TIME-NEUTROSOPHIC SOFT SET AND ITS APPLICATIONS

Shawkat Alkhazaleh *

Department of Mathematics, Faculty of Science and Art, Shaqra University, Saudi Arabia

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Abstract

In 2013 Ayman A. Hazaymeh in his PhD thesis introduced the concept of time-fuzzy soft set as a generalization of fuzzy soft set. In this paper and as a generalization of neutrosophic soft set we introduce the concept of time-neutrosophic soft set and study some of its properties. We also, define its basic operations, complement, union intersection, "AND" and "OR" and study their properties. Also, we give hypothetical application of this concept in decision making problems.
Keywords. neutrosophic set; soft set; neutrosophic soft set; time-neutrosophic soft set

1 General Introduction

In 1995, Smarandache [13] initiated the theory of neutrosophic set as new mathematical tool for handling problems involving imprecise, indeterminacy,and inconsistent data. Molodtsov [1] initiated the theory of soft set as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Presently, work on the soft set theory is progressing rapidly. Maji et al. [2] have also introduced the concept of fuzzy soft set, a more general concept, which is a combination of fuzzy set and soft set and studied its properties. Zou and Xian [10] introduced soft set and fuzzy soft set into the incomplete environment respectively. Alkhazaleh et al. [3] introduced the concept of soft multiset as a generalisation of soft set. They also defined the concepts of fuzzy parameterized interval-valued fuzzy soft set [4] and possibility fuzzy soft set [5] and gave their applications in decision making and medical diagnosis. Alkhazaleh and Salleh [6] introduced the concept of a soft expert set, where the user can know the opinion of all experts in one

*shmk79@gmail.com

model without any operations. Even after any operation the user can know the opinion of all experts. In 2011 Salleh [12] gave a brief survey from soft set to intuitionistic fuzzy soft set. Majumdar and Samanta [8] introduced and studied generalised fuzzy soft set where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Yang et al. [9] presented the concept of interval-valued fuzzy soft set by combining the interval-valued fuzzy set [7, 11] and soft set models. In 2009 Bhowmik and Pal [15] studied the concept of intuitionistic neutrosophic set, and Maji [14] introduced neutrosophic soft set, established its application in decision making, and thus opened a new direction, new path of thinking to engineers, mathematicians, computer scientists and many others in various tests. In 2013 Said and Smarandache [16] defined the concept of intuitionistic neutrosophic soft set and introduced some operations on intuitionistic neutrosophic soft set and some properties of this concept have been established. In many real situations, immediate sensory data is insufficient for decision making. Enriching the state with information about previous actions and situations can disambiguate between situations that would otherwise appear identical, which makes it possible to make correct decisions and also learn the correct decision. Moreover, knowledge of the past can replace the need for unrealistic sensors, such as knowing the exact location in a maze. Using historical information as part of the state representation give us useful information to help us in making better decision, where the time value is not taken into consideration and thus decision making is not very precise. If we want to take the opinions of more than one time (periods), we need to do some operations like union, intersection etc. To solve this problem In 2013 Ayman A. Hazaymeh [17] in his PhD thesis considered a collection of time (periods) and generalized into time-fuzzy soft set (TFSS) and studied some of its properties and explained this concept in decision making problem. In this paper we introduce the concept of time-neutrosophic soft set (TNSS) as a generalization of neutrosophic soft set. We also, define its basic operations, complement, union intersection, "AND" and "OR" and study their properties. Also, we give an application of this concept in decision making problems.

2 Preliminary

In this section we recall some definitions and properties regarding neutrosophic set theory, soft set theory time-fuzzy soft set and neutrosophic soft set theory required in this paper.

Definition 2.1. [13] A neutrosophic set A on the universe of discourse X is defined as $A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$ where T; I; $F : X \to]^-0; 1^+[$ and $^-0 \leqslant T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Molodtsov defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.2. [1] A pair (F,A) is called a *soft set* over U, where F is a mapping

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 2.3. [14] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.4. [14] Let (F,A) and (G,B) be two neutrosophic soft sets over the common universe U. (F,A) is said to be neutrosophic soft subset of (G,B) if $A \subset B$; and $T_F(e)(x) \leq T_G(e)(x)$; $I_F(e)(x) \leq I_G(e)(x)$; $F_F(e)(x) \geq F_G(e)(x)$; $\forall e \in A; x \in U$. We denote it by $(F,A) \subseteq (G,B)$. (F,A) is said to be neutrosophic soft super set of (G,B) if (G,B) is a neutrosophic soft subset of (F,A). We denote it by $(F,A) \supseteq (G,B)$.

Definition 2.5. [14] The complement of a neutrosophic soft set (F,A) denoted by $(F;A)^c$ and is denoted as $(F,A)^c = (F^c, \rceil A)$; where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(\alpha) =$ neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

Definition 2.6. [14] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the union of (H,A) and (G,B) is denoted by '$(H,A) \cup (G,B)$' and is defined by $(H,A) \cup (G,B) = (K,C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$$T_K(e)(m) = T_H(e)(m); \quad \text{if } e \in A - B;$$

$$= T_G(e)(m); \quad \text{if } e \in B - A;$$

$$= max(T_H(e)(m); T_G(e)(m)); \quad \text{if } e \in A \cap B.$$

$$I_K(e)(m) = I_H(e)(m); \quad \text{if } e \in A - B;$$

$$= I_G(e)(m); \quad \text{if } e \in B - A;$$

$$= \frac{I_H(e)(m) + I_G(e)(m))}{2}; \quad \text{if } e \in A \cap B.$$

$$F_K(e)(m) = F_H(e)(m); \quad \text{if } e \in A - B;$$

$$= F_G(e)(m); \quad \text{if } e \in B - A;$$

$$= min(F_H(e)(m); F_G(e)(m)); \quad \text{if } e \in A \cap B.$$

Definition 2.7. [14] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the intersection of (H,A) and (G,B) is denoted by '$(H,A) \cup (G,B)$' and is defined by $(H,A) \cup (G,B) = (K,C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$T_K(e)(m) = min(T_H(e)(m); T_G(e)(m));$

$I_K(e)(m) = \dfrac{I_H(e)(m) + I_G(e)(m))}{2}$ and

$F_K(e)(m) = min(F_H(e)(m); F_G(e)(m)); \quad \forall e \in C.$

Definition 2.8. [14] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the 'AND' operation on them is denoted by '$(H,A)\bigvee(G,B)$' and is defined by $(H,A)\bigvee(G,B) = (K, A\times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A\times B)$ are as follows:

$T_K(\alpha,\beta)(m) = min(T_H(\alpha)(m); T_G(\beta)(m));$

$I_K(\alpha,\beta)(m) = \dfrac{I_H(\alpha)(m) + I_G(\beta)(m))}{2}$ and

$F_K(\alpha,\beta)(m) = max(F_H(\alpha)(m); F_G(\beta)(m)); \quad \forall\alpha \in A, \forall\beta \in B.$

Definition 2.9. [14] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the 'OR' operation on them is denoted by '$(H,A)\bigvee(G,B)$' and is defined by $(H,A)\bigvee(G,B) = (O, A\times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(O, A\times B)$ are as follows:

$T_O(\alpha,\beta)(m) = max(T_H(\alpha)(m); T_G(\beta)(m));$

$I_O(\alpha,\beta)(m) = \dfrac{I_H(\alpha)(m) + I_G(\beta)(m))}{2}$ and

$F_O(\alpha,\beta)(m) = min(F_H(\alpha)(m); F_G(\beta)(m)); \quad \forall\alpha \in A, \forall\beta \in B.$

