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Research Article

A Neutrosophic-Based Approach in Data Envelopment Analysis with Undesirable Outputs

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Data Envelopment Analysis is one of the paramount mathematical methods to compute the general performance of organizations, which utilizes similar sources to produce similar outputs. Original DEA schemes involve crisp information of inputs and outputs that may not always be accessible in real-world applications. Nevertheless, in some cases, the values of the data are information with indeterminacy, impreciseness, vagueness, inconsistent, and incompleteness. Furthermore, the conventional DEA models have been originally formulated solely for desirable outputs. However, undesirable outputs may additionally be present in the manufacturing system, which wishes to be minimized. To tackle the mentioned issues and in order to obtain a reliable measurement that keeps original advantage of DEA and considers the influence of undesirable factors under the indeterminate environments, this paper presents a neutrosophic DEA model with undesirable outputs. The recommended technique is based on the aggregation operator and has a simple construction. Finally, an example is given to illustrate the new model and ranking approach in details.

1. Introduction

All organizations, whether governmental or private, require an effective performance assessment for development, growth, and sustainability in the competitive world of today so that within this growth it can appraise the efficiency and effectiveness of its organizational programs and its human resources processes. In other words, senior executive managers have always been seeking for a solution to ensure that their strategies are executed and, hence, have selected performance assessment methods as tools to implement their strategies. We should be mindful that novel measurement systems have been developed to target the existing strategies, and in this system, the indexes could be viewed as a crucial factor, in the present and future successes of the company. If these factors are alleviated or improved, then the company has launched its strategy.

The Data Envelopment Analysis (DEA) is a nonparametric method to analyse and assess the performance of decision-making units (DMUs), which converts several inputs into several outputs and considers the qualitative and quantitative criterions. Based on the work of [1], Charnes et al. [2] proposed this methodology that is called the CCR model. After that, Banker et al. [3] extended this model and established a new model to measure efficiency that is the so-called BBC model. With DEA, directors can achieve the relative efficiency of a set of DMUs. Note that the production function does not need to be known in this technique. In the recent years, an extensive application of DEA in numerous fields, such as banking institutions [4], insurance industry [5], financial services [6], education [7], sustainability [8], energy [9], and health-care services [10], has been observed. Furthermore, in the past four decades, numerous reports and articles have been published in esteemed global journals verifying that this method is operational, see [11–16].

However, in some cases, the values of the data are often information with indeterminacy, impreciseness, vagueness, inconsistent, and incompleteness. Inaccurate assessments are mainly the outcome of information that is unquantifiable, incomplete, and unavailable. By fuzzy sets [17], it is possible to model the uncertainty in information. In addition, there are several models of DEA with the fuzzy set, see [17–19] and references therein. However, fuzzy sets (FSs) deliberate only the membership function and cannot arrange other parameters of vagueness. Therefore, intuitionistic fuzzy sets (IFSs) have been introduced in [20], see also the Pythagorean fuzzy sets [21–23].

Though IFSs is able to address incomplete data for numerous real-world topics, it cannot handle other natures of uncertainty such as indeterminate information. Hence, Smarandache [24] puts forward an indeterminacy degree of membership as an autonomous element and proposes the neutrosophic set (NS). Then, inspired by sports (winner, loser, and equal), voting (agree, disagree, and abstain), answering (yes, no, and I do not know), decision-making (decision, no decision, and doubt), and acceptance (accept, rejection, and suspension) and since the principle of excluded middle cannot be applied to new logic, he combines three-valued logic with nonstandard analysis. Since then, a series of subclasses of NS have emerged, mainly including interval neutrosophic set [25–27], bipolar neutrosophic set [28, 29], single-valued neutrosophic set [30–33], simplified neutrosophic sets [34–36], neutrosophic structured element [37], multivalued neutrosophic set [38, 39], and neutrosophic linguistic set [40, 41] which have been proposed. Moreover, these concepts utilized in several problems, see [42–51].

