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A Refined Approach for Forecasting Based on Neutrosophic Time Series

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Abstract: This research introduces a neutrosophic forecasting approach based on neutrosophic time series (NTS). Historical data can be transformed into neutrosophic time series data to determine their truth, indeterminacy and falsity functions. The basis for the neutrosophication process is the score and accuracy functions of historical data. In addition**,** neutrosophic logical relationship groups (NLRGs) are determined and a deneutrosophication method for NTS is presented. The objective of this research is to suggest an idea of first-and high-order NTS. By comparing our approach with other approaches, we conclude that the suggested approach of forecasting gets better results compared to the other existing approaches of fuzzy, intuitionistic fuzzy, and neutrosophic time series.

Keywords: neutrosophic time series; triangular neutrosophic number; neutrosophic logical relationship; neutrosophic logical relationship groups

1. Introduction

There are different methods in the literature on fuzzy and intuitionistic fuzzy time series methods to forecast future values. The major difference between traditional and fuzzy time series is that the values of traditional time series are presented in numbers, whereas the values in fuzzy time series are fuzzy sets or linguistic values with real meanings. In intuitionistic fuzzy time series, the values are intuitionistic fuzzy sets or linguistic values. The first method in literature for forecasting future values based on fuzzy time series was introduced by Song and Chissom [1]. They also applied time-variant and time-invariant models for forecasting the enrollment data at the University of Alabama [1,2]. The identification of fuzzy relationship and the defuzzification process in both models were the main steps for calculating forecasted values. In time variant fuzzy time series it is proposed that autocorrelation is dependent due to the time, while in time invariant it is proposed that autocorrelation is independent due to the time.

The term "fuzzy relationship" means a collection of fuzzy sets which are caused only by other sets. In addition, the "defuzzification" process means converting the fuzzy values into crisp ones. Furthermore, a straightforward approach for time series forecasting was presented by Chen [3] by using uncomplicated arithmetic computations. To enhance the accuracy of forecasted outputs, some papers suggested various methods on fuzzy time series (FTS) forecasting [4–7]. A high-order FTS method was also presented by Chen [8] and Singh [9], and a method of bivariate fuzzy time series analysis for the forecasting of a stock index was introduced by Hsu et al. [10]. Furthermore, a framework developed for evaluation and forecasting based on the fuzzy NEAT F-PROMETHEE method was presented by Ziemba and Becker [11] for taking into account the uncertainty of input data, which is particularly burdened with the forecast values of the information and communication technologies development indicators.

The concept of fuzzy set was introduced by Zadeh [12], and it was generalized by Atanassov [13] to intuitionistic fuzzy set (IFS) to make it more suitable to handle ambiguity. The IFS considers both the membership (truth) and non-membership (falsity) degrees. However, the fuzzy set considers only the membership degree. Recently, the IFS was used for handling the fuzzy time series forecasting by Gangwar and Kumar [14] and Wang et al. [15]. In addition, the notion of intuitionistic fuzzy time series (IFTS)was employed in forecasting, as in [16–18]. Several researchers [19,20]proposed forecasting models using a genetic algorithm, or suggested a method of forecasting based on aggregated FTS and particle swarm optimization [21]. A novel method of forecasting based on hesitant fuzzy set was proposed by Bisht and Kumar [22], and fuzzy descriptor models for earthquakes was introduced by Bahrami and Shafiee [23]. A heuristic adaptive-order IFTS forecasting model was presented by Wang et al. [24]. Subsequently, Abhishekh et al. [25,26] presented a weighted type 2 FTS and score function-based IFTS forecasting approach. Moreover**,** Abhishekh and Kumar [27] suggested an approach for forecasting rice production in the area of FTS.

Since the accuracy rates of forecasting in the previous approaches are not good enough in the field of fuzzy and intuitionistic fuzzy time series, we introduce the notion of first- and high-order neutrosophic time series data for this research. Additionally, with the growing need to represent vague and random information, neutrosophic set (NS) theory [28] is an effective extension of fuzzy and intuitionistic fuzzy set theories. Smarandache [29] suggested NSs, which consist of truth membership function, indeterminacy membership function, and falsity membership function, as a better representation of reality. Neutrosophic sets received wide attention, as well as benefitting from various practical applications in diverse fields [30–39]. However, there are only two recent research papers published in the forecasting field (e.g., for stock market analysis). Guan et al. [40] proposed a new forecasting model based on multi-valued neutrosophic sets and two-factor third-order fuzzy logical relationships to forecast the stock market. Subsequently, Guan et al. [41] proposed a new forecasting method based on high-order fluctuation trends and information entropy.