Definition 2.10. [17] Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U, let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F,E)_t \forall t \in T$ is called a *time-fuzzy soft set* $\{T - FSS\}$ over U where F is a mapping given by

$$F_t : A \to I^U.$$

2.1 Time-Neutrosophic Soft Set (TNSS)

Definition 2.11. Let U be an initial universal set and let E be a set of parameters. Let N^U denote the power set of all neutrosophic subsets of U, let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F,E)_t \forall t \in T$ is called a *time-neutrosophic soft set* $\{T - NSS\}$ over U where F is a mapping given by

$$F_t : A \to N^U.$$

Example 2.1. Let $U = \{u_1, u_2, u_3\}$ be a set of universe, $E = \{e_1, e_2, e_3\}$ a set of parameters and $T = \{t_1, t_2, t_3,\}$ be a set of time. Define a function

$$F_t : A \to N^U.$$

as follows:

$$F_1\left(e_1\right)=\left\{\frac{u_1{}^{t_1}}{\langle 0.5;0.2;0.4\rangle},\frac{u_2{}^{t_1}}{\langle 0.3;0.1;0.5\rangle},\frac{u_3{}^{t_1}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_1\left(e_2\right)=\left\{\frac{u_1{}^{t_1}}{\langle 0.7;0.1;0.2\rangle},\frac{u_2{}^{t_1}}{\langle 0.6;0.4;0.2\rangle},\frac{u_3{}^{t_1}}{\langle 0.2;0.2;0.6\rangle}\right\},$$

$$F_1\left(e_3\right)=\left\{\frac{u_1{}^{t_1}}{\langle 0.8;0.2;0.1\rangle},\frac{u_2{}^{t_1}}{\langle 0.6;0.4;0.1\rangle},\frac{u_3{}^{t_1}}{\langle 0.3;0.3;0.5\rangle}\right\},$$

$$F_2\left(e_1\right)=\left\{\frac{u_1{}^{t_2}}{\langle 0.7;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_2\left(e_2\right)=\left\{\frac{u_1{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_2\left(e_3\right)=\left\{\frac{u_1{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_3\left(e_1\right)=\left\{\frac{u_1{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_3}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_3\left(e_2\right)=\left\{\frac{u_1{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_3}}{\langle 0.4;0.2;0.3\rangle}\right\},$$

$$F_3\left(e_3\right)=\left\{\frac{u_1{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_3}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_3}}{\langle 0.4;0.2;0.3\rangle}\right\}.$$

Then we can find the time-neutrosophic soft sets $(F,E)_t$ as consisting of the following collection of approximations:

$$\begin{aligned}
(F,E)_t=\Bigg\{ &\left(e_1,\left\{\frac{u_1{}^{t_1}}{\langle 0.5;0.2;0.4\rangle},\frac{u_2{}^{t_1}}{\langle 0.3;0.1;0.5\rangle},\frac{u_3{}^{t_1}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),\\
&\left(e_2,\left\{\frac{u_1{}^{t_1}}{\langle 0.7;0.1;0.2\rangle},\frac{u_2{}^{t_1}}{\langle 0.6;0.4;0.2\rangle},\frac{u_3{}^{t_1}}{\langle 0.2;0.2;0.6\rangle}\right\}\right),\\
&\left(e_3,\left\{\frac{u_1{}^{t_1}}{\langle 0.8;0.2;0.1\rangle},\frac{u_2{}^{t_1}}{\langle 0.6;0.4;0.1\rangle},\frac{u_3{}^{t_1}}{\langle 0.3;0.3;0.5\rangle}\right\}\right),\\
&\left(e_1,\left\{\frac{u_1{}^{t_2}}{\langle 0.7;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),\\
&\left(e_2,\left\{\frac{u_1{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),\\
&\left(e_3,\left\{\frac{u_1{}^{t_2}}{\langle 0.5;0.3;0.6\rangle},\frac{u_2{}^{t_2}}{\langle 0.3;0.4;0.6\rangle},\frac{u_3{}^{t_2}}{\langle 0.1;0.3;0.5\rangle}\right\}\right),
\end{aligned}$$

$$\left(e_1,\left\{\frac{{u_1}^{t_3}}{\langle 0.7;0.1;0.1\rangle},\frac{{u_2}^{t_3}}{\langle 0.8;0.2;0.1\rangle},\frac{{u_3}^{t_3}}{\langle 0.7;0.1;0.3\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{{u_1}^{t_3}}{\langle 0.1;0.5;0.5\rangle},\frac{{u_2}^{t_3}}{\langle 0.5;0.3;0.2\rangle},\frac{{u_3}^{t_3}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),$$

$$\left(e_3,\left\{\frac{{u_1}^{t_3}}{\langle 0.9;0.4;0.1\rangle},\frac{{u_2}^{t_3}}{\langle 0.7;0.3;0.1\rangle},\frac{{u_3}^{t_3}}{\langle 0.5;0.5;0.1\rangle}\right\}\right)\Bigg\}.$$

Definition 2.12. For two T-NSSs $(F,A)_t$ and $(G,B)_t$ over U, $(F,A)_t$ is called a T-NSS subset of $(G,B)_t$ if

1. $B \subseteq A$,
2. $\forall t \in T, \varepsilon \in B$, $G_t(\varepsilon)$ is neutrosophic soft subset of $F_t(\varepsilon)$.

Definition 2.13. Two T-NSSs $(F,A)_t$ and $(G,B)_t$ over U, are said to be *equal* if $(F,A)_t$ is a T-NSS subset of $(G,A)_t$ and $(G,A)_t$ is a T-NSS subset of $(F,A)_t$.

Example 2.2. Consider Example 2.1 and suppose that the

$$\begin{aligned}(F,E)_t=\Bigg\{&\left(e_1,\left\{\frac{{u_1}^{t_1}}{\langle 0.5;0.2;0.4\rangle},\frac{{u_2}^{t_1}}{\langle 0.3;0.1;0.5\rangle},\frac{{u_3}^{t_1}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),\\
&\left(e_2,\left\{\frac{{u_1}^{t_1}}{\langle 0.7;0.1;0.2\rangle},\frac{{u_2}^{t_1}}{\langle 0.6;0.4;0.2\rangle},\frac{{u_3}^{t_1}}{\langle 0.2;0.2;0.6\rangle}\right\}\right),\\
&\left(e_2,\left\{\frac{{u_1}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{{u_2}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{{u_3}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),\\
&\left(e_3,\left\{\frac{{u_1}^{t_2}}{\langle 0.5;0.3;0.6\rangle},\frac{{u_2}^{t_2}}{\langle 0.3;0.4;0.6\rangle},\frac{{u_3}^{t_2}}{\langle 0.1;0.3;0.5\rangle}\right\}\right),\\
&\left(e_1,\left\{\frac{{u_1}^{t_3}}{\langle 0.7;0.1;0.1\rangle},\frac{{u_2}^{t_3}}{\langle 0.8;0.2;0.1\rangle},\frac{{u_3}^{t_3}}{\langle 0.7;0.1;0.3\rangle}\right\}\right),\\
&\left(e_3,\left\{\frac{{u_1}^{t_3}}{\langle 0.9;0.4;0.1\rangle},\frac{{u_2}^{t_3}}{\langle 0.7;0.3;0.1\rangle},\frac{{u_3}^{t_3}}{\langle 0.5;0.5;0.1\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

$$\begin{aligned}(G,E)_t=\Bigg\{&\left(e_2,\left\{\frac{{u_1}^{t_1}}{\langle 0.5;0.1;0.4\rangle},\frac{{u_2}^{t_1}}{\langle 0.4;0.2;0.4\rangle},\frac{{u_3}^{t_1}}{\langle 0.1;0.1;0.8\rangle}\right\}\right),\\
&\left(e_3,\left\{\frac{{u_1}^{t_2}}{\langle 0.2;0.1;0.8\rangle},\frac{{u_2}^{t_2}}{\langle 0.1;0.1;0.6\rangle},\frac{{u_3}^{t_2}}{\langle 0.2;0.1;0.7\rangle}\right\}\right),\\
&\left(e_1,\left\{\frac{{u_1}^{t_3}}{\langle 0.3;0.1;0.5\rangle},\frac{{u_2}^{t_3}}{\langle 0.2;0.2;0.7\rangle},\frac{{u_3}^{t_3}}{\langle 0.2;0.1;0.5\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

Therefore $(G,E)_t \subseteq (F,E)_t$.

Definition 2.14. A time neutrosophic soft set $(F,A)_t$ over U is said to be semi-null T-NSS denoted by $T_{\sim}\phi$, if $\forall t \in T$, $F_t(e) = \phi$ for at least one e.

