Ultra Neutrosophic Crisp Sets and Relations

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Abstract. In this paper we present a new neutrosophic crisp family generated from the three components' neutrosophic crisp sets presented by Salama [4]. The idea behind Salam's neutrosophic crisp set was to classify the elements of a universe of discourse with respect to an event "A" into three classes: one class contains those elements that are fully supportive to A, another class contains those elements that totally against A, and a third class for those elements that stand in a distance from being with or against A. Our aim here is to study the elements of the universe of discourse which their existence is beyond the three classes of the neutrosophic crisp set given by Salama. By adding more components we will get a four components' neutrosophic crisp sets called the Ultra Neutrosophoc Crisp Sets. Four types of set's operations is defined and the properties of the new ultra neutrosophic crisp sets are studied. Moreover, a definition of the relation between two ultra neutrosophic crisp sets is given.

Key words: Crisp Sets Operations,, Crisp Sets Relations, Fuzzy Sets, Neutrosophic Crisp Sets

1 Introduction

Established by Florentin Smarandache, neutrosophy was presented as the study of origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The main idea was to consider an entity, "A" in relation to its opposite "Non-A", and to that which is neither "A" nor " Non-A ", denoted by "Neut-A". And from then on, neutrosophy became the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics. According to this theory every idea "A" tends to be neutralized and balanced by "neutA" and "nonA" ideas - as a state of equilibrium. In a classical way "A", "neutA", and "antiA" are disjoint two by two. Nevertheless, since in many cases the borders between notions are vague and imprecise, it is possible that "A", "neutA", and "antiA" have common parts two by two, or even all three of them as well.

In [10], [11], [12] , Smarandache introduced the fundamental concepts of neutrosophic sets, that had led Salama et al. to provide a mathematical treatment for the neutrosophic phenomena which already existed in our real world (see for instance [1], [2], [3], [4], [5], [6], [8], [9], and the references therein). Moreover, the work of Salama et al. formed a starting point to construct new branches of neutrosophic mathematics. Hence, neutrosophic set theory turned out to be a generalization of both the classical and fuzzy counterparts.

In [4], Salama introduced the concept of neutrosophic crisp sets as a triple structure of the form, $A_N = \langle A_1, A_2, A_3\rangle$. The three components - A_1, A_2, and A_3 - refers to three classes of the elements of the universe X with respect to an event A. Where A_1 is the class containing those elements that are fully supportive to A, A_3 for those elements that totally against A, and A_2 for those elements that stand in a distance from being with or against A. The three classes are subsets of X. Furthermore, in [7] the authors suggested three different types of such neutrosophic crisp sets depending on whither there is an overlap between the three classes or not, and whither their union covers the universe or not.

The purpose of this paper, is to investigate the elements of the universe which have not been subjected to the classification; those elements belonging to the complement of the union of the three classes. Hence, a fourth component is to be added to the already existed three classes in Salama's sense.

For the purpose of this paper, we will categorize the neutrosophic crisp sets of some universe X, into two categories according to whither the union of the three classes covers the universe or not. The first category will contain all the neutrosophic crisp sets whose components does not cover the universe, whether they are mutually exclusive or they have some common parts in-between; two by two, or even all the three of them. while the second category will contain the remaining neutrosophic crisp sets whose components covers the whole universe.

The remaining of this paper is organized as follows: in (sec. 2) we introduce some basic definition necessary for this work. The concept of the ultra neutrosophic crisp sets is introduced in (sec. 3). Furthermore, four types of ultra neutrosophic crisp sets' operations and its properties are presented in (sec. 4) and (sec. 5). Hence, the product and relation between ultra neutrosophic crisp sets are defined in (sec. 6), (sec. 7) and (sec. 8). Finally, conclusions are drawn and future directions of research are suggested in (sec. 9).

2 Preliminaries

2.1 Neutrosophic Crisp Sets

2.1.1 Definition [4] For any arbitrary universe X, a neutrosophic crisp set A_N is a triple $A_N = \langle A_1, A_2, A_3\rangle$, where $A_i \in P(X), i = 1, 2, 3$.
The three components of A_N represent a classification of the elements of X according to some event A; the subset A_1 contains all the elements of X that are fully supportive to A, A_3 contains those elements that totally against A, and A_2 contains those elements that stand in a distance from being with or against A.