The aim of this research is to enhance accuracy rates of forecasting in the area of fuzzy, intuitionistic fuzzy, and neutrosophic time series (NTS). In this research, we present the notion of forecasting based on first-and high-order NTS data by determining the suitable length of neutrosophic numbers that influence on expected values. We also suggest a neutrosophication of historical time series data, based on the biggest score function (i.e., the maximum value of score function), and define neutrosophic logical relationship groups (NLRGs) for obtaining forecasted outputs. The suggested approach of neutrosophic time series forecasting has been validated and compared with different existing models for showing its superiority.

The remaining parts of this research are organized as follows. The essential concepts of neutrosophic set and neutrosophic time series are briefly presented in Section 2. Section3presents the proposed neutrosophic time series method for the forecasting process. Section 4validates the proposed method by applying it to two numerical examples for showing its effectiveness; a comparison with other existing methods is presented. Finally, Section 5 concludes the research and determines future trends.

2. Some Basic Definitions of Neutrosophic Set and Neutrosophic Time Series

Neutrosophic time series is a concept for solving forecasting problems using neutrosophic concepts. In this section, we present the basic concepts of the neutrosophic set and of the neutrosophic time series (NTS).

Definition 1. *Let X be a finite universal set. A neutrosophic set N in X is an object having the following form:* $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$, *where* $T_N(x): X \to [0, 1]$ *determines the degree of truth membership function,* $I_N(x): X \to [0, 1]$ *determines the degree of indeterminacy, and function* $F_N(x): X \to [0, 1]$

determines the degree of non-membership or falsity function. For every $\boldsymbol{x \in X, 0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+}$ [29].

Definition 2. *A single valued triangular neutrosophic number* $\boldsymbol{\widetilde{N} = \langle (n_1, n_2, n_3); T_{\widetilde{N}}, I_{\widetilde{N}}, F_{\widetilde{N}} \rangle}$ *is a special neutrosophic set on the real number set R whose truth (membership), indeterminacy, and falsity (non-membership) degrees are as follows* [29]:

$$T_{\widetilde{N}}(x) = \begin{cases} \boldsymbol{T_{\widetilde{N}}\left(\frac{x - n_1}{n_2 - n_1}\right)} & \boldsymbol{(n_1 \leq x \leq n_2)} \\ \boldsymbol{T_{\widetilde{N}}} & \boldsymbol{(x = n_2)} \\ \boldsymbol{T_{\widetilde{N}}\left(\frac{n_3 - x}{n_3 - n_2}\right)} & \boldsymbol{(n_2 < x \leq n_3)} \\ \boldsymbol{0} & \textbf{otherwise}, \end{cases} \tag{1}$$

$$I_{\widetilde{N}}(x) = \begin{cases} \frac{(n_2 - x + I_{\widetilde{N}}(x - n_1))}{(n_2 - n_1)} & (n_1 \leq x \leq n_2) \\ I_{\widetilde{N}} & (x = n_2) \\ \frac{(x - n_2 + I_{\widetilde{N}}(n_3 - x))}{(n_3 - n_2)} & (n_2 < x \leq n_3) \\ 1 & \text{otherwise}, \end{cases} \tag{2}$$

$$F_{\widetilde{N}}(x) = \begin{cases} \frac{(n_2 - x + F_{\widetilde{N}}(x - n_1))}{(n_2 - n_1)} & (n_1 \leq x \leq n_2) \\ F_{\widetilde{N}} & (x = n_2) \\ \frac{(x - n_2 + F_{\widetilde{N}}(n_3 - x))}{(n_3 - n_2)} & (n_2 < x \leq n_3) \\ 1 & \text{otherwise}, \end{cases} \tag{3}$$

where $0 \leq T_{\widetilde{N}} \leq 1$, $0 \leq I_{\widetilde{N}} \leq 1$, $0 \leq F_{\widetilde{N}} \leq 1$, $0 \leq T_{\widetilde{N}} + I_{\widetilde{N}} + F_{\widetilde{N}} \leq 3, n_1, n_2, n_3 \in R$, and being the lower, median, and upper values of the triangular neutrosophic number.

