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TOPSIS method for multi-attribute group decision making under single-valued neutrosophic environment

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Abstract A single-valued neutrosophic set is a special case of neutrosophic set. It has been proposed as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets in order to deal with incomplete information. In this paper, a new approach for multi-attribute group decision making problems

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is proposed by extending the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to single-valued neutrosophic environment. Ratings of alternative with respect to each attribute are considered as single-valued neutrosophic set that reflect the decision makers' opinion based on the provided information. Neutrosophic set characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F) which is more capable to catch up incomplete information. Single-valued neutrosophic set based weighted averaging operator is used to aggregate all the individual decision maker's opinion into one common opinion for rating the importance of criteria and alternatives. Finally, an illustrative example is provided in order to demonstrate its applicability and effectiveness of the proposed approach.

Keywords Fuzzy set · Intuitionistic fuzzy set · Multi-attribute group decision making · Neutrosophic set · Single-valued neutrosophic set · TOPSIS

1 Introduction

Multiple attribute decision making (MADM) problems with quantitative or qualitative attribute values have broad applications in the area of operation research, management science, urban planning, natural science and military affairs, etc. The attribute values of MADM problems cannot be expressed always with crisp numbers because of ambiguity and complexity of attribute. In classical MADM methods, such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) developed by Hwang and Yoon [1], PROMETHEE

[2], VIKOR [3], ELECTRE [4], the weight of each attribute and ratings of alternative are presented by crisp numbers. However, in real world, decision maker may prefer to evaluate attributes by using linguistic variables rather than exact values because of partial knowledge about the attribute and lack of information processing capabilities of the problem domain. In such situation, a preference information of alternatives provided by the decision makers may be vague, imprecise or incomplete. Fuzzy set [5] introduced by Zadeh , is one of such tool that utilizes impreciseness in a mathematical form. MADM problem with imprecise information can be modeled quite well by using fuzzy set theory into the field of decision making. Chen [6] extended the TOPSIS method for solving multi-criteria decision making problems in fuzzy environment. However, fuzzy set can only focus on the membership degree of vague parameters or events. It fails to handle non-membership degree and indeterminacy degree of imprecise parameters. In 1986, Atanassov [7] introduced intuitionistic fuzzy set (IFS) characterized by membership and non-membership degrees simultaneously.

Boran et al. [8] extended the TOPSIS method for multi-criteria intuitionistic decision making problem. Pramanik and Mukhopadhyaya [9] studied teacher selection in intuitionistic fuzzy environment. However in IFSs, sum of membership degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems such as problems involving indeterminate information.

Smarandache [10] first introduced the concept of neutrosophic set (NS) from philosophical point of view to handle indeterminate or inconsistent information usually exist in real situation. A neutrosophic set is characterized by a truth membership degree, an indeterminacy membership degree and a falsity membership degree independently. An important feature of NS is that every element of the universe has not only a certain degree of truth (T) but also a falsity degree (F) and indeterminacy degree (I). This set is a generalization of crisp set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set etc. However, NS is difficult to apply directly in real engineering and scientific applications. In order to deal with difficulties, Wang et al. [11] introduced a subclass of NS called single-valued neutrosophic set (SVNS) characterized by truth membership degree, an indeterminacy membership degree and a falsity membership degree. SVNS can be applied quite well in real scientific and engineering fields to handle the uncertainty, imprecise, incomplete, and inconsistent information. Ye [13] studied multi-criteria decision-making problem by using the weighted correlation coefficient of SVNSs. Ye [14] also developed single-valued neutrosophic cross entropy for multi-criteria decision-making problems. Biswas et al. [15] proposed an entropy based grey relational analysis method for solving a multi attribute decision making problem under SVNSs. Biswas et al. [16] also developed a new methodology for solving SVNSs based MADM with unknown weight information. Zhang et al. [18] studied multi-criteria decision making problems under interval neutrosophic sets information . Ye [19] further discussed multi-criteria decision making problem by using aggregation operators for simplified neutrosophic sets. Chi and Liu [17] discussed an extended TOPSIS method for interval neutrosophic set based MADM problems.

The objective of this paper is to extend the concept of TOPSIS method for multi-attribute group decision making (MAGDM) problems into MAGDM with SVNS information. The information provided by different domain experts in MAGDM problems about alternative and attribute values takes the form of single valued neutrosophic set. In a group decision making process, neutrosophic weighted averaging operator needs to be used to aggregate all the decision makers' opinions into a single opinion for rating the selected alternatives.

The remaining part of this paper is organized as follows: section 2 briefly introduces some preliminaries relating to neutrosophic set and the basics of single-valued neutrosophic set. In section 3, basics of TOPSIS method are discussed. Section 4 is devoted to develop TOPSIS method for MADM under simplified neutrosophic environment. In section 5, an illustrative example is provided to show the effectiveness of the proposed approach. Finally, section 6 presents the concluding remarks.

2 Preliminaries of neutrosophic sets and single valued neutrosophic sets

In this section, some basic definitions of neutrosophic set defined by Smarandache [10] have been provided to develop the paper.

2.1 Neutrosophic Set

Neutrosophic set is originated from neutrosophy, a new branch of philosophy which reflects the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [10].

Definition 1 Let X be a universal space of points (objects), with a generic element of X denoted by x. A neutrosophic set $\mathcal{N} \subset X$ is characterized by a truth-membership function $T_{\mathcal{N}}(x)$, an indeterminacy membership function $I_{\mathcal{N}}(x)$ and a falsity-membership function $F_{\mathcal{N}}(x)$. $T_{\mathcal{N}}(x)$, $I_{\mathcal{N}}(x)$ and $F_{\mathcal{N}}(x)$ are real standard or non-standard subsets of $[0^-, 1^+]$, so that all three neutrosophic components $T_{\mathcal{N}}(x) \to [0^-, 1^+]$, $I_{\mathcal{N}}(x) \to [0^-, 1^+]$ and $F_{\mathcal{N}}(x) \to [0^-, 1^+]$.

The set $I_{\mathcal{N}}(x)$ may represent not only indeterminacy but also vagueness, uncertainty, imprecision, error, contradiction, undefined, unknown, incompleteness, redundancy etc, [20,21]. In order to catch up vague information, an indeterminacy membership degree can be split into sub-components, such as"contradiction", "uncertainty", "unknown", etc, [22].

It should be noted that the sum of three independent membership degrees $T_{\mathcal{N}}(x)$, $I_{\mathcal{N}}(x)$ and $F_{\mathcal{N}}(x)$ have no restriction such that [11]

$$0^{-} \leq T_{\mathcal{N}}(x) + I_{\mathcal{N}}(x) + F_{\mathcal{N}}(x) \leq 3^{+}$$

Definition 2 The complement of neutrosophic set A is denoted by A^c and is defined as $T^c_{\mathcal{A}}(x) = 1^{+} \ominus T_{\mathcal{A}}(x)$, $I^c_{\mathcal{A}}(x) = 1^{+} \ominus I_{\mathcal{A}}(x)$, and $F^c_{\mathcal{A}}(x) = 1^{+} \ominus F_{\mathcal{A}}(x)$ for all $x \in X$

Definition 3 A neutrosophic set A is contained in other neutrosophic set B, i.e. $A \subseteq B$ if and only if $inf\, T_A(x) \leq inf\, T_B(x)$, $sup\, T_A(x) \leq sup\, T_B(x)$, $inf\, I_A(x) \geq inf\, I_B(x)$, $sup\, I_A(x) \geq sup\, I_B(x)$, $inf\, F_A(x) \geq inf\, F_B(x)$, $sup\, F_A(x) \geq sup\, F_B(x)$, for all x in X.

