A Decision Making Technique Based on Similarity Measure and Entropy of Bipolar Neutrosophic Sets

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A Decision Making Technique Based on Similarity Measure and Entropy of Bipolar Neutrosophic Sets

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Abstract. In this paper, a new bipolar neutrosophic similarity measure and entropy of bipolar neutrosophic set have been developed. The properties of bipolar neutrosophic similarity and entropy are proved. Finally, an example is presented to demonstrate the effectiveness of bipolar neutrosophic similarity and entropy.

Key words: Neutrosophic Set, Bipolar, Similarity, Entropy

1. Introduction

Florentin Smarandache introduced the idea of neutrosophic set (NS)[1] in 1998 and it is characterised by three independent membership functions like truth membership function T, indeterminacy membership function I, and falsity membership function F. The bipolar neutrosophic set (BNS) was introduced by Ifran.D and Mumtaz Ali in 2015 [2] and the bipolar neutrosophic set is generalized from neutrosophic sets and bipolar fuzzy sets. This emerging technique in fuzzy mathematics is obtained from the origin source of fuzzy sets [3]. Further, many researchers have devloping the concept of bipolar neutrosophic set with their applications.

R.M.Hashim and M.Gulistan [4] applied bipolar neutrosophic sets for planning to build the hospital and they introduced various operators and similarity measures in BNS. Vakkas.U, Irfan and M.Sahin proposed [5] novel similarity measures for BNS with application. S Broumi, et al.[6] introduced bipolar complex neutrosophic set with its similarity measure. Pattern recognition is one of the trending real time applications, in that the researchers dealt with interval valued bipolar neutrosophic (IVBN) [7] set. Further, the IVBN is developed by Surapati P, et al.[8] and they give various correlation coefficient measures to IVBN. The application of neutrosophic bipolar fuzzy sets is explained in [2, 4] with detailed information including some operations and measures.

This paper is orgainzed as follows: The basic concept of neutrosophic set (NS) is explained in section 2; In section 3, bipolar neutrosophic set (BNS) is reviewed with their operations; the strategy is introduced in section 4, the application of the strategy is demonstrated step by step in section 5. Finally, section 6 contains the conclusion of the paper.

2. Preliminaries

This section contains the fundamental concept of neutrosophic sets with their operations.

Definition 2.1. let X be a universe of discourse or a non empty set. Any object in the neutrosophic set B has the form $B =< x, \mu_B(x), \sigma_B(x), \gamma_B(x) > x \in X$, where $\mu_B(x), \sigma_B(x)$ and $\gamma_B(x)$ represent the degree of truth membership, the degree of indeterminacy and the degree of falsity membership respectively of each element $x \in X$ to the set B.

Definition 2.2. Let A_{NS} be a neutrosophic set in X and it is defined as

$$A_{NS} = \{x, < F_A(x), T_A(x), I_A(x) >: x \in X\}$$

where $T_A, I_A, F_A : X \to [0, 1]$ and $T_A(x)-$truth membership value, $I_A(x)-$indeterminacy membership value and $F_A(x)-$ falsity membership value such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3. Let two neutrosophic numbers be $\widetilde{A} =< T_1, I_1, F_1 >$ and $\widetilde{B} =< T_2, I_2, F_2 >$. Then, the operations them are defined as follows:

i. $\lambda\widetilde{A} =< 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda >$
ii. $\widetilde{A}^\lambda =< T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda >$
iii. $\widetilde{A} + \widetilde{B} =< T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2 >$
iv. $\widetilde{A}.\widetilde{B} =< T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2 >$

Definition 2.4. Let A and B be a two neutrosophic sets. Then $A \subseteq B$, if and only if $T_A(x) \leq T_B(x); I_A(x) \leq I_B(x); F_A(x) \geq F_B(x), \forall x \in X$.

