A New Approach for Optimization of Real Life Transportation Problem in Neutrosophic Environment

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Abstract:

Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets and intuitionistic fuzzy sets to represent uncertain, inconsistent and incomplete information about a real world problem. For the first time, this paper attempts to introduce the mathematical representation of a transportation problem in neutrosophic environment. The necessity of the model is discussed. A new method for solving transportation problem with indeterminate and inconsistent information is proposed briefly. A real life example is given to illustrate the efficiency of the proposed method in neutrosophic approach.

1. Introduction

In the present day, problems are there with different types of uncertainties which cannot be solved by classical theory of Mathematics. To deal the problems with imprecise or vague information, Zadeh [1] first introduced the fuzzy set theory in 1965, which is characterized by its membership values. But in many situations, the results or decisions based on the available information are not enough to the level of accuracy. So, several higher order fuzzy sets were introduced to deal such problems. One was the concept of intuitionistic fuzzy set introduced by Atanassov [2] in 1986. Intuitionistic fuzzy sets are suitable to handle problems with imprecision information and are characterized by its membership and non membership values [3]. Hence, both the theories of fuzzy and intuitionistic fuzzy sets were applied in many real life decision making problems.

In due course, any generalization of fuzzy set failed to handle problems with indeterminate or inconsistent information. To overcome this, Smarandache [4], in 1998, introduced neutrosophic sets as an extension of classical sets, fuzzy sets, and intuitionistic fuzzy sets. The components of neutrosophic set namely truth-membership degree, indeterminacy- membership degree, and falsity- membership degree were suitable to represent indeterminacy and inconsistent information. Wang and Smarandache [5] introduced the idea of single valued neutrosophic set in many practical problems. The notion of single valued neutrosophic set was more suitable for solving many real life problems like image processing, medical diagnosis, decision making, water resource management and supply chain management.

Study of optimal transportation model with cost effective manner played a predominant role in supply chain management. Many researchers [6, 7] formulated the mathematical model for transportation problem in various environments. The basic transportation model was introduced by Hitchcock [8] in 1941, in which the transportation constraints were based on crisp values. But in the present world, the transportation parameters like demand, supply and unit transportation cost may be uncertain due to several uncontrolled factors. In this situation, fuzzy transportation problem was formulated and solved by many researchers.

Though, many researchers worked on transportation problem in fuzzy environment, it was Oheigeartaigh [9] who proposed a new method to solve fuzzy transportation problem with triangular fuzzy demand and supply. Chanas and Kutcha [10] proposed a method for optimal solution of fuzzy transportation problem with fuzzy cost coefficients. Jimenez and Verdegay [11] investigated the solution of fuzzy solid transportation problem in which the transportation parameters are trapezoidal fuzzy numbers. Kaur and Kumar[12] proposed a method for solving transportation problem using ranking function. Gani and Razak [13] introduced a two stage cost minimization method for solving fuzzy transportation problem. Pandian and

Natarajan [14] introduced a new method namely, fuzzy zero point method to find the optimal solution for fuzzy transportation problem. Kaur and Kumar [15] solved the fuzzy transportation problem with generalized trapezoidal fuzzy numbers.

Sometimes the membership function in fuzzy set theory was not a suitable one to describe an ambiguous situation of a problem. So, in 1986 Atanassov [2] introduced the concept of intuitionistic fuzzy set theory as an extension of fuzzy set theory, which included both the degree of membership and non membership of each element in the set. In recent research, intuitionistic fuzzy set theory plays an important role in decision making problems [16, 17]. Many researchers [18, 19] used intuitionistic fuzzy approach to solve transportation problems.

In a supply chain optimization, transportation system was the most important economic activity among all the components of business logistics system. Apart from the vagueness or uncertainty in the constraints of the present day transportation model, there exists some indeterminacy due to various factors like unawareness of the problem, imperfection in data and poor forecasting. Intuitionistic fuzzy set theory can handle incomplete information but not indeterminate and inconsistent information. Smarandache [20] proposed a new theory namely neutrosophic logic by adding another independent membership function named as indeterminacy – membership $\mathrm{I}(x)$ along with truth membership $\mathrm{T}(x)$ and falsity $\mathrm{F}(x)$ membership functions. Neutrosophic set is a generalization of intuitionistic fuzzy sets. If hesitancy degree $\mathrm{H}(x)$ of intuitionistic fuzzy set and the indeterminacy – membership degree $\mathrm{I}(x)$ of neutrosophic set are equal, then neutrosophic set will become the intuitionistic fuzzy set.

