IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received February 1, 2020, accepted February 21, 2020, date of publication February 27, 2020, date of current version March 11, 2020.
Digital Object Identifier 10.1109/ACCESS.2020.2976872

A New Group Decision Making Method With Distributed Indeterminacy Form Under Neutrosophic Environment: An Introduction to Neutrosophic Social Choice Theory

SELÇUK TOPAL[1], AHMET ÇEVIK[2], AND FLORENTIN SMARANDACHE[3]
[1]Department of Mathematics, Bitlis Eren University, 13000 Bitlis, Turkey
[2]Gendarmerie and Coast Guard Academy, 06805 Ankara, Turkey
[3]Department of Mathematics, The University of New Mexico–Gallup, Gallup, NM 87301, USA
Corresponding author: Selçuk Topal (s.topal@beu.edu.tr)

ABSTRACT We present a novel social choice theory based multi-criteria decision making method under neutrosophic environment and a new form of truth representation of neutrosophic theory called Distributed Indeterminacy Form (DIF). Our hybrid method consists of classical methods and an aggregation operator used in social choice theory. In addition to this, we also use DIF function to provide a more sensitive indeterminacy approach towards accuracy functions. We also consider reciprocal property for all individuals. This provides, as in intuitionistic fuzzy decision making theory, a consistent decision making for each individual. The solution approach presented in this paper in group decision making is treated under neutrosophic individual preference relations. These new approaches seem to be more consistent with natural human behaviour, hence should be more plausible and feasible. Moreover, the use of a similar approach to develop some *deeper soft* degrees of consensus is outlined. Finally, we give a Python implementation of our work in the Appendix section.

INDEX TERMS Neutrosophic logic, group decision making, neutrosophic preference relations, distributed indeterminacy form, social choice theory, neutrosophic social choice theory.

I. INTRODUCTION

In most cases, it is intricate for decision-makers to accurately reveal a preference when solving multi-criteria decision-making (MCDM) problems with imprecise, vague or incomplete information. Under these conditions, fuzzy sets (fs) [1], where the membership degree is represented by a real number in [0, 1], are viewed as a strong mechanism method for solving MCDM problems [2], as well as reasoning approximation and pattern recognition problems. However, fs cannot cope with particular situations where it is not easy to define the membership degree using a specific value. In order to obviate the absence of knowledge of non-membership degrees, Atanassov [3] introduced intuitionistic fuzzy sets (IFS), an extension of fs. IFS have been widely used in the solution of some significant MCDM problems [4]–[6], including multigranulation [7]–[12], neural networks [13], [14], and medical diagnosis problems [15]. Smarandache [16] introduced neutrosophic logic and neutrosophic sets (NS) and Rivieccio [17] later raised concern about that an NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity and it lies within $]^{-}0, 1^{+}[$, i.e. the non-standard unit interval. Clearly this is an extension of the standard interval [0, 1]. Furthermore, the uncertainty presented here, i.e. the indeterminacy factor, is dependent on the truth and falsity values, whereas the incorporated uncertainty is dependent on the degrees of belongingness and non-belongingness of IFS [18]. Recent studies show that neutrosophy can in fact be used in many applications. Ye [21]–[34], Lui and Wang [35], Lui *et al.* [36], Liu and Li [37], Liu and Shi [38], [39], Liu and Tang [40], Şahin and Liu [41], Chi and Liu [41], Biswas *et al.* [41], Biswas *et al.* [44]–[49], Monda and Pramanik [50]–[54], Peng *et al.* [55], Zhang *et al.* [56], [57], Peng *et al.* [58],

The associate editor coordinating the review of this manuscript and approving it for publication was Alba Amato.

Zhang *et al.* [59], [60], Tian *et al.* [61], [62], Ji *et al.* [63]–[65], Peng and Dai [66], Peng *et al.* [67], Peng and Liu [68], Peng and Dai [69], and Blin and Whinston [70] are some of the significant works on and introduced innovative methods on decision making under fuzzy and neutrosophic environments.

In this study, we propose to distribute the indeterminacy on truth and falsity to be aligned with real life applications and to take into consideration such situations in which uncertainty in social choices have an effective role in truth and falsity. We determine a rational social choice solely by the preferences of individuals in a society. A rational choice is possible only if every individual in the society is rational. Social choice theory investigates solutions to the problem of making a collective decision on a fair and democratic ground. The main purpose and subject area of social choice theory is to study the decision making problem for collectives to make a collective decision in a democratic manner. Of course our main concern will be to devise a method to make a cumulative decision rather than judging how fair the decisions of individuals are. The collective decision will manifest itself in neutrosophic values that the individuals give assignments to the preferences. Every individual is assumed to be able to assign to every preference some neutrosophic comparison value as pairs. We benefit from fuzzy and intuitionistic fuzzy social choice in solving the decision problems concerning neutrosophic social choice. Some well known works in fuzzy social choice and fuzzy decision making can be found in [70]–[75]. As for the intuitionistic fuzzy choice, we refer the reader to [76]–[78]. The advantage of our method is that we take care of Indeterminacy as well into neutrosophic social choice, while the previous methods involving fuzzy and intuitionistic fuzzy into social choice ignored the indeterminacy ? which is not accurate. This paper is about not only a classical decision making paper but also has a paper that considers decision making, truth maker theory and a new accuracy function interpretation (DIF). Addition to these, on the other hand, social choice theory under neutrosophic environment is studied for the first time, so we cannot compare other existing methods to the method in our paper. The comparison method is to cite some papers related to decision making. Many of the computational social choice theories that have been studied are based on rational individuals and their consistent preferences. Knowing the fact that the consistency of these pairwise comparisons forms the main theme, such theories devise appropriate methods based on the winner of the consensus of the group or based on an ordering of the preferences with respect to a priority as a result of voting of each individual. In any social choice, the consensus winner is defined as the choice of the dominant individual or the collective decision of rational individuals. The goal is to determine the best preference picked by the group. For the fuzzy solutions of finding a consensus, we refer the reader to Kacprzyk *et al.* [79]. We introduce a mathematical model for determining a consensus winner as a result of a collective decision, and in case of otherwise, we present a model which orders the preferences with respect to their weights. We also give an example in the last part of the paper to explain the model better. Compared with fuzzy and intuitionistic social choice theories, our model extends the social choice theory to neutrosophic based social choice theory in solving practical decision problems and present a richer language discourse.

II. FUNDAMENTAL DEFINITIONS

In classical set (cs) theory, an element either belongs to a set or not. The membership of elements in a set is interpreted in binary terms according to a divalent case. In fuzzy set theory, introduced by Zadeh [1], a gradual assessment of the membership of elements in a set is permitted by a membership function which takes values in the real unit interval [0, 1]. In fuzzy set theory, classical divalent sets are usually called crisp sets. Fuzzy set theory is a generalization of the classical set theory. IFS are sets whose elements have degrees of membership and non-membership. IFS have been introduced by Atanassov [3] as an extension of the notion of fuzzy set, which itself extends the classical notion of a set. Neutrosophic set theory is a generalization of IFS, CS, FS, paraconsistent set, dialetheist set, paradoxist set, tautological set based on Neutrosophy [16]. An element $x(T, I, F)$ belongs to the set in the following way: it is true in the set with a degree of $t \in [0, 1]$, indeterminate with a degree of $i \in [0, 1]$, and it is false with a degree of $f \in [0, 1]$.

We will now give some definitions of the fundamental concepts related to our study.

Definition 1 [1]: Given a universal set U and a generic element, denoted by x, a *fuzzy set* X in U is a set of ordered pairs defined as

$X = \{(x, \mu_X(x))|x \in U\}$, where $\mu_X : U \longmapsto [0, 1]$ is called the *membership function* of A and $\mu_X(x)$ is the *degree of membership* of the element x in X.

Definition 2 [3]: An *intuitionistic fuzzy set* X over a universe of discourse U is represented as

$X = \{(x, \mu_X(x), \nu_X(x))|x \in U\}$, where $\mu_X : U \longmapsto [0, 1]$ and $\nu_X : U \longmapsto [0, 1]$ are called respectively the *membership function* of A and the *non-membership function* of A for x in X. The *degree of non-membership* of the element x in X is defined as $\mu_X(x) = 1 - \nu_X(x)$.

Definition 3 [16], [19]: Let U be a universe of discourse. A *neutrosophic set* is defined as

$$N = \{(x, T(x), I(x), F(x)) : x \in U\},$$

which is identified by a *truth-membership function* $T_N : U \longmapsto]0^-, 1^+[$, *indeterminacy-membership function* $I_N : U \longmapsto]0^-, 1^+[$ and *falsity-membership function* $F_N : U \longmapsto]0^-, 1^+[$.

