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III

A New Concept of Matrix Algorithm for MST in Undirected Interval Valued Neutrosophic Graph

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Abstract

In this chapter, we introduce a new algorithm for finding a minimum spanning tree (MST) of an undirected neutrosophic weighted connected graph whose edge weights are represented by an interval valued neutrosophic number. In addition, we compute the cost of MST and compare the de-neutrosophied value with an equivalent MST having the detereministic weights. Finally, a numerical example is provided**.**

Keywords

Interval valued Neutrosophic Graph, Score function, Minimum Spanning Tree (MST).

1 Introduction

In order to express the inconsistency and indeterminacy that exist in real-life problems reasonably, Smarandache [3] proposed the concept of neutrosophic sets (NSs) from a philosophical standpoint, which is characterized by three totally independent functions, i.e., a truth-function, an indeterminacy function and a falsity function that are inside the real standard or non-standard unit interval]-0, 1+[. Hence, neutrosophic sets can be regarded as many extended forms of classical fuzzy sets [8] such as intuitionistic fuzzy sets [6], interval-valued intuitionistic fuzzy sets [7] etc. Moreover, for the sake of applying neutrosophic sets in real-world problems efficiently, Smarandache [9] put forward the notion of single valued neutrosophic sets (SVNSs for short) firstly, and then various theoretical operators of single valued neutrosophic sets were defined by Wang et al. [4]. Based on single valued neutrosophic sets, Wang et al. [5] further developed the notion of interval valued neutrosophic sets (IVNSs for short), some of their properties were also explored. Since then, studies of neutrosophic sets and their hybrid extensions have been paid great attention by numerous scholars [19]. Many researchers have proposed a frutiful results on interval valued neutrosophic sets [12,14,16,17,18,20,21-31]

MST is most fundamental and well-known optimization problem used in networks in graph theory. The objective of this MST is to find the minimum weighted spanning tree of a weighted connected graph. It has many real time applications, which includes communication system, transportation problems, image processing, logistics, wireless networks, cluster analysis and so on. The classical problems related to MST [1], the arc lengths are taken and it is fixed so that the decision maker use the crisp data to represent the arc lengths of a connected weighted graph. But in the real world scenarios the arch length represents a parameter which may not have a precise value. For example, the demand and supply, cost problems, time constraints, traffic frequencies, capacities etc., For the road networks, even though the geometric distance is fixed, arc length represents the vehicle travel time which fluctuates due to different weather conditions, traffic flow and some other unexpected factors. There are several algorithms for finding the MST in classical graph theory. These are based on most well-known algorithms such as Prims and Kruskals algorithms. Nevertheless, these algorithms cannot handle the cases when the arc length is fuzzy which are taken into consideration [2].

More recently, some scholars have used neutrosophic methods to find minimum spanning tree in neutrosophic environment. Ye [8] defined a method to find minimum spanning tree of a graph where nodes (samples) are represented in the form of NSs and distance between two nodes represents the dissimilarity between the corresponding samples. Mandal and Basu [9] defined a new approach of optimum spanning tree problems considering the inconsistency, incompleteness and indeterminacy of the information. They considered a network problem with multiple criteria represented by weight of each edge in neutrosophic sets. Kandasamy [11] proposed a double-valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm to cluster the data represented by double-valued neutrosophic information. Mullai [15] discussed the MST problem on a graph in which a bipolar neutrosophic number is associated to each edge as its edge length, and illustrated it by a numerical example. To the end, no research dealt with the cases of interval valued neutrosophic arc lengths.

The main objective of this work is to find the minimum spanning tree of undirected neutrosophic graphs using the proposed matrix algorithm. It would be very much useful and easy to handle the considered problem of interval valued neutrosophic arc lengths using this algorithm.

The rest of the paper is organized as follows. Section 2 briefly introduces the concepts of neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets and the score function of interval valued neutrosophic number. Section 3 proposes a novel approach for finding the minimum spanning tree of interval valued neutrosophic undirected graph. In Section 4, two illustrative examples are presented to illustrate the proposed method. Finally, Section 5 contains conclusions and future work.