2.2 Single-valued neutrosophic Set

Single-valued neutrosophic set is a special case of neutrosophic set. It can be used in real scientific and engineering applications. In the following sections some basic definitions, operations, and properties regarding single valued neutrosophic sets [11] are provided.

Definition 4 Let X be a universal space of points (objects), with a generic element of X denoted by x. A single-valued neutrosophic set (SVNS) $\tilde{\mathcal{N}} \subset X$ is characterized by a truth membership function $T_{\tilde{\mathcal{N}}}(x)$, an indeterminacy membership function $I_{\tilde{\mathcal{N}}}(x)$, and a falsity membership function $F_{\tilde{\mathcal{N}}}(x)$ with $T_{\tilde{\mathcal{N}}}(x)$, $I_{\tilde{\mathcal{N}}}(x)$, $F_{\tilde{\mathcal{N}}}(x) \in [0, 1]$ for all $x \in X$.

It should be noted that for SVNS $\tilde{\mathcal{N}}$, the relation

$$0 \leq T_{\tilde{\mathcal{N}}}(x) + I_{\tilde{\mathcal{N}}}(x) + F_{\tilde{\mathcal{N}}}(x) \leq 3 \quad \text{for all } x \in X$$

holds good. When X is continuous a SVNS $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \int_x \langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle \,|x, \ \text{ for all } x \ \in X.$$

When X is discrete a SVNS $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \sum_x \langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle \,|x, \ \text{ for all } x \ \in X.$$

SVNS has the following pattern: $\tilde{\mathcal{N}}= \{(x|\,\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle)|x \in X\}$.

Thus finite SVNS can be presented by the ordered tetrads:

$\tilde{\mathcal{N}} = \{(x_1|\,\langle T_{\tilde{\mathcal{N}}}(x_1), I_{\tilde{\mathcal{N}}}(x_1), F_{\tilde{\mathcal{N}}}(x_1)\rangle), \ldots, (x_M|\,\langle T_{\tilde{\mathcal{N}}}(x_M), I_{\tilde{\mathcal{N}}}(x_M), F_{\tilde{\mathcal{N}}}(x_M)\rangle)\}$

$\forall x \in X$. For convenience, a SVNS $\tilde{\mathcal{N}} = \{(x|\,\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle)|x \in X\}$ is denoted by the simplified symbol $\tilde{\mathcal{N}} = \langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle$ for all $x \in X$.

Definition 5 Let $\tilde{\mathcal{A}} = \langle T_{\tilde{\mathcal{A}}}(x), I_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{A}}}(x)\rangle$ and $\tilde{\mathcal{B}} = \langle T_{\tilde{\mathcal{B}}}(x), I_{\tilde{\mathcal{B}}}(x), F_{\tilde{\mathcal{B}}}(x)\rangle$ be any two SVNSs, then Wang et al. [11] defined the following set of operations as:

1. $\tilde{\mathcal{A}} \subseteq \tilde{\mathcal{B}}$ if and only if $T_{\tilde{\mathcal{A}}}(x) \leq T_{\tilde{\mathcal{B}}}(x), \ I_{\tilde{\mathcal{A}}}(x) \geq I_{\tilde{\mathcal{B}}}(x), \ F_{\tilde{\mathcal{A}}}(x) \geq F_{\tilde{\mathcal{B}}}(x)$ for all $x \in X$.
2. $\tilde{\mathcal{A}} = \tilde{\mathcal{B}}$ if and only if $\tilde{\mathcal{A}} \subseteq \tilde{\mathcal{B}}$ and $\tilde{\mathcal{B}} \subseteq \tilde{\mathcal{A}}$ for all $x \in X$.
3. $\tilde{\mathcal{A}}^c= \{(x|\,\langle F_{\tilde{\mathcal{A}}}(x), 1 - I_{\tilde{\mathcal{A}}}(x), T_{\tilde{\mathcal{A}}}(x)\rangle)|x \in X\}$ for all $x \in X$. (1)
4. $\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}}= \langle max(T_{\tilde{\mathcal{A}}}(x), T_{\tilde{\mathcal{B}}}(x)), min(I_{\tilde{\mathcal{A}}}(x), I_{\tilde{\mathcal{B}}}(x)), min(F_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{B}}}(x))\rangle$ for all $x \in X$.

5. $\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}} = \langle min(T_{\tilde{\mathcal{A}}}(x), T_{\tilde{\mathcal{B}}}(x)), max(I_{\tilde{\mathcal{A}}}(x), I_{\tilde{\mathcal{B}}}(x)), max(F_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{B}}}(x))\rangle$ for all $x \in X$.

Liu and Wang defined the following set of operations for SVNSs in [12] as:

Definition 6 Let $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ be two SVNSs, then

1. $\tilde{\mathcal{A}} \oplus \tilde{\mathcal{B}} = \langle T_{\tilde{\mathcal{A}}}(x) + T_{\tilde{\mathcal{B}}}(x) - T_{\tilde{\mathcal{A}}}(x).T_{\tilde{\mathcal{B}}}(x), I_{\tilde{\mathcal{A}}}(x).I_{\tilde{\mathcal{B}}}(x), F_{\tilde{\mathcal{A}}}(x).F_{\tilde{\mathcal{B}}}(x)\rangle$ for all $x \in X$. (2)
2. $\tilde{\mathcal{A}} \otimes \tilde{\mathcal{B}} = \langle T_{\tilde{\mathcal{A}}}(x).T_{\tilde{\mathcal{B}}}(x), I_{\tilde{\mathcal{A}}}(x) + I_{\tilde{\mathcal{B}}}(x) - I_{\tilde{\mathcal{A}}}(x).I_{\tilde{\mathcal{B}}}(x), F_{\tilde{\mathcal{A}}}(x) + F_{\tilde{\mathcal{B}}}(x) - F_{\tilde{\mathcal{A}}}(x).F_{\tilde{\mathcal{B}}}(x)\rangle$ for all $x \in X$. (3)
3. $\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}} = \langle max(T_{\tilde{\mathcal{A}}}(x), T_{\tilde{\mathcal{B}}}(x)), min(I_{\tilde{\mathcal{A}}}(x), I_{\tilde{\mathcal{B}}}(x)), min(F_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{B}}}(x))\rangle$ for all $x \in X$.
4. $\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}} = \langle min(T_{\tilde{\mathcal{A}}}(x), T_{\tilde{\mathcal{B}}}(x)), max(I_{\tilde{\mathcal{A}}}(x), I_{\tilde{\mathcal{B}}}(x)), max(F_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{B}}}(x))\rangle$ for all $x \in X$.

2.3 Distance between two SVNSs

Majumdar and Samanta [23] studied similarity and entropy measure by incorporating euclidean distances of neutrosophic sets.