In the DEA literature, there are also some models of DEA with IFSs, see [52–60]. However, in actual activities, there are some indeterminate information. If the indeterminacy factors are ignored, the relative effectiveness of DMUs will still be evaluated using the DEA model established on the basis of determinate values and biased or even wrong information will be obtained, which will bring some errors to management decisions. Therefore, it seems convenient and necessary to consider the neutrosophic DEA model.

As far as we know, there are few studies concerning DEA with neutrosophic information. The utilization of neutrosophic set in DEA can be traced to Edalatpanah [61] and additional investigations have been accessible in [62–67]. However, these neutrosophic DEA methods are formulated solely for desirable outputs and cannot eliminate the influence of undesirable factors on the efficiency evaluation. We know that the undesirable outputs may additionally be present in the manufacturing system that needs to be minimized. For example, in banking industry nonperforming loans/assets and in manufacturing systems, the production of a variety of emissions, pollutions, and industrial waste gas are undesirable outputs [68–73]. Therefore, there is still a need from the neutrosophic DEA method to develop a new model that keeps original advantage and considers the influence of undesirable factors.

Consequently, in this study, we establish a novel method of DEA with undesirable outputs in which all data are single-valued neutrosophic sets (SVNSs). Furthermore, a competent algorithm for solving the new DEA model has been presented. The main contributions of this paper are four folds: (1) we model the indeterminacies in the input and output data using SVNSs; (2) the proposed approach considers the impact of undesirable outputs on the performance of DMUs; (3) we provide a theorem regarding the feasibility and boundedness of the solution of new model; (4) we determine the efficiency scores of the DMUs as crisp values.

The paper unfolds as follows. Some basic knowledge and concepts on DEA and its models are deliberated in Section 2. In Section 3, some knowledge and arithmetic operations on neutrosophic sets are discussed. In Section 4, we propose a new model of neutrosophic DEA with undesirable outputs and establish an algorithm to solve it. In Section 5, the proposed model and the related algorithm are illustrated with a numerical example to ensure their validity and usefulness over the existing models. Finally, conclusions and future direction are offered in Section 6.

2. The Basic Concepts of DEA and its Models

DEA is a mathematical programming methodology that allows performance measurement of homogeneous DMUs that have several inputs and outputs. In DEA approach, there is no need to determine the specific form of the production function, and linear programming is needed to construct a piecewise linear surface (frontier) to cover all data, and then the efficiency of each of the DMUs is calculated from this frontier. The frontier obtained is the efficiency boundary, where the points on it are efficient, and other DMUs that are inside the cover surface are inefficient.

Let DMUO be under consideration; then, the production possibility set of the BCC model proposed by Banker et al. [3] is as follows:

$$T_{\mathrm{BCC}} = \left\{ (x, y) \,\middle|\, \sum_{q=1}^{n} \alpha_q x_q \le x, \sum_{q=1}^{n} \alpha_q y_q \ge y, \sum_{q=1}^{n} \alpha_q = 1, \alpha_q \ge 0, q = 1, \ldots, n \right\}. \tag{1}$$

Therefore, the BCC model is as follows:

$$\begin{aligned} \text{Min} \quad & \eta_o \\ \text{s.t.} \quad & \sum_{q=1}^{n} \alpha_q x_{pq} \le \eta_o x_{p_o}, \quad p = 1, \ldots, m \\ & \sum_{q=1}^{n} \alpha_j y_{sq} \ge y_{so}, \quad s = 1, \ldots, k \\ & \sum_{q=1}^{n} \alpha_q = 1, \quad \alpha_q \ge 0, \; q = 1, \ldots, n. \end{aligned} \tag{2}$$

For the BCC model with undesirable outputs, Guo and Wu [68] proposed

$$\begin{aligned} &\text{Min} \quad \eta_o \\ &\text{s.t} \quad \begin{cases} \sum_{q=1}^{n} \alpha_q x_{pq} \le \eta_o x_{p_\circ}, & p = 1,\ldots,m \\ \sum_{q=1}^{n} \alpha_j y^g_{sq} \ge y^g_{so}, & s = 1,\ldots,k \\ \sum_{q=1}^{n} \alpha_j y^b_{s'q} \le \eta_o y^b_{s'o}, & s' = 1,\ldots,k\prime \\ \sum_{q=1}^{n} \alpha_q = 1, & \alpha_q \ge 0,\ q = 1,\ldots,n. \end{cases} \end{aligned} \tag{3}$$