Definition 2.5. The complement of neutrosophic set A is denoted by A^c and is defined by $A^c = \{< x, F_A(x), 1 - I_A(x), T_A(x) : x \in X\}$

Definition 2.6. Consider two neutrosophic sets A=$\langle T_1(x), I_1(x), F_1(x)\rangle$ and $B = \langle T_2(x), I_2(x), F_2(x)\rangle$
The intersection of A and B is denoted as $\langle A \cap B\rangle (x)$ and is defined by
$= \{< x, min(T_A(x), T_B(x)), max(I_A(x), I_B(x)), max(F_A(x), F_B(x)) >: x \in X\}$
The union of A and B is denoted as $\langle A \cup B\rangle (x)$ and is defined by
$= \{< x, max(T_A(x), T_B(x)), min(I_A(x), I_B(x)), min(F_A(x), F_B(x)) >: x \in X\}$

Definition 2.7. [9] Let A and B be two neurtosophic sets on X and $A, B \in N(X)$. A similarity measure between two neutrosophic sets is a function $S : A(x) \times B(x) \rightarrow [0, 1]$ which is satisfies the following conditions:

i. $0 \leq S(A, B) \leq 1$
ii. $S(A, B) = 1$ iff $A = B$
iii. $S(A, B) = S(B, A)$
iv. If $A \subseteq B \subseteq C$ then $S(A, C) \leq S(A, B)$ and $S(A, C) \leq S(B, C)$ for all $A, B, C \in N(X)$.

Definition 2.8. [9] Let $N(X)$ be all neutrosophic sets on X and $A \in A(X)$. An entrophy on neutrosophic sets is a function $E_N : N(X) \rightarrow [0, 1]$ which is satisfies the following axioms:

i. $E_N(A) = 0$ if A is crisp set
ii. $E_N(A) = 1$ if $(T_A(x), I_A(x), F_A(x)) = (0.5, 0.5, 0.5)$ for all $x \in X$
iii. $E_N(A) \geq E_N(B)$ if $A \subset B$, i.e. $T_A(x) \leq T_B(x), F_A(x) \geq F_B(x)$ and $I_A(x) \leq I_B(x)$ for all $x \in X$
iv. $E_A(x) = E_N(A^c)$ for all $A \in N(X)$.

3. Bipolar Neutrosophic Sets

In this section, a new similarity meaure and entropy for bipolar neutrosophic sets are introduced.

Definition 3.1. [2] A set A of the form

$$A = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\rangle : x \in X\},$$

is called a bipolar neutrosophic set. where $T^+, I^+, F^+ : X \to [0,1]$ and $T^-, I^-, F^- : X \to [-1,0]$. Here $\langle T^+(x), I^+(x), F^+(x)\rangle$ are the positive membership degree of truth, indeterminacy and falsity of an element $x \in X$ and similarly $\langle T^-(x), I^-(x), F^-(x)\rangle$ are the negative membership degree of truth, indeterminacy, falsity of an element $x \in X$ to some counter property of neutrosophic set A.
Example: Let $x_1 \in X$ then $A = \{\langle x_1, 0.6, 0.5, 0.3, -0.5, -0.4, -0.02\rangle\}$ is a bipolar neutrosophic number of X.

Definition 3.2. [2] Let $A = \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x)\right\rangle$ and $B = \left\langle x, T_B^+(x), I_B^+(x), F_B^+(x), T_B^-(x), I_B^-(x), F_B^-(x)\right\rangle$ be two bipolar neutrosophic sets. Then $A \subseteq B$ if and only if

$T_A^+(x) \le T_B^+(x), I_A^+(x) \le I_B^+(x), F_A^+(x) \ge F_B^+(x)$ and
$T_A^-(x) \ge T_B^-(x), I_A^-(x) \ge I_B^-(x), F_A^-(x) \le F_B^-(x)$ for all $x \in X$.

Definition 3.3. [2] Let $A = \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x)\right\rangle$ and $B = \left\langle x, T_B^+(x), I_B^+(x), F_B^+(x), T_B^-(x), I_B^-(x), F_B^-(x)\right\rangle$ be two bipolar neutrosophic sets. The union of two bipolar neutrosophic sets A and B is denoted by $A \cup B$ and defined as:

$$\langle A \cup B\rangle(x) = \Big\{ max\left(T_A^+(x), T_B^+(x)\right), \frac{I_A^+(x) + I_B^+(x)}{2}, min\left(F_A^+(x), F_B^+(x)\right), \\ min\left(T_A^-(x), T_B^-(x)\right), \frac{I_A^-(x) + I_B^-(x)}{2}, max\left(F_A^-(x), F_B^-(x)\right)\Big\}$$