Definition 4 [16], [19]: Let U be a universe of discourse. A *single valued neutrosophic set* is defined as

$$N = \{(x, T(x), I(x), F(x)) : x \in U\},$$

which is identified by *a truth-membership function* $T_N : U \longmapsto [0, 1]$, *indeterminacy-membership function* $I_N : U \longmapsto [0, 1]$ and *falsity-membership function* $F_N : U \longmapsto [0, 1]$ with $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. A *single-valued neutrosophic number* (SVNN) is denoted by $a = (T, I, F)$.

Definition 5 [20]: Let a be a single-valued neutrosophic number. An *accuracy function* H of a single-valued neutrosophic number is represented as follows.

$$H(a) = \frac{1 + T_a - I_a(1 - T_a) - F_a(1 - I_a))}{2}, \qquad (1)$$

where for all a, $H(a) \in [0, 1]$. H is an order relation which gives an accuracy score of information of a. If $H(a_1) = H(a_2)$, then $a_1 = a_2$, that is, they have the same information. If $H(a_1) < H(a_2)$, then a_2 is larger than a_1.

III. ACCURACY FUNCTION AND DISTRIBUTED INDETERMINACY FORM

For a neutrosophic value, the accuracy function H is calculated by the values T, I and F. However, in the process of making a decision, such independent values may not yield results consistent with the decision-making process on objects. Suppose, one has truth, falsity and indeterminacy values applied on a concept. We cannot speak about truth by ignoring indeterminacy. The reason is that we make a decision on the basis of including indeterminacy and the truth-maker gives the values by taking into account the indeterminacy. Sorensen [80]–[82] who published many papers on truth-maker theory, buries the theory of indeterminacy in the truth-maker theory. By a similar approach, we desire to calculate the the accuracy function dependent on T and F, taking the indeterminacy into consideration. The direct application of this idea to neutrosophic decision making helps us to approximate the outcomes with a better precision by distributing the indeterminacy on neutrosophic values. Let H be an accuracy function. This time we reflect the indeterminacy value on the truth and falsity values in the following way: Let $a = (T_a, I_a, F_a)$ be a single valued neutrosophic number with truth value T_a, indeterminacy value I_a, and falsity value F_a. *Distributed Indeterminacy Form* (DIF) of a is defined as $a_{DIF} = (T_a - T_a I_a, 0, F_a - F_a I_a)$. Here, we distribute indeterminacy effect on truth and falsity. In other words, we decrease the power of truth and falsity in proportion to the magnitude of indeterminacy. Our aim here is to determine how the value of truth and falsity is affected by the degree of growth of indeterminacy. Consider the following case for the accuracy function H. Despite that $H(0.5, 0.5, 0.6) = 0.475$, we have that $H(0.5, 0.6, 0.6) = 0.48$. In other words, even though the precision should have been decreased when the indeterminacy increases, we observe the opposite here. This, at first might, may seem contradictory but the case will become clear in a moment. So DIF gives us a method to keep a neutrosophic number as small as possible in the ordering of the preferences in proportional to the increment of the indeterminacy value, provided that the truth or falsity values are fixed.

A. SELF COMPARISON

All comparisons on the same alternative should be assigned a balanced value by rational individuals. The values 0.5, (0.5, 0.5), and (0.5, 0.5, 0.5) are assigned respectively for self-comparison by individuals in fuzzy set, intuitionistic fuzzy set and neutrosophic set. Assigned self comparison of a neutrosophic value a is (0.5, 0.5, 0.5) and outcome of this number under H function is naturally $H(a) = 0.5$. The DIF of this value is $a_{DIF} = (0.25, 0, 0.25)$ and $H(0.25, 0, 0.25) = 0.375$. This in turn gives us a result quite different from self-comparison. One of the most important reasons that we introduce the distributed indeterminacy concept is the effect of indeterminacy over the other two values, i.e truth and falsity. Moreover, we would like to see this effect as a rational assignment in the self-comparison process, so we would like to use the triplet (0.5, 0, 0.5) instead of (0, 5, 0.5, 0.5). As it can be seen, we pull the indeterminacy factor down to zero. Moreover, the DIF of (0.5, 0, 0.5) is equal to itself, that is (0.5, 0, 0.5). Furthermore, the image of (0.5, 0, 0.5) under the function H takes the value 0.5, which is just the appropriate value for the self-comparison process.

IV. RECIPROCAL PROPERTY AND HESITATION FUNCTION

In this section, we will the define reciprocal property and hesitation function for neutrosophy theory by reviewing the properties and the functions in fuzzy and intuitionistic theories.

A. RECIPROCAL PROPERTY IN FUZZY THEORY

Reference [83] A *fuzzy preference relation* $R = (r_{ij})$ on a finite set of alternatives X is a relation in $X \times X$ which is characterised by the membership function $\mu_R : X \times X \longmapsto [0, 1]$. Pairwise comparisons concentrate on two alternatives at a time which enable individuals when giving their preferences. If an individual prefers an alternative x_i to another alternative x_j, then she/he should not simultaneously prefers x_j to x_i. Then, the numerical representation of the comparison of two alternatives is denoted by a reciprocal preference relation R as follows:

$$\begin{aligned} r_{ij} = 1 &\quad \Leftrightarrow \quad x_i \succ x_j \\ r_{ij} = 0 &\quad \Leftrightarrow \quad x_j \succ x_i \\ r_{ij} = 0.5 &\quad \Leftrightarrow \quad x_j \sim x_i \end{aligned}$$

In fuzzy social choice theory, we also see binary crisp preference relations or [0, 1]-valued (fuzzy) preference relations. $x_{ij} = 1$ shows the absolute degree of preference for x_i over x_j. A definite preference for x_i over x_j is $r_{ij} \in (0.5, 1)$. Indifference between x_i and x_j is $r_{ij} = 0.5$. Reciprocal [0, 1]-valued relations ($R = (r_{ij};\ \forall i, j : 0 \leq r_{ij} \leq 1, r_{ij} + r_{ji} = 1$) are widely used in fuzzy set theory for representing preferences.

B. RECIPROCAL PROPERTY AND HESITATION FUNCTION IN INTUITIONISTIC FUZZY THEORY

[76] An *intuitionistic fuzzy preference relation* P on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_P : X \times X \longrightarrow [0, 1]$ and a non-membership function $\nu_P : X \times X \longrightarrow$ such that $0 \leq \mu_P(x_i, x_j) + \nu_P(x_i, x_j) \leq 1$, $\forall (x_i, x_j) \in X \times X$. As in

the case for fuzzy preference relation, an *intuitionistic fuzzy preference relation* is represented by the matrix $P = (p_{ij})$ with $p_{ij} =< \mu_{ij}, \nu_{ij} >, \forall i, j = 1, 2, \ldots, n$. Obviously, when the hesitancy function is the null function we have that $\mu_{ij} + \nu_{ij} = 1$ $(\forall i, j)$, and the intuitionistic fuzzy preference relation $P = (p_{ij})$ is mathematically equivalent to the reciprocal fuzzy preference relation $R = (r_{ij})$, with $r_{ij} = \mu_{ij}$. An intuitionistic fuzzy preference relation is referred to as *reciprocal* when the following additional conditions are imposed:

$$(i)\ \mu_{ii} = \nu_{ii} = 0.5, \quad \forall i \in \{1, \ldots, n\}$$
$$(ii)\ \mu_{ij} = \nu_{ji}, \quad \forall i, j \in \{1, \ldots, n\}.$$

In intuitionistic fuzzy studies, the relations do not need to have reciprocity but must satisfy $r_{ij} \leq 1 - r_{ji}$ due to intuitionistic index. In other words, for an IFS A, $\pi_A(x)$ is determined by the following expression: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the *hesitancy degree* of the element $x \in X$ to the set A, and $\pi_A(x) \in [0, 1], \forall x \in X$.