If we consider the event A in the ordinary sense, each neutrosophic crisp set will be in the form $A_N = \langle A_1, \phi, A_1^c\rangle$, while in the fuzzy sense it will be in the form $A_N = \langle A_1, \phi, A_3 \rangle$, where $A_3 \subseteq A_1^c$. Moreover, in the intuitionistic fuzzy sense $A_N = \langle A_1, (A_1 \cup A_3)^c, A_3 \rangle$.

2.1.2 Definition [4] The complement of a neutrosophic crisp set is defined as:

$$coA_N = \langle coA_1, coA_2, coA_3 \rangle$$

2.1.3 Definition [7] A neutrosophic crisp set $A_N = \langle A_1, A_2, A_3 \rangle$ is called:

- A neutrosophic crisp set of type1, if satisfying that:
  $A_i \cap A_j = \phi$, where $i \neq j$ and $\bigcup_{i=1}^{3} A_i \subset X, \ \forall i, j = 1, 2, 3$
- A neutrosophic crisp set of type2, if satisfying that:
  $A_i \cap A_j = \phi$, where $i \neq j$ and $\bigcup_{i=1}^{3} A_i = X, \ \forall i, j = 1, 2, 3$
- A neutrosophic crisp set of type3, if satisfying that:
  $\bigcap_{i=1}^{3} A_i = \phi$, and $\bigcup_{i=1}^{3} A_i = X, \ \forall i, j = 1, 2, 3$

2.2 Neutrosophic Crisp Sets Operations of Type 1 [7]

For any two neutrosophic crisp sets A_N and B_N, we have that:

$$A_N \subseteq B_N \text{ if } A_1 \subseteq B_1, A_2 \subseteq B_2, \text{ and } A_3 \supseteq B_3,$$
$$A_N = B_N \text{ if and only if } A_i = B_i \text{ for } i = 1, 2, 3$$

hence, we can define the following:

$$A_N \cup B_N = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$$
$$A_N \cap B_N = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$$

2.3 Neutrosophic Crisp Sets Operations of Type 2 [7]

For any two neutrosophic crisp sets A_N and B_N, we have that:

$$A_N \subseteq B_N \text{ if } A_1 \subseteq B_1, A_2 \supseteq B_2, \text{ and } A_3 \supseteq B_3,$$
$$A_N = B_N \text{ if and only if } A_i = B_i \text{ for } i = 1, 2, 3$$

hence, we can define the following:

$$A_N \cup B_N = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$$
$$A_N \cap B_N = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$$

3 Ultra Neutrosophic Crisp Sets

In this section we consider elements in X which do not belong to any of the three classes of the neutrosophic crisp set defined in (2.1.1).

3.1 Definition

Let X be any given universe, the ultra neutrosophic crisp set is defined as:

$$\breve{A} = \langle A_1, A_2, A_3, M_A \rangle, \text{ where } M_A = co(\bigcup_{i=1}^{3} A_i)$$

The family of all ultra neutrosophic crisp sets in X will be denoted by $\breve{\mathfrak{U}}(X)$.

3.2 Definition

The complement of any ultra neutrosophic crisp set $\breve{A}$, is defined as:

$$co\breve{A} = \langle coA_1, coA_2, coA_3, coM_A \rangle$$

4 Ultra Neutrosophic Crisp Sets Operations

4.1 Ultra Operations of Type I

For any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, we have that:

$$\breve{A} \subseteq_I \breve{B} \text{ if } A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \supseteq B_3, \text{ and } M_A \supseteq M_B,$$
$$\breve{A} = \breve{B} \text{ if and only if } A_i = B_i \text{for} i = 1, 2, 3 \text{ and} M_A = M_B$$

hence, we can define the following:

$$\breve{A} \uplus_I \breve{B} = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3, M_A \cap M_B \rangle$$
$$\breve{A} \Cap_I \breve{B} = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3, M_A \cup M_B \rangle$$