Eventhough many scholars applied the notion of neutrosophic theories in multi attribute decision making problems [21, 22, 23] to the best of knowledge, the existing supply chain theories of transportation model do not view in neutrosophic logic. For example, in a given conclusion “The total transportation cost of delivering the goods would be 1000 units”, the supplier cannot conclude immediately that, the precise cost is exactly 1000 units. There may be some neutral part, which is neither truthfulness nor falsity of the statement. This is very closer to our human mind reasoning. In the neutral part, there may be some indeterminacy in deciding unit transportation cost, demand and supply units due to various causes like vehicle routing, road factors, no uniformity in traffic regulations, delivery time of goods, poor demand forecasting, demand mismatches, price fluctuations , lack of trust and so on.

The aim of this paper is to obtain the optimal transportation cost in neutrosophic environment. This paper is well organized as follows. In section 2, the basic concepts of fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets are briefly reviewed. In section 3, the mathematical model of neutrosophic transportation problem is introduced. In section 4, the solution algorithms are developed for solving neutrosophic transportation problem. In section 5, the algorithms are illustrated with suitable real life problems. In section 6, the results are interpreted. Finally, section 7 concludes the paper with future work.

2. Preliminaries

Definition 1 (Fuzzy Set) [1]

Let X be a nonempty set. A fuzzy set $\overline{\mathrm{A}}$ of X is defined as $\overline{\mathrm{A}} = \left\{\langle \mathrm{x}, \mu_{\overline{\mathrm{A}}}(\mathrm{x})\rangle /\ \mathrm{x} \in \mathrm{X}\right\}$ where $\mu_{\overline{\mathrm{A}}}(\mathrm{x})$ is called the membership function which maps each element of X to a value between 0 and 1.

Definition 2 (Fuzzy Number)

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line $\mathbb{R}$ such that:

- There exist atleast one $x \in \mathbb{R}$ with $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3 (Trapezoidal Fuzzy Number) [15]

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number where $a_1, a_2, a_3\ and\ a_4$ are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

Definition 4 (Intuitionistic Fuzzy Set) [2]

Let X be a nonempty set. An intuitionistic fuzzy set $\bar{A}^I$ of X is defined as $\bar{A}^I = \{\langle x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x)\rangle / x \in X\}$ where $\mu_{\bar{A}^I}(x)$ and $\nu_{\bar{A}^I}(x)$ are membership and non membership functions such that $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x): X \to [0,1]$ and $0 \le \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \le 1$ for all $x \in X$.

Definition 5 (Intuitionistic Fuzzy Number) [3]

An intuitionistic fuzzy subset $\tilde{A}^I = \left\{\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)\rangle / x \in \mathbb{R}\right\}$ of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number (IFN) if the following conditions hold:

i. There exists $m \in \mathbb{R}$ such that $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(m) = 0$
ii. $\mu_{\tilde{A}^I}$ is a continuous function from $\mathbb{R} \to [0,1]$ such that $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$ for all $x \in X$.
iii. The membership and non membership functions of $\tilde{A}^I$ are in the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0, & for -\infty < x \le a_1 \\ f(x), & for\ a_1 \le x \le a_2 \\ 1, & for x = a_2 \\ g(x), & for\ a_2 \le x \le a_3 \\ 0, & for\ a_3 \le x < \infty \end{cases} \qquad \nu_{\tilde{A}^I}(x) = \begin{cases} 1, & for -\infty < x \le a_1{}' \\ f'(x), & for\ a_1{}' \le x \le a_2 \\ 0, & for\ x = a_2 \\ g'(x), & for a_2 \le x \le a_3{}' \\ 1, & for\ a_3{}' \le x < \infty \end{cases}$$

where f, f', g, g' are functions from $\mathbb{R} \to [0,1]$, f and g' are strictly increasing functions and g and f' are strictly decreasing functions with the conditions $0 \le f(x) + f'(x) \le 1$ and $0 \le g(x) + g'(x) \le 1$.

