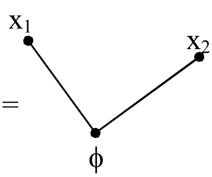
$V_1 =$



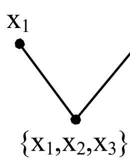
$V_3 =$



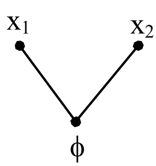
$V_4 =$



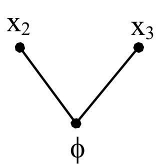
$V_5 =$



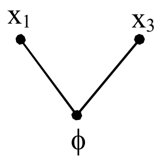
$V_6 =$



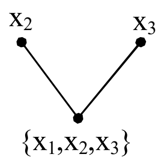
$V_7 =$



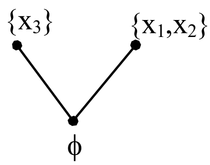
$V_8 =$



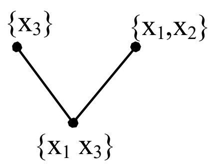
$V_9 =$



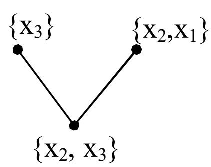
$V_{10} =$



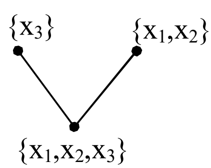
$V_{11} =$



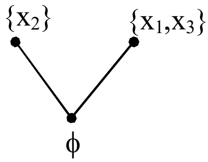
$V_{12} =$



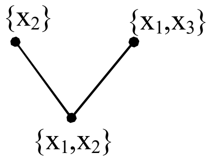
$V_{13} =$



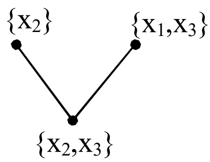
$V_{14} =$



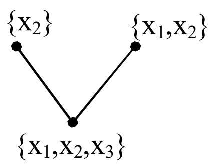
$V_{15} =$



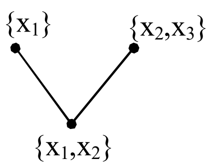
$V_{16} =$



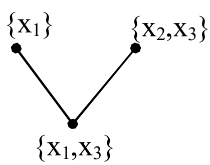
$V_{17} =$



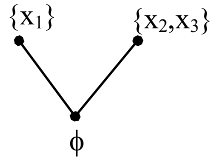
$V_{18} =$



$V_{19} =$



$V_{20} =$



$V_{21} =$

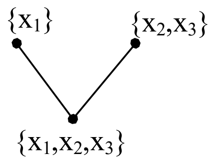


Figure 2.65

Now we proceed onto supply one or two subset vertex graphs of type I using the subsets of P(X) where X = $\{x_1, x_2, x_3, x_4\}$.

We see there is only one empty graph with $\{x_1\}$, $\{x_2\}$, $\{x_3\}$ and $\{x_4\}$ which is the largest graph of type I.

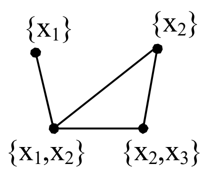


Figure 2.66

This is a subset vertex subgraph of type I with four vertices and four edges.

Consider

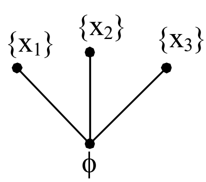


Figure 2.67

This is also a vertex subset graph of type I with four vertices and 3 edges consider.

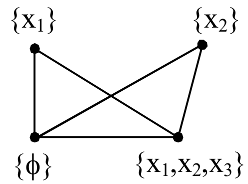


Figure 2.68

This a subset vertex graph of type I with 5 edges.

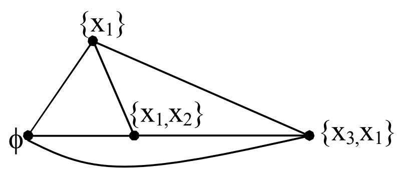


Figure 2.69

This is a subset vertex graph of type I with 6 edges.

Clearly this is a complete subset vertex graph of type I. Thus we have several subset vertex graphs of type I with four vertices which are complete.

The reader is left with the task of finding the number of complete subset vertex graphs of type I which has only four vertices.

Now we give examples of five vertex subset vertex graphs of type I using the power set P(S) where S = $\{x_1, x_2, x_3, x_4\}$.

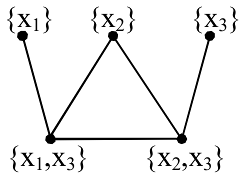


Figure 2.70

This graph with 5 vertices has only 5 edges.

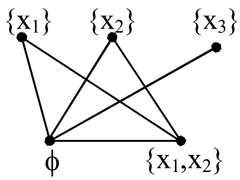


Figure 2.71

We see this graph has 5 vertices and six edges.

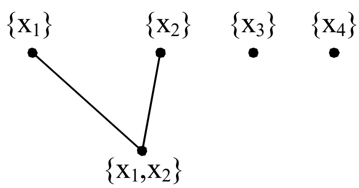


Figure 2.72

This graph has only two edges and five vertices.

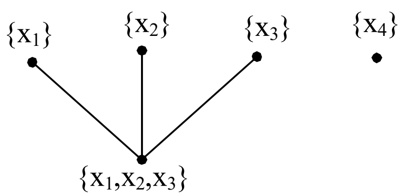


Figure 2.73

This vertex subset graph has 3 edges and five vertices.

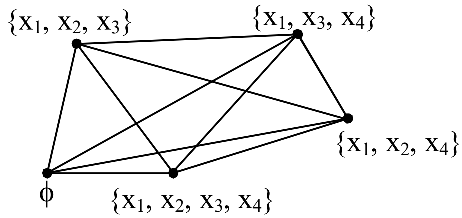


Figure 2.74

This is a complete subset vertex graph of type I with five vertices.

The reader is left with the task of finding the number of complete subset vertex graphs of type I with 5 vertices.

Next we illustrate one or two subset vertex graphs of type I with 6 vertices.

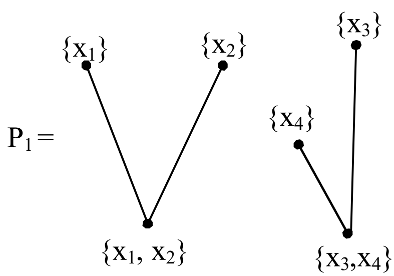


Figure 2.75

This subset vertex graph of type I has four edges and six vertices.

Consider the subset vertex graph P_2 of type I given six vertices.

P_2 =

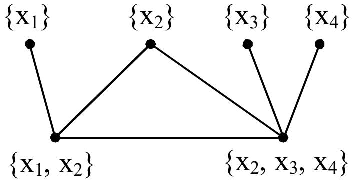


Figure 2.76

P_2 has six vertices and six edges.

Consider the vertex subset graph of type I given by

P_3 =

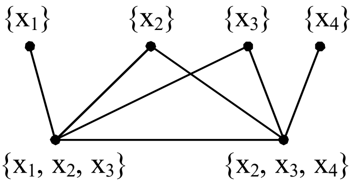


Figure 2.77

This has seven edges.

Consider

P_4 =

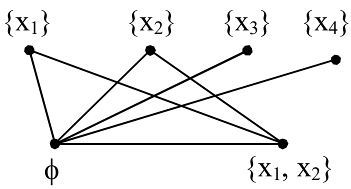


Figure 2.78

This subset vertex graph of type I also has seven edges and six vertices.

P_5 =

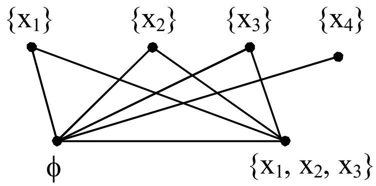


Figure 2.79

We P_5 is a vertex subset graph of type I with 8 edges.

P_6 =

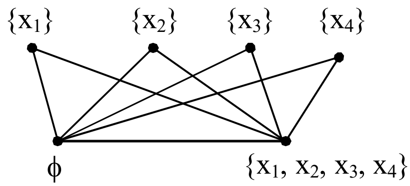


Figure 2.80

This type I subset vertex graph has 9 edges.

Consider

P_7 =

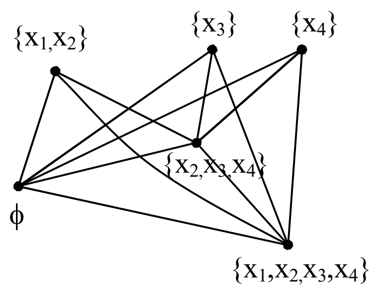


Figure 2.81

This vertex subset graph of type I has 12 edges.

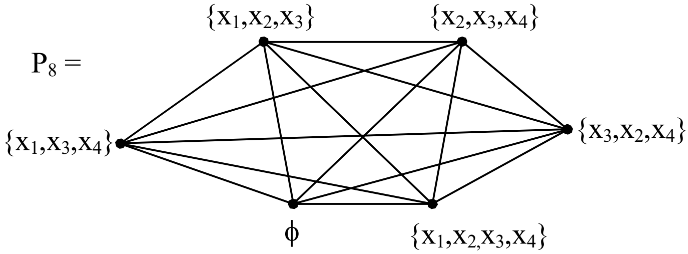


Figure 2.82

The vertex subset graph of type I with six vertices is a complete graph.

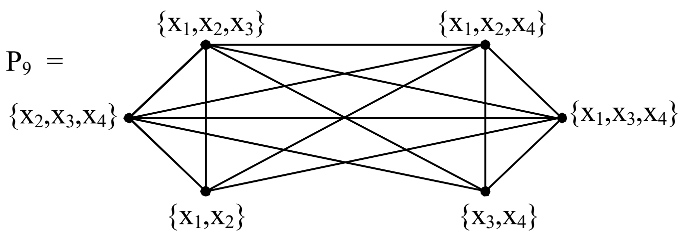


Figure 2.83

Clearly this type I subset vertex graph is not a complete graph. This has only 14 edges.

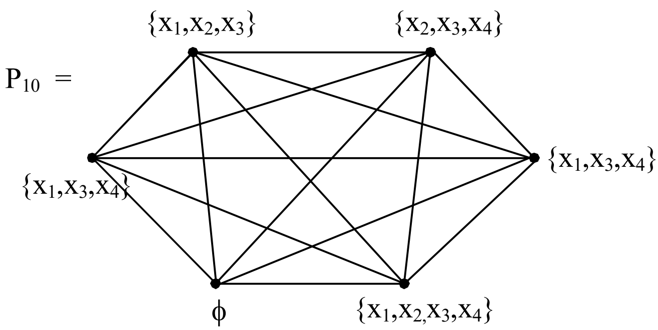


Figure 2.84

Clearly P_{10} is a subset vertex graph of type I which is complete.

Now we proceed onto describe vertex subset graphs of type I which has seven vertices.

$Q_1 =$

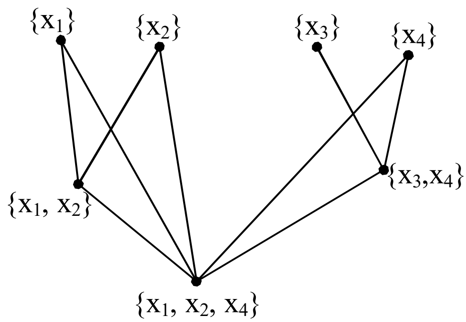


Figure 2.85

This subset vertex graph Q, with seven vertices has 9 edges.

$Q_2 =$

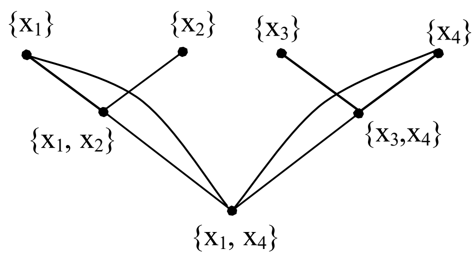


Figure 2.86

This subset graph of type I Q_2 has 8 edges.

Q_3 =

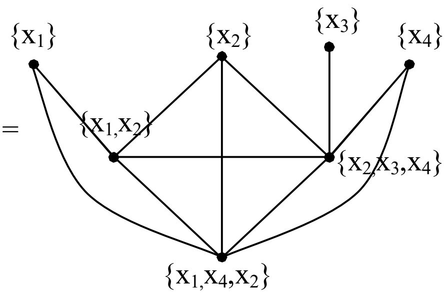


Figure 2.87

This graph Q_3 has 11 edges.

Q_4 =

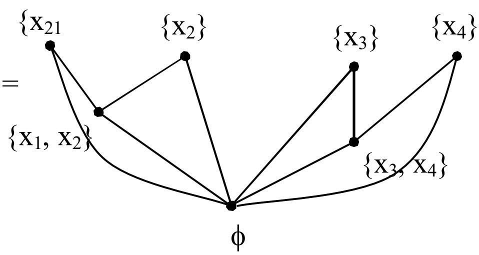


Figure 2.88

Q_4 has only 10 edges.

Q_5 =

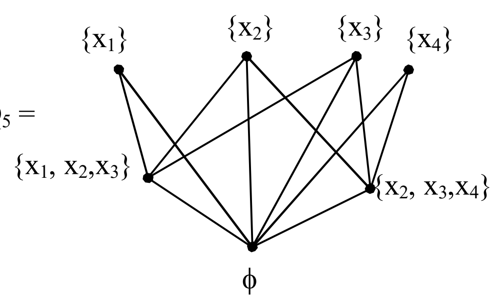


Figure 2.89

The graph Q_5 has 13 edges.

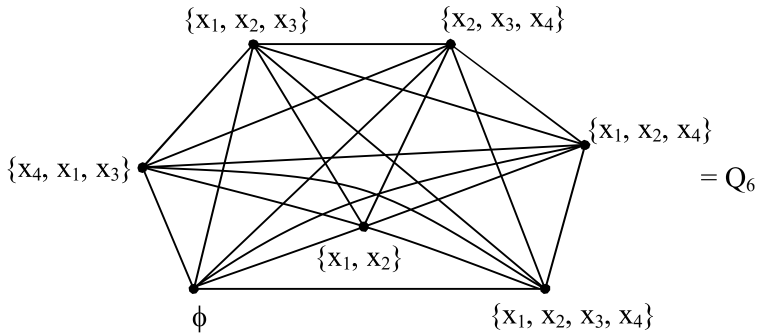


Figure 2.70

Clearly Q_6 is a complete subset vertex graph of type I with seven vertices.

We can have once again a complete vertex graphs of type I using the seven vertices and replacing $\{x_1, x_2\}$ by $\{x_1, x_3\}$ ro $\{x_1, x_4\}$.

Let Q_7 be as follows.

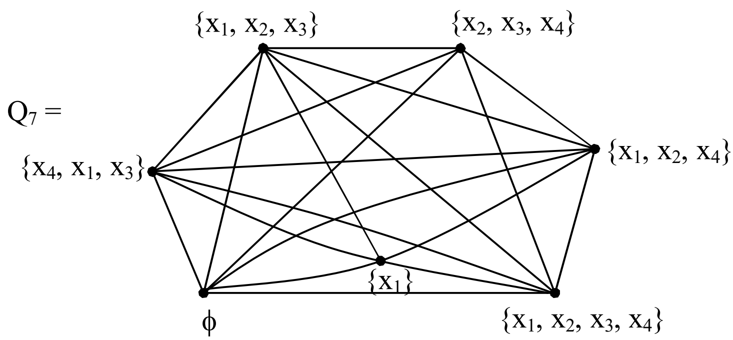


Figure 2.91

Clearly Q_7 is not a complete vertex subset graph of type I.

Now we consider subset vertex graphs with 8 vertices.

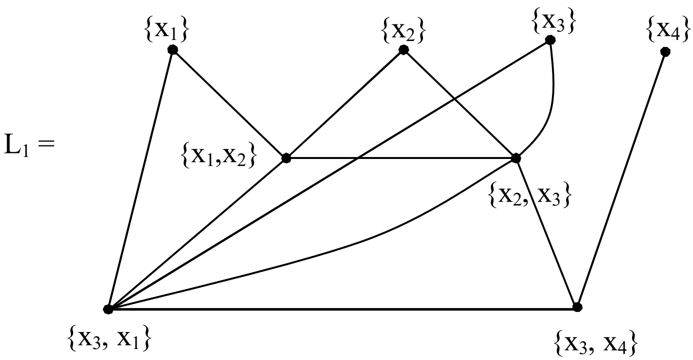


Figure 2.92

This subset vertex graph of type I has 12 edges and eight vertices.

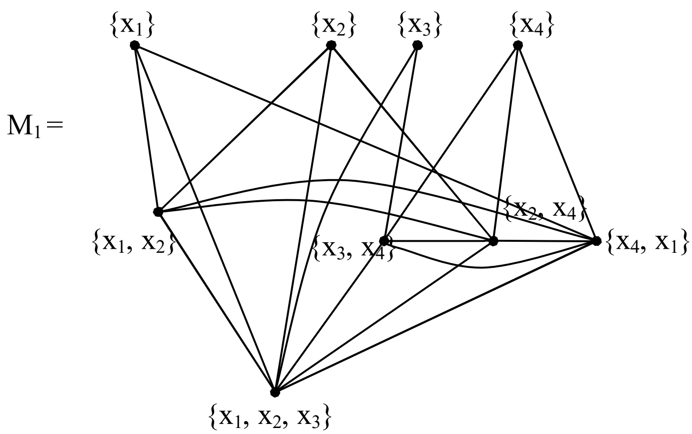


Figure 2.93

M_1 is a subset vertex graph with 9 vertices and 22 edges.

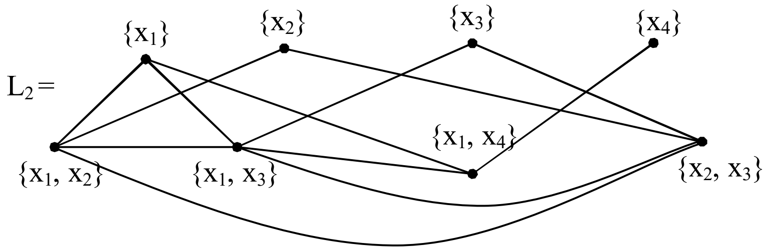


Figure 2.94

The vertex subset graph L_2 has 8 vertices and 12 edges.

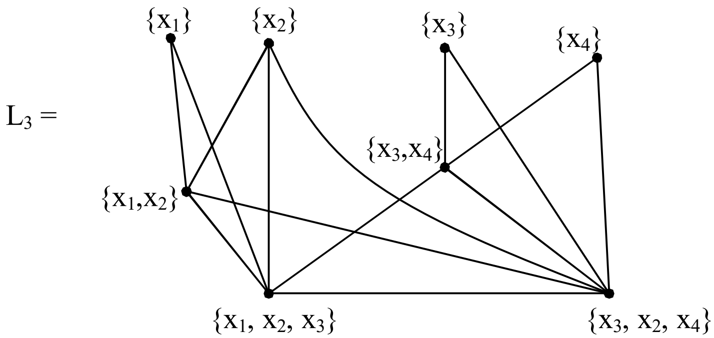


Figure 2.95

This has 13 vertices and eight edges.

Next we proceed on give eight vertices subset graph of type I in the following.

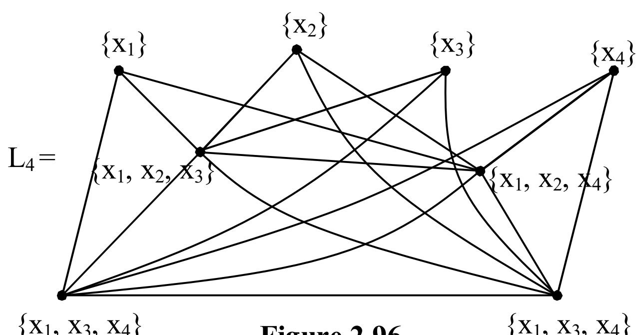


Figure 2.96

There are 18 edges for L_4.

Now we study if any two of the subsets with triples is replaced by ϕ and $\{x_1, x_2, x_2, x_3\}$ how many edges will that vertex subset graph of type I has.

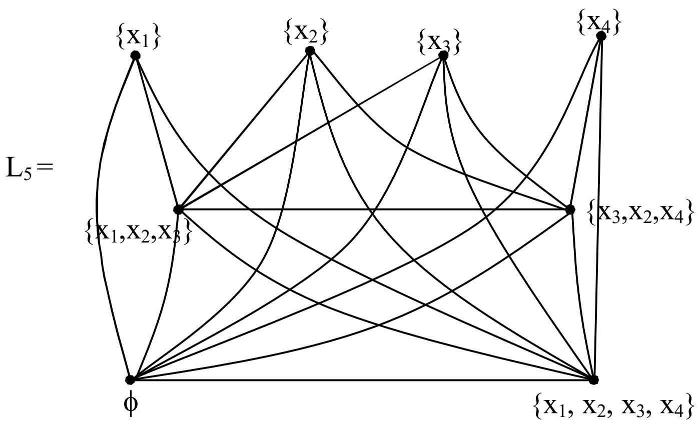


Figure 2.97

There are 20 edges for the subset vertex graphs of type I, L_5 with 8 vertices.

Now we give L_6 the subset vertex graph of type I by the following figure.

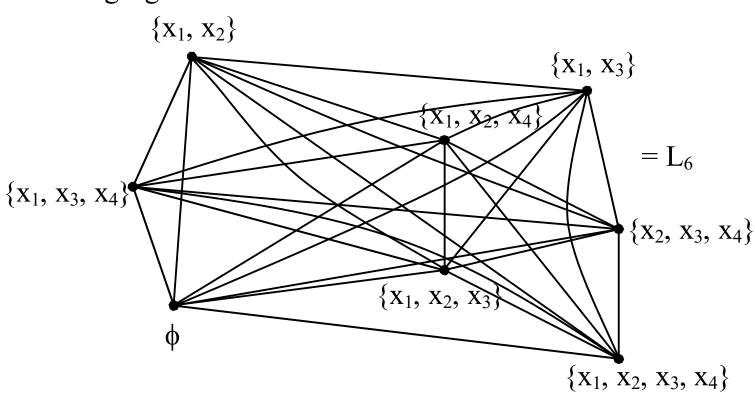


Figure 2.98

Clearly L_6 is a complete subset vertex graph of type I with eight vertices.

Infact we have 9 subset vertex graph of type I which is complete with 8 vertices.

Now we list out 9 vertices subset vertex graphs of type I.

C_1 =

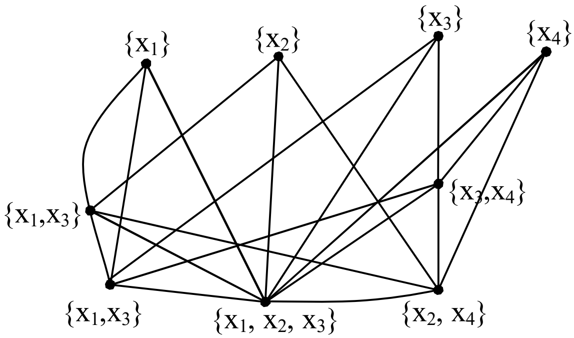


Figure 2.99

There are 21 edges. In fact we claim this is the minimum number of edges we can get with 9 vertices.

We have atleast 3 such vertex subset graphs of type I.

C_2

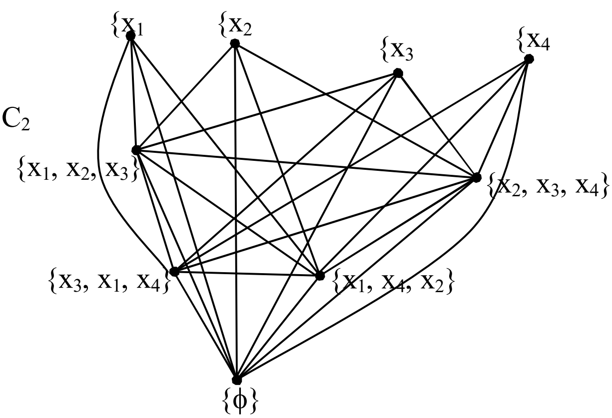


Figure 2.100

There are 26 edges for this subset vertex graph of type I.

There is also another graph similar or isomorphic with this when ϕ is replaced by $\{x_1, x_2, x_3, x_4\}$.

$C_3 =$

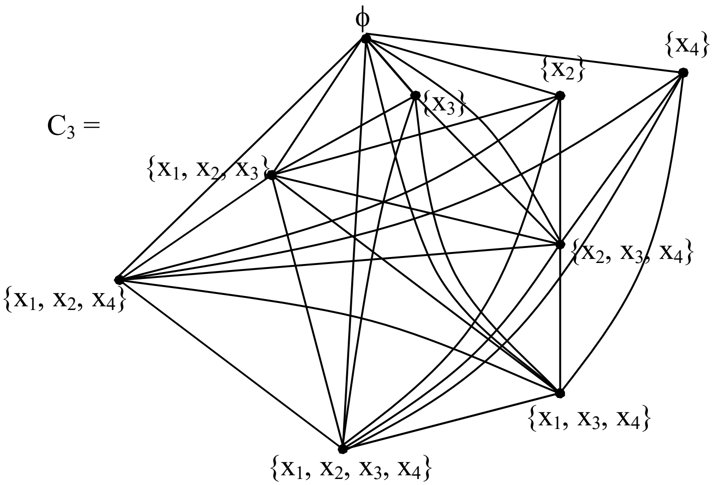


Figure 2.101

Clearly we see this graph C_3 is not complete.

Consider C_4 the vertex subset graph of type I given by the following figure with 9 vertices.

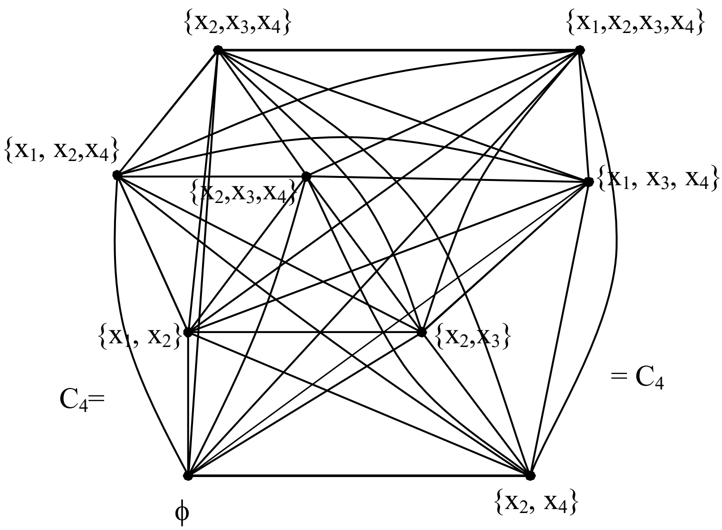


Figure 2.103

It is easily verified C_4 is a complete graph with 9 vertices.

The main criteria being among the 9 vertices set we should have intersection of any pair of vertex subset to be non empty.

Even if one subset vertex is such that its intersection with any other (even only one) vertex subset is empty then the vertex subset graph is not complete.

Now we consider all subset vertex graphs of type I with 10 vertices.

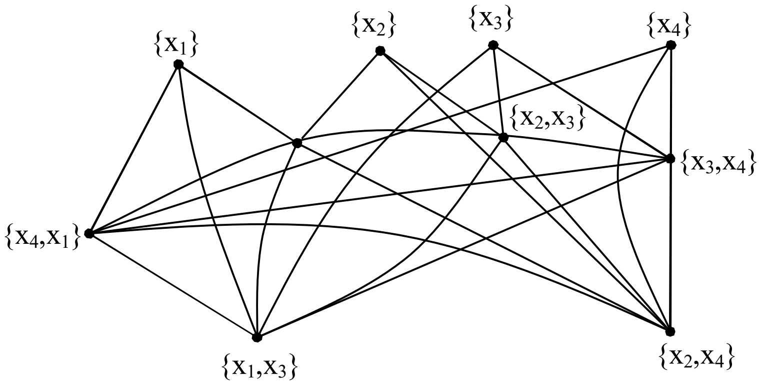


Figure 2.104

We have 26 edges for the subset vertex graph of type I with ten vertices.

Now we proceed onto describe 10 vertices subset graphs of type I.

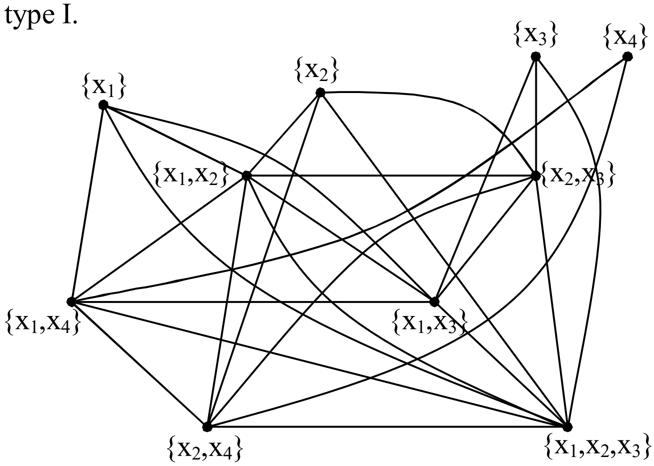


Figure 2.105

This subset vertex graph D_2 has to vertices. Find the number of edges. Which of the graph D_1 or D_2 has more edges ?

Consider D_3 the subset vertex graph of type I with 10 vertices which is as follows.

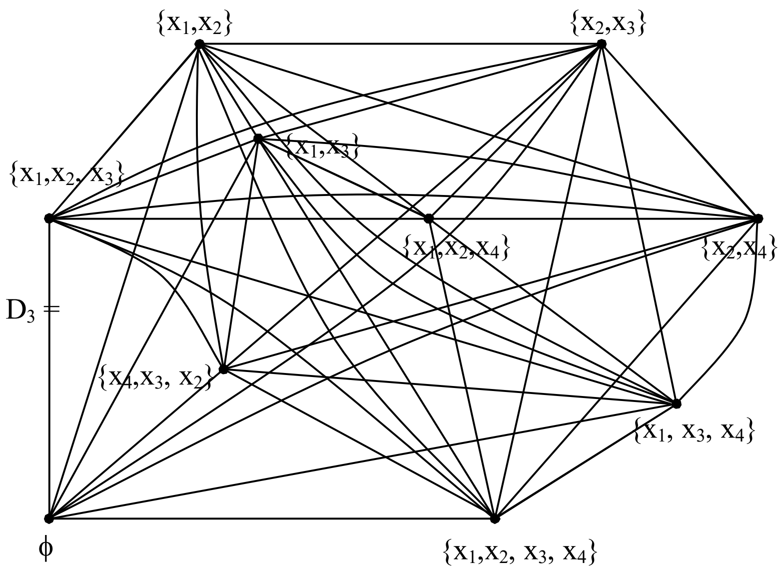


Figure 2.106

We see D_3 is not a complete subset vertex graph of type I.

We see all subset vertex graphs of type I with 10 or more number of vertices can never yield complete subset vertex graphs of type I.

In view of all these we suggest the following problem.

Problem 2.1. Let P(S) be the power set of the set $S = \{x_1, x_2, \ldots, x_n\}$.

Clearly $|P(S)|\ 2^n$; $n \geq 2$.

If G be the collection of all subset vertex graphs of type I with vertices from P(S).

i) The subset vertex graph of type I with 2^n vertices is not complete.

ii) All subset vertex graphs with more than $2^{n-1} + 2$ number of vertices is not a complete subset vertex graph of type I.

iii) We have subset vertex graphs of type I with $2^{n-1} + 1$ number of vertices such that any pair of vertices intersection is nonempty is a complete subset vertex graph of type I.

iv) There are complete subset vertex graphs of type I which has number of vertices to be less than or equal to $2^{n-1} + 1$ $n > 2$.

We now compare these subset vertex graphs of type I built using the power set P(S) with the Boolean algebra associated with P(S).

We know all lattices are graphs but graph in general are not lattices. For rooted trees which are graphs are not even lattices so are not Boolean algebras.

We now first give the Boolean algebra using P(S) and the subset vertex graph of type I with number of vertices to be |P(S)| i.e. the maximum number of vertices using P(S) that |P(S)| itself.

When n = 2 we see P(S) = P({x_1, x_2}) = {ϕ, {x_1}, {x_2}, {x_1, x_2}} is the Boolean algebra of order four. It has only four vertices.

Let G_1 be the type I subset vertex graph with four vertices given b the following figure.

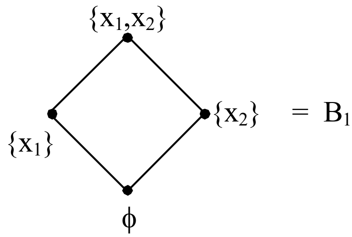


Figure 2.107

Let G_1 be the type I subset vertex graph with four vertices given by the following figure.

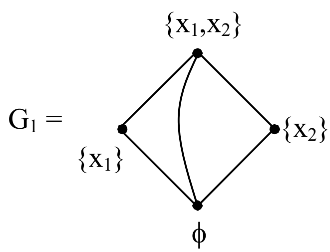


Figure 2.108

Clearly G_1 is not a Boolean algebra of order four. G_1 has five edges. So Boolean algebras in generated are not type I subset vertex graphs except in case of S = {x_1} and P(S) = {ϕ, {x_1}}.

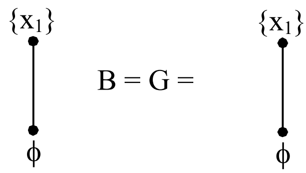


Figure 2.109

Consider the set S = $\{x_1, x_2, x_3\}$. P(S) = $\{\phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, \{x_1, x_2, x_3\}\}$ be the power set of S.

The Boolean algebra B_2 of P(S) is as follows.

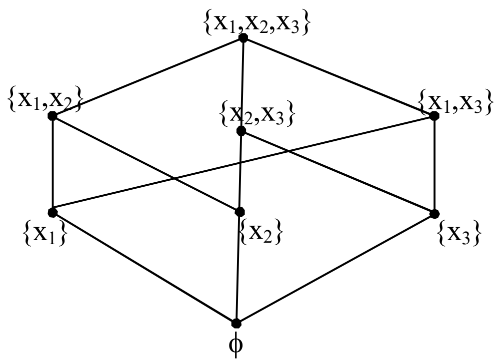


Figure 2.110

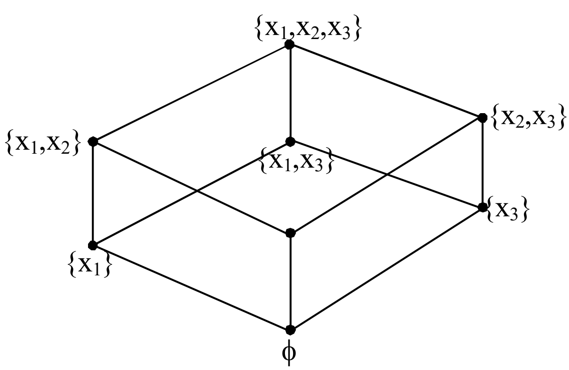


Figure 2.111

Now we give the subset vertex graph of type I, G_2 by the following figure.

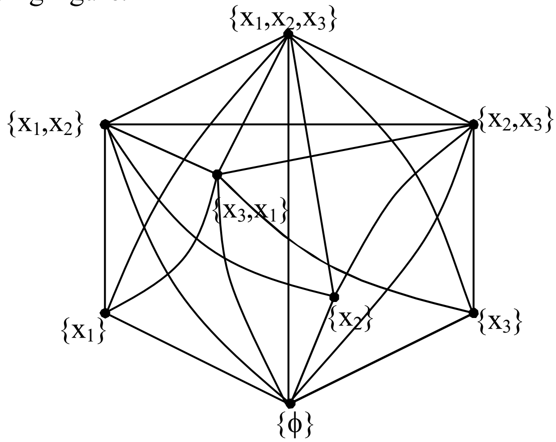


Figure 2.112

Clearly G_2 has 22 edges where as B_2 has only 12 edges and b_2 is Boolean algebra of order 8 and is not subset vertex graph of type I with 8 vertices.

So a Boolean algebra is never a subset vertex graph of type I for any set S.

Another interesting questions is if P = $\{\{x_1, x_2, x_4, x_3\}$ $\{x_1, x_3, x_2\}$, $\{x_1, x_2\}$, $\{x_1\}\}$ be four vertex set we now give the subset vertex graph of the set P in the following.

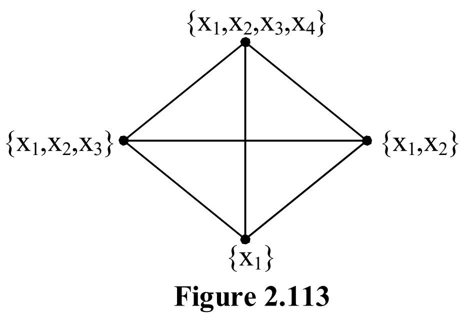


Figure 2.113

Clearly the subset vertex graph of type I is a complete graph.

Now we see the chain lattice associated with the set P is as follows.

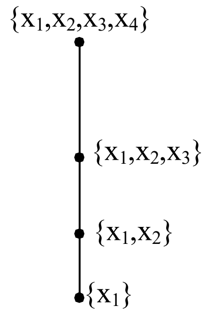


Figure 2.114

Clearly P gives a chain lattice as the set P is totally ordered.

$\{x\} \subseteq \{x_1, x_2\} \subseteq \{x_1, x_2, x_3\} \subseteq \{x_1, x_2, x_3, x_4\}$.

Thus even a totally ordered set in P(S) will give a complete subset vertex graph of type I.

