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Research Article

A New Approach to Operations on Neutrosophic Soft Sets and to Neutrosophic Soft Topological Spaces

Taha Yasin Ozturk[1,*], Cigdem Gunduz Aras[2] and Sadi Bayramov[3]

[1] *Department of Mathematics, Kafkas University, Kars 36100, Turkey*
[2] *Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey*
[3] *Department of Algebra and Geometry, Baku State University, Baku, AZ 1148, Azerbaican*
*__Corresponding author:__ taha36100@kafkas.edu.tr

Abstract. In this study, we re-define some operations on neutrosophic soft sets differently from the studies [3, 9]. On this operations are given interesting examples and them basic properties. In the direction of these newly defined operations, we construct the neutrosophic soft topological spaces differently from the study [3]. Finally, we introduce basic definitions and theorems on neutrosophic soft topological spaces.

Keywords. Neutrosophic soft set; Neutrosophic soft topological space; Neutrosophic soft interior; Neutrosophic soft closure

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1. Introduction

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The treories presented differently from classical methods in studies such as fuzzy set [15], intuitionistic set [7], soft set [11], neutrosophic set [14], etc. Have great importance in this contribution of mathematics in recent years. Many works have been done on these sets by mathematicians in many areas of mathematics [2–6, 8, 12, 13]. In addition, many studies on different combination of these set theories have been presented [1, 9, 10]. One of them is Neurosophic soft sets [14]. Neutrosophic soft topological spaces was presented by Bera in his work [3].

In our study, the intersection, union, AND, OR and difference operations are re-defined on the neutrosophic soft sets in contrast to the studies [3, 9], and the properties related to these operations are presented. Then, considering these newly defined processes, unlike [3], neutrosophic soft topology is reconstructed. In addition, relations between the spaces neutrosophic soft topology, fuzzy soft topology and fuzzy topology are observed. Finally, by defining interior and closure operations, fundamental theorems for neutrosophic soft topological spaces are proved and some examples on the subject are given.

In the preliminaries section, we give fundamental information for the study. In the next section, the operations of union, intersection, difference, AND, OR on neutrosophic soft sets are redefined and their properties are investigated. Then, the next section studies the neutrosophic soft topology and their notions based on these redefined operations. Finally, we provide a conclusion section about these new concepts for our paper.

2. Preliminaries

In this section, we will give some preliminary information for the present study.

Definition 1 ([14])**.** A neutrosophic set A on the universe of discourse X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where T, I, $F : X \to]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2 ([11])**.** Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. A pair (F,E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set (F,E), or as the set of e-approximate elements of the soft set, i.e.,

$$(F,E) = \{(e, F(e)) : e \in E, F : E \to P(X)\}.$$

Firstly, neutrosophic soft set defined by Maji [9] and later this concept has been modified by Deli and Bromi [8] as given below:

Definition 3. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $(\widetilde{F},E)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$ where $\widetilde{F}$ is called approximate function of the neutrosophic soft set $(\widetilde{F},E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,

$$(\widetilde{F},E) = \{(e, \langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\rangle : x \in X) : e \in E\},$$

where $T_{\widetilde{F}(e)}(x)$, $I_{\widetilde{F}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \in [0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{F}(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \le T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \le 3$ is obvious.

Definition 4 ([3])**.** Let $(\widetilde{F},E)$ be neutrosophic soft set over the universe set X. The complement of $(\widetilde{F},E)$ is denoted by $(\widetilde{F},E)^c$ and is defined by:

$$(\widetilde{F},E)^c = \{(e, \langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x)\rangle : x \in X) : e \in E\}.$$

Obvious that, $((\widetilde{F},E)^c)^c = (\widetilde{F},E)$.

Definition 5 ([9])**.** Let $(\widetilde{F},E)$ and $(\widetilde{G},E)$ be two neutrosophic soft sets over the universe set X. $(\widetilde{F},E)$ is said to be neutrosophic soft subset of $(\widetilde{G},E)$ if $T_{\widetilde{F}(e)}(x) \le T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \le I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \ge F_{\widetilde{G}(e)}(x)$, for all $e \in E$, for all $x \in X$. It is denoted by $(\widetilde{F},E) \subseteq (\widetilde{G},E)$.

$(\widetilde{F},E)$ is said to be neutrosophic soft equal to $(\widetilde{G},E)$ if $(\widetilde{F},E)$ is neutrosophic soft subset of $(\widetilde{G},E)$ and $(\widetilde{G},E)$ is neutrosophic soft subset of $(\widetilde{F},E)$. It is denoted by $(\widetilde{F},E) = (\widetilde{G},E)$.

3. A New Approach to Operations on Neutrosophic Soft Sets

In this section, the operations of union, intersection, difference, AND, OR on neutrosophic soft sets are defined differently from the studies [3, 9]. In addition, basic properties of these operations will be presented.

Definition 6. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then their union is denoted by $(\widetilde{F}_1,E) \cup (\widetilde{F}_2,E) = (\widetilde{F}_3,E)$ and is defined by:

$$(\widetilde{F}_3,E) = \{(e, \langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\rangle : x \in X) : e \in E\},$$

where

$$T_{\widetilde{F}_3(e)}(x) = \max\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\},$$
$$I_{\widetilde{F}_3(e)}(x) = \max\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\},$$
$$F_{\widetilde{F}_3(e)}(x) = \min\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\}.$$

Definition 7. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E) = (\widetilde{F}_3,E)$ and is defined by:

$$(\widetilde{F}_3,E) = \{(e, \langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\rangle : x \in X) : e \in E\},$$

where

$$T_{\widetilde{F}_3(e)}(x) = \min\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\},$$
$$I_{\widetilde{F}_3(e)}(x) = \min\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\},$$
$$F_{\widetilde{F}_3(e)}(x) = \max\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\}.$$

Definition 8. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then "$(\widetilde{F}_1,E)$ difference $(\widetilde{F}_2,E)$" operation on them is denoted by $(\widetilde{F}_1,E) \backslash (\widetilde{F}_2,E) = (\widetilde{F}_3,E)$ and is defined by $(\widetilde{F}_3,E)=(\widetilde{F}_1,E)\cap(\widetilde{F}_2,E)^c$ as follows:

$$(\widetilde{F}_3,E)=\{(e,\langle x,T_{\widetilde{F}_3(e)}(x),I_{\widetilde{F}_3(e)}(x),F_{\widetilde{F}_3(e)}(x)\rangle : x\in X):e\in E\},$$

where

$$T_{\widetilde{F}_3(e)}(x)=\min\{T_{\widetilde{F}_1(e)}(x),F_{\widetilde{F}_2(e)}(x)\},$$
$$I_{\widetilde{F}_3(e)}(x)=\min\{I_{\widetilde{F}_1(e)}(x),1-I_{\widetilde{F}_2(e)}(x)\},$$
$$F_{\widetilde{F}_3(e)}(x)=\max\{F_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_2(e)}(x)\}.$$