2 Preliminaries

Definition 2.1 [3] Le ξ be an universal set. The neutrosophic set A on the universal set ξ categorized in to three membership functions called the true $T_A(x)$, indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non-standard subset of $]^-0, 1^+[$ respectively.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup I_A(x) \le 3^+ \quad (1)$$

Definition 2.2 [4] Let ξ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ξ is denoted as following

$$A = \{<x: T_A(x), I_A(x), F_A(x)> x \in \xi\} \quad (2)$$

The functions $T_A(x) \in [0. 1]$, $I_A(x) \in [0. 1]$ and $F_A(x) \in [0. 1]$ are named degree of truth, indeterminacy and falsity membership of x in A, satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + T_A(x) \le 3 \quad (3)$$

Definition 2.3 [5]. An interval valued neutrosophic set A in X is defined as an object of the form

$$\tilde{A} = \{< x, \tilde{t}, \tilde{i}, \tilde{f} >: x \in X\},$$

where $\tilde{t} = [T^L_{\tilde{A}}, T^U_{\tilde{A}}]$, $\tilde{i} = [I^L_{\tilde{A}}, I^U_{\tilde{A}}]$, $\tilde{f} = [F^L_{\tilde{A}}, F^U_{\tilde{A}}]$, and $T_{\tilde{A}}^{L}$, $T_{\tilde{A}}^{U}$ $I_{\tilde{A}}^{L}$, $I_{\tilde{A}}^{U}$, $F_{\tilde{A}}^{L}$, $F_{\tilde{A}}^{U}$:X $\rightarrow$ [0, 1].The interval membership degree where $T_{\tilde{A}}^{L}$, $T_{\tilde{A}}^{U}$ $I_{\tilde{A}}^{L}$, $I_{\tilde{A}}^{U}$, $F_{\tilde{A}}^{L}$, $F_{\tilde{A}}^{U}$ denotes the lower and upper truth membership, lower and upper indeterminate membership and lower and upper false membership of an element $\in$ X corresponding to an interval valued neutrosophic set A where $0 \le T_M^p + I_M^p + F_M^p \le 3$

In order to rank the IVNS, TAN [18] defined the following score function.

Definition 2.4 [18]. Let $\tilde{A} =< \tilde{t}, \tilde{i}, \tilde{f} >$ be an interval valued neutrosophic number , where $\tilde{t} = \left[T_{\tilde{A}}^{L}, T_{\tilde{A}}^{U}\right]$, $\tilde{i} = \left[I_{\tilde{A}}^{L}, I_{\tilde{A}}^{U}\right]$, $\tilde{f} = \left[F_{\tilde{A}}^{L}, F_{\tilde{A}}^{U}\right]$,Then, the score function $s(\tilde{A})$, accuracy function $a(\tilde{A})$ and certainty function $c(\tilde{A})$ of an IVNN can be represented as follows:

(i) $S_{TAN}(\tilde{A}) = \dfrac{(2 + T_{\tilde{A}}^{L} - I_{\tilde{A}}^{L} - F_{\tilde{A}}^{L}) + (2 + T_{\tilde{A}}^{U} - I_{\tilde{A}}^{U} - F_{\tilde{A}}^{U})}{6}$,

$$S(\tilde{A}) \in [0,1] \qquad (4)$$

(ii) $a_{TAN}(\tilde{A}) = \dfrac{(T_{\tilde{A}}^{L} - F_{\tilde{A}}^{L}) - (T_{\tilde{A}}^{U} - F_{\tilde{A}}^{U})}{2}$ $\quad a(\tilde{A}) \in [-1,1]$ (5)

TAN [18] gave an order relation between two IVNNs, which is defined as follows

Let $\tilde{A}_1 =< \tilde{t}_1, \tilde{i}_1, \tilde{f}_1 >$ and $\tilde{A}_2 =< \tilde{t}_2, \tilde{i}_2, \tilde{f}_2 >$ be two interval valued neutrosophic numbers then