So in view of this we have the following theorem.

Theorem 2.1. *Let P(S) be a power set of a finite set S. $J \subseteq P(S)$ be a subset collection of P(S) such that J is a totally ordered set under the set inclusion.*

i) J is a chain lattice.
ii) J is a subset vertex graph of type I which is complete.

Proof is direct and hence left as an exercise to the reader.

Definition 2.1. *Let $T \subseteq P(S)$ be the subset of a set on which the subset vertex graph of type I G is built. We call H a subgraph of G if H itself is a subset vertex graph of type I.*

We will describe this set up by some examples.

Theorem 2.2. *Let P(S) be the power set of a finite set S. If $N \subseteq P(S)$ is a collection of subsets of P(S) such that intersection of every pair of subsets in N is non empty then the vertex subset graph with vertices from N is a complete subset vertex graph of type I.*

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto define subgraphs of a subset vertex graph of type I.

Example 2.3. Let P(S) = {collection of all subsets from the set S = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$}. G be the subset vertex graph of type I with vertex subsets V = $\{\{x_1, x_2\}, \{x_1, x_2, x_3\}$ $\{x_3, x_4, x_5\}$, $\{x_4, x_3, x_5, x_6\}$, $\{x_6, x_2, x_3, \phi\}$ given by the following figure

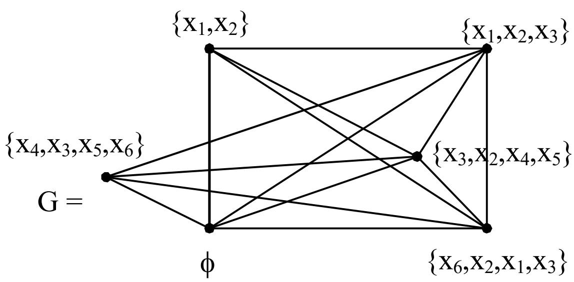


Figure 2.115

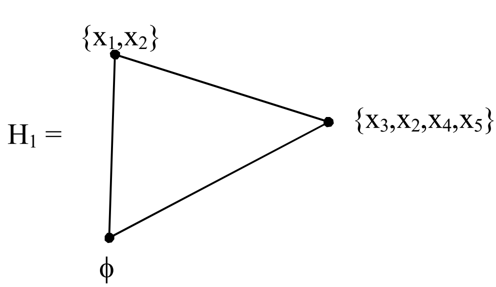


Figure 2.116

is a subset vertex subgraph of type I given by vertex subset $\{x_1, x_2\}$, ϕ, $\{x_3, x_2, x_4, x_5\}\}$. Clearly H_1 is a complete graph and it is a triad.

However G is not a complete subset vertex graph of type I.

Consider

H_2 =

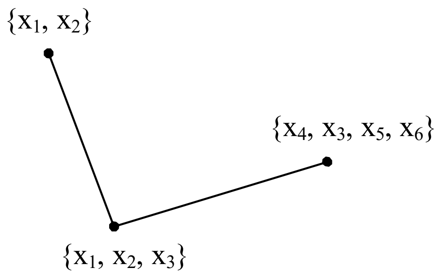


Figure 2.117

H_2 is a subset vertex subgraph of type I, H_2 is a forbidden triad. H_2 is not complete.

H_3 =

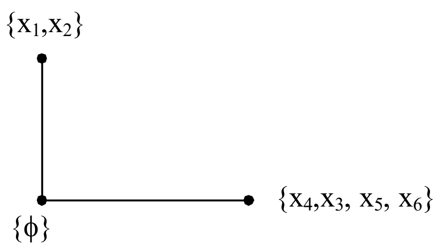


Figure 2.118

H_3 is also a subgraph of G which is again a forbidden triad.

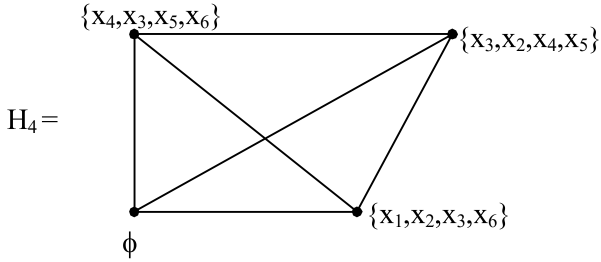


Figure 2.119

Clearly H_4 is a complete subset vertex subgraph of type I.

We have 5 complete subset vertex subgraphs with four vertices. Consider

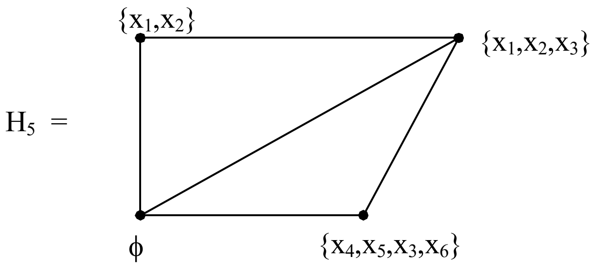


Figure 2.120

Clearly H_5 is subset vertex subgraph of type I with four vertices and is not complete.

We have two subset vertex subgraphs with five vertices which is complete is given in the following.

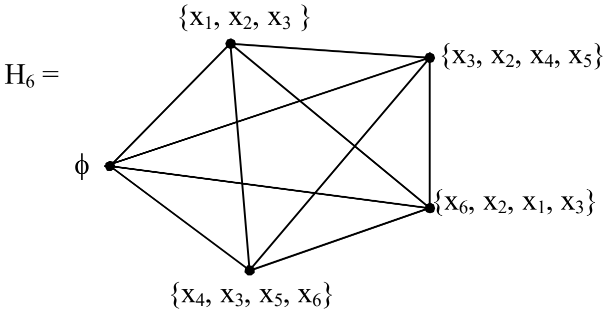


Figure 2.121

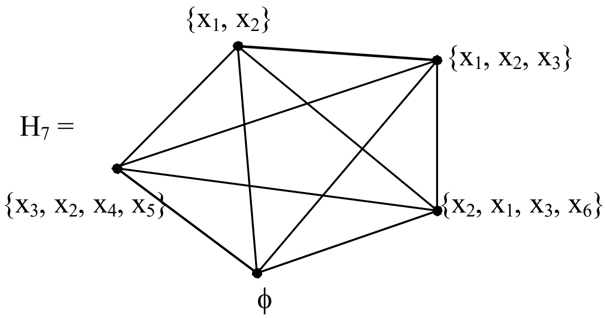


Figure 2.122

We see there are four subset vertex graphs of type I with five vertices which are not complete whenever the vertex subset $\{x_1, x_2\}$ is present with the vertex subset $\{x_4, x_5, x_3, x_6\}$ certainly the resulting subgraph is not complete.

Let us now consider the subset W = $\{\{x_1, x_2\}\ \{x_3\}, \{x_2, x_3\}, \{x_1\}, \{x_5\}, \{x_5, x_1\}, \{x_5, x_3\}, \{x_2, x_3, x_5, x_1\}, \{x_6, x_1\}\} \subseteq$ P(S)

The subset vertex graph of type I associated with W is as follows.

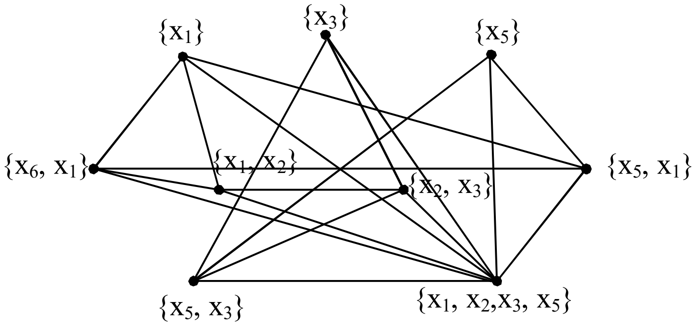


Figure 2.123

Thus K is a subset vertex subgraph of type I with 9 vertices.

Now we have the concept of subset vertex subgraphs of type I.

Next we find all the subset vertex subgraphs using P(S), (S = $\{x_1, x_2\}$) the power set of S

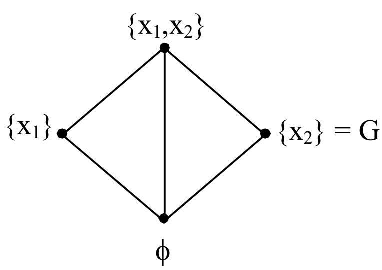
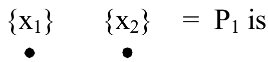


Figure 2.124

We find all the vertex subset subgraphs of G.

We do not member the trivial vertex subset subgraphs. Consider



The empty subset vertex subgraph of type I.

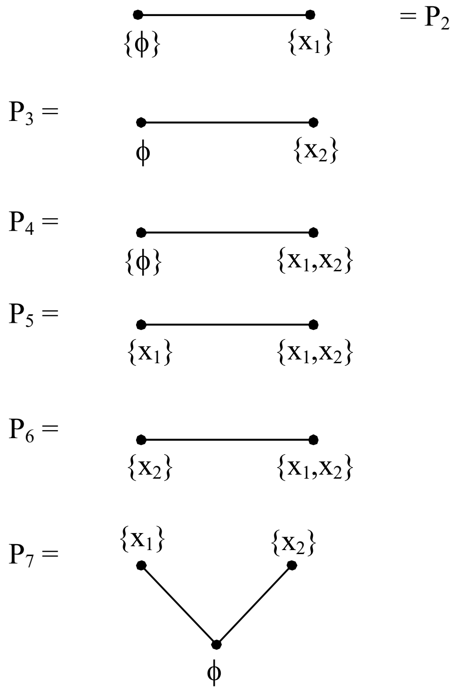


Figure 2.125

For bidden triad when we have 3 subsets vertices

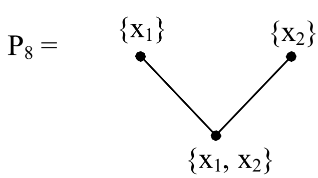


Figure 2.126

Once again this subset vertex subgraph of G is a forbidden triad.

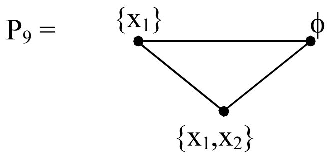


Figure 2.127

This subset vertex subgraph is a complete graph.

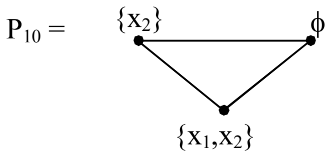


Figure 2.128

P_{10} is again a subset vertex subgraph which is complete.

Thus we have 10 subset vertex subgraphs of type I for G.

Consider the vertex subset graph H of P(S) with |P(S)| = 8 given by the following figure.

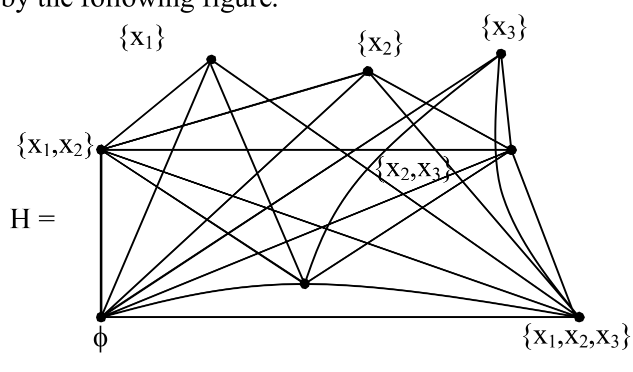
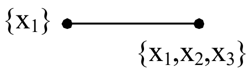


Figure 2.129

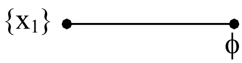
We list out the all subset vertex subgraphs of H which are nontrivial and empty.

$= S_1$

$= S_2$

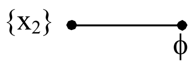


$= S_3$

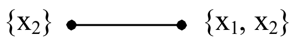


$= S_4$

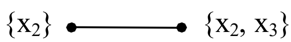
$S_5 =$



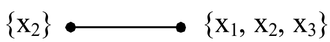
$S_6 =$



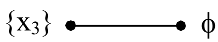
$S_7 =$



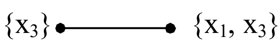
$S_8 =$



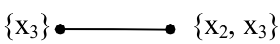
$S_9 =$



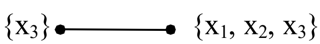
$S_{10} =$



$S_{11} =$



$S_{12} =$



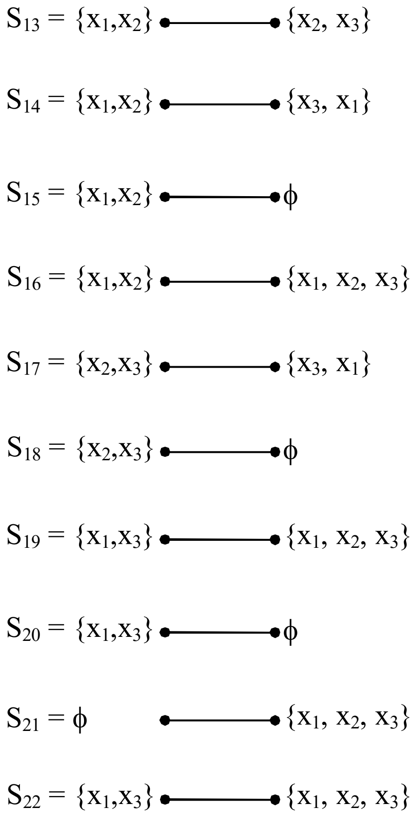
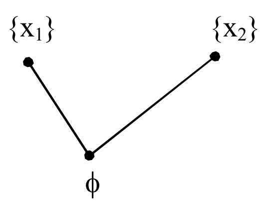


Figure 2.130

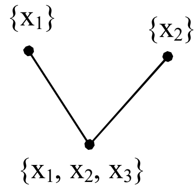
There are only 22 subset vertex subgraphs of type I which are non directional dyads.

We now list all vertex subset subgraphs with 3 subset vertices which are non empty.

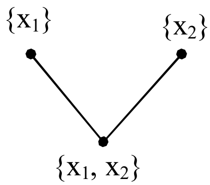
$L_1 =$



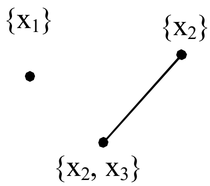
$L_2 =$



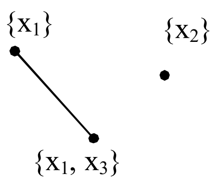
$L_3 =$



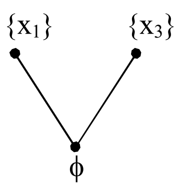
$L_4 =$



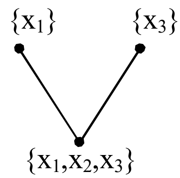
$L_5 =$



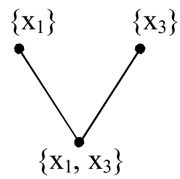
$L_6 =$



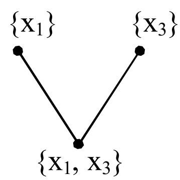
$L_7 =$



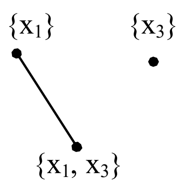
$L_8 =$



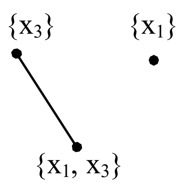
$L_8 =$



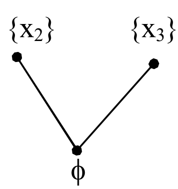
$L_9 =$



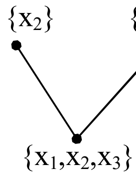
$L_{10} =$



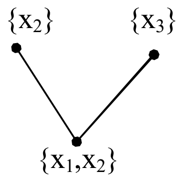
$L_{11} =$



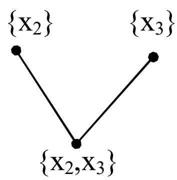
$L_{12} =$



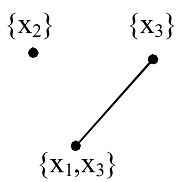
$L_{13} =$



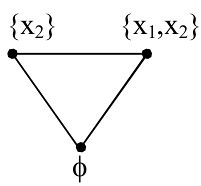
$L_{14} =$



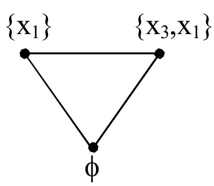
$L_{15} =$



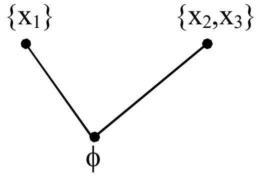
$L_{16} =$



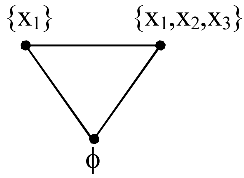
$L_{17} =$



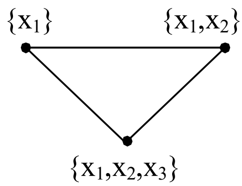
$L_{18} =$



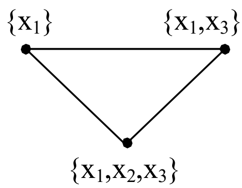
$L_{19} =$



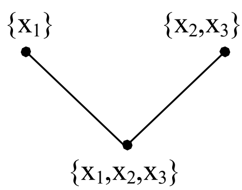
$L_{20} =$



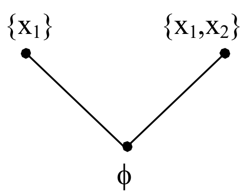
$L_{21} =$



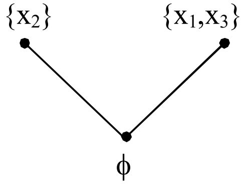
$L_{22} =$



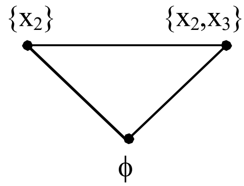
$L_{23} =$



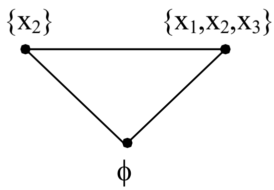
$L_{24} =$



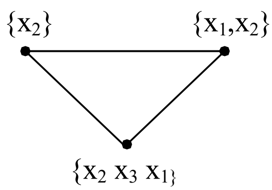
$L_{25} =$



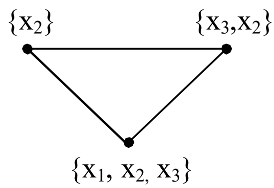
$L_{26} =$



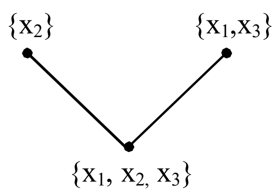
$L_{27} =$

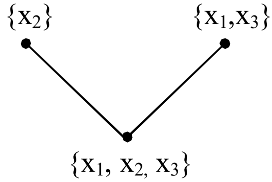


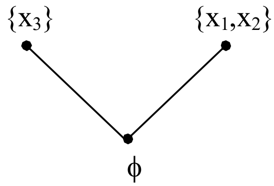
$L_{28} =$

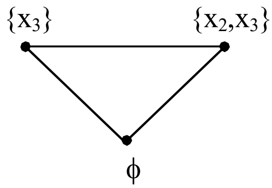


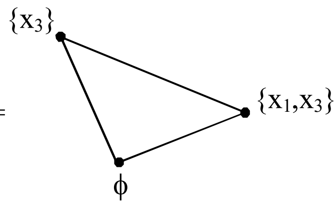
$L_{29} =$

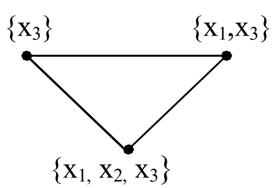


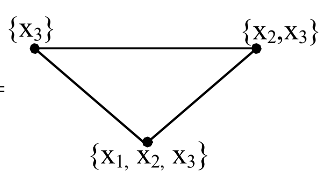
$L_{29} =$ 

$L_{30} =$ 

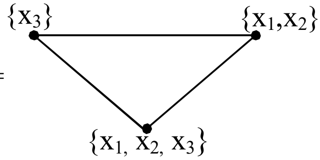
$L_{31} =$ 

$L_{32} =$ 

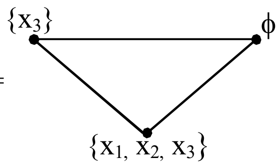
$L_{33} =$ 

$L_{34} =$ 

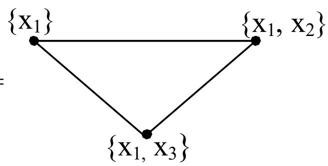
$L_{35} =$



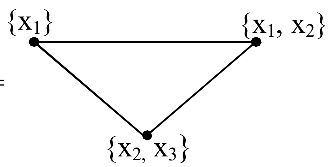
$L_{36} =$



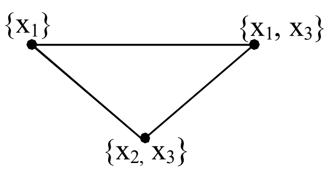
$L_{37} =$



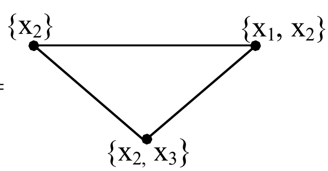
$L_{38} =$



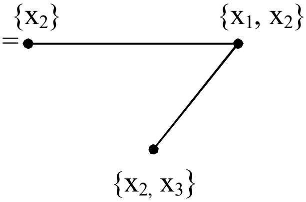
$L_{39} =$



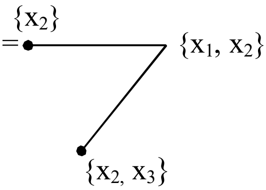
$L_{40} =$



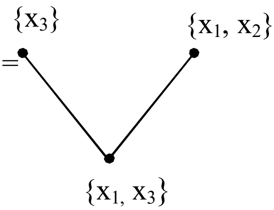
$L_{41} =$



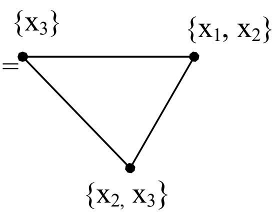
$L_{42} =$



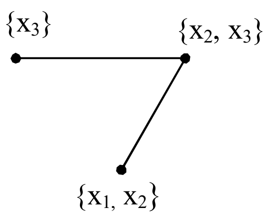
$L_{43} =$



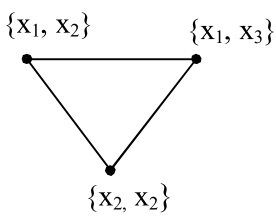
$L_{44} =$



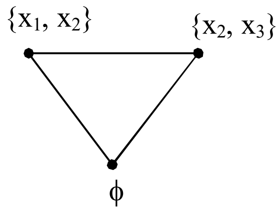
$L_{45} =$



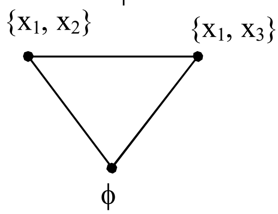
$L_{46} =$



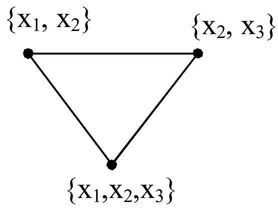
L_{47}=



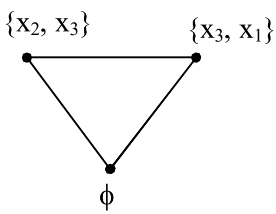
L_{48}=



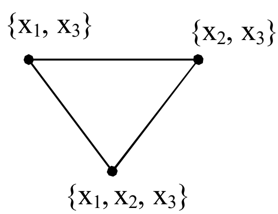
L_{49}=



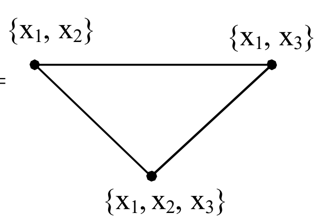
L_{50}=



L_{51}=



L_{52}=



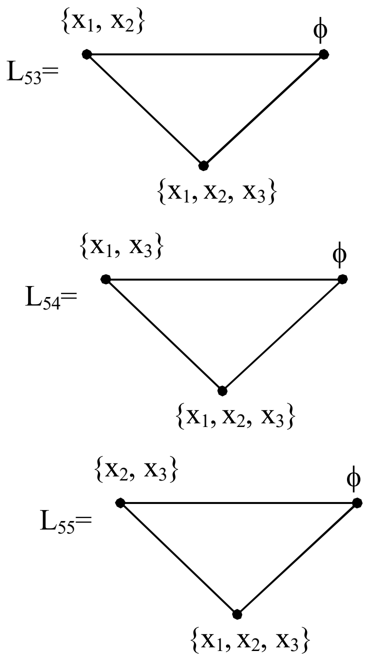


Figure 2.131

Here we have found several cliques of the given graph for they happen to be one of the challenging problems in social networks.

Next we find all subset vertex subgraphs of P(S) with four vertex subsets.

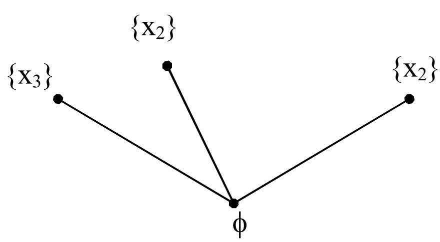


Figure 2.132

This is a rooted tree with one layer is a subset vertex subgraph of type I. Interested reader can find all subgraphs of H for this can be compared with usual graph and subset vertex subgraphs of type II which will be defined in chapter III of this book. We see even if

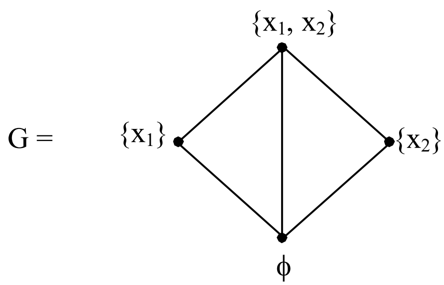


Figure 2.133

is a subset vertex graph of type I and usual classical graph with 4 vertices.

Infact G_0 has more classical subgraphs than the subset vertex graph of type I given by G.

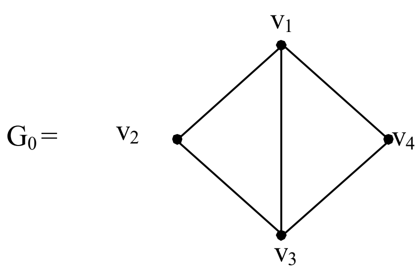


Figure 2.134

However we do not have loops in this situation for we do not use the fact if $P \in P(S)$; $P \cap P = P$.

As this condition is not imposed on subset vertex graphs of type I there are no loops. Clearly these subset vertex graphs are not directed.

We describe the matrix associated with the subset vertex graphs of type I by some examples.

Example 2.4. Let S ={x_1, x_2, x_3, x_4, x_5} be the given set. P(S) be the power set of S.

We describe a few subset vertex graphs of type I and their associated matrices (connection matrices).

Clearly o(P(S)) = 2^5 = 32.

Let V = {v_1 = {x_1, x_3, x_2}, v_2 = {x_4, x_5}, v_3 = {x_1, x_2, x_5, x_4}, v_4 = {x_1}, v_5 = {x_2, x_5, x_1}, v_6 = {x_2, x_3, x_5}, v_7 = {x_3, x_5, x_4}} be the vertex subset contained in P(S). Number nodes in V is 7.

Now we describe in the following the subset vertex graph of type I G, with these seven nodes in the following.

G =

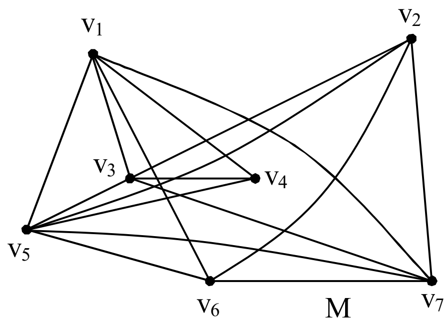


Figure 2.135

The connection matrix associated with G is

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

We find $M \times M = M^2$

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 5 & 4 & 4 & 2 & 4 & 3 & 3 \\ 4 & 4 & 3 & 2 & 3 & 3 & 3 \\ 4 & 3 & 6 & 2 & 5 & 4 & 4 \\ 2 & 2 & 2 & 3 & 2 & 3 & 3 \\ 4 & 3 & 5 & 2 & 6 & 4 & 4 \\ 3 & 3 & 4 & 3 & 4 & 5 & 4 \\ 3 & 3 & 4 & 3 & 4 & 4 & 5 \end{bmatrix} \end{array}.$$

We are sure if we proceed on to find M^3, M^4 and so on the entries would become only very large.

It is further pertinent to note that none of these powers of M ever represent any sort of vertex subset graph of type I. Any sort of thresholding will not be useful for it will not yield any relation.

We put forth some problems for the reader.

Problem 2.2. Let P(S) be a power set of the set S. Let V $\subseteq$ P(S) be the subset of P(S). Does there exist a V in P(S) such that it adjacency matrix yield again a vertex subset graph of type I?

Now we continue with the same problem.

We consider W = $\{v_1 = \{x_1\}, \{x_2\} = v_2, v_3 = \{x_3\}, v_4 = \{x_4, x_5\}, v_5 = \{x_1, x_2, x_4\}\} \subseteq$ P(S) and find the related subset vertex graph of type I.

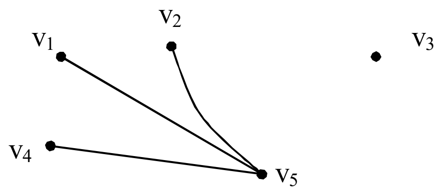


Figure 2.136

Next we proceed onto give examples of start subset vertex graph in the following.

Let P(S) = {Power set of S = $\{x_1, x_2, \ldots, x_{10}\}$}. Consider the collection of subsets of $T_1 \subseteq$ P(S) where = $\{\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_4, x_5\}, \{x_5, x_6\}$m $\{x_6, x_7\}, \{x_7, x_8\}\}$.

The subset vertex graph associated with the vertex set T_1 is a line vertex subset graph given by the following figure.

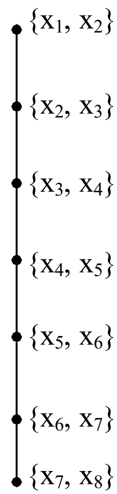


Figure 2.137

Clearly G_1 is a subset vertex line graph of type I.

Now we consider the $T_2 = \{\{x, x_2,\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_4, x_5\}, \{x_5, x_6\}, \{x_6, x_7\}, \{x_7, x_8\}, \{x_8, x_9\}, \{x_9, x_1\}\} \subseteq P(S)$.

Now the subset vertex graph G_2 of type I associated with the vertex set T_2 is as follows.

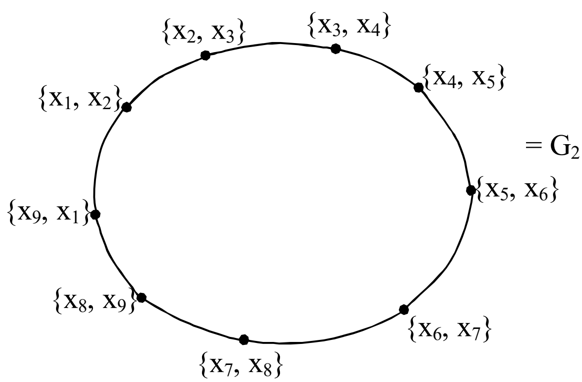


Figure 2.138

Clearly G_2 is a subset vertex circle graph of type I.

This has nodes.

Consider the set $T_3 = \{\{x_1\}, \{x_2\}, \ldots, \{x_{10}\}, \{x_1, x_2, x_3, x_4, \ldots, x_{10}\}\} \subseteq P(S)$ be the subset of P(S).

Now we give the subset vertex graph G_3 of type I associated with the vertex set T_3.

$G_3 =$

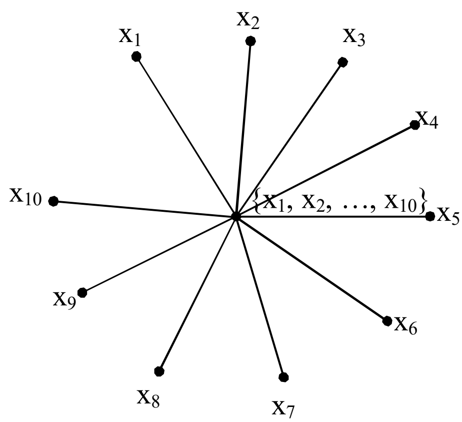


Figure 2.139

Clearly G_3 is a subset vertex star graph of type I, with 11 nodes.

In view of this we have the following interesting result.

Theorem 2.3 *Let P(S) = Power set of S where* $S = \{x_1, x_2, \ldots, x_n\}$. $|P(S)| = 2^n$. *There is a subset vertex star graph of type I with n + 1 vertices with vertex set from P(S).*

Proof. Let $B = \{\{x_1\}, \{x_2\}, \ldots, \{x_n\}, \{x_1, x_2, \ldots, x_n\}\} \subseteq P(S)$. Now the subset vertex graph with vertex set as B is a star subset vertex graph of type I with n + 1 nodes / vertices.

Clearly |B| = n + 1 and the subset vertex stat graph of type I is as follows.

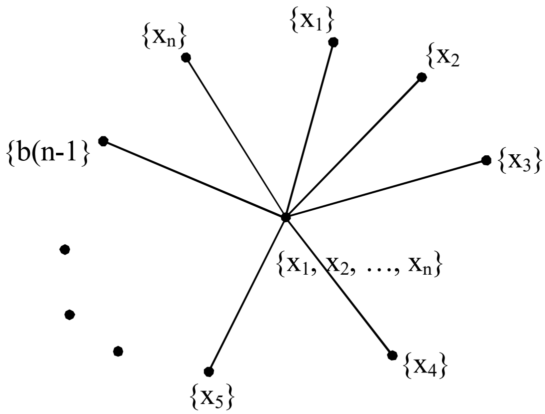


Figure 2.140

Hence the result.

There is also a star graph with $\frac{n}{2}$ + 1 nodes if n is even and $\left(\frac{n}{2}+1\right)$ subset vertex star graph of type I given by the graphs B_1 and B_2 respectively.

B_1 =

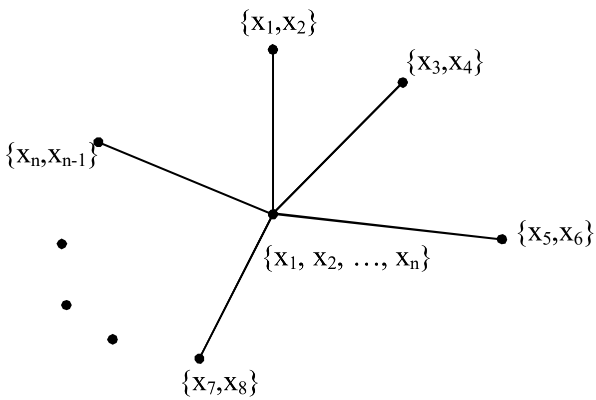


Figure 2.141

If n is even.

If n is odd then B_2 the vertex subset star graph of type I given in the following.

B_2 =

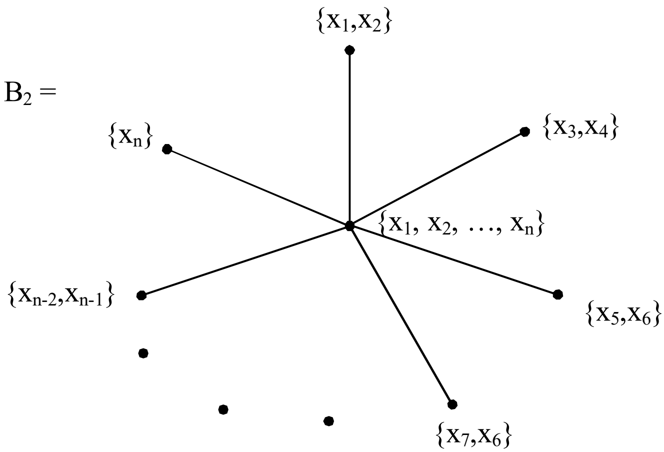


Figure 2.142

B_2 has $\left(\frac{n+1}{2}+1\right)$ vertices and B_1 has $\frac{n}{2}+1$ vertices.

We will illustrate this situation by some examples.

Let S = $\{x_1, x_2, \ldots, x_3\}$ be the given set and P(S) the power set of S.

$A_1 = \{\{x_1, x_2\}, \{x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}, \{x_9, x_{10}\}, \{x_{11}, x_{12}\}, \{x_3\}, \{x_1, x_2, \ldots, x_{13}\}\} \subseteq P(S)$.

The subset vertex graph of type with vertex set A_1 is as follows.

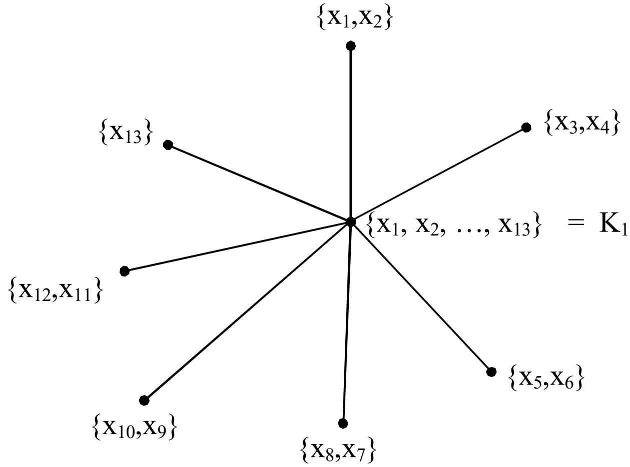


Figure 2.143

Clearly K_1 has 8 vertices nodes.

