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A Novel TOPSIS-MABAC Method for Multi-attribute Decision Making with Interval Neutrosophic Set

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ABSTRACT

Interval neutrosophic Set is a useful tool to describe the indeterminate, inconsistent, and incomplete information. This paper presents the application of the new TOPSIS-MABAC model with interval neutrosophic number in multi-attribute decision-making problem. In this model, the combined weight of attributes is obtained based on TOPSIS method while the best alternatives by MABAC method. Firstly, some definitions of INS are given in this paper. Secondly, the objective attribute weights are determined by TOPSIS method, and then a combined attribute weight is proposed. Finally an extended MABAC method is developed to rank the alternatives in multi-attribute decision-making problem and an illustrative examples are given to demonstrate the practicality and effectiveness of this new method.

1. INTRODUCTION

Multi-attribute decision making (MADM) problem [1] is an important part of modern decision science. Because of the fuzziness of human thinking and the complexity and uncertainty of objective things, it is difficult for a decision maker to express the evaluation value of an attribute with a crisp value. For this reason, fuzzy value is a better choice to describe these fuzzy information.

Fuzzy set (FS) is characterized by membership function and was firstly proposed by Zadeh [2]. On this basis, Atanassov [3], [4] proposed the intuitionistic fuzzy set (IFS) with membership function and non-membership function, and used it to solve

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some decision problems. Then some aggregation operators based on these were proposed by Xu [5-6] and some methods for MADM with IFS were proposed in [7-8]. Furthermore, Atanassov and Gargov [4,9] extended the membership function and non-membership function to interval numbers and proposed interval-value intuitionistic fuzzy set (IVIFS). But IFS and IVIFS can only deal with incomplete information, but not uncertain and inconsistent information.

Therefore, Smarandache [10], [11] firstly proposed Neutrosophic Set (NS), however NS was mainly put forward from a philosophical viewpoint. So Wang et al. [12] proposed Single-valued Neutrosophic Set (SVNS) with the corresponding properties and operation rules. Similar to IVIFS, Wang et al. [13] proposed Interval Neutrosophic Set (INS) and gave the set-theoretic operators of INS.

In this paper, we propose the TOPSIS-MABAC method, which is a combined method under interval neutrosophic environment for solving MADM problem. The specific arrangements of this article are structured as follows. In section 2, we briefly introduce some concepts and definitions of INS. In Section 3, we propose TOPSIS method to determine the objective attribute weights and the combined weights, and then use MABAC method to obtain the best alternative. In Section 4, we give an example to illustrate the application of proposed method. In Section 5, we make a conclusion.

2. PRELIMINARIES OF NEUTROSOPHIC

2.1 Neutrosophic Set

Definition 1: Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$, that is $T_A(x): X \to]^-0,1^+[$, $I_A(x): X \to]^-0,1^+[$ and $F_A(x): X \to]^-0,1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$,and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

2.2 Interval Neutrosophic Set

Definition 2: Let X be a space of points (objects), with a generic element in X denoted by x. An Interval neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, then A can be denoted by

$$A=\left\{\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle \middle| x\in X\right\},$$

Where $T_A(x)=\left[T_A^L(x),T_A^U(x)\right]$, $I_A(x)=\left[I_A^L(x),I_A^U(x)\right]$, $F_A(x)=\left[F_A^L(x),F_A^U(x)\right]\subseteq[0,1]$ for every x in X ,and $0\le \sup T_A(x)+\sup I_A(x)+\sup F_A(x)\le 3$.

For convenience, we refer to $A=\langle T_A,I_A,F_A\rangle=\left\langle\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right]\right\rangle$ as INN.

Definition 3: The complement of A is denoted by A^c and is defined as

$$A^c=\left\langle\left[F_A^L,F_A^U\right],\left[1-I_A^U,1-I_A^L\right],\left[T_A^L,T_A^U\right]\right\rangle$$

Definition 4: Let A and B be two INNs, then the normalized Euclidean distance between A and B is

$$d(A,B)=\sqrt{\frac{1}{6}\left\{\left(T_A^L-T_B^L\right)^2+\left(T_A^U-T_B^U\right)^2+\left(I_A^L-I_B^L\right)^2+\left(I_A^U-I_B^U\right)^2+\left(F_A^L-F_B^L\right)^2+\left(F_A^U-F_B^U\right)^2\right\}} \qquad (1)$$

Definition 5: Let A be an INN, a score function S of A is:

$$S(A)=\frac{4+T_A^L-I_A^L-F_A^L+T_A^U-I_A^U-F_A^U}{6}$$

Definition 6: Let A be an INN, an accuracy function H of A is:

$$H(A)=\frac{\left(T_A^L+T_A^U\right)-\left(F_A^L+F_A^U\right)}{2}$$

Definition 7: Let A and B be two INNs, $S(A)$ and $S(B)$ be the score functions, $H(A)$ and $H(B)$ be the accuracy functions, then if $S(A)<S(B)$, then $A<B$; if $S(A)=S(B)$,then

(1) if $H(A)=H(B)$,then $A=B$;

(2) if $H(A)<H(B)$,then $A<B$.

