

# A Lattice Theoretic Look: A Negated Approach to Adjectival (Intersective, Neutrosophic and Private) Phrases and More

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## ABSTRACT

*This paper is an extended version of "A Lattice Theoretic Look: A Negated Approach to Adjectival (Intersective, Neutrosophic and Private) Phrases" in INISTA 2017. Firstly, some new negations of intersective adjectival phrases and their set-theoretic semantics such as non-red non-cars and red non-cars are presented. Secondly, a lattice structure is built on positive and negative nouns and their positive and negative intersective adjectival phrases. Thirdly, a richer lattice is obtained from previous one by adding neutrosophic prefixes neut and anti to intersective adjectival phrases. Finally, the richest lattice is constructed via extending the previous lattice structures by private adjectives (fake, counterfeit). These lattice classes are called Neutrosophic Linguistic Lattices (NLL). In the last part of the paper (Section 4 does not take place in the paper introduced in INISTA 2017), noun and adjective based positive and negative sub-lattices of NLL are introduced.*

**KEYWORDS:** Logic of natural languages; neutrosophy; pre-orders, orders and lattices; adjectives; noun phrases; negation

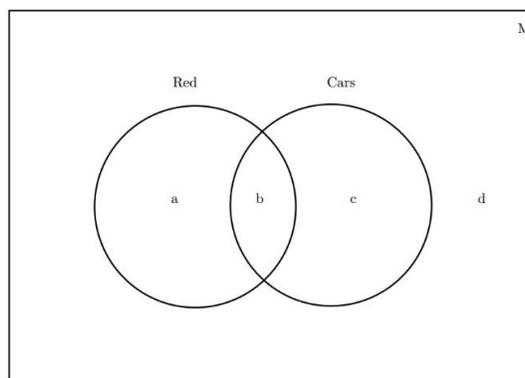
## 1. INTRODUCTION

Lattice theory, one of the fundamental sub-fields of the foundations of mathematics and mathematical logic, is a powerful tool of many areas such as Linguistics, Chemistry, Physics, and Information Science. In information science, it is essential to make data understandable and meaningful. Mathematical structures are the most effective tools for transferring human natural phrases and sentences to computer environment as meaningful data. Especially, with a set theoretical view, lattice applications of mathematical models in linguistics are a common occurrence. Fundamentally, Natural Logic (Moss, 2010), (van Benthem, 2008) is a human reasoning discipline that explores inference patterns and logics in natural language. These patterns and logics are constructed on relations between syntax and semantics of sentences and phrases. In order to explore and identify the entailment relations among sentences by mathematical structures, it is first necessary to determine the relations between words and clauses themselves. We would like to find new connections between natural logic and neutrosophic by discovering the phrases and neutrosophic clauses. In this sense, we will associate phrases and negated phrases to neutrosophic concepts. Recently, a theory called Neutrosophy, introduced by Smarandache (Smarandache, 1998, 2004, 2015) has widespread mathematics, philosophy and applied sciences coverage. Mathematically, it offers a system which is an extension of intuitionistic fuzzy system. Neutrosophy considers an entity,  $A$  in relation to its opposite,  $anti-A$  and that which is *not*  $A$ ,  $non-A$ , and that which is neither  $A$  nor  $anti-A$ , denoted by  $neut-A$ . Up to section 3.3, we will obtain various negated versions of phrases (intersective adjectival) because Neutrosophy considers opposite property of concepts and we would like to associate the phrases and Neutrosophic phrases. We will present the first *NLL* in section 3.3. Notice that all models and interpretations of phrases will be finite throughout the paper. The research problem of this paper is to put forth lattice structures of neutrosophic phrases for purpose of exploring relations between the phrases on the mathematical level. The results of the paper may help to prove soundness and completeness theorems of possible logics obtained by sentences formed by neutrosophic phrases. The original contribution of this paper is that none of the

lattices and sub-lattices of such phrases have never been studied. Relevant studies have not gone beyond simple names and adjectives (intersective and private). The phrases allow us to study the details of lattice theory, in addition to lattices such as sub-lattices and even ideals and filters because the expressive power of neutrosophic phrases present much richer structures.