Definition 2.15. A time neutrosophic soft set $(F,A)_t$ over U is said to be null T-NSS denoted by T_ϕ, if $\forall t \in T$, $F_t(e) = \phi \ \forall e$.

Definition 2.16. A time neutrosophic soft set $(F,A)_t$ over U is said to be semi-absolute T-NSS denoted by $T_{\sim}A$, if $\forall t \in T$, $F_t(e) = \bar{1}$ for at least one e.

Definition 2.17. A time neutrosophic soft set $(F,A)_t$ over U is said to be absolute T-NSS denoted by T_A, if $\forall t \in T$, $F_t(e) = \bar{1} \ \forall e$.

Example 2.3. Consider Example 2.1. Let

$$\begin{aligned}
(F,E)_t = \Bigg\{ & \left(e_1, \left\{ \frac{u_1{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_2{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_3{}^{t_1}}{\langle 0;0;0\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_2{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_3{}^{t_1}}{\langle 0;0;0\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_2{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_3{}^{t_1}}{\langle 0;0;0\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{u_1{}^{t_2}}{\langle 0.7;0.2;0.3\rangle}, \frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}, \frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}, \frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}, \frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_2}}{\langle 0.5;0.3;0.6\rangle}, \frac{u_2{}^{t_2}}{\langle 0.3;0.4;0.6\rangle}, \frac{u_3{}^{t_2}}{\langle 0.1;0.3;0.5\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{u_1{}^{t_3}}{\langle 0.7;0.1;0.1\rangle}, \frac{u_2{}^{t_3}}{\langle 0.8;0.2;0.1\rangle}, \frac{u_3{}^{t_3}}{\langle 0.7;0.1;0.3\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_3}}{\langle 0.1;0.5;0.5\rangle}, \frac{u_2{}^{t_3}}{\langle 0.5;0.3;0.2\rangle}, \frac{u_3{}^{t_3}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_3}}{\langle 0.9;0.4;0.1\rangle}, \frac{u_2{}^{t_3}}{\langle 0.7;0.3;0.1\rangle}, \frac{u_3{}^{t_3}}{\langle 0.5;0.5;0.1\rangle} \right\}\right) \Bigg\}.
\end{aligned}$$

Then $(F,E)_t = T_{\sim}\phi$.

Let

$$\begin{aligned}
(F,E)_t = \Bigg\{ & \left(e_1, \left\{ \frac{u_1{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_2{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_3{}^{t_1}}{\langle 0;0;0\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_2{}^{t_1}}{\langle 0;0;0\rangle}, \frac{u_3{}^{t_1}}{\langle 0;0;0\rangle} \right\}\right),
\end{aligned}$$

$$\left(e_3,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_1,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_3,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_1,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right),$$

$$\left(e_3,\left\{\frac{u_1{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_2{}^{t_1}}{\langle 0;0;0\rangle},\frac{u_3{}^{t_1}}{\langle 0;0;0\rangle}\right\}\right)\Big\}.$$

Then $(F,E)_t = T_\phi$.

Let

$$(F,E)_t = \Big\{\left(e_1,\left\{\frac{u_1{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_2{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_3{}^{t_1}}{\langle 1;1;0\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_2{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_3{}^{t_1}}{\langle 1;1;0\rangle}\right\}\right),$$

$$\left(e_3,\left\{\frac{u_1{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_2{}^{t_1}}{\langle 1;1;0\rangle},\frac{u_3{}^{t_1}}{\langle 1;1;0\rangle}\right\}\right),$$

$$\left(e_1,\left\{\frac{u_1{}^{t_2}}{\langle 0.7;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_2{}^{t_2}}{\langle 0.4;0.2;0.3\rangle},\frac{u_3{}^{t_2}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),$$

$$\left(e_3,\left\{\frac{u_1{}^{t_2}}{\langle 0.5;0.3;0.6\rangle},\frac{u_2{}^{t_2}}{\langle 0.3;0.4;0.6\rangle},\frac{u_3{}^{t_2}}{\langle 0.1;0.3;0.5\rangle}\right\}\right),$$

$$\left(e_1,\left\{\frac{u_1{}^{t_3}}{\langle 0.7;0.1;0.1\rangle},\frac{u_2{}^{t_3}}{\langle 0.8;0.2;0.1\rangle},\frac{u_3{}^{t_3}}{\langle 0.7;0.1;0.3\rangle}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_3}}{\langle 0.1;0.5;0.5\rangle},\frac{u_2{}^{t_3}}{\langle 0.5;0.3;0.2\rangle},\frac{u_3{}^{t_3}}{\langle 0.4;0.2;0.3\rangle}\right\}\right),$$

$$\left(e_3, \left\{ \frac{u_1{}^{t_3}}{\langle 0.9; 0.4; 0.1\rangle}, \frac{u_2{}^{t_3}}{\langle 0.7; 0.3; 0.1\rangle}, \frac{u_3{}^{t_3}}{\langle 0.5; 0.5; 0.1\rangle} \right\}\right) \Bigg\}.$$

Then $(F,A)_t = T_{\sim}A$.

Let

$$\begin{aligned}
(F,E)_t = \Bigg\{ & \left(e_1, \left\{ \frac{u_1{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_1}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_1}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_1}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_1}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{u_1{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_2}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_2}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_2}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_2}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{u_1{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_3}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{u_1{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_3}}{\langle 1;1;0\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{u_1{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_2{}^{t_3}}{\langle 1;1;0\rangle}, \frac{u_3{}^{t_3}}{\langle 1;1;0\rangle} \right\}\right) \Bigg\}.
\end{aligned}$$

Then $(F,A)_t = T_A$.

3 Basic Operations

In this section we introduce the definitions of complement, union and intersection of T-NSS, derive some properties and give some examples.

3.1 Complement

Definition 3.1. The *complement* of T-NSS $(F,A)_t$ is denoted by $\widetilde{c}(F,A)_t$ $\forall t \in T$ where $\widetilde{c}$ denotes a neutrosophic soft complement.

Example 3.1. Consider Example 2.1, we have

$$
\begin{aligned}
\widetilde{c}(F,E)_t = \Bigg\{ & \left(e_1, \left\{\frac{{u_1}^{t_1}}{\langle 0.4;0.2;0.5\rangle}, \frac{{u_2}^{t_1}}{\langle 0.5;0.1;0.3\rangle}, \frac{{u_3}^{t_1}}{\langle 0.3;0.2;0.4\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_1}}{\langle 0.2;0.1;0.7\rangle}, \frac{{u_2}^{t_1}}{\langle 0.2;0.4;0.6\rangle}, \frac{{u_3}^{t_1}}{\langle 0.6;0.2;0.2\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_1}}{\langle 0.1;0.2;0.8\rangle}, \frac{{u_2}^{t_1}}{\langle 0.1;0.4;0.6\rangle}, \frac{{u_3}^{t_1}}{\langle 0.5;0.3;0.3\rangle}\right\}\right), \\
& \left(e_1, \left\{\frac{{u_1}^{t_2}}{\langle 0.3;0.2;0.7\rangle}, \frac{{u_2}^{t_2}}{\langle 0.3;0.2;0.4\rangle}, \frac{{u_3}^{t_2}}{\langle 0.3;0.2;0.4\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_2}}{\langle 0.3;0.2;0.4\rangle}, \frac{{u_2}^{t_2}}{\langle 0.3;0.2;0.4\rangle}, \frac{{u_3}^{t_2}}{\langle 0.3;0.2;0.4\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_2}}{\langle 0.6;0.3;0.5\rangle}, \frac{{u_2}^{t_2}}{\langle 0.6;0.4;0.3\rangle}, \frac{{u_3}^{t_2}}{\langle 0.1;0.3;0.5\rangle}\right\}\right), \\
& \left(e_1, \left\{\frac{{u_1}^{t_3}}{\langle 0.1;0.1;0.7\rangle}, \frac{{u_2}^{t_3}}{\langle 0.1;0.2;0.8\rangle}, \frac{{u_3}^{t_3}}{\langle 0.3;0.1;0.7\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_3}}{\langle 0.5;0.5;0.1\rangle}, \frac{{u_2}^{t_3}}{\langle 0.2;0.3;0.5\rangle}, \frac{{u_3}^{t_3}}{\langle 0.3;0.2;0.4\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_3}}{\langle 0.1;0.4;0.9\rangle}, \frac{{u_2}^{t_3}}{\langle 0.1;0.3;0.7\rangle}, \frac{{u_3}^{t_3}}{\langle 0.1;0.5;0.5\rangle}\right\}\right)\Bigg\}.
\end{aligned}
$$

Proposition 3.1. *If $(F,A)_t$ is a T-NSS over U, and by using the neutrosophic complement we have:*

1. $\widetilde{c}(\widetilde{c}(F,A)_t) = (F,A)_t$,
2. $\widetilde{c}(T_{\sim}\phi) = (T_{\sim}A)$,
3. $\widetilde{c}(T_\phi) = (T_A)$,
4. $\widetilde{c}(T_{\sim}A) = (T_{\sim}\phi)$,
5. $\widetilde{c}(T_A) = (T_\phi)$.