4.2 Ultra Operations of Type II

For any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, we have that:

$$\breve{A} \subseteq_{II} \breve{B} \text{ if } A_1 \subseteq B_1, A_2 \supseteq B_2, A_3 \supseteq B_3, \text{ and } M_A \supseteq M_B,$$
$$\breve{A} = \breve{B} \text{ if and only if } A_i = B_i \text{for} i = 1, 2, 3 \text{ and} M_A = M_B$$

hence, we can define the following:

$$\breve{A} \uplus_{II} \breve{B} = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3, M_A \cap M_B \rangle$$
$$\breve{A} \Cap_{II} \breve{B} = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3, M_A \cup M_B \rangle$$

4.3 Ultra Operations of Type III

For any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, we have that:

$$\breve{A} \subseteq_{III} \breve{B} \text{ if } A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \supseteq B_3 \text{ and } M_A \subseteq M_B,$$
$$\breve{A} = \breve{B} \text{ if and only if } A_i = B_i \text{for} i = 1,2,3 \text{ and} M_A = M_B$$

hence, we can define the following:

$$\breve{A} \uplus_{III} \breve{B} = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3, M_A \cup M_B \rangle$$
$$\breve{A} \Cap_{III} \breve{B} = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3, M_A \cap M_B \rangle$$

4.4 Ultra Operations of Type IV

For any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, we have that:

$$\breve{A} \subseteq_{IV} \breve{B} \text{ if } A_1 \subseteq B_1, A_2 \supseteq B_2, A_3 \supseteq B_3 \text{ and } M_A \subseteq M_B,$$
$$\breve{A} = \breve{B} \text{ if and only if } A_i = B_i \text{for} i = 1,2,3 \text{ and} M_A = M_B$$

hence, we can define the following:

$$\breve{A} \uplus_{IV} \breve{B} = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3, M_A \cup M_B \rangle$$
$$\breve{A} \Cap_{IV} \breve{B} = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3, M_A \cap M_B \rangle$$

4.5 Definition

- The difference between any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, is defined as $\breve{A} \setminus \breve{B} = \breve{A} \Cap co\breve{B}$.
- The symmetric difference between any two ultra neutrosophic crisp sets $\breve{A}$ and $\breve{B}$, is defined as $\breve{A} \bigoplus \breve{B} = \breve{A} \setminus \breve{B} \uplus \breve{B} \setminus \breve{A}$.

5 Properties of Ultra Neutrosophic Crisp Sets

Knowing that the four components A_1, A_2, A_3, and M_A are crisp subsets of the universe X, we can prove that for all the Types (I, II, III, and IV) the ultra neutrosophic crisp sets operations verify the following properties:

1. Associative laws: $\breve{A} \uplus (\breve{B} \uplus \breve{C}) = (\breve{A} \uplus \breve{B}) \uplus \breve{C}$
   $\breve{A} \Cap (\breve{B} \Cap \breve{C}) = (\breve{A} \Cap \breve{B}) \Cap \breve{C}$
2. Commutative laws: $\breve{A} \uplus \breve{B} = \breve{B} \uplus \breve{A}$
   $\breve{A} \Cap \breve{B} = \breve{B} \Cap \breve{A}$
3. Distributive laws: $\breve{A} \uplus (\breve{B} \Cap \breve{C}) = (\breve{A} \uplus \breve{B}) \Cap (\breve{A} \uplus \breve{C})$
   $\breve{A} \Cap (\breve{B} \uplus \breve{C}) = (\breve{A} \Cap \breve{B}) \uplus (\breve{A} \Cap \breve{C})$
4. Idempotent laws: $\breve{A} \uplus \breve{A} = \breve{A}$
   $\breve{A} \Cap \breve{A} = \breve{A}$
5. Absorption laws: $\breve{A} \uplus (\breve{A} \Cap \breve{B}) = \breve{A}$
   $\breve{A} \Cap (\breve{A} \uplus \breve{B}) = \breve{A}$
6. Involution law: $co(co\breve{A}) = \breve{A}$
7. DeMorgan's laws: $co(\breve{A} \uplus \breve{B}) = co\breve{A} \Cap co\breve{B}$
   $co(\breve{A} \Cap \breve{B}) = co\breve{A} \uplus co\breve{B}$