Definition 9. Let $\{(\widetilde{F}_i,E)|i\in I\}$ be a family of neutrosophic soft sets over the universe set X. Then

$$\bigcup_{i\in I}(\widetilde{F}_i,E)=\{(e,\langle x,\sup[T_{\widetilde{F}_i(e)}(x)]_{i\in I},\sup[I_{\widetilde{F}_i(e)}(x)]_{i\in I},\inf[F_{\widetilde{F}_i(e)}(x)]_{i\in I}\rangle : x\in X):e\in E\},$$
$$\bigcap_{i\in I}(\widetilde{F}_i,E)=\{(e,\langle x,\inf[T_{\widetilde{F}_i(e)}(x)]_{i\in I},\inf[I_{\widetilde{F}_i(e)}(x)]_{i\in I},\sup[F_{\widetilde{F}_i(e)}(x)]_{i\in I}\rangle : x\in X):e\in E\}.$$

Definition 10. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then "AND" operation on them is denoted by $(\widetilde{F}_1,E)\wedge(\widetilde{F}_2,E)=(\widetilde{F}_3,E\times E)$ and is defined by:

$$(\widetilde{F}_3,E\times E)=\{((e_1,e_2),\langle x,T_{\widetilde{F}_3(e_1,e_2)}(x),I_{\widetilde{F}_3(e_1,e_2)}(x),F_{\widetilde{F}_3(e_1,e_2)}(x)\rangle : x\in X):(e_1,e_2)\in E\times E\},$$

where

$$T_{\widetilde{F}_3(e_1,e_2)}(x)=\min\{T_{\widetilde{F}_1(e_1)}(x),T_{\widetilde{F}_2(e_2)}(x)\},$$
$$I_{\widetilde{F}_3(e_1,e_2)}(x)=\min\{I_{\widetilde{F}_1(e_1)}(x),I_{\widetilde{F}_2(e_2)}(x)\},$$
$$F_{\widetilde{F}_3(e_1,e_2)}(x)=\max\{F_{\widetilde{F}_1(e_1)}(x),F_{\widetilde{F}_2(e_2)}(x)\}.$$

Definition 11. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then "OR" operation on them is denoted by $(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E)=(\widetilde{F}_3,E\times E)$ and is defined by:

$$(\widetilde{F}_3,E\times E)=\{((e_1,e_2),\langle x,T_{\widetilde{F}_3(e_1,e_2)}(x),I_{\widetilde{F}_3(e_1,e_2)}(x),F_{\widetilde{F}_3(e_1,e_2)}(x)\rangle : x\in X):(e_1,e_2)\in E\times E\},$$

where

$$T_{\widetilde{F}_3(e_1,e_2)}(x)=\max\{T_{\widetilde{F}_1(e_1)}(x),T_{\widetilde{F}_2(e_2)}(x)\},$$
$$I_{\widetilde{F}_3(e_1,e_2)}(x)=\max\{I_{\widetilde{F}_1(e_1)}(x),I_{\widetilde{F}_2(e_2)}(x)\},$$
$$F_{\widetilde{F}_3(e_1,e_2)}(x)=\min\{F_{\widetilde{F}_1(e_1)}(x),F_{\widetilde{F}_2(e_2)}(x)\}.$$

Definition 12.

1. A neutrosophic soft set $(\widetilde{F},E)$ over the universe set X is said to be null neutrosophic soft set if $T_{\widetilde{F}(e)}(x)=0$, $I_{\widetilde{F}(e)}(x)=0$, $F_{\widetilde{F}(e)}(x)=1$; for all $e\in E$, for all $x\in X$. It is denoted by $0_{(X,E)}$.
2. A neutrosophic soft set $(\widetilde{F},E)$ over the universe set X is said to be absolute neutrosophic soft set if $T_{\widetilde{F}(e)}(x)=1$, $I_{\widetilde{F}(e)}(x)=1$, $F_{\widetilde{F}(e)}(x)=0$; for all $e\in E$, for all $x\in X$. It is denoted by $1_{(X,E)}$.

Clearly, $0^c_{(X,E)}=1_{(X,E)}$ and $1^c_{(X,E)}=0_{(X,E)}$.

Proposition 1. *Let $(\widetilde{F}_1,E)$, $(\widetilde{F}_2,E)$ and $(\widetilde{F}_3,E)$ be neutrosophic soft sets over the universe set X. Then,*

1. $(\widetilde{F}_1,E)\cup[(\widetilde{F}_2,E)\cup(\widetilde{F}_3,E)]=[(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)]\cup(\widetilde{F}_3,E)$ *and* $(\widetilde{F}_1,E)\cap[(\widetilde{F}_2,E)\cap(\widetilde{F}_3,E)]=[(\widetilde{F}_1,E)\cap(\widetilde{F}_2,E)]\cap(\widetilde{F}_3,E)$;
2. $(\widetilde{F}_1,E)\cup[(\widetilde{F}_2,E)\cap(\widetilde{F}_3,E)]=[(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)]\cap[(\widetilde{F}_1,E)\cup(\widetilde{F}_3,E)]$ *and* $(\widetilde{F}_1,E)\cap[(\widetilde{F}_2,E)\cup(\widetilde{F}_3,E)]=[(\widetilde{F}_1,E)\cap(\widetilde{F}_2,E)]\cup[(\widetilde{F}_1,E)\cap(\widetilde{F}_3,E)]$;
3. $(\widetilde{F}_1,E)\cup 0_{(X,E)}=(\widetilde{F}_1,E)$ *and* $(\widetilde{F}_1,E)\cap 0_{(X,E)}=0_{(X,E)}$;
4. $(\widetilde{F}_1,E)\cup 1_{(X,E)}=1_{(X,E)}$ *and* $(\widetilde{F}_1,E)\cap 1_{(X,E)}=(\widetilde{F}_1,E)$.

Proof. Straightforward. □

Proposition 2. *Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then,*

1. $[(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)]^c=(\widetilde{F}_1,E)^c\cap(\widetilde{F}_2,E)^c$;
2. $[(\widetilde{F}_1,E)\cap(\widetilde{F}_2,E)]^c=(\widetilde{F}_1,E)^c\cup(\widetilde{F}_2,E)^c$.

Proof. 1. For all $e\in E$ and $x\in X$,

$$(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)=\{\langle x,\max\{T_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_2(e)}(x)\},\max\{I_{\widetilde{F}_1(e)}(x),I_{\widetilde{F}_2(e)}(x)\},\min\{F_{\widetilde{F}_1(e)}(x),F_{\widetilde{F}_2(e)}(x)\}\rangle\}$$
$$[(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)]^c=\{\langle x,\min\{F_{\widetilde{F}_1(e)}(x),F_{\widetilde{F}_2(e)}(x)\},1-\max\{I_{\widetilde{F}_1(e)}(x),I_{\widetilde{F}_2(e)}(x)\},\max\{T_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_2(e)}(x)\}\rangle\}.$$

Now,

$$(\widetilde{F}_1,E)^c=\{\langle x,F_{\widetilde{F}_1(e)}(x),1-I_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_1(e)}(x)\rangle\},$$
$$(\widetilde{F}_2,E)^c=\{\langle x,F_{\widetilde{F}_2(e)}(x),1-I_{\widetilde{F}_2(e)}(x),T_{\widetilde{F}_2(e)}(x)\rangle\}.$$

Then,

$$\begin{aligned}(\widetilde{F}_1,E)^c\cap(\widetilde{F}_2,E)^c&=\{\langle x,\min\{F_{\widetilde{F}_1(e)}(x),F_{\widetilde{F}_2(e)}(x)\},\min\{(1-I_{\widetilde{F}_1(e)}(x)),(1-I_{\widetilde{F}_2(e)}(x))\},\max\{T_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_2(e)}(x)\}\rangle\}\\&=\{\langle x,\min\{F_{\widetilde{F}_1(e)}(x),F_{\widetilde{F}_2(e)}(x)\},1-\max\{I_{\widetilde{F}_1(e)}(x),I_{\widetilde{F}_2(e)}(x)\},\max\{T_{\widetilde{F}_1(e)}(x),T_{\widetilde{F}_2(e)}(x)\}\rangle\}.\end{aligned}$$

Therefore, $[(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E)]^c=(\widetilde{F}_1,E)^c\cap(\widetilde{F}_2,E)^c$.