Consider $A_2 = \{\{x_1, x_2, x_3\}\ \{x_4, x_5, x_6\}, \{x_7, x_8, x_9\}, \{x_{10}, x_{11}, x_{12}\}, x_{13}, \{x_1\ x_2, \ldots, x_{13}\} \subseteq P(S)$ be a vertex subset of P(S). The subset vertex graph K_2 associated with vertex set A_2 is as follows.

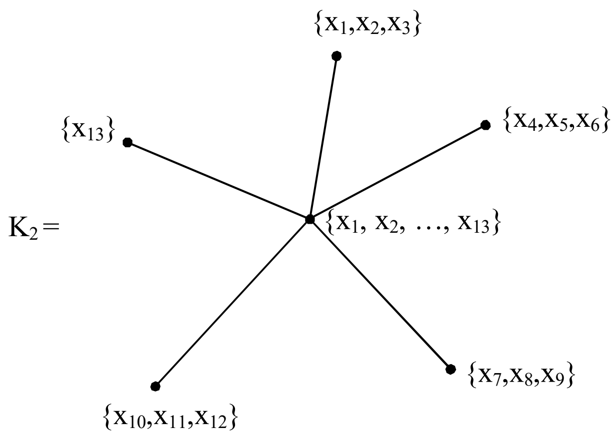


Figure 2.144

Clearly K_2 is a subset vertex star graph of type I with 6 nodes / vertices.

Consider

$$A_3 = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6, x_7, x_8\}, \{x_9, x_{10}, x_{11}, x_{12}\}, \{x_{13}\}, \{x_1, x_2, \ldots, x_{13}\}\} \subseteq P(S).$$

The subset vertex graph K_3 with A_3 set as vertices is as follows.

$K_3 =$

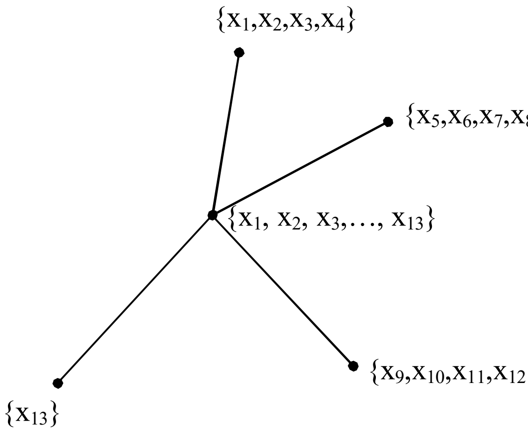


Figure 2.145

Clearly K_3 is a subset vertex star graph with 5 vertices.

Consider $A_4 = \{\{x_1\}, \{x_2, x_3\}, \{x_9, x_5, x_6\}, \{x_7, x_8, x_9, x_{10}\}, \{x_{11}\}, \{x_{12}, x_{13}\}, \{x_1, x_2, \ldots, x_{13}\}\}$ be the subset in P(S).

The subset vertex graph K_4 associated with the vertex set A_4 is as follows.

$K_4 =$

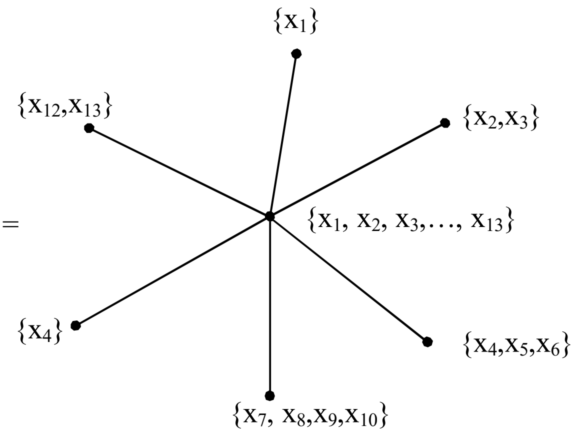


Figure 2.146

We see K_4 is a subset vertex graph of type I which is a star graph with seven vertices.

The reader is left with the task of finding all subset vertex star graphs of type I with seven nodes.

Consider $A_5 = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5, x_6, x_7\}, \{x_8, x_9, x_{10}, x_{11}\}, \{x_{12}, x_{13}\}, \{x_1, x_2, x_3, \ldots, x_{13}\}\} \subseteq P(S)$ be a subset of P(S). The subset vertex graph K_5 associated with A_5 is as follows.

$K_5 =$

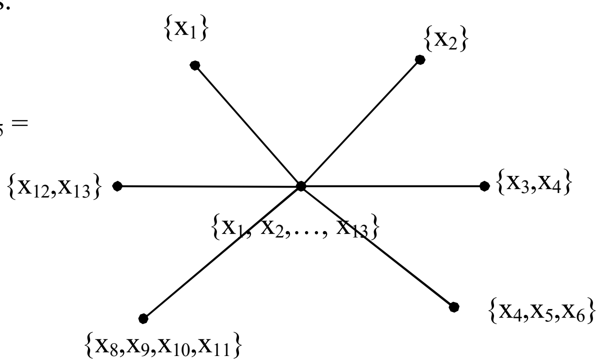


Figure 2.147

Clearly K_5 as again a subset vertex star graph of type I with seven vertices.

Consider

$$A_6 = \{\{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}, \{x_{11}, x_{13}, x_{12}\}, \{x_1, x_2, \ldots, x_{13}\}\} \subseteq P(S)$$

be the subset of P(S).

The vertex subset graph K_6 associated with A_6 is as follows.

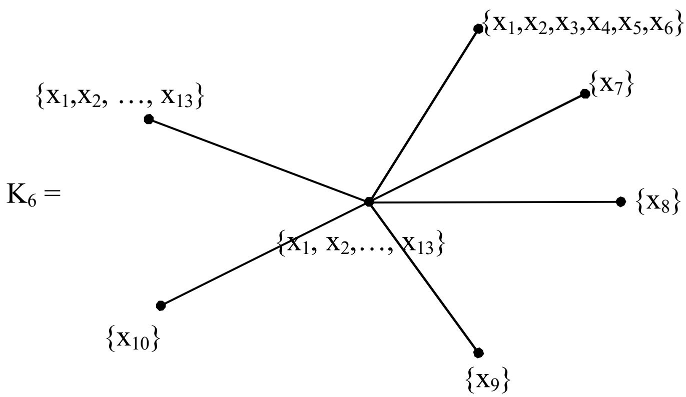


Figure 2.148

Let

$$A_7 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}, x_{11}, x_{12}\}, \{x_{13}\}, \{x_1, x_2, x_3, \ldots, x_{13}\}\} \subseteq P(S)$$

be the subset of type I using A_7 as the vertex set is as follows.

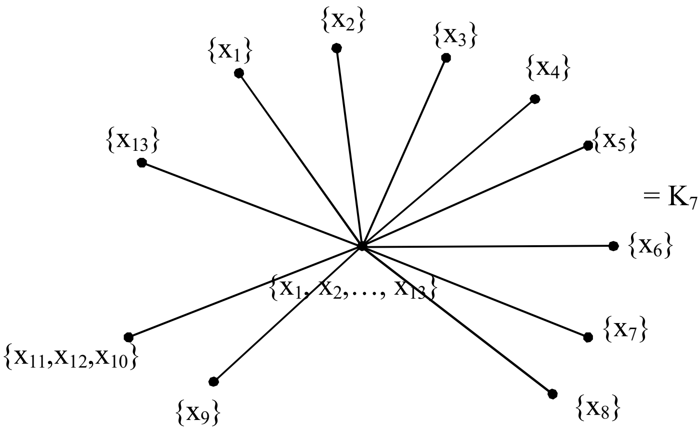


Figure 2.149

K_7 is a vertex subset star graph of type I with 12 vertices / nodes.

Let $A_8 = \{\{x_1, x_2, x_3, x_4\}, \{x_5, x_6, x_7, x_8, x_9\}, \{x_{10}, x_{11}, x_{12}\}, \{x_{13}\}, \{x_1, x_2, \ldots, x_{13}\}\} \subseteq P(S)$ be the subset of P(S0.

The subset vertex graph K_8 with A_8 as its vertex set is as follows.

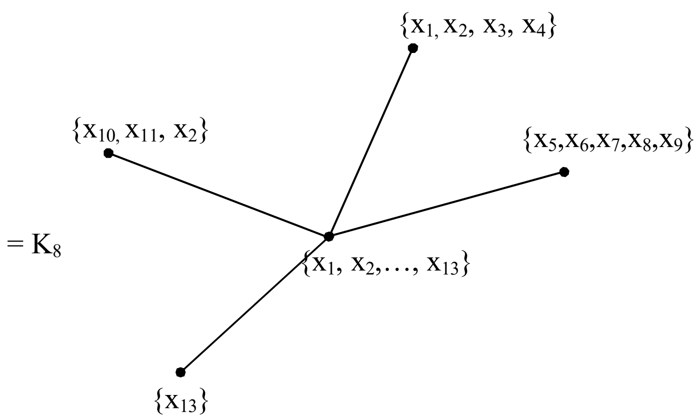


Figure 2.150

K_8 is a vertex subset star graph with 5 nodes.

We have dealt elaborately for in social networks we have nodes which are very powerful (or powerful nodes) and infact. They determine the total network.

It is left as an open problem /conjecture.

Conjecture 2.1. Let S = $\{x_1, x_2 \ldots, x_n\}$ be the set of n elements P(S) be the power set of S.

Find all subset vertex star graphs of type I with vertex set from P(S).

Note in case S = $\{x_1, x_2\}$ we have P(S) = $\{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ be the power set of S. The star subset vertex graphs of type I using the vertex set from P(S) are as follows.

G_1 =

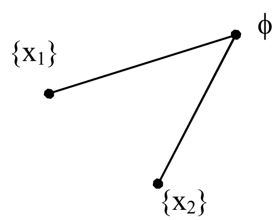


Figure 2.151

G_2 =

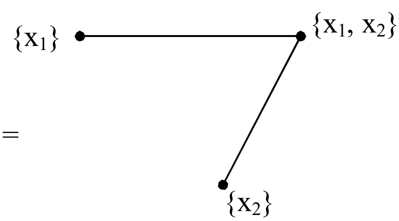
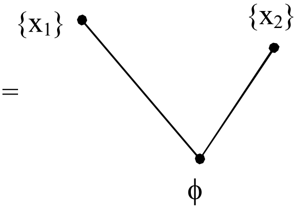


Figure 2.152

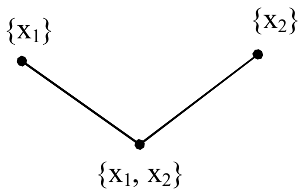
We find when S = $\{x_1, x_2, x_3\}$ and P(S) = $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2], \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$ be the power set of S.

The state subset vertex graph of type I or subset vertex start graphs of type I are as follows.

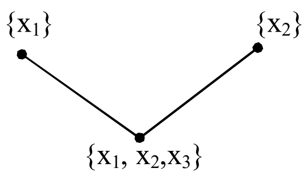
P_1 =



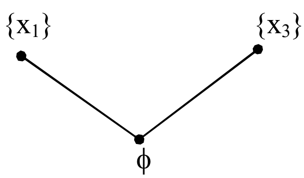
P_2 =



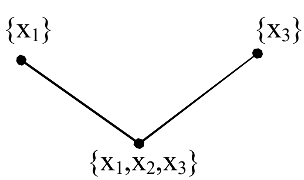
P_3 =



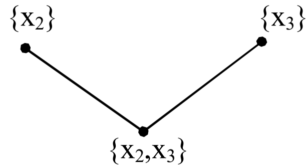
P_4 =



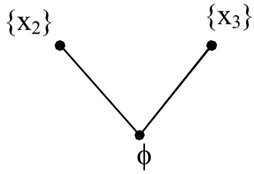
P_5 =



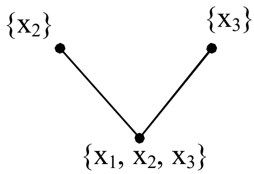
$P_6 =$



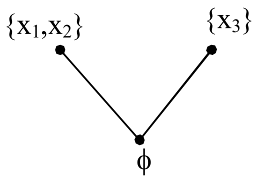
$P_7 =$



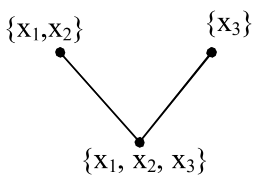
$P_8 =$



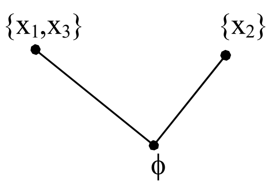
$P_9 =$



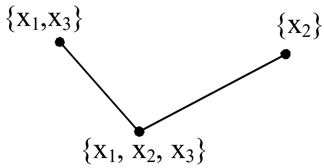
$P_{10} =$



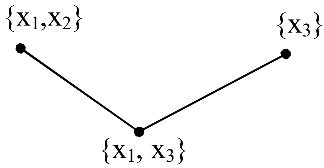
$P_{11} =$



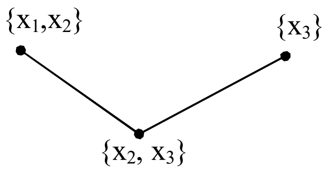
$P_{12} =$



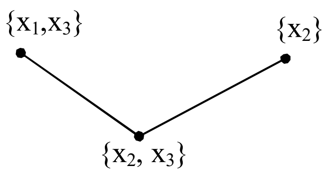
$P_{13} =$



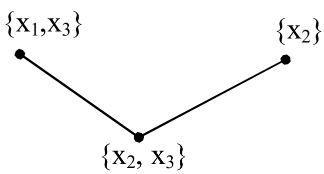
$P_{14} =$



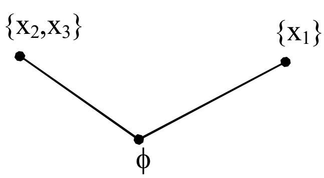
$P_{15} =$



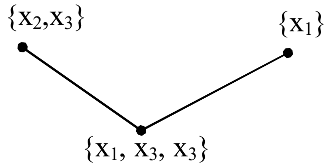
$P_{16} =$



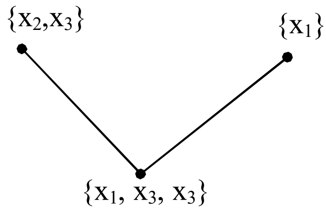
$P_{17} =$



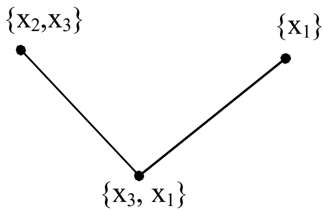
P_{18} =



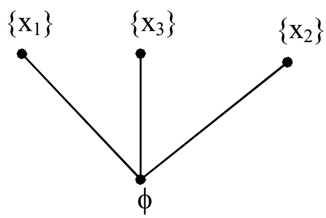
P_{19} =



P_{20} =



P_{21} =



P_{22} =

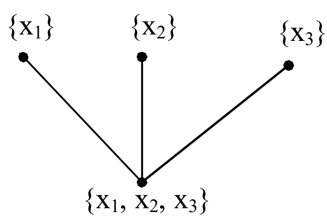


Figure 2.153

There are 22 subset vertex star graphs of type I.

Thus for S = $\{x_1, x_2\}$ we have only two such subset vertex star graphs of type I.

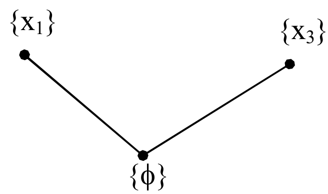
Now for S = $\{x_1, x_2, x_3\}$ we the only 22 such subset vertex star graphs of type I.

S = $\{x_1, x_2, x_3, x_4\}$ be the set.

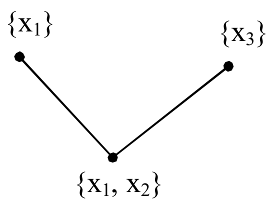
Now P(S) = $\{\{x_1\}, \{x_2\}, \{x_4\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\} \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}, \phi\}$.

The vertex subset star graphs of type I associated with P(S); where |P(S)| = 16 is as follows.

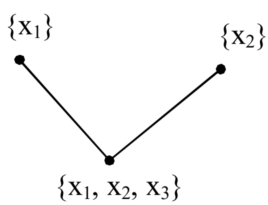
T_1 =



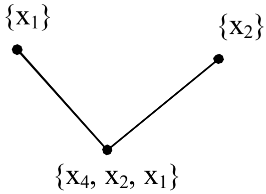
T_2 =



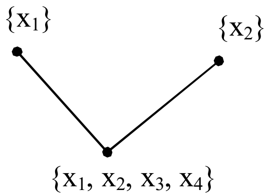
T_3 =



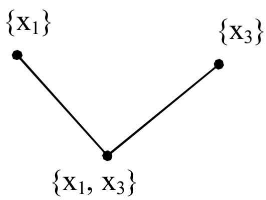
$T_4 =$



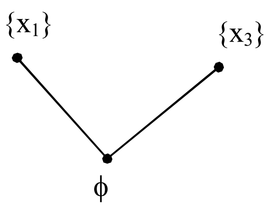
$T_5 =$



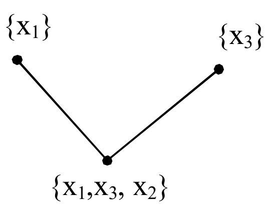
$T_6 =$



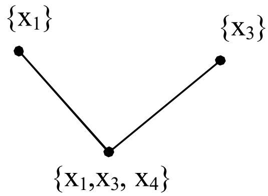
$T_7 =$



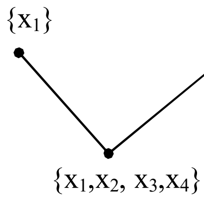
$T_8 =$



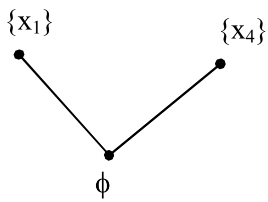
$T_9 =$



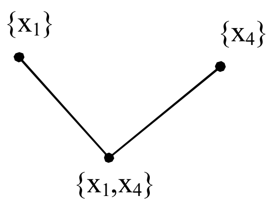
$T_{10} =$



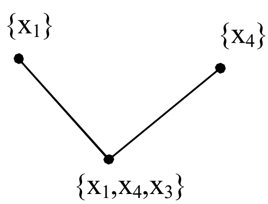
$T_{11} =$



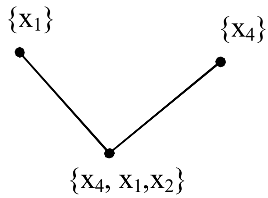
$T_{12} =$



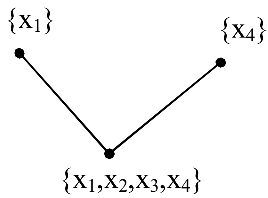
$T_{13} =$



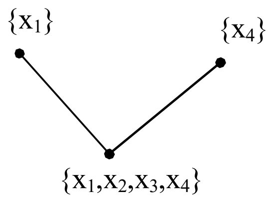
$T_{14} =$



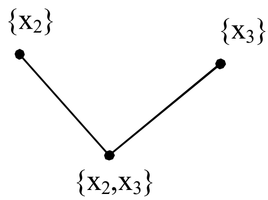
$T_{15} =$



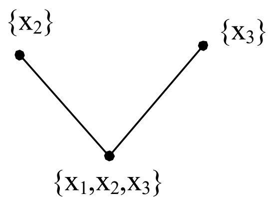
$T_{16} =$



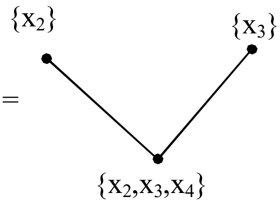
$T_{17} =$



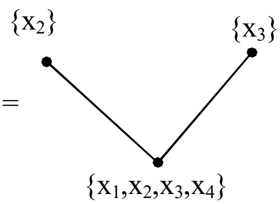
$T_{18} =$



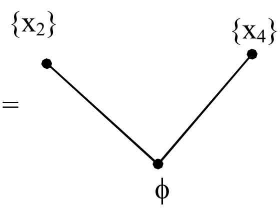
$T_{19} =$



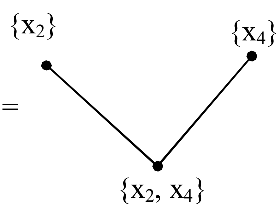
$T_{20} =$



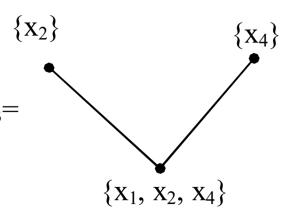
$T_{21} =$



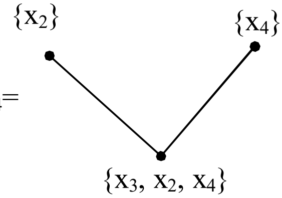
$T_{22} =$



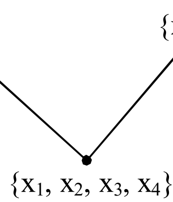
$T_{23} =$



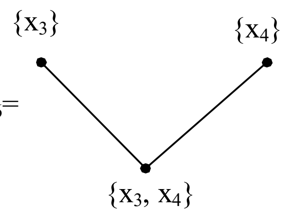
$T_{24}=$



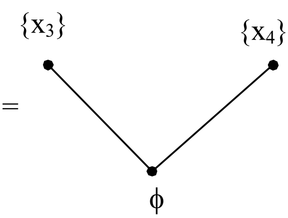
$T_{25}=$



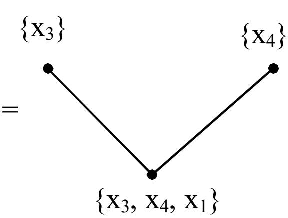
$T_{26}=$



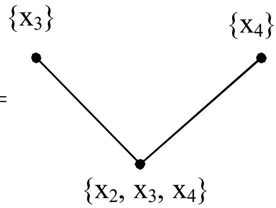
$T_{27}=$



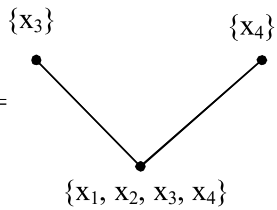
$T_{28}=$



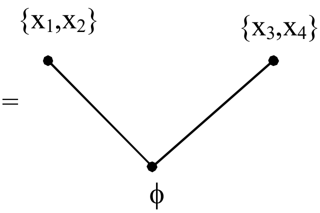
$T_{29} =$



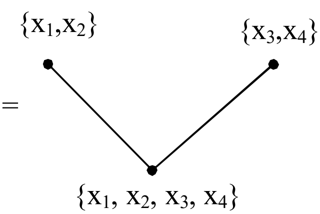
$T_{30} =$



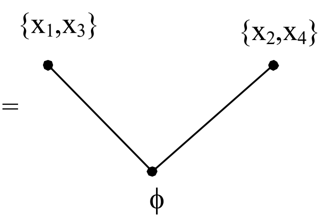
$T_{31} =$



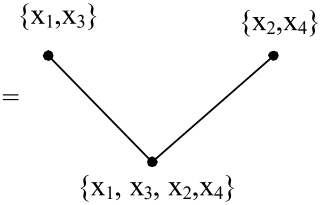
$T_{32} =$



$T_{33} =$



$T_{34} =$



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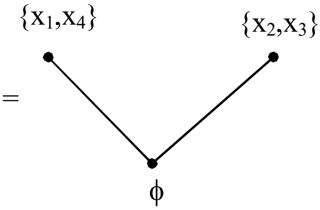

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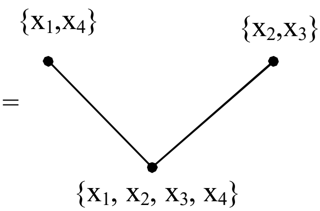
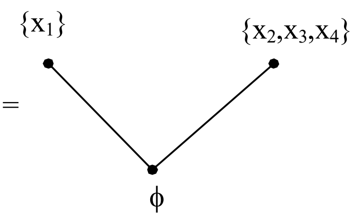
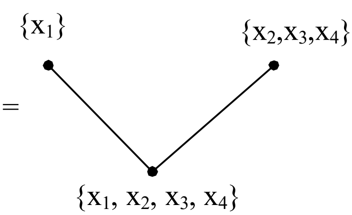


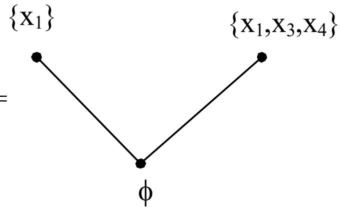
$T_{37} =$



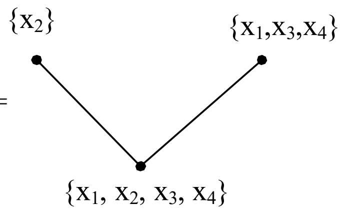
$T_{38} =$



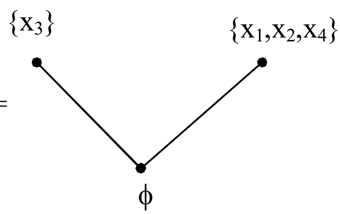
$T_{39} =$



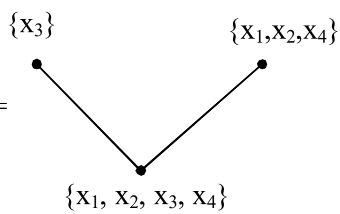
$T_{40} =$



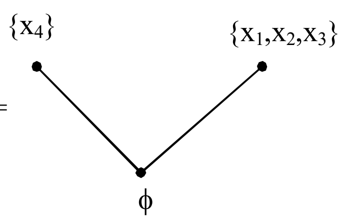
$T_{41} =$



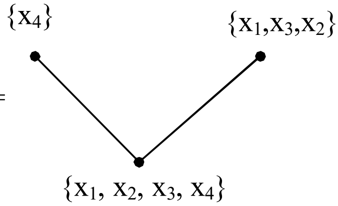
$T_{42} =$



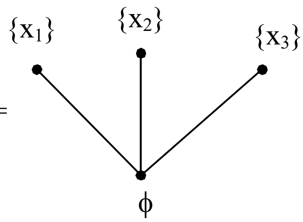
$T_{43} =$



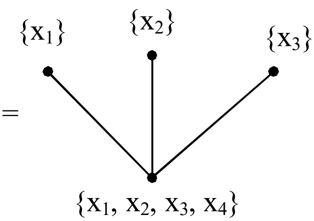
$T_{44} =$



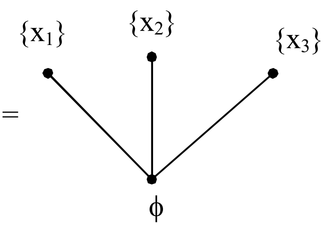
$T_{45} =$



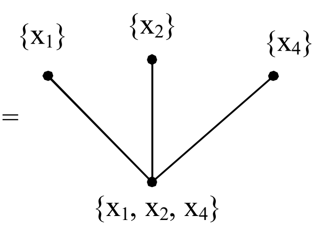
$T_{46} =$



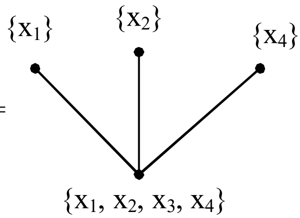
$T_{47} =$



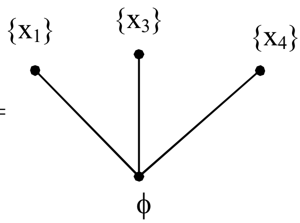
$T_{48} =$



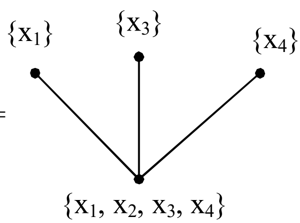
$T_{49} =$



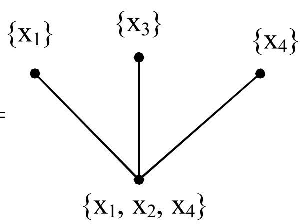
$T_{50} =$



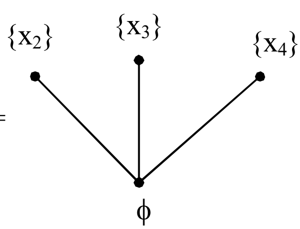
$T_{51} =$



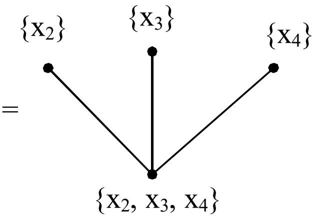
$T_{53} =$



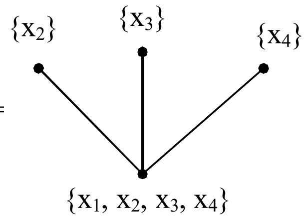
$T_{54} =$



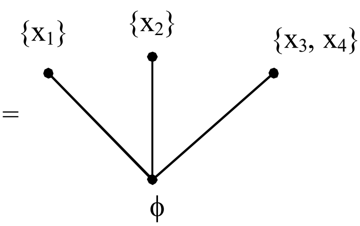
$T_{55} =$



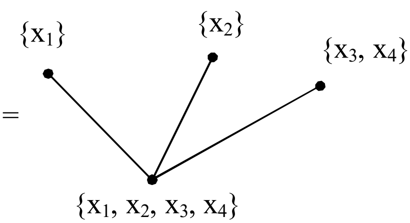
$T_{56} =$



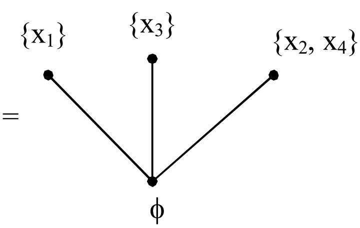
$T_{57} =$



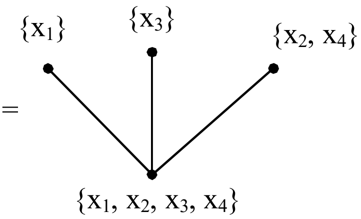
$T_{58} =$



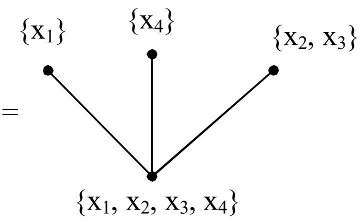
$T_{59} =$



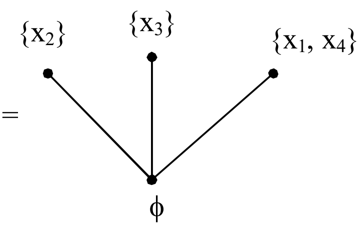
T_{60} =



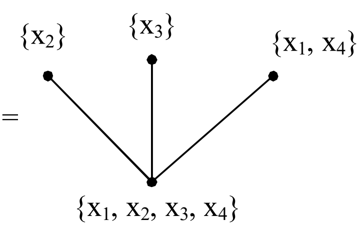
T_{62} =



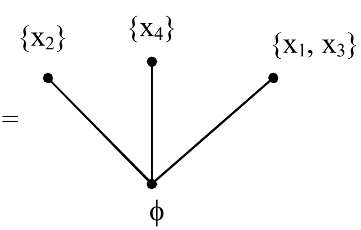
T_{63} =



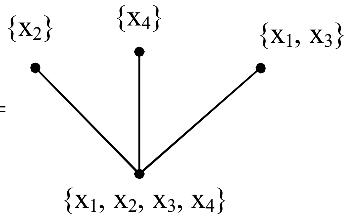
T_{64} =



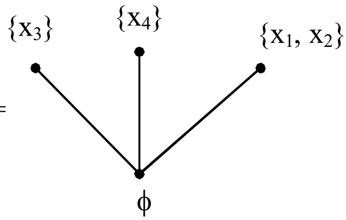
T_{65} =



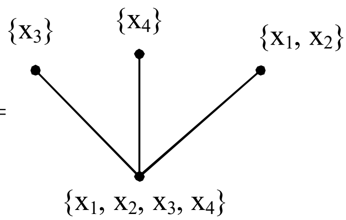
$T_{66} =$



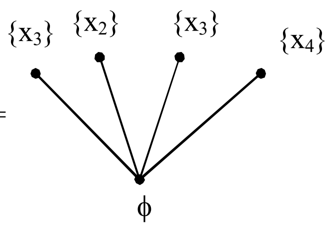
$T_{67} =$



$T_{68} =$



$T_{69} =$



$T_{70} =$

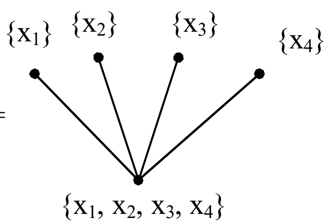


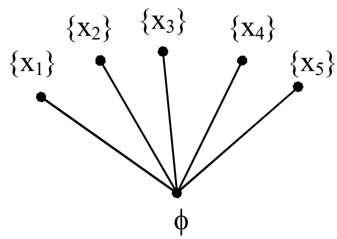
Figure 2.154

There are several subset vertex star graphs of type I built using P(S) where S = $\{x_1, x_2, x_3, x_4\}$.

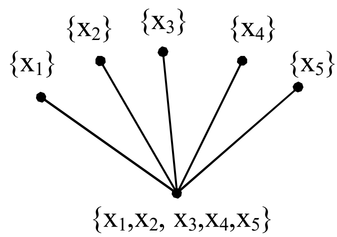
Let S = $\{x_1, x_2, x_3, x_4, x_5\}$ be the set with 5 nodes.

P(S) be the power set of S. The subset vertex star graphs associated with vertex set from P(S) are as follows.

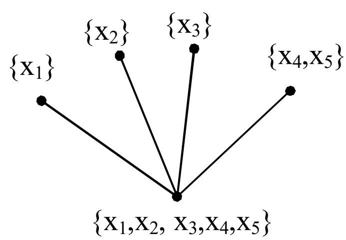
S_1 =



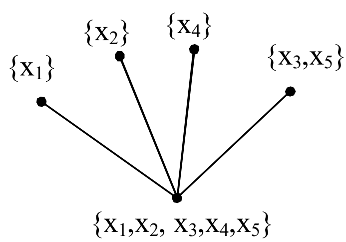
S_2 =



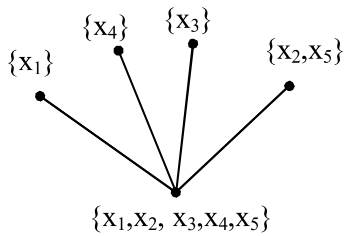
S_3 =



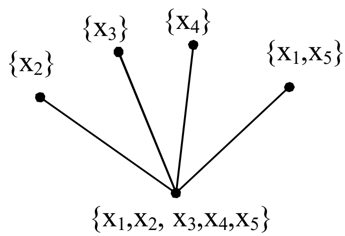
S_4 =



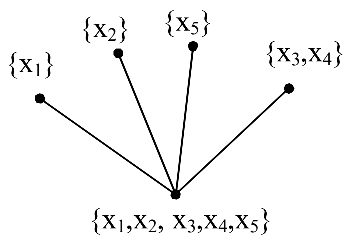
$S_5 =$


{x1}
{x4}
{x3}
{x2,x5}
{x1,x2, x3,x4,x5}

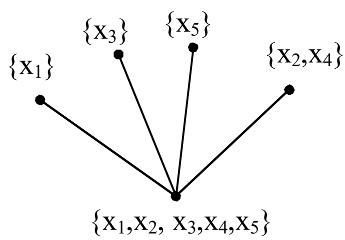
$S_6 =$


{x2}
{x3}
{x4}
{x1,x5}
{x1,x2, x3,x4,x5}

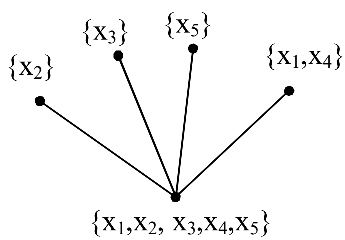
$S_7 =$


{x1}
{x2}
{x5}
{x3,x4}
{x1,x2, x3,x4,x5}

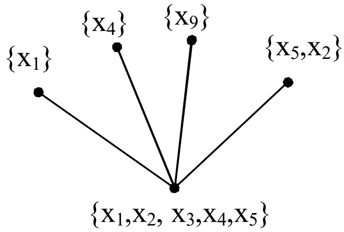
$S_8 =$


{x1}
{x3}
{x5}
{x2,x4}
{x1,x2, x3,x4,x5}

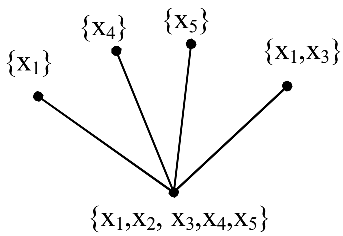
$S_9 =$


{x2}
{x3}
{x5}
{x1,x4}
{x1,x2, x3,x4,x5}

S_{10} =



S_{11} =



S_{12} =

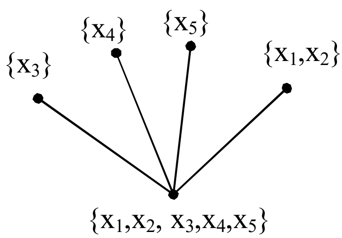
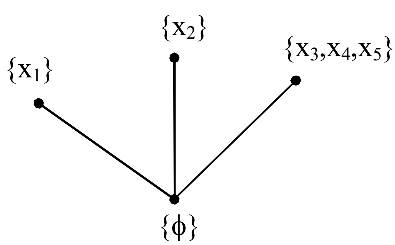


Figure 2.155

S_{13} to S_{22} will be those subset vertex star graphs with 5 nodes in which $\{x_1, x_2, x_3, x_4, x_5\}$ will be replaced in graphs S_3 to S_{12} by the set ϕ.