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$
ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $a(\tilde{A}_1) \succ a(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$
iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$

Definition 2.5 [17]: Let $\tilde{A} = \left\langle \left[T_{\tilde{A}}^{L}, T_{\tilde{A}}^{U}\right], \left[I_{\tilde{A}}^{L}, I_{\tilde{A}}^{U}\right], \left[F_{\tilde{A}}^{L}, F_{\tilde{A}}^{U}\right] \right\rangle$ be an IVNN, the score function S of $\tilde{A}$ is defined as follows

$$S_{RIDVAN}\left(\tilde{A}\right) = \frac{1}{4}\left(2 + T_A^L + T_A^U - 2I_A^L - 2I_A^U - F_A^L - F_A^L\right),\ S\left(\tilde{A}\right) \in [-1,1]. \quad (6)$$

Definition 2.6 [17]: Let $\tilde{A}=\left\langle\left[T_{\tilde{A}}^{L},T_{\tilde{A}}^{U}\right],\left[I_{\tilde{A}}^{L},I_{\tilde{A}}^{U}\right],\left[F_{\tilde{A}}^{L},F_{\tilde{A}}^{U}\right]\right\rangle$ be an IVNN, the accuracy function H of $\tilde{A}$ is defined as follows

$$H_{RIDVAN}\left(\tilde{A}\right)=\frac{1}{2}\begin{pmatrix}T_{A}^{L}+T_{A}^{U}-2 \\ I_{A}^{U}(1-T_{A}^{U})-I_{A}^{L}(1-T_{A}^{L})-F_{A}^{U}(1-I_{A}^{L})-F_{A}^{L}(1-I_{A}^{U})\end{pmatrix}, H\left(\tilde{A}\right)\in[-1,1]. \quad (7)$$

To rank any two IVNNs $\tilde{A}=\left\langle\left[T_{\tilde{A}}^{L},T_{\tilde{A}}^{U}\right],\left[I_{\tilde{A}}^{L},I_{\tilde{A}}^{U}\right],\left[F_{\tilde{A}}^{L},F_{\tilde{A}}^{U}\right]\right\rangle$ and $\tilde{B}=\left\langle\left[T_{\tilde{B}}^{L},T_{\tilde{B}}^{U}\right],\left[I_{\tilde{B}}^{L},I_{\tilde{B}}^{U}\right],\left[F_{\tilde{B}}^{L},F_{\tilde{B}}^{U}\right]\right\rangle$,

Ridvan [17] introduced the following method.

Definition 2.7 [17]: Let $\tilde{A}$ and $\tilde{B}$ be two IVNNs, $S\left(\tilde{A}\right)$ and $S\left(\tilde{B}\right)$ be scores of $\tilde{A}$ and $\tilde{B}$ respectively, and $H\left(\tilde{A}\right)$ and $H\left(\tilde{B}\right)$ be accuracy values of $\tilde{A}$ and $\tilde{B}$ respectively, then

i. If $S(\tilde{A})>S(\tilde{B})$ then $\tilde{A}$ is larger than $\tilde{B}$, denoted $\tilde{A}>\tilde{B}$.
ii. If $S(\tilde{A})=S(\tilde{B})$ then we check their accuracy values and decide as follows:
(a) If $H(\tilde{A})=H(\tilde{B})$, then $\tilde{A}=\tilde{B}$.
(b) However, if $H(\tilde{A})>H(\tilde{B})$, then $\tilde{A}$ is larger than $\tilde{B}$, denoted $\tilde{A}>\tilde{B}$.