Definition 7 (Euclidean distance)

Let $\tilde{\mathcal{A}} = \{(x_1| \langle T_{\tilde{\mathcal{A}}}(x_1), I_{\tilde{\mathcal{A}}}(x_1), F_{\tilde{\mathcal{A}}}(x_1)\rangle, \ldots, (x_n| \langle T_{\tilde{\mathcal{A}}}(x_n), I_{\tilde{\mathcal{A}}}(x_n), F_{\tilde{\mathcal{A}}}(x_n)\rangle\}$

and $\tilde{\mathcal{B}} = \{(x_1| \langle T_{\tilde{\mathcal{B}}}(x_1), I_{\tilde{\mathcal{B}}}(x_1), F_{\tilde{\mathcal{B}}}(x_1)\rangle, \ldots, (x_n| \langle T_{\tilde{\mathcal{B}}}(x_n), I_{\tilde{\mathcal{B}}}(x_n), F_{\tilde{\mathcal{B}}}(x_n)\rangle\}$ be two SVNSs for $x_i \in X$, where $i = 1, 2, \ldots, n$. Then the Euclidean distance between two SVNSs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ can be defined as follows:

$$D_{Eucl}(\tilde{\mathcal{A}},\tilde{\mathcal{B}})=\sqrt{\sum_{i=1}^{n}\left\{\begin{array}{r}\left(T_{\tilde{\mathcal{A}}}(x_1)-T_{\tilde{\mathcal{B}}}(x_1)\right)^2+\left(I_{\tilde{\mathcal{A}}}(x_1)-I_{\tilde{\mathcal{B}}}(x_1)\right)^2\\ +\left(F_{\tilde{\mathcal{A}}}(x_1)-F_{\tilde{\mathcal{B}}}(x_1)\right)^2\end{array}\right\}} \tag{4}$$

and the normalized Euclidean distance between two SVNSs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ can be defined as follows:

$$D_{Eucl}^{N}(\tilde{\mathcal{A}},\tilde{\mathcal{B}})=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left\{\begin{array}{r}\left(T_{\tilde{\mathcal{A}}}(x_1)-T_{\tilde{\mathcal{B}}}(x_1)\right)^2+\left(I_{\tilde{\mathcal{A}}}(x_1)-I_{\tilde{\mathcal{B}}}(x_1)\right)^2\\ +\left(F_{\tilde{\mathcal{A}}}(x_1)-F_{\tilde{\mathcal{B}}}(x_1)\right)^2\end{array}\right\}} \tag{5}$$

Definition 8 (Deneutrosophication of SVNS) Deneutrosophication of SVNS $\tilde{\mathcal{N}}$ can be defined as a process of mapping $\tilde{\mathcal{N}}$ into a single crisp output $\psi^* \in X$ i.e. $f : \tilde{\mathcal{N}} \to \psi^*$ for $x \in X$. If $\tilde{\mathcal{N}}$ is discrete set then the vector of tetrads $\tilde{\mathcal{N}} = \{(x|\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle)|x \in X\}$ is reduced to a single scalar quantity $\psi^* \in X$ by deneutrosophication. The obtained scalar quantity $\psi^* \in X$ best represents the aggregate distribution of three membership degrees of neutrosophic element $\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\rangle$.

3 TOPSIS

TOPSIS method is used to determine the best alternative from the concepts of the compromise solution. The best compromise solution should have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution. The procedures of TOPSIS can be described as follows. Let $A=\{A_1, A_2, \dots A_m\}$ be the set of alternatives , $C=\{C_1, C_2, \dots C_n\}$ be the set of criteria and $D=\{d_{ij}\}$, $i = 1, 2, \dots, m$, $j =$

$1, 2, \ldots, n$, be the performance ratings with the criteria weight vector $W = \{w_j | j = 1, 2, \ldots, n\}$. TOPSIS method is presented with these following steps:

Step 1. Normalization the decision matrix

The normalized value d_{ij}^N is calculated as follows:

- For benefit criteria (larger the better), $d_{ij}^N = (d_{ij} - d_j^-)/(d_j^+ - d_j^-)$, where $d_j^+ = max_i(d_{ij})$ and $d_j^- = min_i(d_{ij})$ or setting d_j^+ is the aspired or desired level and d_j^- is the worst level.
- For cost criteria (smaller the better), $d_{ij}^N = (d_j^- - d_{ij})/(d_j^- - d_j^+)$.

Step 2. Calculation of weighted normalized decision matrix

In the weighted normalized decision matrix, the modified ratings are calculated as the following way:

$$v_{ij} = w_j \times d_{ij}^N \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n. \tag{6}$$

where w_j is the weight of the j-th criteria such that $w_j \geq 0$ for $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$.

Step 3. Determination of the positive and the negative ideal solutions

The positive ideal solution (PIS) and the negative ideal solution(NIS) are derived as follows:

$$\begin{aligned} \text{PIS} = A^{+} &= \left\{v_1^{+}, v_2^{+}, \ldots v_n^{+}, \right\} \\ &= \left\{ \left(\max_j v_{ij} | j \in J_1 \right), \left(\min_j v_{ij} | j \in J_2 \right) | j = 1, 2, \ldots, n \right\} \end{aligned} \tag{7}$$

and

$$\begin{aligned} \text{NIS} = A^{-} &= \left\{v_1^{-}, v_2^{-}, \ldots v_n^{-}, \right\} \\ &= \left\{ \left(\min_j v_{ij} | j \in J_1 \right), \left(\max_j v_{ij} | j \in J_2 \right) | j = 1, 2, \ldots, n \right\} \end{aligned} \tag{8}$$

where J_1 and J_2 are the benefit and cost type criteria respectively.

Step 4. Calculate the separation measures for each alternative from the PIS and the NIS.

The separation values for the PIS can be measured by using the n-dimensional Euclidean distance which is given as:

$$D_i^{+} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{+}\right)^2} \quad i = 1, 2, \ldots m. \tag{9}$$

Similarly, separation values for the NIS is

$$D_i^{-} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{-}\right)^2} \quad i = 1, 2, \ldots m. \tag{10}$$

Step 5. Calculation of the relative closeness coefficient to the positive ideal solution.

The relative closeness coefficient for the alternative A_i with respect to A^+ is

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \text{ for } \quad i = 1, 2, \ldots m. \tag{11}$$

Step 6. Ranking the alternatives

According to relative closeness coefficient to the ideal alternative, larger value of C_i indicates the better alternative A_i.

4 TOPSIS Method for multi-attribute decision making with single-valued neutrosophic information.

Consider a multi-attribute decision-making problem with m alternatives and n attributes. Let A={$A_1, A_2, ..., A_m$} be a discrete set of alternatives, and C={$C_1, C_2, ..., C_n$} be the set of attributes. The rating provided by the decision maker describes the performance of alternative A_i against attribute C_j. Let us also assume that $W = \{w_1, w_2 \ldots, w_n\}$ be the weight vector assigned for the attributes C_1, C_2, ..., C_n by the decision makers. The values associated with the alternatives for MADM problems can be presented in the following decision matrix

$$\mathbf{D} = \langle d_{ij} \rangle_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \dots & \dots & \dots & \dots \\ d_{1m} & d_{2m} & \dots & d_{mn} \end{pmatrix} \end{array} \quad (12)$$

Step 1. Determination of the most important attribute

Generally, there are many criteria or attributes in decision-making problems, where some of them are important and others may not be so important. So it is crucial to select the proper criteria or attributes for decision-making situation. The most important attributes may be chosen with the help of expert opinions or by some others method that are technically sound.

Step 2. Construction of decision matrix with SVNSs

It is assumed that the rating of each alternative with respect to each attribute is expressed as SVNS for MADM problem.The neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix:

$$\mathbf{D}_{\tilde{\mathbf{N}}} = \left\langle d_{ij}^{s} \right\rangle_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} \tag{13}$$

$$= \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \ldots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \ldots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \ldots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{pmatrix} \end{array} \tag{14}$$

In the matrix $\mathbf{D}_{\tilde{\mathbf{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$, T_{ij},I_{ij} and F_{ij} denote the degree of truth membership value, indeterminacy membership value and falsity membership value of alternative A_i with respect to attribute C_j satisfying the following properties under the single-valued neutrosophic environment:

1. $0 \le T_{ij} \le 1$; $0 \le I_{ij} \le 1$; $0 \le F_{ij} \le 1$.
2. $0 \le T_{ij} + I_{ij} + F_{ij} \le 3$ for i = 1,2,..., n and j = 1,2,..., m.