S_{13} =



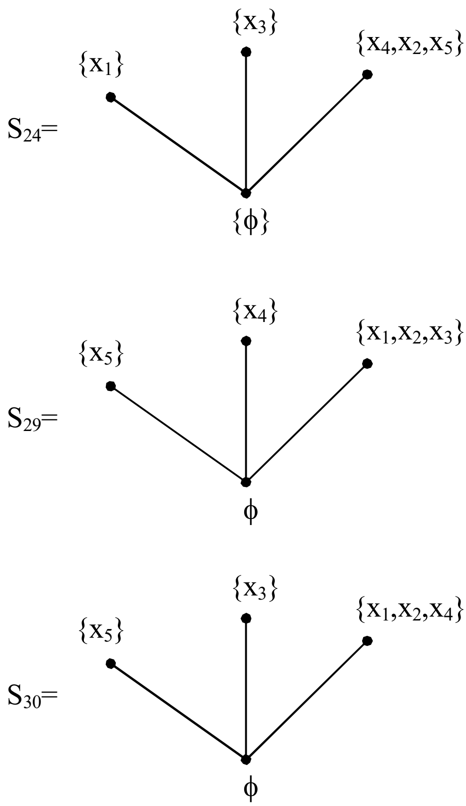


Figure 2.156

and so on.

It is left for the reader to find the number of subset vertex star graphs of type I associated with P(S) where S = $\{x_1, x_2, x_3, x_4, x_5\}$. In case $|S| = 2$ and $|P(S)| = 4 = 2^2$ we have only two subset vertex star graphs of type I.

When $|S| = 3$ that is $|P(S)| = 8 = 2^3$ we have 22 subset vertex star graphs of type I when $|S| = 4$ that $|P(S)| = 16$ we have 70 such vertex subset star graphs of type I.

Further we have for any set $S = \{x_1, \ldots, x_n\}$ there are only two subset vertex star graphs of type I of with $(n + 1)$ nodes which only subset vertex star graphs with largest number of nodes.

We have found the least number of nodes required to form a subset vertex star graphs of type I is three.

There are only two subset vertex star graphs in case $S = \{x_1, x_2\}$ when $S = \{x_1, x_2, x_3\}$ we have there are 20 subset vertex star graphs which 3 nodes. We call subset vertex star graphs with 3 nodes as smallest subset vertex star graphs of type I.

Further in case $S = \{x_1, x_2, x_3, x_4\}$; we have 44 subset vertex star graphs with 3 nodes that is there are 44 subset vertex star graphs which are smallest subset vertex star graphs of P(S) when $S = \{x_1, x_2, x_3, x_4\}$.

It is left as an open problem for the researchers to find the number of smallest subset vertex star graphs of type I (that is star graphs with 3 nodes) from the set P(S) where $S = \{x_1, x_2, \ldots, x_n\}$; $n \geq 5$.

Now we study the line graph built using P(S).

We will first illustrate this situation by some examples.

Example 2.5. Let $S = \{x_1, x_2\}$ be the set P(S) the power set of S.

The subset vertex line graph of type I of P(S) are as follows.

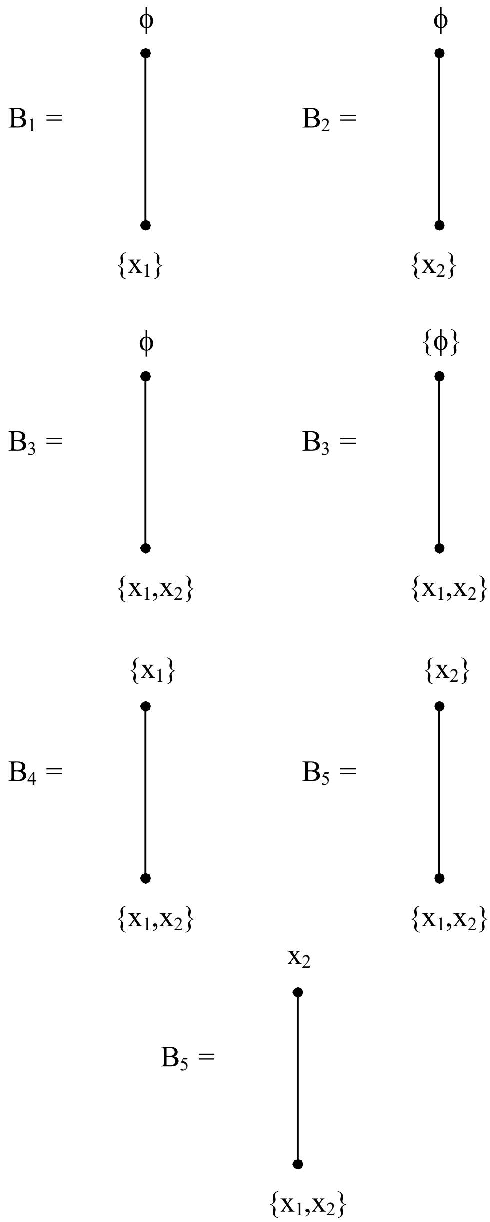

$B_1 =$
ϕ
$\{x_1\}$
$B_2 =$
ϕ
$\{x_2\}$
$B_3 =$
ϕ
$\{x_1,x_2\}$
$B_3 =$
$\{\phi\}$
$\{x_1,x_2\}$
$B_4 =$
$\{x_1\}$
$\{x_1,x_2\}$
$B_5 =$
$\{x_2\}$
$\{x_1,x_2\}$
$B_5 =$
x_2
$\{x_1,x_2\}$

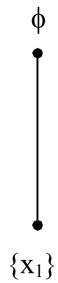
Figure 2.154

There are only 5 subset vertex line graphs of type I using P(S) where S = $\{x_1, x_2\}$.

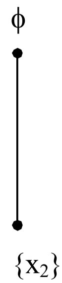
Let S = $\{x_1, x_2, x_3\}$ be the set P(S) be the power set of S.

We find all subset vertex line graphs of type I using this P(S).

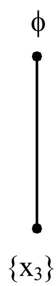
C_1 =



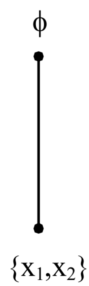
C_4 =



C_3 =

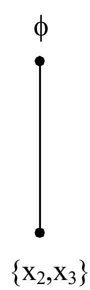


C_4 =

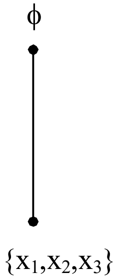


C_5 =

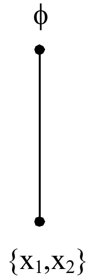
C_6 =



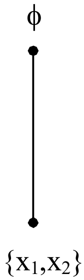
$C_7 =$



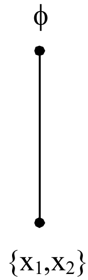
$C_8 =$



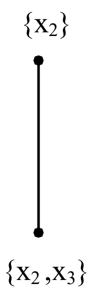
$C_9 =$



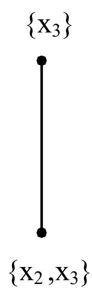
$C_{10} =$



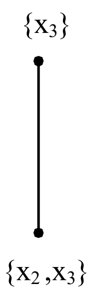
$C_{11} =$



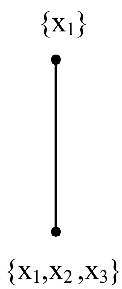
$C_{12} =$



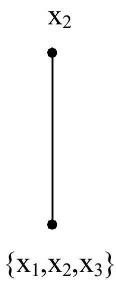
$C_{13} =$



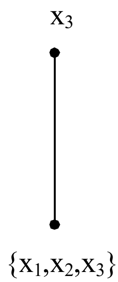
$C_{14} =$



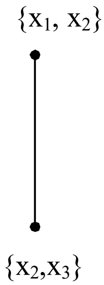
$C_{15} =$



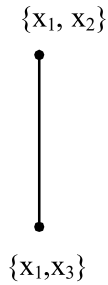
$C_{16} =$



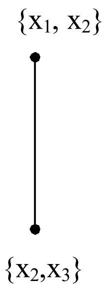
$C_{17} =$



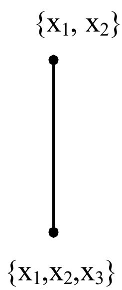
$C_{18} =$



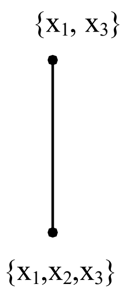
$C_{19} =$



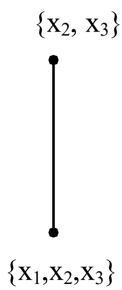
$C_{20} =$



$C_{21} =$



$C_{22} =$



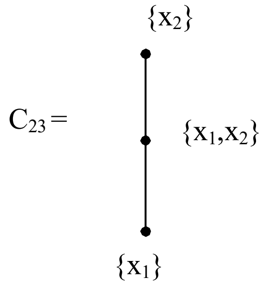


Figure 2.158

C_{23} is a subset vertex line graph or subset vertex star graph of type I.

All star graphs with 3 nodes are trivially line graphs with 3 nodes.

So there are 20 such subset vertex line graphs of type I . So 23 to 42 are subset vertex line / star graphs of type I.

Next we proceed onto describe line graphs with 4 nodes built using P(S) where S = $\{x_1, x_2, x_3\}$

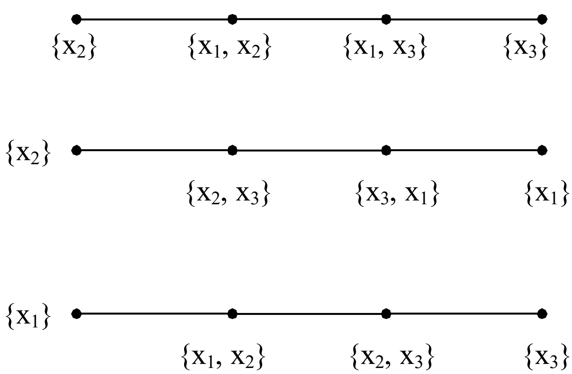


Figure 2.159

We have only 3 subset vertex line graph of type I in this P(S). There are no other subset vertex line graphs with four nodes.

We just conjecture that in case of P(S) where S = $\{x_1, x_2, x_3, x_4, \ldots, x_n\}$.

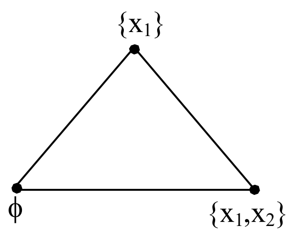
i) We can have only subset vertex line graphs of maximum length n + 1 that is with n + 1 nodes.

ii) There can be only n such subset vertex line graphs of type I with n + 1 nodes; n finite.

Next we study the possible existence of subset vertex circle graphs of type I in P(S).

When S = $\{x_1, x_2\}$ the power set P(S) has the following circle graphs.

T_1 =



T_2 =

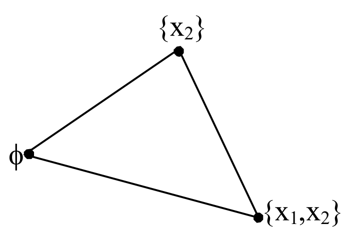
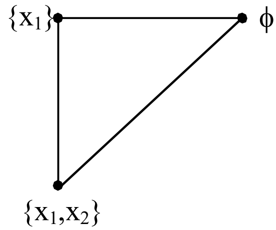


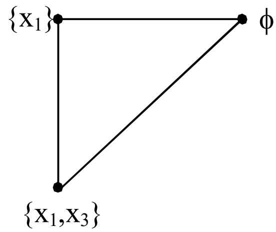
Figure 2.160

When S = $\{x_1, x_2, x_3\}$ are find all the subset vertex circle graphs that can be got using subsets of P(S).

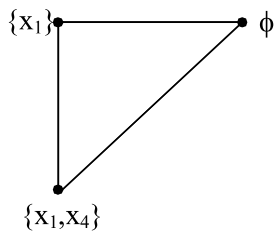
$L_1 =$



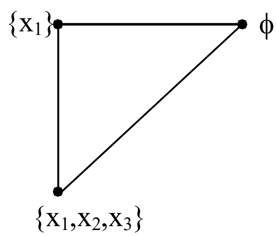
$L_2 =$



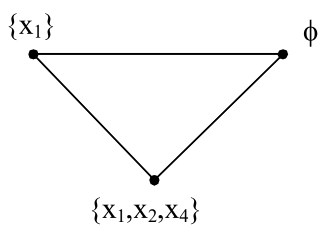
$L_3 =$

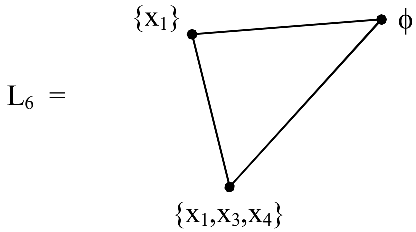


$L_4 =$



$L_5 =$





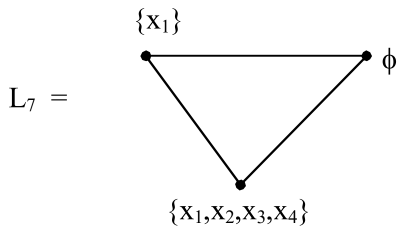


Figure 2.161

We will get 6 more such subset vertex circle graphs if ϕ is replaced by $\{x_1, x_2, x_3, x_4\}$.

Thus there are 13 subset vertex circle graphs with three nodes.

Similarly we will 13 × 3 more subset vertex circle graphs using replacing $\{x_1\}$ by $\{x_2\}$ or $\{x_3\}$ or $\{x_4\}$.

There are several subset vertex circle graphs with 3 nodes.

Interested reader can find the number of subset vertex circle graphs with 3 nodes with vertices from P(S) where S = $\{x_1, x_2, x_3, x_4\}$.

Now we give the subset vertex circle graphs of this P(S) with four nodes.

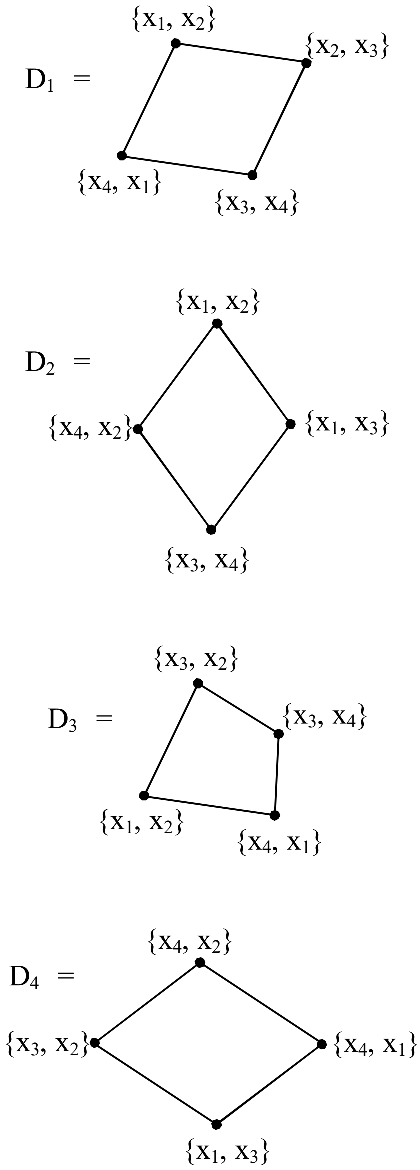

D1 =
{x1, x2}
{x2, x3}
{x4, x1}
{x3, x4}
D2 =
{x1, x2}
{x4, x2}
{x1, x3}
{x3, x4}
D3 =
{x3, x2}
{x3, x4}
{x1, x2}
{x4, x1}
D4 =
{x4, x2}
{x3, x2}
{x4, x1}
{x1, x3}

Figure 2.162

We have just given subset vertex circle graphs of type I of order 4.

The reader is left with the task of finding all circle subgraphs of order 3, order 4 and so on. Also find the largest subset vertex circle graph of with vertex set from this power set P(S).

Let K be a subset vertex graph given by the following

K =

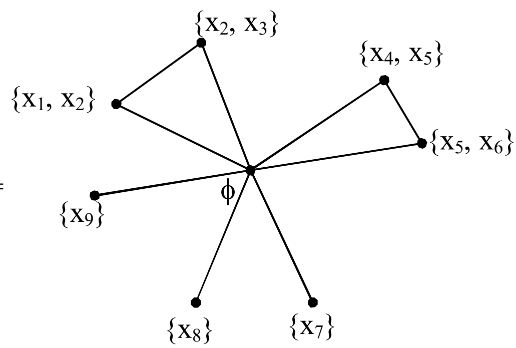


Figure 2.163

with vertex entries from the power set P(S) where S = $\{x_1, x_2, \ldots, x_9\}$. For this K calculate the Global Clustering Coefficient and Local Clustering Coefficient. Using this P(S) for the vertex sets of the subset vertex graphs how many triangle-line graphs can be drawn?

The following observations are pertinent in case of subset vertex graphs.

Suppose S = $\{x_1, x_2\}$ and P(S) be the subset vertex graph G with |P(S)| vertices.

Then we have G given by the following figure.

G =

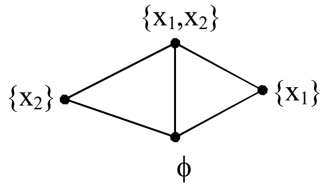
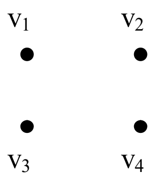


Figure 2.164

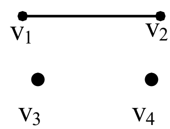
So this is the only vertex subset graph of type I.

Clearly it is not directed by the very definition of subset vertex graphs of type I what is important in general given four vertices v_1, v_2, v_3 and v_4 we can have the following graphs none of which are subset vertex graphs of type I.

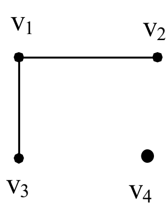
P_1 =



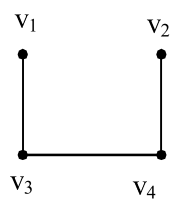
P_2 =



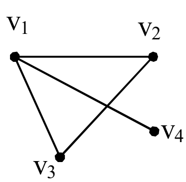
P_3 =



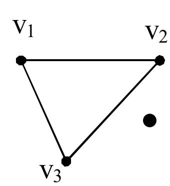
P_4 =



P_5 =



P_6 =



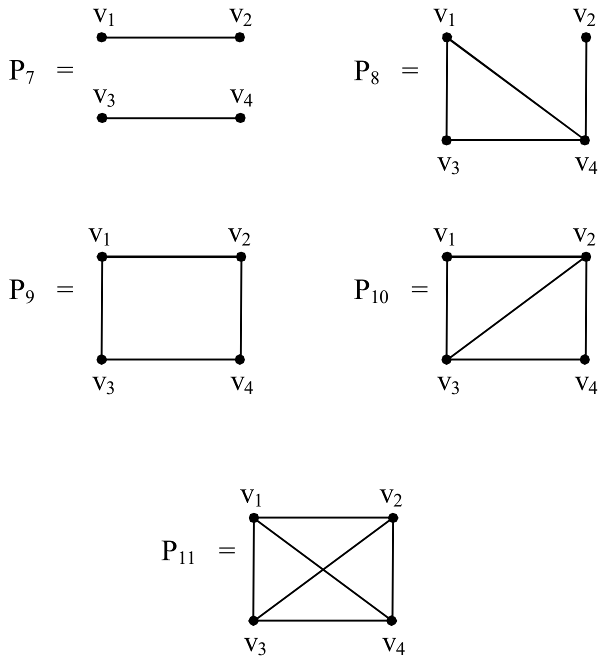


Figure 2.165

and so on.

But for the given set of vertices $\{\{x_1\}, \{x_2\}, \{x_1, x_2\}, \phi\}$ we can have only one subset vertex graph of type I.

So if precisely one is interested in getting a unique graph one can clearly choose the vertex subset graph of type I.

Now consider $P(S) = \{\{x_1\},\{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_1\}, \{x_1, x_2, x_3\}\}$ to be the power set of S. The subset vertex graph of type I given with these 8 vertices is as follows.

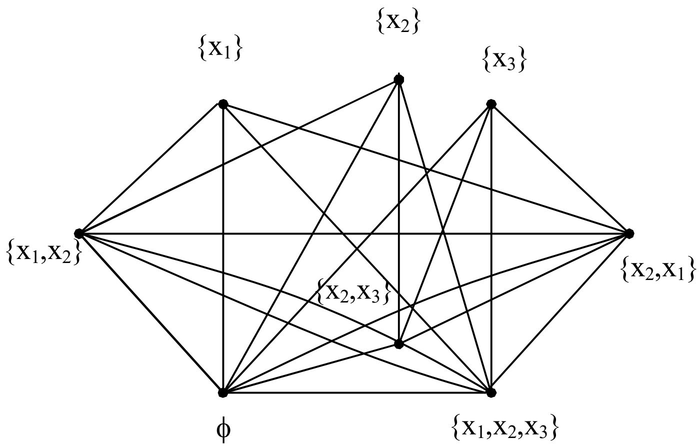


Figure 2.166

There are 24 edges for this 8 node subset vertex graphs of type I. This is the only one such graph we can get using these 8 nodes infact it is unique.

However in case of classical graphs we can have several such graphs using 8 nodes we just display a few of them using $\{v_1, v_2, v_3,\ldots, v_8\}$ as the vertex set by the following figures.

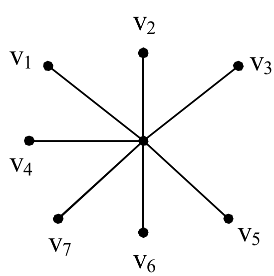
$G_1 =$ 

Figure 2.167

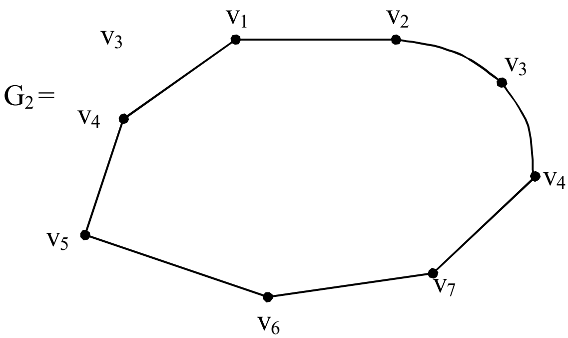


Figure 2.168

G_1 is a star graph with 8 nodes.

G_2 is a circle graph with 8 nodes.

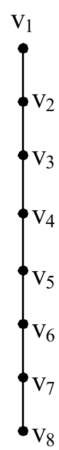


Figure 2.169

G_3 is a line graph with 8 nodes.

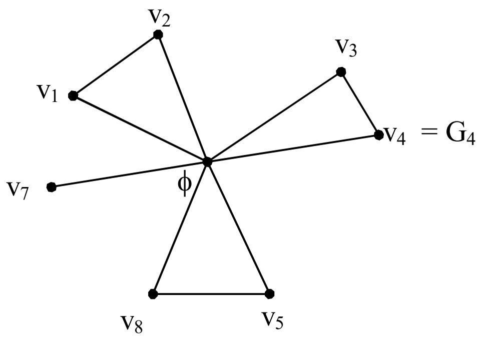


Figure 2.170

G_4 is a triangle - line graph with 8 vertices.

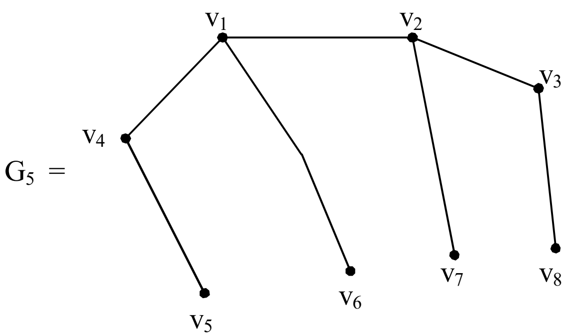


Figure 2.171

G_5 is a graph with 7 edges.

G_6 =

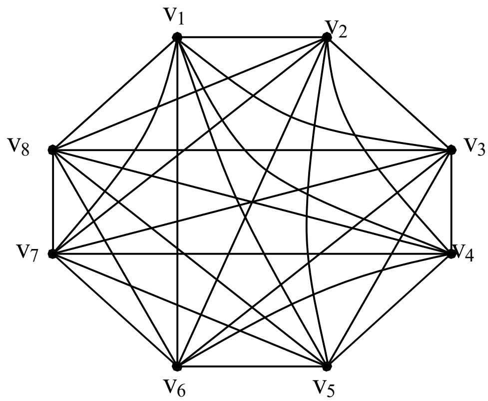


Figure 2.172

This is a complete graph with 8 vertices. However in P(S) with |P(S)| = 8 we do not have a complete subset vertex graph of type I with eight vertices.

Consider the set $\{\{x_1, x_2\}, \{x_1\ x_3\}\ \{x_2, x_3\}, \{\phi\}, \{x_1, x_2, x_3\}\} = V$.

Let K_1 be the subset vertex graph of type I with 5 vertices given by the following figure.

K_1 =

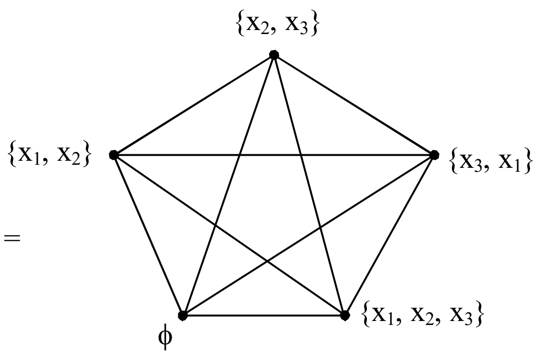


Figure 2.173

We see K_1 is a complete subset vertex graph of type I. Infact this is the largest subset vertex graph of type I which is complete.

It is left as a open conjecture to find the number of nodes of a vertex subset graph of type I constructed using P(S) where $S = \{x_1, x_2, \ldots, x_n\}$ so that the subset vertex graph is complete.

For n = 2 we have 2 complete subset vertex graphs with three nodes.

For n = 3 we get only one complete subset vertex graph with 5 vertices or 5 nodes and 2 complete subset vertex graphs of type I with 5 nodes or vertices.

For n = 4 we have only one vertex subset graphs with six nodes to be complete which is the largest complete vertex subset graph.

We have also subset vertex graphs with five nodes is also complete.

For n = 5 we have the largest complete vertex subset graph has 17 nodes.

Ofcourse there are subset vertex graphs which are complete with nodes less than 17.

When n = 6 we have the largest complete vertex subset graph has only 23 node.

When n = 7 we have the largest complete vertex subset graph has 65 nodes.

Thus the reader is left with the open problem of finding the largest complete subset vertex subset graph of type I.

We suggest a few problems for the researcher;

Problems

1. Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set with six nodes. P(S) the power set of S.

   i) Find all subset vertex line graphs of type I with vertex set from P(S).

   ii) What is the highest number of nodes a subset vertex line graph can have?

   iii) Find all subset vertex star graphs of type I using the vertex set from P(S).

   iv) What is the largest subset vertex star graph using the vertex set from P(S)?

   v) How many subset vertex circle graphs can be built with vertex set from P(S)?

   vi) Give any other interesting properties associated with PS)

2. Study questions (i) to (vi) of problem (1) for P(S) where S = $\{x_1, x_2, \ldots, x_{12}\}$.

3. Let P(S) be the power set associated with S = $\{x_1, x_2, x_3, x_4\}$. Give a subset vertex graph using vertex set from P(S) which has the greatest diameter.

4. Show none of the subset vertex graphs are Boolean algebras.

5. Prove no Boolean algebra of order greater than or equal to four is a vertex subset graph of P(S).

6. Find the total number of edges built using the power set P(S) of the subset vertex graph with 2^n vertices where S = $\{x_1, x_2, \ldots, x_n\}$

7. Show for the vertex subset graph with vertex set $\{\phi\}$, $\{x_1\}$, $\{x_2\}$ and $\{x_1, x_2\}$ the graph has only 5 edges.

8. Find all complete vertex subset graphs of type I using the vertex set from P(S) = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$.

9. How many subset vertex complete graph are these in problem 8.

10. Find all subgraphs of the following subset vertex graph of type I given in the following figure.

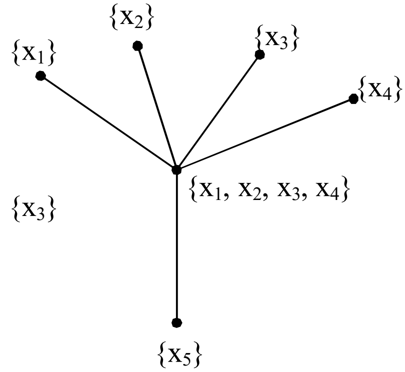


Figure 2.174

where the vertex sets are from P(S) with S = $\{x_1, x_2, x_3, x_4, x_5\}$.

11. Let P(S) be the power set of S, where S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

(a) Construct the vertex subset graph G_1 of type I using the vertex set $V_1 = \{x_1, x_2\}, \{x_1, x_3, x_9\}, \{x_1, x_2, x_8, x_9, x_7\}, \{x_7, x_4, x_5\}, \{x_6, x_9\}, \{x_2, x_3, x_9\}, \{x_8, x_6, x_3\}, \{x_8, x_1, x_7\}\}$.

i) Find the number edges of G_1

ii) Find all subgraphs of G_1

iii) Is G_1 a subset vertex star graph?

iv) Does G_1 have a vertex subset star graph?

b) Find all subset vertex line graphs using the vertex set from P(S).

c) What is the largest number of nodes associated with the subset vertex cycle graph of type I?

12. Given the subset vertex graph G with vertex set from P(S) where S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ by the following figure.

i) Find all subset vertex subgraphs of G.

ii) Find the degrees of all nodes.

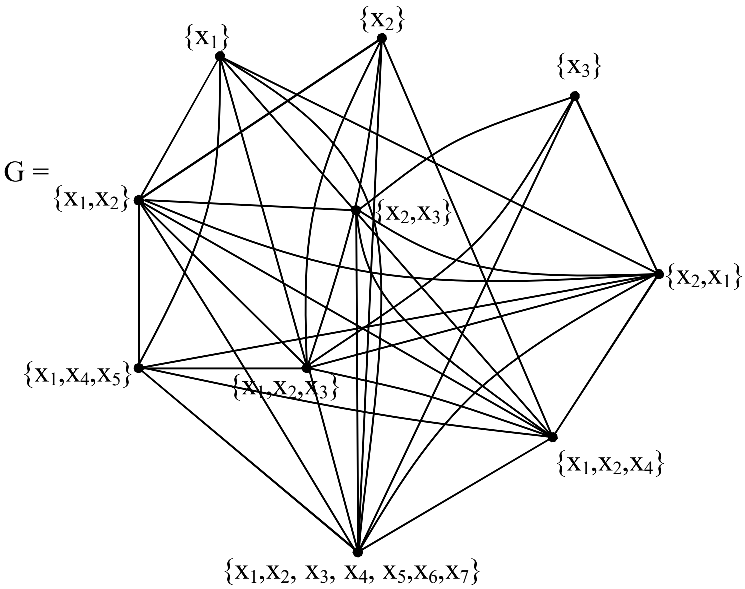


Figure 2.175

iii) Calculate the density of the subset vertex graph of type I.

iv) How many usual or classical graphs can be constructed using these 10 vertices?

v) Prove there can be only one graph with the any fixed number of vertices from P(S).

vi) Draw all line subset vertex graphs using this P(S).

vii) Find all star subset vertex graphs using this P(S).

viii) Find all subset vertex circle graphs using this P(S).

ix) For G be the subset vertex graph of type I.

a) Calculate the Global clustering coefficient?

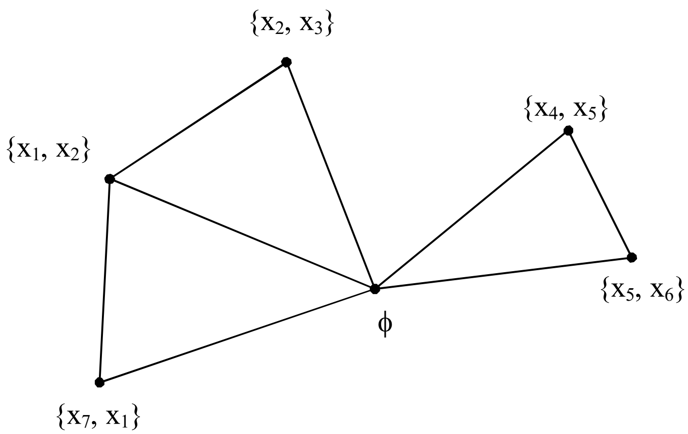


Figure 2.176

b) Calculate the local clustering coefficient og G.

c) Find the diameter of G.

13. Let P(S) be the power set of S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$.

i) How many complete vertex subset graphs of type I can be got using the vertex set from P(S)?

ii) Prove there is only one largest subset vertex graph which is complete of type I. Find the number of vertices of it.

iii) Is the nodes of the largest complete vertex subset graph in (ii) can have more than 80 vertices.

14. Let $S = \{x_1, \ldots, x_n\}$, n a finite positive integer; P(S) be the power set of S. It is an open conjecture to find the number of nodes of the largest complete subset vertex graph of type I using P(S).

15. Let P(S) be as in problem 14.

    Find the largest subset vertex line graph using P(S).

16. Find the largest subset vertex star graph using the P(S) in problem 14.

17. Find the largest subset vertex circle graph using vertex set from P(S) in problem 14.

18. Let $S = \{x_1, x_2, \ldots, x_{12}\}$ be the given set. P(S) be the power set of S.

    a) Let T =

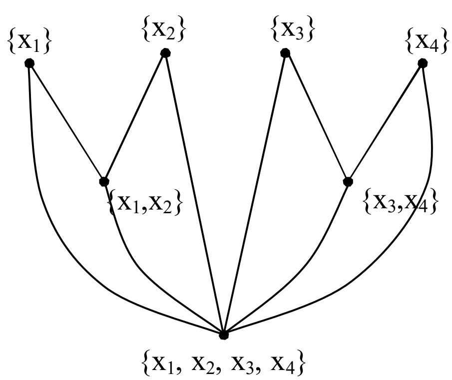


Figure 2.177

be the subset vertex graph of type I we call such type of subset vertex graphs as wrapped trees.

b) Find all subset vertex graphs which are wrapped trees.

c)

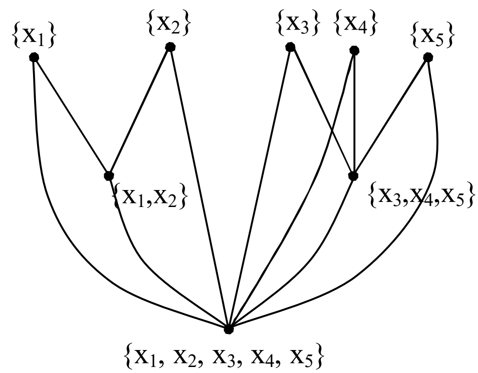


Figure 2.178

d) Can we say if there are n nodes of the wrapped tree in the first layer then there are only n wrappers.

e) What is the biggest subset vertex wrapped tree of type I using this P(S).

19. Can a subset vertex wrapped tree of type I have more than 3 layers?

20. What are the special features associated with subset vertex graphs in general?

21. When will subset vertex graphs be preferred to usual classical graphs?

22. What are the real world problems these vertex subset graphs can find applications?

23. What are the advantages of the uniqueness of these subset vertex graphs of type I in the place of classical graphs where several such graphs exists?

24. Can we have the concept of lattice in the subset vertex graphs of type I using P(S).

25. Can these subset vertex graphs of type I be useful / adopted in social net work analysis?

26. Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the P(S) be the power set of S.

    i) Let G be a

G =

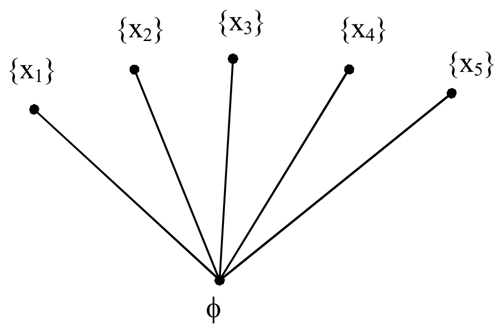


Figure 2.179

subset vertex star graph of type I. Prove all subset vertex subgraphs of order greater than or equal to four which are non empty are again subset vertex star graphs.

    ii) Let H be the subset vertex graph of type I given by the following figure;

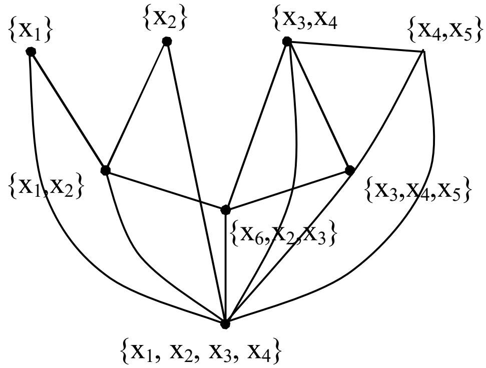


Figure 2.180

Is H a subset vertex wrapped tree of type I? Justify your claim.

iii) Let K be the subset vertex graph of type I given by the following figure

K =

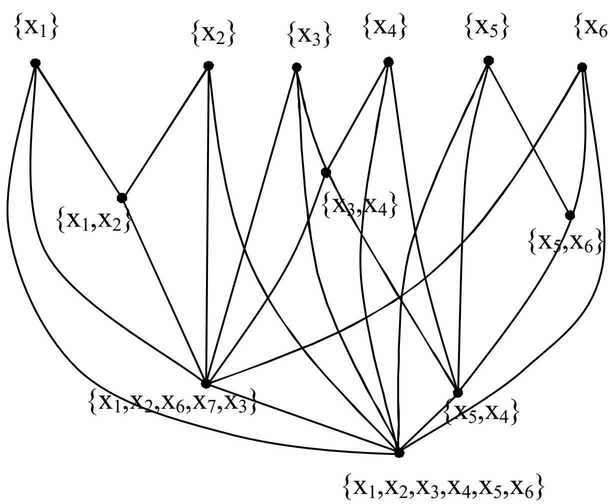


Figure 2.181

We define K as the double layered wrapped up tree

(a) Can K have a subset vertex wrapped up tree of type I?