## 2. NEGATING INTERSECTIVE ADJECTIVAL PHRASES

Phrases such as *red cars* can be interpreted the intersection of the set of *red things* with the set of *cars* and get the set of *red cars*. In the sense of model-theoretic semantics, the interpretation of a phrase such as *red cars* would be the intersection of the interpretation of *cars* with a set of *red individuals* (the region *b* in Figure 1). Such adjectives are called intersective adjectives or intersecting adjectives. As to negational interpretation, Keenan and Faltz told that “similarly, intersective adjectives, like common nouns, are negatable by non-: non-Albanian (cf. non-student) “in their book (Keenan & Faltz, 2012). In this sense, *non-red cars* would interpret the intersection of the of *non-red things* and the set of *cars*. Negating intersective adjectives without nouns (*red things*) would be complements of the set of *red things*, in other words, *non-red things*. We mean by “*non-red things*”: the things are which are *not red*. Remark that the conceptual field of “*non-red things*” does not guarantee that *these individuals* have to have a color property or something else. It is changeable under incorporating situations, but we will might say something about it in another paper. On the other hand, negating nouns (*cars*) would be complements of the set of *cars*, in other words, *non-cars*. We mean by non-cars that the things are which are not cars. Adhering to the spirit of intersective adjectivity, we can explore new meanings and their interpretations from negated intersective adjectival phrases by intersecting negated (or not) adjectives with negated (or not) nouns. As was in the book, *non-red cars* is the intersection the set of things that are not red with cars. In other words, *non-red cars* are the cars but not red (the region *c* in Figure 1). Another candidate for the negated case, *non-red non-cars* refers to intersect the set of non-red things (things that are not red) with non-cars (the region *d* in Figure 1). The last one, *red non-cars* has meaning that is the set of intersection of the set of red things and the set of non-cars (the region *a* in Figure 1).  $\overline{red}x$  is called *noun level partially semantic complement*.  $\overline{red}x$  is called *adjective level partially semantic complement*.  $\overline{red}x$  is called *full phrasal semantic complement*. In summary, we obtain *non-red cars*, *red non-cars* and *non-red non-cars* from *red cars* we already had.



**Fig. 1: An example of *cars* and *red* in a discourse universe**

The intersective theory and conjunctives suit well into Boolean semantics (Keenan & Faltz, 2012), (Roelofsen, 2013) which proposes very close relationship between *and* and *or* in natural language, as conjunction and disjunction in propositional and predicate logics that have been applied to natural language semantics. In these logics, the relationship between conjunction and disjunction corresponds to the relationship between the set-theoretic notions of intersection and union (Champollion, 2016), (Hardegree, 1994). On the other hand, correlative conjunctions might help to interpret negated intersective adjectival phrases within Boolean semantics because the conjunctions are paired conjunctions (neither/nor, either/or, both/and,) that link words, phrases, and clauses. We might reassessment those negated intersective adjectival phrases in perspective of correlative conjunctions. “*neither A nor B* “and “*both non-A and non-*

" $B$ " can be used interchangeably where  $A$  is an intersective adjective and  $B$  is a noun. Therefore, we say "neither red (things) nor pencils" and "both non-red (things) and non-pencils" equivalent sentences. An evidence for the interchangeability comes from equivalent statements in propositional logic, that is,  $\neg(R \vee C)$  is logically equivalent to  $\neg R \wedge \neg C$  (Champollion, 2016). Other negated statements would be  $\neg R \wedge C$  and  $R \wedge \neg C$ . Semantically,  $\neg R \wedge \neg C$  is full phrasal semantic complement of  $R \vee C$ , and also  $\neg R \wedge C$  and  $R \wedge \neg C$  are partially semantic complements of  $R \vee C$ . We will explore full and partially semantic complements of several adjectival phrases. We will generally negate the phrases and nouns by adding prefix "non", "anti" and "neut". We will use interpretation function  $[[\ ]]$  from set of phrases ( $Ph$ ) to power set of universe ( $P(M)$ ) (set of individuals) to express phrases with understanding of a set-theoretic viewpoint. Hence,  $[[p]] \subseteq M$  for every  $p \in Ph$ . For an adjective (negated or not) and a plural noun  $n$  (negated or not,  $a n$  will be interpreted as  $[[a]] \cap [[n]]$ ). If  $n$  is a positive plural noun,  $non-n$  is interpreted as  $[[non - n]] = [[n]] = M \setminus [[n]]$  and, similarly, if  $a$  is a positive adjective,  $non-a$  is interpreted as