In Model (3), we assume that there are n DMUs that use inputs $x_{pq}\,(p = 1,\ldots,m)$, to achieve desirable outputs (good) $y^g_{sq}\,(s = 1,\ldots,k)$ and undesirable outputs (bad) $y^b_{s'q}\,(s' = 1,\ldots,k\prime)$ After calculation, if $\eta_o = 1$, then $\mathrm{DMU_o}$ is efficient; else, it is inefficient.

3. Neutrosophic Sets

Smarandache has given a number of actual examples for potential applications of NS; nevertheless, usage of NSs in real applied problems is difficult. Hence, NSs reduced into a type of SVNSs that will preserve the processes of the NSs. Here, we will discuss some basic definitions related to neutrosophic sets and SVNSs, respectively [30–33].

Definition 1. In universal U, a NS is distinct with truth, falsity, and indeterminacy membership functions of x in the real nonstandard $]^-0, 1^+[$, where the summation of them belong to [0, 3]. If these functions are singleton in the [0, 1], then a SVNS ψ is denoted by

$$\psi = \left\{\left(x,\ \hbar_\psi(x),\ \ell_\psi(x),\ \lambda\!\!\!^-_\psi(x)\right) \mid x \in U\right\}. \tag{4}$$

Furthermore, $\psi = \left\langle \hbar_\psi, \ell_\psi, \lambda\!\!\!^-_\psi \right\rangle$ is called the single-valued neutrosophic number (SVNN) if U has just one component.

Definition 2. Suppose that C and D be two SVNSs, then $C \subseteq D$ if and only if $\hbar_C(x) \le \hbar_D(x)$, $\ell_C(x) \ge \ell_D(x)$, and $\lambda\!\!\!^-_C(x) \ge \lambda\!\!\!^-_D(x)$.

Definition 3. LetMandNbe two SVNNs. Then, equations (5) to (8) are true:

$$\text{(i)}\ M \oplus N = \langle \hbar_M + \hbar_N - \hbar_M\hbar_N, \ell_M\ell_N, \lambda\!\!\!^-_M\lambda\!\!\!^-_N \rangle, \tag{5}$$

$$\text{(ii)}\ M \otimes N = \langle \hbar_M\hbar_N, \ell_M + \ell_N - \ell_M\ell_N, \lambda\!\!\!^-_M + \lambda\!\!\!^-_N - \lambda\!\!\!^-_M\lambda\!\!\!^-_N \rangle, \tag{6}$$

$$\text{(iii)}\ \alpha \mathrm{M} = \langle 1 - (1 - t\hbar_{\mathrm{M}} n(x))^\alpha, (\ell_{\mathrm{M}}(x))^\alpha, \cdot (\lambda\!\!\!^-_{\mathrm{M}}(x))^\alpha \rangle, \quad \alpha > 0, \tag{7}$$

$$\text{(iv)}\ \mathrm{M}^\alpha = \langle (\hbar_{\mathrm{M}}(x))^\alpha, 1 - (1 - \ell_{\mathrm{M}}(x))^\alpha, 1 - (1 - \lambda\!\!\!^-_{\mathrm{M}}(x))^\alpha \rangle, \quad \alpha > 0. \tag{8}$$

Definition 4. Suppose that $\mathrm{M}_q = \left\langle \hbar_{\mathrm{M}_q}, \ell_{\mathrm{M}_q}, \lambda\!\!\!^-_{\mathrm{M}_q} \right\rangle$ $(q = 1,\ldots,n)$ be a collection SVNNs. Then, the related weighted arithmetic average operator is

$$F_W(\mathrm{M}_1,\ldots,\mathrm{M}_n) = \sum_{q=1}^{n} \omega_q \mathrm{M}_q, \tag{9}$$

and the aggregation operator is defined as follows:

$$F_W(\mathrm{M}_1,\ldots,\mathrm{M}_n) = \left\langle 1 - \prod_{q=1}^{n}\left(1 - \hbar_{\mathrm{M}_q}(x)\right)^{\omega_q}, \prod_{q=1}^{n}\left(\ell_{\mathrm{M}_q}(x)\right)^{\omega_q}, \prod_{q=1}^{n}\left(\lambda\!\!\!^-_{\mathrm{M}_q}(x)\right)^{\omega_q} \right\rangle, \tag{10}$$

where $W = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weight vector of M_q,$\omega_q \in [0, 1]$ and $\sum_{q=1}^{n} \omega_q = 1$.