(b) Will every subset vertex double layered wrapped up tree have a subgraph which is a subset vertex single layered wrapped up tree? Justify with examples.

27. Can subset vertex wrapped trees be used in data mining.

28. Prove in many cases subset vertex wrapped trees is advantages as the first layered node can directly reach the root without traversing other nodes, which can save both time and economy (we also call the wrapped tree subset vertex graph as lotus tree subset vertex graphs).

29. Find all subset vertex wrapped trees of type I using the power set P(S) where S = $\{x_1, x_2, \ldots, x_{27}\}$.

30. For the above problem what is the largest number of layers and the largest number of nodes.

31. Consider the subset vertex wrapped tree given by the following figure G with vertices from P(S), S = $\{x_1, x_2, \ldots, x_8\}$.

G =

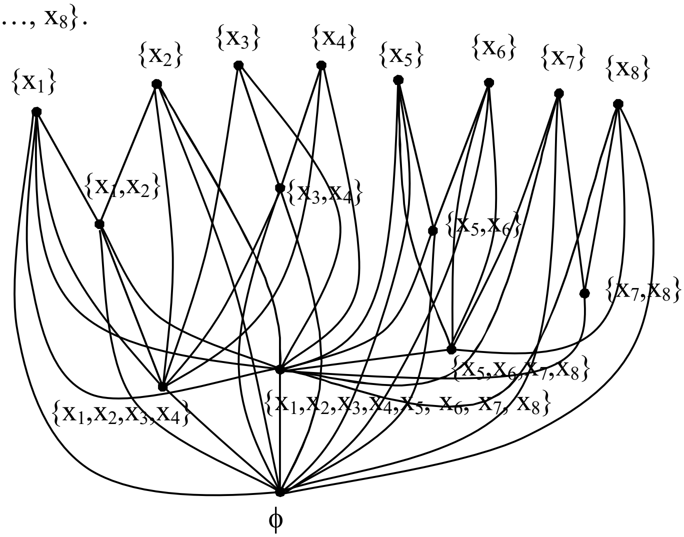


Figure 2.182

i) What is the number of layers in G?

ii) Does G contain subset vertex wrapped tree subgraphs with 2 or 3 or 4 layer? Justify.

iii) What is the largest number of nodes associated with the subset vertex wrapped trees of type I using vertex set from this P(S)?

iv) Which of the subset vertex wrapped tree G has highest degree?

v) Which of the subset vertex wrapped tree g has the least degree?

vi) Construct all possible subset vertex wrapped trees associated with P(S) . S = $\{x_1, x_2, \ldots, x_8\}$

32. Discuss all possible applications of subset vertex graph of type I.

33. Describe the special applications of subset vertex graphs which are wrapped trees.

34. Can we solve problems related to classical graph theory in case of subset vertex graphs of type I?

35. Enumerate all NP hard problems in classical graph theory which can have solution in case of subset vertex graph of type I.

36. Given P(S) where S = $\{x_1, \ldots, x_n\}$ how many subset vertex graph of type I can be constructed? (we do not include empty graphs).

37. Prove given a set vertices from P(S) one can have one and only vertex subset graph of type I using this M.

38. Prove if G is a graph with 5 vertices then there is only one subgraph of type I associated with every subset of the vertex set of G. Find all subset vertex subgraphs of type I for this subset vertex graph G given below.

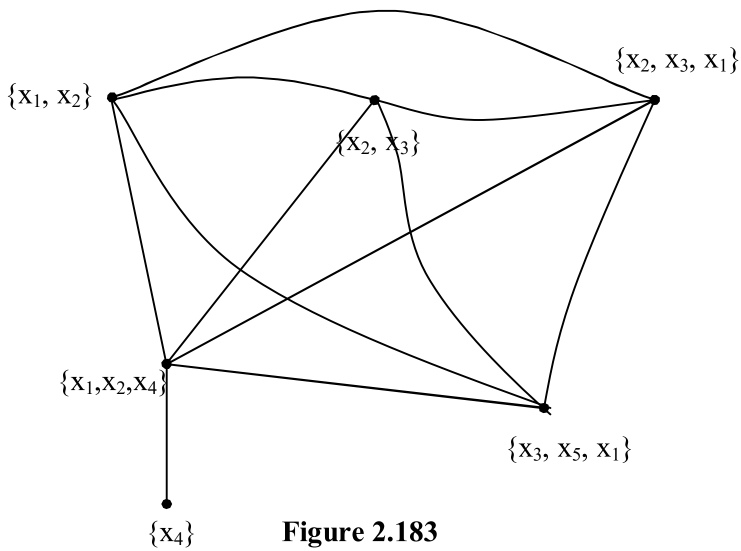


Figure 2.183

39. Let P(S) be the power set of the set $S = \{x_1, x_2, \ldots, x_9\}$.
    a) Find all subset vertex graphs of type I constructed using vertex set V from P(S) which are such that the degree of each vertex is the same.
    b) How many such subset vertex graphs exist?
    c) Prove there is only one complete subset vertex graph of type I with 11 vertices.

40. Generalize the problem (39) for any n, $n \geq 3$.

41. Obtain all special features and probable applications of subset vertex graphs of type I.

42. Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set with six elements. P(S) the power set of S.
    (a) Find all cycles that can be built using elements from P(S) for vertex set
    (b) Find all subset vertex star graphs of type I that can be got using P(S).
    (c) Find all subset vertex line graphs of type I that can be built using the subsets of the power set P(S) as vertex sets.
    (d) Can these subset vertex graphs of type I yield rooted trees with more number of layers?
43. Consider the following subset vertex graphs of type I

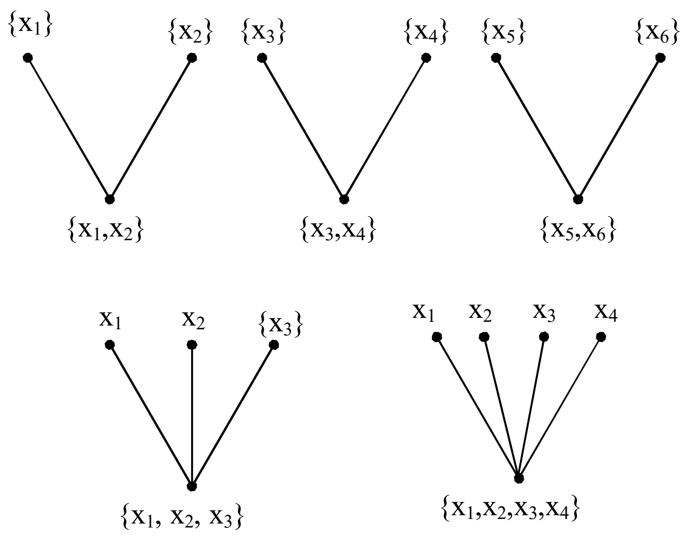


Figure 2.184

are 1 layered subset vertex rooted trees of type I?

Prove or disprove it is impossible to have subset vertex rooted trees of type I with more than two layers.

Chapter Three

SUBSET VERTEX GRAPHS OF TYPE II AND THEIR APPLICATIONS

In this chapter we proceed onto define the notion of subset vertex graphs of type II. In the earlier chapter the notion of subset vertex graphs of type I was defined and their properties were discovered and described. It is pertinent to keep on record that subset vertex graph of type I are not directed. Secondly given a set of vertices from subsets of a power set P(S) the graph is uniquely determined and there is one and only subset vertex graph of type I.

However, when subset vertex graphs are directed and they are defined differently. Further one will try to find the relation existing between type I graphs and type II graphs provided a relation exists.

Throughout this chapter we denote by P(S) the power set of the finite set S; S can be finite or infinite, but in this chapter we discuss and develope subset vertex graphs of type II using only finite set S next proceed onto define type II subset vertex graphs. These graphs are also built using subsets of a set S as vertices.

Let S be the set with some n elements, P(S) be the power set of S ($n < \infty$).

We say if $A, B \in P(S)$ there is a edge joining from A to B if $A \subset B$ then we get the graph as .

Figure 3.1

If $B \subset A$ then the map is from .

Figure 3.2

Clearly these subset vertex graphs of type II are only directed. Infact we can define them injective subset vertex graphs. However we call them only as subset vertex graphs of type II.

They are not subset vertex type I graphs.

So if we have

$A = \{x_1, x_2, x_3\}$ and $B = \{x_4, x_1\}$ then we do not have a edge in case of type II subset vertex graphs but we have a edge between the vertices A and B in subset vertex type I graphs.

Figure 3.3

This is the marked difference between type I and type II subset vertex graphs.

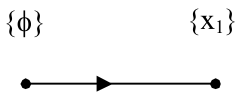
We first provide some examples of type II subset graphs in the following.

Example 3.1. Let $S = \{x_1, x_2\}$ be the set $P(S) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ power set of S.

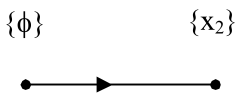
We see

$\{x_1\}$, $\{x_2\}$ $= P_1$ are just

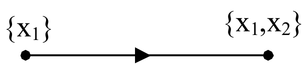
Point subset vertex graph



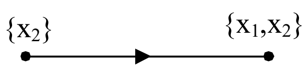
$= P_2$



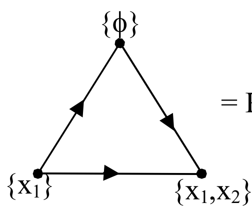
$= P_3$



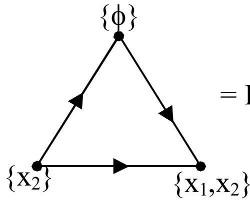
$= P_4$



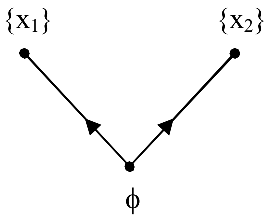
$= P_5$



$= P_6$



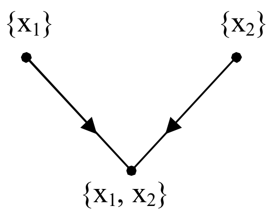
$= P_7$

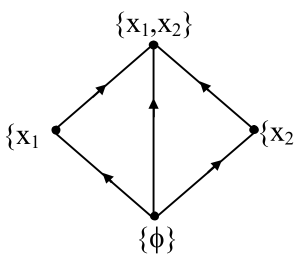


$= P_8$

$P_9 =$

$P_{10} =$





$= P_{11}$

Figure 3.4

We make the assumption that the empty set is always the subset of every other subset of P(S).

Consider the subset vertex graph of $\{\{x_1\}, \{x_2\}, \{x_1, x_2\}$ and $\phi\}$.

All the graphs given by P_1, P_2, …, P_{11} are subset vertex graphs of type II.

They are directed and a directed edge exists if and only if $A \subset B$.

$$A \bullet\!\longrightarrow\!\bullet B$$

Figure 3.5

We see if A = B then the graph is only a single point given by

$\bullet A = B \ \bullet$

We see in case of $P(S) = P(\{x_1, x_2\}) = \{\phi, \{x_1\}, \{x_2\}, \{x_2, x_1\}\}$ we get only 11 subset vertex graphs of type II.

We cannot get any other than these.

Now in the following example we give the subset vertex graph of type II using $S = \{x_1, x_2, x_3\}$.

$P(S) = \{\phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}$ be the power set of S.

First we list out the point set vertex graphs of type I

$P_1 =$ $\overset{\{x_1\}}{\bullet}$ $\overset{\{x_2\}}{\bullet}$

$P_2 =$ $\overset{\{x_1\}}{\bullet}$ $\overset{\{x_3\}}{\bullet}$

$P_3 =$ $\overset{\{x_1\}}{\bullet}$ $\overset{\{x_2,x_3\}}{\bullet}$

$P_4 = \quad \{x_2\} \qquad \{x_3\}$

$P_5 = \quad \{x_2\} \qquad \{x_1,x_3\}$

$P_6 = \quad \{x_3\} \qquad \{x_2,x_1\}$

Figure 3.6

Thus P(S) when |S| = 3 we get seven point set vertex graphs of type II and they are also point set vertex graphs of type I in fact they are classical point graphs.

We list out all type II vertex subset graphs with two vertices and one edge.

It is pertinent to observe that the set ϕ and $\{x_1, x_2, x_3\}$ do not contribute to point subset vertex graphs.

Consider the subset vertex graphs of type II with two vertices and one edge.

$P_8 = \quad \{\phi\} \longrightarrow \{x_1\}$

$P_9 = \quad \{\phi\} \longrightarrow \{x_3\}$

$P_{10} = \quad \{\phi\} \longrightarrow \{x_3\}$

$P_{11} = \quad \{\phi\} \longrightarrow \{x_1,x_2\}$

$P_{12} = \quad \{\phi\} \longrightarrow \{x_1,x_3\}$

$$P_{13} = \{\phi\} \longrightarrow \{x_2,x_3\}$$

$$P_{14} = \{\phi\} \longrightarrow \{x_1,x_2,x_3\}$$

$$P_{15} = \{x_1\} \longrightarrow \{x_1,x_2\}$$

$$P_{16} = \{x_1\} \longrightarrow \{x_1,x_3\}$$

$$P_{17} = \{x_1\} \longrightarrow \{x_1,x_2,x_3\}$$

$$P_{18} = \{x_2\} \longrightarrow \{x_1,x_2\}$$

$$P_{19} = \{x_2\} \longrightarrow \{x_2,x_3\}$$

$$P_{20} = \{x_2\} \longrightarrow \{x_1,x_2,x_3\}$$

$$P_{21} = \{x_3\} \longrightarrow \{x_2,x_3\}$$

$$P_{22} = \{x_3\} \longrightarrow \{x_3,x_1\}$$

$$P_{23} = \{x_3\} \longrightarrow \{x_3,x_2,x_3\}$$

$$P_{24} = \{x_1,x_2\} \longrightarrow \{x_1,x_2,x_3\}$$

$$P_{25} = \{x_1,x_3\} \longrightarrow \{x_1,x_2,x_3\}$$

$$P_{26} = \{x_2,x_3\} \longrightarrow \{x_1,x_2,x_3\}.$$

Figure 3.7

Thus we have 19 subset vertex graphs of type II with one edge and two vertices. These will be also known as dyads which are directed or have relational ties in only one direction. However both of the vertices are actor set and not different.

The subset vertex graphs of type II given by figures. P_8 to P_{26} are directional dyads [ ].

Now we proceed onto describe all subset vertex graphs of type II with three vertices and one edge in the following.

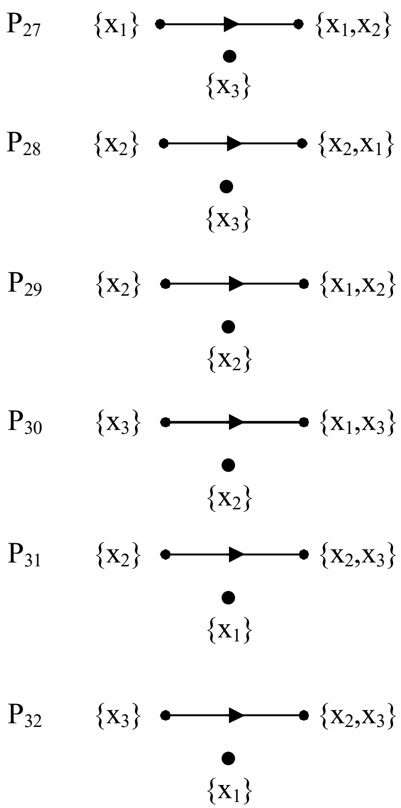


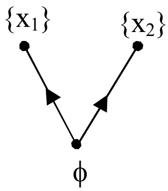
Figure 3.8

We can say these graphs can also be realized as forbidden triads.

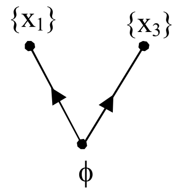
There are six forbidden triads which are subset vertex graphs of type II.

Next we proceed on to describe vertex subset graphs of type II which have three vertices and only two edges.

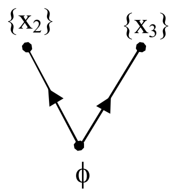
$P_{33} =$



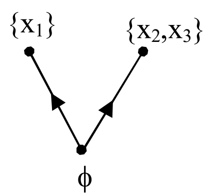
$P_{34} =$



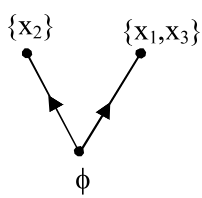
$P_{36} =$



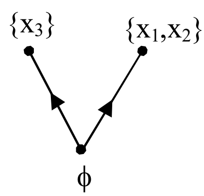
$P_{37} =$



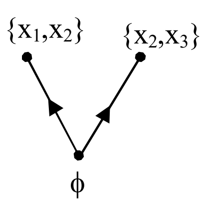
$P_{38} =$



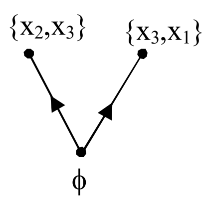
$P_{39} =$



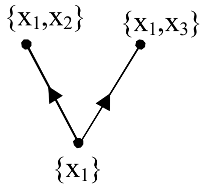
$P_{40} =$



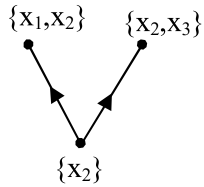
$P_{41} =$



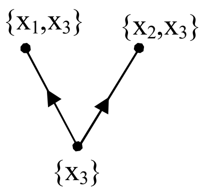
$P_{43} =$



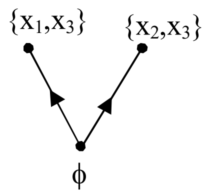
$P_{44} =$



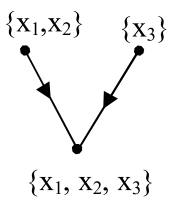
$P_{45} =$



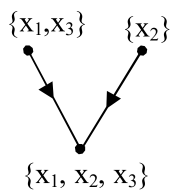
$P_{42} =$



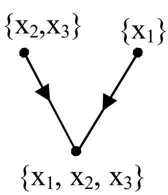
$P_{46} =$



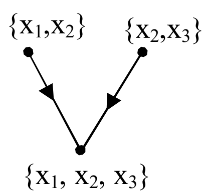
$P_{47} =$



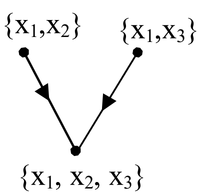
$P_{48} =$



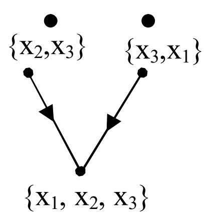
$P_{49} =$



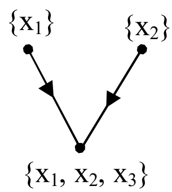
$P_{50} =$



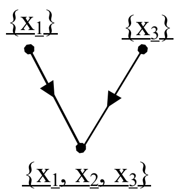
$P_{51} =$



$P_{52} =$



$P_{53} =$



$P_{54} =$

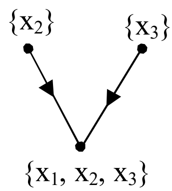
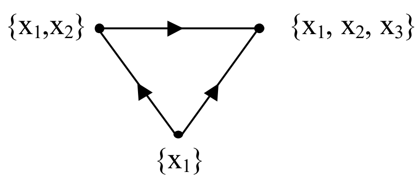


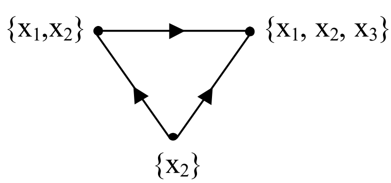
Figure 3.9

Next we proceed onto give all type II vertex subset graphs with three vertices and three edges triads formed from the subsets of P(S) where S = $\{x_1, x_2, x_3\}$.

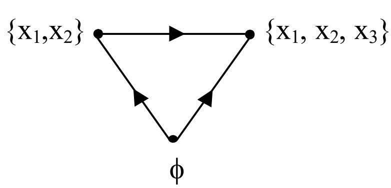
$P_{55} =$



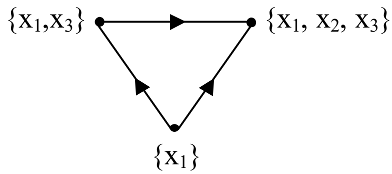
$P_{56} =$



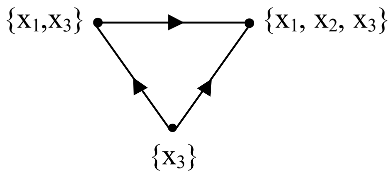
$P_{57} =$



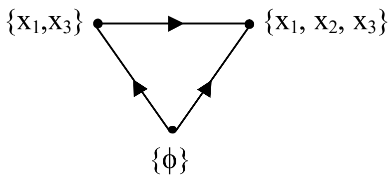
$P_{58} =$



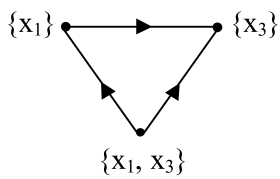
$P_{59} =$



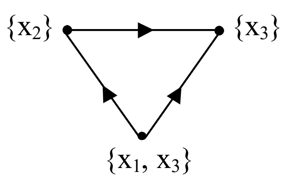
$P_{60} =$



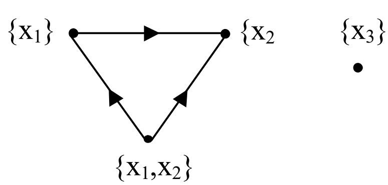
$P_{61} =$



$P_{62} =$



$P_{63} =$



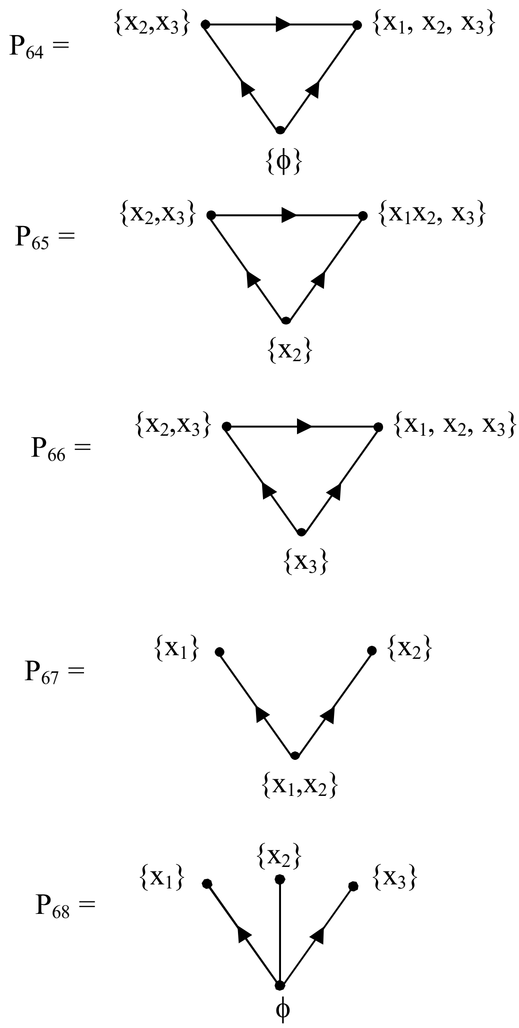
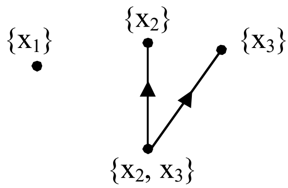


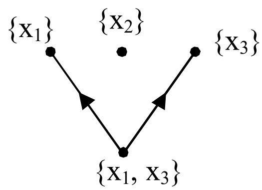
Figure 3.10

This subset vertex graph of type II is a star graph with four vertices.

$P_{69} =$



$P_{70} =$



$P_{71} =$

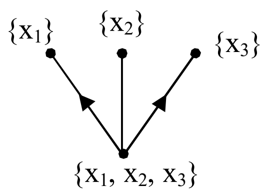
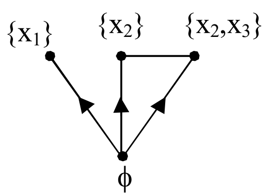


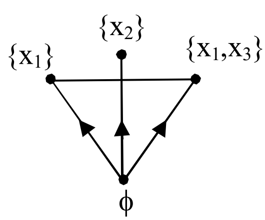
Figure 3.11

This is again a subset vertex type II star graph with four nodes.

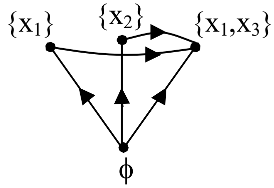
$P_{72} =$



$P_{73} =$



$P_{74} =$



$P_{75} =$

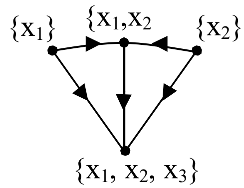


Figure 3.12

Observe in case 74 and 75 the direction of the 3 edges are different.

P_{76}

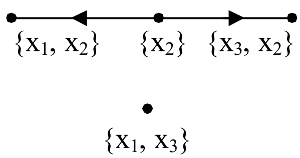


Figure 3.13

We see the subset vertex graph of type I using the four vertices $\{\{x_2\}, \{x_2, x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}$ is a different from the subset vertex graph of type II given us P_{76}.

A =

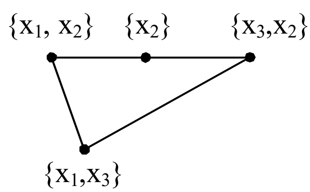


Figure 3.14

We see the graph A which is a subset vertex graph of type I is closed graph but P_{76} is not so, apart from being directed.

$P_{77} =$

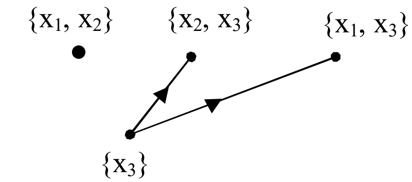


Figure 3.15

$P_{78} =$

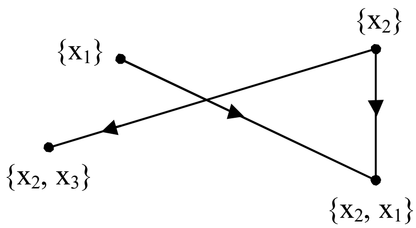


Figure 3.16

$P_{79} =$

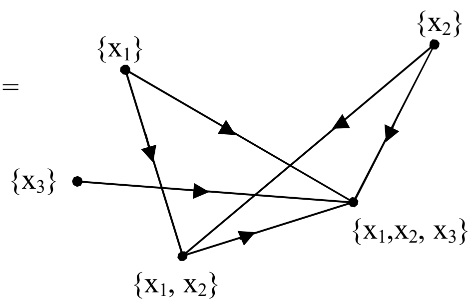


Figure 3.17

$P_{80} =$

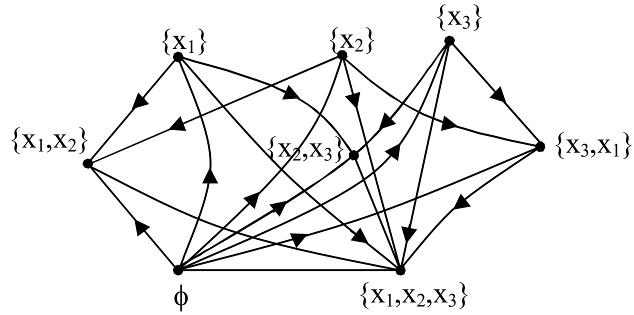


Figure 3.18

G =

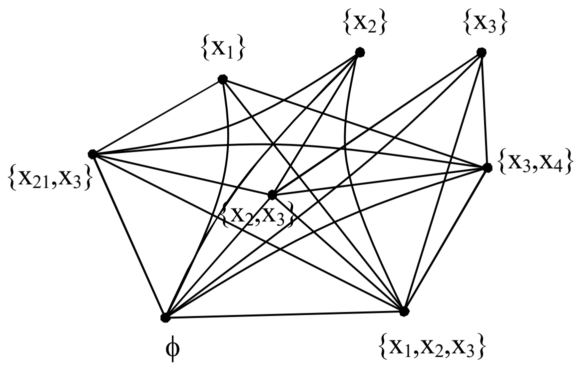


Figure 3.19

We can observe the subset vertex graphs of type I and type II are distinct.

The number of edges for these two subset vertex graphs as follows.

P_{80} has only 19 edges where the subset vertex graph of type I given by G has only 22 edges. This is the difference between them apart from P_{80} being directed.

Now we give the Boolean algebra B of P(S) where |P(S)| = 8 which is given in the following. The Boolean algebra B is given by the following figure.

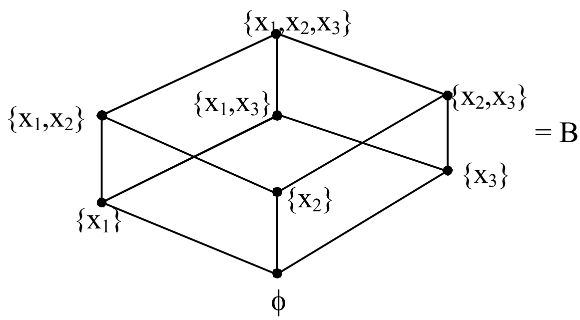


Figure 3.20

Clearly the Boolean algebra has only 12 edges.

Thus in all cases as the subsets are related by some sort of relation depending on which such edges can be defined. This is clearly evident from P_{80}, G and B, the subset vertex graph of type II, subset vertex graph of type I and the Boolean algebra.

We see depending on the relation by which the edges are defined the graphs are different, however the graphs in all cases are distinct. Thus depending on the need one can use the model to study the problem be it a social networking or any other relevant situation.

Consider P(S) the power set of S where S = $\{x_1, x_2, x_3, x_4\}$. The subset A = $\{\{x_1, x_2\},\{x_1\}, \{x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_4\}, \{x_2, x_4\}, \{x_3, x_2\}\}$ be the vertex set of the graph of G_1, subset vertex graph of type I; G_1 is as follows.

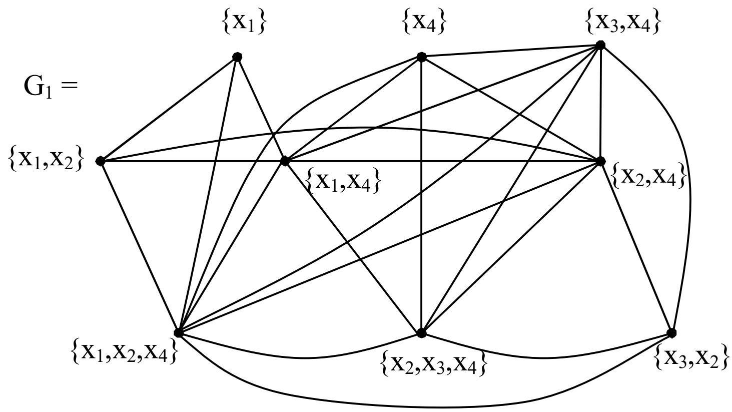


Figure 3.21

The subset vertex graph G_2 of type II for the same set of vertices A is given by the following figure.

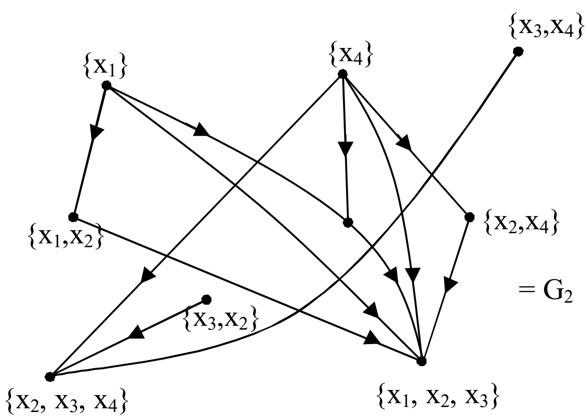


Figure 3.22

We now analyse the vertex subset graphs G_1 and G_2 of type I and type II. The maximum degree in case of G_1 is related with the vertex $\{x_1, x_2, x_4\}$, which is the maximum degree of 8.

Thus the greatest degree is 8 followed by 7 taken by the vertices $\{x_2, x_4\}$, and $\{x_1, x_4\}$ respectively.

Thus interested reader can study the degree, diameter density of the vertex subset graphs of G_1 and G_2 type I and type II and compare them.

Let us consider the vertex set B = $\{\{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}\}$.

We find the vertex subset graphs of both types.

F_1 =

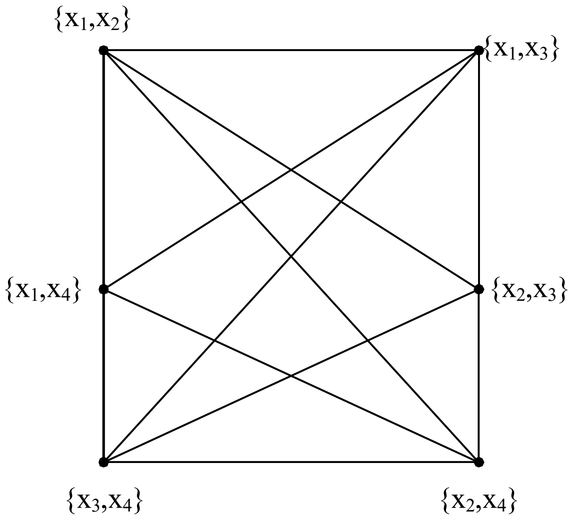


Figure 3.23

Let F_1 denote the subset vertex graph of type I.

Clearly F_1 is a pseudo complete subset vertex graph as degree of each vertex is equal to 4. Clearly F_1 is not a complete subset vertex graph as the degree of each node is not five and as the number of vertices of F_1 is 6.

We see F_2 the subset vertex graph of type II is just a empty graph given in the following.

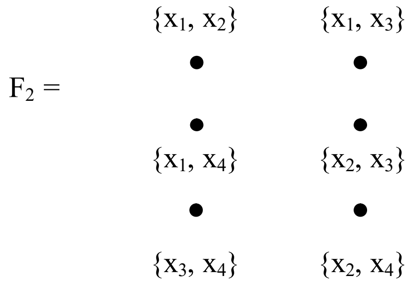


Figure 3.24

Next we proceed onto describe the subset vertex graphs of type I and type II respectively by the following graphs E_1 and E_2.

Let P = $\{\{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}\}$ be the vertex subset of P(S).

The subset vertex graph K_1 of type I given by the set P is as follows.

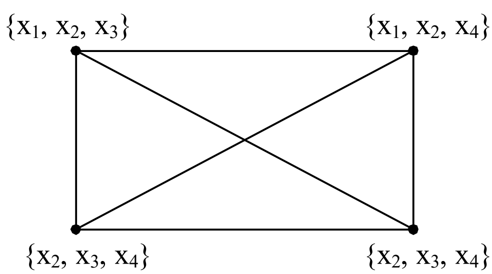


Figure 3.25

The type II subset vertex graph K_2 using the vertex set P is just the empty graph given by

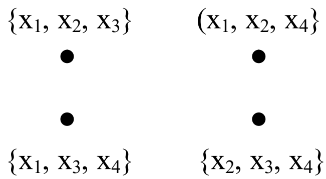


Figure 3.26

Thus both in some cases behave in a very odd way.

In view of all these we put forth the following result.

Theorem 3.1. *Let $S = \{x_1, x_2, x_3, x_4, \ldots, x_n\}$ be the finite set and $P(S)$ be the power set of S.*

i) *Using vertex set with $(m - 1)$ elements each m, $(m = 3, 4, \ldots$, up to $n)$; $3 \leq m \leq n$ we see the subset vertex graph of type I is either complete or pseudo complete and incase of subset vertex graph II it is always a empty graph.*

ii) *When $m = n$ we get a complete subset vertex graph of type I.*

Proof is direct and hence left as an exercise to the reader.

We now make a definition of the pseudo complete graph abstractly.

Definition 3.1. *Let $S = \{x_1, x_2, \ldots, x_n\}$ and $P(S)$ be the power set of S. Let V be the vertex set of S, $V \subseteq P(S)$ with $|V| = nC_m$ $(m = 2, 3, \ldots, n - 1)$ then the subset vertex graph G of type I with* *vertex set V is defined as a pseudo complete graph of type I and G has less than (n – 1)edges adjacent with each vertex depending on m, $2 \leq m \leq n - 2$ when $m = n - 1$ we get a complete subset graph of type I with n vertices and (n – 1) edges associated with each vertex.*

We proceed onto describe more examples which will illustrate the associated properties of the subset vertex type II graph.