Definition 6 (Trapezoidal Intuitionistic Fuzzy Number) [16]

A trapezoidal Intuitionistic fuzzy number is denoted by $\tilde{A}^I = (a_1, a_2, a_3, a_4), (a_1{}', a_2, a_3, a_4{}')$ where $a_1{}' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4{}'$ with membership and non membership functions are defined as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases} \qquad \nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1{}'}, & for\ a_1{}' \le x \le a_2 \\ 0, & for\ a_2 \le x \le a_3 \\ \dfrac{x-a_3}{a_4{}'-x}, & for\ a_3 \le x \le a_4{}' \\ 1, & otherwise \end{cases}$$

Definition 7 (Neutrosophic Set) [4]

Let X be a nonempty set. Then a neutrosophic set $\bar{A}^N$ of X is defined as $\bar{A}^N = \{\langle x, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x)\rangle / x \in X, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) \in]^-0, 1^+[\}$ where $T_{\bar{A}^N}(x)$, $I_{\bar{A}^N}(x)$ and $F_{\bar{A}^N}(x)$ are truth membership function, a indeterminacy- membership function and a falsity-membership function and there is no restriction on the sum of $T_{\bar{A}^N}(x)$, $I_{\bar{A}^N}(x)$ and $F_{\bar{A}^N}(x)$, so $^-0 \le T_{\bar{A}^N}(x) + I_{\bar{A}^N}(x) + F_{\bar{A}^N}(x) \le 3^+$ and $]^-0, 1^+[$ is a non-standard unit interval.

But it is difficult to apply neutrosophic set theories in real life problems directly. So Wang introduced single valued neutrosophic set as a subset of neutrosophic set and the definition is as follows.

Definition 8 (Single Valued Neutrosophic Set) [5]

Let X be a nonempty set. Then a single valued neutrosophic set $\overline{A}_S^N$ of X is defined as $\overline{A}_S^N = \{\langle x, T_{\overline{A}^N}(x), I_{\overline{A}^N}(x), F_{\overline{A}^N}(x)\rangle / x \in X\}$ where $T_{\overline{A}^N}(x)$, $I_{\overline{A}^N}(x)$ and $F_{\overline{A}^N}(x) \in [0,1]$ for each $x \in X$ and $0 \leq T_{\overline{A}^N}(x) + I_{\overline{A}^N}(x) + F_{\overline{A}^N}(x) \leq 3$.

Definition 9 (Single Valued Trapezoidal Neutrosophic Number)

Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a special neutrosophic set on the real **line set** $\mathbb{R}$, whose truth- membership, indeterminacy-membership and falsity-membership functions are given as follows.

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}}\left(\dfrac{x-a_1}{a_2-a_1}\right), & for\ a_1 \leq x \leq a_2 \\ w_{\tilde{a}}, & for\ a_2 \leq x \leq a_3 \\ w_{\tilde{a}}\left(\dfrac{a_4-x}{a_4-a_3}\right), & for\ a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+u_{\tilde{a}}(x-a_1)}{a_2-a_1}, & for\ a_1 \leq x \leq a_2 \\ u_{\tilde{a}}, & for\ a_2 \leq x \leq a_3 \\ \frac{x-a_3+u_{\tilde{a}}(a_4-x)}{a_4-a_3}, & for\ a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$ and

$$\lambda_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2-x+y_{\tilde{a}}(x-a_1)}{a_2-a_1}, & for\ a_1 \leq x \leq a_2 \\ y_{\tilde{a}}, & for\ a_2 \leq x \leq a_3 \\ \dfrac{x-a_3+y_{\tilde{a}}(a_4-x)}{a_4-a_3}, & for\ a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

Where $w_{\tilde{a}}, u_{\tilde{a}}$ and $y_{\tilde{a}}$ denote the maximum truth-membership degree, minimum-indeterminacy membership degree and minimum falsity- membership degree respectively. A single valued trapezoidal neutrosophic number $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ may express an ill defined quantity about a, which is approximately equal to $[a_2, a_3]$.

Definition 10 (Arithmetic Operations on Single Valued Trapezoidal Neutrosophic Numbers)

Let $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $k \neq 0$ then

i) $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$

ii) $\tilde{a} - \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$

iii) $$\tilde{a}\tilde{b} = \begin{cases} \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle\ if\ a_4 > 0, b_4 > 0 \\ \langle(a_1b_4, a_2b_3, a_3b_2, a_4b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle\ if\ a_4 < 0, b_4 > 0 \\ \langle(a_4b_4, a_3b_3, a_2b_2, a_1b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle\ if\ a_4 < 0, b_4 < 0 \end{cases}$$

iv) $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1 \,/\, b_4, a_2 \,/\, b_3, a_3 \,/\, b_2, a_4 \,/\, b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle \; if \; a_4 > 0, b_4 > 0 \\ \langle (a_4/b_4, a_3 \,/\, b_3, a_2 \,/\, b_2, a_1 \,/\, b_1); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle \; if \; a_4 < 0, b_4 > 0 \\ \langle (a_4 \,/\, b_1, a_3 \,/\, b_2, a_2 \,/\, b_3, a_1/b_4); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle \; if \; a_4 < 0, b_4 < 0 \end{cases}$

v) $k\tilde{a} = \begin{cases} \langle (ka_1, ka_2, ka_3, ka_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \; if \; k > 0 \\ \langle (ka_4, ka_3, ka_2, ka_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \; if \; k < 0 \end{cases}$

vi) $\tilde{a}^{-1} = \langle \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ where $\tilde{a} \neq 0$