The ratings of each alternative over the attributes are best illustrated by the neutrosophic cube [24] proposed by Dezert in 2002. The vertices of neutrosophic cube are $(0, 0, 0)$, $(1, 0, 0)$,$(1, 0, 1)$,$(0, 0, 1)$,$(0, 1, 0)$,$(1, 1, 0)$,$(1, 1, 1)$ and $(0, 1, 1)$. The area of ratings in neutrosophic cube are classified in three categories namely, 1. highly acceptable neutrosophic ratings, 2. tolerable neutrosophic rating and 3. unacceptable neutrosophic ratings.

Definition 9 (Highly acceptable neutrosophic ratings)

The subcube(Λ) of a neutrosophic cube(Δ) (i.e. $\Lambda \subset \Delta$) represents the area of highly acceptable neutrosophic ratings (U)for decision making. Vertices of Λ are defined with these eight points $(0.5,0,0)$,$(1,0,0)$,$(1,0,0.5)$,$(0.5,0,0.5)$, $(0.5,0,0.5)$,$(1,0,0.5)$,$(1,0.5,0.5)$ and $(0.5,0.5,0.5)$. U includes all the ratings of alternative considered with the above average truth membership degree, below average indeterminacy degree and below average falsity membership degree for multi-attribute decision making. Therefore, U has a great contribution in decision making process and can be defined as

$$U = \langle T_{ij}, I_{ij}, F_{ij} \rangle \quad \text{where } \; 0.5 < T_{ij} < 1, \; 0 < I_{ij} < 0.5 \; \text{ and } 0 < F_{ij} < 0.5.$$

for i=$1,2,\ldots,m$ and j=$1,2,\ldots,n$.

Definition 10 (Unacceptable neutrosophic ratings) The area Γ of unacceptable neutrosophic ratings V is defined by the ratings which are characterized by 0% membership degree, 100% indeterminacy degree and 100% falsity membership degree. Thus the set of unacceptable ratings V can be considered as the set of all ratings whose truth membership value is zero.

$$V = \langle T_{ij}, I_{ij}, F_{ij} \rangle \quad \text{where } \; T_{ij} = 0, \; 0 < I_{ij} \leq 1 \; \text{ and } 0 < F_{ij} \leq 1.$$

for i=$1,2,\ldots,m$ and j=$1,2,\ldots,n$.

Consideration of V should be avoided in decision making process.

Definition 11 (Tolerable neutrosophic ratings) Excluding the area of highly acceptable ratings and unacceptable ratings from a neutrosophic cube, tolerable neutrosophic rating area Θ $(=\Delta \cap \neg\Lambda \cap \neg\Gamma)$can be determined. The tolerable neutrosophic rating (Z) considered with below average truth membership degree, above average indeterminacy degree and above average falsity membership degree are taken in decision making process. Z can be defined by the following expression

$$Z = \langle T_{ij}, I_{ij}, F_{ij} \rangle \quad \text{where } \; 0 < T_{ij} < 0.5,\; 0.5 < I_{ij} < 1 \; \text{ and } 0.5 < F_{ij} < 1.$$

for i=$1, 2, \ldots, m$ and j=$1, 2, \ldots, n$.

Definition 12 Fuzzification of SVNS $\tilde{\mathcal{N}} = \{(x| \langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \rangle)|x \in X\}$ can be defined as a process of mapping$\tilde{\mathcal{N}}$ into fuzzy set $\tilde{F} = \{x|\mu_{\tilde{F}}(x)|x \in X\}$ i.e. $f : \tilde{\mathcal{N}} \rightarrow \tilde{F}$ for $x \in X$. The representative fuzzy membership degree $\mu_{\tilde{F}}(x) \in [0,\; 1]^1$ of the vector tetrads $\{(x| \langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \rangle)|x \in X\}$ is defined from the concept of neutrosophic cube. It can be obtained by determining the root mean square of $1 - T_{\tilde{\mathcal{N}}}(x)$, $I_{\tilde{\mathcal{N}}}(x)$ and $F_{\tilde{\mathcal{N}}}(x)$ for all $x \in X$. Therefore the equivalent fuzzy membership degree is as:

$$\mu_{\tilde{F}}(x) = \begin{cases} 1 - \sqrt{\{(1 - T_{\tilde{\mathcal{N}}}(x))^2 + I_{\tilde{\mathcal{N}}}(x)^2 + F_{\tilde{\mathcal{N}}}(x)^2\}/3}, & \text{for } \forall x \in U \cup Z \\ 0, & \text{for } \forall x \in V \end{cases} \tag{15}$$

Step 3. Determination of the weights of decision makers

Let us assume that the group of p decision makers having their own decision weights. Thus the importance of the decision makers in a committee may not be equal to each other. Let us assume that the importance of each decision maker is considered with linguistic variables and expressed it by neutrosophic numbers.

Let E_k=$\langle T_k, I_k, F_k \rangle$ be a neutrosophic number defined for the rating of k-th decision maker. Then, according to Eq.(15) the weight of the k-th decision maker can be written as :

$$\psi_k = \frac{1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}}{\sum_{k=1}^{p}\left(1 - \sqrt{\{(1 - T_k(x))^2 + (I_k(x))^2 + (F_k(x))^2\}/3}\right)} \tag{16}$$

and $\sum_{k=1}^{p} \psi_k = 1$

Step 4. Construction of aggregated single-valued neutrosophic decision matrix based on decision makers' assessments

Let $D^{(k)}$= $(d_{ij}^{(k)})_{m\times n}$ be the single-valued neutrosophic decision matrix of the $k-th$ decision maker and Ψ= $(\psi_1, \psi_2 \ldots, \psi_p)^T$ be the weight vector of decision maker such that each $\psi_k \in [0, 1]$. In the group decision making process, all the individual assessments need to be fused into a group opinion to make an aggregated neutrosophic decision matrix. This aggregated matrix can be obtained by using single-valued neutrosophic weighted averaging (SVNWA) aggregation operator proposed by Ye[19] for SVNSs as follows:

$D = (d_{ij})_{m\times n}$ where,

$$\begin{aligned} d_{ij} &= SVNSWA_{\Psi}\left(d_{ij}^{(1)}, d_{ij}^{(2)}, \ldots, d_{ij}^{(p)}\right) \\ &= \psi_1 d_{ij}^{(1)} \oplus \psi_2 d_{ij}^{(2)} \oplus \cdots \oplus \psi_{(p)} d_{ij}^{(p)} \\ &= \left\langle 1 - \prod_{k=1}^{p}\left(1 - T_{ij}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(I_{ij}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(F_{ij}^{(p)}\right)^{\psi_k} \right\rangle \end{aligned} \tag{17}$$

Therefore the aggregated neutrosophic decision matrix is defined as follows:

$$\mathbf{D} = \langle d_{ij} \rangle_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} \tag{18}$$

$$= \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \dots \\ A_m \end{array} & \begin{pmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \dots & \dots & \dots & \dots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{pmatrix} \end{array} \tag{19}$$

where, d_{ij}= $\langle T_{ij}, I_{ij}, F_{ij} \rangle$ is the aggregated element of neutrosophic decision matrix D for $i = 1, 2, \dots m$ and $j = 1, 2, \dots n$.