2. It is obtained in a similar way. □

Proposition 3. *Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ be two neutrosophic soft sets over the universe set X. Then,*

1. $[(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E)]^c=(\widetilde{F}_1,E)^c\wedge(\widetilde{F}_2,E)^c$;
2. $[(\widetilde{F}_1,E)\wedge(\widetilde{F}_2,E)]^c=(\widetilde{F}_1,E)^c\vee(\widetilde{F}_2,E)^c$.

Proof. 1. For all $(e_1,e_2)\in E\times E$ and $x\in X$,

$$(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E)=\{\langle x,\max\{T_{\widetilde{F}_1(e_1)}(x),T_{\widetilde{F}_2(e_2)}(x)\},\max\{I_{\widetilde{F}_1(e_1)}(x),I_{\widetilde{F}_2(e_2)}(x)\},\min\{F_{\widetilde{F}_1(e_1)}(x),F_{\widetilde{F}_2(e_2)}(x)\}\rangle\},$$
$$[(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E)]^c=\{\langle x,\min\{F_{\widetilde{F}_1(e_1)}(x),F_{\widetilde{F}_2(e_2)}(x)\},1-\max\{I_{\widetilde{F}_1(e_1)}(x),I_{\widetilde{F}_2(e_2)}(x)\},\max\{T_{\widetilde{F}_1(e_1)}(x),T_{\widetilde{F}_2(e_2)}(x)\}\rangle\}.$$

On the other hand,

$$(\widetilde{F}_1,E)^c=\{\langle x,F_{\widetilde{F}_1(e_1)}(x),1-I_{\widetilde{F}_1(e_1)}(x),T_{\widetilde{F}_1(e_1)}(x)\rangle:e_1\in E\},$$

$$(\widetilde{F}_2,E)^c = \{\langle x, F_{\widetilde{F}_{2(e_2)}}(x), 1-I_{\widetilde{F}_{2(e_2)}}(x), T_{\widetilde{F}_{2(e_2)}}(x)\rangle : e_2 \in E\}.$$

Then,

$$\begin{aligned}
&(\widetilde{F}_1,E)^c \wedge (\widetilde{F}_2,E)^c \\
&= \{\langle x, \min\{F_{\widetilde{F}_{1(e_1)}}(x), F_{\widetilde{F}_{2(e_2)}}(x)\}, \min\{(1-I_{\widetilde{F}_{1(e_1)}}(x)),(1-I_{\widetilde{F}_{2(e_2)}}(x))\}, \max\{T_{\widetilde{F}_{1(e_1)}}(x), T_{\widetilde{F}_{2(e_2)}}(x)\}\rangle\} \\
&= \{\langle x, \min\{F_{\widetilde{F}_{1(e_1)}}(x), F_{\widetilde{F}_{2(e_2)}}(x)\}, 1-\max\{I_{\widetilde{F}_{1(e_1)}}(x), I_{\widetilde{F}_{2(e_2)}}(x)\}, \max\{T_{\widetilde{F}_{1(e_1)}}(x), T_{\widetilde{F}_{2(e_2)}}(x)\}\rangle\}.
\end{aligned}$$

Hence, $[(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E)]^c = (\widetilde{F}_1,E)^c \wedge (\widetilde{F}_2,E)^c$. □

Example 1. Suppose that, the universe set X given by $X = \{x_1,x_2,x_3,x_4\}$ and the set of parameters $E = \{e_1,e_2\}$. Let us consider neutrosophic soft sets $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ over the universe set X as follows:

$$(\widetilde{F}_1,E) = \left\{\begin{array}{l} e_1 = \{\langle x_1,0.3,0.7,0.6\rangle, \langle x_2,0.4,0.3,0.8, \langle x_3,0.6,0.4,0.5\rangle, \langle x_4,0.2,0.5,0.4\rangle\}, \\ e_2 = \{\langle x_1,0.4,0.6,0.8\rangle, \langle x_2,0.3,0.7,0.2\rangle, \langle x_3,0.3,0.3,0.7\rangle, \langle x_4,0.1,0.4,0.9\rangle\} \end{array}\right\},$$

$$(\widetilde{F}_2,E) = \left\{\begin{array}{l} e_1 = \{\langle x_1,0.6,0.6,0.8\rangle, \langle x_2,0.2,0.9,0.3\rangle, \langle x_3,0.1,0.2,0.4\rangle, \langle x_4,0.5,0.4,0.3\rangle\}, \\ e_2 = \{\langle x_1,0.7,0.9,0.5\rangle, \langle x_2,0.4,0.2,0.3\rangle, \langle x_3,0.5,0.5,0.4\rangle, \langle x_4,0.4,0.3,0.6\rangle\} \end{array}\right\}.$$

Then,

$$(\widetilde{F}_1,E)\cup(\widetilde{F}_2,E) = \left\{\begin{array}{l} e_1 = \{\langle x_1,0.6,0.7,0.6\rangle, \langle x_2,0.4,0.9,0.3\rangle, \langle x_3,0.6,0.4,0.4\rangle, \langle x_4,0.5,0.5,0.3\rangle\}, \\ e_2 = \{\langle x_1,0.7,0.9,0.5\rangle, \langle x_2,0.4,0.7,0.2\rangle, \langle x_3,0.5,0.5,0.4\rangle, \langle x_4,0.4,0.4,0.6\rangle\} \end{array}\right\},$$

$$(\widetilde{F}_1,E)\cap(\widetilde{F}_2,E) = \left\{\begin{array}{l} e_1 = \{\langle x_1,0.3,0.6,0.8\rangle, \langle x_2,0.2,0.3,0.8\rangle, \langle x_3,0.1,0.2,0.5\rangle, \langle x_4,0.2,0.4,0.4\rangle\}, \\ e_2 = \{\langle x_1,0.4,0.6,0.8\rangle, \langle x_2,0.3,0.2,0.3\rangle, \langle x_3,0.3,0.3,0.7\rangle, \langle x_4,0.1,0.3,0.9\rangle\} \end{array}\right\},$$