The intersection of two bipolar neutrosophic sets A and B is denoted by $A \cap B$ and defined as:

$$\langle A \cap B\rangle(x) = \Big\{ min\left(T_A^+(x), T_B^+(x)\right), \frac{I_A^+(x) + I_B^+(x)}{2}, max\left(F_A^+(x), F_B^+(x)\right), \\ max\left(T_A^-(x), T_B^-(x)\right), \frac{I_A^-(x) + I_B^-(x)}{2}, min\left(F_A^-(x), F_B^-(x)\right)\Big\}$$

Definition 3.4. [2] Let $A = \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x)\right\rangle$ be a bipolar neutrosophic set in X. Then complement of A is denoted by A^c and is defined by: A^c
$=\{\langle 1^+ - T^+(x), 1^+ - I^+(x), 1^+ - F^+(x), 1^{-1} - T^-(x), 1^{-1} - I^-(x), 1^{-1} - F^-(x)\rangle\}$

Definition 3.5. [2] Let $\widetilde{a}_j = \left\langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-\right\rangle, j = 1, 2, 3 \ldots n$ be a family of bipolar neutrosophic numbers. A function $A_w : \Omega_n \to \Omega$ is called bipolar neutrosophic weighted average operator if it satisfies

$$\begin{aligned} A_w(\widetilde{a}_1, \widetilde{a}_1, \widetilde{a}_1, \ldots, \widetilde{a}_n) &= \sum_{j=1}^{n} w_j \widetilde{a}_j \\ &= \Big\langle 1 - \prod_{j=1}^{n} \left(1 - T_j^+\right)^{w_j}, \prod_{j=1}^{n} I_j^+, \prod_{j=1}^{n} F_j^+, -\prod_{j=1}^{n} \left(-T_j^-\right)^{w_j}, \\ &\quad -\left(1 - \prod_{j=1}^{n} \left(1 - (-I_j^-)\right)^{w_j}\right), -\left(1 - \prod_{j=1}^{n} \left(1 - (-F_j^-)\right)^{w_j}\right) \Big\rangle \end{aligned} \tag{1}$$

Definition 3.6. Similarity measure of bipolar neutrosophic sets
Let A, B be two bipolar neutrosophic sets in X. The similarity measure between the bipolar neutrosophic sets A and B is defined by:

$$S(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{\begin{array}{c}\left|T_A^+(x_i) - T_B^+(x_i)\right| + \left|T_A^-(x_i) - T_B^-(x_i)\right| + \left|I_A^+(x_i) - I_B^+(x_i)\right| \\ + \left|I_A^-(x_i) - I_B^-(x_i)\right| + \left|F_A^+(x_i) - F_B^+(x_i)\right| + \left|F_A^-(x_i) - F_B^-(x_i)\right|\end{array}}{6}\right] \tag{2}$$

We shall prove that this bipolar similarity measure satisfies the fundamental properties of the definition 2.7

Proof:
This bipolar similarity measure satisfies the properties $(1-3)$ of definition 2.7 and it is obivious. We prove the property 4 by following:
Let $A \subseteq B \subseteq C$, Then we have

$$T_A^+(x_i) \leq T_B^+(x_i) \leq T_C^+(x_i), I_A^+(x_i) \leq I_B^+(x_i) \leq I_C^+(x_i), F_A^-(x_i) \geq F_B^-(x_i) \geq F_C^-(x_i),$$

$$T_A^-(x_i) \geq T_B^-(x_i) \geq T_C^-(x_i), I_A^-(x_i) \geq I_B^-(x_i) \geq I_C^-(x_i), F_A^-(x_i) \leq F_B^-(x_i) \leq F_C^-(x_i).$$

It follows that
$\left|T_A^+(x_i) - T_B^+(x_i)\right| + \left|T_A^-(x_i) - T_B^-(x_i)\right| \leq \left|T_A^+(x_i) - T_C^+(x_i)\right| + \left|T_A^-(x_i) - T_C^-(x_i)\right|,$
$\left|I_A^+(x_i) - I_B^+(x_i)\right| + \left|I_A^-(x_i) - I_B^-(x_i)\right| \leq \left|I_A^+(x_i) - I_C^+(x_i)\right| + \left|I_A^-(x_i) - I_C^-(x_i)\right|,$
$\left|F_A^+(x_i) - F_B^+(x_i)\right| + \left|F_A^-(x_i) - F_B^-(x_i)\right| \leq \left|F_A^+(x_i) - F_C^+(x_i)\right| + \left|F_A^-(x_i) - F_C^-(x_i)\right|.$
Then