Step 5. Determination of the attribute weights

In the decision-making process, decision makers may feel that all attributes are not equal importance. Thus every decision maker may have their very own opinion regarding attribute weights. To obtain the grouped opinion of the chosen attribute, all the decision makers' opinions for the importance of each attribute need to be aggregated. Let w_k^j=$(w_j^{(1)}, w_j^{(2)} \dots, w_j^{(p)})$ be the neutrosophic number(NN) assigned to the attribute C_j by the $k-th$ decision maker. Then the combined weight W=$\{w_1, w_2 \dots, w_n\}$ of the attribute can be determined by using SVNWA aggregation operator [19].

$$
\begin{aligned}
w_j &= SVNWA_{\Psi}\left(w_j^{(1)}, w_j^{(2)}, \ldots, w_j^{(p)}\right) \\
&= \psi_1 w_j^{(1)} \oplus \psi_2 w_j^{(2)} \oplus \cdots \oplus \psi_{(p)} w_j^{(p)} \\
&= \left\langle 1 - \prod_{k=1}^{p}\left(1 - T_j^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(I_j^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(F_j^{(p)}\right)^{\psi_k} \right\rangle
\end{aligned} \tag{20}
$$

$$
W = \{w_1, w_2, \ldots, w_n\} \tag{21}
$$

where, $w_j = \langle T_j, I_j, F_j \rangle$ for $j = 1, 2, \ldots n$.

Step 6. Aggregation of the weighted neutrosophic decision matrix

In this section, the obtained weights of attribute and aggregated neutrosophic decision matrix need to be further fused to make the aggregated weighted neutrosophic decision matrix.

The aggregated weighted neutrosophic decision matrix can be obtained by using the multiplication formula (3) of two neutrosophic sets as:

$$
\mathbf{D} \otimes \mathbf{W} = \mathbf{D^w} = \left\langle d_{ij}^{w_j} \right\rangle_{m \times n} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle_{m \times n} \tag{22}
$$

$$
= \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array}
\begin{array}{c}
\begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\
\begin{pmatrix}
\langle T_{11}^{w_1}, I_{11}^{w_1}, F_{11}^{w_1} \rangle & \langle T_{12}^{w_2}, I_{12}^{w_2}, F_{12}^{w_2} \rangle & \ldots & \langle T_{1n}^{w_n}, I_{1n}^{w_n}, F_{1n}^{w_n} \rangle \\
\langle T_{21}^{w_1}, I_{21}^{w_1}, F_{21}^{w_1} \rangle & \langle T_{22}^{w_2}, I_{22}^{w_2}, F_{22}^{w_2} \rangle & \ldots & \langle T_{2n}^{w_n}, I_{2n}^{w_n}, F_{2n}^{w_n} \rangle \\
\ldots & \ldots & \ldots & \ldots \\
\langle T_{m1}^{w_1}, I_{m1}^{w_1}, F_{m1}^{w_1} \rangle & \langle T_{m2}^{w_2}, I_{m2}^{w_2}, F_{m2}^{w_2} \rangle & \ldots & \langle T_{mn}^{w_n}, I_{mn}^{w_n}, F_{mn}^{w_n} \rangle
\end{pmatrix}
\end{array} \tag{23}
$$

Here, $d_{ij}^{w_j} = \langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ is an element of the aggregated weighted neutrosophic decision matrix $\mathbf{D^w}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Step 7. Determination of the relative positive ideal solution (RPIS) and the relative negative ideal solution (RNIS) for SVNSs

Let $\mathbf{D}_{\tilde{\mathbf{N}}} = \langle d_{ij}^{w} \rangle_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$ be a SVNS based decision matrix, where, T_{ij}, I_{ij} and F_{ij} are the membership degree, indeterminacy degree and non-membership degree of evaluation for the attribute C_j with respect to the alternative A_i.

In practical, two types of attributes namely, benefit type attribute and cost type attribute exist in multi-attribute decision making problem.

Definition 13 Let J_1 and J_2 be the benefit type attribute and cost type attribute respectively. $Q_{\tilde{N}}^{+}$ is the relative neutrosophic positive ideal solution (RNPIS) and $Q_{\tilde{N}}^{-}$ is the relative neutrosophic negative ideal solution (RNNIS). Then $Q_{\tilde{N}}^{+}$ can be defined as follows:

$$Q_{\tilde{N}}^{+} = \left[d_1^{w+}, d_2^{w+}, \ldots, d_n^{w+} \right] \tag{24}$$

where, $d_j^{w+} = \langle T_j^{w+}, I_j^{w+}, F_j^{w+} \rangle$ for $j = 1, 2, \ldots, n$.

$$T_j^{w+} = \left\{ \left(\max_i \{T_{ij}^{w_j}\} | j \in J_1 \right), \left(\min_i \{T_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{25}$$

$$I_j^{w+} = \left\{ \left(\min_i \{I_{ij}^{w_j}\} | j \in J_1 \right), \left(\max_i \{I_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{26}$$

$$F_j^{w+} = \left\{ \left(\min_i \{F_{ij}^{w_j}\} | j \in J_1 \right), \left(\max_i \{F_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{27}$$

and $Q^-_{\tilde{N}}$ can be defined by

$$Q^-_{\tilde{N}} = \left[d_1^{w-}, d_2^{w-}, \ldots, d_n^{w-}\right] \tag{28}$$

where, $d_j^{w-} = \left\langle T_j^{w-}, I_j^{w-}, F_j^{w-} \right\rangle$ for $j = 1, 2, \ldots, n$.

$$T_j^{w-} = \left\{ \left(\min_i \{T_{ij}^{w_j}\} | j \in J_1 \right), \left(\max_i \{T_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{29}$$

$$I_j^{w-} = \left\{ \left(\max_i \{I_{ij}^{w_j}\} | j \in J_1 \right), \left(\min_i \{I_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{30}$$

$$F_j^{w-} = \left\{ \left(\max_i \{F_{ij}^{w_j}\} | j \in J_1 \right), \left(\min_i \{F_{ij}^{w_j}\} | j \in J_2 \right) \right\} \tag{31}$$

Step 8. Determination of the distance measure of each alternative from the RNPIS and the RNNIS for SVNSs

Similar to Eq.(5), the normalized Euclidean distance measure of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the RNPIS $\left\langle T_j^{w+}, I_j^{w+}, F_j^{w+} \right\rangle$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ can be written as follows:

$$D_{Eu}^{i+}\left(d_{ij}^{w_j}, d_j^{w+}\right) = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left\{ \begin{array}{r} \left(T_{ij}^{w_j}(x_j) - T_j^{w+}(x_j)\right)^2 + \left(I_{ij}^{w_j}(x_j) - I_j^{w+}(x_j)\right)^2 \\ + \left(F_{ij}^{w_j}(x_j) - F_j^{w+}(x_j)\right)^2 \end{array} \right\}} \tag{32}$$

similarly, the normalized Euclidean distance measure of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the RNNIS $\left\langle T_j^{w-}, I_j^{w-}, F_j^{w-} \right\rangle$ can be written as:

$$D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right) = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left\{ \begin{array}{r} \left(T_{ij}^{w_j}(x_j) - T_j^{w-}(x_j)\right)^2 + \left(I_{ij}^{w_j}(x_j) - I_j^{w-}(x_j)\right)^2 \\ + \left(F_{ij}^{w_j}(x_j) - F_j^{w-}(x_j)\right)^2 \end{array} \right\}} \tag{33}$$

Step 9. Determination of the relative closeness co-efficient to the neutrosophic ideal solution for SVNSs

The relative closeness coefficient of each alternative A_i with respect to the neutrosophic positive ideal solution $Q^+_{\tilde{N}}$ is defined as follows:

$$C_i^* = \frac{D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right)}{D_{Eu}^{i+}\left(d_{ij}^{w_j}, d_j^{w-}\right) + D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right)} \tag{34}$$

where, $0 \leq C_i^* \leq 1$.