3. TOPSIS-MABAC METHOD FOR INTERVAL NEUTROSOPHIC MADM PROBLEM

Let $A=\{A_1,A_2,\cdots,A_m\}$ be a set of alternatives, $C=\{C_1,C_2,\cdots,C_n\}$ be a series of attributes, and $w=\{w_1,w_2,\cdots,w_n\}$ be the subjective weight of the attribute, w_j is the weight of the j-th attribute where $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$. The interval neutr-
osophic number $a_{ij}=\left\langle\left[T_{ij}^{L},T_{ij}^{U}\right],\left[I_{ij}^{L},I_{ij}^{U}\right],\left[F_{ij}^{L},F_{ij}^{U}\right]\right\rangle$ is the evaluated value of A_i under C_j, then the decision matrix $A=\left(a_{ij}\right)_{m\times n}$ is obtained. To get the optimal alter-
Native (s), we propose the TOPSIS-MABAC method with INN.

Step 1: Normalization of the initial decision matrix. That is, normalized the matrix $A=\left(a_{ij}\right)_{m\times n}$ into

$R=\left(r_{ij}\right)_{m\times n}=\left(\left\langle\left[T_{ij}^{NL},T_{ij}^{NU},\right],\left[I_{ij}^{NL},I_{ij}^{NU},\right],\left[F_{ij}^{NL},F_{ij}^{NU},\right]\right\rangle\right)_{m\times n}$, where

$$r_{ij}=\begin{cases}a_{ij}, & C_j \textit{ is the benefit}-\textit{type attribuite};\\ a_{ij}^{c}, & C_j \textit{ is the } \cos t-\textit{type attribuite.}\end{cases}.$$

Step 2: Calculating the combined weight.

According to the normalized decision matrix, we can define the positive ideal so-
lution (PIS) and negative ideal solution (NIS) as following:

$$\begin{cases}PIS=R^{+}=(R_1^{+},R_2^{+},\cdots,R_n^{+})\\ NIS=R^{-}=(R_1^{-},R_2^{-},\cdots,R_n^{-})\end{cases}\tag{2}$$

Where $\begin{cases}R_j^{+}=\left(\left[\max\limits_i T_{ij}^{NL},\max\limits_i T_{ij}^{NU},\right],\left[\min\limits_i I_{ij}^{NL},\min\limits_i I_{ij}^{NU},\right],\left[\min\limits_i F_{ij}^{NL},\min\limits_i F_{ij}^{NU},\right]\right)\\ R_j^{-}=\left(\left[\min\limits_i T_{ij}^{NL},\min\limits_i T_{ij}^{NU},\right],\left[\max\limits_i I_{ij}^{NL},\max\limits_i I_{ij}^{NU},\right],\left[\max\limits_i F_{ij}^{NL},\max\limits_i F_{ij}^{NU},\right]\right)\end{cases}$

for $j=1,2,\cdots,n$.

①Determination of objective weight vector $\omega^{o}=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$.

For the PIS, the closer distance between A_i and R^{+} is, the better A_i is. So we assume that the weight distance between A_i and R^{+} under C_j is

$e_i^+(\omega)=\sum_{j=1}^{n}d(r_{ij},R_j^+)\omega_j^+$ .So $e^+(\omega)=\sum_{i=1}^{m}\sum_{j=1}^{n}d(r_{ij},R_j^+)\omega_j^+$ represents the sum of the weight distance between all the alternatives and PIS. Therefore, we can establish the model as follows:

$$\begin{cases}\min \quad e^+(\omega)=\sum_{i=1}^{m}\sum_{j=1}^{n}d(r_{ij},R_j^+)\omega_j^+ \\ s.t. \quad \sum_{j=1}^{n}\omega_j^+=1,\omega_j^+>0,\ j=1,2,\cdots,n\end{cases}.$$

By constructing the Lagrange function, we can get $\omega_j^+=\dfrac{\sum_{i=1}^{m}d(r_{ij},R_j^+)}{\sum_{j=1}^{n}\sum_{i=1}^{m}d(r_{ij},R_j^+)}$. (3)

Similarly we can get $\omega_j^-=\dfrac{\sum_{i=1}^{m}d(r_{ij},R_j^-)}{\sum_{j=1}^{n}\sum_{i=1}^{m}d(r_{ij},R_j^-)}$. (4)

So the objective weight is $\omega^o=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$, where $\omega_j=\frac{1}{2}\left(\omega_j^++\omega_j^-\right)$.