Definition 11 (Score and Accuracy Functions of Single Valued Trapezoidal Neutrosophic Number)

One can compare any two single valued trapezoidal neutrosophic numbers based on the score and accuracy functions. Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_a, u_a, y_a \rangle$ be a single valued trapezoidal neutrosophic number, then

i) Score function $S(\tilde{a}) = \frac{1}{16}[a_1 + a_2 + a_3 + a_4] \times [\mu_{\tilde{a}} + (1 - \nu_{\tilde{a}}) + (1 - \lambda_{\tilde{a}})]$ and

ii) Accuracy function $A(\tilde{a}) = \frac{1}{16}[a_1 + a_2 + a_3 + a_4] \times [\mu_{\tilde{a}} + (1 - \nu_{\tilde{a}}) + (1 + \lambda_{\tilde{a}})]$

Definition 12 (Comparison of Single Valued Trapezoidal Neutrosophic Number)

Let $\tilde{a}$ and $\tilde{b}$ be any two single valued trapezoidal neutrosophic numbers, then

i) If $S(\tilde{a}) < S(\tilde{b})$ $then$ $\tilde{a} < \tilde{b}$

ii) If $S(\tilde{a}) = S(\tilde{b})$ and if

1. $A(\tilde{a}) < A(\tilde{b})$ then $\tilde{a} < \tilde{b}$
2. $A(\tilde{a}) > A(\tilde{b})$ then $\tilde{a} > \tilde{b}$
3. $A(\tilde{a}) = A(\tilde{b})$ then $\tilde{a} = \tilde{b}$

Example

Let $\tilde{a} = \langle (4,8,10,16); 0.5,0.3,0.6 \rangle$ and $\tilde{b} = \langle (3,7,11,14); 0.4,0.5,0.0.6 \rangle$ be two single valued trapezoidal neutrosophic numbers, then

i) $\tilde{a} + \tilde{b} = \langle 7,15,21,30; 0.4,0.5,0.6 \rangle$

ii) $\tilde{a} - \tilde{b} = \langle -10, -3,3,13; 0.4,0.5,0.6 \rangle$

iii) $\tilde{a}\tilde{b}$=$\langle 12,56,110,224; 0.4,0.5,0.6 \rangle$

iv) $\tilde{a}/\tilde{b} = \langle \frac{4}{14}, \frac{8}{11}, \frac{10}{7}, \frac{16}{3}; 0.4,0.5,0.6 \rangle$

v) $3\tilde{a} = \langle 12,24,30,48; .4,0.5,0.6 \rangle$

vi) $\tilde{a}^{-1} = \langle \frac{1}{16}, \frac{1}{10}, \frac{1}{8}, \frac{1}{4}; 0.4,0.5,0.6 \rangle$

vii) $S(\tilde{a}) = \frac{1}{16}[4 + 8 + 10 + 16] \times [0.5 + (1 - 0.3) + (1 - 0.6)] = 3.8$

viii) $A(\tilde{a}) = \frac{1}{16}[4 + 8 + 10 + 16] \times [0.5 + (1 - 0.3) + (1 + 0.6)] = 6.65$

3. Introduction of Transportation Problem in Neutrosophic Environment

3.1. Mathematical Formulation

3.1.1. Model I

In this model, a transportation problem is introduced in a single valued neutrosophic environment. Consider a transportation problem with ‘m’ sources and ‘n’ destinations in which the decision maker is indeterminate about the precise values of transportation cost from i^{th} source to j^{th} destination, but there is no uncertainty about the demand and supply of the product with the following assumptions and constraints.

Distribution Assumptions:
i is the source index for all $i = 1, 2,3, \dots, \mathrm{m}$
j is the destination index for all $j = 1, 2,3, \dots, \mathrm{n}$

Transportation parameters:
x_{ij} is the number of units of the product transported from i^{th} source to j^{th} destination.
$\tilde{\mathrm{C}}_{\mathrm{ij}}^{\mathrm{N}}$ is the neutrosophic cost of one unit quantity transported from i^{th} source to j^{th} destination.
x_{i} is the total availability of the product at the source i.
x_{j} is the total demand of the product at the destination j.

Transportation constraints:

Supply constraints: $\sum_{\mathrm{j=0}}^{\mathrm{n}} x_{\mathrm{ij}} = a_{\mathrm{i}}$ for all sources i.