$$\sum_{i=1}^{n}\left[1 - \mu_s(A,B)\right] \geq \sum_{i=1}^{n}\left[1 - \mu_s(A,C)\right]$$

where

$$\mu_s(A,B) = \left(\left|T_A^+(x_i) - T_B^+(x_i)\right| + \left|T_A^-(x_i) - T_B^-(x_i)\right| + \left|I_A^+(x_i) - I_B^+(x_i)\right| \right.$$
$$\left. + \left|I_A^-(x_i) - I_B^-(x_i)\right| + \left|F_A^+(x_i) - F_B^+(x_i)\right| + \left|F_A^-(x_i) - F_B^-(x_i)\right|\right)/6,$$

$$\mu_s(A,C) = \left(\left|T_A^+(x_i) - T_C^+(x_i)\right| + \left|T_A^-(x_i) - T_C^-(x_i)\right| + \left|I_A^+(x_i) - I_C^+(x_i)\right| \right.$$
$$\left. + \left|I_A^-(x_i) - I_C^-(x_i)\right| + \left|F_A^+(x_i) - F_C^+(x_i)\right| + \left|F_A^-(x_i) - F_C^-(x_i)\right|\right)/6$$

Therefore it satisfies that $S(A,B) \geq S(A,C)$. Similarly, $S(B,C) \geq S(A,C)$. Hence the proof is complete.

Definition 3.7. Entropy of bipolar neutrosophic sets
Let $N_B(X)$ be bipolar neutrosophic set on X and $A \in N_B(X)$. An entropy on bipolar neutrosophic sets is a function $E_{N_B} : N_B(X) \to [0,1]$ which is satisfies the following properties:

i. $E_B(A) = 0$ if A is crisp set
ii. $E_B(A) = 1$ iff $\langle T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x)\rangle$
$= \langle 0.5, 0.5, 0.5, -0.5, -0.5, -0.5\rangle$
iii. $E_B(A) \geq E_B(B)$ if A is more uncertain than B.
i.e. $T_A^+(x) - (1 - T_A^+(x)) + F_A^+(x) - (1 - F_A^+(x)) \leq T_B^+(x) - (1 - T_B^+(x)) + F_B^+(x) - (1 - F_B^+(x))$ and $\left|I_A^+(x) - (1 - I_A^+(x))\right| + \left|I_A^-(x) - (1^- - I_A^-(x))\right| \leq \left|I_B^+(x) - (1 - I_B^+(x))\right| + \left|I_B^-(x) - (1^- - I_B^-(x))\right|$
iv. $E_B(A) = E_B(A^c) \quad \forall A \in N_B(X)$

Considering the fact that uncertainty in a bipolar neutrosophic set occurs due to partial belongingness, non-belongingness and indeterminacy over belongingness, an entropy measure E_B of bipolar neutrosophic set A is proposed here as follows:

$$E_B(A) = 1 - \frac{1}{n}\left[\sum_{x_i \in X} \frac{\begin{array}{c}\left(T_A^+(x_i) + (1 - T_A^-(x_i)) + F_A^+(x_i) + (1 - F_A^-(x_i))\right) * \\ \left(\left|I_A^+(x) - (1 - I_A^+(x))\right| + \left|I_A^-(x) - (1^- - I_A^-(x))\right|\right)\end{array}}{2}\right] \quad (3)$$

4. Algorithm for the Proposed Method

Step 1 For a MADM problem, a bipolar neutrosophic decision matrix $(BNDM)$ $A = (a_{ij})_{m\times n}$ is constructed.

Step 2 Compute bipolar neutrosophic average operator vaules for each alternative over attributes by $\widetilde{A}_i = A_W(\widetilde{a}_1, \widetilde{a}_2, \widetilde{a}_3, \ldots, \widetilde{a}_n)$

Step 3 Fix absoulte ideal point for bipolar neutrosophic sets

$$R_w^* = (T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-) = (1, 0, 0, 0, -1, -1), \forall j = 1, 2, \ldots n$$

Step 4 Calculate bipolar similarity measure between bipolar neutrosophic average operator vaules $\widetilde{A}_i$ and bipolar neutrosophic fixed ideal point R_w^*.

$$S(\widetilde{A}_i, R_w^*), \forall i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n$$

Step 5 Calculate the information entropy $E_B(\widetilde{A}_i)$ corresponding to all bipolar neutrosophic average operator vaules $\widetilde{A}_i = A_W(\widetilde{a}_1, \widetilde{a}_2, \widetilde{a}_3, \ldots, \widetilde{a}_n)$.