Proof. The proof is straightforward from Definition 3.1. □

3.2 Union

Definition 3.2. The *union* of two T-NSSs $(F,A)_t$ and $(G,B)_t$ over U, is the T-NSS $(H,C)_t$, denoted by $(F,A)_t \widetilde{\cup} (G,B)_t$, such that $C = A \cup B \subset E$ and is defined as follows

$$
H_t(\varepsilon) = \begin{cases} F_t(\varepsilon), & if\, \varepsilon \in A - B, \\ G_t(\varepsilon), & if\, \varepsilon \in B - A, \\ F_t(\varepsilon) \widetilde{\cup}\, G_t(\varepsilon), & if\, \varepsilon \in A \cap B, \end{cases}
$$

where $\tilde{\cup}$ denoted the neutrosophic soft union.

Example 3.2. Consider Example 2.1. Suppose $(F,A)_t$ and $(G,B)_t$ are two time-neutrosophic soft sets over U such that

$$
\begin{aligned}
(F,A)_t = \Bigg\{ & \left(e_1, \left\{\frac{{u_1}^{t_1}}{\langle 0.1;0.5;0.6\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.3;0.4\rangle}, \frac{{u_3}^{t_1}}{\langle 0.3;0.3;0.6\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_1}}{\langle 0.7;0.4;0.3\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.5;0.3\rangle}, \frac{{u_3}^{t_1}}{\langle 0.4;0.4;0.4\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_2}}{\langle 0.1;0.5;0.7\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.6;0.2\rangle}, \frac{{u_3}^{t_2}}{\langle 0.8;0.2;0.1\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_3}}{\langle 0.5;0.2;0.4\rangle}, \frac{{u_2}^{t_3}}{\langle 0.3;0.3;0.6\rangle}, \frac{{u_3}^{t_3}}{\langle 0.1;0.2;0.9\rangle}\right\}\right) \Bigg\}.
\end{aligned}
$$

$$
\begin{aligned}
(G,B)_t = \Bigg\{ & \left(e_1, \left\{\frac{{u_1}^{t_1}}{\langle 0.4;0.3;0.6\rangle}, \frac{{u_2}^{t_1}}{\langle 0.1;0.7;0.6\rangle}, \frac{{u_3}^{t_1}}{\langle 0.5;0.4;0.4\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_2}}{\langle 0.6;0.2;0.3\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.7;0.3\rangle}, \frac{{u_3}^{t_2}}{\langle 0.2;0.5;0.7\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_3}}{\langle 0.7;0.5;0.2\rangle}, \frac{{u_2}^{t_3}}{\langle 0.1;0.5;0.8\rangle}, \frac{{u_3}^{t_3}}{\langle 0.9;0.5;0.1\rangle}\right\}\right) \Bigg\}.
\end{aligned}
$$

By using neutrosophic union we can easily verify that $(F,A)_t \tilde{\cup} (G,B)_t = (H,C)_t$ where

$$
\begin{aligned}
(H,C)_t = \Bigg\{ & \left(e_1, \left\{\frac{{u_1}^{t_1}}{\langle 0.4;0.4;0.6\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.5;0.4\rangle}, \frac{{u_3}^{t_1}}{\langle 0.5;0.35;0.4\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_1}}{\langle 0.7;0.4;0.3\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.5;0.3\rangle}, \frac{{u_3}^{t_1}}{\langle 0.4;0.4;0.4\rangle}\right\}\right), \\
& \left(e_2, \left\{\frac{{u_1}^{t_2}}{\langle 0.1;0.5;0.7\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.6;0.2\rangle}, \frac{{u_3}^{t_2}}{\langle 0.8;0.2;0.1\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_2}}{\langle 0.6;0.2;0.3\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.7;0.3\rangle}, \frac{{u_3}^{t_2}}{\langle 0.2;0.5;0.7\rangle}\right\}\right), \\
& \left(e_3, \left\{\frac{{u_1}^{t_3}}{\langle 0.7;0.35;0.2\rangle}, \frac{{u_2}^{t_3}}{\langle 0.3;0.4;0.6\rangle}, \frac{{u_3}^{t_3}}{\langle 0.9;0.35;0.1\rangle}\right\}\right) \Bigg\}.
\end{aligned}
$$

Proposition 3.2. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-FSSs over U, then*

1. $(F,A)_t \tilde{\cup} ((G,B)_t \tilde{\cup} (H,C)_t) = ((F,A)_t \tilde{\cup} (G,B))_t \tilde{\cup} (H,C)_t$,
2. $(F,A)_t \tilde{\cup} (F,A)_t = (F,A)_t$.

Proof. The proof is straightforward from Definition 3.2. $\square$

3.3 Intersection

Definition 3.3. The *intersection* of two T-NSSs $(F,A)_t$ and $(G,B)_t$ over U, is the T-NSS $(H,C)_t$, denoted by $(F,A)_t \widetilde{\cap} (G,B)_t$, such that $C = A \cup B \subset E$ and is defined as follows

$$H_t(\varepsilon) = \begin{cases} F_t(\varepsilon), & if\, \varepsilon \in A - B, \\ G_t(\varepsilon), & if\, \varepsilon \in B - A, \\ F_t(\varepsilon) \tilde{\cap} G_t(\varepsilon), & if\, \varepsilon \in A \cap B, \end{cases}$$

where $\tilde{\cap}$ denoted the neutrosophic soft intersection.

Example 3.3. Consider Example 3.2. By using basic neutrosophic intersection we can easily verify that $(F,A)_t \widetilde{\cap} (G,B)_t = (H,C)_t$ where

$$\begin{aligned} (H,C)_t = \Bigg\{ & \left(e_1, \left\{\frac{{u_1}^{t_1}}{\langle 0.1;0.4;0.6\rangle}, \frac{{u_2}^{t_1}}{\langle 0.1;0.5;0.6\rangle}, \frac{{u_3}^{t_1}}{\langle 0.3;0.35;0.6\rangle}\right\}\right), \\ & \left(e_2, \left\{\frac{{u_1}^{t_1}}{\langle 0.7;0.4;0.3\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.5;0.3\rangle}, \frac{{u_3}^{t_1}}{\langle 0.4;0.4;0.4\rangle}\right\}\right), \\ & \left(e_2, \left\{\frac{{u_1}^{t_2}}{\langle 0.1;0.5;0.7\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.6;0.2\rangle}, \frac{{u_3}^{t_2}}{\langle 0.8;0.2;0.1\rangle}\right\}\right), \\ & \left(e_3, \left\{\frac{{u_1}^{t_2}}{\langle 0.6;0.2;0.3\rangle}, \frac{{u_2}^{t_2}}{\langle 0.4;0.7;0.3\rangle}, \frac{{u_3}^{t_2}}{\langle 0.2;0.5;0.7\rangle}\right\}\right), \\ & \left(e_3, \left\{\frac{{u_1}^{t_3}}{\langle 0.5;0.35;0.4\rangle}, \frac{{u_2}^{t_3}}{\langle 0.1;0.4;0.8\rangle}, \frac{{u_3}^{t_3}}{\langle 0.1;0.35;0.9\rangle}\right\}\right) \Bigg\}. \end{aligned}$$

Proposition 3.3. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-NSSs over U, then*

1. $(F,A)_t \widetilde{\cap} ((G,B)_t \widetilde{\cap} (H,C)_t) = ((F,A)_t \widetilde{\cap} (G,B))_t \widetilde{\cap} (H,C)_t$,
2. $(F,A)_t \widetilde{\cap} (F,A)_t = (F,A)_t$.