Example 3.2. Let S = $\{x_1, x_2, x_3, x_4, x_5\}$ be the set with 5 elements P(S) the power set of S, |S| = 32.

Consider V = $\{\{x_1\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}\}$ be the vertex set the subset vertex graph with the vertex set V is as follows.

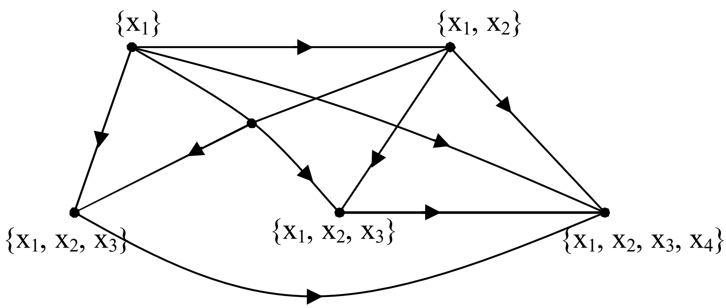


Figure 3.27

We redraw this type II vertex subset graph G as follows.

G =

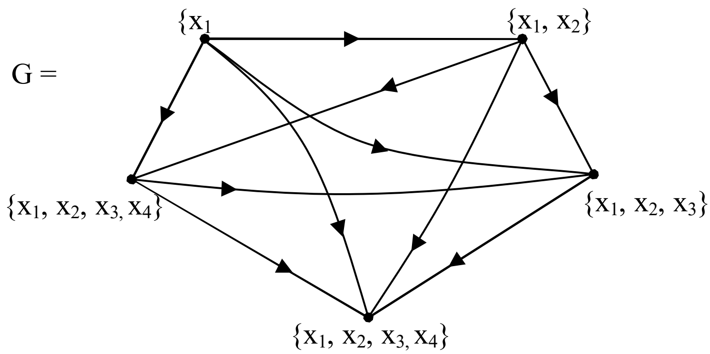


Figure 3.28

This is complete or tournament subset vertex type II graph which is directed. From the vertex $\{x_1\}$ all the edges go outside there is no edge which is incoming. All edges (arrows) are out coming. The vertex $\{x_1, x_2\}$ has only one incoming arrow and 3 arrows are outgoing.

For the vertex $\{x_1, x_2, x_3\}$; 3 arrows are outgoing and one arrow is incoming.

For the above $\{x_1, x_2, x_3, x_4\}$ we see all the four arrows are only incoming. Thus G is a tournament.

We see

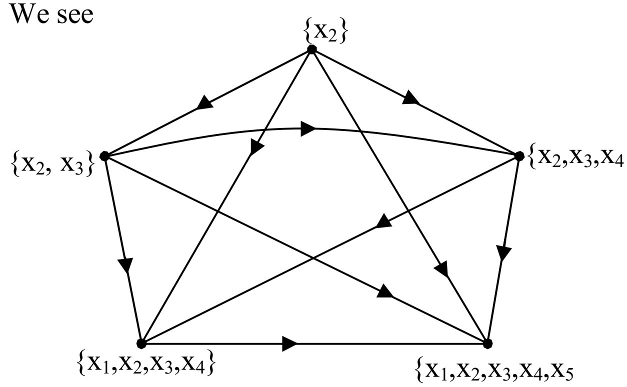


Figure 3.29

This is again a subset vertex directed graph of type II which is a subset vertex tournament.

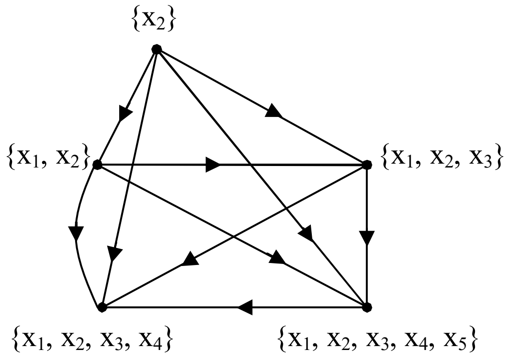


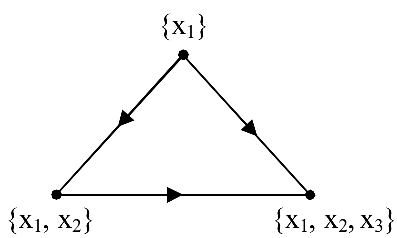
Figure 3.30

This is again a subset vertex directed graph of type II which is a tournament.

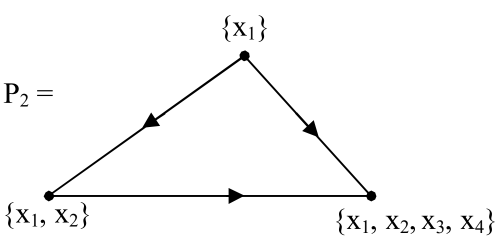
The reader is left with the task of finding all subset vertex tournaments with 5 vertices.

We find all subgraphs of this vertex subset directed graphs whose vertex set is greater than or equal to three.

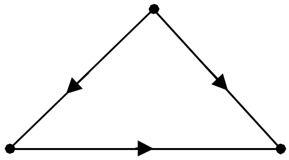
$P_1 =$



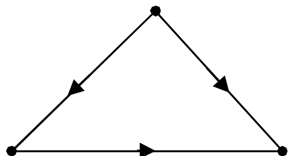
$P_2 =$



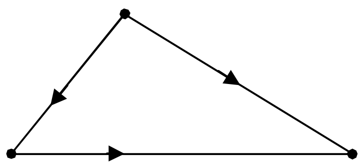
$P_3 =$


$\{x_1\}$
$\{x_1, x_2\}$
$\{x_1, x_2, x_3, x_4, x_5\}$

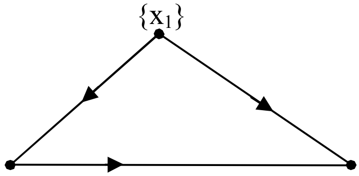
$P_4 =$


$\{x_1\}$
$\{x_1, x_2, x_3\}$
$\{x_1, x_2, x_3, x_4\}$

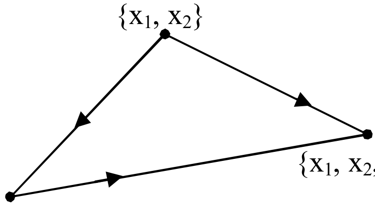
$P_5 =$


$\{x_1\}$
$\{x_1, x_2, x_3\}$
$\{x_1, x_2, x_3, x_4, x_5\}$

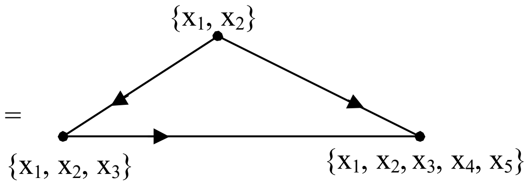
$P_6 =$


$\{x_1\}$
$\{x_1, x_2, x_3, x_4\}$
$\{x_1, x_2, x_3, x_4, x_5\}$

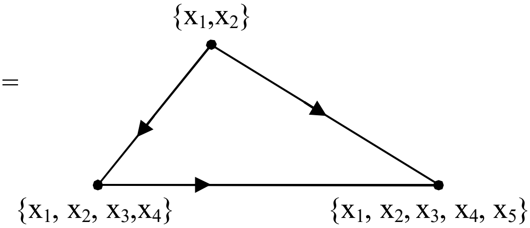
$P_7 =$


$\{x_1, x_2\}$
$\{x_1, x_2, x_3, x_4\}$
$\{x_1, x_2, x_3\}$

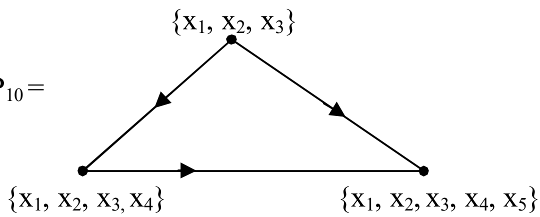
$P_8 =$



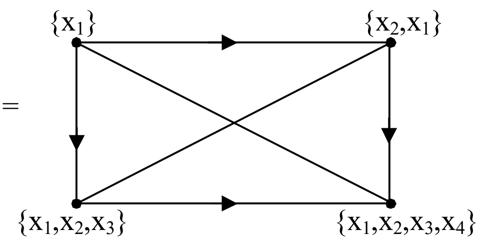
$P_9 =$



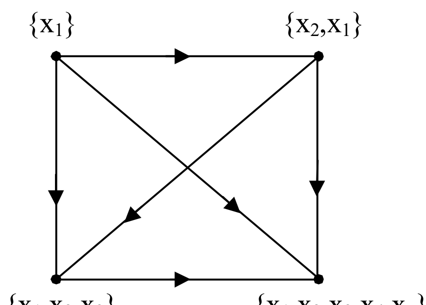
$P_{10} =$



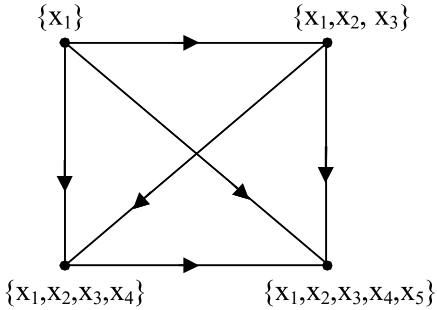
$P_{11} =$



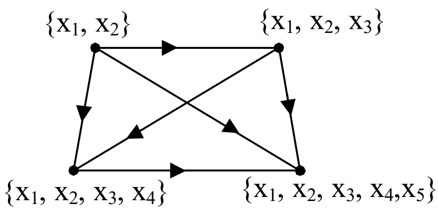
$P_{12} =$



$P_{13} =$



$P_{14} =$



$P_{15} =$

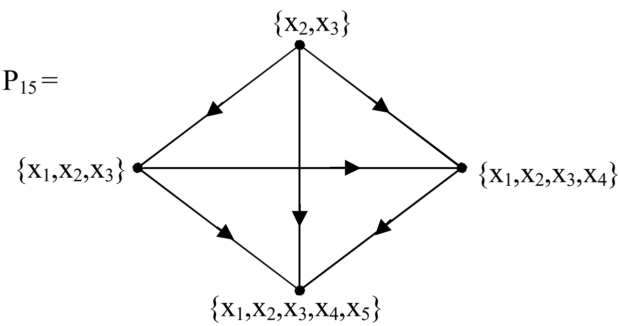


Figure 3.32

All vertex subset subgraph of type II with cardinality of vertex set greater than or equal to three are all subset vertex tournaments.

There are 15 subset vertex subgraphs of type II which are tournaments.

Now we define near tournament in case of subset vertex graphs of type II which are tournaments.

We first describe them by following figures.

G =

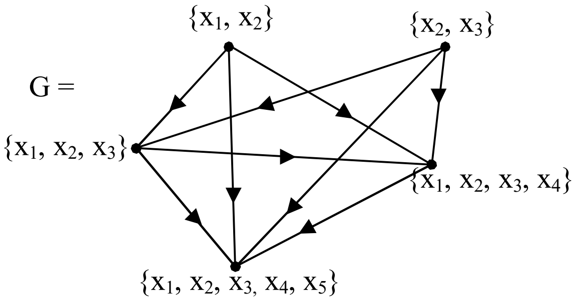
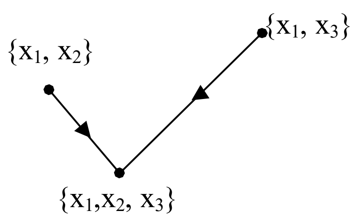


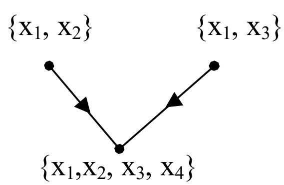
Figure 3.33

We find all subset vertex subgraphs of type I whose vertex set is greater than or equal to three.

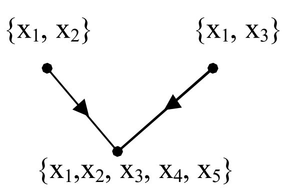
X_1 =



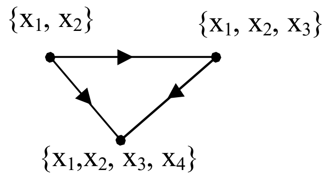
X_2 =



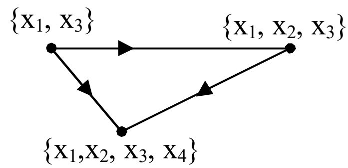
X_3 =



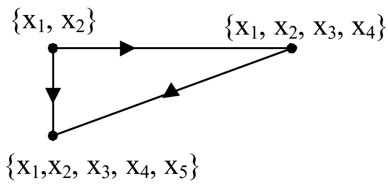
X_4 =



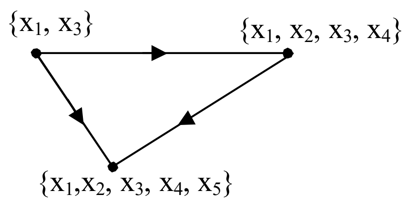
X_5 =



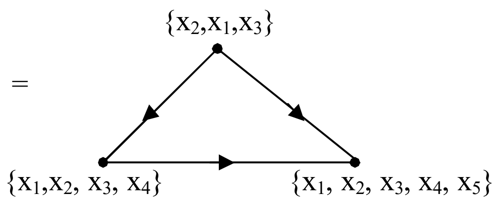
X_6 =



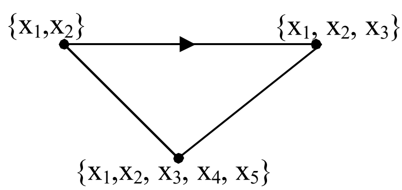
X_7 =



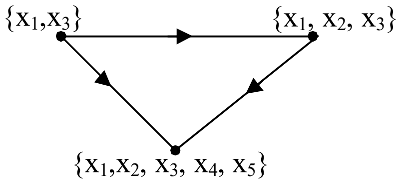
X_8 =



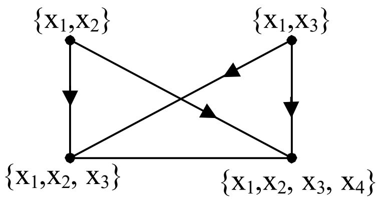
X_9 =



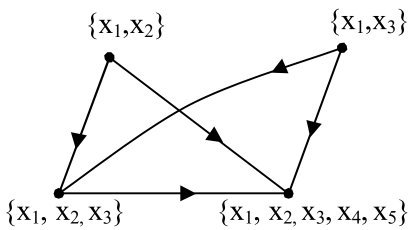
X_{10} =



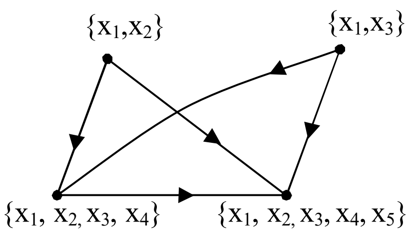
X_{11} =



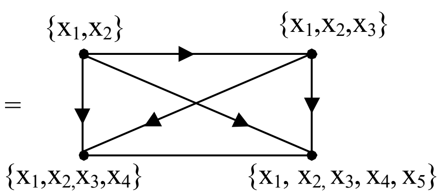
X_{12} =



X_{13} =



X_{14} =



X_{15} =

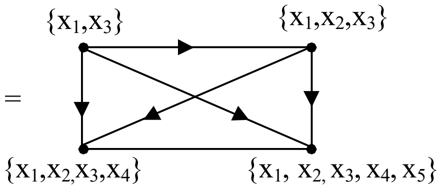


Figure 3.34

We see of the 15 subset vertex subgraphs of the subset vertex graph G of type II satisfy the following conditions.

9 of the subset vertex graphs of the subset vertex graph G of type II are subset vertex tournaments.

Some of the subset vertex subgraphs of type II are not tournaments. The subset vertex graph G_2 is not a tournament. Thus we make the following definition.

Definition 3.2 *Let P(S) be the power set of the set S = {S = {1, 2, ..., n}; $(2 \leq n < \text{infinity})$. A subset vertex graph of type II with m vertices, $3 \leq m < 2^n$ is said to be a pseudo semi subset vertex tournament if there are at least $\frac{m}{2}$ vertices (m even) and $\frac{m+1}{2}$ vertices (if m is odd) for which there are exactly $(m - 1)$ edges incident to it and the rest have $\frac{m}{2}$ edges or more incident to them.*

We will provide examples of them.

Consider S = $\{x_1, x_2, \ldots, x_{18}\}$ be the set with 18 elements P(S) be the power set of S.

We will proceed onto give examples of pseudo semi subset vertex tournaments of different types built using the vertex set from P(S).

G =

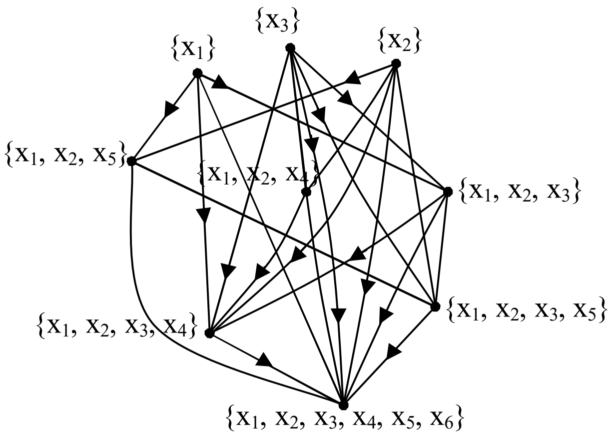


Figure 3.35

Clearly G with vertex set as $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2, x_5\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_5\}\}$ is not a subset vertex pseudo semi tournament of type II.

It is only a directed subset vertex type II graph with V as its vertex set. So all subset vertex graphs of type I built using the subsets of P(S) as vertex set need not in general be a tournament or a pseudo semi tournament.

Let V $=\{\{x_1, x_2\}, \{x_2, x_3\}.\{x_3, x_1\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_5\}, \{x_1, x_2, x_3, x_6\}, \{x_1, x_2, x_3, x_7\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}\}$ be vertex set in P(S). The subset vertex graph G of type II associated with V is as follows.

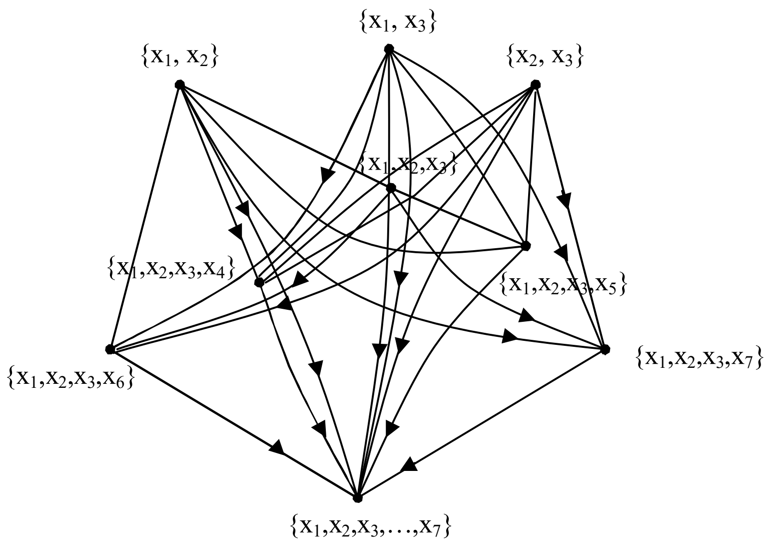


Figure 3.36

We put the table relating the vertices and edges to verify whether this subset vertex graph to G of type II is a tournament or a quasi semi tournament or neither.

Vertex set	Number of edges
$\{x_1, x_2\}$	6
$\{x_1, x_3\}$	6
$\{x_2, x_3\}$	6
$\{x_1, x_2, x_3\}$	8
$\{x_1, x_2, x_3, x_4\}$	5
$\{x_1, x_2, x_3, x_5\}$	5
$\{x_1, x_2, x_3, x_6\}$	5
$\{x_1, x_2, x_3, x_7\}$	5
$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$	8

Clearly G is neither a subset vertex tournament nor a pseudo semi subset vertex tournament.

Consider the vertex set V = {{x_1}, {x_1, x_2}, {x_1, x_2, x_3}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5, x_6}, {x_1, x_2, x_3, x_4, x_5, x_6, x_7}} $\subseteq$ P(S). Let H be the subset vertex graph of type II associated with V given by the following figure.

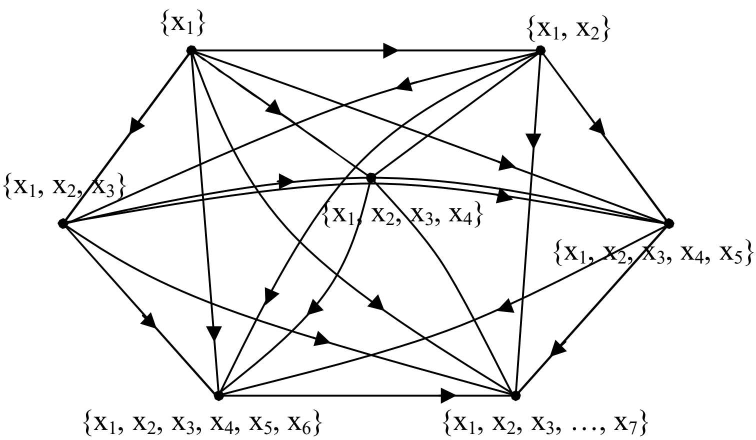


Figure 3.37

The table associated with this subset vertex graph H of type II is as follows.

S. No.	Vertex set	Number of edges
1	{x_1}	6
2	{x_1, x_2}	6
3	{x_1, x_2, x_3}	6
4	{x_1, x_2, x_3, x_4}	6
5	{x_1, x_2, x_3, x_4, x_5}	6
6	{x_1, x_2, x_3, x_4, x_5, x_6}	6
7	{x_1, x_2, x_3, x_4, x_5, x_6, x_7}	6

Clearly we see H is a vertex subset type II graph which is a vertex subset tournament.

In view of all these we make the following observations.

First we observe the vertex set V associated with the graph H can be ordered by the inclusion relation $\subseteq$.

So that V is a chain given in the following.

$\{x_1\} \subseteq \{x_1, x_2\} \subseteq \{x_1, x_2, x_3\} \subseteq \{x_1, x_2, x_3, x_4\} \subseteq \{x_1, x_2, x_3, x_4, x_5\} \subseteq \{x_1, x_2, x_3, x_4, x_5, x_6\} \subseteq \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ … I

Clearly I is a chain of length 7 or is a totally ordered set by the inclusion relation.

Thus a chain of length 7 contributes to a subset vertex graph H of type II which is a subset vertex tournament or a complete directed graph and for each vertex we have exactly $(7 - 1) = 6$ edges incident to it.

In view of all these fact we put forth the following theorem.

Theorem 3.2. *Let $P(S)$ be the power set of a set $S = \{x_1, x_2, \ldots, x_n\}$. Let V be a vertex set which is a chain of length m (that is $|V| = m$). The vertex subset graph G of type II associated with V is a subset vertex tournament or a directed complete graph which has $(m - 1)$ edges incident with each vertex of G.*

Proof is direct and hence left as an exercise to the reader.

It is left as an open problem for the researchers to find the total number of subset vertex graphs of type II with vertex set from P(S); |S| = n; $3 \leq n < \infty$ which are directed complete graphs or subset vertex tournament.

Consider the vertex set of P(S) where |S| = 9 where V $= \{\{x_1, x_2\}, \{x_2, x_4\}, \{x_1, x_2, x_4\}, \{x_1, x_2, x_4, x_5\}, \{x_1, x_2, x_4, x_5, x_3\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}\} \subseteq P(S)$.

Clearly V is not a totally ordered set under inclusion relation.

$\{x_1, x_2\}$ $\qquad\qquad$ $\{x_2, x_4\}$

$\subseteq$ $\qquad\qquad$ $\subseteq$

$\{x_1, x_2, x_4\}$

$\subseteq$

$\{x_1, x_2, x_4, x_5\}$

$\subseteq$

$\{x_1, x_2, x_4, x_5, x_3\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, x_5, x_6\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, \ldots, x_7\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, \ldots, x_7, x_8\}$

Figure 3.38

The subset vertex graph K of type II related with the vertex set V is as follows.

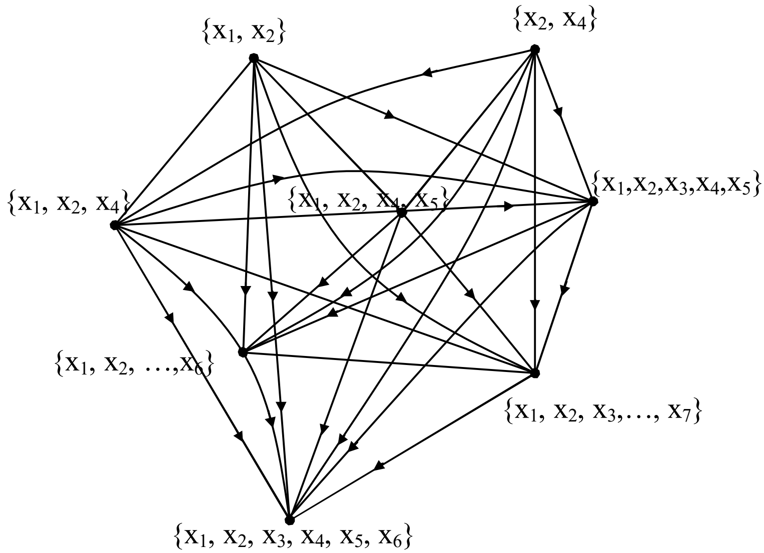


Figure 3.39

Now we give the vertex edge table associated with the subset vertex graph of type II in the following.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1, x_2\}$	6
2	$\{x_2, x_4\}$	6
3	$\{x_1, x_2, x_4\}$	7
4	$\{x_1, x_2, x_4, x_5\}$	7
5	$\{x_1, x_2, x_3, x_4, x_5\}$	7
6	$\{x_1, x_2, x_3, x_4, x_5, x_6\}$	7
7	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$	7
8	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$	7

We see the subset vertex graph of type II is a pseudo semi tournament or a subset vertex tournament.

` Consider the following vertex set V = $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{x_1, x_2, x_3, x_4, \ldots, x_8\}\} \subseteq P(S)$.

This V is not a totally ordered set under the inclusion relation. This is only partially ordered and is given in the following.

$\{x_1\}$ $\qquad$ $\{x_2\}$ $\qquad$ $\{x_3\}$

$\subseteq$ $\qquad$ $\subseteq$ $\qquad$ $\subseteq$

$\{x_1, x_2, x_3\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, x_5\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, x_5, x_6\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

Figure 3.40

The subset vertex graph M of type II with vertex set V is as follows.

M =

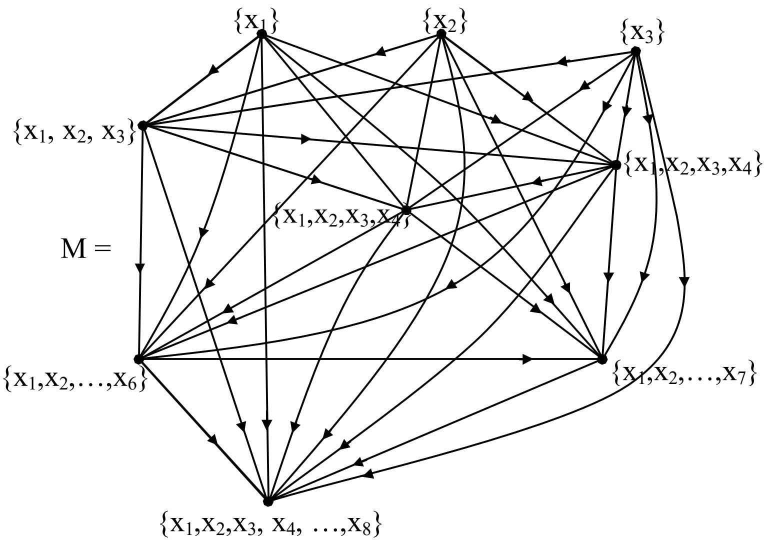


Figure 3.41

The table associated with the subset vertex graph M of type I is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1\}$	6
2	$\{x_2\}$	6
3	$\{x_3\}$	6
4	$\{x_1, x_2, x_3\}$	8
5	$\{x_1, x_2, x_3, x_4\}$	8
6	$\{x_1, x_2, x_3, x_4, x_5\}$	8
7	$\{x_1, x_2, x_3, x_4, x_6\}$	8
8	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$	8
9	$\{x_1, x_2, \ldots, x_8\}$	8

Clearly M is not a subset vertex pseudo semi quasi tournament.

We see M has subgraphs which are tournaments or subset vertex quasi semi tournaments.

We give yet another example where $V_1 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1,x_2\}, \{x_1, x_3\}, \{x_3, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{x_1, x_2, \ldots, x_8\}\} \subseteq P(S)$.

Let P be the subset vertex graph of type II given by the following figure using V_1 as the vertex set.

P =

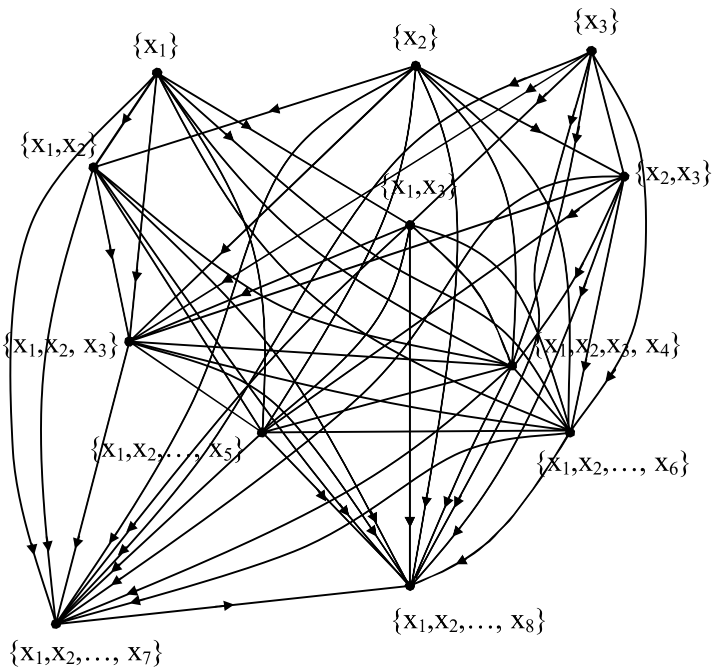


Figure 3.42

The partial ordered relation of the set V is described in the following.

$\{x_1\}$ $\quad$ $\{x_2\}$ $\quad$ $\{x_1, x_3\}$ $\quad$ $\{x_3\}$

$\subseteq$ $\quad$ $\subseteq$ $\quad$ $\subseteq$ $\quad$ $\subseteq$

$\{x_1, x_2\}$ $\quad$ $\{x_2, x_3\}$

$\subseteq$ $\quad$ $\subseteq$

$\{x_1, x_2, x_3\}$

$\subseteq$

$\{x_1, x_2, x_3, x_4\}$

$\subseteq$

$\{x_1,x_2, \ldots, x_5\}$

$\subseteq$

$\{x_1, x_2, \ldots, x_6\}$

$\subseteq$

$\{x_1, x_2,\ldots, x_7\}$

$\subseteq$

$\{x_1, x_2, .., x_8\}$

Figure 3.43

The table related with the subset graph P of type II is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1\}$	8
2	$\{x_2\}$	8
3	$\{x_3\}$	8
4	$\{x_1, x_2\}$	8
5	$\{x_1, x_3\}$	8
6	$\{x_2, x_3\}$	8
7	$\{x_1, x_2, x_3\}$	11
8	$\{x_1, x_2, x_3, x_4\}$	11
9	$\{x_1, x_2, x_3, x_4, x_5\}$	11
10	$\{x_1, x_2, x_3, \ldots, x_6\}$	11
11	$\{x_1, x_2, x_3, \ldots, x_7\}$	11
12	$\{x_1, x_2, x_3, \ldots, x_8\}$	11

Cleary from the table it is clear P is a subset vertex quasi semi tournament of type II. This has subgraphs which are both tournament as well as pseudo semi tournaments.

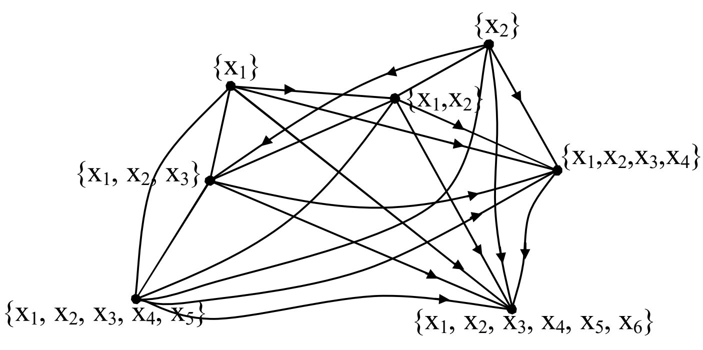


Figure 3.44

Clearly T_1 is a subgraph of P which is only a pseudo semi tournament subset vertex subgraph.

The table associated with T_1 is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1\}$	5
2	$\{x_2\}$	5
3	$\{x_1, x_2\}$	6
4	$\{x_1, x_2, x_3\}$	6
5	$\{x_1, x_2, x_3, x_4\}$	6
6	$\{x_1, x_2, x_3, x_4\}$	6
7	$\{x_1, x_2, \ldots, x_6\}$	6

Let T_2 be the subset vertex graph of the graph P given in the following.

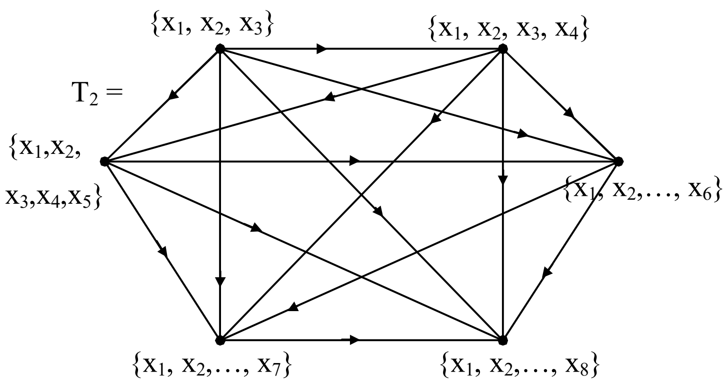


Figure 3.45

The table associated with the subset vertex subgraph of P is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1, x_2, x_3\}$	5
2	$\{x_1, x_2, x_3, x_4\}$	5
3	$\{x_1, x_2, x_3, x_4, x_5\}$	5
4	$\{x_1, x_2, \ldots, x_6\}$	5
5	$\{x_1, x_2, \ldots, x_7\}$	5
6	$\{x_1, x_2, \ldots, x_8\}$	5

Clearly from this table and the figure of T_2 it is clear. T_2 is a subset vertex sub tournament of type II. Further T_2 is a subset vertex complete directed graph.

We also see for the vertex $\{x_1, x_2, x_3\}$ all the edges are out coming and on the contrary all the edges of the vertex $\{x_1, x_2, \ldots, x_8\}$ are incoming.

Thus T_2 is the biggest subset vertex tournament of P that is T_2 is the directed complete graph of the pseudo semi subset vertex tournament P.

Consider the subset vertex subgraph of P given by the following vertex set W = $\{\{x_1\}, \{x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, \ldots, x_7\}, \{x_1, x_2, x_3, x_4, x_5, \ldots, x_8\} \subseteq P(S)$.

The subset vertex graph W of type II contributed by W is as follows.

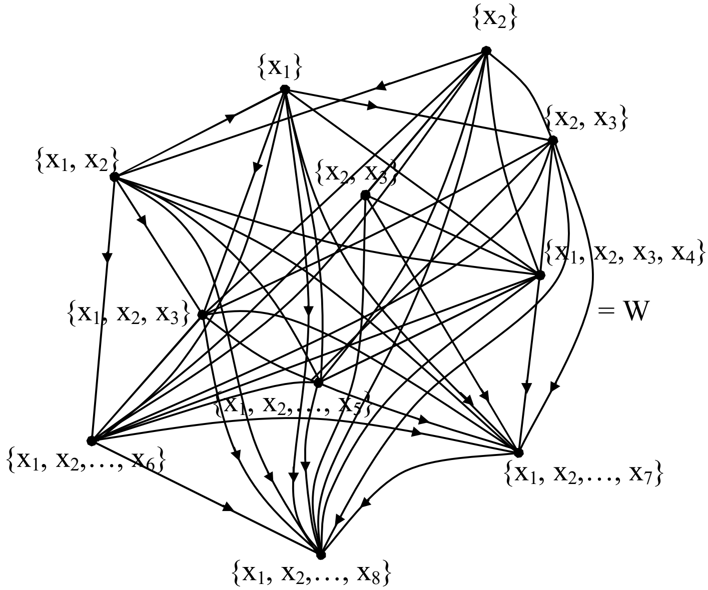


Figure 3.46

In this table we give the following relation between number of edges and incident with each node.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1\}$	8
2	$\{x_2\}$	8
3	$\{x_1, x_2\}$	8
4	$\{x_2, x_3\}$	8
5	$\{x_3, x_1\}$	8
6	$\{x_1, x_2, x_3\}$	10
7	$\{x_1, x_2, x_3, x_4\}$	10
8	$\{x_1, x_2, x_3, x_4, x_5\}$	10
9	$\{x_1, x_2, x_3, \ldots, x_6\}$	10
10	$\{x_1, x_2, \ldots, x_7\}$	10
11	$\{x_1, x_2, \ldots, x_8\}$	10

Clearly W is a subset vertex graph of type II which is a subset vertex pseudo semi tournament.