$[[non - a]] = [[a]] = M \setminus [[a]]$ . When we will add *non-to* both nouns and adjectives as prefix, "anti" and "neut" will be added in front of only adjectives. Some adjectives themselves have negational meaning such as *fake*. Semantics of phrases with *anti*, *neut* and *fake* will be mentioned in next sections.

### 3. LATTICE THEORETIC LOOK

We will give some fundamental definitions before we start to construct lattice structures from these adjectival phrases. A lattice is an algebraic structure that consists of a partially ordered set in which every two elements have a unique supremum (a least upper bound or join) and a unique infimum (a greatest lower bound or meet) (Davey & Priestley, 2002). The most classical example is on sets by interpreting set intersection as meet and union as join. For any set  $A$ , the power set of  $A$  can be ordered via subset inclusion to obtain a lattice bounded by  $A$  and the empty set. We will give two new definitions in subsection 3.2 to begin constructing lattice structures.

**Remark 1.** We will use the letter  $a$  and *red* for intersective adjectives, and the letter  $x$ ,  $n$  and *cars* for common plural nouns in the name of abbreviation and space saving throughout the paper.

#### 3.1 Individuals

Each element of  $[[a x]]$  and  $[[\bar{a} x]]$  is a distinct individual and belongs to  $[[x]]$ . It is already known that  $[[a x]] \cap [[\bar{a} x]] = \emptyset$  and  $[[a x]] \cup [[\bar{a} x]] = [[x]]$ . It means that no common elements exist in  $[[a x]]$  and  $[[\bar{a} x]]$ . Hence, every element of these sets can be considered as individual objects such as Larry, John, Meg, ... etc. Uchida and Cassimatis (Uchida & Cassimatis, 2014) already gave a lattice structure on power set of all of individuals (a domain or a universe).

#### 3.2. Lattice $\mathcal{L}_{IA}$

Intersective adjectives (*red*) provide some properties for nouns (*cars*). Excluding (complementing) a property from an intersective adjective phrase also provide another property for nouns. In this direction, "*red*" is a property for a noun, "*non-red*" is another property for the noun as well. *red* and *non-red* have discrete meaning and sets as can be seen in Figure 1. Naturally, every set of restricted objects with a

property (*red cars*) is a subset of those objects without the properties (*cars*).  $[[red x]]$  and  $[[\bar{red} x]]$  are always subsets of  $[[x]]$ . Neither  $[[red x]] \leq_* [[\bar{red} x]]$  nor  $[[\bar{red} x]] \leq_* [[red x]]$  since

$[[red x]] \cap [[\bar{red} x]]$  by assuming  $[[\bar{red} x]] \neq \emptyset$  and  $[[red x]] \neq \emptyset$ . Without loss of generality, for