Definition 3. *Let X and Y be two finite universal sets. A neutrosophic relation R from X to Y is a neutrosophic set in the direct product space X to Y:*

$$R = \{\langle \boldsymbol{(x, y)}, \boldsymbol{T_N(x, y)}, \boldsymbol{I_N(x, y)}, \boldsymbol{F_N(x)} \rangle | \boldsymbol{(x, y) \in X \times Y}\}$$

where $0^- \leq T_N(x, y) + I_N(x, y) + F_N(x, y) \leq 3^+$, $\forall (x, y) \in X \times Y$ for $T_N(x, y) \to [0,1]$, $I_N(x, y) \to [0,1]$, and $F_N(x, y) \to [0,1]: X \times Y \to [0,1]$.

Definition 4. *Let* $\boldsymbol{X(t)(t = 1, 2, \ldots,)}$, *a subset of* $\boldsymbol{R}$, *be the universe of discourse on which neutrosophic sets* $\boldsymbol{f_i(t) = \langle T_N(x, y), I_N(x, y), F_N(x) \rangle (i = 1, 2, \ldots)}$ *are defined.* $\boldsymbol{F(t) = \{f_1(x), f_2(x), \ldots\}}$ *is a collection of* $\boldsymbol{f_i(t)}$ *and it defines a neutrosophic time series on* $\boldsymbol{X(t)(t = 0, 1, 2, \ldots)}$.

Definition 5. *If there exists a neutrosophic relationship* $\boldsymbol{R(t-1, t)}$, *such that* $\boldsymbol{F(t) = F(t-1) \times R(t-1, t)}$, *where '×' represents an operator, then* $\boldsymbol{F(t)}$ *is said to be caused by* $\boldsymbol{F(t-1)}$. *The relationship between* $\boldsymbol{F(t)}$ *and* $\boldsymbol{F(t-1)}$ *is symbolized by* $\boldsymbol{F(t-1) \to F(t)}$.

Definition 6. *Let* $\boldsymbol{F(t)}$ *caused by* $\boldsymbol{F(t-1)}$ *only and symbolized by* $\boldsymbol{F(t-1) \to F(t)}$; *consequently, a neutrosophic relationship exists between* $\boldsymbol{F(t)}$ *and* $\boldsymbol{F(t-1)}$ *that is denoted as* $\boldsymbol{F(t) = F(t-1) \times R(t-1, t)}$, *since* $\boldsymbol{R}$ *is a first-order model of* $\boldsymbol{F(t)}$. *The* $\boldsymbol{F(t)}$ *is a time-invariant neutrosophic time series if* $\boldsymbol{R(t-1,t)}$ *is independent of time* $\boldsymbol{t}$, $\boldsymbol{R(t,t-1)=R(t-1,t-2)}\ \forall\ \boldsymbol{t}$. *Otherwise,* $\boldsymbol{F(t)}$ *is called a time-variant neutrosophic time series.*

Definition 7. *Let* $\boldsymbol{F(t-1)=\widetilde{N}_i}$ *and* $\boldsymbol{(t)=\widetilde{N}_j}$; *a neutrosophic logical relationship (NLR) can be defined as* $\boldsymbol{\widetilde{N}_i\rightarrow\widetilde{N}_j}$, *where* $\boldsymbol{\widetilde{N}_i}$, $\boldsymbol{\widetilde{N}_j}$ *are the current and next state of NLR. Since* $\boldsymbol{F(t)}$ *is occurred by more than one neutrosophic set* $\boldsymbol{F(t-n)}$, $\boldsymbol{F(t-n+1)},\ldots\boldsymbol{F(t-1)}$, *then the neutrosophic relationship is represented by* $\boldsymbol{\widetilde{N}_{i1},\widetilde{N}_{i2},\ldots,\widetilde{N}_{in}\rightarrow\widetilde{N}_j}$, *where* $\boldsymbol{F(t-n)=\widetilde{N}_{i1}}$, $\boldsymbol{F(t-n+1)=\widetilde{N}_{i2}}$. *The relationship is called high-order neutrosophic time series model.*

3. Neutrosophic Time Series Forecasting Algorithm

Because a neutrosophic set plays a significant role in decision-making and data analysis problems by handling vague, inconsistent, and incomplete information [30–39], we propose in this section an enhanced approach of forecasting using the concept of neutrosophic time series (NTS).

The stepwise method of the suggested algorithm of neutrosophic time series forecasting is dependent on historical time series data.