4. Proposed Model

In the DEA literature, there are some models of DEA with neutrosophic information, see [61–67]. Nevertheless, these models work solely for desirable outputs and cannot consider undesirable outputs. Hence, there is a requisite to develop a new model that keeps original advantage of neutrosophic DEA models and considers the influence of undesirable outputs. In this section, we propose an input-oriented BCC model with undesirable outputs in which all data are SVNNs. Furthermore, we propose an algorithm for solving the new DEA model. Moreover, a theorem regarding the feasibility and boundedness of the solution of the new model has been provided. For the qth DMU, consider the input $\tilde{x}_{pq} = \left\langle \hbar_{x_{pq}}, \ell_{x_{pq}}, \lambda\!\!\!^-_{x_{pq}} \right\rangle$, desirable outputs $\tilde{y}^g_{sq} = \left\langle \hbar_{y^g_{sq}}, \ell_{y^g_{sq}}, \lambda\!\!\!^-_{y^g_{sq}} \right\rangle$, and undesirable outputs $\tilde{y}^b_{sq} = \left\langle \hbar_{y^b_{sq}}, \ell_{y^b_{sq}}, \lambda\!\!\!^-_{y^b_{sq}} \right\rangle$ which are the SVNNs. Thus, by combinations of the BCC model under the single-valued neutrosophic environment with undesirable outputs, we have the following model:

$$\begin{aligned} &\text{Min} \quad \eta_o \\ &\text{s.t.} \quad \begin{cases} \sum_{q=1}^{n} \alpha_q \tilde{x}_{pq} \le \eta_o \tilde{x}_{p_\circ}, & p = 1,\ldots,m \\ \sum_{q=1}^{n} \alpha_j \tilde{y}^g_{sq} \ge \tilde{y}^g_{so}, & s = 1,\ldots,k \\ \sum_{q=1}^{n} \alpha_j \tilde{y}^b_{s'q} \le \eta_o \tilde{y}^b_{s'o}, & s' = 1,\ldots,k' \\ \sum_{q=1}^{n} \alpha_q = 1, & \alpha_q \ge 0, q = 1,\ldots,n. \end{cases} \end{aligned} \tag{11}$$

Step 1. Construct the problem based on Model (10).

Step 2. According to Definitions 3 and 4, transform the SVNBCC-UO model of Step 1 into

$$\begin{array}{ll}
\text{Min} & \eta_o \\
& \left\langle 1-\prod_{q=1}^{n}(1-\hbar_{x_{pq}})^{\alpha_q}, \prod_{q=1}^{n}(\ell_{x_{pq}})^{\alpha_q}, \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{x_{pq}})^{\alpha_q}\right\rangle \le \left\langle 1-(1-\hbar_{x_{po}})^{\eta_o}, (\ell_{x_{po}})^{\eta_o}, (\lambda\!\!\!^{-}_{x_{po}})^{\eta_o}\right\rangle, \quad p=1,\ldots,m \\
& \left\langle 1-\prod_{q=1}^{n}(1-\hbar_{y^g_{sq}})^{\alpha_q}, \prod_{q=1}^{n}(\ell_{y^g_{sq}})^{\alpha_q}, \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{y^g_{sq}})^{\alpha_q}\right\rangle \ge \left\langle \hbar_{y^g_{so}}, \ell_{y^g_{so}}, \lambda\!\!\!^{-}_{y^g_{so}}\right\rangle, \quad s=1,\ldots,k \\
\text{S.t.} & \left\langle 1-\prod_{q=1}^{n}(1-\hbar_{y^b_{s'q}})^{\alpha_q}, \prod_{q=1}^{n}(\ell_{y^b_{s'q}})^{\alpha_q}, \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{y^b_{s'q}})^{\alpha_q}\right\rangle \le \left\langle 1-(1-\hbar_{y^b_{s'o}})^{\eta_o}, (\ell_{y^b_{s'o}})^{\eta_o}, (\lambda\!\!\!^{-}_{y^b_{s'o}})^{\eta_o}\right\rangle, \quad s'=1,\ldots,k' \\
& \sum_{q=1}^{n}\alpha_q = 1 \\
& \alpha_q \ge 0, \quad q=1,\ldots,n.
\end{array}$$