$$(\widetilde{F}_1,E)\backslash(\widetilde{F}_2,E) = \left\{\begin{array}{l} e_1 = \{\langle x_1,0.3,0.4,0.6\rangle, \langle x_2,0.3,0.1,0.8\rangle, \langle x_3,0.4,0.4,0.5\rangle, \langle x_4,0.2,0.5,0.5\rangle\}, \\ e_2 = \{\langle x_1,0.4,0.1,0.8\rangle, \langle x_2,0.3,0.7,0.4\rangle, \langle x_3,0.3,0.3,0.7\rangle, \langle x_4,0.1,0.4,0.9\rangle\} \end{array}\right\}$$

$$(\widetilde{F}_1,E)\wedge(\widetilde{F}_2,E) = \left\{\begin{array}{l} (e_1,e_1) = \{\langle x_1,0.3,0.6,0.8\rangle, \langle x_2,0.2,0.3,0.8\rangle, \langle x_3,0.1,0.2,0.5\rangle, \langle x_4,0.2,0.4,0.4\rangle\}, \\ (e_1,e_2) = \{\langle x_1,0.3,0.7,0.6\rangle, \langle x_2,0.4,0.2,0.8\rangle, \langle x_3,0.5,0.4,0.5\rangle, \langle x_4,0.2,0.3,0.6\rangle\}, \\ (e_2,e_1) = \{\langle x_1,0.4,0.6,0.8\rangle, \langle x_2,0.2,0.7,0.3\rangle, \langle x_3,0.1,0.2,0.7\rangle, \langle x_4,0.1,0.4,0.9\rangle\}, \\ (e_2,e_2) = \{\langle x_1,0.4,0.6,0.8\rangle, \langle x_2,0.3,0.2,0.3\rangle, \langle x_3,0.3,0.3,0.7\rangle, \langle x_4,0.1,0.3,0.9\rangle\} \end{array}\right\},$$

$$(\widetilde{F}_1,E)\vee(\widetilde{F}_2,E) = \left\{\begin{array}{l} (e_1,e_1) = \{\langle x_1,0.6,0.7,0.6 rangle, \langle x_2,0.4,0.9,0.3\rangle, \langle x_3,0.6,0.4,0.4\rangle, \langle x_4,0.5,0.5,0.3\rangle\}, \\ (e_1,e_2) = \{\langle x_1,0.7,0.9,0.5\rangle, \langle x_2,0.4,0.3,0.3\rangle, \langle x_3,0.6,0.5,0.4\rangle, \langle x_4,0.4,0.5,0.4\rangle\}, \\ (e_2,e_1) = \{\langle x_1,0.6,0.6,0.8\rangle, \langle x_2,0.3,0.9,0.2\rangle, \langle x_3,0.3,0.3,0.4\rangle, \langle x_4,0.5,0.4,0.3\rangle\}, \\ (e_2,e_2) = \{\langle x_1,0.7,0.9,0.5\rangle, \langle x_2,0.4,0.7,0.2\rangle, \langle x_3,0.5,0.5,0.4\rangle, \langle x_4,0.4,0.4,0.6\rangle\} \end{array}\right\}.$$

4. Neutrosophic Soft Topological Spaces

In this section, the neutrosophic soft topology based on the redefined operations of the neutrosophic soft union and intersection; the neutrosophic soft null and absolute set above will be defined differently from the study [3].

Definition 13. Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X and $\overset{NSS}{\tau} \subset NSS(X,E)$. Then $\overset{NSS}{\tau}$ is said to be a neutrosophic soft topology on X if

1. $0_{(X,E)}$ and $1_{(X,E)}$ belongs to $\overset{NSS}{\tau}$
2. the union of any number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$
3. the intersection of finite number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$.

Then $(X, \overset{NSS}{\tau}, E)$ is said to be a neutrosophic soft topological space over X. Each members of $\overset{NSS}{\tau}$ is said to be neutrosophic soft open set.

Definition 14. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be a neutrosophic soft set over X. Then $(\widetilde{F}, E)$ is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Proposition 4. *Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X. Then*

1. $0_{(X,E)}$ *and* $1_{(X,E)}$ *are neutrosophic soft closed sets over* X
2. *the intersection of any number of neutrosophic soft closed sets is a neutrosophic soft closed set over* X
3. *the union of finite number of neutrosophic soft closed sets is a neutrosophic soft closed set over* X.

Proof. It is easily obtained from the definition neutrosophic soft topological space and Proposition 3. □

Definition 15. Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X.

1. If $\overset{NSS}{\tau} = \{0_{(X,E)}, 1_{(X,E)}\}$, then $\overset{NSS}{\tau}$ is said to be the neutrosophic soft in discrete topology and $(X, \overset{NSS}{\tau}, E)$ is said to be a neutrosophic soft indiscrete topological space over X.
2. If $\overset{NSS}{\tau} = NSS(X,E)$, then $\overset{NSS}{\tau}$ is said to be the neutrosophic soft discrete topology and $(X, \overset{NSS}{\tau}, E)$ is said to be a neutrosophic soft discrete topological space over X.

Proposition 5. *Let $(X, \overset{NSS}{\tau_1}, E)$ and $(X, \overset{NSS}{\tau_2}, E)$ be two neutrosophic soft topological spaces over the same universe set X. Then $(X, \overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}, E)$ is neutrosophic soft topological space over X.*

Proof. 1. Since $0_{(X,E)}, 1_{(X,E)} \in \overset{NSS}{\tau_1}$ and $0_{(X,E)}, 1_{(X,E)} \in \overset{NSS}{\tau_2}$, then $0_{(X,E)}, 1_{(X,E)} \in \overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}$.

2. Suppose that $\{(\widetilde{F}_i, E) | i \in I\}$ be a family of neutrosophic soft sets in $\overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}$.
Then $(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1}$ and $(\widetilde{F}_i, E) \in \overset{NSS}{\tau_2}$ for all $i \in I$, so $\underset{i\in I}{\cup}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1}$ and $\underset{i\in I}{\cup}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_2}$.
Thus $\underset{i\in I}{\cup}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}$.

3. Let $\{(\widetilde{F}_i, E) | i = \overline{1,n}\}$ be a family of the finite number of neutrosophic soft sets in $\overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}$.
Then $(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1}$ and $(\widetilde{F}_i, E) \in \overset{NSS}{\tau_2}$ for $i = \overline{1,n}$, so $\overset{n}{\underset{i=1}{\cap}}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1}$ and $\overset{n}{\underset{i=1}{\cap}}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_2}$.
Thus $\overset{n}{\underset{i=1}{\cap}}(\widetilde{F}_i, E) \in \overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}$. □

Remark 1. The union of two neutrosophic soft topologies over X may not be a neutrosophic soft topology on X.