3.1. The Proposed Method of Forecasting Based on First-Order NTS Data

Step 1: By depending on the range of the existing data set, determine the universe of discourse U as follows:

- Select the largest D_l and the smallest D_s from all available data D_v, then

$$U = [D_s - D_1,\ D_l + D_2] \tag{4}$$

where D_1 and D_2 are two proper positive numbers assigned by experts in the problem domain. So, we can define D_1, D_2 as the values by which the range of the universe of discourse is less than the specified value of D_s for the first (i.e., D_1) or greater than the specified value of D_l for the latter (i.e., D_2).

Step 2: Create a partition of the universe of discourse, to m triangular neutrosophic numbers as follows:

- Decide the suitable length (Le) of available time series data:
    - o Among the value D_{v-1}, D_v, calculate all absolute differences and take the average of these differences.
    - o Consider half the average as the initial length.
    - o According to the obtained result, use the base mapping table [42] to determine the base for the length of intervals.
    - o Round the result to determine the appropriate length of neutrosophic numbers.
    - o For example: if we have these time series data 30,50,80,120,100,70, then the absolute differences will be 20,30,40,20,30, and the average of these values = 28. Then, half of the average will be 14 and this is the initial value of length. By using the base mapping table [42], the base for length = 10 because 14 locates in the range $[11-100]$ and by rounding the length 14 by the base ten, the result will equal 10. Here, the appropriate length of neutrosophic numbers equals 10.

- Compute the number of triangular neutrosophic numbers (m) as follows:

$$m = \frac{D_l + D_2 - D_s + D_1}{le} \tag{5}$$

Step 3: According to the numbers of triangular neutrosophic numbers on the universe of discourse and determined length (le), begin to construct the triangular neutrosophic numbers. The triangular neutrosophic numbers are $\widetilde{N}_1,\widetilde{N}_2,\ldots,\widetilde{N}_m$.

As we illustrated in Definition 2, each triangular neutrosophic number consists of two parts which are the value of the triangular neutrosophic number (lower, median, upper) and the degree of confirmation (truth/membership degree T, indeterminacy degree I, falsity/non-membership degree F). The initial value of T, I, F must be determined by experts according to the existing problem.

Step 4: Make a neutrosophication process of the existing data:
For $i, j = 1,2, \dots, v$ (the end of data):

Rule 1: Use this equation to calculate the score degree, and if the score degree of two neutrosophic numbers is not equal for any data, then choose the maximum value of the score degree:

$$SC_{\tilde{N}_j}(x_i) = 2 + T_{\tilde{N}_j}(x_i) - I_{\tilde{N}_j}(x_i) - F(x_i) \tag{6}$$

Then, select $SC_{\tilde{N}_k} = max\ (SC_{\tilde{N}_k}, SC_{\tilde{N}_k}, \dots, SC_{\tilde{N}_k})$ for $x_i, i = 1,2 \dots, n,\ 1 \leq k \leq n$, and assign the neutrosophic number $\tilde{N}_k$ to x_i.

Rule 2: If two neutrosophic numbers have the same score degree, then use the following equation to calculate the score degree, and select the minimum accuracy degree:

$$AC_{\tilde{N}_j}(x_i) = 2 + T_{\tilde{N}_j}(x_i) - I_{\tilde{N}_j}(x_i) + F(x_i) \tag{7}$$

Furthermore, $AC_{\tilde{N}_k} = min\ (AC_{\tilde{N}_k}, AC_{\tilde{N}_k}, \dots, AC_{\tilde{N}_k})$ for $x_i, i = 1,2 \dots, n,\ 1 \leq k \leq n$; assign the neutrosophic number $\tilde{N}_k$ to x_i.

Step 5: Construct the neutrosophic logical relationships (NLRs) as follows:
If $\tilde{N}_j, \tilde{N}_k$ are the neutrosophication values of year $n$and year $n+1$, respectively, then the NLR is symbolized as $\tilde{N}_j \to \tilde{N}_k$.

Step 6: Based on the NLR, begin to establish the neutrosophic logical relationship groups (NLRGs).

Step 7: Calculate the forecasted values as follows:

Rule 1: If the neutrosophication value of $data_i$ is $\tilde{N}_k$ and it is not caused by any other neutrosophication values and, by looking at the NLRG of this value, you cannot find the value which it depends on (i.e., $\neq\to \tilde{N}_k$), then the forecasted value in this case will equal—(i.e., leave it empty). The ≠ symbol means no value.