Definition 2.8 [12]: Let $\tilde{A}=\left\langle\left[T_{\tilde{A}}^{L},T_{\tilde{A}}^{U}\right],\left[I_{\tilde{A}}^{L},I_{\tilde{A}}^{U}\right],\left[F_{\tilde{A}}^{L},F_{\tilde{A}}^{U}\right]\right\rangle$ be an IVNN, the score function S of $\tilde{A}$ is defined as follows

$$S_{NANCY}\left(\tilde{A}\right)=\frac{4+\left(T_{A}^{L}+T_{A}^{U}-2I_{A}^{L}-2I_{A}^{U}-F_{A}^{L}-F_{A}^{L}\right)\left(4-T_{A}^{L}+T_{A}^{U}-F_{A}^{L}-F_{A}^{L}\right)}{8},\ S\left(\tilde{A}\right)\in[0,1].$$

Remark 2.9: In neutrosophic mathematics, the zero sets are represented by the following form 0_N={<x, [0, 0], [1, 1], [1,1])> :x∈ X}.

3 The proposed algorithm

The following algorithm is a new concept of finding the MST of undirected interval valued neutrosophic graph using the matrix approach.

Algorithm:

Input: the weight matrix M $=\left[W_{ij}\right]_{n\times n}$ for the undirected weighted interval valued neutrosophic graph *G*.

Output: Minimum cost Spanning tree *T* of *G*.

Step 1: Input interval valued neutrosophic adjacency matrix *A*.

Step 2: Convert the interval valued neutrosophic matrix into a score matrix $\left[S_{ij}\right]_{n\times n}$ using the score function.

Step 3: Iterate step 4 and step 5 until all (n-1) entries matrix of S are either marked or set to zero or other words all the nonzero elements are marked.

Step 4: Find the weight matrix M either columns-wise or row-wise to determine the unmarked minimum entries S_{ij} which is the weight of the corresponding edge e_{ij} in M.

Step 5: If the corresponding edge e_{ij} of selected S_{ij} produces a cycle with the previous marked entries of the score matrix S then set $S_{ij} = 0$ else mark S_{ij} .

Step 6: Construct the graph *T* including only the marked entries from the score matrix S which shall be desired minimum cost spanning tree of *G*.

4 Practical example

4.1 Example 1

In this section, a numerical example of IVNMST is used to demonstrate of the proposed algorithm. Consider the following graph G= (V, E) shown Figure 1, with fives nodes and seven edges. Various steps involved in the construction of the minimum cost spanning tree are described as follow –

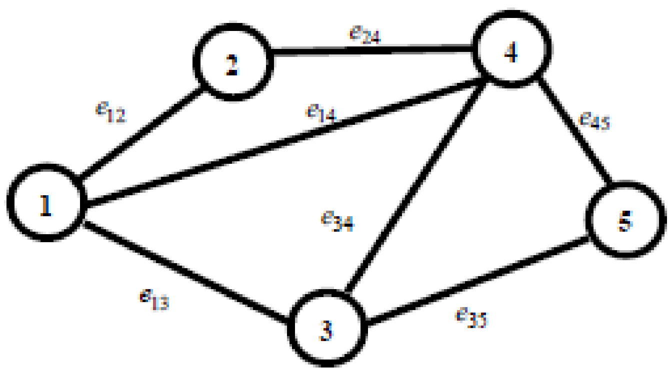


Fig.1. Undirected interval valued neutrosophic graphs

Table 1.

e_{ij}	**Edge length**
e_{12}	<[.3, .4], [.1, .2], <[.2, .4]>
e_{13}	<[.4, .5], [.2, .6], <[.4, .6]>
e_{14}	<[.1, .3], [.6, .8], <[.8, .9]>
e_{24}	<[.4, .5], [.8, .9], <[.3, .4]>
e_{34}	<[.2, .4], [.3, .4], <[.7, .8]>
e_{35}	<[.4, .5], [.6, .7], <[.5, .6]>
e_{45}	<[.5, .6], [.4, .5], <[.3, .4]