negative (complement) of the noun  $x$  and the intersective adjective  $red$  (positive and negative) are  $\bar{x}$ ,  $red \bar{x}$  and  $\bar{red} \bar{x}$ .  $[[red \bar{x}]]$  and  $[[\bar{red} \bar{x}]]$  are always subsets of  $[[\bar{x}]]$ . Neither  $[[red \bar{x}]] \leq_* [[\bar{red} \bar{x}]]$  nor  $[[\bar{red} \bar{x}] \leq_* [[red \bar{x}]]$  since  $[[red \bar{x}]] \cap [[\bar{red} \bar{x}]] = \emptyset$  by assuming  $[[red \bar{x}]] \neq \emptyset$  and  $[[\bar{red} \bar{x}]] \neq \emptyset$ . On the other hand,  $[[x]] \cap [[\bar{x}]] = \emptyset$  and  $[[x]] \overset{*}{\cup} [[\bar{x}]] = M$  ( $M$  is the universe of discourse) and also  $[[red \bar{x}]]$ ,  $[[\bar{red} \bar{x}]]$ ,  $[[red \bar{x}]]$  and  $[[\bar{red} \bar{x}]]$  are by two discrete. We do not allow  $[[red \bar{x}]] \overset{*}{\cup} [[\bar{red} \bar{x}]]$  and  $[[red \bar{x}]] \overset{*}{\cup} [[\bar{red} \bar{x}]]$  and  $[[\bar{red} \bar{x}]] \overset{*}{\cup} [[red \bar{x}]]$  and  $[[\bar{red} \bar{x}]] \overset{*}{\cup} [[red \bar{x}]]$  to take places in the lattice in Figure 2 because we try to build the lattice from phrases only in our language. To do this, we define a set operation  $\overset{*}{\cup}$  and an order relation  $\leq_*$  as follows:

**Definition 2.** We define a binary set operator  $\overset{*}{\cup}$  for our languages as the follow: Let  $S$  be a set of sets and  $A, B \in S$ .  $A \overset{*}{\cup} B = C \Leftrightarrow C$  is the smallest set which includes both  $A$  and  $B$ , and also  $C \in S$ .

**Definition 3.** We define a partial order  $\leq_*$  on sets as the follow:

$$A \leq_* B \text{ if } B = A \overset{*}{\cup} B$$

$$A \leq_* B \text{ if } A = A \cap B$$

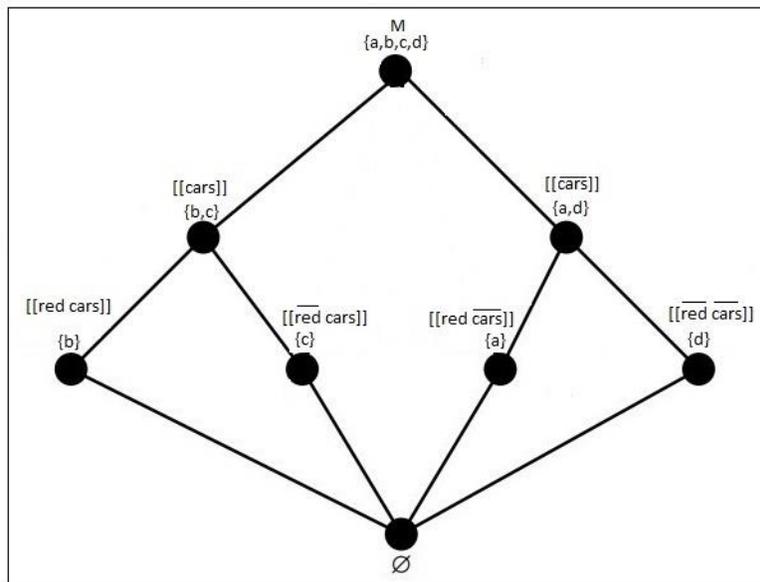
**Example 4.** Let  $A = \{1, 2\}, B = \{2, 3\}, C = \{1, 2, 4\}, D = \{1, 2, 3, 4\}$  and  $S = \{A, B, C, D\}$ .

$$A \overset{*}{\cup} A = A, A \overset{*}{\cup} C = C, A \overset{*}{\cup} B = D, B \overset{*}{\cup} C = D, C \overset{*}{\cup} D = D, C \leq_* C, A \leq_* C, A \leq_* D, B \leq_* D, C \leq_* D.$$

Notice that  $\leq_*$  is a reflexive, transitive relation (pre-order) and  $\overset{*}{\cup}$  is a reflexive, symmetric relation.