Proof. The proof is straightforward from Definition 3.3. □

Proposition 3.4. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-NSSs over U, then*

1. $(F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cap} \left((F,A)_t \widetilde{\cup} (H,C)_t\right)$,
2. $(F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cup} \left((F,A)_t \widetilde{\cap} (H,C)_t\right)$.

Proof. The proof is straightforward from Definitions 3.3 and 3.2. □

Proposition 3.5. *If $(F,A)_t$ and $(G,B)_t$ are two T-NSSs over U, then*

1. $\left((F,A)_t \widetilde{\cup} (G,B)_t\right)^c = (F,A)_t^c \widetilde{\cap} (G,B)_t^c$,
2. $\left((F,A)_t \widetilde{\cap} (G,B)_t\right)^c = (F,A)_t^c \widetilde{\cup} (G,B)_t^c$.

Proof. The proof is straightforward from Definitions 3.3 and 3.2. □

4 AND and OR Operations

In this section, we introduce the definitions of AND and OR operations for T-NSS, derive their properties, and give some examples.

Definition 4.1. If $(F,A)_t$ and $(G,B)_t$ are two T-NSSs over U then "$(F,A)_t$ AND $(G,B)_t$" denoted by $(F,A)_t \wedge (G,B)_t$, is defined by

$$(F,A)_t \wedge (G,B)_t = (H,A \times B)_t$$

such that $H(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcap} G(\beta)_t, \forall (\alpha,\beta) \in A \times B$, where $\widetilde{\bigcap}$ is time-neutrosophic soft intersection.

Example 4.1. Consider Example 2.1. Let $(F,A)_t$ and $(G,B)_t$ are two T-NSSs over U such that

$$\begin{aligned}(F,A)_t = \Bigg\{ & \left(e_1, \left\{\frac{u_1{}^{t_1}}{\langle 0.1;0.5;0.6\rangle}, \frac{u_2{}^{t_1}}{\langle 0.4;0.3;0.4\rangle}, \frac{u_3{}^{t_1}}{\langle 0.3;0.3;0.6\rangle}\right\}\right), \\ & \left(e_2, \left\{\frac{u_1{}^{t_1}}{\langle 0.7;0.4;0.3\rangle}, \frac{u_2{}^{t_1}}{\langle 0.4;0.5;0.3\rangle}, \frac{u_3{}^{t_1}}{\langle 0.4;0.4;0.4\rangle}\right\}\right), \\ & \left(e_2, \left\{\frac{u_1{}^{t_2}}{\langle 0.1;0.5;0.7\rangle}, \frac{u_2{}^{t_2}}{\langle 0.4;0.6;0.2\rangle}, \frac{u_3{}^{t_2}}{\langle 0.8;0.2;0.1\rangle}\right\}\right), \\ & \left(e_3, \left\{\frac{u_1{}^{t_3}}{\langle 0.5;0.2;0.4\rangle}, \frac{u_2{}^{t_3}}{\langle 0.3;0.3;0.6\rangle}, \frac{u_3{}^{t_3}}{\langle 0.1;0.2;0.9\rangle}\right\}\right) \Bigg\}.\end{aligned}$$

$$\begin{aligned}(G,B)_t = \Bigg\{ & \left(e_1, \left\{\frac{u_1{}^{t_1}}{\langle 0.4;0.3;0.6\rangle}, \frac{u_2{}^{t_1}}{\langle 0.1;0.7;0.6\rangle}, \frac{u_3{}^{t_1}}{\langle 0.5;0.4;0.4\rangle}\right\}\right), \\ & \left(e_3, \left\{\frac{u_1{}^{t_2}}{\langle 0.6;0.2;0.3\rangle}, \frac{u_2{}^{t_2}}{\langle 0.4;0.7;0.3\rangle}, \frac{u_3{}^{t_2}}{\langle 0.2;0.5;0.7\rangle}\right\}\right), \\ & \left(e_3, \left\{\frac{u_1{}^{t_3}}{\langle 0.7;0.5;0.2\rangle}, \frac{u_2{}^{t_3}}{\langle 0.1;0.5;0.8\rangle}, \frac{u_3{}^{t_3}}{\langle 0.9;0.5;0.1\rangle}\right\}\right) \Bigg\}.\end{aligned}$$

Then we can easily verify that $(F,A) \wedge (G,B) = (H,A \times B)$ where:

$$\begin{aligned}(H,A \times B) = \Bigg\{ & \left(\left(e_1^{t_1}, e_1^{t_1}\right), \left\{\frac{u_1{}^{t_{1,1}}}{\langle 0.4;0.4;0.6\rangle}, \frac{u_2{}^{t_{1,1}}}{\langle 0.4;0.5;0.4\rangle}, \frac{u_3{}^{t_{1,1}}}{\langle 0.5;0.35;0.4\rangle}\right\}\right), \\ & \left(\left(e_1^{t_1}, e_3^{t_2}\right), \left\{\frac{u_1{}^{t_{1,2}}}{\langle 0.6;0.35;0.3\rangle}, \frac{u_2{}^{t_{1,2}}}{\langle 0.4;0.5;0.3\rangle}, \frac{u_3{}^{t_{1,2}}}{\langle 0.3;0.4;0.4\rangle}\right\}\right), \\ & \left(\left(e_1^{t_1}, e_3^{t_3}\right), \left\{\frac{u_1{}^{t_{1,3}}}{\langle 0.7;0.5;0.2\rangle}, \frac{u_2{}^{t_{1,3}}}{\langle 0.4;0.4;0.4\rangle}, \frac{u_3{}^{t_{1,3}}}{\langle 0.9;0.4;0.1\rangle}\right\}\right), \\ & \left(\left(e_2^{t_1}, e_1^{t_1}\right), \left\{\frac{u_1{}^{t_{1,1}}}{\langle 0.7;0.35;0.3\rangle}, \frac{u_2{}^{t_{1,1}}}{\langle 0.4;0.6;0.3\rangle}, \frac{u_3{}^{t_{1,1}}}{\langle 0.5;0.4;0.4\rangle}\right\}\right),\end{aligned}$$

$$\left(\left(e_2^{t_1},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{1,2}}}{\langle 0.7;0.3;0.3\rangle},\frac{u_2{}^{t_{1,2}}}{\langle 0.4;0.6;0.3\rangle},\frac{u_3{}^{t_{1,2}}}{\langle 0.4;0.45;0.4\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_1},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{1,3}}}{\langle 0.7;0.45;0.2\rangle},\frac{u_2{}^{t_{1,3}}}{\langle 0.4;0.5;0.3\rangle},\frac{u_3{}^{t_{1,3}}}{\langle 0.9;0.45;0.1\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_2},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{2,1}}}{\langle 0.4;0.4;0.6\rangle},\frac{u_2{}^{t_{2,1}}}{\langle 0.4;0.65;0.2\rangle},\frac{u_3{}^{t_{2,1}}}{\langle 0.8;0.3;0.1\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_2},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{2,2}}}{\langle 0.6;0.35;0.3\rangle},\frac{u_2{}^{t_{2,2}}}{\langle 0.4;0.65;0.2\rangle},\frac{u_3{}^{t_{2,2}}}{\langle 0.8;0.35;0.1\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_2},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{2,3}}}{\langle 0.7;0.5;0.2\rangle},\frac{u_2{}^{t_{2,3}}}{\langle 0.4;0.55;0.2\rangle},\frac{u_3{}^{t_{2,3}}}{\langle 0.9;0.35;0.1\rangle}\right\}\right),$$

$$\left(\left(e_3^{t_3},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{3,1}}}{\langle 0.5;0.25;0.4\rangle},\frac{u_2{}^{t_{3,1}}}{\langle 0.3;0.5;0.6\rangle},\frac{u_3{}^{t_{3,1}}}{\langle 0.5;0.3;0.4\rangle}\right\}\right),$$

$$\left(\left(e_3^{t_3},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{3,2}}}{\langle 0.6;0.2;0.3\rangle},\frac{u_2{}^{t_{3,2}}}{\langle 0.4;0.5;0.3\rangle},\frac{u_3{}^{t_{3,2}}}{\langle 0.2;0.35;0.7\rangle}\right\}\right),$$

$$\left.\left(\left(e_3^{t_3},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{3,3}}}{\langle 0.7;0.35;0.2\rangle},\frac{u_2{}^{t_{3,3}}}{\langle 0.3;0.4;0.6\rangle},\frac{u_3{}^{t_{3,3}}}{\langle 0.9;0.35;0.1\rangle}\right\}\right)\right\}.$$