Proof

For explanation, we will show the proof of the first associative law for type I, the proof of the first distributive law for type II, the proof of the first absorption law for type III, and the proof of the first DeMorgan's law for typeIV. using the definitions

i) $\breve{A} \uplus_{\mathrm{I}} (\breve{B} \uplus_{\mathrm{I}} \breve{C}) = \langle A_1 \cup (B_1 \cup C_1), A_2 \cup (B_2 \cup C_2), A_3 \cap (B_3 \cap C_3), M_A \cap (M_B \cap M_C)\rangle$
$= \langle (A_1 \cup B_1) \cup C_1, (A_2 \cup B_2) \cup C_2, (A_3 \cap B_3) \cap C_3, (M_A \cap M_B) \cap M_C)\rangle$
$= (\breve{A} \uplus_{\mathrm{I}} \breve{B}) \uplus_{\mathrm{I}} \breve{C}$

ii) $\breve{A} \uplus_{\mathrm{II}} (\breve{B} \Cap_{\mathrm{II}} \breve{C}) = \langle A_1 \cup (B_1 \cap C_1), A_2 \cap (B_2 \cup C_2), A_3 \cap (B_3 \cup C_3, M_A \cap (M_B \cup M_C)\rangle$
$= \langle (A_1 \cup B_1) \cap (A_1 \cup C_1), (A_2 \cap B_2) \cup (A_2 \cap C_2), (A_3 \cap B_3) \cup (A_3 \cap C_3),$
$(M_A \cap M_B) \cup (M_A \cap M_C)\rangle$
$= (\breve{A} \uplus_{\mathrm{II}} \breve{B}) \Cap_{\mathrm{II}} (\breve{A} \uplus_{\mathrm{II}} \breve{C})$

iii) $\breve{A} \uplus_{\mathrm{III}} (\breve{A} \Cap_{\mathrm{III}} \breve{B}) = \langle A_1 \cup (A_1 \cap B_1), A_2 \cup (A_2 \cap B_2), A_3 \cap (A_3 \cup B_3), M_A \cup (M_A \cap M_B)\rangle$
$= \langle A_1, A_2, A_3, M_A\rangle$
$= \breve{A}$

iv) $co(\breve{A} \uplus_{\mathrm{IV}} \breve{B}) = \langle co(A_1 \cup B_1), co(A_2 \cap B_2), co(A_3 \cap B_3), co(M_A \cup M_B)\rangle$
$= \langle coA_1 \cap coB_1, coA_2 \cup coB_2, coA_3 \cup coB_3, coM_A \cap coM_B)\rangle$
$= co\breve{A} \Cap_{\mathrm{IV}} co\breve{B}$

Note that: the same procedure can be applied to prove any of the laws given in (5) for all types: I, II, II, and IV.

5.1 Proposition

Let $\breve{A}_i$, $i \in J$, be an arbitrary family of ultra neutrosophic crisp sets on X; then we have the following:

1. Type I: ${}_{i\in J_I}\breve{A}_i = \langle \bigcap_{i\in J} A_{i1}, \bigcap_{i\in J} A_{i2}, \bigcup_{i\in J} A_{i3}, \bigcup_{i\in J} M_{Ai} \rangle$
   ${}_{i\in J}\breve{A}_i = \langle \bigcup_{i\in J} A_{i1}, \bigcup_{i\in J} A_{i2}, \bigcap_{i\in J} A_{i3}, \bigcap_{i\in J} M_{Ai} \rangle$
2. Type II: ${}_{i\in J}\breve{A}_i = \langle \bigcap_{i\in J} A_{i1}, \bigcup_{i\in J} A_{i2}, \bigcup_{i\in J} A_{i3}, \bigcup_{i\in J} M_{Ai} \rangle$
   ${}_{i\in J}\breve{A}_i = \langle \bigcup_{i\in J} A_{i1}, \bigcap_{i\in J} A_{i2}, \bigcap_{i\in J} A_{i3}, \bigcap_{i\in J} M_{Ai} \rangle$
3. Type III: ${}_{i\in J}\breve{A}_i = \langle \bigcap_{i\in J} A_{i1}, \bigcap_{i\in J} A_{i2}, \bigcup_{i\in J} A_{i3}, \bigcap_{i\in J} M_{Ai} \rangle$
   ${}_{i\in J}\breve{A}_i = \langle \bigcup_{i\in J} A_{i1}, \bigcup_{i\in J} A_{i2}, \bigcap_{i\in J} A_{i3}, \bigcup_{i\in J} M_{Ai} \rangle$
4. Type IV: ${}_{i\in J}\breve{A}_i = \langle \bigcap_{i\in J} A_{i1}, \bigcup_{i\in J} A_{i2}, \bigcup_{i\in J} A_{i3}, \bigcap_{i\in J} M_{Ai} \rangle$
   ${}_{i\in J}\breve{A}_i = \langle \bigcup_{i\in J} A_{i1}, \bigcap_{i\in J} A_{i2}, \bigcap_{i\in J} A_{i3}, \bigcup_{i\in J} M_{Ai} \rangle$