The interval valued neutrosophic adjacency matrix A is computed below:

$$A= \begin{bmatrix} 0 & \boldsymbol{e_{12}} & \boldsymbol{e_{13}} & \boldsymbol{e_{14}} & 0 \\ \boldsymbol{e_{12}} & 0 & 0 & \boldsymbol{e_{24}} & 0 \\ \boldsymbol{e_{13}} & 0 & 0 & \boldsymbol{e_{34}} & \boldsymbol{e_{35}} \\ \boldsymbol{e_{14}} & \boldsymbol{e_{24}} & \boldsymbol{e_{34}} & 0 & \boldsymbol{e_{45}} \\ 0 & 0 & \boldsymbol{e_{35}} & \boldsymbol{e_{45}} & 0 \end{bmatrix}$$

Applying the score function proposed by Tan [18], we get the score matrix:

$$S= \begin{bmatrix} 0 & 0.633 & 0.517 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & 0.583 & 0 \end{bmatrix}$$

In this matrix, the minimum entries 0.217 is selected and the corresponding edge (1, 4) is marked by the green color. Repeat the procedure until termination (Figure 2).

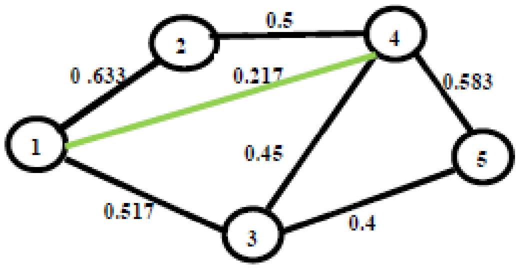


Fig.2. Marked interval valued neutrosophic graphs

The next non-zero minimum entries 0.4 is marked and corresponding edges (3, 5) are also colored (Figure 3).

$$S = \begin{bmatrix} 0 & 0.633 & 0.517 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & 0.583 & 0 \end{bmatrix}$$

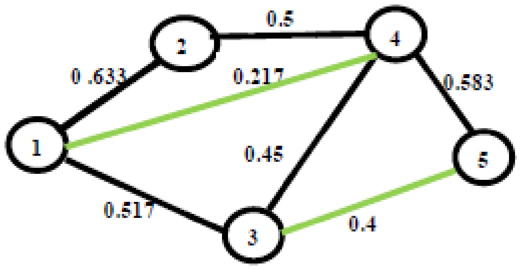


Fig. 3. Marked interval valued neutrosophic graphs in next iteration

$$S = \begin{bmatrix} 0 & 0.633 & 0.517 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & 0.583 & 0 \end{bmatrix}$$

The next non-zero minimum entries 0.45 is marked. The corresponding marked edges are portrayed in Figure 4.

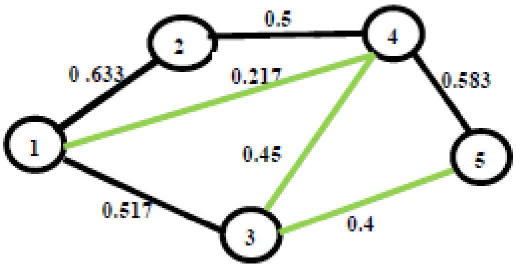


Fig. 4. Marked interval valued neutrosophic graphs in next iteration

$$S=\begin{bmatrix} 0 & 0.633 & 0.517 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & 0.583 & 0 \end{bmatrix}$$

The next non-zero minimum entries 0. 5 is marked. The corresponding marked edges are portrayed in Figure 5.

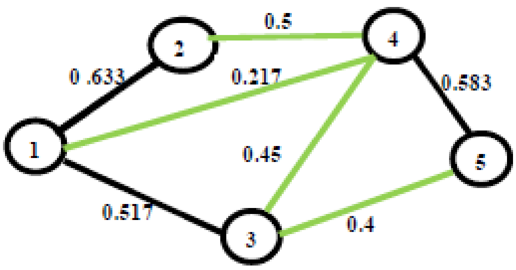


Fig. 5. Marked interval valued neutrosophic graphs in next iteration

$$S=\begin{bmatrix} 0 & 0.633 & \cancel{0.517}\ 0 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & 0.583 & 0 \end{bmatrix}$$

The next minimum non-zero element 0.517 is marked. However, while drawing the edges, it produces the cycle so we delete and mark it as 0 instead of 0.517 (Figure 6).