Figure 3 illustrates a diagram on  $cars$  and  $red$ . The diagram does not contain sets  $\{b, d\}, \{a, b\}, \{a, c\}$  and  $\{c, d\}$  because the sets do not represent linguistically any phrases in the language. Because of this reason,

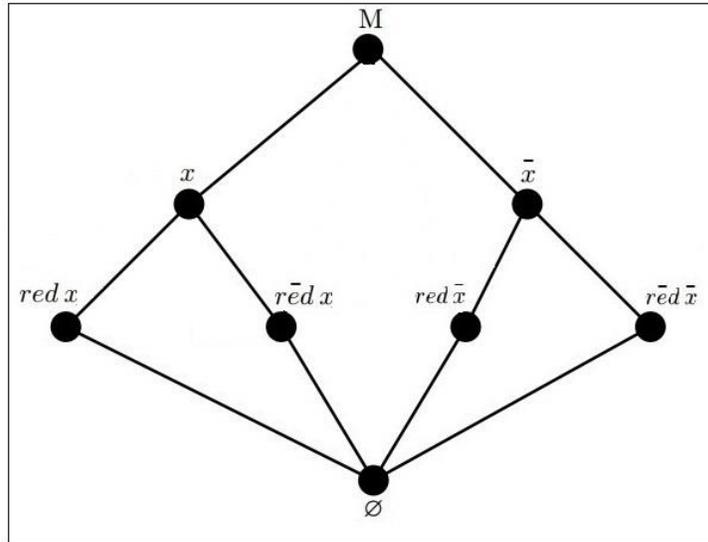
$\{a\} \overset{*}{\cup} \{c\}$  and  $\{a\} \overset{*}{\cup} \{b\}$  and  $\{d\} \overset{*}{\cup} \{c\}$  and  $\{a, b, c, d\} = M$ . This structure builds a lattice up by  $\overset{*}{\cup}$  and  $\cap$  that is the classical set intersection operation.



**Fig. 2: Lattice on cars and red**

$\mathcal{L}_{IA} = (L, \emptyset, \cap, \cup)$  is a lattice where  $L = \{M, x, \bar{x}, red\ x, red\ \bar{x}, red\ x, red\ \bar{x}\}$ . Remark that

$\mathcal{L}_{IA} = (L, \emptyset, \cap, \cup) = (L, \emptyset, \leq_*)$ . We call this lattice briefly  $\mathcal{L}_{IA}$ .



**Fig. 3: Hasse Diagram of lattice of  $\mathcal{L}_{IA} = (L, \emptyset, \cap, \cup)$**

### 3.3 Lattice $\mathcal{L}_{IA}^N$

In this section, we present first NLL. [4] Let A be the color white. Then,

$$non - A = \{black, red, yellow, blue, \dots\},$$

anti-A points at black, and

$$neut - A = \{red, yellow, blue, \dots\}.$$

In our interpretation base, anti-black cars ( $black\ cars^a$ ) is a specific set of cars which is a subset of set non-black cars ( $black\ cars^{\bar{}}$ ). neut-black cars ( $black\ cars^n$ ) is a subset of  $black\ cars^{\bar{}}$  which is obtained by excluding sets black cars and  $black\ cars^a$  from  $black\ cars^{\bar{}}$ . Similarly, anti-black cars ( $black\ cars^a$ ) is a specific set of  $cars^{\bar{}}$  which is a subset of set non-black non-cars ( $black\ cars^{\bar{}}$ ).  $neut - black\ cars^{\bar{}}$  ( $black\ cars^n$ ) is a subset of  $black\ cars^{\bar{}}$  which is obtained by excluding sets of  $black\ cars^{\bar{}}$  and  $black\ cars^a$  from  $black\ cars^{\bar{}}$ . The new structure represents an extended lattice equipped with  $\leq_*$  as can be seen in Figure 4. We call this lattice  $\mathcal{L}_{IA}^N$ .