We see W has both subgraphs which are subset vertex tournaments as well as subset vertex pseudo semi tournaments. The removal of one vertex results only in a subset vertex semi quasi tournament given by the following figure.

$P_1 =$

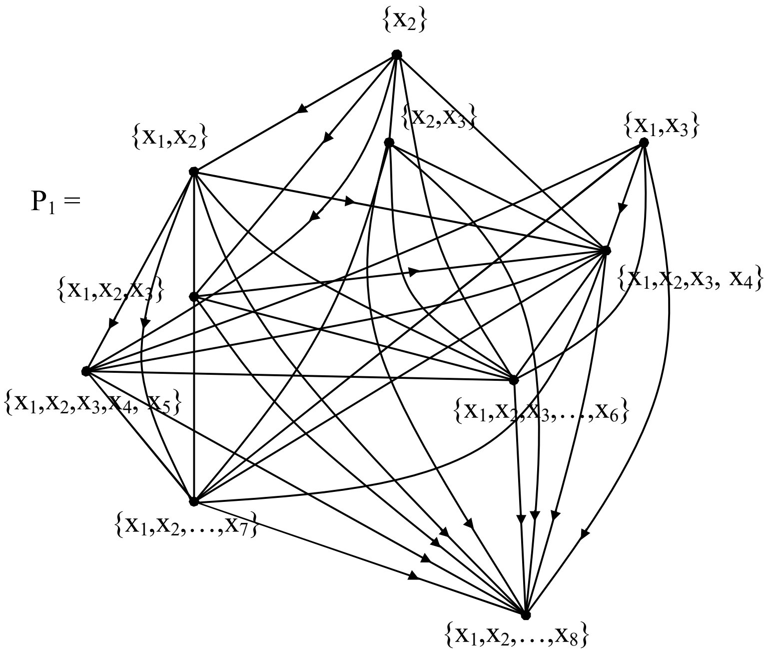


Figure 3.47

Clearly P_1 is only a quasi semi tournament and is not a tournament.

The table associated with this subset vertex quasi semi tournament P_1 is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_2\}$	6
2	$\{x_1, x_2\}$	6
3	$\{x_1, x_3\}$	6
4	$\{x_2, x_3\}$	6
5	$\{x_1, x_2, x_3\}$	9
6	$\{x_1, x_2, x_3, x_4\}$	9
7	$\{x_1, x_2, \ldots, x_5\}$	9
8	$\{x_1, x_2, \ldots, x_6\}$	9
9	$\{x_1, x_2, \ldots, x_7\}$	9
10	$\{x_1, x_2, \ldots, x_8\}$	9

Let P_2 be the subset vertex subgraph got from W which is given by the following figure

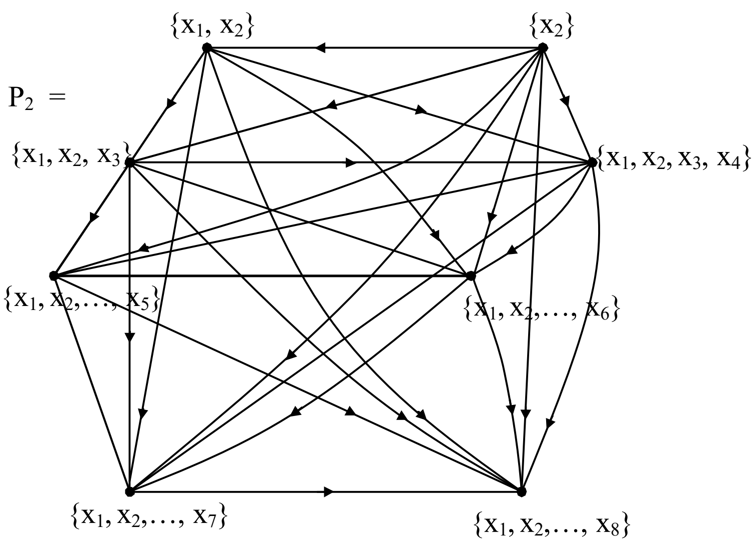


Figure 3.48

Clearly P_2 is a subset vertex tournament.

The table associated with P_2 is as follows.

S. No.	**Vertex set**	**Number of edges incident to the vertex**
1	$\{x_2\}$	7
2	$\{x_1, x_2\}$	7
3	$\{x_1, x_2, x_3\}$	7
4	$\{x_1, x_2, x_3, x_4\}$	7
5	$\{x_1, x_2, x_3, x_4, x_5\}$	7
6	$\{x_1, x_2, \ldots, x_6\}$	7
7	$\{x_1, x_2, \ldots, x_7\}$	7
8	$\{x_1, x_2, \ldots, x_8\}$	7

Thus from the vertex subset pseudo semi tournament if the vertices $\{\{x_1\}, \{x_1, x_3\}$ and $\{x_2, x_3\}\}$ is removed the resultant subset vertex subgraph is a subset vertex tournament.

Now we want to study whether the removal of three vertices are needed to be the subset vertex pseudo semi tournament or tournament or can we say is that set minimal to make it tournament. To this end we add one more vertex arbitrarily say $\{x_2, x_3\}$ to the vertex set of P_2 and give the related graph P_3 in the following.

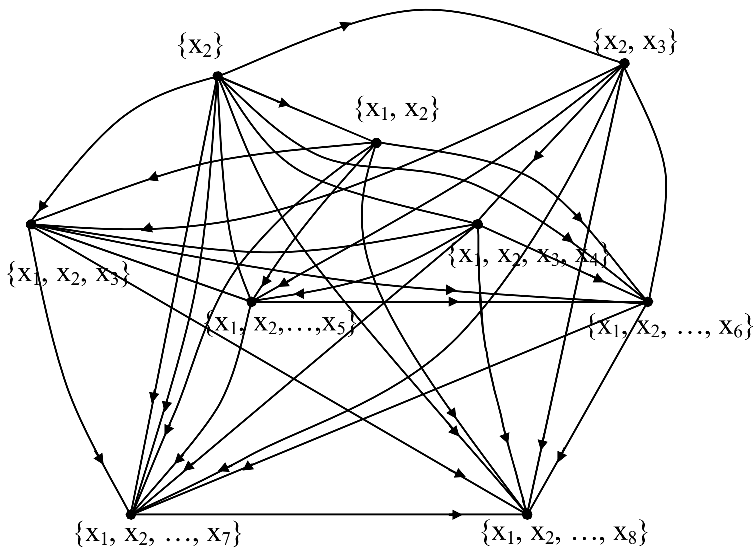


Figure 3.49

Now we give the table related with P_2.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_2\}$	8
2	$\{x_2, x_3\}$	7
3	$\{x_1, x_2, x_3\}$	8
4	$\{x_1, x_2, x_3, x_4\}$	8
5	$\{x_1, x_2, \ldots, x_5\}$	8
6	$\{x_1, x_2, \ldots, x_6\}$	8
7	$\{x_1, x_2, \ldots, x_7\}$	8
8	$\{x_1, x_2, x_3, \ldots, x_8\}$	8
9	$\{x_1, x_2\}$	7

It is pertinent to keep on record that we can have many subsets of the vertex set of W which can give subset vertex biggest tournaments.

For instance

A = $\{\{x_2\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, \ldots, x_7\}$ and $\{x_1, x_2, x_3, \ldots, x_8\}\}$

$A_2 = \{\{x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, \ldots, x_7\}$ and $\{x_1, x_2, x_3, \ldots, x_8\}\}$

$A_3 = \{\{x_1\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, \ldots, x_5\}, \{x_1, x_2, \ldots, x_6\}, \{x_1, x_2, \ldots, x_7\}, \{x_1, x_2, \ldots, x_8\}\}$ and $A_4 = \{\{x_1\}, \{x_3, x_1\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \{x_1, x_2, \ldots, x_5\}, \{x_1, x_2, \ldots, x_5, x_6\}, \{x_1, x_2, \ldots, x_7\}$ and $\{x_1, x_2, \ldots, x_8\}\}$. These are the only vertex subsets which can contribute to the biggest subset vertex tournaments built using the subset vertex set from the vertex subset of W and no more.

Now in view of all these we make the following definition.

Definition 3.3. *Let P(S) be a power set of a set S, where $S = \{x_1, x_2, \ldots, x_m\}$. W be a vertex subset in P(S) of order n ($|W| = n$; $n < 2^m$) such that the subset vertex graph associated with W is a subset vertex pseudo semi tournament B. If $T \subseteq W$ is a vertex subset in W that is the smallest one when removed from W yields a subset vertex subgraph Z using the vertex subset $W \setminus T$ which is a subset vertex tournament or subset vertex complete directed graph then we define this subset vertex* *complete directed graph to be the pseudo semi hyper tournament of B with W and vertex subset T as the discarded or cut of vertex subset of the subset vertex pseudo semi tournament B.*

In the first place we first put forth the following open problems which can also be viewed as open conjectures.

Conjecture 3.1. Let P(S) be the power set of the set S = $\{x_1, x_2, \ldots, x_n\}$; $2 < n < \infty$.

i) How many subset vertex graphs of type II exist using the vertex set from P(S) which are subset vertex tournaments?

ii) How many subset vertex graphs of type II exist using vertex set from P(S) which are subset vertex pseudo semi tournaments?

iii) What is the order of the biggest complete directed graph using the vertex subsets of P(S)?

iv) Find the largest subset vertex pseudo semi tournament A using the vertex subsets of P(S).

v) Find the order of the pseudo hyper semi tournament of A.

vi) What is the order of the discarded or cut of vertex subset of the subset vertex pseudo semi hyper tournament A in question (iv).

We try to describe the subset vertex graphs of type II using P(S) where S = 2, 3, 4.

Let P(S) = {{x_1}, {x_2}, {x_1, x_2}, ϕ} be the power set of S = {x_1, x_2}.

Using the vertex set as P(S) we have the following subset vertex graph of type II.

T_1 =

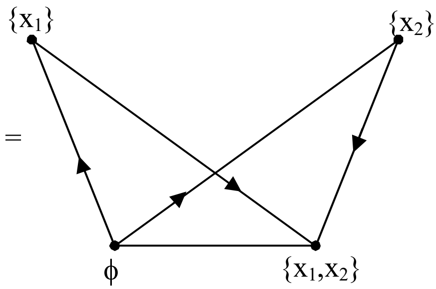


Figure 3.50

Clearly T_1 is only a subset vertex semi quasi tournament. The table associated with T_1 is as follows.

S. No.	Vertex set	Number of edges incident to the vertex
1	{x_1}	2
2	{x_2}	2
3	{x_1, x_2}	3
4	ϕ	3

P(S) \ {x_1} and P(S) \ {x_2} gives subset vertex tournament of order three or the subset vertex directed complete graphs given in the following.

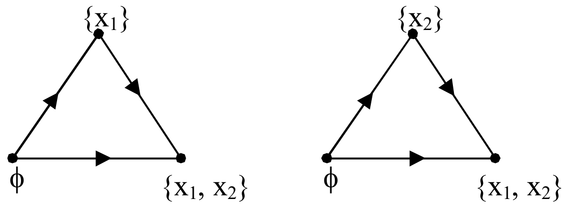


Figure 3.51

Now we give the subset vertex graph T_2 of type II using P(S) to be the vertex set where S = $\{x_1, x_2, x_3\}$.

T_2 =

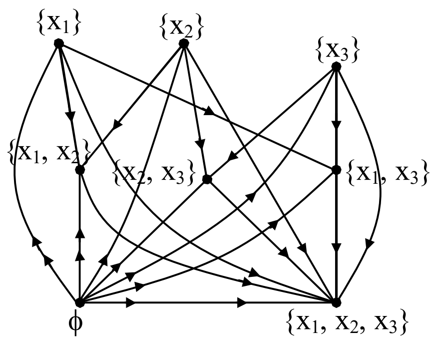


Figure 3.52

Clearly T_2 is not a subset vertex pseudo semi tournament. The table associated with T_2 is given in the following.

S. No.	Vertex set	Number of edges incident to the vertex
1	$\{x_1\}$	4
2	$\{x_2\}$	4
3	$\{x_3\}$	4
4	$\{x_1, x_2\}$	4
5	$\{x_1, x_3\}$	4
6	$\{x_2, x_3\}$	4
7	$\{x_1, x_2, x_3\}$	7
8	$\{\phi\}$	7

Clearly looking at the table we can easily conclude that T_2 is not a subset vertex pseudo semi tournament.

So we find the largest subset vertex pseudo semi tournament that can be built using the elements from the vertex set $B \subseteq P(S)$.

Consider $B = \{\{x_1\}, \{x_3, x_1\}, \{x_1, x_2, x_3\}, \{\phi\}\} \subseteq P(S)$.

The subset vertex graph D_1 of type II associated with the vertex subset B is as follows.

$D_1 =$

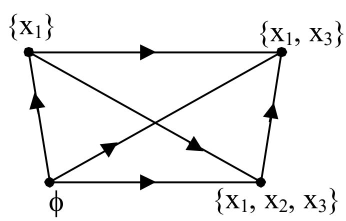


Figure 3.53

Clearly D_1 is a complete directed subset vertex graph or the subset vertex tournament.

We have the following to be subset vertex tournaments of order 4 given by the following figure.

$D_2 =$

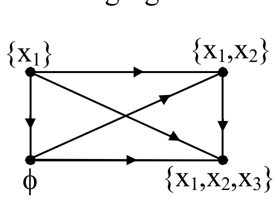


Figure 3.54

$D_3 =$

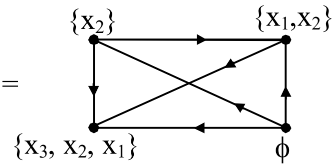


Figure 3.55

$D_4 =$

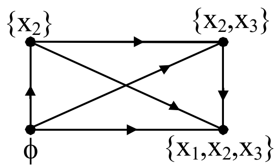
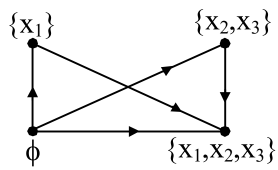


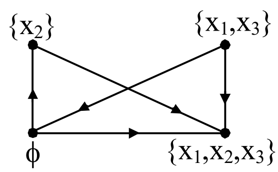
Figure 3.56

Now we give the subset vertex pseudo semi tournaments of order four

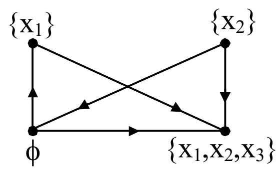
$C_1 =$



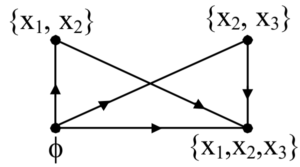
$C_2 =$



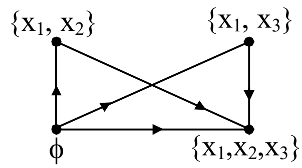
$C_3 =$



$C_4 =$



$C_5 =$



$C_6 =$

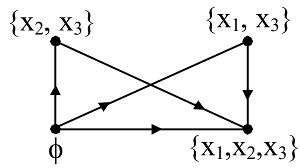


Figure 3.57

Clearly C_1 to C_6 are only subset vertex pseudo semi tournaments if any one vertex with two edges adjacent to it is removed then these C_i contain subset vertex tournaments of order B; $1 \leq i \leq 6$.

Now we try to work with five vertices from the set P(S) and describe them by the following figures.

$F_1 =$

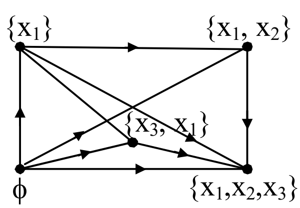


Figure 3.58

The table associated with the vertex subset graph which is not a quasi semi tournament F_1 is as follows.

S. No.	Vertex set	Number of edges incident with each vertex
1	$\{x_1\}$	4
2	$\{x_1, x_2\}$	3
3	$\{x_3, x_1\}$	3
4	$\{\phi\}$	5
5	$\{x_1, x_2, x_3\}$	5

Since the edges are distributed non uniformly F_1 is not a vertex subset pseudo semi tournament.

Next we proceed onto study P(S) the power set of S = $\{x_1, x_2, x_3, x_4\}$; $|P(S)| = 16$.

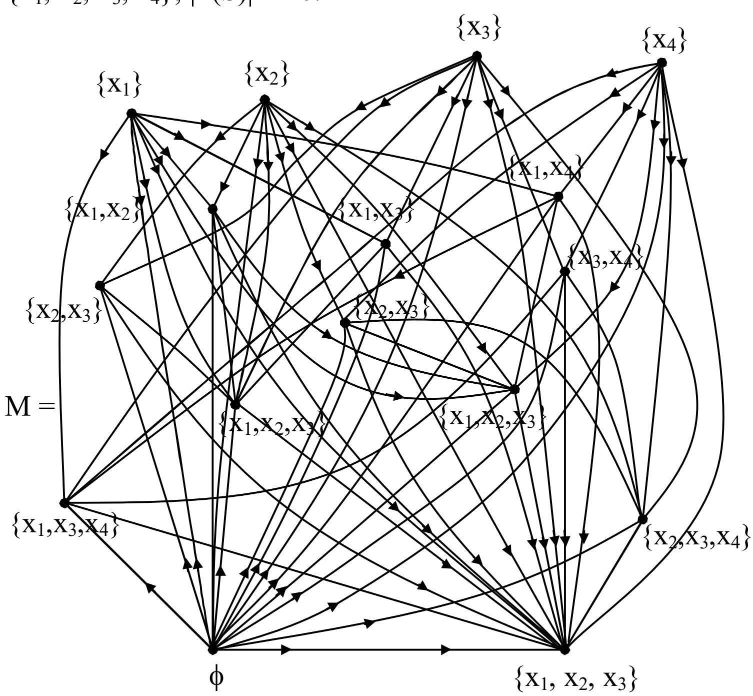


Figure 3.59

M is the subset vertex graph of type II taking all the 16 nodes from P(S).

The table associated with M is as follows.

S. No.	**Vertex set**	**Number of lines incident with each vertex**
1	$\{x_1\}$	8
2	$\{x_2\}$	8
3	$\{x_3\}$	8
4	$\{x_4\}$	8
5	$\{x_1, x_2\}$	6
6	$\{x_1, x_3\}$	6
7	$\{x_1, x_4\}$	6
8	$\{x_2, x_3\}$	6
9	$\{x_2, x_4\}$	6
10	$\{x_3, x_4\}$	6
11	$\{x_1, x_2, x_3\}$	8
12	$\{x_1, x_2, x_4\}$	8
13	$\{x_1, x_3, x_4\}$	8
14	$\{x_2, x_3, x_4\}$	8
15	$\{x_1, x_2, x_3, x_4\}$	15
16	ϕ	15

We see M is not a subset vertex pseudo semi tournament only a subset vertex graph of type II.

In view of all these we put forth the following result.

Theorem 3.2. *Let P(S) be the power set of the set S; S = {x_1, x_2, …, x_n}. The subset vertex graph of type II with |P(S)| = 2^n vertices is never a subset vertex semi pseudo tournament.*

Proof. Follows from the simple fact that the nodes {x_1, …, x_n} and ϕ have the maximum number of edges incident to it; viz (n – 1). No other node has (n – 1) edges incident to it.

Further the nodes which are vertices have varying degrees hence the claim. Only when n = 2 the power set P(S) contributes to pseudo semi tournament.

Further the nodes {x_1}, {x_2}, …, {x_n} have only 2^{n-1} nodes incident to it.

Next we proceed onto study the star graph got using P(S).

We see if P(S) is the power set of S where S = {x_1, x_2} then we have two subset vertex star graphs given by

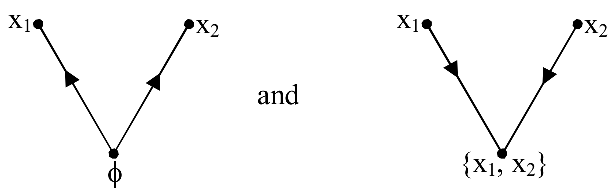


Figure 3.60

Clearly both are distinct.

Consider P(S) the power set of S = {x_1, x_2, x_3}.

The star graphs are

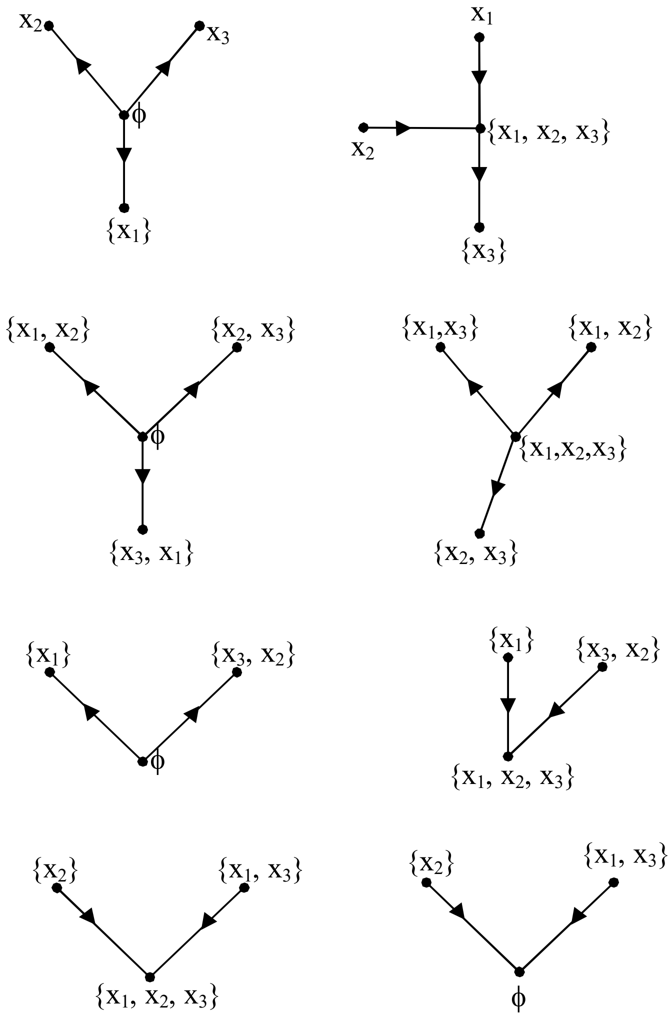


Figure 3.61

The star graph with four nodes is the largest. We see further we cannot have more than four nodes.

We make the following observations.

The star graphs are such that they are dual in a very special way for all nodes in one case is inward and in other all the nodes are outwards. Consider the power set P(S) of the set S where S = $\{x_1, x_2, x_3, x_4\}$.

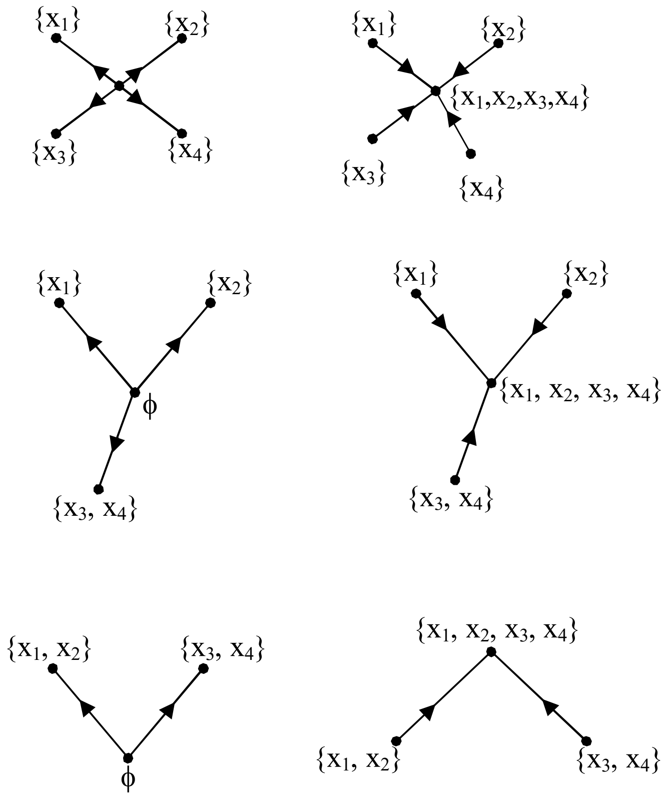


Figure 3.62

Let P(S) be the power set of the set S = $\{x_1, x_2, x_3, x_4, x_5\}$. The subset vertex star graph of type II is as follows.

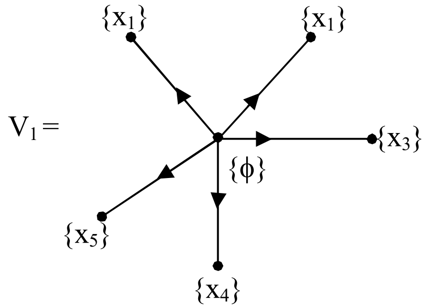


Figure 3.63

Star graph with all edges outward from the central nodes.

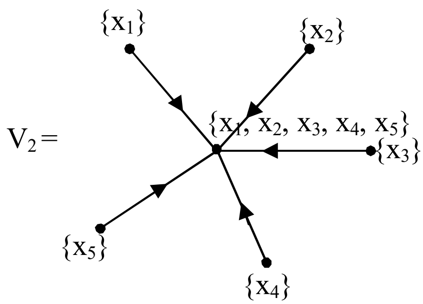


Figure 3.64

This is also subset vertex star graph V_2 where all the edges get inwardly compressed where as V_1 gets dispersed from the empty centre.

At this juncture authors ponder over the philosophy of life when nothingness or emptiness is at the centre all arrows fly out where as when everything is present in the center the arrows get compressed at the center.

For these type of notion will be described in the forthcoming book on philosophy and mathematics.

Can we also find wheels in subset vertex graphs of type II?

Does there exist line graphs in case of subset vertex graphs of type II of length greater than two?

Can there be subset vertex circle graphs of type II?

Suppose we have a vertex set V = $\{\{x_1\}, \{x_2, x_1\}, \{x_1, x_2, x_3\}\}$.

To get the subset vertex graph of type I.

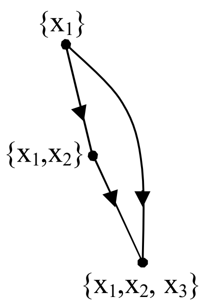


Figure 3.65

That is

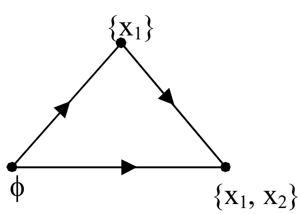


Figure 3.66

Consider the following.

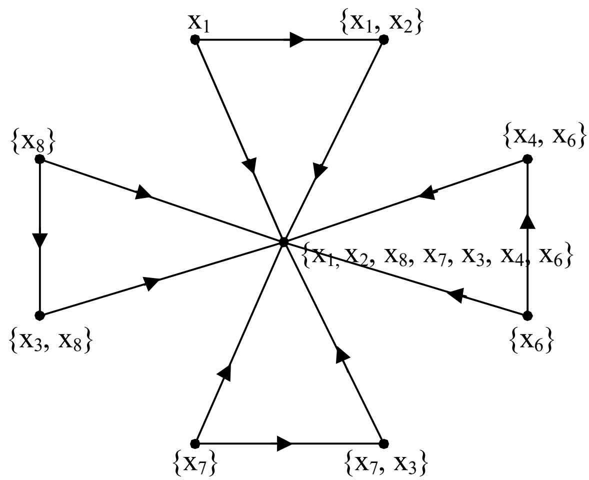


Figure 3.67

Such subset vertex graphs are possible and there are subgraphs which are complete with 3 nodes. Infact there are 4 complete directed subgraphs are tournaments.

We see these subset vertex graphs of type II happens to be directed given a set of vertices not only the graph is unique the lines connecting them are unique in direction.

We have seen the presence of tournaments and the largest tournament in the subset vertex graphs of type II.

We have also left open problems in this direction.

These new class of graphs will certainly be a loon to social network analysis more so in the community network analysis.

These subset vertex graphs are abundant in cliques (or tournaments which h are more number of vertices which directly depends on the size of the set S considered in P(S).

Thus it is left as an open problem to find the number of subset vertex tournaments of these subsets vertex graphs of type II using P(S) with |S| = n.

We have also said the size of the tournament depends on the length of the chain contributed by the subset vertex set.

Let us consider the following subset vertex graph of type II given by the following figure.

$M_1 =$

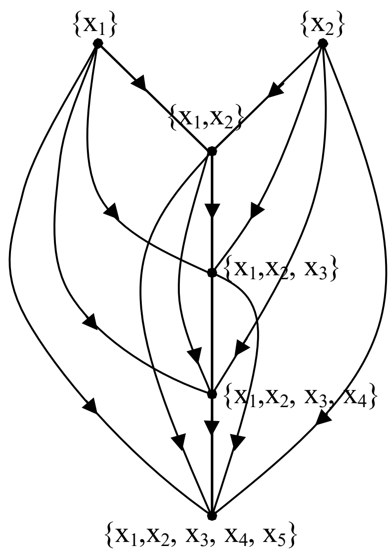


Figure 3.68

This can also be drawn little differently by not following the chain given in the figure

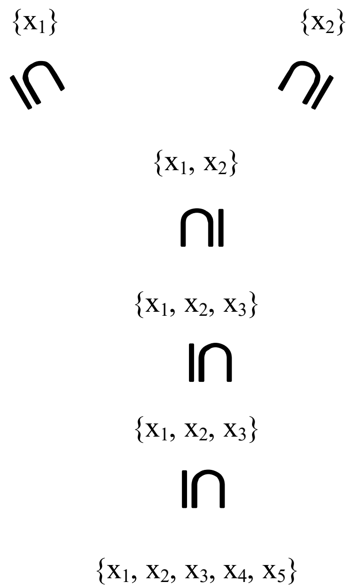


Figure 3.69

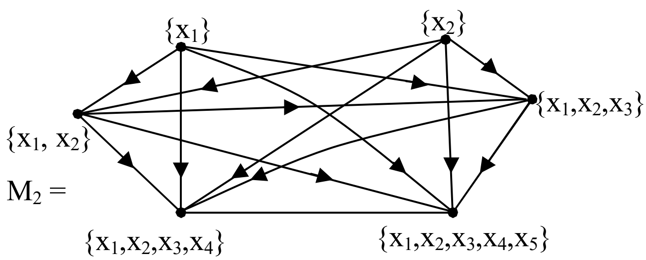


Figure 3.70

We define the subset vertex graph of type II.

They are subset vertex semi quasi tournaments.

Next we proceed onto study groups of graphs which are weakly lined. We first describe them by examples.

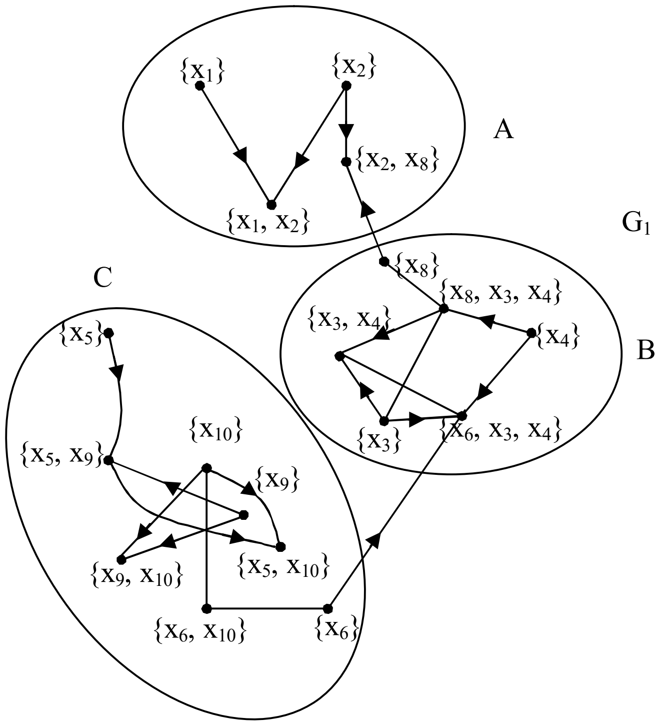


Figure 3.71

We see the figure is a subset vertex graph of type II.

This has 3 entities A, B, C where all of the three graphs are subset vertex graph of type II they are weakly linked for B links with A however A has no links directed towards B.

C links B and B is not linked directed towards C, but A and C are linked via B only. Now we call B as the Bonding graphs of A and C.

As there is no link between A and C we all such subset vertex graphs as open linked graphs in the same direction for $C \to B \to A$.

Now we give the graph as a subset vertex graph of type I we use the same set of vertices and describe the graph in the following.

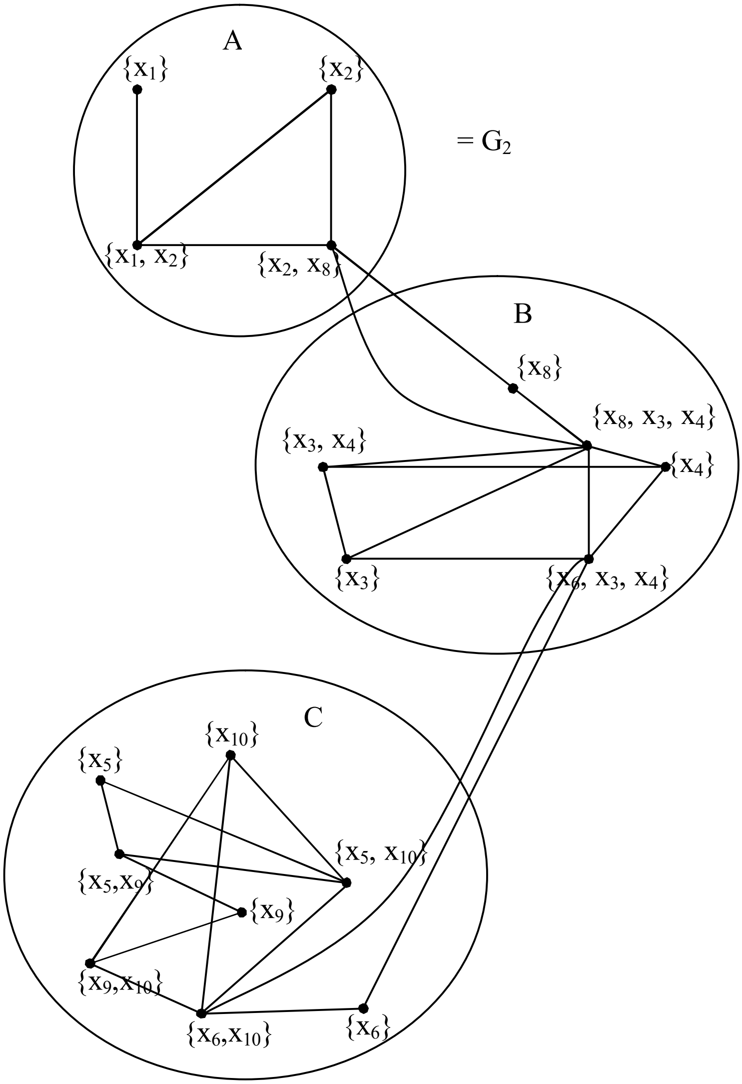


Figure 3.72

G_2 is a subset vertex graph of type I. Clearly when we try to compare G_1 with G_2 the following are to be noted.

In the first place G_2 is such that it has three components A, B and C. Only A and B are linked through 2 edges and B and C are again linked by 2 edges, however A and C are not linked as in case of G_1. The links here are strong for there are two edges but G_2 is also only a open linked graph. As in case of G_1 we cannot say B is linked to A and A has no outward link with B. Similarly in case of C.

However such things cannot be spelt out in case of G_2 if the vertex $\{x_8\}$ is removed in B the link between A and B are not removed however if vertex $\{x_2, x_8\}$ is removed in A the total links (two links) between A and B is removal.

Similarly if in the graph B the vertex $\{x_6, x_3, x_4\}$ is removed then the two links between B and C are removed. However if $\{x_6\}$ vertex from C is removed still there is one link between the subset vertex graphs C and B.

We have made two observations not only the subset vertex graphs of type I and type II are distinct but the very linking is directed in case of type II subset vertex graphs but also the number of links are more in case o f subset vertex graphs of type I, though we have used only the same vertex set. This sort of linking in case of subset vertex graphs is open if A is linked to B and B is linked to C with A not linked to C. However in both cases the subset vertex graphs A, B and C are open linked. A natural question would be if A, B and C are three subsets graphs of type I and type II. Can A, B and C the three subsets graphs is close linked of type I and open linked of type II and vice versa?

To this effect we first define what is close linked.

Let A, B, C be three subset vertex graphs of type I (or type II).

We say A, B, C are close linked if A is linked with B, B is linked with C and C is linked with A. We call this also as circle linked graphs.

After describing by a example we will proceed onto excavate more properties.

We have only discussed about only three subset vertex graphs of type I and type II we can have n of them and the linking can also be different.