C. RECIPROCAL PROPERTY AND HESITATION FUNCTION IN NEUTROSOPHY THEORY

Let $S = \{s_1, s_2, s_3, \ldots, s_n\}$ be a set of alternatives (or options) and m be a set of individuals. Each individual declares his or her own preferences over S which are represented by an individual neutrosophic preference relation R_k such that

$$N_{R_k} : S \times S \longmapsto [0, 1] \times [0, 1] \times [0, 1]$$

which is traditionally represented by a matrix $R_k = [r_{ij}^k = N_{R_k}(r_i^k, r_j^k)], i, j = 1, 2, 3, .., n; k = 1, 2, 3, \ldots, m$.

$$R_k = \begin{bmatrix} (0.5, 0.5, 0.5) & r_{12}^k & r_{13}^k & r_{14}^k \\ r_{21}^k & (0.5, 0.5, 0.5) & r_{23}^k & r_{24}^k \\ r_{31}^k & r_{32}^k & (0.5, 0.5, 0.5) & r_{33}^k \\ r_{41}^k & r_{42}^k & r_{43}^k & (0.5, 0.5, 0.5) \end{bmatrix}$$

The matrix above shows that neutrosophic preferences of an individual k are among s_1, s_2, s_3, s_4. Also that $N_{R_k}(s_1, s_1) = N_{R_k}(s_2, s_2) = N_{R_k}(s_3, s_3) = N_{R_k}(s_4, s_4) = (0.5, 0.5, 0.5)$, $N_{R_k}(s_1, s_2) = r_{12}^k$, $N_{R_k}(s_3, s_4) = r_{34}^k$, etc. We require that there is no larger outcome when an alternative is compared to itself. Almost all studies in the literature on decision making assign no value or assign zero degree to their underlying discourse for self-comparisons. We follow a entirely computational approach here. On the other hand though, zeros given in other previous studies may lead us have a false perception to compare any s_i. For a neutrosophic preference function mu, if $mu(s_i, s_j) = 0$, then s_i is definitely larger than s_j. If we had a rational individual, $mu(s_i, s_i)$ would have been 0.5, since if we do self-comparison, an alternative can not have any advantage over itself. We use the H function in Definition 2.5 for preciseness and to act as a neutrosophic index of SVNNs. If $i = j$, then we take $N_{R_k}(s_i, s_j)$ to be $(0.5, 0.5, 0.5)$ without DIF, and $(0.5, 0, 0.5)$ with DIF. So, we have the following matrix:

$$R_k = \begin{bmatrix} (0.5, 0, 0.5) & r_{12}^k & r_{13}^k & r_{14}^k \\ r_{21}^k & (0.5, 0, 0.5) & r_{23}^k & r_{24}^k \\ r_{31}^k & r_{32}^k & (0.5, 0, 0.5) & r_{33}^k \\ r_{41}^k & r_{42}^k & r_{43}^k & (0.5, 0, 0.5) \end{bmatrix}$$

The function H (called *neutrosophic index* or *neutrosophic hesitation function*) assigns each a_{ij} neutrosophic value to a number in [0, 1].

We have that

$$H(a_{ij}) = \frac{1 + T(a_{ij}) - I(a_{ij})(1 - T(a_{ij})) - F(a_{ij})(1 - I(a_{ij}))}{2} \tag{2}$$

Now, we have a new matrix $R_k^H = [H(r_{ij}^k) = H^k(N_{R_k}(s_i, s_j))]$, where $i, j = 1, 2, 3, \ldots, n$; $k = 1, 2, 3, \ldots, m$. More explicitly,

$$R_k^H = \begin{bmatrix} H((0.5, 0, 0.5)) & H(r_{12}^k) & H(r_{13}^k) & H(r_{14}^k) \\ H(r_{21}^k) & H((0.5, 0, 0.5)) & H(r_{23}^k) & H(r_{24}^k) \\ H(r_{31}^k) & H(r_{32}^k) & H((0.5, 0, 0.5)) & H(r_{34}^k) \\ H(r_{41}^k) & H(r_{42}^k) & H(r_{43}^k) & H((0.5, 0, 0.5)) \end{bmatrix}$$

We find it more appropriate to use the notion of *hesitation* in order to have consistency between the choosers (individuals) and their preference. Here, we benefit from the IFS. In utilizing IFS, we provide a hybrid account of the neutrosophic accuracy function by hesitation. We adopt intuitionistic index in our study since we use the function H as a solid index throughout the paper. Not every $H^k(r_{ij})$ needs to be reciprocal, i.e. $H^k(r_{ij}) \neq 1 - H^k(r_{ji})$ but should be quasi-reciprocal. That is, $H(r_{ij}^k) \leq 1 - H(r_{ji}^k)$, for each $i, j = 1, \ldots, n$. If k is not quasi-reciprocal, we call k an *irrational individual*. If $i = j$, then we just take $N_{Rk}(a_i, a_j) = (0.5, 0.5, 0.5)$ since $H((0.5, 0.5, 0.5)) = 0.5$ irrespective of DIF. Furthermore, when we consider DIF, the neutrosophic value of the assignment made by a rational individual on the same preference is $(0.5, 0, 0.5)$ from now on, and $H((0.5, 0, 0.5)) = 0.5$ as desired.

$$DIF(R_k) = \begin{bmatrix} (0.5, 0, 0.5) & DIF(r_{12}^k) & DIF(r_{13}^k) & DIF(r_{14}^k) \\ DIF(r_{21}^k) & (0.5, 0, 0.5) & DIF(r_{23}^k) & DIF(r_{24}^k) \\ DIF(r_{31}^k) & DIF(r_{32}^k) & (0.5, 0, 0.5) & DIF(r_{33}^k) \\ DIF(r_{41}^k) & DIF(r_{42}^k) & DIF(r_{43}^k) & (0.5, 0, 0.5) \end{bmatrix}$$

R_i : preference matrix of the ith individual,

$DIF(R_i)$: DIF of preference matrix of the ith individual,

R_i^H : range of preference matrix of the ith individual under H function,

$r_k^H(ij)$: represents the element at the row i and column j of R_i^H for individual k,

$h^k(ij)$: distribution of the kth individual's votes for each pairwise comparison of alternative's value is determined through 0.5 derived from R_i^H,

$[[h^k]]$: the matrix obtained by each element of $h^k(ij)$,
$[[H_{ij}]]$: matrix of the group vote,
A_k : the degree for preference k assigned by the group,
a_{ij}^k : majority determination value for preference k of the group (the element at the row i and column j of $[[h^k]]$),
H_{ij}^k : majority determination value for preference k of the group under H function,

$$h^k(ij) = \begin{cases} 1, & r_k^H(ij) > 0.5 \\ 0, & otherwise \end{cases}$$

$H_{\pi_{ij}}$: average majority determination value of the group under H function,
H_π : consensus winner determination matrix,
$C(s_i)$: social aggregation function for the alternative (preference) s_i,

Example 6: Suppose that there are three experts m_1, m_2, m_3 and four facilities s_1, s_2, s_3, s_4 in the same business industry. We assume that all experts are rational and so we assume all neutrosophic values satisfy quasi-reciprocal property. We also take the self-comparison value to be (0.5, 0, 0.5). Each expert assigns some neutrosophic opinion value by comparing the facilities in pairs as follows:

R_{m_i} is the set of assigned values (preferences) by m_i to pairs in the facilities where $1 \leq i \leq 3$.

$$\begin{aligned} R_{m_1} &= \{(s_1, s_1) = (0.5, 0, 0.5), (s_1, s_2) = (0.45, 0.24, 0.27), \\ &(s_1, s_3) = (0.31, 0.14, 0.66), (s_1, s_4) = (0.8, 0.3, 0), \\ &(s_2, s_1) = (0.1, 0.45, 0.52), (s_2, s_2) = (0.5, 0, 0.5), \\ &(s_2, s_3) = (0.48, 0.26, 0.37), (s_2, s_4) = (0.2, 0.7, 0.8), \\ &(s_3, s_1) = (0.61, 0.43, 0.71), (s_3, s_2) = (0.31, 0, 0.71), \\ &(s_3, s_3) = (0.5, 0, 0.5), (s_3, s_4) = (0.76, 0.23, 0.27), \\ &(s_4, s_1) = (0.1, 0.6, 0.9), (s_4, s_2) = (0.81, 0.55, 0.33), \\ &(s_4, s_3) = (0.11, 0.32, 0.59), (s_4, s_4) = (0.5, 0, 0.5)\} \end{aligned}$$

$$\begin{aligned} R_{m_2} &= \{(s_1, s_1) = (0.5, 0, 0.5), (s_1, s_2) = (0.2, 0.4, 0.7), \\ &(s_1, s_3) = (0.21, 0.55, 0.95), (s_1, s_4) = (0.4, 0.5, 0.3), \\ &(s_2, s_1) = (0.29, 0.53, 0.38), (0.29, 0.53, 0.38), \\ &(s_2, s_2) = (0.5, 0, 0.5), (s_2, s_3) = (0.62, 0.45, 0.16), \\ &(s_2, s_4) = (0.2, 0.7, 0.8), (s_3, s_1) = (0.72, 0.15, 0.18), \\ &(s_3, s_2) = (0.11, 0.13, 0.79), (s_3, s_3) = (0.5, 0, 0.5), \\ &(s_3, s_4) = (0.51, 0.45, 0.53), (s_4, s_1) = (0.15, 0.35, 0.23), \\ &(s_4, s_2) = (0.81, 0.55, 0.33), (s_4, s_3) = (0.17, 0.57, 0.36), \\ &(s_4, s_4) = (0.5, 0, 0.5)\} \end{aligned}$$

