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The Characteristic Function of a Neutrosophic Set

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Abstract. The purpose of this paper is to introduce and study the characteristic function of a neutrosophic set. After given the fundamental definitions of neutrosophic set operations generated by the characteristic function of a neutrosophic set (for short), we obtain several properties, and discussed the relationship between neutrosophic sets generated by Ng and others. Finally, we introduce the neutrosophic topological spaces generated by . Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Neutrosophic Topology; Characteristic Function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [2-13]. In this paper we introduce definitions of neutrosophic sets by characteristic function. After given the fundamental definitions of neutrosophic set operations by , we obtain several properties, and discussed the relationship between neutrosophic sets and others. Added to, we introduce the neutrosophic topological spaces generated by Ng .

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7- 9], Hanafy, Salama et al. [2- 13] and Demirci in [1].

3 Neutrosophic Sets generated by Ng

We shall now consider some possible definitions for basic concepts of the neutrosophic sets generated by and its operations.

3.1 Definition

Let X is a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A=\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle$ where $\mu_A(x),\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x\in X$ to the set A .and let $g_A : X\times[0,1]\to[0,1]=I$ be reality function, then $Ng_A(\lambda)=Ng_A(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$is said to be the characteristic function of a neutrosophic set on X if

$$Ng_A(\lambda)=\begin{cases}1 \text{ if } \mu_A(x)=\lambda_1,\sigma_{A(x)}=\lambda_2,\nu_A(x)=\lambda_3\\ 0 \quad \text{otherwise}\end{cases}$$

Where $\lambda=(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$. Then the object $G(A)=\langle x,\mu_{G(A)}(x),\sigma_{G(A)}(x),\nu_{G(A)}(x)\rangle$ is a neutrosophic set generated by where

$$\mu_{G(A)}=\sup_{\lambda_1}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\sigma_{G(A)}=\sup_{\lambda_2}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\nu_{G(A)}=\sup_{\lambda_3}\{Ng_A(\lambda)\wedge\lambda\}$$

3.1 Proposition

1) $A\subseteq^{Ng} B\Leftrightarrow G(A)\subseteq G(B)$.

2) $A =^{Ng} B \Leftrightarrow G(A) = G(B)$

3.2 Definition

Let A be neutrosophic set of X. Then the neutrosophic complement of A generated by denoted by A^{Ngc} iff $[G(A)]^c$ may be defined as the following:

(Ng^{c1}) $\langle x, \mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x)\rangle$

(Ng^{c2}) $\langle x, \nu_A(x), \sigma\ {}_A(x), \mu_A(x)\rangle$

(Ng^{c3}) $\langle x, \nu_A(x), \sigma^c{}_A(x), \mu_A(x)\rangle$

3.1 Example. Let $X = \{x\}$, $A = \langle x, 0.5, 0.7, 0.6\rangle$, $Ng_A = 1$, $Ng_A = 0$. Then $G(A) = (\langle x, 0.5, 0.7, 0.6\rangle)$

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the $G(0_N)$ and $G(1_N)$ as follows $G(0_N)$ may be defined as:

i) $G(0_N) = \langle x, 0, 0, 1\rangle$

ii) $G(0_N) = \langle x, 0, 1, 1\rangle$

iii) $G(0_N) = \langle x, 0, 1, 0\rangle$

iv) $G(0_N) = \langle x, 0, 0, 0\rangle$

$G(1_N)$ may be defined as:

i) $G(1_N) = \langle x, 1, 0, 0\rangle$

ii) $G(1_N) = \langle x, 1, 0, 1\rangle$

iii) $G(1_N) = \langle x, 1, 1, 0\rangle$

iv) $G(1_N) = \langle x, 1, 1, 1\rangle$

We will define the following operations intersection and union for neutrosophic sets generated by Ng denoted by $\cap^{Ng}$ and $\cup^{Ng}$ respectively.

3.3 Definition. Let two neutrosophic sets $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and

$B = \langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle$ on X, and

$G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$,

$G(B) = \langle x, \mu_{G(B)}(x), \sigma_{G(B)}(x), \nu_{G(B)}(x)\rangle$. Then $A \cap^{Ng} B$ may be defined as three types:

i) Type $G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$

ii) Type II:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$.

ii) Type III:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \times \mu_{G(B)}, \sigma_{G(A)}(x) \times \sigma_{G(B)}(x), \nu_{G(A)}(x) \times \nu_{G(B)}(x)\rangle$

$A \cup^{Ng} B$ may be defined as two types:

Type I :

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$ ii) Type II:

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$

3.4 Definition

Let a neutrosophic set $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and $G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$. Then

(1) $[\ ]^{Ng} A = \langle x : \mu_{G(A)}(x), \sigma_{G(A)}(x), 1 - \nu_{G(A)}(x)\rangle$

(2) $<>^{Ng} A =$

$\langle x : 1 - \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$

3.2 Proposition

For all two neutrosophic sets A and B on X generated by Ng, then the following are true

1) $(A \cap B)^{cNg} = A^{cNg} \cup B^{cNg}$.