6 The Ultra Cartesian Product of Ultra Neutrosophic Crisp Sets

Consider any two ultra neutrosophic crisp sets, A on X and B on Y; where $\breve{A} = \langle A_1, A_2, A_3, M_A \rangle$ and $\breve{B} = \langle B_1, B_2, B_3, M_B \rangle$
The ultra cartesian product of $\breve{A}$ and $\breve{B}$ is defined as the quadruple structure:

$$\breve{A} \times \breve{B} = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3, M_A \times M_B \rangle$$

where each component is a subset of the cartesian product $X \times Y$;

$$A_i \times B_i = \{(a_i, b_i) : a_i \in A_i \text{ and } b_i \in B_i\}, \forall i = 1, 2, 3 \text{ and}$$
$$M_A \times M_B = \{(m_a, m_b) : m_a \in M_A \text{ and } m_b \in M_B\}$$

6.1 Corollary

In general if $\breve{A} \neq \breve{B}$, then $\breve{A} \times \breve{B} \neq \breve{B} \times \breve{A}$

7 Ultra Neutrosophic Crisp Relations

An ultra neutrosophic crisp relation $\breve{R}$ from an ultra neutrosophic crisp set $\breve{A}$ to $\breve{B}$, namely $\breve{R} : \breve{A} \to \breve{B}$, is defined as a quadruple structure of the form $\breve{R} = \langle R_1, R_2, R_3, R_M \rangle$, where $R_i \subseteq A_i \times B_i$, $\forall i = 1, 2, 3$ and $R_M \subseteq M_A \times M_B$,
that is

$$R_i = \{(a_i, b_i) : a_i \in A_i \text{ and } b_i \in B_i\}$$
$$R_M = \{(m_a, m_b) : m_a \in M_A \text{ and } m_b \in M_B\}$$

7.1 Domain and Range of Ultra Neutrosophic Crisp Relations

For any ultra neutrosophic crisp relation $\breve{R} : \breve{A} \to \breve{B}$, we define the following:

- The ultra domain of $\breve{R}$, is defined as:
  $uDom(\breve{R}) = \langle dom(R_1), dom(R_2), dom(R_3), dom(R_M) \rangle$

- The ultra range of $\breve{R}$, is defined as:
  $uRng(\breve{R}) = \langle rng(R_1), rng(R_2), rng(R_3), rng(R_M) \rangle$

- The Domain of $\breve{R}$, is defined as:
  $Dom(\breve{R}) = dom(R_1) \cup dom(R_2) \cup dom(R_3) \cup dom(R_M)$

- The Range of $\breve{R}$, is defined as:
  $Rng(\breve{R}) = rng(R_1) \cup rng(R_2) \cup rng(R_3) \cup rng(R_M)$

7.2 Corollary

From the definitions given in 7.1, one may notice that for any ultra neutrosophic crisp relation $\breve{R} : \breve{A} \to \breve{B}$, we have:

- The domain of $\breve{R}$ is a crisp subset of X, namely, $Dom(\breve{R}) \subseteq X$.

- The range of $\breve{R}$ is a crisp subset of Y, namely, $Rng(\breve{R}) \subseteq Y$.