②Determination of combined weight $\bar{\omega}=\left(\bar{\omega}_1,\bar{\omega}_2,\cdots,\bar{\omega}_n\right)$, where $\bar{\omega}_j=\lambda\omega_j+\left(1-\lambda\right)w_j$, for $j=1,2,\cdots,n$,and $0\le\lambda\le 1$.

Step 3: Calculation of weighted normalized decision matrix $V=\left(v_{ij}\right)_{m\times n}=\left\langle\left[\bar{T}_{ij}^L,\ \bar{T}_{ij}^U\right],\left[\bar{I}_{ij}^L,\bar{I}_{ij}^U\right],\left[\bar{F}_{ij}^L,\bar{F}_{ij}^U\right]\right\rangle$, $i=1,2,\cdots,n,\ j=1,2,\cdots,m$, where

$$v_{ij}=\bar{\omega}_j\Box r_{ij}=\left\langle\left[1-\left(1-T_{ij}^{NL}\right)^{\bar{\omega}_j},1-\left(1-T_{ij}^{NU}\right)^{\bar{\omega}_j}\right],\right.$$
$$\left.\left[\left(I_{ij}^{NL}\right)^{\bar{\omega}_j},\left(I_{ij}^{NU}\right)^{\bar{\omega}_j}\right],\left[\left(F_{ij}^{NL}\right)^{\bar{\omega}_j},\left(F_{ij}^{NU}\right)^{\bar{\omega}_j}\right]\right\rangle \tag{5}$$

Step 4: Determination of the border approximation area matrix $G=\left(g_j\right)_{1\times n}$.

$$g_j=\prod_{i=1}^{m}\left(v_{ij}\right)^{\frac{1}{m}}=\left\langle\left[\prod_{i=1}^{m}\left(\bar{T}_{ij}^L\right)^{\frac{1}{m}},\prod_{i=1}^{m}\left(\bar{T}_{ij}^U\right)^{\frac{1}{m}}\right],\left[1-\prod_{i=1}^{m}\left(1-\bar{I}_{ij}^L\right)^{\frac{1}{m}},1-\prod_{i=1}^{m}\left(1-\bar{I}_{ij}^U\right)^{\frac{1}{m}}\right],\right.$$
$$\left.\left[1-\prod_{i=1}^{m}\left(1-\bar{F}_{ij}^L\right)^{\frac{1}{m}},1-\prod_{i=1}^{m}\left(1-\bar{F}_{ij}^U\right)^{\frac{1}{m}}\right]\right\rangle \tag{6}$$

Step 5: Determination of the distance matrix $D=\left(d_{ij}\right)_{m\times n}$, where

$$d_{ij}=\begin{cases} d\left(v_{ij},g_j\right) & if\ v_{ij}>g_j \\ 0 & if\ v_{ij}=g_j \\ -d\left(v_{ij},g_j\right) & if\ v_{ij}<g_j \end{cases}$$

Step 6: Calculation of Q_i, where $Q_i=\sum_{j=1}^{n} d_{ij}$.

Step 7: Ranking the alternatives according to the value of Q_i .The value of Q_i is larger, the alternative is better.

4. NUMERICAL EXAMPLE

There is an investment company which wants to invest a number of money in a best option. There are four possible alternatives to invest the money: A_1, A_2, A_3 and A_4. The investment company should take a decision according to the following three attribute: C_1, C_2 and C_3, where C_1 and C_2 are benefit-type attributes and C_3 is a cost-type attribute .The objective weight vector of the attribute is given by $w=\left(0.35, 0.25, 0.4\right)$. The four possible alternatives are to be evaluated under the above three attribute by the form of INNs, as shown in the following interval neutrosophic decision matrix A:

$$A=\begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \end{bmatrix}$$

Then we use the proposed method to obtain the best alternative.