Step 10. Ranking the alternatives

According to the relative closeness co-efficient values larger the values of C_i^* reflects the better alternative A_i for $i = 1, 2, \ldots, m$.

5 Numerical example

Let us suppose that a group of four decision makers (DM_1,DM_2, DM_3, DM_4) intend to select the most suitable tablet from the four initially chosen tablet (A_1, A_2, A_2, A_4) by considering six attributes namely: Features C_1, Hardware C_2, Display C_3, Communication C_4, Affordable Price C_5, Customer care C_6. Based on the proposed approach discussed in section 4, the considered problem is solved by the following steps:

Step 1. Determination of the weights of decision makers.

The importance of four decision makers in a selection committee may not be equal to each other according their status. Their decision powers are considered as linguistic terms expressed in Table -1. The importance of each decision

Table 1 Linguistic terms for rating of attributes and decision makers.

Linguistic Terms	SVNNs
Very Good / Very Important (VG/VI)	$\langle 0.90, 0.10, 0.10\rangle$
Good / Important(G /I)	$\langle 0.80, 0.20, 0.15\rangle$
Fair / Medium(F/M)	$\langle 0.50, 0.40, 0.45\rangle$
Bad / Unimportant (B / UI)	$\langle 0.35, 0.60, 0.70\rangle$
Very Bad / Very Unimportant (VB/VUI)	$\langle 0.10, 0.80, 0.90\rangle$

maker expressed by linguistic term with its corresponding SVNN shown in Table-2. The weight of decision maker is determined with the help of Eq.(16) as follows:

$$\psi_1 = \frac{1 - \sqrt{(0.01 + 0.01 + 0.01)/3}}{\left(4 - \sqrt{0.03/3} - \sqrt{0.1025/3} - \sqrt{0.6125/3} - \sqrt{0.1025/3}\right)} = 0.292$$

Similarly, other three weights of decision ψ_2= 0.265, ψ_3= 0.178 and ψ_4= 0.265 can be obtained. Thus the weight vector of the four decision maker is:

$$\Psi = (0.292, 0.265, 0.178, 0.265) \tag{35}$$

Table 2 Importance of decision makers expressed with SVNNs.

	DM-1	DM-2	DM-3	DM-4
LT	VI	I	M	I
$\widetilde{W}$	$\langle 0.90, 0.10, 0.10\rangle$	$\langle 0.80, 0.20, 0.15\rangle$	$\langle 0.50, 0.40, 0.45\rangle$	$\langle 0.80, 0.20, 0.15\rangle$

Table 3 Linguistic terms for rating the candidates with SVNNs.

Linguistic terms	SVNNs
Extremely Good/High (EG/EH)	$\langle 1.00, 0.00, 0.00 \rangle$
Very Good/High (VG/VH)	$\langle 0.90, 0.10, 0.05 \rangle$
Good/High (G/H)	$\langle 0.80, 0.20, 0.15 \rangle$
Medium Good/High (MG/MH)	$\langle 0.65, 0.35, 0.30 \rangle$
Medium/Fair (M/F)	$\langle 0.50, 0.50, 0.45 \rangle$
Medium Bad/Medium Law (MB/ML)	$\langle 0.35, 0.65, 0.60 \rangle$
Bad/Law (B/L)	$\langle 0.20, 0.75, 0.80 \rangle$
Very Bad/Low (VB/VL)	$\langle 0.10, 0.85, 0.90 \rangle$
Very Very Bad/low (VVB/VVL)	$\langle 0.05, 0.90, 0.95 \rangle$

Step-2. Construction of the aggregated neutrosophic decision matrix based on the assessments of decision makers.

The linguistic term along with SVNNs is defined in Table-3 to rate each alternative with respect to each attribute. The assessment values of each alternative A_i $(i = 1, 2, 3, 4)$ with respect to each attribute C_j $(j = 1, 2, 3, 4, 5, 6)$ provided by four decision makers are listed in Table-4. Then the aggregated neutrosophic decision matrix can be obtained by fusing all the decision makers' opinion with the help of aggregation operator [19] as in Table-5.

By using Eq.(17), the aggregated value of the four decision makers' assessment values are arbitrarily chosen as an illustration for the alternative A_1 with

Table 4 Assessments of alternatives and attribute weights given by four decision makers.

Alternatives (A_i)	Decision Makers	C_1	C_2	C_3	C_4	C_5	C_6
A_1	DM-1	VG	G	G	G	G	VG
	DM-2	VG	VG	G	G	G	VG
	DM-3	G	VG	G	G	VG	G
	DM-4	G	G	G	G	G	G
A_2	DM-1	M	G	M	G	G	M
	DM-2	G	MG	G	G	MG	G
	DM-3	G	M	G	G	M	M
	DM-4	M	G	M	G	M	M
A_3	DM-1	VG	VG	G	G	VG	VG
	DM-2	G	VG	VG	G	G	VG
	DM-3	VG	G	G	MG	G	MG
	DM-4	VG	VG	G	G	MG	G
A_4	DM-1	M	VG	G	G	VG	M
	DM-2	M	M	G	G	M	G
	DM-3	G	G	G	G	M	VG
	DM-4	G	M	M	G	G	VG
Weights	DM-1	VI	VI	I	M	I	I
	DM-2	I	VI	I	I	M	M
	DM-3	M	I	M	M	I	M
	DM-4	M	VI	M	I	VI	I

Table 5 Aggregated neutrosophic decision matrix.

$$\begin{array}{cccc} & C_1 & C_2 & C_3 \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \multicolumn{3}{c}{\begin{pmatrix} \langle 0.864, 0.136, 0.081\rangle & \langle 0.853, 0.147, 0.092\rangle & \langle 0.800, 0.200, 0.150\rangle \\ \langle 0.667, 0.333, 0.277\rangle & \langle 0.727, 0.273, 0.219\rangle & \langle 0.667, 0.333, 0.277\rangle \\ \langle 0.880, 0.120, 0.067\rangle & \langle 0.887, 0.113, 0.064\rangle & \langle 0.834, 0.166, 0.112\rangle \\ \langle 0.667, 0.333, 0.277\rangle & \langle 0.735, 0.265, 0.195\rangle & \langle 0.768, 0.232, 0.180\rangle \end{pmatrix}} \end{array}$$

$$\begin{array}{cccc} & C_4 & C_5 & C_6 \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \multicolumn{3}{c}{\begin{pmatrix} \langle 0.704, 0.296, 0.241\rangle & \langle 0.823, 0.177, 0.123\rangle & \langle 0.864, 0.136, 0.081\rangle \\ \langle 0.744, 0.256, 0.204\rangle & \langle 0.652, 0.348, 0.293\rangle & \langle 0.608, 0.392, 0.336\rangle \\ \langle 0.779, 0.221, 0.170\rangle & \langle 0.811, 0.189, 0.109\rangle & \langle 0.850, 0.150, 0.092\rangle \\ \langle 0.727, 0.273, 0.221\rangle & \langle 0.791, 0.209, 0.148\rangle & \langle 0.808, 0.192, 0.127\rangle \end{pmatrix}} \end{array} \qquad (39)$$

respect to the attribute C_1 and shown in Eq.(36), Eq.(37), and Eq.(38).

$$T_{11} = 1 - (1-0.90)^{0.292} \times (1-0.90)^{0.265} \times (1-0.80)^{0.178} \times (1-0.80)^{0.265} = 1 - 0.1359 = 0.8641. \qquad (36)$$

$$I_{11} = (0.10)^{0.292} \times (0.10)^{0.265} \times (0.20)^{0.178} \times (0.20)^{0.265} = 0.1359. \qquad (37)$$

$$F_{11} = (0.05)^{0.292} \times (0.05)^{0.265} \times (0.15)^{0.178} \times (0.15)^{0.265} = 0.0813. \qquad (38)$$

Step-3. Determine the weights of attribute.