- The ultre domain of $\breve{R}$ is a quadruple structure whose components are crisp subsets of X; furthermore, $dom(R_i) \subseteq A_i, i = 1, 2, 3$ and $dom(R_M) \subseteq M_A$

- The ultre range of $\breve{R}$ is a quadruple structure whose components are crisp subsets of Y; furthermore, $rng(R_i) \subseteq B_i, i = 1, 2, 3$ and $rng(R_M) \subseteq M_B$

7.3 Definition

An ultra neutrosophic crisp inverse relation $\breve{R}^{-1}$ is an ultra neutrosophic crisp relation from an ultra neutrosophic crisp set $\breve{B}$ to $\breve{A}$, $\breve{R}^{-1} : \breve{B} \to \breve{A}$, and to be defined as a quadruple structure of the form:
$\breve{R}^{-1} = \langle R_1^{-1}, R_2^{-1}, R_3^{-1}, R_M^{-1} \rangle$, where $R_i^{-1} \subseteq B_i \times A_i, \forall i = 1, 2, 3$ and $R_M \subseteq M_B \times M_A$, that is:

$$R_i^{-1} = \{(b_i, a_i) : (a_i, b_i) \in R_i\}$$
$$R_M^{-1} = \{(m_b, m_a) : (m_a, m_b) \in R_M\}$$

7.4 Corollary

or any ultra neutrosophic crisp relation $\breve{R} : \breve{A} \to \breve{B}$, we have that :

$$Dom(\breve{R}^{-1}) = Rng(\breve{R}) \quad Rng(\breve{R}^{-1}) = Dom(\breve{R})$$
$$uDom(\breve{R}^{-1}) = uRng(\breve{R}) \quad uRrng(\breve{R}^{-1}) = uDom(\breve{R})$$

8 Composition of Ultra Neutrosophic Crisp Relations

Consider the three ultra neutrosophic crisp sets: $\breve{A}$ of X, $\breve{B}$ of Y and $\breve{C}$ of Z; and, the two ultra neutrosophic crisp relations: $\breve{R} : \breve{A} \to \breve{B}$ and $\breve{S} : \breve{B} \to \breve{C}$; where $\breve{R} = \langle R_1, R_2, R_3, R_M \rangle$, and $\breve{S} = \langle S_1, S_2, S_3, S_M \rangle$. The composition of $\breve{R}$ and $\breve{S}$, is denoted and defined as:
$\breve{R}\breve{S} = \langle R_1 \circ S_1, R_2 \circ S_2, R_3 \circ S_3, R_M \circ S_M \rangle$ such that,
$R_i \circ S_i : A_i \to C_i$, where, $R_i \circ S_i = \{(a_i, c_i) : \exists b_i \in B_i,\ (a_i, b_i) \in R_i \text{ and } (b_i, c_i) \in S_i\}$;
$R_M \circ S_M : M_A \to M_C$, where, $R_M \circ S_M = \{(m_a, m_c) : \exists m_b \in M_B,\ (m_a, m_b) \in R_M \text{ and } (m_b, m_c) \in S_M\}$

8.1 Corollary

For any two ultra neutrosophic crisp relations: $\breve{R} : \breve{A} \to \breve{B}$ and $\breve{S} : \breve{B} \to \breve{C}$;

$$uDom(\breve{R} \odot \breve{S}) \subseteq uDom(\breve{R})$$
$$uRng(\breve{R} \odot \breve{S}) \subseteq uRng(\breve{S})$$

8.2 Corollary

Consider the three ultra neutrosophic crisp relations: $\breve{R} : \breve{A} \to \breve{B}$, $\breve{S} : \breve{B} \to \breve{C}$, and $\breve{K} : \breve{C} \to \breve{D}$; we have that:

$$\breve{R} \odot (\breve{S} \odot \breve{K}) = (\breve{R} \odot \breve{S}) \odot \breve{K}$$

9 Conclusion and Future Work

In this paper we have presented a new concept of neutrosophic crisp sets, called "The Ultra Neutrosophic Crisp Sets", as a quadrable structure. The first three components represent a classification of the universe of discourse with respect to some event; while the fourth component deals with the elements which have not been subjected to that classification. While the elements of the first and the third are considered to be well defined, there is a blurry about the behavior of the elements in both second and fourth components. Consequently, four types of set's operations were established and the properties of the new ultra neutrosophic crisp sets were studied according to different expectations about the performance of the second and the fourth components. Moreover, the definition of the relation between two ultra neutrosophic crisp sets were given. Finally, the concepts of product and composition of ultra neutrosophic crisp sets were introduced.

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