Rule 2: If the neutrosophication value of $data_i$ is $\tilde{N}_k$ and it is caused by $\tilde{N}_j$ ($\tilde{N}_j \to \tilde{N}_k$), then look at NLRG of $\tilde{N}_j$, and

- If NLRG of $\tilde{N}_j$ is empty (i.e., $\tilde{N}_j \to \emptyset$, or $\tilde{N}_j \to \tilde{N}_j$), then the forecasted value is the middle value of $\tilde{N}_j$.
- If NLRG of $\tilde{N}_j$ is one-to-one (i.e., $\tilde{N}_j \to \tilde{N}_k$), then the forecasted value is the middle value of $\tilde{N}_k$.
- If NLRG of $\tilde{N}_j$ is one-to-many (i.e., $\tilde{N}_j \to \tilde{N}_{k1}, \tilde{N}_{k2}, \dots, \tilde{N}_{kn}$), then the forecasted value is the average of the middle values of $\tilde{N}_{k1}, \tilde{N}_{k2}, \dots, \tilde{N}_{kn}$.

Step 8: Use the following equations to calculate the forecasting error:

$$\text{Root mean square error (RMSE)} = \sqrt{\frac{\sum_{i=1}^{n}(\text{Forecast}_i - \text{Actual}_i)^2}{n}}, \tag{8}$$

$$\text{Forecasting error} = \frac{|\text{Forecast} - \text{Actual}|}{Actual} \times 100, \tag{9}$$

$$\text{Average forecasting error (AFE) (\%)} = \frac{\text{Sum of forecasting error}}{number\ of\ errors} \times 100. \tag{10}$$

3.2. The Proposed Method of Forecasting Based on High-Order NTS Data

We can also apply the proposed method of forecasting based on high-order NTS data:

- All steps from 1 to 4 are the same as previously, but in step 5 we begin to construct the neutrosophic logical relationships (NLRs) of the nth order NTS, where $n \geq 2$.
- Based on the NLR of the nth order, NTS begin to establish the neutrosophic logical relationship groups (NLRGs).
- Calculate the forecasted values as follows:

- Rule 1: If the neutrosophication values of $data_i$ is $\widetilde{N}_l$ and it is not caused by any other neutrosophication values and, by looking at the NLRG of this value, you cannot find the values which it depends on (i.e., $\neq\rightarrow \widetilde{N}_l$), then the forecasted value in this case will equal— (i.e., leave it empty). The ≠ symbol means no value.
- Rule 2: If the neutrosophication value of $data_i$ is $\widetilde{N}_l$ and it is caused by $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik}$ (i.e., $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik} \rightarrow \widetilde{N}_l$), then look at the NLRG of $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik}$, and
  - If $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik} \rightarrow \emptyset$, then the forecasted value at this year is the average of the middle value of $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik}$.
  - If $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik} \rightarrow \widetilde{N}_j$, then the forecasted value at this year is the middle value of $\widetilde{N}_j$.
  - If $\widetilde{N}_{in}$, $\widetilde{N}_{i(n-1)}$, …, $\widetilde{N}_{ik} \rightarrow \widetilde{N}_j, \widetilde{N}_{j1}, \widetilde{N}_{j2}$, then the forecasted value at this year is the average of the middle value of $\widetilde{N}_j, \widetilde{N}_{j1}, \widetilde{N}_{j2}$.

4. Numerical Examples

In this section, we solve two numerical examples and compare outputs with other existing methods for verifying the applicability and superiority of the suggested method.

4.1. Numerical Example 1

In this example, the suggested approach is implemented on the benchmarking time series data of student enrollments at the University of Alabama from year 1971 to 1992 adopted from [26]. The steps are as follows:

Step 1: Let the two proper positive numbers D_1 and D_2 be 5 and 13, determined by the expert. By selecting the largest and the smallest observation from all available data which are presented in Table 1, then $D_l = 19{,}337$ and $D_s = 13{,}055$, respectively. Consequently, the universe of discourse $U = [13{,}055 - 5, 19{,}337 + 13] = [13{,}050, 19{,}350]$.

Step 2: Create a partition of the universe of discourse, to m triangular neutrosophic numbers, as follows:

- Determine the suitable length (Le) of available time series data:
  - From Table 1, the average of absolute differences = 510.3.
  - The initial length = $\frac{510.3}{2} = 255.15$.
  - By using the base mapping table [42], the base for length of intervals = 100, since it is located in the range [101,1000].
  - By rounding 255.15 with regard to base 100, then the appropriate length of neutrosophic numbers = 300.
- Compute the number of triangular neutrosophic numbers (m) as follows:

$$m = \frac{19350 - 13050}{300} = 21.$$

Then, we can partition U into 21 triangular neutrosophic numbers with length = 300.