Example 2. Let $X = \{x_1, x_2, x_3\}$ be an initial universe set, $E = \{e_1, e_2\}$ be a set of parameters and

$\overset{NSS}{\tau_1} = \{0_{(X,E)}, 1_{(X,E)}, (\widetilde{F}_1, E), (\widetilde{F}_2, E), (\widetilde{F}_3, E)\}$ and

$\overset{NSS}{\tau_2} = \{0_{(X,E)}, 1_{(X,E)}, (\widetilde{F}_2, E), (\widetilde{F}_4, E)\}$

be two neutrosophic soft topologies over X. Here, the neutrosophic soft sets $(\widetilde{F}_1, E)$, $(\widetilde{F}_2, E)$, $(\widetilde{F}_3, E)$ and $(\widetilde{F}_4, E)$ over X are defined as following:

$$(\widetilde{F}_1, E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1, 0.9, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.6, 0.5\rangle, \langle x_3, 0.4, 0.5, 0.3\rangle\}, \\ e_2 = \{\langle x_1, 0.7, 0.3, 0.4\rangle, \langle x_2, 0.6, 0.6, 0.2\rangle, \langle x_3, 0.6, 0.4, 0.5\rangle\} \end{array} \right\},$$

$$(\widetilde{F}_2, E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1, 0.7, 0.4, 0.5\rangle, \langle x_2, 0.4, 0.5, 0.5\rangle, \langle x_3, 0.3, 0.3, 0.4\rangle\}, \\ e_2 = \{\langle x_1, 0.6, 0.2, 0.4\rangle, \langle x_2, 0.5, 0.4, 0.3\rangle, \langle x_3, 0.4, 0.1, 0.5\rangle\} \end{array} \right\},$$

$$(\widetilde{F}_3, E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1, 0.5, 0.3, 0.6\rangle, \langle x_2, 0.3, 0.4, 0.7\rangle, \langle x_3, 0.2, 0.2, 0.5\rangle\}, \\ e_2 = \{\langle x_1, 0.4, 0.1, 0.5\rangle, \langle x_2, 0.4, 0.3, 0.4\rangle, \langle x_3, 0.1, 0.1, 0.6\rangle\} \end{array} \right\},$$

$$(\widetilde{F}_4, E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1, 0.8, 0.5, 0.4\rangle, \langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_3, 0.7, 0.6, 0.2\rangle\}, \\ e_2 = \{\langle x_1, 0.7, 0.3, 0.3\rangle, \langle x_2, 0.6, 0.5, 0.1\rangle, \langle x_3, 0.7, 0.4, 0.3\rangle\} \end{array} \right\}.$$

Since $(\widetilde{F}_1, E) \cup (\widetilde{F}_4, E) \notin \overset{NSS}{\tau_1} \cup \overset{NSS}{\tau_2}$, then $\overset{NSS}{\tau_1} \cup \overset{NSS}{\tau_2}$ is not a neutrosophic soft topology over X.

Proposition 6. *Let* $(X, \overset{NSS}{\tau}, E)$ *be a neutrosophic soft topological space over* X *and* $\overset{NSS}{\tau} = \{(\widetilde{F}_i, E) : (\widetilde{F}_i, E) \in NSS(X,E)\} = \{[e, \widetilde{F}_i(e)]_{e\in E} : (\widetilde{F}_i, E) \in NSS(X,E)\}$ *where* $\widetilde{F}_i(e) = \{\langle x, T_{\widetilde{F}_i(e)}(x), I_{\widetilde{F}_i(e)}(x), F_{\widetilde{F}_i(e)}(x)\rangle : x \in X\}$. *Then*

$$\tau_1 = \{[T_{\widetilde{F}_i(e)}(X)]_{e\in E}\},$$

$$\tau_2 = \{[I_{\widetilde{F}_i(e)}(X)]_{e\in E}\},$$

$$\tau_3 = \{[F_{\widetilde{F}_i(e)}(X)]^c_{e\in E}\}$$

define fuzzy soft topologies on X.

Proof. 1. $0_{(X,E)}, 1_{(X,E)} \in \overset{NSS}{\tau} \Rightarrow 0, 1 \in \tau_1$, $0, 1 \in \tau_2$ and $0, 1 \in \tau_3$

2. Suppose that $\{(\widetilde{F}_i, E) | i \in I\}$ be a family of neutrosophic soft sets in $\overset{NSS}{\tau}$. Then$\{[T_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I}$ is a family of fuzzy soft sets in τ_1, $\{[I_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I}$ is a family of fuzzy soft sets in τ_2 and $\{[F_{\widetilde{F}_i(e)}(X)]^c_{e\in E}\}_{i\in I}$ is a family of fuzzy soft sets in τ_3. Since $\overset{NSS}{\tau}$ is a neutrosophic soft topology, then $\underset{i\in I}{\cup}(\widetilde{F}_i, E) \in \overset{NSS}{\tau}$. That is,

$$\underset{i\in I}{\cup}(\widetilde{F}_i, E) = \{\langle \sup[T_{\widetilde{F}_i(e)}(X)]_{e\in E}, \sup[I_{\widetilde{F}_i(e)}(X)]_{e\in E}, \inf[F_{\widetilde{F}_i(e)}(X)]_{e\in E}\rangle\}_{i\in I} \in \overset{NSS}{\tau}.$$

Therefore,

$$\{\sup[T_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I} \in \tau_1,$$

$$\{\sup[I_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I} \in \tau_2,$$

$$\{\sup[F_{\widetilde{F}_i(e)}(X)]^c_{e\in E i\in I}\}_{i\in I} \in \tau_3.$$

3. Suppose that $\{(\widetilde{F}_i, E) | i = \overline{1,n}\}$ be a family of finite neutrosophic soft sets in $\overset{NSS}{\tau}$. Then$\{[T_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i=\overline{1,n}}$ is a family of fuzzy soft sets in τ_1, $\{[I_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i=\overline{1,n}}$ is a family

of fuzzy soft sets in τ_2 and$\{[F_{\widetilde{F}_i(e)}(X)]^c_{e\in E}\}_{i=\overline{1,n}}$ is a family of fuzzy soft sets in τ_3. Since $\overset{NSS}{\tau}$ is a neutrosophic soft topology, then $\bigcap_{i=1}^{n}(\widetilde{F}_i,E)\in\overset{NSS}{\tau}$. That is,

$$\bigcap_{i=1}^{n}(\widetilde{F}_i,E)=\{\langle\min[T_{\widetilde{F}_i(e)}(X)]_{e\in E},\min[I_{\widetilde{F}_i(e)}(X)]_{e\in E},\max[F_{\widetilde{F}_i(e)}(X)]_{e\in E}\rangle\}_{i=\overline{1,n}}\in\overset{NSS}{\tau}.$$

Therefore,

$$\{\min[T_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I}\in\tau_1,\quad\{\min[I_{\widetilde{F}_i(e)}(X)]_{e\in E}\}_{i\in I}\in\tau_2,\qquad\{\min[F_{\widetilde{F}_i(e)}(X)]^c_{e\in E}\}_{i\in I}\in\tau_3.$$

This completes the proof. □

Remark 2. Generally, converse of the above proposition is not true.