2) $(A \cup B)^{cNg} = A^{cNg} \cap B^{cNg}$.

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic subsets generated by Ng as follows:

3.3 Proposition.

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic subsets in X generated by two types , then

a) $\cap^{Ng} A_j$ may be defined as :

1) Type I :

$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

2) Type II:

$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

b) $\cup^{Ng} A_j$ may be defined as :

1) $G(\cup A_j) = \langle \vee \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x)\rangle$ or

2) $G(\cup A_j) = \left\langle \vee \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x) \right\rangle$.

3.4 Definition

Let f. X → Y be a mapping .

(i) The image of a neutrosophic set A generated by on X under f is a neutrosophic set B on Y generated by , denoted by f (A) whose reality function g_B : Y x I → I=[0, 1] satisfies the property

$$\mu_{G(B)} = \sup_{\lambda_1} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\sigma_{G(B)} = \sup_{\lambda_2} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\nu_{G(B)} = \sup_{\lambda_3} \{Ng_A(\lambda) \wedge \lambda\}$$

(ii) The preimage of a neutrosophic set B on Y generated by under f is a neutrosophic set A on X generated by , denoted by f^{-1} (B) , whose reality function g_A : X x [0, 1] →[0, 1] satisfies the property G(A) = G (B) o f

3.4 Proposition

Let $\{A_j : j \in J\}$ and $\{B_j : j \in J\}$ be families of neutrosophic sets on X and Y generated by , respectively. Then for a function f: X → Y, the following properties hold:

(i) If $A_j \subseteq^{Ng} A_k$;i , j∈J, then f (A_j) $\subseteq^{Ng}$ f(A_k)

(ii) If $B_j \subseteq^{Ng} B_k$, for j , K ∈ J, then

$f^{-1}(B_j) \subseteq^{Ng} f^{-1}(B_K)$

(iii) $f^{-1}\left(\underset{jEJ}{\cup}^{Ng} B_j \right) =^{Ng} \underset{jEJ}{\cup}^{Ng} f^{-1}(B_j)$

3.5 Proposition

Let A and B be neutrosophic sets on X and Y generated by , respectively. Then, for a mappings f : X → Y , we have :

(i) $A \subseteq^{Ng} f^{-1}(f(A))$ (if f is injective the equality holds) .

(ii) $f(f^{-1}(B)) \subseteq^{Ng} B$ (if f is surjective the equality holds) .

(iii) $[f^{-1}(B)]^{Ngc} \subseteq^{Ng} f^{-1}(B^{Ngc})$.

3.5 Definition . Let X be a nonempty set, Ψ a family of neutrosophic sets generated by and let us use the notation

$$G(\Psi) = \{ G(A) : A \in \Psi \} .$$

If (X , G (Ψ)= $N\tau$) is a neutrosophic topological space on X is Salama's sense [3] , then we say that Ψ is a neutrosophic topology on X generated by and the pair (X , Ψ) is said to be a neutrosophic topological space generated by (ngts , for short). The elements in Ψ are called genuine neutrosophic open sets. also , we define the family

$$G(\Psi^c) = \{ 1- G(A) : A \in \Psi \} .$$

3.6 Definition

Let (X , Ψ) be a ngts . A neutrosophic set C in X generated by is said to be a neutrosophic closed set generated by , if 1- G(C) ∈ G (Ψ) = $N\tau$.

3.7 Definition

Let (X , Ψ) be a ngts and A a neutrosophic set on X generated by . Then the neutrosophic interior of A generated by , denoted by, ngintA, is a set characterized by G(intA) = $\underset{G(\Psi)}{\text{int}}$ G(A) , where $\underset{G(\Psi)}{\text{int}}$ denotes the interior operation in neutrosophic topological spaces generated by .Similarly, the neutrosophic closure of A generated by , denoted by nglA , is a neutrosophic set characterized by G(ngclA)= $\underset{G(\psi)}{cl}$ G(A) , where $\underset{G(\psi)}{cl}$ denotes the closure operation in neutrosophic topological spaces generated by .

The neutrosophic interior gnint(A) and the genuine neutrosophic closure gnclA generated by can be characterized by :

gnintA $=^{Ng} \cup^{Ng}$ { U : U ∈ Ψ and U $\subseteq^{Ng}$ A }

gnclA $=^{Ng} \cap^{Ng}$ { C : C is neutrosophic closed generated by and A $\subseteq^{Ng}$ C }

Since : G (gnint A) = ∪ { G (U) : G (U) ∈ G (Ψ) , G (U) ⊆ G (A) }

G (gncl A) = ∩ { G (C) : G (C) ∈ G (Ψ^c) , G (A) ⊆ G(C) }.

3.6 Proposition . For any neutrosophic set A generated by on a NTS (X , Ψ) , we have

(i) $\text{cl } A^{Ngc} = {}^{Ng}(\text{ int } A)^{Ngc}$

(ii) $\text{Int } A^{Ngc} = {}^{Ng}(\text{ cl } A)^{Ngc}$

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Received: April 23rd, 2014. Accepted: May 4th, 2014.