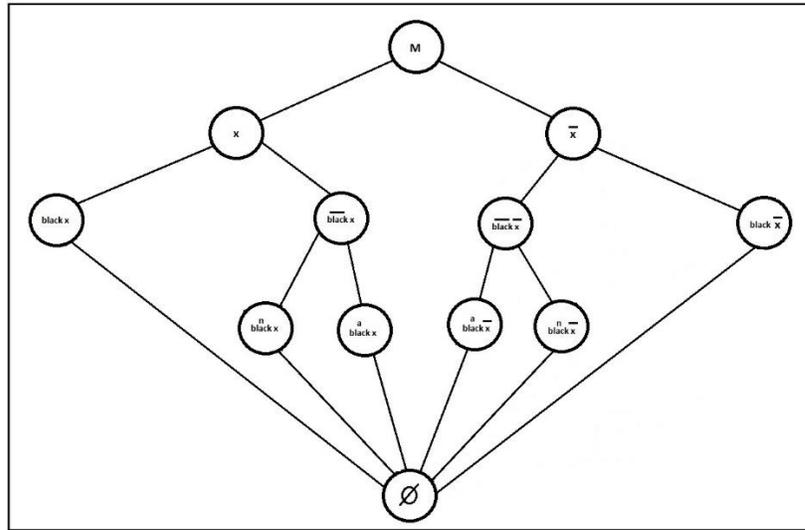


Fig. 4: The Lattice  $\mathcal{L}_{IA}^N$

### 3.4 Lattice $\mathcal{L}_{IA}^N(F)$

Another *NLL* is an extended version of  $\mathcal{L}_{IA}^N$  by private adjectives. Those adjectives have negative effects on nouns such fake and counterfeit. The adjectives are representative elements of, called private, a special class of adjectives (Chatzikyriakidis & Luo, 2013), (Partee, 2007), (Hoffher & Matushansky 2010).

Chatzikyriakidis and Luo (Chatzikyriakidis & Luo, 2013), treated transition from the adjectival phrase to noun as *Private Adj(N) ⇒ ¬N* in inferential base. Furthermore, they gave an equivalence: “

*real\_gun(g)* if and only if “ $\neg$ *fake\_gun(g)* where  $[[g \text{ is a real gun}]] = \text{real\_gun}(g)$  and

$[[f \text{ is not a real gun}]] = \neg \text{real\_gun}(f)$  in order to constitute a modern type-theoretical setting.

Considering these facts, fake car is not a car (real) and plural form: fake cars are not cars. Hence, set of fake cars is a subset of set of non-cars in our treatment. On the one hand, compositions with private adjectives and intersective adjectival phrases do not affect the intersective adjectives negatively but nouns as usual. Then, interpretation of “fake red cars” would be intersection of set of red things and set of non-cars. Applying “non” to private adjectival phrases,

non-fake cars are cars(real),  $[[\text{non} - \text{fake cars}]] = [[\text{cars}]]$  whereas  $[[\text{fake cars}]] \subseteq [[\text{non} - \text{cars}]]$ .

*non-fake cars* will be not given a place in the lattice. Remark that phrase “non-fake non-cars” is ambiguous since fake is not a intersective adjective. We will not consider this phrase in our lattice.  $x$  is incomparable

both  $\overline{\text{black } x}$  and  $\overline{\text{black } \overline{x}}$  except  $\overline{x}$  as can be seen in Figure 5. So, we cannot determine that set of *fake cars* is a subset or superset of a set of any adjectival phrases. But we know that

$[[\text{fake cars}]] \subseteq [[\text{non} - \text{cars}]]$ . Then, we can see easily  $[[\text{fake black cars}]] \subseteq [[\text{blacks non} - \text{cars}]]$  by

using  $[[\text{fake cars}]] \cap [[\text{black things}]] \subseteq [[\text{cars}]] \cap [[\text{black things}]]$ . Without loss of generality, set of

*fake black cars* is a subset of set *black non-cars* and also set of fake *non-black cars* is a subset of set *non-black non-cars*. Continuing with neut and anti, set of *fake neut black cars* is a subset of set of neut *black non-cars* and also *fake anti-black cars* is a subset of set of *anti-black non-cars*. These phrases build the