Step 6 Utilize the formula $Z_i(w) = E_B(\widetilde{A}_i) * S(\widetilde{A}_i, R_w^*)$ to obtain the alternative values.

Step 7 Choose the alternatives A_i according to values of $Z_i(w), i = 1, 2, ..m$.

5. Illustrative example of the proposed method

Higher education could be a major challenges in India. Particularly, rural students of the country are facing most of the challenges. One of the challenges in education is academic performance of the students and it plays a significant role in development of induvial and further in the growth of the country. So as to search out the most important challenges, we have a tendency to use fuzzy approach based on bipolar neutrosophic similarity measure and bipolar neutrosophic entropy to the problem. We catagorized a number of the factors that are directly or indirectly faced by the students and we have listed the factors as follows:

Alternatives

A_1 - Lack of poper transportation.

A_2 - People those who are belonging to remote rural areas have meager income.

A_3 - Lack of proper infrastructure at rural colleges.

A_4 - Number of higher education sector is less in rural areas.

A_5 - Low level of awareness.

A_6 - Minimal resources to guide students.

Attributes

C_1 - Being absent frequently / coming late to class.

C_2 - Not studying / not reviewing lessons at home.

C_3 - Having problem with teacher.

C_4 - Due to addiction dislike to study.

C_5 - Not understanding the teacher's language.

C_6 - Having no plan for future.

C_7 - Feeling shy / feeling uncomfortable / not focusing.

In this decision making problem we have taken seven attributes to evaluate the alternatives. First we get expert opinion for the above problems in terms of bipolar neutrosophic values. TABLE 1. and TABLE 2. show the expert values in bipolar neutrosophic decision matrix.

TABLE 1.

DM	C_1	C_2	C_3	C_4
A_1	(0.8,0.4,0.3, -0.3,-0.7,-0.7)	(0.6,0.4,0.5, -0.3,-0.2,-0.5)	(0.3,0.6,0.7, -0.1,-0.3,-0.5)	(0.2,0.5,0.6, -0.3,-0.6,-0.7)
A_2	(0.1,0.6,0.8, -0.6,-0.4,-0.5)	(0.3,0.6,0.7, -0.5,-0.5,-0.6)	(0.2,0.7,0.8, -0.7,-0.4,-0.2)	(0.6,0.5,0.3, -0.4,-0.3,-0.6)
A_3	(0.2,0.7,0.8, -0.9,-0.3,-0.4)	(0.6,0.4,0.8, -0.5,-0.3,-0.2)	(0.4,0.6,0.8, -0.5,-0.6,-0.2)	(0.3,0.7,0.6, -0.6,-0.5,-0.5)
A_4	(0.2,0.8,0.9, -0.7,-0.3,-0.2)	(0.3,0.6,0.8, -0.6,-0.5,-0.3)	(0.1,0.7,0.9, -0.8,-0.4,-0.2)	(0.3,0.5,0.6, -0.8,-0.5,-0.5)
A_5	(0.6,0.4,0.4, -0.5,-0.3,-0.5)	(0.8,0.3,0.1, -0.2,-0.7,-0.8)	(0.5,0.5,0.6, -0.4,-0.6,-0.3)	(0.9,0.2,0.2, -0.2,-0.9,-0.8)
A_6	(0.4,0.5,0.7, -0.6,-0.4,-0.3)	(0.6,0.4,0.4, -0.3,-0.7,-0.6)	(0.8,0.3,0.2, -0.3,-0.2,-0.7)	(0.3,0.5,0.8, -0.7,-0.6,-0.3)

TABLE 2.