$$\begin{aligned} R_{m_3} &= \{(s_1, s_1) = (0.5, 0, 0.5), (s_1, s_2) = (0.3, 0.45, 0.7), \\ &(s_1, s_3) = (0.1, 0.85, 0.78), (s_1, s_4) = (0.4, 0.5, 0.3), \\ &(s_2, s_1) = (0.36, 0.51, 0.39), (s_2, s_2) = (0.5, 0, 0.5), \\ &(s_2, s_3) = (0.62, 0.45, 0.16), (s_2, s_4) = (0.1, 0.8, 0.21), \\ &(s_3, s_1) = (0.92, 0.1, 0.16), (s_3, s_2) = (0.11, 0.13, 0.79), \\ &(s_3, s_3) = (0.5, 0, 0.5), (s_3, s_4) = (0.23, 0.45, 0.74), \\ &(s_4, s_1) = (0.15, 0.35, 0.23), (s_4, s_2) = (0.6, 0.2, 0.1), \\ &(s_4, s_3) = (0.57, 0.57, 0.36), (s_4, s_4) = (0.5, 0, 0.5)\} \end{aligned}$$

$$\begin{aligned} R_{m_4} &= \{(s_1, s_1) = (0.5, 0, 0.5), (s_1, s_2) = (0.2, 0.4, 0.7), \\ &(s_1, s_3) = (0.25, 0.87, 0.38), (s_1, s_4) = (0.4, 0.5, 0.3), \\ &(s_2, s_1) = (0.29, 0.53, 0.38), (s_2, s_2) = (0.5, 0, 0.5), \\ &(s_2, s_3) = (0.62, 0.45, 0.16), (s_2, s_4) = (0.34, 0.66, 0.21), \\ &(s_3, s_1) = (0.73, 0.87, 0.56), (s_3, s_2) = (0.14, 0.19, 0.79), \\ &(s_3, s_3) = (0.5, 0, 0.5), (s_3, s_4) = (0.21, 0.45, 0.66), \\ &(s_4, s_1) = (0.16, 0.35, 0.23), (s_4, s_2) = (0.6, 0.4, 0.8), \\ &(s_4, s_3) = (0.68, 0.57, 0.36), (s_4, s_4) = (0.5, 0, 0.5)\} \end{aligned}$$

We now represent each R_{m_i} in matrix form and then calculate their distributed indeterminacy forms $DIF(R_{m_i})$.

$$R_{m_1} = \begin{bmatrix} (0.5, 0, 0.5) & (0.45, 0.24, 0.27) & (0.31, 0.14, 0.66) & (0.8, 0.3, 0) \\ (0.1, 0.45, 0.52) & (0.5, 0, 0.5) & (0.48, 0.26, 0.37) & (0.2, 0.7, 0.8) \\ (0.61, 0.43, 0.71) & (0.31, 0, 0.71) & (0.5, 0, 0.5) & (0.76, 0.23, 0.27) \\ (0.1, 0.6, 0.9) & (0.81, 0.55, 0.33) & (0.11, 0.32, 0.59) & (0.5, 0, 0.5) \end{bmatrix}$$

$$DIF(R_{m_1}) = \begin{bmatrix} (0.5, 0.5, 0.5) & (0.342, 0, 0.2052) & (0.2666, 0, 0.5676) & (0.56, 0, 0) \\ (0.055, 0, 0.286) & (0.5, 0, 0.5) & (0.3552, 0, 0.2738) & (0.06, 0, 0.24) \\ (0.3477, 0, 0.4047) & (0.31, 0, 0.71) & (0.5, 0, 0.5) & (0.5852, 0, 0.2079) \\ (0.04, 0, 0.36) & (0.3645, 0, 0.1485) & (0.0748, 0, 0.4012) & (0.5, 0, 0.5) \end{bmatrix}$$

$$R_{m_2} = \begin{bmatrix} (0.5, 0, 0.5) & (0.2, 0.4, 0.7) & (0.21, 0.55, 0.95) & (0.4, 0.5, 0.3) \\ (0.29, 0.53, 0.38) & (0.5, 0, 0.5) & (0.62, 0.45, 0.16) & (0.2, 0.7, 0.8) \\ (0.72, 0.15, 0.18) & (0.11, 0.13, 0.79) & (0.5, 0, 0.5) & (0.51, 0.45, 0.53) \\ (0.15, 0.35, 0.23) & (0.81, 0.55, 0.33) & (0.17, 0.57, 0.36) & (0.5, 0, 0.5) \end{bmatrix}$$

$$DIF(R_{m_2}) = \begin{bmatrix} (0.5, 0.5, 0.5) & (0.12, 0, 0.42) & (0.0945, 0, 0.4275) & (0.2, 0, 0.15) \\ (0.1363, 0, 0.1786) & (0.5, 0, 0.5) & (0.341, 0, 0.088) & (0.06, 0, 0.24) \\ (0.612, 0, 0.153) & (0.0957, 0, 0.6873) & (0.5, 0, 0.5) & (0.2805, 0, 0.2915) \\ (0.0975, 0, 0.1495) & (0.3645, 0, 0.1485) & (0.0731, 0, 0.1548) & (0.5, 0, 0.5) \end{bmatrix}$$

$$R_{m_3} = \begin{bmatrix} (0.5, 0, 0.5), (0.3, 0.45, 0.7), (0.76, 0.35, 0.38), (0.4, 0.5, 0.3) \\ (0.36, 0.51, 0.39), (0.5, 0, 0.5), (0.62, 0.45, 0.16), (0.46, 0.46, 0.21) \\ (0.92, 0.86, 0.35), (0.11, 0.13, 0.79), (0.5, 0, 0.5), (0.23, 0.45, 0.74) \\ (0.15, 0.35, 0.23), (0.6, 0.4, 0.8), (0.57, 0.57, 0.36), (0.5, 0, 0.5) \end{bmatrix}$$

$$DIF(R_{m_3}) = \begin{bmatrix} (0.5, 0, 0.5) & (0.165, 0, 0.385) & (0.494, 0, 0.247) & (0.2, 0, 0.15) \\ (0.1764, 0, 0.1911) & (0.5, 0, 0.5) & (0.341, 0, 0.088) & (0.2484, 0, 0.1134) \\ (0.1288, 0, 0.049) & (0.0957, 0, 0.6873) & (0.5, 0, 0.5) & (0.1265, 0, 0.407) \\ (0.0975, 0, 0.1495) & (0.36, 0, 0.48) & (0.2451, 0, 0.1548) & (0.5, 0, 0.5) \end{bmatrix}$$

$$R_{m_4} = \begin{bmatrix} (0.5, 0, 0.5), (0.2, 0.4, 0.7), (0.51, 0.35, 0.38), (0.4, 0.5, 0.3) \\ (0.29, 0.53, 0.38), (0.5, 0, 0.5), (0.62, 0.45, 0.16), (0.34, 0.66, 0.21) \\ (0.73, 0.87, 0.56), (0.14, 0.19, 0.79), (0.5, 0, 0.5), (0.21, 0.45, 0.66) \\ (0.16, 0.35, 0.23), (0.6, 0.4, 0.8), (0.68, 0.57, 0.36), (0.5, 0, 0.5) \end{bmatrix}$$

$$DIF(R_{m_4}) = \begin{bmatrix} (0.5, 0, 0.5) & (0.12, 0, 0.42) & (0.3315, 0, 0.247) & (0.2, 0, 0.15) \\ (0.1363, 0, 0.1786) & (0.5, 0, 0.5) & (0.341, 0, 0.088) & (0.1156, 0, 0.0714) \\ (0.0949, 0, 0.0728) & (0.1134, 0, 0.6399) & (0.5, 0, 0.5) & (0.1155, 0, 0.363) \\ (0.104, 0, 0.1495) & (0.36, 0, 0.48) & (0.2924, 0, 0.1548) & (0.5, 0, 0.5) \end{bmatrix}$$

Now we apply the H function to $DIF(R_i)$ and then obtain R_i^H.