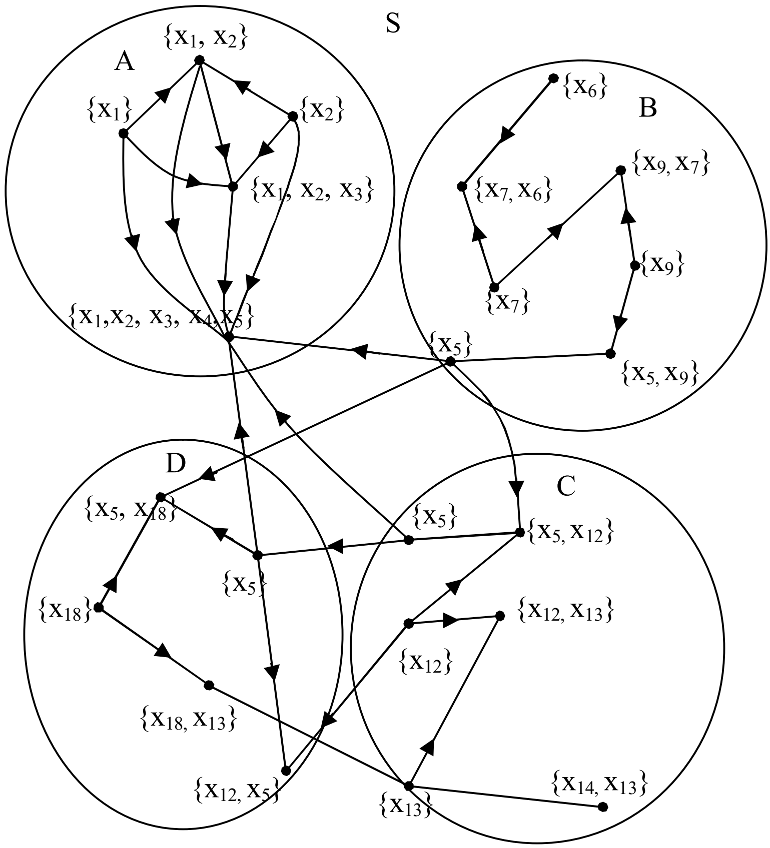


Figure 3.73

Clearly S is a linked subset graph of type II. It is not open linked or close linked. It is just linked as A is linked to B that is B → A B is linked to C, B → C, C is linked to A, C → A C is linked to D, C → D D is linked to A, D → A B is linked to D. However D is not linked to B. We call such type of links as networks or linked networks.

For the same set of vertices we now describe the subset vertex type I linked graphs.

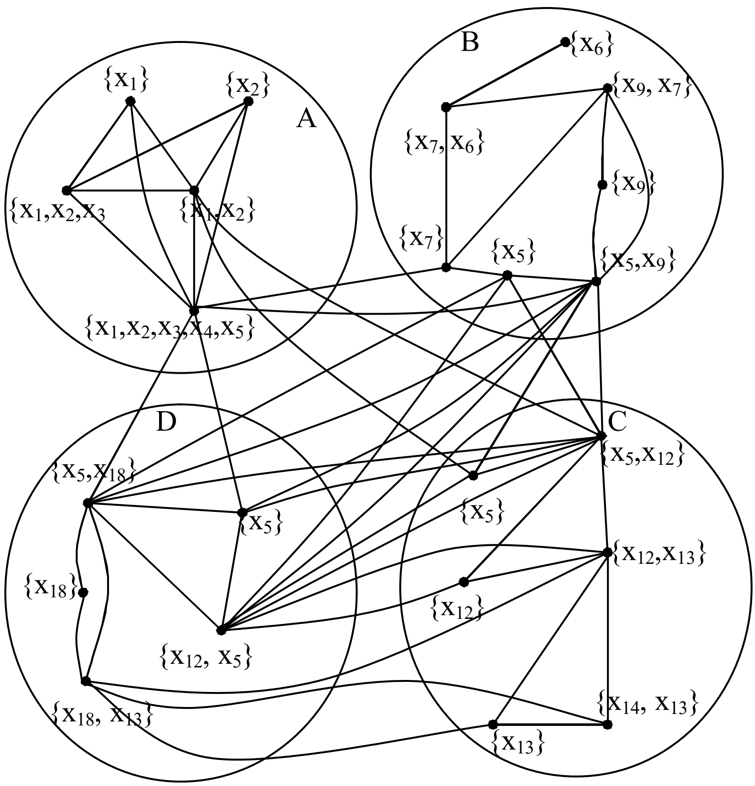


Figure 3.74

We see there are several links connecting the graphs C and D also the graphs B and C and B and D.

There are 6 links from A to B, C and D.

There are 10 links from B to A, C and D.

There are 17 links from D to A, B and C.

Finally there are 15 links from C to A, B and D.

It is easily compared that there is only few links in type II subset vertex graphs when compared with type I vertex subset graphs.

Further we see these types of subset vertex linked graphs of type I and type II can be used to study and analyse community networks.

At times the community networks may be very weak if the edges connecting the different sets of graphs are very few, otherwise the conclusion would be they are very strong ties between communities.

Further if the ties between communities that is the graphs A, B, C and D in this case we say they ties or the relationship between the groups is highly dependent. If the ties are less they (those groups) lives are fairly independent.

However it is pertinent to keep on record that links of the groups of communities become fixed automatically as the graph is the subset vertex graph of type I or type II. Further we have got some more properties of these graphs which can be easily exploited, for instance we can have the groups to enjoy subset vertex graphs of type I and the networking can be type II and vice versa.

Thus according to need and situation we can have such type of group community graphs.

We will illustrate this situation by some simple examples.

However the following facts are to be observed.

i) If the researcher/investigator is interested about to get the strongest bonds between the groups then they can study using type I graphs for linking.

ii) If they want to analyse the community network with very weak links to wants to study the independence of the groups the researcher can choose to study them using the subset vertex type II relation (graphs) for links.

Further one can also make a study of both and derive, the comparison of them and functioning of them in case of high interdependence and independence.

Thus we can have several types of community networks using type I and type II subset vertex graphs which is described in the following.

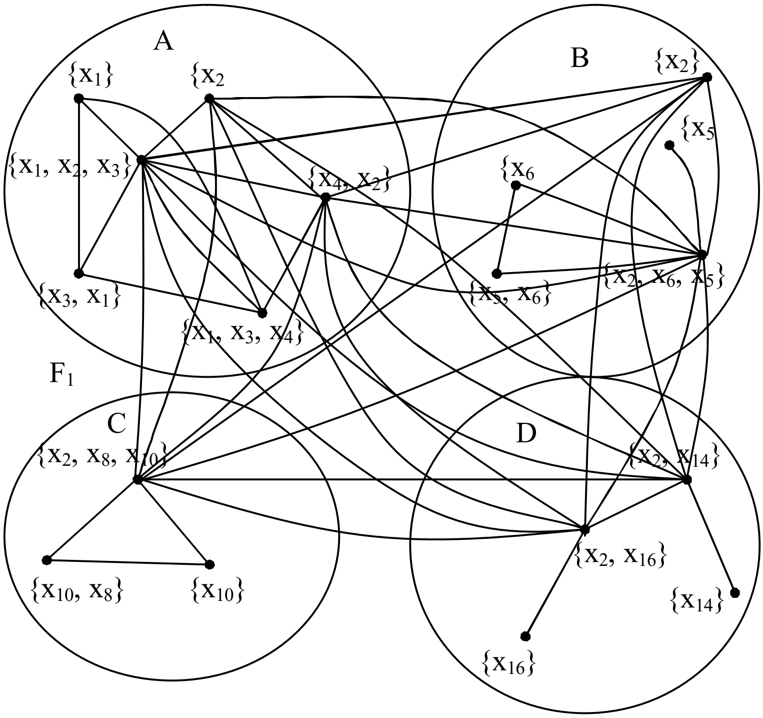


Figure 3.75

Let F_1 be the subset vertex graph of type I where the four groups of the social network use only type I graph and the related links are also only type I graphs.

Thus using type I subset vertex graphs property both among groups as well as in links we see the groups are highly interdependent among themselves. Only though the only element x_2 is present in all the four groups which forms vertex sets. Might be that node can be poverty or one specific type of works by which they earn money or so on.

Now F_2 will describe the same four groups with varying type of relation namely we will use both for the groups as well as for the links we will use subset vertex type II graphs.

We will compare them relative to type I subset graphs representing community networks both among internalities in groups and among the links among graphs.

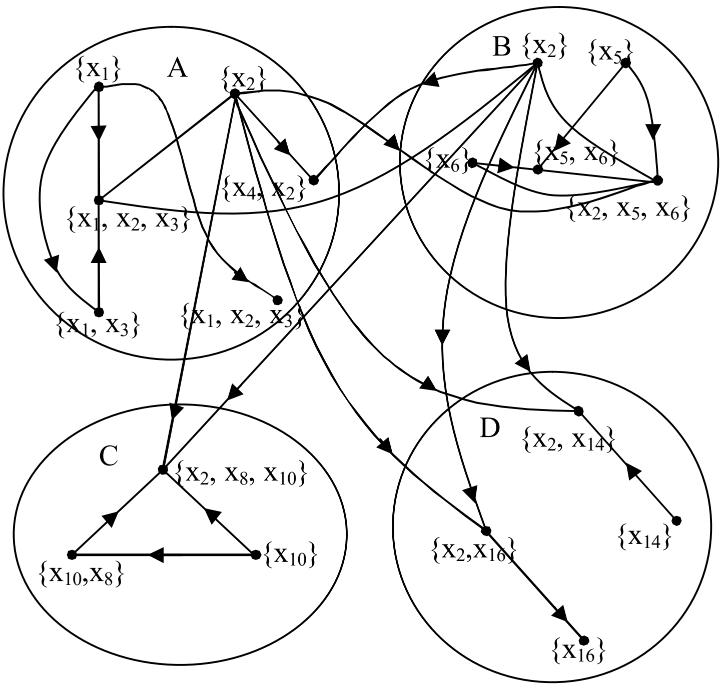


Figure 3.76

We see there is no dependence between the groups D and C.

Further it is very clear from the direction of the edges A is dependent on B and B is also dependent on A. But A and B are interdependent on each other.

Yet A and C are not interdependent A is only dependent on C and C is not dependent on A. Likewise B is dependent on D and D is not dependent on B.

When we compare the community network of F_1 with F_2 the direction of the edge that is which group is dependent on the other or vice versa cannot be easily calculated.

Further the groups D and C are dependent in F_1 where as they are independent in F_2.

Such study may show more relations enjoyed by all the groups.

However we are not in a position to know about the dependence of one group on the other in case of F_1 but in case of F_2 it is very clear.

Next we proceed on to describe the community network where the groups enjoy the graphs of vertex subsets enjoy type I property and the links are only subset vertex of type II.

Let F_3 be the community network with groups enjoying the subset vertex graphs of type I and the links enjoying the subset vertex graphs of type II.

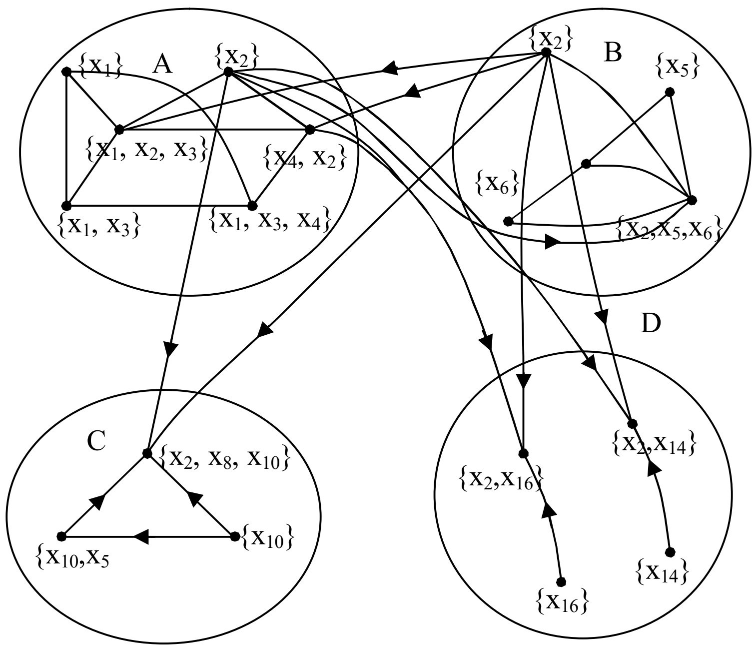


Figure 3.77

We see from the graphs F_1, F_2 and F_3; F_3 has made changes only in groups from F_2 and the changes in links of F_2 remain the same and the changes in the links from F_2. WE see such study is interesting and innovative.

Only if the researcher is interested in studying the internal structure of the groups with directed links he can use this method. Similarly one can vary the groups structure by imposing on it the type I or type II and study the social community network. Or we can find directed links for some pair and for some pair only other. Thus one can get several such community networks for the given pair.

Working with them using these types of graphs is beneficial as the working yields only a fixed graph using type I or type II. This property is most advantageous as one need not work for analyzing all possible types of graphs using the given set of vertices.

Such study is not only innovative but can save both time and economy. Also the job of approximations can be resolved by this method. Once the nodes are appropriately fixed all other related properties fall in place in a unique manner. Further we have to build a procedure to work with the adjacency matrix associated with them.

Several things are deviant from the usual graphs. They are not one to one relation between a matrix and a graph, given a matrix a graph exist and vice versa. But in case of vertex subset graphs of type I and type II we can have only one matrix and given a matrix we cannot arbitrarily make a graph for the entries $v_1, \ldots, v_m$ are subsets from P(S) for some finite set S.

There is lot to be done in this direction and in the opinion of the authors this field will be a powerful of one for any researcher in social networking and community detection.

By appropriately fixing the vertices from the subsets of the set P(S) for some desired or needed set of attributes / nodes / actors under study.

The graphs obtained will be fixed be it type I or type II which is a big advantage as the human element or bias can be avoided.

Certainly these graphs can be used in biological networks, social nets and other appropriate networks the main advantage being that they are unique so the cliques contributed by them are unique.

Further very advantageously we can use these graphs to show growth of communities for a time period and also this will yield automatic fixing of links and edges among the s^{th} period to $(s + 1)^{th}$ period. The same procedure can show the non growth or determination in the aforesaid period and the interlinking links and edges.

Finally this can also exhibit the merging and splitting of the evolution of communities.

We will just describe this situation by some examples.

We first show the growth in the evaluation of two communities.

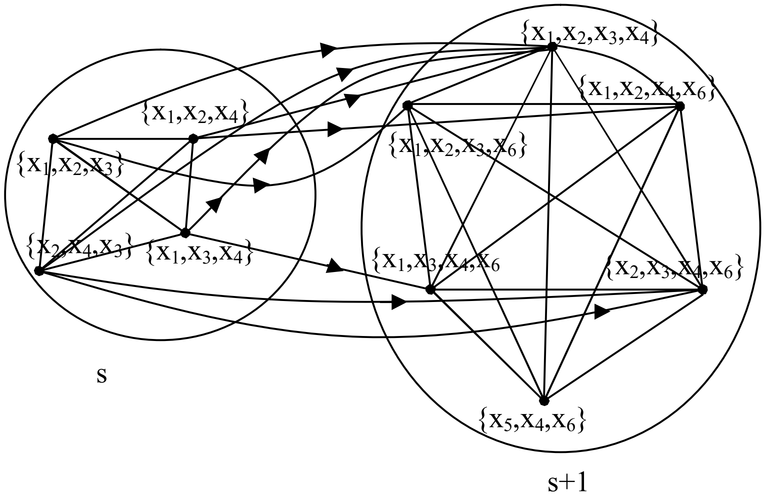


Figure 3.78

Now we can also give in addition the growth of each unit (vertex). We can say the growth is uniform and the x_6 factor is the one which has influenced most of the units so on and so forth.

Now we will briefly describe the contribution or determination or retardation of communities by social networking using the vertex subset graphs of type I and type II by the following graphs.

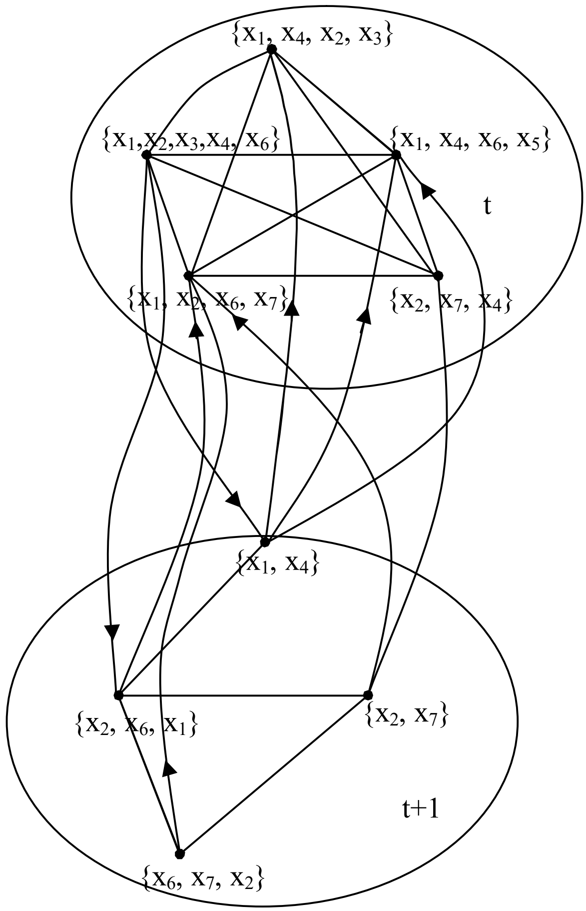


Figure 3.79

We see from the map the deteriorated units are as follows.

Thus by this new classes of graphs we can study the retardation of the communities or the growth of the communities. For such type of studies these graphs are not only apt, also can give the most accurate result.

Next we briefly describe the merging of the communities using the subset graphs of type I or type II in the following.

Let A and B be the two graphs of communities represented by the following subset vertex graphs of type I is the subset vertex graph of type I representing one group.

A =

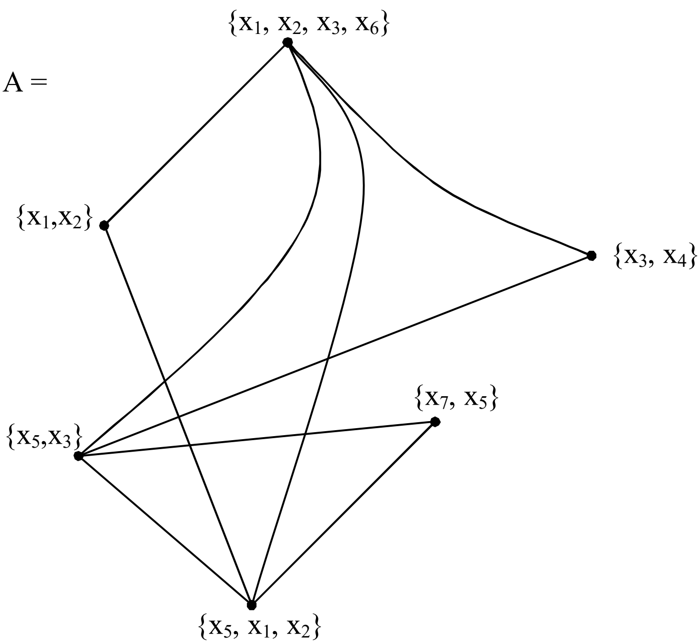


Figure 3.80

Let B be the subset graph of type I representing another group which is as follows.

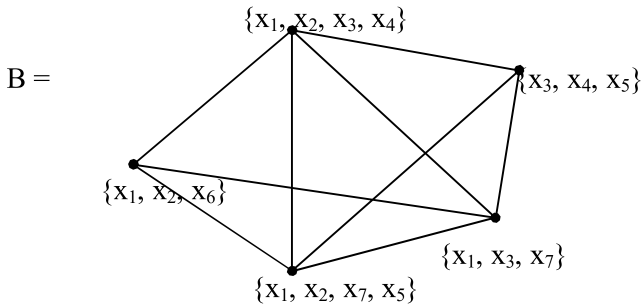


Figure 3.81

We now give the merger of the two community groups given by the following figure.

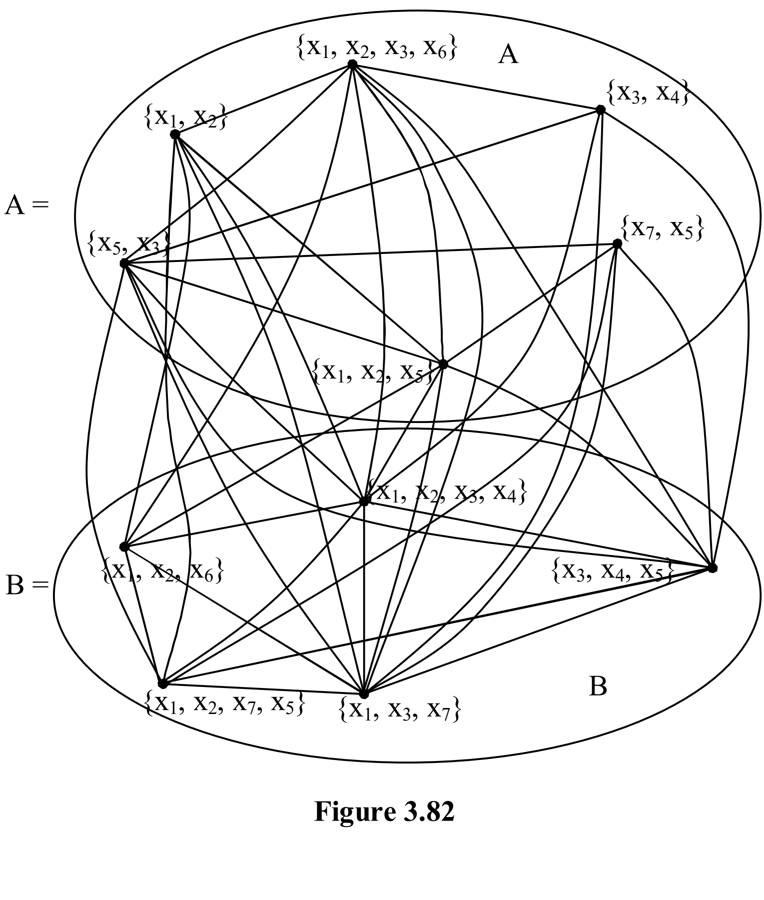


Figure 3.82

The merging becomes automatic and is not fixed. It occurs in a very natural way. In fact in this case they merge in a very strong way using type I method.

If however we use type II merging of the two group graphs then the merging will not have these many strong bonds.

We will also describe first the merging of the subset vertex graphs of type I using type II merging in the following. However we will next show how the two groups graphs are under type II relational ties.

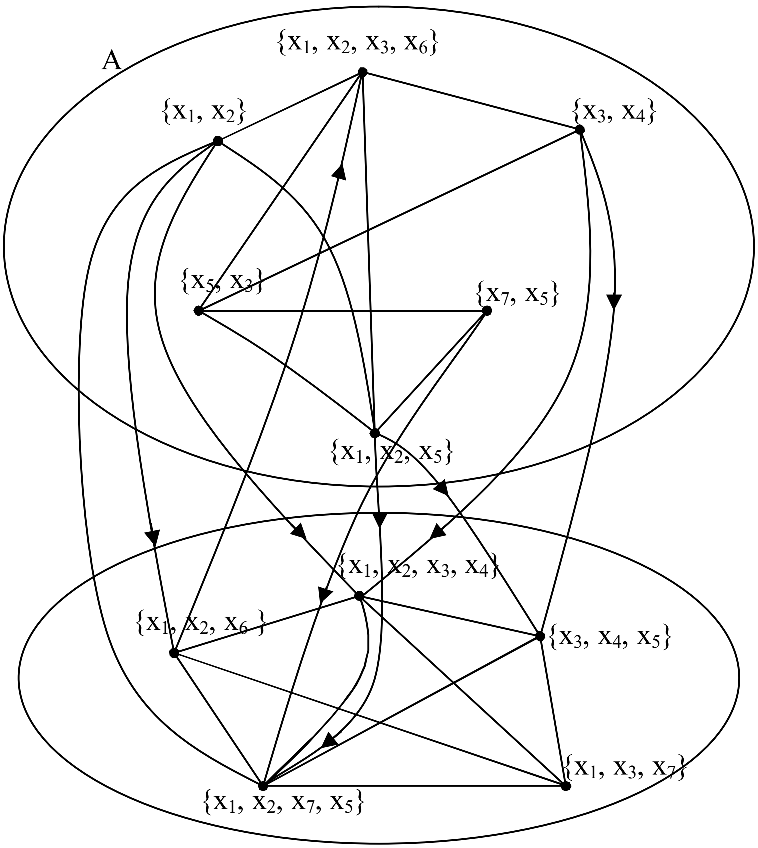


Figure 3.83

Clearly when we use type II merging the number of ties become less in comparison with type I merging which is evident by the very inspection of the figures.

Interested reader can work on with merging when the groups A and B enjoy type II graph property and so on.

On similar lines one can work with splitting of a subset vertex graph of type I (and type II) independently.

Further several of the properties enjoyed by graphs in general can be derived in this case with a very special restriction given a vertex subset with entries from P(S) there exists only one graph be is type I (or be it type II).

Already we have recorded the adjacency matrix for a type I graph is always symmetric where as for type II graphs as it is directed the adjacency matrix is non symmetric.

Further in both the cases the diagonal elements are assumed to be zero.

Finally given a matrix we cannot find either type I or type II graphs. As for a given set of vertices from P(S) there is only one graph of type I (or type II) and only one adjacency matrix symmetric (non symmetric) associated with this. This will be just exhibited by an example each in the following.

G =

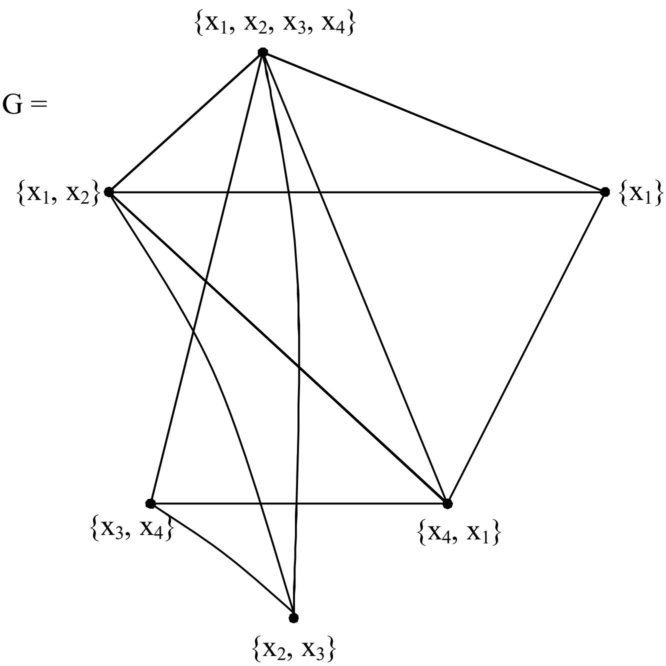


Figure 3.84

The adjacency matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ \{x_1,x_2,x_3,x_4\} \\ \{x_1,x_2\} \\ \{x_1\} \\ \{x_3,x_4\} \\ \{x_4,x_1\} \\ \{x_2,x_3\} \end{array} \begin{array}{c} \begin{array}{ccc} \{x_1,x_2,x_3,x_4\} & \{x_1,x_2\} & \{x_1\} \end{array} \\ \left[\begin{array}{ccc} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right. \end{array}$$

$$\begin{array}{c} \begin{matrix} \{x_3, x_4\} & \{x_4, x_1\} & \{x_2, x_3\} \end{matrix} \\ \left. \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix} \right] \end{array}.$$

Clearly the matrix M which will be known as the subset vertex type I matrix is symmetric.

Now let G_1 be the subset vertex graph of type II given by the following figure.

$G_1 =$

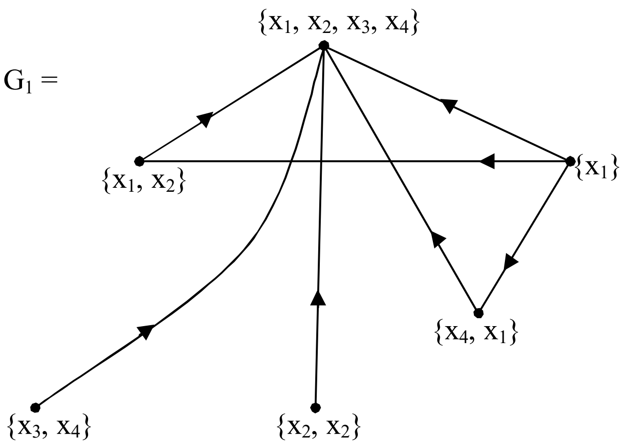


Figure 3.85

Now we proceed on the give the adjacency matrix M_1 of type II of the subset vertex graph of type II G_1 in the following.

$$\begin{array}{c} \\ \{x_1,x_2,x_3,x_4\} \\ \{x_1,x_2\} \\ \{x_1\} \\ \{x_3,x_4\} \\ \{x_2,x_3\} \\ \{x_4,x_1\} \end{array} \begin{array}{c} \begin{array}{cccccc} \{x_1,x_2,x_3,x_4\} & \{x_1,x_2\} & \{x_1\} & \{x_3,x_4\} & \{x_2,x_3\} & \{x_4,x_1\} \end{array} \\ \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Clearly M_1 is not a symmetric subset matrix of type II.

Interested reader can work with other properties associated with these matrices. Study in this direction however is a matter of routine.

Almost all properties associated with graph matrices can be derived in case of subset vertex matrix of type I and type II with appropriate modifications.

The main advantage of using these graphs is the nature of them is that for a given set of vertices the subset vertex graph be it type I or type II they are unique.

Further type II subset vertex graphs are directed where as subset vertex graph of type I are not directed. In most cases type I graphs always has more edges them type II graphs.

Sometimes for the same set of vertices type II subset vertex graphs may be empty.

We can also use these both types of graphs in social networking and community networking.

Finally authors wish to state when set theoretic concepts are used in the study of topological spaces. Only very recently algebraic structure using subsets of a set was defined and developed [ ].

However using subsets of a power set P(S) in graph theory is totally absent, though Boolean algebras was constructed which is a form of graph. But the Boolean algebra was not used in studying any form of social networks or community net.

So in this book authors have ventured to define two types of subset vertex graphs in case of type I subset vertex graphs two distinct vertices v_1, v_2 are connected by an edge if $v_1 \cap v_2 \neq \phi$.

This type I graphs are not directed with the further assumption that there is no loop connecting the vertex with itself.

The type II vertex subset graphs are defined as follows for two distinct vertices v_1 and v_2 an edge exist if $v_1 \subset v_2$ $(v_1 \rightarrow v_2)$

or $v_2 \subset v_1$ $(v_2 \rightarrow v_1)$

if $v_1 \not\subseteq v_2$ so edge exists.

Thus subset vertex graphs of type II are always directed. We cannot say any form of relation between them for one is directed and the other one undirected. It is pertinent to keep on record that only S is a finite set it is a complex set, neutrosophic set or the complex neutrosophic set, still the edges defined will only follow the above principle, hence one can define them appropriately. However it is to be noted the edges are only real. Thus it is a challenging and open problem for researchers to define complex edge or neutrosophic edge or complex - neutrosophic edge with some vital definitions by imposing conditions.

Several interesting properties are enjoyed by them which show their distinct nature.

In the following we suggest some problems for the reader. Some of the problems are exercises and some of them are at research level.

Problems

1. Show that all subset vertex type II graphs are directed.

2. Let P(S) = {The power set of S = $\{x_1, x_2, \ldots, x_9\}$}.

   i) Find all subset vertex graphs of type II using this P(S).

   ii) Can we have type II subset vertex graphs which are chains?

   iii) Is it possible to construct subset vertex graphs of type II which are star graphs?

   iv) How many subset vertex graphs of type II using this P(S) are tournaments?

   v) How many vertex subset graphs of type II are circles? (Justify your answer).

   vi) Compare the subset vertex graphs of type I with type II for some fixed vertex set V in P(S).

3. Let P(S) be the power set of S, where S = $\{x_1, x_2, \ldots, x_n\}$. Study questions (i) to (vi) of problem (2) for this P(S). (The answers are generalized ones and some of them are really difficult to be solved).

4. Let P(S) be the power set of S, S = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$.

i) For V = $\{\{x_1\}, \{x_2\}, \{x_2\}, \{x_3\}, \{x_1, x_4\}, \{x_2, x_5\}, \{x_3, x_6\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_5\}, \{x_2, x_3, x_6\}\} \subseteq P(S)$.

Construct the subset vertex graphs of type I and type II and compare them.

ii) Can the vertex set V contribute to a subset vertex directed complete graph of type II? Justify?

iii) Can the vertex set V lead to a quasi semi subset vertex tournament.

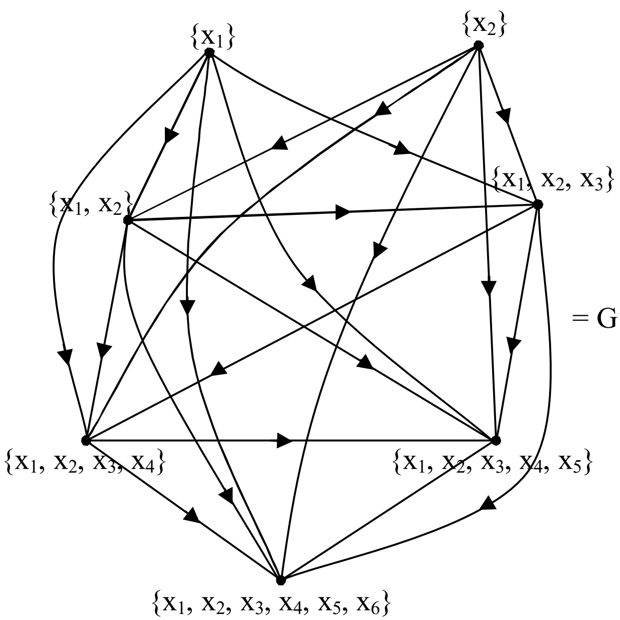


Figure 3.86

Let G given by the above figure be a subset vertex graph of type II.

a) (a) Prove G is a semi quasi tournament.

b) Can G have subgraphs which are tournaments?

c) Obtain any other special or striking properties enjoyed by them.

5. Illustrate some applications of type II graphs.

6. Compare a type I subset vertex graph with a subset vertex type II graph.

7. Prove both subset vertex type I graphs as well a subset vertex type II graphs can be used in the construction of community networks.

8. Implement the subset vertex graphs in real world problem for community networks.

9. Given P(S) is a power set of S, where S = $\{x_1, x_2, \ldots, x_{18}\}$.

Let A, B, C and D be the four groups of communities given by the following sets of vertices V_1, V_2, V_3 and V_4 where

$V_1 = \{\{x_1, x_2\}, \{x_1, x_2, x_6\}, \{x_1, x_2, x_8, x_{12}\},$
$\{x_4, x_2, x_{12}, x_{18}, x_{10}\}, \{x_{10}, x_8\}, \{x_6, x_{12}, x_8, x_{17}\}\},$

$V_2 = \{\{x_{14}, x_2, x_9, x_{17}\}, \{x_5, x_{15}\}, \{x_{15}\}, \{x_{15}, x_2\},$
$\{x_9, x_{15}, x_5\}\ \{x_5, x_{17}\}\},$

$V_3 = \{\{x_9, x_3\}, \{x_3\}, \{x_9, x_2\}, \{x_7, x_9, x_2, x_3\},$
$\{x_3\}, \{x_2, x_3\}\}$ and

$V_4 = \{\{x_2, x_{11}, x_{13}\}\}, \{x_{11}\}, \{x_2, x_{13}\},$
$\{x_9, x_{11}, x_3, x_1, x_{11}\}\}.$

i) Find the subset vertex graphs of type I for the four groups with type I links.

ii) Find the subset vertex graphs of type I for the four groups with type II links.

iii) Using vertex set V_1 find the subset vertex graphs type I and type II and compare them.

iv) Find the subset vertex graphs of type II for the four groups with type II links.

v) For the vertex set V_1 and V_3 use the subset vertex graph of type I and for V_1 to V_2 use link of type II and for V_3 to V_4 use link type I and for other use only type II links and give the graphs.
   a. Compare them.
   b. Adopt this in a real world problem of social networking.

10. Can there be a social networking in a real world problem which gives a tournament?

11. Does there exists a community network which has no cliques?

12. Find all subset vertex graphs of type I which has K-cliques using P(S) where S = $\{x_1, x_2, \ldots, x_n\}$.

13. How many subset vertex graphs of type I have cliques where S = $\{x_1, x_2, \ldots, x_{10}\}$?

14. How many subset vertex graphs of type II have cliques where S = $\{x_1, x_2, \ldots, x_{10}\}$?

15. Compare the results in problems (13) and (14).

16. Obtain any other special and interesting features enjoyed by subset vertex graphs of type I with that of usual graphs.

17. Prove using type I and type II subset graphs growth or retardation of two communities can be determined uniquely by appropriately fixing the vertex sets from P(S).

18. Prove the technique of merging in the case of subset graphs of type I and type II the answers are not arbitrary but very unique.

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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Authors in this book define a new class of graphs called subset vertex graphs whose vertices are subsets from a power set P(S) of S. A special way of defining them yields only one subset vertex graph for a given set of subset vertices. This new phenomenon can reduce the ambiguity or arbitrariness in fixing the edges in general in any network and in particular social networks.

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