DM	C_5	C_6	C_7
A_1	(0.1,0.7,0.8, -0.3,-0.4,-0.5)	(0.1,0.6,0.7, -0.8,-0.3,-0.4)	(0.3,0.4,0.6, -0.5,-0.5,-0.7)
A_2	(0.1,0.7,0.9, -0.9,-0.3,-0.2)	(0.8,0.4,0.1, -0.3,-0.5,-0.8)	(0.7,0.5,0.3, -0.2,-0.4,-0.5)
A_3	(0.1,0.7,0.9, -0.8,-0.4,-0.2)	(0.2,0.5,0.8, -0.7,-0.4,-0.3)	(0.5,0.3,0.4, -0.4,-0.8,-0.5)
A_4	(0.1,0.8,0.9, -0.9,-0.3,-0.2)	(0.4,0.4,0.5, -0.5,-0.3,-0.7)	(0.2,0.7,0.9, -0.8,-0.2,-0.3)
A_5	(0.2,0.7,0.8, -0.9,-0.4,-0.3)	(0.8,0.3,0.3, -0.3,-0.7,-0.6)	(0.4,0.5,0.7, -0.7,-0.6,-0.3)
A_6	(0.7,0.3,0.5, -0.4,-0.8,-0.6)	(0.9,0.3,0.2, -0.2,-0.7,-0.8)	(0.4,0.5,0.7, -0.8,-0.7,-0.4)

Weighted values of attributes is $C_1 = 0.2, C_2 = 0.15, C_3 = 0.1, C_4 = 0.1, C_5 = 0.15, C_6 = 0.2, C_7 = 0.1$. Then bipolar neutrosophic average operator values for each alternatives over attributes is calculated by definition 3.2 and the values are as follows:

$$\widetilde{A_1} = \langle 0.446, 0.502, 0.556, -0.344, -0.461, -0.577\rangle$$
$$\widetilde{A_2} = \langle 0.476, 0.554, 0.433, -0.472, -0.415, -0.557\rangle$$
$$\widetilde{A_3} = \langle 0.329, 0.544, 0.738, -0.643, -0.465, -0.331\rangle$$
$$\widetilde{A_4} = \langle 0.248, 0.620, 0.755, -0.691, -0.358, -0.393\rangle$$
$$\widetilde{A_5} = \langle 0.677, 0.384, 0.350, -0.397, -0.626, -0.572\rangle$$
$$\widetilde{A_6} = \langle 0.676, 0.384, 0.426, -0.398, -0.632, -0.583\rangle$$

Using definition 3.6, the bipolar neutrosophic similarity measure between bipolar neutrosophic average operator vaules $\widetilde{A}_i$ and bipolar neutrosophic fixed ideal point R^*_w are as follows:

$$S(\widetilde{A}_1, R^*_w) = 0.514,\ S(\widetilde{A}_2, R^*_w) = 0.498,$$
$$S(\widetilde{A}_3, R^*_w) = 0.366,\ S(\widetilde{A}_4, R^*_w) = 0.322,$$
$$S(\widetilde{A}_5, R^*_w) = 0.624,\ S(\widetilde{A}_6, R^*_w) = 0.614.$$

The information entropy $E_B(\widetilde{A}_i)$ corresponding to all bipolar neutrosophic average operator vaules $\widetilde{A}_i$ is calculated by the definition 3.7

$$E_B(\widetilde{A}_1) = 0.085,\ E_B(\widetilde{A}_2) = 0.262,$$
$$E_B(\widetilde{A}_3) = 0.166,\ E_B(\widetilde{A}_4) = 0.502,$$
$$E_B(\widetilde{A}_5) = 0.498,\ E_B(\widetilde{A}_6) = 0.525.$$

By applying the formula $Z_i(w) = E_B(\widetilde{A}_i) * S(\widetilde{A}_i, R^*_w)$, we get the following values for each alternatives

$$Z_1(w) = 0.044, Z_2(w) = 0.131, Z_3(w) = 0.061,$$
$$Z_4(w) = 0.162, Z_5(w) = 0.311, Z_6(w) = 0.322.$$

6. CONCLUSION

This paper contributes a new similarity measure between two bipolar neutrosophic sets and also introduces a new procedure to find the entropy of bipolar neutrosophic set and vality of the definition 3.6 and 3.7 have been verified with their properties. From the analysis of the problem, according to the proposed method, the ranking order of the six alternatives is $A_6 > A_5 > A_4 > A_2 > A_3 > A_1$. Hence, the important challenge in higher education is A_6 - the minimal resources to guide students.

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