The linguistic terms shown in Table-1 are used to evaluate each attribute. The importance of each attribute for every decision maker are rated with linguistic terms shown in Table-4. Four decision makers' opinions need to be aggregated to determine the combined weight of each attribute.The fused attribute weight vector is determined by using Eq.(20) as follows:

$$W = \begin{bmatrix} \langle 0.755, 0.222, 0.217\rangle, \langle 0.887, 0.113, 0.107\rangle, \\ \langle 0.765, 0.226, 0.182\rangle, \langle 0.692, 0.277, 0.251\rangle, \\ \langle 0.788, 0.200, 0.180\rangle, \langle 0.700, 0.272, 0.244\rangle \end{bmatrix} \tag{40}$$

Step-4. Construction of the aggregated weighted neutrosophic decision matrix

After obtaining the combined weights of attribute and the ratings of alternative, the aggregated weighted neutrosophic decision matrix shown in Table 6, can be formed. For example, the element of aggregated weighted decision matrix for the alternative A_1 with respect to attribute C_1, is determined by the following Eq. (41).

$$\langle T_{11}^w, I_{11}^w, F_{11}^w \rangle = \left\langle \begin{array}{r} 0.864 \times 0.755, 0.136 + 0.222 - 0.136 \times 0.222, 0.081 \\ +0.217 - 0.081 \times 0.217 \end{array} \right\rangle \tag{41}$$

$$= \langle 0.65232, 0.3278, 0.2804 \rangle$$

Step-5. Determination of the neutrosophic relative positive ideal solution and the neutrosophic relative negative ideal solution.

The NRPIS can be calculated from the aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by using Eq. (24)

Table 6 Aggregated weighted neutrosophic decision matrix.

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle 0.652, 0.328, 0.280\rangle & \langle 0.757, 0.243, 0.289\rangle & \langle 0.612, 0.381, 0.305\rangle \\ \langle 0.504, 0.481, 0.434\rangle & \langle 0.645, 0.355, 0.388\rangle & \langle 0.510, 0.484, 0.409\rangle \\ \langle 0.664, 0.315, 0.269\rangle & \langle 0.787, 0.213, 0.267\rangle & \langle 0.638, 0.354, 0.274\rangle \\ \langle 0.504, 0.481, 0.434\rangle & \langle 0.652, 0.348, 0.370\rangle & \langle 0.588, 0.406, 0.329\rangle \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} C_4 & C_5 & C_6 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle 0.487, 0.491, 0.432\rangle & \langle 0.649, 0.342, 0.281\rangle & \langle 0.605, 0.371, 0.305\rangle \\ \langle 0.515, 0.402, 0.404\rangle & \langle 0.514, 0.478, 0.420\rangle & \langle 0.426, 0.557, 0.498\rangle \\ \langle 0.539, 0.437, 0.378\rangle & \langle 0.639, 0.351, 0.269\rangle & \langle 0.595, 0.381, 0.314\rangle \\ \langle 0.503, 0.474, 0.417\rangle & \langle 0.623, 0.367, 0.301\rangle & \langle 0.566, 0.412, 0.340\rangle \end{pmatrix} \end{array} \tag{42}$$

as

$$Q^{+}_{\tilde{N}} = \begin{bmatrix} \langle 0.664, 0.315, 0.269\rangle, \langle 0.787, 0.213, 0.267\rangle, \\ \langle 0.638, 0.354, 0.274\rangle, \langle 0.539, 0.437, 0.378\rangle, \\ \langle 0.649, 0.342, 0.269\rangle, \langle 0.605, 0.371, 0.305\rangle \end{bmatrix} \tag{43}$$

where, $d_1^{w+} = \left\langle T_1^{w+}, I_1^{w+}, F_1^{w+} \right\rangle$ is calculated as

$$\begin{aligned} T_1^{w+} &= max\,\{0.652, 0.504, 0.664, 0.504\} && = 0.664 \\ I_1^{w+} &= min\,\{0.328, 0.481, 0.315, 0.481\} && = 0.315 \\ F_1^{w+} &= min\,\{0.280, 0.434, 0.269, 0.434\} && = 0.269 \end{aligned}$$

and others. Similarly, the NRNIS can be calculated from aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by

Table 7 Distance measure and relative closeness co-efficient of each alternative.

Alternatives (A_i)	D^{i+}_{Eucl}	D^{i-}_{Eucl}	C^*_i
A_1	0.0283	0.1281	0.8190
A_2	0.3472	0.0490	0.1158
A_3	0.0224	0.1382	**0.8605**
A_4	0.0900	0.0831	0.4801

using Eq. (28) as

$$Q^-_{\tilde{N}} = \begin{bmatrix} \langle 0.504, 0.481, 0.434\rangle, \langle 0.645, 0.355, 0.388\rangle, \\ \langle 0.510, 0.484, 0.409\rangle, \langle 0.487, 0.491, 0.432\rangle, \\ \langle 0.514, 0.478, 0.420\rangle, \langle 0.426, 0.557, 0.498\rangle \end{bmatrix} \tag{44}$$

where, $d^{w-}_1 = \left\langle T^{w-}_1, I^{w-}_1, F^{w-}_1 \right\rangle$ is calculated as

$$T^{w-}_1 = min\{0.652, 0.504, 0.664, 0.504\} = 0.504$$

$$I^{w-}_1 = max\{0.328, 0.481, 0.315, 0.481\} = 0.481$$

$$F^{w-}_1 = max\{0.280, 0.434, 0.269, 0.434\} = 0.434$$

and other components are similarly calculated.

Step-6. Determination of the distance measure of each alternative from the RNPIS and the RNNIS and relative closeness co-efficient

Normalized Euclidean distance measures defined in Eq.(32) and Eq. (33) are used to determine the distances of each alternative from the RNPIS and the RNNIS. With these distances relative closeness co-efficient is calculated by using Eq. (34). These results are listed in Table-7.

Step-7. Ranking the alternatives

According to the values of relative closeness coefficient of each alternative shown in Table-7, the ranking order of four alternatives is

$$A_3 \succ A_1 \succ A_4 \succ A_2.$$

Thus A_3 is the best alternative tablet to buy.

6 Conclusions

This paper is devoted to present a new TOPSIS based approach for MAGDM under simplified neutrosophic environment. In the evaluation process, the ratings of each alternative with respect to each attribute are given as linguistic variables characterized by single valued neutrosophic numbers. Neutrosophic aggregation operator is used to aggregate all the opinions of decision makers. Neutrosophic positive ideal and neutrosophic negative ideal solution are defined from aggregated weighted decision matrix. Euclidean distance measure is used to determine the distances of each alternative from positive as well as negative ideal solutions for relative closeness co-efficient of each alternative. However the author hope that the concept presented in this paper may open up new avenue of research in competitive neutrosophic decision making arena. TOPSIS method with neutrosophic set information has enormous chance of success for multi-attribute decision making problems. In future, the proposed approach can be used for dealing with decision making problems such as personal selection in academia, project evaluation, supplier selection, manufacturing systems, and many other areas of management systems.