Demand constraints: $\sum_{\mathrm{i=0}}^{\mathrm{m}} x_{\mathrm{ij}} = b_{\mathrm{j}}$ for all destinations j and $\sum_{\mathrm{i=0}}^{\mathrm{m}} a_{\mathrm{i}} = \sum_{\mathrm{j=0}}^{\mathrm{n}} b_{\mathrm{j}}$.

Non negativity constraints: $x_{\mathrm{ij}} \geq 0 \ \forall\, i, j$.

Now the mathematical formulation of the problem is given by,

$$\text{Minimize } \tilde{\mathrm{Z}}^{\mathrm{N}} = \sum_{\mathrm{i=0}}^{\mathrm{m}} \sum_{\mathrm{j=0}}^{\mathrm{n}} x_{\mathrm{ij}}\, \tilde{\mathrm{c}}_{\mathrm{ij}}^{\mathrm{N}}$$

$$\text{subject to } \sum_{\mathrm{j=0}}^{\mathrm{n}} x_{\mathrm{ij}} = a_{\mathrm{i}},\ \mathrm{i} = 1,2, \dots, \mathrm{m}$$

$$\sum_{\mathrm{i=0}}^{\mathrm{m}} x_{\mathrm{ij}} = b_{\mathrm{j}},\ \mathrm{j} = 1,2, \dots, \mathrm{n} \quad \text{and } x_{\mathrm{ij}} \geq 0 \ \forall\, i, j.$$

3.1.2. Model II

In this model the decision maker will not be sure about the unit transportation costs, supply and the demand units. So the mathematical formulation of the problem becomes,

$$\text{Minimize } \tilde{Z}^{N} = \sum_{i=0}^{m} \sum_{j=0}^{n} \tilde{x}_{ij}^{N}\, \tilde{c}_{ij}^{N}$$

$$\text{subject to } \sum_{j=0}^{n} \tilde{x}_{ij}^{N} = \tilde{a}_{i}^{N},\ i = 1,2, \dots, m$$

$$\sum_{i=0}^{m} \tilde{x}_{ij}^{N} = \tilde{b}_{j}^{N},\ j = 1,2, \dots, n \quad \text{and } \tilde{x}_{ij}^{N} \geq 0 \ \forall i, j.$$

4. Procedure for Proposed Algorithms Based on Neutrosophic Numbers

4.1. Basic Assumptions of the proposed Algorithms:

1. Requirement Assumption: The entire supply units from each source must be distributed to destinations.
2. Feasible Solution Assumption: The neutrosophic transportation problem will have feasible solution if and only if $\sum_{\mathrm{i=0}}^{\mathrm{m}} a_{\mathrm{i}} = \sum_{\mathrm{j=0}}^{\mathrm{n}} b_{\mathrm{j}}$, $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.
3. Cost Assumption: The total transportation cost depends only on the number of units transported and the unit transportation cost but not on other factors like distance and mode of transport.
4. Input Assumption: The parameters of the problem will be represented by either crisp or trapezoidal neutrosophic numbers.

4.2. Neutrosophic Initial Basic Feasible Solution for Model I

Step 1: Calculate the score value of each neutrosophic cost $\tilde{c}_{ij}^{N}$ and replace all the neutrosophic costs by its score value to obtain the classical transportation problem.
Step 2: For each row and column of the table obtained in Step 1, Calculate the difference between minimum and next to minimum of the transportation costs and denote it as Penalty.

Step 3: In the row/column, corresponding to maximum penalty, make the maximum allotment in the cell having the minimum transportation cost.
Step 4: If the maximum penalty corresponding to more than one row or column , select the top most row and the extreme left column.
Repeat the above procedure until all the supplies are fully exhausted and all the demands are satisfied.

4.3. Neutrosophic Optimal Solution for Model I

Step 1: Convert each neutrosophic cost into crisp value by the score function and obtain the classical transportation problem.
Step 2: Choose the minimum in each row and subtract it from the corresponding row entries. Do the same procedure for each column. Now there will be atleast one zero in each row and column in the resultant table.
Step 3: Verify whether, the demand of each column is less than the sum of supplies whose reduced costs are zero in that column and supply of each row is less than the sum of demands whose reduced costs in that row are zero. If so, go to Step 5, otherwise go to Step 4.
Step 4: Draw the minimum number of horizontal and vertical lines that cover all the zeros in the reduced table and revise the table as follows:

(i) Find the least element from the uncovered entries
(ii) Subtract it from all the uncovered entries and add it to the entries at the intersection of any two lines.