Step 3. By Definition 2, transform Step 2 into

$$\begin{array}{ll}
\text{Min} & \eta_o \\
& \prod_{q=1}^{n}(1-\hbar_{x_{pq}})^{\alpha_q} \ge (1-\hbar_{x_{po}})^{\eta_o}, \quad p=1,2,\ldots,m \\
& \prod_{q=1}^{n}(\ell_{x_{pq}})^{\alpha_q} \ge (\ell_{x_{po}})^{\eta_o}, \quad p=1,2,\ldots,m \\
& \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{x_{pq}})^{\alpha_q} \ge (\lambda\!\!\!^{-}_{x_{po}})^{\eta_o}, \quad p=1,2,\ldots m \\
& \prod_{q=1}^{n}(1-\hbar_{y^g_{sq}})^{\alpha_q} \le (1-\hbar_{y^g_{so}}), \quad s=1,\ldots,k \\
& \prod_{q=1}^{n}(\ell_{y^g_{sq}})^{\alpha_q} \le \ell_{y^g_{so}}, \quad s=1,\ldots,k \\
\text{s.t.} & \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{y^g_{sq}})^{\alpha_q} \le \lambda\!\!\!^{-}_{y^g_{so}}, \quad s=1,\ldots,k \\
& \prod_{q=1}^{n}(1-\hbar_{y^b_{s'q}})^{\alpha_q} \ge (1-\hbar_{y^b_{s'o}})^{\eta_o}, \quad s'=1,\ldots,k' \\
& \prod_{q=1}^{n}(\ell_{y^b_{s'q}})^{\alpha_q} \ge (\ell_{y^b_{s'o}})^{\eta_o}, \quad s'=1,\ldots,k' \\
& \prod_{q=1}^{n}(\lambda\!\!\!^{-}_{y^b_{s'q}})^{\alpha_q} \ge (\lambda\!\!\!^{-}_{y^b_{s'o}})^{\eta_o}, \quad s'=1,\ldots,k' \\
& \sum_{q=1}^{n}\alpha_q = 1, \\
& \alpha_q \ge 0, \quad q=1,\ldots,n.
\end{array}$$

Step 4. Convert Step 3 into the following linear model:

$$\begin{array}{ll}
\text{Min} & \eta_o \\
& \sum_{q=1}^{n}\alpha_q \ln(1-\hbar_{x_{pq}}) \ge \eta_o \ln(1-\hbar_{x_{po}}), \quad p=1,2,\ldots,m \\
& \sum_{q=1}^{n}\alpha_q \ln(\ell_{x_{pq}}) \ge \eta_o \ln(\ell_{x_{po}}), \quad p=1,2,\ldots,m \\
& \sum_{q=1}^{n}\alpha_q \ln(\lambda\!\!\!^{-}_{x_{pq}}) \ge \eta_o \ln(\lambda\!\!\!^{-}_{x_{po}}), \quad p=1,2,\ldots,m \\
& \sum_{q=1}^{n}\alpha_q \ln(1-\hbar_{y^g_{sq}}) \le \ln(1-\hbar_{y^g_{so}}), \quad s=1,\ldots,k \\
& \sum_{q=1}^{n}\alpha_q \ln(\ell_{y^g_{sq}}) \le \ln(\ell_{y^g_{so}}), \quad s=1,\ldots,k \\
\text{s.t} & \sum_{q=1}^{n}\alpha_q \ln(\lambda\!\!\!^{-}_{y^g_{sq}}) \le \ln(\lambda\!\!\!^{-}_{y^g_{so}}), \quad s=1,\ldots,k \\
& \sum_{q=1}^{n}\alpha_j \ln(1-\hbar_{y^b_{s'q}}) \ge \eta_o \ln(1-\hbar_{y^b_{s'o}}), \quad s'=1,\ldots,k' \\
& \sum_{q=1}^{n}\alpha_j \ln(\ell_{y^b_{s'q}}) \ge \eta_o \ln(\ell_{y^b_{s'o}}), \quad s'=1,\ldots,k' \\
& \sum_{q=1}^{n}\alpha_j \ln(\lambda\!\!\!^{-}_{y^b_{s'q}}) \ge \eta_o \ln(\lambda\!\!\!^{-}_{y^b_{s'o}}), \quad s'=1,\ldots,k' \\
& \sum_{q=1}^{n}\alpha_q = 1 \\
& \alpha_q \ge 0, \quad q=1,\ldots,n.
\end{array}$$