$$R_{m_1}^H = \begin{bmatrix} 0.5 & 0.5684 & 0.3495 & 0.78 \\ 0.3844 & 0.5 & 0.5407 & 0.41 \\ 0.4715 & 0.3 & 0.5 & 0.6886 \\ 0.34 & 0.608 & 0.3368 & 0.5 \end{bmatrix}$$

$$h^{m_1}(ij) = \begin{cases} 1, & r_{m_1}^H(ij) > 0.5 \\ 0, & otherwise \end{cases}$$

$$[[h^{m_1}]] = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$R_{m_2}^H = \begin{bmatrix} 0.5 & 0.35 & 0.3335 & 0.525 \\ 0.4788 & 0.5 & 0.6265 & 0.41 \\ 0.7295 & 0.2041 & 0.5 & 0.4945 \\ 0.474 & 0.474 & 0.4591 & 0.5 \end{bmatrix}$$

$$h^{m_2}(ij) = \begin{cases} 1, & r^H_{m_2}(ij) > 0.5 \\ 0, & otherwise \end{cases}$$

$$[[h^{m_2}]] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^H_{m_3} = \begin{bmatrix} 0.5 & 0.39 & 0.6234 & 0.525 \\ 0.4926 & 0.5 & 0.6265 & 0.5675 \\ 0.5399 & 0.2041 & 0.5 & 0.35975 \\ 0.474 & 0.4399 & 0.54515 & 0.5 \end{bmatrix}$$

$$h^{m_3}(ij) = \begin{cases} 1, & r^H_{m_3}(ij) > 0.5 \\ 0, & otherwise \end{cases}$$

$$[[h^{m_3}]] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R^H_{m_4} = \begin{bmatrix} 0.5 & 0.35 & 0.5422 & 0.525 \\ 0.4788 & 0.5 & 0.6265 & 0.5221 \\ 0.511 & 0.2367 & 0.5 & 0.3762 \\ 0.477 & 0.439 & 0.5688 & 0.5 \end{bmatrix}$$

$$h^{m_4}(ij) = \begin{cases} 1, & r^H_{m_4}(ij) > 0.5 \\ 0, & otherwise \end{cases}$$

$$[[h^{m_4}]] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The next step is to collect and compare the preferences. To do this, we add the columns of $[[H_{ij}]]$ and divide it to number of the alternatives.

$$A_k = \frac{1}{m}\sum [[H_{ik}]]$$

such that $1 \leq k \leq m$

$$H_{\pi_{ij}} = \begin{cases} \frac{1}{m}\sum_{k=1}^{m} a^k_{ij}, & i \neq j \\ 0, & i = j \end{cases}$$

such that $i, j = 1, 2, \ldots n$ and $k = 1, 2, \ldots, m$.

$$H_{\pi_{12}} = \frac{a^{m_1}_{12} + a^{m_2}_{12} + a^{m_3}_{12} + a^{m_4}_{12}}{4} = \frac{1+0+0+0}{4} = \frac{1}{4},$$
$$H_{\pi_{13}} = \frac{1}{2}, \quad H_{\pi_{14}} = 1, \; H_{\pi_{21}} = 0, H_{\pi_{23}} = 1, H_{\pi_{24}} = \frac{1}{2},$$
$$H_{\pi_{31}} = \frac{3}{4}, \quad H_{\pi_{32}} = 0, \; H_{\pi_{34}} = \frac{1}{4}, H_{\pi_{41}} = 0, H_{\pi_{42}} = \frac{1}{4},$$
$$H_{\pi_{43}} = \frac{1}{2}$$

$$H_\pi = \begin{bmatrix} - & \frac{1}{4} & \frac{1}{2} & 1 \\ 0 & - & 1 & \frac{1}{2} \\ \frac{3}{4} & 0 & - & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & - \end{bmatrix}$$

We now define the notion of a consensus winner.

Definition 7 [74]: $s_i \in W$ is called a *consensus winner* if and only if $\forall s_j \neq s_i : r_{ij} > 0.5$, where $r_{ij} \in H_\pi$.
In our example above, there is no winner because there are multiple numbers greater than 0.5. If there is a consensus winner, it must be unique and the set W must be a singleton since the reciprocal property must hold. Of course, it is easy to define that α-consensus winner for different α-values. So we define a *social aggregation average function* C to calculate the order of s_i in the group to the extent that individuals are not against option s_i.

$$C(s_i) = \frac{1}{m-1}\sum_{i \neq j} r_{ij}, \tag{3}$$

where $i, j = 1, 2, \ldots, m$.

$$C(s_1) = \frac{7}{12}, \quad C(s_2) = \frac{6}{12}, \; C(s_3) = \frac{5}{12}, C(s_4) = \frac{3}{12}.$$

So, $C(s_1) > C(s_2) > C(s_3) > C(s_4)$.

V. CONCLUSION

The main aim of this paper is to bring into attention the interplay between neutrosophy and social choice theory. Within the framework of this intention, we have taken inheritance from studies on fuzzy and intuitionistic fuzzy social choice theory and developed *the neutrosophic based social choice theory*. First we defined the *DIF*, which was used in Sorensen's truth-maker theory to distribute the indeterminacy on truth and falsity values for certain neutrosophic calculations. We believe that the notion of *DIF* gives a new insight, breath and different perspectives for neutrosophic studies. Through *DIF*, we emphasize hesitation and reciprocal characteristics in self-comparisons and other pairwise comparisons to define a consistent decision maker. We determine a consensus winner if exists. In case of otherwise, we obtain orders of the given alternatives by defining a social aggregation average function. Finally we give in the Appendix, a Python implementation of an algorithm computing the output in the order of $\frac{n}{11}$ seconds, where n is the input size (the number of matrices), when executed in a mid-end computer.

A. FURTHER RESEARCH DIRECTIONS

Some future researches to extend and diversify this work may include the following ideas:

- studying the quantifiers *most, at most,* etc [86],
- considering interval valued neutrosophic sets [87],
- considering bipolar valued neutrosophic sets [88],
- introducing different forms of DIF depending on underlying models,
- presenting several forms of aggregation operators [89],
- applications on plithogenic sets [90].
- applications on Maclaurin symmetric mean, q-rung orthopair 2-tuple linguistic aggregation and continuous interval-valued Pythagorean operators [91]- [93].

B. AUTHOR CONTRIBUTIONS

Conceptualization, S.T.; Methodology, S.T.,; Validation, S.T., A.C., F.S.; Investigation, S.T.; Resources, S.T., A.C., F.S.; Writing-Original Draft Preparation, S.T. and A.C; Writing-Review and Editing, S.T., A.C., F.S.; Supervision, F.S.