Again check the condition at Step 3.
Step 5: Select a cell (x, y) such that whose reduced cost is maximum in the reduced cost table. If the maximum exists at more than one cell, then select any one.
Step 6: Select a cell in x - row and y - column, which is the only cell whose reduced cost is zero. And then allot the maximum possible units in it. If such cell does not occur for the maximum cost, go for next maximum. If such cell does not occur for any value, then choose any cell at random whose reduced cost is zero.
Step 7: Revise the reduced table by omitting fully exhausted row and fully satisfied column and repeat Step 5 and Step 6 again.
Repeat the procedure until all the supply units are fully used and all the demand units are fully received.

4.4. Neutrosophic Solution for Model II:

Initial basic feasible solution and the optimum solution of Model II will be obtained by the procedure in the above algorithms 4.1 and 4.2 without altering the neutrosophic demand and supply units. Subtraction of single valued trapezoidal neutrosophic numbers is applied to modify the neutrosophic demand and supply units in each iteration.

5. Illustrative Example

5.1. Model I

Consider a transportation problem in which the peanuts are initially stored at three sources namely O1, O2, O3 and are transported to peanut butter manufacturing company located at four different destinations namely D1, D2, D3 and D4 with trapezoidal neutrosophic unit transportation cost and crisp demand and supply as given in Table 1. Obtain the optimal transportation of peanuts to minimize the total transportation cost.

TABLE 1: Input data for Neutrosophic Transportation Problem

	D1	D2	D3	D4	Supply

O1	(3,5,6,8); 0.6,0.5,0.4	(5,8,10,14); 0.3,0.6,0.6	(12,15,19,22); 0.6,0.4,0.5	(14,17,21,28); 0.8,0.2.0.6	26
O2	(0,1,3,6); 0.7,0.5,0.3	(5,7,9,11); 0.9,0.7,0.5	(15,17,19,22); 0.4,0.8,0.4	(9,11,14,16); 0.5,0.4,0.7	24
O3	(4,8,11,15); 0.6,0.3,0.2	(1,3,4,6); 0.6,0.3,0.5	(5,7,8,10); 0.5,0.4,0.7	(5,9,14,19); 0.3,0.7,0.6	30
Demand	17	23	28	12	

5.1.1. Neutrosophic Initial Basic Feasible Solution for Model I

Now by score function of trapezoidal neutrosophic number, calculate the score value of each neutrosophic cost to obtain the crisp transportation problem. (Here score values are rounded off to the nearest integer). The results are given in Table 2. Then calculate the penalty for each row and each column which is presented in Table 3.

TABLE 2: Crisp Transportation Problem

	D1	D2	D3	D4	Supply
O1	2	3	7	10	26
O2	1	3	6	4	24
O3	5	2	3	3	30
Demand	17	23	28	12	

TABLE 3: Tabular Representation with Penalties

	D1	D2	D3	D4	Supply	Penalty
O1	2	3	7	10	26	1
O2	1	3	6	4	24	2
O3	5	2	3	3	30	1
Demand	17	23	28	12		
Penalty	1	1	3*	1		

In the above Table 3, the highest penalty 3(marked with *) occurs at column 3. Now allot the maximum possible units 28 in the minimum cost cell (3,3) and revise the supply units corresponding to row 3. Then the penalties are to be revised in Table 4.

TABLE 4: First Allotment with Penalties

	D1	D2	D3	D4	Supply	Penalty
O1	2	3	-	10	26	1
O2	1	3	-	4	24	2
O3	5	2	**28**	3	2	1
Demand	17	23	-	12		
Penalty	1	1	-	1		

Proceeding the neutrosophic initial basic feasible solution algorithm and after few iterations we get the complete allotment transportation units as given in Table 5.

Table 5: Table with Complete Allotment

	D1	D2	D3	D4	Supply
O1	-	**23**	-	**3**	-
O2	**17**	-	-	**7**	-
O3	-	-	**28**	**2**	-
Demand	-	-	-	-	

The initial basic feasible solution is,

$x_{12} = 23, x_{14} = 3,\ x_{21} = 17,\ x_{24} = 7,\ x_{33} = 28,\ x_{34} = 2$

Hence the minimum total neutrosophic cost is,

Minimize $\tilde{Z}^{N} = \sum_{i=0}^{3}\sum_{j=0}^{4} x_{ij}\ \tilde{c}_{ij}^{N}$

$= 23(5,8,10,14);0.3,0.6,0.6 + 3\ (14,17,21,28);0.8,0.2.0.6 +$

$17\ (0,1,3,6);0.7,0.5,0.3 + 7\ (9,11,14,16);0.5,0.4,0.7 +$

$28\ (5,7,8,10);0.5,0.4,0.7 + 2(5,9,14,19);0.3,0.7,0.6$

$= (370,543,694,938);\ 0.3,0.7,0.7$

5.1.2 Neutrosophic Optimum Solution for Model I

Consider the neutrosophic optimal solution for the transportation problem given in Table 1. After applying the steps 1 and 2 of the optimal solution algorithm we obtain Table 6.