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Table 1: Linguistic terms for rating of attributes and Decision Makers.

Linguistic Terms	SVNNs
Very Good / Very Important (VG/VI)	$\langle 0.90, 0.10, 0.10\rangle$
Good / Important(G /I)	$\langle 0.80, 0.20, 0.15\rangle$
Fair / Medium(F/M)	$\langle 0.50, 0.40, 0.45\rangle$
Bad / Unimportant (B / UI)	$\langle 0.35, 0.60, 0.70\rangle$
Very Bad / Very Unimportant (VB/VUI)	$\langle 0.10, 0.80, 0.90\rangle$

Table 2: Importance of decision makers expressed with SVNNs.

	DM-1	DM-2	DM-3	DM-4
LT	VI	I	M	I
$\widetilde{W}$	$\langle 0.90, 0.10, 0.10\rangle$	$\langle 0.80, 0.20, 0.15\rangle$	$\langle 0.50, 0.40, 0.45\rangle$	$\langle 0.80, 0.20, 0.15\rangle$

Table 3: Linguistic terms for rating the candidates with SVNNs.

Linguistic terms	SVNNs
Extremely Good/High (EG/EH)	$\langle 1.00, 0.00, 0.00\rangle$
Very Good/High (VG/VH)	$\langle 0.90, 0.10, 0.05\rangle$
Good/High (G/H)	$\langle 0.80, 0.20, 0.15\rangle$
Medium Good/High (MG/MH)	$\langle 0.65, 0.35, 0.30\rangle$
Medium/Fair (M/F)	$\langle 0.50, 0.50, 0.45\rangle$
Medium Bad/Medium Law (MB/ML)	$\langle 0.35, 0.65, 0.60\rangle$
Bad/Law (B/L)	$\langle 0.20, 0.75, 0.80\rangle$
Very Bad/Low (VB/VL)	$\langle 0.10, 0.85, 0.90\rangle$
Very Very Bad/low (VVB/VVL)	$\langle 0.05, 0.90, 0.95\rangle$

Table 4: Assessments of alternatives and attribute weights given by four decision makers.

Alternatives (A_i)	Decision Makers	C_1	C_2	C_3	C_4	C_5	C_6
A_1	DM-1	VG	G	G	G	G	VG
	DM-2	VG	VG	G	G	G	VG
	DM-3	G	VG	G	G	VG	G
	DM-4	G	G	G	G	G	G
A_2	DM-1	M	G	M	G	G	M
	DM-2	G	MG	G	G	MG	G
	DM-3	G	M	G	G	M	M
	DM-4	M	G	M	G	M	M
A_3	DM-1	VG	VG	G	G	VG	VG
	DM-2	G	VG	VG	G	G	VG
	DM-3	VG	G	G	MG	G	MG
	DM-4	VG	VG	G	G	MG	G
A_4	DM-1	M	VG	G	G	VG	M
	DM-2	M	M	G	G	M	G
	DM-3	G	G	G	G	M	VG
	DM-4	G	M	M	G	G	VG
Weights	DM-1	VI	VI	I	M	I	I
	DM-2	I	VI	I	I	M	M
	DM-3	M	I	M	M	I	M
	DM-4	M	VI	M	I	VI	I

Table 5: Aggregated neutrosophic decision matrix.

$$
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3 \\ A_4
\end{array}
\begin{array}{c}
\begin{array}{ccc} \qquad C_1 \qquad & \qquad C_2 \qquad & \qquad C_3 \qquad \end{array} \\
\begin{pmatrix}
\langle 0.864, 0.136, 0.081\rangle & \langle 0.853, 0.147, 0.092\rangle & \langle 0.800, 0.200, 0.150\rangle \\
\langle 0.667, 0.333, 0.277\rangle & \langle 0.727, 0.273, 0.219\rangle & \langle 0.667, 0.333, 0.277\rangle \\
\langle 0.880, 0.120, 0.067\rangle & \langle 0.887, 0.113, 0.064\rangle & \langle 0.834, 0.166, 0.112\rangle \\
\langle 0.667, 0.333, 0.277\rangle & \langle 0.735, 0.265, 0.195\rangle & \langle 0.768, 0.232, 0.180\rangle
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3 \\ A_4
\end{array}
\begin{array}{c}
\begin{array}{ccc} \qquad C_4 \qquad & \qquad C_5 \qquad & \qquad C_6 \qquad \end{array} \\
\begin{pmatrix}
\langle 0.704, 0.296, 0.241\rangle & \langle 0.823, 0.177, 0.123\rangle & \langle 0.864, 0.136, 0.081\rangle \\
\langle 0.744, 0.256, 0.204\rangle & \langle 0.652, 0.348, 0.293\rangle & \langle 0.608, 0.392, 0.336\rangle \\
\langle 0.779, 0.221, 0.170\rangle & \langle 0.811, 0.189, 0.109\rangle & \langle 0.850, 0.150, 0.092\rangle \\
\langle 0.727, 0.273, 0.221\rangle & \langle 0.791, 0.209, 0.148\rangle & \langle 0.808, 0.192, 0.127\rangle
\end{pmatrix}
\end{array}
$$

Table 6: Aggregated weighted neutrosophic decision matrix.

$$
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3 \\ A_4
\end{array}
\begin{array}{c}
\begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\
\begin{pmatrix}
\langle 0.652, 0.328, 0.280 \rangle & \langle 0.757, 0.243, 0.289 \rangle & \langle 0.612, 0.381, 0.305 \rangle \\
\langle 0.504, 0.481, 0.434 \rangle & \langle 0.645, 0.355, 0.388 \rangle & \langle 0.510, 0.484, 0.409 \rangle \\
\langle 0.664, 0.315, 0.269 \rangle & \langle 0.787, 0.213, 0.267 \rangle & \langle 0.638, 0.354, 0.274 \rangle \\
\langle 0.504, 0.481, 0.434 \rangle & \langle 0.652, 0.348, 0.370 \rangle & \langle 0.588, 0.406, 0.329 \rangle
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3 \\ A_4
\end{array}
\begin{array}{c}
\begin{array}{ccc} \quad C_4 \quad & \quad C_5 \quad & \quad C_6 \quad \end{array} \\
\begin{pmatrix}
\langle 0.487, 0.491, 0.432 \rangle & \langle 0.649, 0.342, 0.281 \rangle & \langle 0.605, 0.371, 0.305 \rangle \\
\langle 0.515, 0.402, 0.404 \rangle & \langle 0.514, 0.478, 0.420 \rangle & \langle 0.426, 0.557, 0.498 \rangle \\
\langle 0.539, 0.437, 0.378 \rangle & \langle 0.639, 0.351, 0.269 \rangle & \langle 0.595, 0.381, 0.314 \rangle \\
\langle 0.503, 0.474, 0.417 \rangle & \langle 0.623, 0.367, 0.301 \rangle & \langle 0.566, 0.412, 0.340 \rangle
\end{pmatrix}
\end{array}
$$

Table 7: Distance measure and relative closeness co-efficient of each alternative.

Alternatives (A_i)	D^{i+}_{Eucl}	D^{i-}_{Eucl}	C_i^*
A_1	0.0283	0.1281	0.8190
A_2	0.3472	0.0490	0.1158
A_3	0.0224	0.1382	**0.8605**
A_4	0.0900	0.0831	0.4801