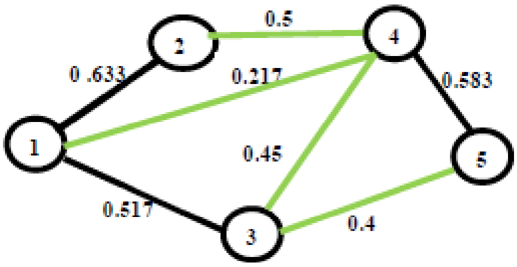


Fig. 6. Cycle {1, 3, 4}

$$S=\begin{bmatrix} 0 & 0.633 & \cancel{0.517}\ 0 & 0.217 & 0 \\ 0.633 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & \cancel{0.583}\ 0 & 0 \end{bmatrix}$$

The next minimum non-zero element 0.583 is marked. However, while drawing the edges, it produces the cycle so we delete and mark it as 0 instead of 0.583 (Figure 7).

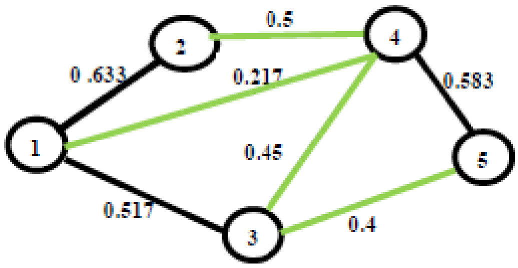


Fig. 7. Cycle {3, 4, 5}

$$S=\begin{bmatrix} 0 & 0.633 & \cancel{0.517}\ 0 & 0.217 & 0 \\ \cancel{0.633}\ 0 & 0 & 0 & 0.5 & 0 \\ 0.517 & 0 & 0 & 0.45 & 0.4 \\ 0.217 & 0.5 & 0.45 & 0 & 0.583 \\ 0 & 0 & 0.4 & \cancel{0.583}\ 0 & 0 \end{bmatrix}$$

The next minimum non-zero element 0.633 is marked. However, while drawing the edges, it produces the cycle so we delete and mark it as 0 instead of 0.633 (Figure 8).

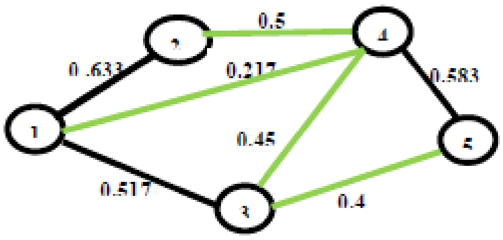


Fig. 8. Marked edges in the next round

Finally, we get the final path of minimum cost of spanning tree of G is portrayed in Figure 9.