TABLE 6: Table with zero Point

	D1	D2	D3	D4	Supply
O1	0	1	4	7	26
O2	0	2	4	2	24
O3	3	0	0	0	30
Demand	17	23	28	12	

Now the Table 6 does not satisfy the optimal solution condition stated in Step 3 of the algorithm 4.2. So proceed to Step 4, and get the revised costs as given in Table 7.

TABLE 7: Modified Table with zero Point

	D1	D2	D3	D4	Supply
O1	0	0	3	6	26
O2	0	1	3	1	24
O3	4	0	0	0	30
Demand	17	23	28	12	

As per allotment rules given in Step 5 to Step 7, one can get the complete allotment schedule which is presented in Table 8.

TABLE 8: Table with complete allocation

	D1	D2	D3	D4	Supply
O1	**3**	**23**			-

O2	**14**			**10**	-
O3			**28**	**2**	-
Demand	-	-	-	-	

The optimal solution is

$x_{11} = 3,\ x_{12} = 23,\ x_{21} = 14,\ x_{24} = 10,\ x_{33} = 28,\ x_{34} = 2$

Hence the minimum total neutrosophic cost is

Minimize $\tilde{Z}^{N} = \sum_{i=0}^{3}\sum_{j=0}^{4} x_{ij}\, \tilde{c}_{ij}^{N} = (364,537,682,908); 0.3,0.7,0.7$

= 3(3,5,6,8);0.6,0.5,0.4 + 23(5,8,10,14);0.3,0.6,0.6 + 14(0,1,3,6);0.7,0.5,0.3 + 10(9,11,14,16);0.5,0.4,0.7 + 28(5,7,8,10);0.5,0.4,0.7 + 2(5,9,14,19);0.3,0.7,0.6

= (364,537,682,908); 0.3,0.7,0.7

5.2. Model II

Consider a problem for Model II with single valued neutrosophic trapezoidal cost, demand and supply given in Table 9.

TABLE 9: Input data for Neutrosophic Transportation Problem

	D1	D2	D3	D4	Supply
O1	(3,5,6,8); 0.6,0.5,0.4	(5,8,10,14); 0.3,0.6,0.6	(12,15,19,22); 0.6,0.4,0.5	(14,17,21,28); 0.8,0.2.0.6	(22,26,28,32); 0.7,0.3,0.4
O2	(0,1,3,6); 0.7,0.5,0.3	(5,7,9,11); 0.9,0.7,0.5	(15,17,19,22); 0.4,0.8,0.4	(9,11,14,16); 0.5,0.4,0.7	(17,22,27,31); 0.6,0.4,0.5
O3	(4,8,11,15) ;0.6,0.3,0.2	(1,3,4,6); 0.6,0.3,0.5	(5,7,8,10); 0.5,0.4,0.7	(5,9,14,19); 0.3,0.7,0.6	(21,28,32,37); 0.8,0.2,0.4
Demand	(13,16,18,21); 0.5,0.5,0.6	(17,21,24,28); 0.8,0.2,0.4	(24,29,32,35); 0.9,0.5,0.3	(6,10,13,15); 0.7,0.3,0.4	

For the above Table 9, calculate the score value of each neutrosophic cost to get crisp cost and consider the demand and supply units as it is. It is presented in Table 10.

TABLE 10: Neutrosophic Transportation Problem with Crisp Cost

	D1	D2	D3	D4	Supply
O1	2	3	7	10	(22,26,28,32); 0.7,0.3,0.4
O2	1	3	6	4	(17,22,27,31); 0.6,0.4,0.5
O3	5	2	3	3	(21,28,32,37); 0.8,0.2,0.4
Demand	(13,16,18,21); 0.5,0.5,0.6	(17,21,24,28); 0.8,0.2,0.4	(24,29,32,35); 0.9,0.5,0.3	(6,10,13,15); 0.7,0.3,0.4	

Here, the arithmetic operations of single valued neutrosophic trapezoidal numbers are applied to modify the neutrosophic demand and supply in each iteration. Proceed the neutrosophic optimal solution method and after few iterations the optimal solution in terms of single valued neutrosophic trapezoidal numbers is obtained as follows:

$x_{11} = (-6,2,7,15); 0.7,0.3.0.4,\quad x_{12} = (17,21,24,28); 0.8,0.2,0.4,\quad x_{21} = (-8,6,19,33); 0.6,0.5,0.5$

$x_{24} = (-2,8,16,25); 0.7,0.5,0.4,\quad x_{33} = (26,29,32,35); 0.9,0.5,0.3,\quad x_{34} = (-6,0,6,14); 0.8,0.5,0.4$

Minimize $\tilde{Z}^{N} = \sum_{i=0}^{3}\sum_{j=0}^{4} \tilde{x}_{ij}^{N}\, \tilde{c}_{ij}^{N} = (149,475,903,1726); 0.3,0.7,0.7$

= (3,5,6,8);0.6,0.5,0.4 × (-6,2,7,15);0.7,0.3,0.4 + (5,8,10,14);0.3,0.6,0.6 × 17,21,24,28);0.8,0.2,0.4 + (0,1,3,6);0.7,0.5,0.3 × (-8,6,19,33);0.6,0.5,0.5 +(9,11,14,16);0.5,0.4,0.7 × (-2,8,16,25);0.7,0.5,0.4+ (5,7,8,10);0.5,0.4,0.7 × (26,29,32,35);0.9,0.5,0.3 + (5,9,14,19);0.3,0.7,0.6 × (-6,0.6,14);0.8,0.5,0.4

= (149, 475, 903, 1726); 0.3, 0.7, 0.7

6. Results and Discussions

In the example 5.1, the neutrosophic optimum solution is better than the neutrosophic initial basic feasible solution $(370,543,694,938); 0.3,0.7,0.7$. In the optimum solution, the total minimum transportation cost will be greater than 364 and less than 908. And for the total minimum transportation cost lies between 537 to 682, the overall level of acceptance or satisfaction or the truthfulness is 30%. Also for the remaining values of total minimum transportation cost, the degree of truthfulness is $\mu(x) \times 100$ where bx denotes the total cost and $\mu(x)$ and is given by,

$$\mu(x) = \begin{cases} 0.3\left(\dfrac{x-364}{537-364}\right), & for\ \ 364 \le x \le 537 \\ 0.3, & for\ 537 \le x \le 682 \\ 0.3\left(\dfrac{908-x}{908-682}\right), & for\ \ 682 \le x \le 908 \\ 0, & otherwise \end{cases}$$

In the optimum solution, the degrees of indeterminacy and falsity are same. Hence, degree of indeterminacy and falsity for the minimum transportation cost are

$$\nu(x) = \begin{cases} \dfrac{537-x+0.7(x-364)}{537-364}, & for\ \ 364 \le x \le 537 \\ 0.7, & for\ 537 \le x \le 682 \\ \dfrac{x-682+0.7(908-x)}{908-682}, & for\ \ 682 \le x \le 908 \\ 1, & otherwise \end{cases}$$

$$\lambda(x) = \begin{cases} \dfrac{537-x+0.7(x-364)}{537-364}, & for\ \ 364 \le x \le 537 \\ 0.7, & for\ 537 \le x \le 682 \\ \dfrac{x-682+0.7(908-x)}{908-682}, & for\ \ 682 \le x \le 908 \\ 1, & otherwise \end{cases}$$

respectively. Hence, a decision maker can conclude that the total neutrosophic cost from the range 364 to 908, with its truth degree, indeterminacy degree and falsity degree. Based on the above result, he may schedule the transportation and budget constraints.

7. Conclusion

Neutrosophic sets being a generalization of intuitionistic fuzzy sets provide an additional possibility to represent the indeterminacy along with the uncertainty. Though, there are many transportation problems that have been studied with different types of input data, this research has investigated the solutions of transportation problems in neutrosophic environment. Two different models in neutrosophic environment were considered in the study. The arithmetic operations on single valued neutrosophic trapezoidal numbers are employed to find the solutions. The solution procedures are illustrated with day today problems. Though, the proposed algorithms concretely analyses the solutions of neutrosophic transportation problems, they are some limitations in predicting the solutions of qualitative and complex data. The computational complexity in handling higher dimensional problems will be overcome by genetic algorithm approach. In future, the research will be extended to deal multiobjective solid transportation problems in environment. The researchers will be interested to overcome the above stated limitations. Further, the approaches of transportation problems on fuzzy and intuitionistic fuzzy logic may be extended to neutrosophic logic.

Conflict of Interests

The two authors mentioned that there is no conflict of ideas regarding the publication of the paper.

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