Step 5. Using Step 4, get the optimal efficiency of each DMU.

ALGORITHM 1: Solution of the SVNBCC-UO model.

We called this model as single-valued neutrosophic BCC with undesirable outputs (SVNBCC-UO). Now, we present an algorithm to solve Model (11).

In the following, we show that our model is feasible and bounded.

Theorem 1. *The model of SVNBCC-UO is feasible and bounded. Furthermore, its optimal objective function is 1.*

Proof. From Algorithm 1, we can transform the model SVNBCC-UO into Step 4 in Algorithm 1. So, with the solution $\alpha_q = \begin{cases} 1, & q=o, \\ 0, & else, \end{cases}$ and $\eta_o = 1$, it is easy to see that the Step 4 in Algorithm 1 is always feasible. Thus, regardless of the values of inputs and outputs, there is always at least one feasible solution for Step 4 in Algorithm 1. Because the above solution is feasible along with the objective function of Step 4

Table 1: Input information of the 15 DMUs.

DMUS	Input 1	Input 2	Input 3
1	⟨0.6, 0.4, 0.3⟩	⟨0.5, 0.4, 0.3⟩	⟨0.7, 0.3, 0.4⟩
2	⟨0.3, 0.1, 0.2⟩	⟨0.6, 0.5, 0.1⟩	⟨0.6, 0.4, 0.5⟩
3	⟨0.4, 0.2, 0.1⟩	⟨0.2, 0.6, 0.3⟩	⟨0.8, 0.2, 0.1⟩
4	⟨0.5, 0.5, 0.2⟩	⟨0.2, 0.3, 0.1⟩	⟨0.5, 0.2, 0.2⟩
5	⟨0.3, 0.2, 0.4⟩	⟨0.2, 0.8, 0.7⟩	⟨0.7, 0.2, 0.7⟩
6	⟨0.5, 0.2, 0.2⟩	⟨0.7, 0.5, 0.1⟩	⟨0.7, 0.3, 0.2⟩
7	⟨0.7, 0.1, 0.2⟩	⟨0.4, 0.4, 0.2⟩	⟨0.7, 0.3, 0.2⟩
8	⟨0.2, 0.6, 0.5⟩	⟨0.1, 0.7, 0.2⟩	⟨0.3, 0.9, 0.6⟩
9	⟨0.6, 0. 3, 0.3⟩	⟨0.5, 0.2, 0.5⟩	⟨0.9, 0.2, 0.3⟩
10	⟨0.4, 0.7, 0.7⟩	⟨0.8, 0.2, 0.6⟩	⟨0.6, 0.2, 0.3⟩
11	⟨0.7, 0.3, 0.3⟩	⟨0.4, 0.1, 0.3⟩	⟨0.5, 0.1, 0.4⟩
12	⟨0.6, 0.3, 0.5⟩	⟨0.5, 0.5, 0.3⟩	⟨0.6, 0., 0.3⟩
13	⟨0.5, 0.2, 0.4⟩	⟨0.9, 0.3, 0.3⟩	⟨0.8, 0.1, 0.4⟩
14	⟨0.4, 0.3, 0.1⟩	⟨0.8, 0.4, 0.1⟩	⟨0.9, 0.2, 0.3⟩
15	⟨0.6, 0.4, 0.3⟩	⟨0.5, 0.1, 0.1⟩	⟨0.6, 0.1, 0.1⟩

Table 2: Output information of the 15 DMUs.