Definition 4.2. If $(F,A)_t$ and $(G,B)_t$ are two T-NSSs over U then "$(F,A)_t$ OR $(G,B)_t$" denoted by $(F,A)_t \vee (G,B)_t$, is defined by

$$(F,A)_t \vee (G,B)_t = (O,A\times B)_t$$

such that $O(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcup} G(\beta)_t, \forall(\alpha,\beta)\in A\times B$, where $\widetilde{\bigcup}$ is time-neutrosophic soft union.

Example 4.2. Consider Example 4.1 we have U Then we can easily verify that $(F,A)\vee(G,B) = (O,A\times B)$ where:

$$(O,A\times B) = \left\{\left(\left(e_1^{t_1},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{1,1}}}{\langle 0.1;0.4;0.6\rangle},\frac{u_2{}^{t_{1,1}}}{\langle 0.1;0.5;0.6\rangle},\frac{u_3{}^{t_{1,1}}}{\langle 0.3;0.35;0.6\rangle}\right\}\right),\right.$$

$$\left(\left(e_1^{t_1},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{1,2}}}{\langle 0.1;0.35;0.6\rangle},\frac{u_2{}^{t_{1,2}}}{\langle 0.4;0.5;0.4\rangle},\frac{u_3{}^{t_{1,2}}}{\langle 0.2;0.4;0.7\rangle}\right\}\right),$$

$$\left(\left(e_1^{t_1},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{1,3}}}{\langle 0.1;0.5;0.6\rangle},\frac{u_2{}^{t_{1,3}}}{\langle 0.1;0.4;0.8\rangle},\frac{u_3{}^{t_{1,3}}}{\langle 0.3;0.4;0.6\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_1},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{1,1}}}{\langle 0.4;0.35;0.6\rangle},\frac{u_2{}^{t_{1,1}}}{\langle 0.1;0.6;0.6\rangle},\frac{u_3{}^{t_{1,1}}}{\langle 0.4;0.4;0.4\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_1},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{1,2}}}{\langle 0.6;0.3;0.3\rangle},\frac{u_2{}^{t_{1,2}}}{\langle 0.4;0.6;0.3\rangle},\frac{u_3{}^{t_{1,2}}}{\langle 0.2;0.45;0.7\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_1},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{1,3}}}{\langle 0.7;0.45;0.3\rangle},\frac{u_2{}^{t_{1,3}}}{\langle 0.1;0.5;0.8\rangle},\frac{u_3{}^{t_{1,3}}}{\langle 0.8;0.45;0.1\rangle}\right\}\right),$$

$$\left(\left(e_2^{t_2}, e_1^{t_1}\right), \left\{\frac{u_1{}^{t_{2,1}}}{\langle 0.1;0.4;0.7\rangle}, \frac{u_2{}^{t_{2,1}}}{\langle 0.1;0.65;0.6\rangle}, \frac{u_3{}^{t_{2,1}}}{\langle 0.5;0.3;0.4\rangle}\right\}\right),$$
$$\left(\left(e_2^{t_2}, e_3^{t_2}\right), \left\{\frac{u_1{}^{t_{2,2}}}{\langle 0.1;0.35;0.7\rangle}, \frac{u_2{}^{t_{2,2}}}{\langle 0.4;0.65;0.3\rangle}, \frac{u_3{}^{t_{2,2}}}{\langle 0.2;0.35;0.7\rangle}\right\}\right),$$
$$\left(\left(e_2^{t_2}, e_3^{t_3}\right), \left\{\frac{u_1{}^{t_{2,3}}}{\langle 0.1;0.5;0.7\rangle}, \frac{u_2{}^{t_{2,3}}}{\langle 0.1;0.55;0.8\rangle}, \frac{u_3{}^{t_{2,3}}}{\langle 0.8;0.35;0.1\rangle}\right\}\right),$$
$$\left(\left(e_3^{t_3}, e_1^{t_1}\right), \left\{\frac{u_1{}^{t_{3,1}}}{\langle 0.4;0.25;0.6\rangle}, \frac{u_2{}^{t_{3,1}}}{\langle 0.1;0.5;0.6\rangle}, \frac{u_3{}^{t_{3,1}}}{\langle 0.1;0.3;0.9\rangle}\right\}\right),$$
$$\left(\left(e_3^{t_3}, e_3^{t_2}\right), \left\{\frac{u_1{}^{t_{3,2}}}{\langle 0.5;0.2;0.4\rangle}, \frac{u_2{}^{t_{3,2}}}{\langle 0.3;0.5;0.6\rangle}, \frac{u_3{}^{t_{3,2}}}{\langle 0.1;0.35;0.9\rangle}\right\}\right),$$
$$\left(\left(e_3^{t_3}, e_3^{t_3}\right), \left\{\frac{u_1{}^{t_{3,3}}}{\langle 0.5;0.35;0.4\rangle}, \frac{u_2{}^{t_{3,3}}}{\langle 0.1;0.4;0.8\rangle}, \frac{u_3{}^{t_{3,3}}}{\langle 0.2;0.35;0.7\rangle}\right\}\right)\Bigg\}.$$

Proposition 4.1. *If (F,A) and (G,B) are two T-NSSs over U, then*

1. $((F,A)_t \wedge (G,B)_t)^c = (F,A)_t^c \vee (G,B)_t^c$
2. $((F,A)_t \vee (G,B)_t)^c = (F,A)_t^c \wedge (G,B)_t^c$

Proof. The proof is straightforward from Definitions 4.1, 4.2 and 3.1. □

Proposition 4.2. *If (F,A), (G,B) and (H,C) are three T-NSSs over U, then*

1. $(F,A)_t \wedge ((G,B)_t \wedge (H,C)_t) = ((F,A)_t \wedge (G,B)_t) \wedge (H,C)_t$,
2. $(F,A)_t \vee ((G,B)_t \vee (H,C)_t) = ((F,A)_t \vee (G,B)_t) \vee (H,C)_t$,
3. $(F,A)_t \vee ((G,B)_t \wedge (H,C)_t) = ((F,A)_t \vee (G,B)_t) \wedge ((F,A)_t \vee (H,C)_t)$,
4. $(F,A)_t \wedge ((G,B)_t \vee (H,C)_t) = ((F,A)_t \wedge (G,B)_t) \vee ((F,A)_t \wedge (H,C)_t)$.

Proof. The proof is straightforward from Definitions 4.1 and 4.2. □

4.1 An Application of Time-Neutrosophic Soft in Decision Making

In this section, we provide hypothetical application of the time-neutrosophic soft set theory in a decision making problem which demonstrate that this method can be successfully applied to problems of many fields that contain uncertainty. We suggest the following algorithm to solving time-neutrosophic soft based decision making problems. We note here that we will use the abbreviation (MA) for Maji's Algorithm.