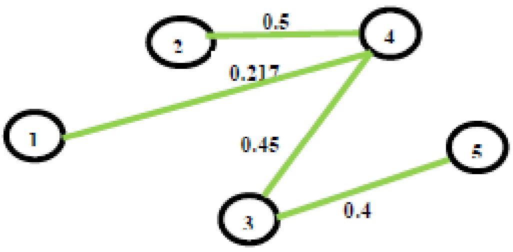
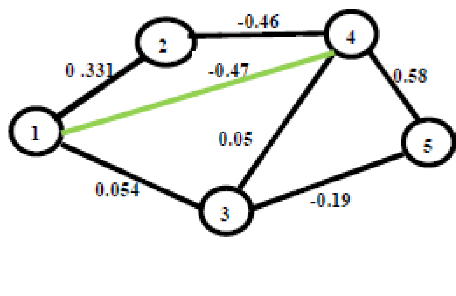
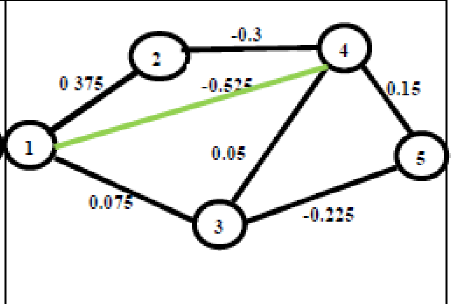
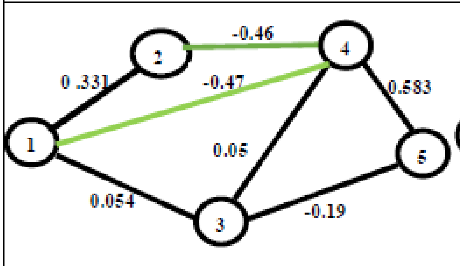
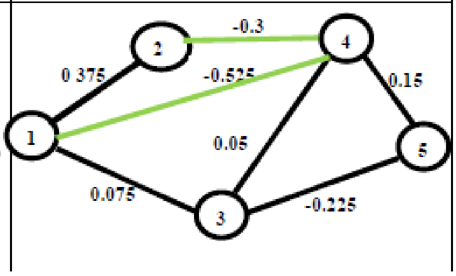
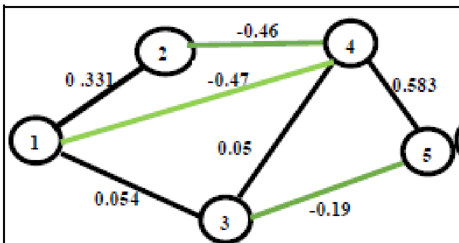
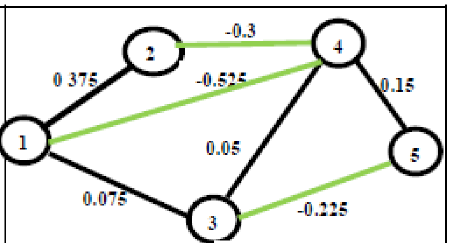
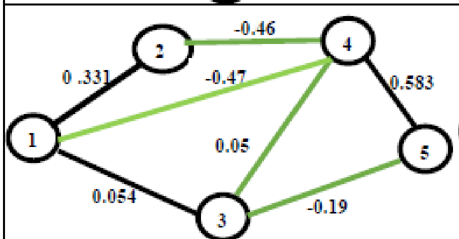
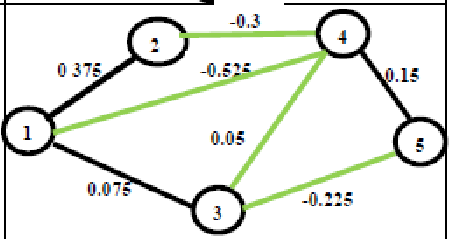
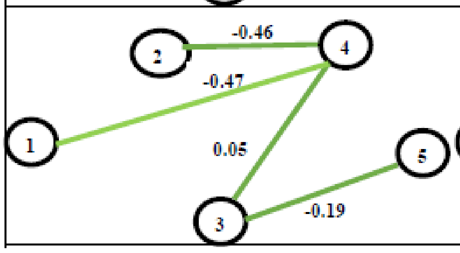
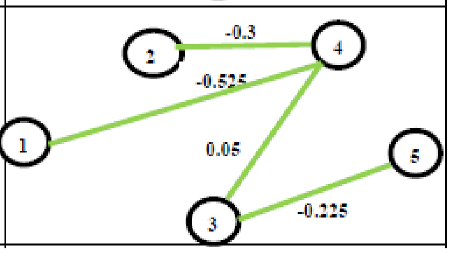


Fig. 9. Final path of minimum cost of spanning tree of the graph

And thus, the crisp minimum cost spanning tree is 1.567 and the final path of minimum cost of spanning tree is{2, 4},{4, 1},{4, 3},{3, 5}. The procedure is termination.

4.2 Example 2

The score function is used in machine learning involved in manipulating probabilities. Here the score functions in the proposed algorithm plays a vital role in identifying the minimum spanning tree of undirected interval valued neutrosophic graphs. Also based on the order of polynomial time computation the score function used are approaching towards different MST for an Neutrosophic graph. We compare our proposed method with these scoring methods used by different researchers and hence compute the MST of undirected interval valued neutrosophic graphs.