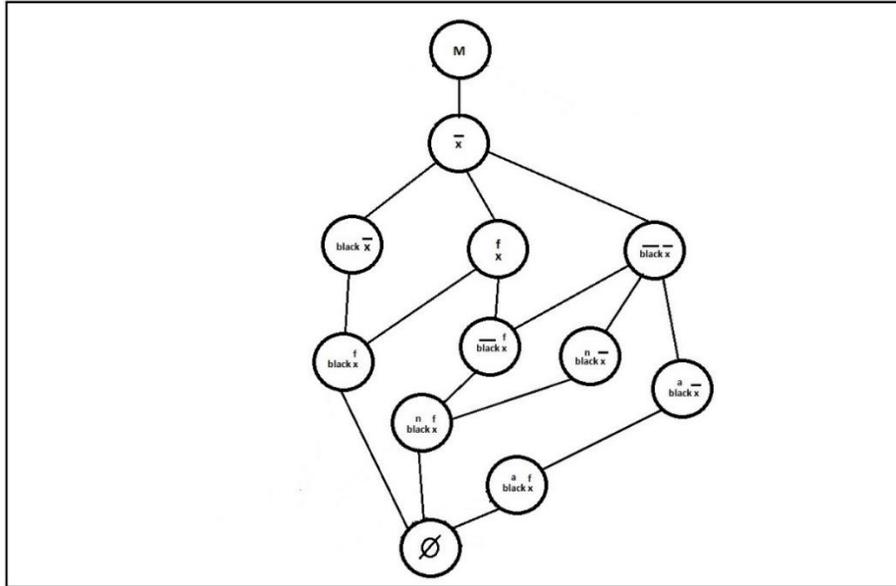
lattice  $\mathcal{L}_{IA}^N(F)$  in Fig. 5.



**Definition 7.** Noun based negative sub-lattice (NBPSL): An **NBPSL** is a sub-lattice of  $\mathcal{L}_{IA}^N(F)$  which consists of negative noun phrases, and  $M$  and  $\emptyset$  only.

**Remark 8.**  $x$  is a positive noun and both  $\overset{f}{x}$  and  $\overset{-}{x}$  are negative nouns.

**Remark 9.** As can be seen in Fig. 7, elements of the biggest **NBNSL** sub-lattice of  $\mathcal{L}_{IA}^N(F)$  consists of  $M, \overset{f}{x}, \overset{-}{x}, \overset{f}{black\ x}, \overset{-}{black\ x}, \overset{n}{black\ x}, \overset{-}{black\ x}, \overset{f}{black\ x}, \overset{-}{black\ x}$  and  $\emptyset$ .

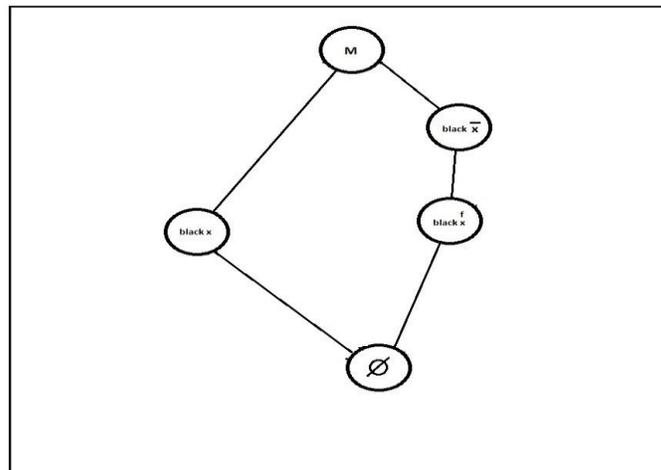


**Fig. 7: The biggest NBNSL sub-lattice of  $\mathcal{L}_{IA}^N(F)$**

**Definition 10.** Adjective based positive sub-lattice (ABPSL): An **ABPSL** is a sub-lattice of  $\mathcal{L}_{IA}^N(F)$  which consists of noun phrases with positive adjectives, and  $M$  and  $\emptyset$  only.

**Remark 11.**  $black$  is a positive adjective.  $\overset{-}{black}$ ,  $\overset{a}{black}$  and  $\overset{n}{black}$  are negative adjectives.

**Remark 12.** As can be seen in Fig. 8, elements of the biggest **ABPSL** sub-lattice of  $\mathcal{L}_{IA}^N(F)$  consists of  $M, \overset{f}{black\ x}, \overset{-}{black\ x}, \overset{a}{black\ x}, \overset{n}{black\ x}$  and  $\emptyset$ .