Definition 4.3. [14] Comparison Matrix. It is a matrix whose rows are labeled by the object names $h_1; h_2; ..., h_n$ and the columns are labeled by the parameters $e_1; e_2; ..., e_m$: The entries c_{ij} are calculated by $c_{ij} = a + b - c$, where 'a' is the integer calculated as 'how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in U$, 'b'is the integer calculated as 'how many times $I_{h_i}(e_j)$ exceeds or equal to $I_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in U$, and 'c' is the integer 'how many times $F_{h_i}(e_j)$ exceeds or equal to $F_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in U$,.

Definition 4.4. [14] Score of an Object. The score of an object h_i is S_i and is calculated as $S_i = \sum_j c_{ij}$

Example 4.3. Suppose that the Ministry of Agriculture want to evaluate agricultural lands for the establishment of certain agricultural project in one of these lands through specific parameters for five previous time periods and these parameters are mentioned below. Let $U = \{u_1,\ u_2,\ u_3 u_4\}$, be a set of lands, there may be five parameters. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of decision parameters to evaluate lands. For i = 1, 2, 3, 4, 5, the parameters e_i $(i = 1,2,3,4,5,6)$ stand for "Humidity rate", "Rainfall", "soil pH", "Groundwater rate", "Temperature" and let $T = \{t_1, t_2, t_3\}$. From the findings of this study, it will be clear to identify the best land which satisfies the above mentioned parameters. We note that when evaluating the soil pH 1 means that the acidity in the lower levels and 0 means that the acidity in the highest levels

$$
\begin{aligned}
(F,E)_t = \Bigg\{ & \left(e_1, \left\{ \frac{{u_1}^{t_1}}{\langle 0.5;0.2;0.4\rangle}, \frac{{u_2}^{t_1}}{\langle 0.3;0.1;0.5\rangle}, \frac{{u_3}^{t_1}}{\langle 0.4;0.2;0.3\rangle}, \frac{{u_4}^{t_1}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{{u_1}^{t_1}}{\langle 0.7;0.1;0.2\rangle}, \frac{{u_2}^{t_1}}{\langle 0.6;0.4;0.2\rangle}, \frac{{u_3}^{t_1}}{\langle 0.2;0.2;0.6\rangle}, \frac{{u_4}^{t_1}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{{u_1}^{t_1}}{\langle 0.8;0.2;0.1\rangle}, \frac{{u_2}^{t_1}}{\langle 0.6;0.4;0.1\rangle}, \frac{{u_3}^{t_1}}{\langle 0.3;0.3;0.5\rangle}, \frac{{u_4}^{t_1}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_4, \left\{ \frac{{u_1}^{t_1}}{\langle 0.4;0.4;0.4\rangle}, \frac{{u_2}^{t_1}}{\langle 0.1;0.3;0.4\rangle}, \frac{{u_3}^{t_1}}{\langle 0.5;0.6;0.4\rangle}, \frac{{u_4}^{t_1}}{\langle 0.5;0.3;0.4\rangle} \right\}\right), \\
& \left(e_5, \left\{ \frac{{u_1}^{t_1}}{\langle 0.9;0.1;0.1\rangle}, \frac{{u_2}^{t_1}}{\langle 0.4;0.4;0.1\rangle}, \frac{{u_3}^{t_1}}{\langle 0.6;0.3;0.2\rangle}, \frac{{u_4}^{t_1}}{\langle 0.3;0.2;0.7\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{{u_1}^{t_2}}{\langle 0.7;0.2;0.3\rangle}, \frac{{u_2}^{t_2}}{\langle 0.5;0.1;0.2\rangle}, \frac{{u_3}^{t_2}}{\langle 0.5;0.3;0.4\rangle}, \frac{{u_4}^{t_2}}{\langle 0.5;0.3;0.4\rangle} \right\}\right), \\
& \left(e_2, \left\{ \frac{{u_1}^{t_2}}{\langle 0.6;0.1;0.4\rangle}, \frac{{u_2}^{t_2}}{\langle 0.5;0.3;0.5\rangle}, \frac{{u_3}^{t_2}}{\langle 0.3;0.3;0.4\rangle}, \frac{{u_4}^{t_2}}{\langle 0.7;0.3;0.1\rangle} \right\}\right), \\
& \left(e_3, \left\{ \frac{{u_1}^{t_2}}{\langle 0.5;0.3;0.6\rangle}, \frac{{u_2}^{t_2}}{\langle 0.3;0.4;0.6\rangle}, \frac{{u_3}^{t_2}}{\langle 0.1;0.3;0.5\rangle}, \frac{{u_4}^{t_2}}{\langle 0.4;0.2;0.3\rangle} \right\}\right), \\
& \left(e_4, \left\{ \frac{{u_1}^{t_2}}{\langle 0.4;0.2;0.7\rangle}, \frac{{u_2}^{t_2}}{\langle 0.5;0.4;0.3\rangle}, \frac{{u_3}^{t_2}}{\langle 0.4;0.3;0.3\rangle}, \frac{{u_4}^{t_2}}{\langle 0.7;0.2;0.1\rangle} \right\}\right), \\
& \left(e_5, \left\{ \frac{{u_1}^{t_2}}{\langle 0.2;0.4;0.6\rangle}, \frac{{u_2}^{t_2}}{\langle 0.7;0.4;0.2\rangle}, \frac{{u_3}^{t_2}}{\langle 0.4;0.3;0.5\rangle}, \frac{{u_4}^{t_2}}{\langle 0.3;0.3;0.5\rangle} \right\}\right), \\
& \left(e_1, \left\{ \frac{{u_1}^{t_3}}{\langle 0.7;0.1;0.1\rangle}, \frac{{u_2}^{t_3}}{\langle 0.8;0.2;0.1\rangle}, \frac{{u_3}^{t_3}}{\langle 0.7;0.1;0.3\rangle}, \frac{{u_4}^{t_3}}{\langle 0.4;0.2;0.3\rangle} \right\}\right),
\end{aligned}
$$

$$\left(e_2, \left\{ \frac{{u_1}^{t_3}}{\langle 0.1;0.5;0.5\rangle}, \frac{{u_2}^{t_3}}{\langle 0.5;0.3;0.2\rangle}, \frac{{u_3}^{t_3}}{\langle 0.4;0.2;0.3\rangle}, \frac{{u_4}^{t_3}}{\langle 0.4;0.2;0.3\rangle} \right\}\right),$$

$$\left(e_3, \left\{ \frac{{u_1}^{t_3}}{\langle 0.9;0.4;0.1\rangle}, \frac{{u_2}^{t_3}}{\langle 0.7;0.3;0.1\rangle}, \frac{{u_3}^{t_3}}{\langle 0.5;0.5;0.1\rangle}, \frac{{u_4}^{t_3}}{\langle 0.4;0.2;0.3\rangle} \right\}\right),$$

$$\left(e_4, \left\{ \frac{{u_1}^{t_3}}{\langle 0.3;0.5;0.4\rangle}, \frac{{u_2}^{t_3}}{\langle 0.6;0.3;0.1\rangle}, \frac{{u_3}^{t_3}}{\langle 0.5;0.2;0.2\rangle}, \frac{{u_4}^{t_3}}{\langle 0.5;0.3;0.3\rangle} \right\}\right),$$

$$\left(e_5, \left\{ \frac{{u_1}^{t_3}}{\langle 0.1;0.6;0.6\rangle}, \frac{{u_2}^{t_3}}{\langle 0.6;0.2;0.3\rangle}, \frac{{u_3}^{t_3}}{\langle 0.5;0.3;0.4\rangle}, \frac{{u_4}^{t_3}}{\langle 0.5;0.3;0.4\rangle} \right\}\right) \Bigg\}.$$

4.2 Algorithm

Our goal is to convert the time-neutrosophic soft set to neutrosophic soft set then we apply Maji's algorithm to find the optimal decision. We can use the following algorithm to convert the time-neutrosophic soft set to neutrosophic soft set and find the decision. We can satisfy our goal as follows:

1. Input the tabular representation of $(F,E)_t$.
2. Find the tabular representation of $F(E)$, where $F(E)$ is defined as follows:

$$F(e) = \left\{ \frac{u}{\langle T(e), I(e), F(e)\rangle} : u \in U, e \in E \right\} \tag{1}$$

   such that
   $T(e) = \dfrac{\sum_{i=1}^{n} \alpha_{t_i} T_{t_i}(e)}{n \max_{i=1}^{n} \alpha_i(e)}$,
   $I(e) = \dfrac{\sum_{i=1}^{n} \alpha_{t_i} I_{t_i}(e)}{n \max_{i=1}^{n} \alpha_i(e)}$,
   $F(e) = \dfrac{\sum_{i=1}^{n} \alpha_{t_i} F_{t_i}(e)}{n \max_{i=1}^{n} \alpha_i(e)}$,
   where $n = |T|$ and α_{t_i} the weight of t_i.