Example 3. Let $X=\{x_1,x_2,x_3\}$ be a initial universe, $E=\{e_1,e_2\}$ be a set of parameters and

$$\overset{NSS}{\tau}=\{0_{(X,E)},1_{(X,E)},(\widetilde{F}_1,E),(\widetilde{F}_2,E),(\widetilde{F}_3,E)\}$$

be a family of neutrosophic soft sets over X. Here, the neutrosophic soft sets $(\widetilde{F}_1,E)$, $(\widetilde{F}_2,E)$ and $(\widetilde{F}_3,E)$ over X are defined as following:

$$(\widetilde{F}_1,E)=\left\{\begin{array}{l}e_1=\{\langle x_1,0.7,0.3,0.2\rangle,\langle x_2,0.5,0.4,0.6\rangle,\langle x_3,0.5,0.3,0.1\rangle\},\\ e_2=\{\langle x_1,0.4,0.5,0.3\rangle,\langle x_2,0.2,0.3,0.5\rangle,\langle x_3,0,0.4,0.5\rangle\}\end{array}\right\},$$

$$(\widetilde{F}_2,E)=\left\{\begin{array}{l}e_1=\{\langle x_1,0.6,0.5,0.3\rangle,\langle x_2,0.4,0.6,0.6\rangle,\langle x_3,0.3,0.5,0.6\rangle\},\\ e_2=\{\langle x_1,0.3,0.6,0.5\rangle,\langle x_2,0.1,0.7,0.6\rangle,\langle x_3,0,0.6,0.5\rangle\}\end{array}\right\},$$

$$(\widetilde{F}_3,E)=\left\{\begin{array}{l}e_1=\{\langle x_1,0.8,0.4,0.5\rangle,\langle x_2,0.5,0.5,0.8\rangle,\langle x_3,0.5,0.4,0.7\rangle\},\\ e_2=\{\langle x_1,0.5,0.5,0.7\rangle,\langle x_2,0.4,0.6,0.7\rangle,\langle x_3,0.6,0.5,0.9\rangle\}\end{array}\right\}.$$

Then,

$$\tau_1=\{\langle T_{\widetilde{F}_{0_{(X,E)}}(e)}(X),T_{\widetilde{F}_{1_{(X,E)}}(e)}(X),T_{\widetilde{F}_1(e)}(X),T_{\widetilde{F}_2(e)}(X),T_{\widetilde{F}_3(e)}(X)\rangle_{e\in E}\},$$

$$\tau_2=\{\langle I_{\widetilde{F}_{0_{(X,E)}}(e)}(X),I_{\widetilde{F}_{1_{(X,E)}}(e)}(X),I_{\widetilde{F}_1(e)}(X),I_{\widetilde{F}_2(e)}(X),I_{\widetilde{F}_3(e)}(X)\rangle_{e\in E}\},$$

$$\tau_3=\{\langle F_{\widetilde{F}_{0_{(X,E)}}(e)}(X),F_{\widetilde{F}_{1_{(X,E)}}(e)}(X),F_{\widetilde{F}_1(e)}(X),F_{\widetilde{F}_2(e)}(X),F_{\widetilde{F}_3(e)}(X)\rangle_{e\in E}\},$$

are fuzzy soft topologies on X. For example,

$$\tau_1=\left\{\begin{array}{l}\langle(0,0,0),(1,1,1),(0.7,0.5,0.5),(0.6,0.4,0.3),(0.8,0.5,0.5)\rangle_{e_1},\\ \langle(0,0,0),(1,1,1),(0.4,0.2,0),(0.3,0.1,0),(0.5,0.4,0.6)\rangle_{e_2}\end{array}\right\}$$

and so on.

On the other hand, since $(\widetilde{F}_2,E)\cap(\widetilde{F}_3,E)\notin\overset{NSS}{\tau}$, $\overset{NSS}{\tau}$ is not a neutrosophic soft topology on X.

Proposition 7. *Let $(X,\overset{NSS}{\tau},E)$ be a neutrosophic soft topological space over X. Then*

$$\tau_{1_e}=\{[T_{\widetilde{F}(e)}(X)]:(\widetilde{F},E)\in\overset{NSS}{\tau}\},$$

$$\tau_{2_e}=\{[I_{\widetilde{F}(e)}(X)]:(\widetilde{F},E)\in\overset{NSS}{\tau}\},$$

$$\tau_{3_e}=\{[F_{\widetilde{F}(e)}(X)]^c:(\widetilde{F},E)\in\overset{NSS}{\tau}\}$$

for each $e\in E$, define fuzzy topologies on X.

Proof. Straightforward. □

Remark 3. Generally, converse of the above proposition is not true.

Example 4. Let us consider the Example 3. Then,

$$\tau 1_{e_1} = \{T_{\tilde{F}_{0_{(X,E)}}(e_1)}(X), T_{\tilde{F}_{1_{(X,E)}}(e_1)}(X), T_{\tilde{F}_1(e_1)}(X), T_{\tilde{F}_2(e_1)}(X), T_{\tilde{F}_3(e_1)}(X)\},$$
$$\tau 2_{e_1} = \{I_{\tilde{F}_{0_{(X,E)}}(e_1)}(X), I_{\tilde{F}_{1_{(X,E)}}(e_1)}(X), I_{\tilde{F}_1(e_1)}(X), I_{\tilde{F}_2(e_1)}(X), I_{\tilde{F}_3(e_1)}(X)\},$$
$$\tau 3_{e_1} = \{F_{\tilde{F}_{0_{(X,E)}}(e_1)}(X), F_{\tilde{F}_{1_{(X,E)}}(e_1)}(X), F_{\tilde{F}_1(e_1)}(X), F_{\tilde{F}_2(e_1)}(X), F_{\tilde{F}_3(e_1)}(X)\},$$

are fuzzy topologies on X. For example,

$$\tau 1_{e_1} = \{(0,0,0),(1,1,1),(0.7,0.5,0.5),(0.6,0.4,0.3),(0.8,0.5,0.5)\}$$

and so on. Here, $\{\tau 1_{e_1}, \tau 2_{e_1}, \tau 3_{e_1}\}$ and $\{\tau 1_{e_2}, \tau 2_{e_2}, \tau 3_{e_2}\}$ are fuzzy tritopology on X. But $\overset{NSS}{\tau}$ is not a neutrosophic soft topology on X.

Definition 16. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X,E)$ be a neutrosophic soft set. Then, the neutrosophic soft interior of $(\tilde{F}, E)$, denoted $(\tilde{F}, E)^\circ$, is defined as the neutrosophic soft union of all neutrosophic soft open subsets of $(\tilde{F}, E)$.

Clearly, $(\tilde{F}, E)^\circ$ is the biggest neutrosophic soft open set that is contained by $(\tilde{F}, E)$.

Example 5. Let us consider the neutrosophic soft topology $\overset{NSS}{\tau_1}$ given in Example 2. Suppose that an any $(\tilde{F}, E) \in NSS(X,E)$ is defined as following:

$$(\tilde{F}, E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1, 0.8, 0.5, 0.2\rangle, \langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_3, 0.4, 0.4, 0.3\rangle\}, \\ e_2 = \{\langle x_1, 0.8, 0.4, 0.1\rangle, \langle x_2, 0.7, 0.6, 0.2\rangle, \langle x_3, 0.8, 0.4, 0.4\rangle\} \end{array} \right\}.$$

Then $0_{(X,E)}$, $(\tilde{F}_2, E)$, $(\tilde{F}_3, E) \subseteq (\tilde{F}, E)$. Therefore, $(\tilde{F}, E)^\circ = 0_{(X,E)} \cup (\tilde{F}_2, E) \cup (\tilde{F}_3, E) = (\tilde{F}_2, E)$.