**Fig. 8: The biggest ABPSL sub-lattice of  $\mathcal{L}_{IA}^N(F)$**



applications in pattern recognition. *Journal of Intelligent & Fuzzy Systems*, 30(4), 1957-1963.

Çevik, A. (2012). *Hesaplanabilirlik Kuramı ve Turing Derecelerine Giriş*, Gaziosmanpaşa Üniversitesi Bilimsel Araştırma Dergisi, Volume 1, 1-20.

Çevik, A. (2013a). Antibasis theorems for classes and the jump hierarchy. *Archive for Mathematical Logic*, 1-6.

Çevik, A. (2013b). *On the possibility of supertasks*, *Beytulhikme An International Journal of Philosophy*, 5, 171-180.

Çevik, A. (2016). choice classes. *Mathematical Logic Quarterly*, 62(6), 563-574.

Champollion, L. (2016). Ten men and women got married today: Noun coordination and the intersective theory of conjunction. *Journal of Semantics*, 33(3), 561-622.

Chatzikyriakidis, S., & Luo, Z. (2013). *Adjectives in a modern type-theoretical setting*. In *Formal Grammar* (pp. 159-174). Springer Berlin Heidelberg.

Davey, B. A., & Priestley, H. A. (2002). *Introduction to lattices and order*. Cambridge university press.

Deli, I., Ali, M., & Smarandache, F. (2015, August). Bipolar neutrosophic sets and their application based on multi-criteria decision-making problems. In *Advanced Mechatronic Systems (ICAMechS), 2015 International Conference on* (pp. 249-254). IEEE.

Hardegree, G. M. (1994). *Symbolic logic: A first course*. McGraw-Hill.

Hoffner, P. C., & Matushansky, O. (Eds.). (2010). *Adjectives: formal analyses in syntax and semantics* (Vol. 153). John Benjamins Publishing.

Keenan, E. L., & Faltz, L. M. (2012). *Boolean semantics for natural language* (Vol. 23). Springer Science & Business Media.

Partee, B. (2007). Compositionality and coercion in semantics: The dynamics of adjective meaning. *Cognitive foundations of Interpretation*, 145-161.

Roelofsen, F. (2013). Algebraic foundations for the semantic treatment of inquisitive content. *Synthese*, 190(1), 79-102.

Smarandache, F. (1998). Invisible paradox. *Neutrosophy/Neutrosophic Probability, Set, and Logic*", Am. Res. Press, Rehoboth, 22-23.

Smarandache, F. (2004). Matter, antimatter, and unmatter. CDS-CERN (pp. 173-177). EXT-2005-142.

Smarandache, F. (2015). Neutrosophic actions, prevalence order, refinement of neutrosophic entities, and neutrosophic literal logical operators. A Publication of Society for Mathematics of Uncertainty, 11.

Thanh, N.D., Ali, M. & Son, (2017). A Novel Clustering Algorithm in a Neutrosophic Recommender System for Medical Diagnosis, L.H. Cogn Comput 9: 526. <https://doi.org/10.1007/s12559-017-9462-8>

- Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, 10(4), 106.
- Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. *Measurement*. Volume 124, August 2018, Pages 47-55
- Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.
- Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
- Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, 86, 12-29.
- Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*, 33(6), 4055-4066.
- Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. *Symmetry* 2018, 10, 116.
- Uchida, H., & Cassimatis, N. L. (2014). Quantifiers as Terms and Lattice-Based Semantics. <https://pdfs.semanticscholar.org/9dbc/eb5c1175338f3b8dff285fb727bbdbcd8325.pdf>.
- van Benthem, J. (2008). A brief history of natural logic. London College Publications 9781904987444.
- van Rooij, R. (2010). Extending syllogistic reasoning. In *Logic, Language and Meaning* (pp. 124-132). Springer, Berlin, Heidelberg.
- Winter, Y., & Zwarts, J. (2012) On the event semantics of nominals and adjectives: The one argument hypothesis, *Proceedings of Sinn and Bedeutung*, 16.