3. Use Maji's algorithm for $F(E)$.
   - Input the Neutrosophic Soft Set $F(E)$.
   - Input P, the choice parameters of Ministry of Agriculture which is a subset of A
   - Consider the NSS (H,P) and write it in tabular form.
   - Compute the comparison matrix of the NSS (H,P).

- Compute the score S_i of u_i; $\forall i$.
- Find $S_k = \max_i S_i$.
- If k has more than one value then any one of u_i could be the preferable choice.

Then we have the following results shown in Table 1.

Table 1: Representation of $(F,E)_t$

U	u_1	u_2	u_3	u_4
(e_1,t_1)	$\langle 0.5;0.2;0.4\rangle$	$\langle 0.3;0.1;0.5\rangle$	$\langle 0.4;0.2;0.3\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_2,t_1)	$\langle 0.7;0.1;0.2\rangle$	$\langle 0.6;0.4;0.2\rangle$	$\langle 0.2;0.2;0.6\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_3,t_1)	$\langle 0.8;0.2;0.1\rangle$	$\langle 0.6;0.4;0.1\rangle$	$\langle 0.3;0.3;0.5\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_4,t_1)	$\langle 0.4;0.4;0.4\rangle$	$\langle 0.1;0.3;0.4\rangle$	$\langle 0.5;0.6;0.4\rangle$	$\langle 0.5;0.3;0.4\rangle$
(e_5,t_1)	$\langle 0.9;0.1;0.1\rangle$	$\langle 0.4;0.4;0.1\rangle$	$\langle 0.6;0.3;0.2\rangle$	$\langle 0.3;0.2;0.7\rangle$
(e_1,t_2)	$\langle 0.7;0.2;0.3\rangle$	$\langle 0.5;0.1;0.2\rangle$	$\langle 0.5;0.3;0.4\rangle$	$\langle 0.5;0.3;0.4\rangle$
(e_2,t_2)	$\langle 0.6;0.1;0.4\rangle$	$\langle 0.5;0.3;0.5\rangle$	$\langle 0.3;0.3;0.4\rangle$	$\langle 0.7;0.3;0.1\rangle$
(e_3,t_2)	$\langle 0.5;0.3;0.6\rangle$	$\langle 0.3;0.4;0.6\rangle$	$\langle 0.1;0.3;0.5\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_4,t_2)	$\langle 0.4;0.2;0.7\rangle$	$\langle 0.5;0.4;0.3\rangle$	$\langle 0.4;0.3;0.3\rangle$	$\langle 0.7;0.2;0.1\rangle$
(e_5,t_2)	$\langle 0.2;0.4;0.6\rangle$	$\langle 0.7;0.4;0.2\rangle$	$\langle 0.4;0.3;0.5\rangle$	$\langle 0.3;0.3;0.5\rangle$
(e_1,t_3)	$\langle 0.7;0.1;0.1\rangle$	$\langle 0.8;0.2;0.1\rangle$	$\langle 0.7;0.1;0.3\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_2,t_3)	$\langle 0.1;0.5;0.5\rangle$	$\langle 0.5;0.3;0.2\rangle$	$\langle 0.4;0.2;0.3\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_3,t_3)	$\langle 0.9;0.4;0.1\rangle$	$\langle 0.7;0.3;0.1\rangle$	$\langle 0.5;0.5;0.1\rangle$	$\langle 0.4;0.2;0.3\rangle$
(e_4,t_3)	$\langle 0.3;0.5;0.4\rangle$	$\langle 0.6;0.3;0.1\rangle$	$\langle 0.5;0.2;0.2\rangle$	$\langle 0.5;0.3;0.3\rangle$
(e_5,t_3)	$\langle 0.1;0.6;0.6\rangle$	$\langle 0.6;0.2;0.3\rangle$	$\langle 0.5;0.3;0.4\rangle$	$\langle 0.5;0.3;0.4\rangle$

Next by using relation 1 and suppose that $\alpha_{t_1} = 0.3$, $\alpha_{t_2} = 0.5$ and $\alpha_{t_3} = 0.8$, we compute the $F(E)$ to convert the time-neutrosophic soft set to neutrosophic soft set, to illustrate this step we calculate $F(e_1)$ for u_1 as show below.

$$F_{u_1}(e_1) = \left\{ \frac{u_1}{\langle T(e_1), I(e_1), F(e_1)\rangle} \right\} \tag{2}$$

Where

$$\begin{aligned} T(e_1) &= \frac{0.3*0.5+0.5*0.7+0.8*0.7}{3*\max\{0.3,0.5,0.8\}} \\ &= 1.06 \diagup 2.4 \\ &= 0.44. \\ I(e_1) &= \frac{0.3*0.2+0.5*0.2+0.8*0.1}{3*\max\{0.3,0.5,0.8\}} \\ &= 0.24 \diagup 2.4 \end{aligned}$$

$$= 0.1.$$

$$F(e_1) = \frac{0.3*0.4+0.5*0.3+0.8*0.1}{3*\max\{0.6,0.7,0.9\}}$$
$$= 0.35 \diagup 2.4$$
$$= 0.14.$$

Then

$$F_{u_1}(e_1) = \left\{ \frac{u_1}{\langle 0.44, 0.1, 0.14 \rangle} \right\}$$

The converting for u_i with all parameters can be done by similar way. The results of converting are shown in Table 2.

Table 2: Representation of $F(E)$

U	u_1	u_2	u_3	u_4
e_1	$\langle 0.44, 0.1, 0.14 \rangle$	$\langle 0.41, 0.1, 0.14 \rangle$	$\langle 0.39, 0.12, 0.22 \rangle$	$\langle 0.29, 0.15, 0.22 \rangle$
e_2	$\langle 0.24, 0.2, 0.28 \rangle$	$\langle 0.35, 0.21, 0.2 \rangle$	$\langle 0.22, 0.15, 0.26 \rangle$	$\langle 0.33, 0.15, 0.16 \rangle$
e_3	$\langle 0.5, 0.22, 0.17 \rangle$	$\langle 0.37, 0.23, 0.17 \rangle$	$\langle 0.23, 0.27, 0.2 \rangle$	$\langle 0.27, 0.13, 0.2 \rangle$
e_4	$\langle 0.23, 0.26, 0.33 \rangle$	$\langle 0.32, 0.22, 0.15 \rangle$	$\langle 0.31, 0.2, 0.18 \rangle$	$\langle 0.38, 0.15, 0.17 \rangle$
e_5	$\langle 0.19, 0.3, 0.34 \rangle$	$\langle 0.4, 0.2, 0.15 \rangle$	$\langle 0.33, 0.2, 0.26 \rangle$	$\langle 0.27, 0.19, 0.33 \rangle$

The comparison-matrix of the above resultant-time neutrosophic soft are shown below in Table 3.

Table 3: Comparison Matrix of TNSS

U	e_1	e_2	e_3	e_4	e_4
u_1	3	0	3	0	0
u_2	2	5	3	4	5
u_3	0	-1	0	0	3
u_4	0	3	-2	2	-1

Next we compute the score for each u_i as shown below, (Table 4)

Table 4: Score table of TNSS

U	S_i
u_1	3
u_2	19
u_3	2
u_4	2

From the above score table, clearly that the maximum score is 19, scored by u_2.
Decision Ministry of Agriculture will select the land u_2.

5 Conclusion

In this chapter we have introduced the concept of time-neutrosophic soft set and studied some of its properties. The complement, union and intersection operations have been defined on the time-neutrosophic soft set. A application of this theory in solving a decision making problem is given.

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