Theorem 1. *Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X,E)$. $(\tilde{F}, E)$ is a neutrosophic soft open set iff $(\tilde{F}, E) = (\tilde{F}, E)^\circ$.*

Proof. Let $(\tilde{F}, E)$ be a neutrosophic soft open set. Then the biggest neutrosophic soft open set that is contained by $(\tilde{F}, E)$ is equal to $(\tilde{F}, E)$. Hence, $(\tilde{F}, E) = (\tilde{F}, E)^\circ$.

Conversely, it is known that $(\tilde{F}, E)^\circ$ is a neutrosophic soft open set and if $(\tilde{F}, E) = (\tilde{F}, E)^\circ$, then $(\tilde{F}, E)$ is a neutrosophic soft open set. □

Theorem 2. *Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}_1, E), (\tilde{F}_2, E) \in NSS(X,E)$. Then,*

1. $[(\tilde{F}_1, E)^\circ]^\circ = (\tilde{F}_1, E)^\circ$,
2. $(0_{(X,E)})^\circ = 0_{(X,E)}$ *and* $(1_{(X,E)})^\circ = 1_{(X,E)}$,
3. $(\tilde{F}_1, E) \subseteq (\tilde{F}_2, E) \Rightarrow (\tilde{F}_1, E)^\circ \subseteq (\tilde{F}_2, E)^\circ$,
4. $[(\tilde{F}_1, E) \cap (\tilde{F}_2, E)]^\circ = (\tilde{F}_1, E)^\circ \cap (\tilde{F}_2, E)^\circ$,
5. $(\tilde{F}_1, E)^\circ \cup (\tilde{F}_2, E)^\circ \subseteq [(\tilde{F}_1, E) \cup (\tilde{F}_2, E)]^\circ$.

Proof. 1. Let $(\widetilde{F}_1,E)^\circ = (\widetilde{F}_2,E)$. Then $(\widetilde{F}_2,E) \in \overset{NSS}{\tau}$ iff $(\widetilde{F}_2,E) = (\widetilde{F}_2,E)^\circ$. So, $[(\widetilde{F}_1,E)^\circ]^\circ = (\widetilde{F}_1,E)^\circ$.

2. Straighforward.

3. It is known that $(\widetilde{F}_1,E)^\circ \subseteq (\widetilde{F}_1,E) \subseteq (\widetilde{F}_2,E)$ and $(\widetilde{F}_2,E)^\circ \subseteq (\widetilde{F}_2,E)$. Since $(\widetilde{F}_2,E)^\circ$ is the biggest neutrosophic soft open set contained in $(\widetilde{F}_2,E)$ and so, $(\widetilde{F}_1,E)^\circ \subseteq (\widetilde{F}_2,E)^\circ$.

4. Since $(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E) \subseteq (\widetilde{F}_1,E)$ and $(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E) \subseteq (\widetilde{F}_2,E)$,
then $[(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ \subseteq (\widetilde{F}_1,E)^\circ$ and $[(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ \subseteq (\widetilde{F}_2,E)^\circ$ and so,
$[(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ \subseteq (\widetilde{F}_1,E)^\circ \cap (\widetilde{F}_2,E)^\circ$.

On the other hand, since $(\widetilde{F}_1,E)^\circ \subseteq (\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)^\circ \subseteq (\widetilde{F}_2,E)$, then $(\widetilde{F}_1,E)^\circ \cap (\widetilde{F}_2,E)^\circ \subseteq (\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)$. Besides, $[(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ \subseteq (\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)$ and it is the biggest neutrosophic soft open set. Therefore, $(\widetilde{F}_1,E)^\circ \cap (\widetilde{F}_2,E)^\circ \subseteq [(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ$. Thus, $[(\widetilde{F}_1,E) \cap (\widetilde{F}_2,E)]^\circ = (\widetilde{F}_1,E)^\circ \cap (\widetilde{F}_2,E)^\circ$.

5. Since $(\widetilde{F}_1,E) \subseteq (\widetilde{F}_1,E) \cup (\widetilde{F}_2,E)$ and $(\widetilde{F}_2,E) \subseteq (\widetilde{F}_1,E) \cup (\widetilde{F}_2,E)$, then $(\widetilde{F}_1,E)^\circ \subseteq [(\widetilde{F}_1,E) \cup (\widetilde{F}_2,E)]^\circ$ and $(\widetilde{F}_2,E)^\circ \subseteq [(\widetilde{F}_1,E) \cup (\widetilde{F}_2,E)]^\circ$. Therefore, $(\widetilde{F}_1,E)^\circ \cup (\widetilde{F}_2,E)^\circ \subseteq [(\widetilde{F}_1,E) \cup (\widetilde{F}_2,E)]^\circ$. □

Definition 17. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F},E) \in NSS(X,E)$ be a neutrosophic soft set. Then, the neutrosophic soft closure of $(\widetilde{F},E)$, denoted $\overline{(\widetilde{F},E)}$, is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of $(\widetilde{F},E)$.

Clearly, $\overline{(\widetilde{F},E)}$ is the smallest neutrosophic soft closed set that containing $(\widetilde{F},E)$.

Example 6. Let us consider the neutrosophic soft topology $\overset{NSS}{\tau_1}$ given in Example 2. Suppose that an any $(\widetilde{F},E) \in NSS(X,E)$ is defined as following:

$$(\widetilde{F},E) = \left\{ \begin{array}{l} e_1 = \{\langle x_1,0.2,0.5,0.9\rangle, \langle x_2,0.5,0.3,0.7\rangle, \langle x_3,0.2,0.4,0.6\rangle\}, \\ e_2 = \{\langle x_1,0.1,0.4,0.8\rangle, \langle x_2,0.1,0.3,0.7\rangle, \langle x_3,0.3,0.4,0.8\rangle\} \end{array} \right\}.$$

Obviously, $0^c_{(X,E)}$, $1^c_{(X,E)}$, $(\widetilde{F}_1,E)^c$, $(\widetilde{F}_2,E)^c$ and $(\widetilde{F}_3,E)^c$ are all neutrosophic soft closed sets over $(X, \overset{NSS}{\tau_1}, E)$. They are given as following:

$$0^c_{(X,E)} = 1_{(X,E)},\ 1^c_{(X,E)} = 0_{(X,E)}$$

$$(\widetilde{F}_1,E)^c = \left\{ \begin{array}{l} e_1 = \{\langle x_1,0.3,0.6,0.9\rangle, \langle x_2,0.5,0.4,0.5\rangle, \langle x_3,0.3,0.5,0.4\rangle\}, \\ e_2 = \{\langle x_1,0.4,0.7,0.7\rangle, \langle x_2,0.2,0.4,0.6\rangle, \langle x_3,0.5,0.6,0.6\rangle\} \end{array} \right\},$$

$$(\widetilde{F}_2,E)^c = \left\{ \begin{array}{l} e_1 = \{\langle x_1,0.5,0.6,0.7\rangle, \langle x_2,0.5,0.5,0.4\rangle, \langle x_3,0.4,0.7,0.3\rangle\}, \\ e_2 = \{\langle x_1,0.4,0.8,0.6\rangle, \langle x_2,0.3,0.6,0.5\rangle, \langle x_3,0.5,0.9,0.4\rangle\} \end{array} \right\},$$

$$(\widetilde{F}_3,E)^c = \left\{ \begin{array}{l} e_1 = \{\langle x_1,0.6,0.7,0.5\rangle, \langle x_2,0.7,0.6,0.3\rangle, \langle x_3,0.5,0.8,0.2\rangle\}, \\ e_2 = \{\langle x_1,0.5,0.9,0.4\rangle, \langle x_2,0.4,0.7,0.4\rangle, \langle x_3,0.6,0.9,0.1\rangle\} \end{array} \right\}.$$

Then $1^c_{(X,E)}$, $(\widetilde{F}_1,E)^c$, $(\widetilde{F}_2,E)^c, (\widetilde{F}_3,E)^c \supseteq (\widetilde{F},E)$. Therefore, $\overline{(\widetilde{F},E)} = 1^c_{(X,E)} \cap (\widetilde{F}_1,E)^c \cap (\widetilde{F}_2,E)^c \cap (\widetilde{F}_3,E)^c = (\widetilde{F}_1,E)^c$.