Using the score function proposed by Nancy [12], we get the score matrix	Using the score function proposed by RIDVAN [17], we get the score matrix
$S=\begin{bmatrix} 0 & 0.331 & 0.054 & -0.47 & 0 \\ 0.331 & 0 & 0 & -0.46 & 0 \\ 0.517 & 0 & 0 & 0.05 & -0.19 \\ -0.47 & -0.46 & 0.05 & 0 & 0.115 \\ 0 & 0 & -0.19 & 0.115 & 0 \end{bmatrix}$	$S=\begin{bmatrix} 0 & 0.375 & 0.075 & -0.525 & 0 \\ 0.375 & 0 & 0 & -0.3 & 0 \\ 0.075 & 0 & 0 & 0.05 & -0.225 \\ -0.525 & -0.3 & 0.05 & 0 & 0.15 \\ 0 & 0 & -0.225 & 0.15 & 0 \end{bmatrix}$
	
	
	
	
	

The crisp minimum cost spanning tree is -1.07 and the final path of minimum cost of spanning tree is{2, 4},{4, 1},{4, 3},{3, 5}. The procedure is termination.	The crisp minimum cost spanning tree is -1 and the final path of minimum cost of spanning tree is{2, 4},{4, 1},{4, 3},{3, 5}. The procedure is termination.

5 Comparative study

In what follows we compare the proposed method presented in section 4 with other existing methods including the algorithm proposed by Mullai et al [15] as follow

Iteration 1:

Let $C_1 = \{1\}$ and $\overline{C}_1 = \{2, 3, 4, 5\}$

Iteration 2:

Let $C_2 = \{1, 4\}$ and $\overline{C}_2 = \{2, 3, 5\}$

Iteration 3:

Let $C_3 = \{1, 4, 3\}$ and $\overline{C}_3 = \{2, 5\}$

Iteration 4:

Let $C_4 = \{1, 3, 4, 5\}$ and $\overline{C}_4 = \{2\}$

Finally, the interval valued neutrosophic minimal spanning tree is

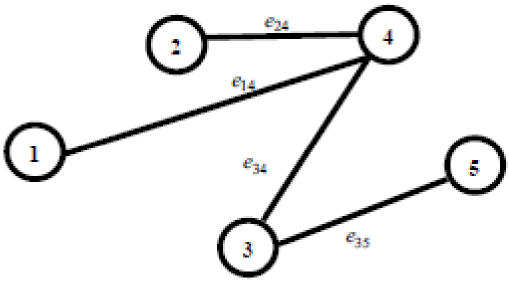


Fig .10. IVN minimal spanning tree obtained by Mullai's algorithm.

So, it can be seen that the interval valued neutrosophic minimal spanning tree {2, 4},{4, 1},{4, 3},{3, 5}.obtained by Mullai's algorithm, After deneutrosophication of edges'weight using the score function, is the same as the path obtained by proposed algorithm. The difference between the proposed algorithm and Mullai's algorithm is that our approach is based on Matrix approch, which can be easily implemented in Matlab, whereas the Mullai's algorithm is based on the comparison of edges in each iteration of the algorithm and this leads to high computation.

7 Conclusions and Future Work

This article analyse about the minimum spanning tree problem where the edges weights are represented by interval valued neutrosophic numbers. In the proposed algorithm, many examples were investigated on MST. The main objective of this study is to focus on algorithmic approach of MST in uncertain environment by using neutrosophic numbers as arc lengths. In addition, the algorithm we use is simple enough and more effective for real time environment. This work could be extended to the case of directed neutrosophic graphs and other kinds of neutrosophic graphs such as bipolar and interval valued bipolar neutrosophic graphs. In future, the proposed algorithm could be implemented to the real time scenarios in transportation and supply chain management in the field of operations research. On the other hand, graph interpretations (decision trees) of syllogistic logics and bezier curves in neutrosophic world could be considered and implemented as the real-life applications of natural logics and geometries of data [31-36].

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