Step1: Normalized the decision matrix and get the normalized matrix:

$$R=\begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.4,0.5],[0.7,0.8],[0.7,0.9]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.8,0.9],[0.5,0.8],[0.3,0.6]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.7,0.9],[0.6,0.8],[0.4,0.5]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.8,0.9],[0.6,0.7],[0.6,0.7]\rangle \end{bmatrix}$$

Step2: Calculating the combined weight. According to Eq.(2), we can get R^+ and R^-.

$R_1^+ = \langle [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] \rangle$, $R_1^- = \langle [0.3, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle$,
$R_2^+ = \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle$, $R_2^- = \langle [0.4, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle$,
$R_3^+ = \langle [0.8, 0.9], [0.5, 0.7], [0.3, 0.5] \rangle$, $R_3^- = \langle [0.4, 0.5], [0.7, 0.8], [0.7, 0.9] \rangle$.
According to Eq.(3) and Eq.(4),we can get

$$\omega_1^+ = 0.4, \omega_2^+ = 0.2, \omega_3^+ = 0.4,$$
$$\omega_1^- = 0.3, \omega_2^- = 0.2, \omega_3^- = 0.5$$

Then the objective weight is obtained and denoted as $\omega^o = (0.35, 0.2, 0.45)$, In this section, we set $\lambda = 0.7$, so the combined weight is $\bar{\omega} = (0.35, 0.215, 0.435)$.

Step 3: Calculation of the weighted normalized decision matrix $V = (v_{ij})_{m \times n}$ which is shown in Table I.

TABLE I. THE WEIGHTED DECISION MATRIX.

	C_1
A_1	$([0.1637, 0.2154], [0.5693, 0.6561], [0.6561, 0.7256])$
A_2	$([0.2744, 0.3439], [0.4467, 0.5693], [0.5693, 0.6561])$
A_3	$([0.1174, 0.2744], [0.5693, 0.6561], [0.6561, 0.7256])$
A_4	$([0.3439, 0.4307], [0, 0.4467], [0.4467, 0.5693])$
	C_2
A_1	$([0.1040, 0.1788], [0.6095, 0.7719], [0.7075, 0.8212])$
A_2	$([0.1788, 0.2281], [0.6095, 0.7075], [0.7075, 0.7719])$
A_3	$([0.1385, 0.1788], [0.7075, 0.7719], [0.7719, 0.8212])$
A_4	$([0.1788, 0.2281], [0.6095, 0.7075], [0.6095, 0.7719])$
	C_3
A_1	$([0.1993, 0.2603], [0.8563, 0.9075], [0.8563, 0.9552])$
A_2	$([0.5035, 0.6327], [0.7379, 0.9075], [0.5923, 0.8007])$
A_3	$([0.4077, 0.6327], [0.8007, 0.9075], [0.6713, 0.7397])$
A_4	$([0.5035, 0.6327], [0.8007, 0.8563], [0.8007, 0.8563])$

Step 4: Calculating the border approximation area matrix $G=\left(g_j\right)_{1\times n}$ .According to Eq.(6),we can get:

$$g_1=\left\langle[0.2064,0.3059],[0.4340,0.5903],[0.5903,0.6750]\right\rangle$$

$$g_2=\left\langle[0.1465,0.2020],[0.6367,0.7417],[0.7045,0.7980]\right\rangle$$

$$g_3=\left\langle[0.3788,0.5067],[0.8033,0.8967],[0.7511,0.8648]\right\rangle$$

Step 5: Calculating the distance matrix D.

$$D=\begin{bmatrix} -0.0812 & -0.0275 & -0.1387 \\ 0.0353 & 0.0311 & 0.1046 \\ -0.0800 & -0.0410 & 0.0809 \\ 0.2142 & 0.0472 & 0.0761 \end{bmatrix}$$

Step 6: Calculating $Q_i=\sum_{j=1}^{n} d_{ij}$, $i=1,2,\cdots,4$ .We can get

$$Q_1=-0.2474,Q_2=0.1710,Q_3=-0.0431,Q_4=0.3375.$$

According the value of Q_i , we can get $A_4>A_2>A_3>A_1$, so A_4 is the optimal alternative.

5. CONCLUSIONS

The aim of this paper is to introduce a new approach for MADM with interval n-eutrosophic set. At the beginning of this article, we briefly introduce some concepts and definitions of INS, and then propose TOPSIS method for determining the objective attribute weights and the combined weights. Next, in order to get the best alternative(s), we combined the MABAC method with the combined weights. Finally, we give an example to illustrate the application of proposed method. From the results we can see that this new method is useful for multi-attribute decision making problem.

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