Theorem 3. *Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\widetilde{F},E) \in NSS(X,E)$. $(\widetilde{F},E)$ is neutrosophic soft closed set iff $(\widetilde{F},E) = \overline{(\widetilde{F},E)}$.*

Proof. Straightforward. □

Theorem 4. *Let $(X,\overset{NSS}{\tau},E)$ be a neutrosophic soft topological space over X and $(\tilde F_1,E),(\tilde F_2,E)\in NSS(X,E)$. Then,*

1. $\overline{[\overline{(\tilde F_1,E)}]}=\overline{(\tilde F_1,E)}$,
2. $\overline{(0_{(X,E)})}=0_{(X,E)}$ *and* $\overline{(1_{(X,E)})}=1_{(X,E)}$
3. $(\tilde F_1,E)\subseteq(\tilde F_2,E)\Rightarrow\overline{(\tilde F_1,E)}\subseteq\overline{(\tilde F_2,E)}$,
4. $\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}=\overline{(\tilde F_1,E)}\cup\overline{(\tilde F_2,E)}$,
5. $\overline{[(\tilde F_1,E)\cap(\tilde F_2,E)]}\subseteq\overline{(\tilde F_1,E)}\cap\overline{(\tilde F_2,E)}$.

Proof. 1. Let $\overline{(\tilde F_1,E)}=(\tilde F_2,E)$. Then, $(\tilde F_2,E)$ is a neutrosophic soft closed set. Hence, $(\tilde F_2,E)$ and $\overline{(\tilde F_2,E)}$ are equal. Therefore, $\overline{[\overline{(\tilde F_1,E)}]}=\overline{(\tilde F_1,E)}$.

2. Straightforward.

3. It is known that $(\tilde F_1,E)\subseteq\overline{(\tilde F_1,E)}$ and $(\tilde F_2,E)\subseteq\overline{(\tilde F_2,E)}$ and so, $(\tilde F_1,E)\subseteq(\tilde F_2,E)\subseteq\overline{(\tilde F_2,E)}$. Since $\overline{(\tilde F_1,E)}$ is the smallest neutrosophic soft closed set containing $(\tilde F_1,E)$, then $\overline{(\tilde F_1,E)}\subseteq\overline{(\tilde F_2,E)}$.

4. Since $(\tilde F_1,E)\subseteq(\tilde F_1,E)\cup(\tilde F_2,E)$ and $(\tilde F_2,E)\subseteq(\tilde F_1,E)\cup(\tilde F_2,E)$, then $\overline{(\tilde F_1,E)}\subseteq\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}$ and $\overline{(\tilde F_2,E)}\subseteq\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}$ and so, $\overline{(\tilde F_1,E)}\cup\overline{(\tilde F_2,E)}\subseteq\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}$.

Conversely, since $(\tilde F_1,E)\subseteq\overline{(\tilde F_1,E)}$ and $(\tilde F_2,E)\subseteq\overline{(\tilde F_2,E)}$, then $(\tilde F_1,E)\cup(\tilde F_2,E)\subseteq\overline{(\tilde F_1,E)}\cup\overline{(\tilde F_2,E)}$. Besides, $\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}$ is the smallest neutrosophic soft closed set that containing $(\tilde F_1,E)\cup(\tilde F_2,E)$. Therefore, $\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}\subseteq\overline{(\tilde F_1,E)}\cup\overline{(\tilde F_2,E)}$. Thus, $\overline{[(\tilde F_1,E)\cup(\tilde F_2,E)]}=\overline{(\tilde F_1,E)}\cup\overline{(\tilde F_2,E)}$.

5. Since $(\tilde F_1,E)\cap(\tilde F_2,E)\subseteq\overline{(\tilde F_1,E)}\cap\overline{(\tilde F_2,E)}$ and $\overline{[(\tilde F_1,E)\cap(\tilde F_2,E)]}$ is the smallest neutrosophic soft closed set that containing $(\tilde F_1,E)\cap(\tilde F_2,E)$, then $\overline{[(\tilde F_1,E)\cap(\tilde F_2,E)]}\subseteq\overline{(\tilde F_1,E)}\cap\overline{(\tilde F_2,E)}$. □

Theorem 5. *Let $(X,\overset{NSS}{\tau},E)$ be a neutrosophic soft topological space over X and $(\tilde F,E)\in NSS(X,E)$. Then,*

1. $[\overline{(\tilde F,E)}]^c=[(\tilde F,E)^c]^\circ$,
2. $[(\tilde F,E)^\circ]^c=\overline{[(\tilde F,E)^c]}$.

Proof. 1. $\overline{(\tilde F,E)}=\cap\{(\tilde G,E)\in\overset{NSS}{\tau^c}:(\tilde G,E)\supseteq(\tilde F,E)\}$

$\Longrightarrow[\overline{(\tilde F,E)}]^c=[\cap\{(\tilde G,E)\in\overset{NSS}{\tau^c}:(\tilde G,E)\supseteq(\tilde F,E)\}]^c=\cup\{(\tilde G,E)^c\in\overset{NSS}{\tau}:(\tilde G,E)^c\subseteq(\tilde F,E)^c\}=[(\tilde F,E)^c]^\circ$.

2. $(\tilde F,E)^\circ=\cup\{(\tilde G,E)\in\overset{NSS}{\tau}:(\tilde G,E)\subseteq(\tilde F,E)\}$

$\Longrightarrow[(\tilde F,E)^\circ]^c=[\cup\{(\tilde G,E)\in\overset{NSS}{\tau}:(\tilde G,E)\subseteq(\tilde F,E)\}]^c=\cap\{(\tilde G,E)^c\in\overset{NSS}{\tau^c}:(\tilde G,E)^c\supseteq(\tilde F,E)^c\}=\overline{[(\tilde F,E)^c]}$. □

5. Conclusion

In this paper, we re-introduce some operations of neutrosophic soft set and the concept of neutrosophic soft topological spaces. Finally, we investigate the properties of neutrosophic soft topological spaces and the relationships between neutrosophic soft topology and fuzzy topology;

fuzzy soft topology. We hope that, the results of this study may help in the investigation of neutrosophic soft continuous function spaces and in many researches.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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