APPENDIX

A Python implementation [84], [85] of the group decision making method with distributed indeterminacy form under neutrosophic environment is as follows:

```
from __future__ import division
from collections import defaultdict
import math
import sys


R1=[ [ (0.5,0,0.5),(0.45,0.24,0.27), (0.31,0.14,0.66) , (0.8,0.3,0)],
     [(0.1,0.45,0.52) , (0.5,0,0.5), (0.48,0.26,0.37) , (0.2,0.7,0.8)],
     [(0.61,0.43,0.71) , (0.31,0,0.71) , (0.5,0,0.5)  , (0.76,0.23,0.27)],
     [(0.1,0.6,0.9) , (0.81,0.55,0.33) , (0.11,0.32,0.59) , (0.5,0,0.5)]]


R2=[ [ (0.5,0,0.5),(0.2,0.4,0.7), (0.21,0.55,0.95) , (0.4,0.5,0.3)],
     [(0.29,0.53,0.38) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.2,0.7,0.8)],
     [(0.72,0.15,0.18) , (0.11,0.13,0.79)  , (0.5,0,0.5)  , (0.51,0.45,0.53)],
     [(0.15,0.35,0.23) , (0.81,0.55,0.33) , (0.17,0.57,0.36) , (0.5,0,0.5)] ]


R3=[ [ (0.5,0,0.5),(0.3,0.45,0.7), (0.1,0.85,0.78) , (0.4,0.5,0.3)],
     [(0.36,0.51,0.39) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.1,0.8,0.21)],
     [(0.92,0.1,0.16) , (0.11,0.13,0.79)  , (0.5,0,0.5)  , (0.23,0.45,0.74)],
     [(0.15,0.35,0.23) , (0.6,0.2,0.1) , (0.57,0.57,0.36) , (0.5,0,0.5)] ]


R4=[ [ (0.5,0,0.5),(0.2,0.4,0.7), (0.25,0.87,0.38) , (0.4,0.5,0.3)],
     [(0.29,0.53,0.38) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.34,0.66,0.21)],
     [(0.73,0.87,0.56) , (0.14,0.19,0.79)  , (0.5,0,0.5)  , (0.21,0.45,0.66)],
     [(0.16,0.35,0.23) , (0.6,0.4,0.8) , (0.68,0.57,0.36) , (0.5,0,0.5)] ]


AllTogether= {'R1': [[ (0.5,0,0.5),(0.45,0.24,0.27), (0.31,0.14,0.66) , (0.8,0.3,0)],
                     [(0.1,0.45,0.52) , (0.5,0,0.5), (0.48,0.26,0.37), (0.2,0.7,0.8)],
     [(0.61,0.43,0.71) , (0.31,0,0.71) , (0.5,0,0.5)  , (0.76,0.23,0.27)],
                     [(0.1,0.6,0.9) , (0.81,0.55,0.33) , (0.21,0.32,0.59) , (0.5,0,0.5)]],
    'R2': [[ (0.5,0,0.5),(0.2,0.4,0.7), (0.21,0.55,0.95) , (0.4,0.5,0.3)],
           [(0.29,0.53,0.38) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.83,0.46,0.21)],
    [(0.72,0.15,0.18) , (0.11,0.13,0.79)  , (0.5,0,0.5)  , (0.51,0.45,0.53)],
           [(0.15,0.35,0.23) , (0.6,0.4,0.8) , (0.47,0.57,0.36) , (0.5,0,0.5)]],
    'R3': [[ (0.5,0,0.5),(0.3,0.45,0.7), (0.76,0.35,0.38) , (0.4,0.5,0.3)],
           [(0.36,0.51,0.39) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.46,0.46,0.21)],
           [(0.92,0.86,0.35) , (0.11,0.13,0.79)  , (0.5,0,0.5)  , (0.23,0.45,0.74)],
           [(0.15,0.35,0.23) , (0.6,0.4,0.8) , (0.57,0.57,0.36) , (0.5,0,0.5)]],
    'R4': [[ (0.5,0,0.5),(0.2,0.4,0.7), (0.51,0.35,0.38) , (0.4,0.5,0.3)],
           [(0.29,0.53,0.38) , (0.5,0,0.5), (0.62,0.45,0.16) , (0.34,0.66,0.21)],
           [(0.73,0.87,0.56) , (0.14,0.19,0.79)  , (0.5,0,0.5)  , (0.21,0.45,0.66)],
           [(0.16,0.35,0.23) , (0.6,0.4,0.8) , (0.68,0.57,0.36) , (0.5,0,0.5)]]}


def AccuracyFunction(T,I,F):

    HV= (1+ T - I*(1-T)-F*(1-I))/2
    return HV


def DIF(T,I,F):
    T1=math.fabs(T-I*T)
    F1=math.fabs(F-I*F)

    DIFi='('+str(T1)+','+str(0)+','+str(F1)+')'
    return DIFi


def AccuracyIntedeteminacyDistubition(T,I,F):
    T1=math.fabs(T-I*T)
    F1=math.fabs(F-I*F)

    ID=AccuracyFunction(T1,I,F1)
    return ID


def RationalityChecker(R):
    columnR=len(R)
    idn=0

    rowR=len(R[0])
    for i in range(0,rowR-1):
        if R[i][i] != (0.5, 0, 0.5):
            print '(,',i,i,") is not (0.5, 0, 0.5), so, s ",i, ' is not rational agent'
            idn=1


    for i in range(0,rowR):
        for j in range(0,rowR):
            if i !=j:
                t1=R[i][j][0]
                i1=R[i][j][1]
                f1=R[i][j][2]
                A1=AccuracyIntedeteminacyDistubition(t1,i1,f1)

                t2=R[j][i][0]
                i2=R[j][i][1]
                f2=R[j][i][2]

                A2=AccuracyIntedeteminacyDistubition(t2,i2,f2)

                if A1 > 1-A2 : # A1~must be less than~or~equal to 1-A2
                    idn=1

                    print R[i][j], ' and ', R[j][i], ' does not satisfy hesitation
                    property'

    return idn


def RHcreation(K):

    global RHtogether
    RHtogether= defaultdict()
    for i in K.keys():

            columnAll=len(K[i])

            rowAll1=len(K[i][0])
            rowAll2=len(K[i][0])

            for j in range(0,rowAll1):


                for k in range(0,rowAll2):

                    t1=K[i][j][k][0]
                    i1=K[i][j][k][1]
                    f1=K[i][j][k][2]
                    A=AccuracyIntedeteminacyDistubition(t1,i1,f1)


                    if i not in  RHtogether.keys():
                        RHtogether[i]=[A]

                    else:
                        RHtogether[i].extend([A])

            number= int(math.sqrt(len(RHtogether[i])))
            m=0
            new_list=[]
            while m<len(RHtogether[i]):
                new_list.append( RHtogether[i][m:m + number])
                m+= number

            RHtogether[i]=new_list


    return RHtogether


def OneZero(K):
    global H
    H=defaultdict()
    for i in K.keys():
            columnAllin=len(K[i])

            rowAll=len(K[i][0])
            for j in range(0,columnAllin):
```

```
            for k in range(0,rowAll):
                if K[i][j][k]>0.5:
                    if i not in H:
                        H[i]=[1]
                    else:
                        H[i].append(1)
                else:
                    if i not in H:
                        H[i]=[0]
                    else:
                        H[i].append(0)
        number= int(math.sqrt(len(H[i])))
        m=0
        new_list=[]
        while m<len(H[i]):
            new_list.append( H[i][m:m + number])
            m+= number

        H[i]=new_list

    return H

def H_pi_ij(K):
    global Hpij
    Hpij= defaultdict()
    columnAllin112=len(H)


    for i in range(0,columnAllin112):



        Topij=0

        for j in range(0,columnAllin112):
            Topij=0

            for k in H.keys():

                if i != j:

                    Topij = Topij + H[k][i][j]

                else:
                    Topij=0

            aij=str(i+1)+str(j+1)
            TopijAvarage= Topij/len(H)

            if aij not in Hpij.keys():
                TopijAvarage= Topij/len(H)
                Hpij[aij]=TopijAvarage
            else:
                Hpij[aij]=TopijAvarage
    return Hpij


def Alternative_Ordinary(Hpij):
    global ORD
    ORD= defaultdict()

    Number_Of_Alternatives=int(math.sqrt(len(Hpij)))
    for i in range(1,Number_Of_Alternatives+1):
        istr=str(i)

        Top=0
        for k in Hpij.keys():

            if istr==k[1]:

                Top=Top+Hpij[k]

        TopJavarage= Top/Number_Of_Alternatives


        if istr not in ORD.keys():
            istA='Alternative '+istr
            ORD[istA]=TopJavarage

        else:
            ORD[istA]=TopJavarage
    return ORD
def GroupDecisionWithID(m):
    for i in AllTogether.keys():
        if RationalityChecker(AllTogether[i])==1:
            print 'inconsistent agent'


    Step1=RHcreation(m)
    Step2=OneZero(Step1)
    Step3=H_pi_ij(Step2)
    Step4=Alternative_Ordinary(Step3)
    return Step4
```

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Manage. Sci.*, vol. 17, no. 4, p. B-141, 1970.

[3] K. T. Atanassov, "Intuitionistic fuzzy sets," in *Intuitionistic Fuzzy Sets*. Heidelberg, Germany: Physica, 1999, pp. 1–137.

[4] H.-W. Liu and G.-J. Wang, "Multi-criteria decision-making methods based on intuitionistic fuzzy sets," *Eur. J. Oper. Res.*, vol. 179, no. 1, pp. 220–233, May 2007.

[5] Z. Xu, "Intuitionistic fuzzy multiattribute decision making: An interactive method," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 3, pp. 514–525, Jun. 2012.

[6] J.-Q. Wang and H.-Y. Zhang, "Multicriteria decision-making approach based on Atanassov's intuitionistic fuzzy sets with incomplete certain information on weights," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 3, pp. 510–515, Jun. 2013.

[7] J. Zhan and B. Sun, "Covering-based intuitionistic fuzzy rough sets and applications in multi-attribute decision-making," *Artif. Intell. Rev.*, vol. 53, pp. 671–701, Dec. 2018.

[8] J. Zhan, B. Sun, and J. C. R. Alcantud, "Covering based multigranulation (I,T)-fuzzy rough set models and applications in multi-attribute group decision-making," *Inf. Sci.*, vol. 476, pp. 290–318, Feb. 2019.

[9] L. Zhang, J. Zhan, and Z. Xu, "Covering-based generalized IF rough sets with applications to multi-attribute decision-making," *Inf. Sci.*, vol. 478, pp. 275–302, Apr. 2019.

[10] K. Zhang, J. Zhan, and Y. Yao, "TOPSIS method based on a fuzzy covering approximation space: An application to biological nano-materials selection," *Inf. Sci.*, vol. 502, pp. 297–329, Oct. 2019.

[11] L. Zhang, J. Zhan, Z. Xu, and J. C. R. Alcantud, "Covering-based general multigranulation intuitionistic fuzzy rough sets and corresponding applications to multi-attribute group decision-making," *Inf. Sci.*, vol. 494, pp. 114–140, Aug. 2019.

[12] K. Zhang, J. Zhan, and W.-Z. Wu, "Novel fuzzy rough set models and corresponding applications to multi-criteria decision-making," *Fuzzy Sets Syst.*, vol. 383, pp. 92–126, Mar. 2020.

[13] L. Li, J. Yang, and W. Wu, "Intuitionistic fuzzy hopfield neural network and its stability," *Neural Netw. World*, vol. 21, no. 5, pp. 461–472, 2011.

[14] S. Sotirov, E. Sotirova, and D. Orozova, "Neural network for defining intuitionistic fuzzy sets in e-learning," *Notes Intuitionistic Fuzzy Sets*, vol. 15, no. 2, pp. 33–36, 2009.