DMUS	Output 1(good)	Output 2 (good)	Output 3 (bad)
1	⟨0.5, 0.5, 0.7⟩	⟨0.5, 0.4, 0.4⟩	⟨0.5, 0.2, 0.1⟩
2	⟨0.2, 0.7, 0.5⟩	⟨0.7, 0.2, 0.5⟩	⟨0.4, 0.1, 0.1⟩
3	⟨0.3, 0.5, 0.8⟩	⟨0.6, 0.7, 0.5⟩	⟨0.6, 0.2, 0.1⟩
4	⟨0.2, 0.5, 0.7⟩	⟨0.3, 0.5, 0.6⟩	⟨0.5, 0.2, 0.1⟩
5	⟨0.8, 0.4, 0.3⟩	⟨0.8, 0.5, 0.1⟩	⟨0.3, 0.1, 0.6⟩
6	⟨0.3, 0.6, 0.7⟩	⟨0.1, 0.6, 0.7⟩	⟨0.8, 0.1, 0.2⟩
7	⟨0.5, 0.2, 0.6⟩	⟨0.5, 0.5, 0.4⟩	⟨0.9, 0.2, 0.3⟩
8	⟨0.3, 0.3, 0.6⟩	⟨0.7, 0.4, 0.7⟩	⟨0.3, 0.3, 0.3⟩
9	⟨0.6, 0.4, 0.4⟩	⟨0.5, 0.5, 0.5⟩	⟨0.4, 0.2, 0.1⟩
10	⟨0.2, 0.3, 0.7⟩	⟨0.5, 0.3, 0.3⟩	⟨0.5, 0.4, 0.1⟩
11	⟨0.3, 0.8, 0.5⟩	⟨0.2, 0.7, 0.6⟩	⟨0.5, 0.2, 0.2⟩
12	⟨0.6, 0.1, 0.4⟩	⟨0.2, 0.2, 0.3⟩	⟨0.1, 0.5, 0.5⟩
13	⟨0.4, 0.7, 0.8⟩	⟨0.4, 0.8, 0.6⟩	⟨0.7, 0.1, 0.1⟩
14	⟨0.5, 0.5, 0.6⟩	⟨0.5, 0.3, 0.5⟩	⟨0.5, 0.2, 0.4⟩
15	⟨0.2, 0.4, 0.8⟩	⟨0.7, 0.4, 0.6⟩	⟨0.4, 0.1, 0.1⟩

Table 3: The efficiencies of the DMUs.

DMUS	Efficiency (η^*)	Rank
DMU1	0.8329	6
DMU2	0.7504	8
DMU3	0.9451	4
DMU4	0.8197	7
DMU5	1.0000	**1**
DMU6	0.5822	12
DMU7	0.7430	9
DMU8	1.0000	**1**
DMU9	0.9821	2
DMU10	1.0000	**1**
DMU11	0.9463	3
DMU12	1.0000	**1**
DMU13	0.7270	10
DMU14	0.8987	5
DMU15	0.6549	11

in Algorithm 1 is minimization, the best value regarding the objective function is certainly lower than or equal to 1. □

5. Numerical Experiment

In this section, the proposed SVNBCC-UO and the related algorithm are illustrated with a numerical example to ensure their validity and usefulness over the existing models.

Example 1. Consider an efficiency problem with 15 DMUs, three inputs, two desirable outputs, and one undesirable output that all information are presented as SVNNs (see Tables 1 and 2).

Here, we used Algorithm 1 for solving the SVNBCC-UO model. Suppose that DMU_1 is under consideration, so based on Algorithm 1 we have the following.