[15] T. K. Shinoj and S. J. John, "Intuitionistic fuzzy multisets and its application in medical diagnosis," *World Acad. Sci., Eng. Technol.*, vol. 6, no. 1, pp. 1418–1421, 2012.

[16] F. Smarandache, *Neutrosophy, Neutrosophic Probability, Set, and Logic*. Rehoboth, NM, USA: American Research Press, 1998, pp. 104–106.

[17] U. Rivieccio, "Neutrosophic logics: Prospects and problems," *Fuzzy Sets Syst.*, vol. 159, no. 14, pp. 1860–1868, Jul. 2008.

[18] P. Majumdar and S. K. Samanta, "On similarity and entropy of neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 26, no. 3, pp. 1245–1252, 2014.

[19] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multispace Multistruct.*, vol. 4, pp. 410–413, Oct. 2010.

[20] R. Şahin, "Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment," 2014, *arXiv:1412.5202*. [Online]. Available: http://arxiv.org/abs/1412.5202

[21] J. Ye, "Another form of correlation coefficient between single valued neutrosophic sets and its multiple attribute decision-making method," *Neutrosophic Sets Syst.*, vol. 1, no. 1, pp. 8–12, 2013.

[22] J. Ye, "Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment," *Int. J. Gen. Syst.*, vol. 42, no. 4, pp. 386–394, May 2013.

[23] J. Ye, "Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 27, no. 5, pp. 2231–2241, 2014.

[24] J. Ye, "A multi criteria decision-making method using aggregation operators for simplified neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 26, no. 5, pp. 2459–2466, 2014.

[25] J. Ye, "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making," *J. Intell. Fuzzy Syst.*, vol. 26, no. 1, pp. 165–172, 2014.

[26] J. Ye, "Single valued neutrosophic cross-entropy for multicriteria decision making problems," *Appl. Math. Model.*, vol. 38, no. 3, pp. 1170–1175, Feb. 2014.

[27] J. Ye, "Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment," *J. Intell. Fuzzy Syst.*, vol. 27, no. 6, pp. 2927–2935, 2014.

[28] J. Ye, "Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making," *Int. J. Fuzzy Syst.*, vol. 16, no. 2, pp. 204–211, 2014.

[29] J. Ye, "Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 27, no. 5, pp. 2453–2462, 2014.

[30] J. Ye, "Trapezoidal neutrosophic set and its application to multiple attribute decision-making," *Neural Comput. Appl.*, vol. 26, no. 5, pp. 1157–1166, Dec. 2014.

[31] J. Ye, "An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers," *J. Intell. Fuzzy Syst.*, vol. 28, no. 1, pp. 247–255, 2015.

[32] J. Ye, "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment," *J. Intell. Syst.*, vol. 24, no. 1, pp. 23–36, Jan. 2014.

[33] J. Ye, "Projection and bidirectional projection measures of single-valued neutrosophic sets and their decision-making method for mechanical design schemes," *J. Exp. Theor. Artif. Intell.*, vol. 29, no. 4, pp. 731–740, Nov. 2016.

[34] J. Ye, "Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers," *Neural Comput. Appl.*, vol. 28, no. 5, pp. 1021–1029, 2017.

[35] P. Liu and Y. Wang, "Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted bonferroni mean," *Neural Comput. Appl.*, vol. 25, nos. 7–8, pp. 2001–2010, Aug. 2014.

[36] P. Liu, Y. Chu, Y. Li, and Y. Chen, "Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making," *Int. J. Fuzzy Syst.*, vol. 16, no. 2, pp. 242–255, 2014.

[37] P. Liu and H. Li, "Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators," *Neural Comput. Appl.*, vol. 28, no. 1, pp. 179–194, 2017.

[38] P. Liu and L. Shi, "Some neutrosophic uncertain linguistic number heronian mean operators and their application to multi-attribute group decision making," *Neural Comput. Appl.*, vol. 28, no. 5, pp. 1079–1093, Nov. 2015.

[39] P. Liu and L. Shi, "The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making," *Neural Comput. Appl.*, vol. 26, no. 2, pp. 457–471, Oct. 2014.

[40] P. Liu and G. Tang, "Some power generalized aggregation operators based on the interval neutrosophic sets and their application to decision making," *J. Intell. Fuzzy Syst.*, vol. 30, no. 5, pp. 2517–2528, Apr. 2016.

[41] R. Şahin and P. Liu, "Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information," *Neural Comput. Appl.*, vol. 27, no. 7, pp. 2017–2029, Jul. 2015.

[42] P. Chi and P. Liu, "An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set," *Neutrosophic Sets Syst.*, vol. 1, no. 1, pp. 63–70, 2013.

[43] P. Biswas, S. Pramanik, and B. C. Giri, "A new methodology for neutrosophic multi-attribute decision making with unknown weight information," *Neutrosophic Sets Syst.*, vol. 3, pp. 42–52, 2014.

[44] P. Biswas, S. Pramanik, and B. C. Giri, "Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments," *Neutrosophic Sets Syst.*, vol. 2, pp. 102–110, May 2014.

[45] P. Biswas, S. Pramanik, and B. C. Giri, "Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers," *Neutrosophic Sets Syst.*, vol. 8, pp. 46–56 Apr. 2014.

[46] P. Biswas, S. Pramanik, and B. C. Giri, "Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making," *Neutrosophic Sets Syst.*, vol. 12, pp. 127–138, 2016.

[47] P. Biswas, S. Pramanik, and B. C. Giri, "Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making," *Neutrosophic sets Syst.*, vol. 12, pp. 20–40, 2016.

[48] P. Biswas, S. Pramanik, and B. C. Giri, "TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment," *Neural Comput. Appl.*, vol. 27, no. 3, pp. 727–737, Apr. 2015.

[49] S. Pramanik, P. Biswas, and B. C. Giri, "Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment," *Neural Comput. Appl.*, vol. 28, no. 5, pp. 1163–1176, Dec. 2015.

[50] K. Mondal and S. Pramanik, "Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment," *Neutrosophic Sets Syst.*, vol. 6, pp. 28–34, 2014.

[51] K. Mondal and S. Pramanik, "Neutrosophic tangent similarity measure and its application to multiple attribute decision making," *Neutrosophic sets Syst.*, vol. 9, pp. 80–87, 2015.

[52] K. Mondal and S. Pramanik, "Neutrosophic decision making model of school choice," *Neutrosophic Sets Syst.*, vol. 7, pp. 62–68, Feb. 2015.

[53] K. Mondal and S. Pramanik, "Rough neutrosophic multi-attribute decision-making based on grey relational analysis," *Neutrosophic Sets Syst.*, vol. 7, pp. 8–17, Feb. 2015.

[54] K. Mondal and S. Pramanik, "Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decesion making," *J. New Theory*, no. 8, pp. 41–50, 2015.

[55] J.-J. Peng, J.-Q. Wang, H.-Y. Zhang, and X.-H. Chen, "An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets," *Appl. Soft Comput.*, vol. 25, pp. 336–346, Dec. 2014.

[56] H.-Y. Zhang, J.-Q. Wang, and X.-H. Chen, "Interval neutrosophic sets and their application in multicriteria decision making problems," *Sci. World J.*, vol. 2014, pp. 1–15, 2014.

[57] H.-Y. Zhang, P. Ji, J.-Q. Wang, and X.-H. Chen, "An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision-making problems," *Int. J. Comput. Intell. Syst.*, vol. 8, no. 6, pp. 1027–1043, Oct. 2015.

[58] J.-J. Peng, J.-Q. Wang, J. Wang, H.-Y. Zhang, and X.-H. Chen, "Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems," *Int. J. Syst. Sci.*, vol. 47, no. 10, pp. 2342–2358, Jan. 2015.

[59] H. Zhang, J. Wang, and X. Chen, "An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets," *Neural Comput. Appl.*, vol. 27, no. 3, pp. 615–627, Apr. 2015.

[60] H.-Y. Zhang, P. Ji, J.-Q. Wang, and X.-H. Chen, "A neutrosophic normal cloud and its application in decision-making," *Cognit. Comput.*, vol. 8, no. 4, pp. 649–669, Mar. 2016.

[61] Z.-P. Tian, H.-Y. Zhang, J. Wang, J.-Q. Wang, and X.-H. Chen, "Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets," *Int. J. Syst. Sci.*, vol. 47, no. 15, pp. 3598–3608, Oct. 2015.

[62] Z.-P. Tian, J. Wang, J.-Q. Wang, and H.-Y. Zhang, "Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development," *Group Decis. Negotiation*, vol. 26, no. 3, pp. 597–627, Apr. 2016.

[63] P. Ji, H.-Y. Zhang, and J.-Q. Wang, "Fuzzy decision-making framework for treatment selection based on the combined QUALIFLEX–TODIM method," *Int. J. Syst. Sci.*, vol. 48, no. 14, pp. 3072–3086, Aug. 2017.