First, we construct the model using Steps 1–3. Then, by means of Step 4, we construct the *Min* η_1 with the constraints of Step 4 in Algorithm 1. For example, by Step 4 in Algorithm 1 for Input 1, we have the following relation:

$$\begin{aligned}
&\alpha_1 \ln(1-0.6) + \alpha_2 \ln(1-0.3) + \alpha_3 \ln(1-0.4) + \alpha_4 \ln(1-0.5) \\
&+\alpha_5 \ln(1-0.3) + \alpha_6 \ln(1-0.5) + \alpha_7 \ln(1-0.7) + \alpha_8 \ln(1-0.2) \\
&+\alpha_9 \ln(1-0.6) + \alpha_{10} \ln(1-0.4) + \alpha_{11} \ln(1-0.7) + \alpha_{12} \ln(1-0.6) \\
&+\alpha_{13} \ln(1-0.5) + \alpha_{14} \ln(1-0.4) + \alpha_{15} \ln(1-0.6) \ge \eta_1 \ln(1-0.6).
\end{aligned} \tag{12}$$

So, after calculations with Matlab, we get $\eta_1^* = 0.9068$ for DMU_1. Correspondingly, the technical efficiencies of each $\mathrm{DMU_i}$ are measured using the proposed algorithm and are presented in Table 3.

To validate the presented efficiencies, these efficiencies were compared with the efficiencies obtained by the existing neutrosophic DEA methods and are given in Figure 1. In this figure, we can see the impact of undesirable outputs on the performance of DMUs. From Figure 1, we can see that the efficiencies of DMUs are found to be smaller for SVNBCC-UO compared to the existing neutrosophic DEA methods. It is interesting that DMUs 1–4, 6–7, and 9 are efficient in the existing neutrosophic DEA methods; however, they are actually inefficient with the efficiency scores of 0.8329, 0.7504, 0.9451, 0.8197, 0.5822, 0.7430, and 0.9821 using

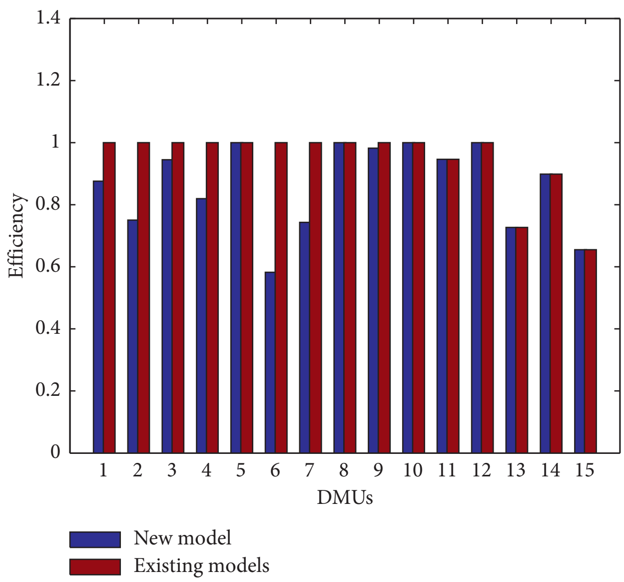

Figure 1: Comparison of the new model and the existing models.

SVNBCC-UO, respectively. Therefore, SVNBCC-UO is more *realistic* than the existing neutrosophic DEA methods. Therefore, if we solve the mentioned problem with the existing methods, we cannot get to a reliable evaluation.

6. Conclusions and Future Work

This paper, established a new strategy to solve a neutrosophic data envelopment analysis model with undesirable outputs. In comparison with the existing neutrosophic DEA methods, the significant characteristic of the new DEA method is that it can handle the undesirable outputs simply and effectively. A numerical experiment and comparison results with the existing models have been demonstrated to display the competence of the presented method. The proposed methodology has created hopeful consequences from computing and performance facets. It is worth mentioning that the uncertainty, ambiguity, and indeterminacy in this paper are limited to single-valued neutrosophic numbers. Nevertheless, the other types of numbers such as bipolar NSs and interval-valued neutrosophic numbers, Pythagorean fuzzy set, and q-rung orthopair fuzzy set can also be used to indicate variables characterizing the core in world-wide problems. As for future research, we intend to extend the proposed approach to these kinds of tools.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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