[64] P. Ji, H.-Y. Zhang, and J.-Q. Wang, "A fuzzy decision support model with sentiment analysis for items comparison in e-commerce: The case study of http://PConline.com," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 10, pp. 1993–2004, Oct. 2019.

[65] P. Ji, H.-Y. Zhang, and J.-Q. Wang, "Selecting an outsourcing provider based on the combined MABAC–ELECTRE method using single-valued neutrosophic linguistic sets," *Comput. Ind. Eng.*, vol. 120, pp. 429–441, Jun. 2018.

[66] X. Peng and J. Dai, "Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function," *Neural Comput. Appl.*, vol. 29, no. 10, pp. 939–954, Sep. 2016.

[67] X. Peng, J. Dai, and H. Yuan, "Interval-valued fuzzy soft decision making methods based on MABAC, similarity measure and EDAS," *Fundamenta Informaticae*, vol. 152, no. 4, pp. 373–396, May 2017.

[68] X. Peng and C. Liu, "Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set," *J. Intell. Fuzzy Syst.*, vol. 32, no. 1, pp. 955–968, Jan. 2017.
[69] X. Peng and J. Dai, "Algorithms for interval neutrosophic multiple attribute decision-making based on MABAC, similarity measure, and EDAS," *Int. J. Uncertainty Quantification*, vol. 7, no. 5, pp. 395–421, 2017.
[70] J. M. Blin and A. B. Whinston, "Fuzzy sets and social choice," *J. Cybern.*, vol. 3, no. 4, pp. 28–36, 1973.
[71] D. Dubois and J.-L. Koning, "Social choice axioms for fuzzy set aggregation," *Fuzzy Sets Syst.*, vol. 43, no. 3, pp. 257–274, Oct. 1991.
[72] B. Dutta, "Fuzzy preferences and social choice," *Math. Social Sci.*, vol. 13, no. 3, pp. 215–229, Jun. 1987.
[73] G. Richardson, "The structure of fuzzy preferences: Social choice implications," *Social Choice Welfare*, vol. 15, no. 3, pp. 359–369, May 1998.
[74] H. Nurmi, "Approaches to collective decision making with fuzzy preference relations," *Fuzzy Sets Syst.*, vol. 6, no. 3, pp. 249–259, Nov. 1981.
[75] B. Richard and S. Maurice, "Social choice with fuzzy preferences," in *Handbook of Social Choice and Welfare*, vol. 2. Amsterdam, The Netherlands: Elsevier, 2011, pp. 367–389.
[76] E. Szmidt and J. Kacprzyk, "Intuitionistic fuzzy sets in group decision making," *Notes IFS*, vol. 2, no. 1, pp. 15–32, 1996.
[77] Z. Yue, "Deriving decision maker's weights based on distance measure for interval-valued intuitionistic fuzzy group decision making," *Expert Syst. Appl.*, vol. 38, no. 9, pp. 11665–11670, 2011.
[78] Z. Yue and Y. Jia, "An application of soft computing technique in group decision making under interval-valued intuitionistic fuzzy environment," *Appl. Soft Comput.*, vol. 13, no. 5, pp. 2490–2503, May 2013.
[79] J. Kacprzyk, H. Nurmi, and M. Fedrizzi, Eds., *Consensus Under Fuzziness*, vol. 10. Cham, Switzerland: Springer, 2012.
[80] R. Sorensen, *Vagueness and Contradiction*. Oxford, U.K.: Oxford Univ. Press, 2001.
[81] R. Sorensen, "Prěcis of vagueness and contradiction," *Philosophy Phenomenol. Res.*, vol. 71, no. 3, pp. 678–685, Nov. 2005.
[82] R. Sorensen, "A reply to critics," *Philosophy Phenomenol. Res.*, vol. 71, no. 3, pp. 712–728, Nov. 2005.
[83] M. Dasgupta and R. Deb, "Fuzzy choice functions," *Social Choice Welfare*, vol. 8, pp. 171–182, Apr. 1991.
[84] Python Software Foundation. *Python Language Reference, Version 2.7.* Accessed: Apr. 20, 2019. [Online]. Available: http://www.python.org
[85] G. van Rossum, "Python tutorial," CWI, Amsterdam, The Netherlands, Tech. Rep. CS-R9526, May 1995.
[86] R. R. Yager, "Connectives and quantifiers in fuzzy sets," *Fuzzy Sets Syst.*, vol. 40, no. 1, pp. 39–75, Mar. 1991.
[87] H. Wang, F. Smarandache, R. Sunderraman, and Y. Q. Zhang, "Interval neutrosophic sets and logic: Theory and applications in computing," 2005, *arXiv:cs/0505014*. [Online]. Available: https://arxiv.org/abs/cs/0505014
[88] I. Deli, M. Ali, and F. Smarandache, "Bipolar neutrosophic sets and their application based on multi-criteria decision making problems," in *Proc. Int. Conf. Adv. Mech. Syst. (ICAMechS)*, Aug. 2015, pp. 249–254.
[89] J. M. Merigó, M. Casanovas, and L. Martínez, "Linguistic aggregation operators for linguistic decision making based on the Dempster–Shafer theory of evidence," *Int. J. Uncertainty, Fuzziness Knowl.-Based Syst.*, vol. 18, no. 03, pp. 287–304, Nov. 2011.
[90] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics* (Infinite Study). Brussels, Belgium: Pons, Jan. 2017, pp. 14–16.
[91] H. Garg, "Hesitant pythagorean fuzzy Maclaurin symmetric mean operators and its applications to multiattribute decision-making process," *Int. J. Intell. Syst.*, vol. 34, no. 4, pp. 601–626, 2019.
[92] L. Wang, H. Garg, and N. Li, "Interval-valued *q*-rung orthopair 2-tuple linguistic aggregation operators and their applications to decision making process," *IEEE Access*, vol. 7, pp. 131962–131977, 2019.
[93] L. Wang and N. Li, "Continuous interval-valued pythagorean fuzzy aggregation operators for multiple attribute group decision making," *J. Intell. Fuzzy Syst.*, vol. 36, no. 6, pp. 6245–6263, Jun. 2019.

SELÇUK TOPAL received the M.Sc. and Ph.D. degrees in mathematics (Foundations of Mathematics and Mathematical Logic) from Ege University, Turkey. He was a Visiting Scholar with Indiana (USA) and Calabria (Italy) Universities. He is currently an Associate Professor of mathematics with Bitlis Eren University, Turkey. My general area of interest is applied logic: the study of mathematical and conceptual tools for use in computer science, linguistics, artificial intelligence, and other areas. Details to follow: In general, he is interested in exploring logics and formal semantics in fragments of natural languages and algebraic structures, proof search algorithms, and implementations of the logics. These explorations allow us to discover new algebraic structures, graph algorithms and new calculations. Studying topological, algebraic and geometric applications of neutrosophic sets, rough sets, and fuzzy sets. Computational Logic (Ontology-based systems, analysis of artificial intelligence, and logic systems) Python, C++, Haskell programming languages. Mathematical investigations in quantum mechanics. Data analysis, computational models and decision making algorithms of uncertain data. Applications of the analysis in medicine. Recently, he is interested in blockchain techs, linear logic, data science, and logical methods in deep learning.

AHMET ÇEVIK received the Ph.D. degree in mathematical logic from the Department of Pure Mathematics, University of Leeds, in 2014. He was a Research Assistant with the Department of Computer Engineering, Atilim University, from 2007 to 2010. He was a Postdoc Visitor with the Department of Mathematics, UC Berkeley, from 2015 to 2016. He has been giving lectures with the Department of Mathematics, Middle East Technical University, the Department of Philosophy, Middle East Technical University, and the Department of Computer Engineering, Middle East Technical University, since 2015. He is currently working as an Assistant Professor with the Gendarmerie and Coast Guard Academy, Ankara. His research interests are logic, philosophy of mathematics, and theoretical computer science.

FLORENTIN SMARANDACHE received the M.Sc. degree in mathematics and computer science from the University of Craiova, Romania, and the Ph.D. degree in mathematics from the State University of Kishinev. He held a postdoctoral position in applied mathematics at the Okayama University of Sciences, Japan. He is currently a Professor of mathematics with The University of New Mexico–Gallup, USA. He has been the Founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics, since 1995, and has published 100 articles on neutrosophic physics, superluminal and instantaneous physics, unmatter, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, law of included multiple-middle, multispace, degree of dependence and independence between the neutrosophic components, refined neutrosophic set, neutrosophicover-under-off-set, plithogenicset, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, DSmT, and so on to many peer-reviewed international journals and many books. He presented articles and plenary lectures to many international conferences around the world.