Step 3: According to the number of triangular neutrosophic numbers on the universe of discourse and determined length (le), begin to construct the triangular neutrosophic numbers as follows:

$$\widetilde{N}_1 = \langle 13050, 13350, 13650; 0.90, 0.10, 0.10 \rangle,$$

$$\widetilde{N}_2 = \langle 13350, 13650, 13950; 0.80, 0.20, 0.10 \rangle,$$

$$\widetilde{N}_3 = \langle 13650, 13950, 14250; 0.90, 0.20, 0.10 \rangle,$$

$$\widetilde{N}_4 = \langle 13950, 14250, 14550; 0.85, 0.15, 0.10 \rangle,$$

$$\widetilde{N}_5 = \langle 14250, 14550, 14850; 0.75, 0.10, 0.30 \rangle,$$

$$\widetilde{N}_6 = \langle 14550, 14850, 15150; 0.90, 0.10, 0.10 \rangle,$$

$$\widetilde{N}_7 = \langle 14850{,}15150{,}15450; 0.60{,}0.30{,}0.40\rangle,$$
$$\widetilde{N}_8 = \langle 15150{,}15450{,}15750; 0.80{,}0.20{,}0.20\rangle,$$
$$\widetilde{N}_9 = \langle 15450{,}15750{,}16050; 0.70{,}0.20{,}0.30\rangle,$$
$$\widetilde{N}_{10} = \langle 15750{,}16050{,}16350; 0.90{,}0.10{,}0.30\rangle,$$
$$\widetilde{N}_{11} = \langle 16050{,}16350{,}16650; 0.85{,}0.10{,}0.15\rangle,$$
$$\widetilde{N}_{12} = \langle 16350{,}16650{,}16950; 0.80{,}0.20{,}0.20\rangle,$$
$$\widetilde{N}_{13} = \langle 16650{,}16950{,}17250; 0.90{,}0.10{,}0.30\rangle,$$
$$\widetilde{N}_{14} = \langle 16950{,}17250{,}17550; 0.90{,}0.10{,}0.30\rangle,$$
$$\widetilde{N}_{15} = \langle 17250{,}17550{,}17850; 0.75{,}0.10{,}0.30\rangle,$$
$$\widetilde{N}_{16} = \langle 17550{,}17850{,}18150; 0.65{,}0.20{,}0.35\rangle,$$
$$\widetilde{N}_{17} = \langle 17850{,}18150{,}18450; 0.90{,}0.10{,}0.10\rangle,$$
$$\widetilde{N}_{18} = \langle 18150{,}18450{,}18750; 0.90{,}0.10{,}0.10\rangle,$$
$$\widetilde{N}_{19} = \langle 18450{,}18750{,}19050; 0.60{,}0.20{,}0.30\rangle,$$
$$\widetilde{N}_{20} = \langle 18750{,}19050{,}19350; 0.90{,}0.10{,}0.10\rangle,$$
$$\widetilde{N}_{21} = \langle 19050{,}19350{,}19350; 0.90{,}0.10{,}0.10\rangle.$$

Step 4: Make a neutrosophication of the available time series data:

The first value of actual enrollments is 13,055 which is located only in the range of triangular neutrosophic number $\widetilde{N}_1$, then the neutrosophication value of 13,055 is $\widetilde{N}_1$ as in Table 1.

Also, the second value of actual enrollments (i.e., 13,563) locates in the range of triangular neutrosophic numbers $\widetilde{N}_1 = \langle 13050{,}13350{,}13650; 0.90{,}0.10{,}0.10\rangle$ and $\widetilde{N}_2 = \langle 13350{,}13650{,}13950; 0.80{,}0.20{,}0.10\rangle$.

Then, we must select the highest score degree of 13,563 as follows:

The membership, indeterminacy, and non-membership degrees of this value are calculated by using Equations (1)–(3) as follows:

$$T_{\widetilde{N}_1}(13563) = 0.261,\ \mathrm{I}_{\widetilde{N}_1}(13563) = 0.739,\ \mathrm{F}_{\widetilde{N}_1}(13563) = 0.739.$$

We must also calculate membership, indeterminacy, and non-membership degrees of 13,563 according to $\widetilde{N}_2 = \langle 13350{,}13650{,}13950; 0.80{,}0.20{,}0.10\rangle$ as follows:

$$T_{\widetilde{N}_2}(13563) = 0.568,\ \mathrm{I}_{\widetilde{N}_2}(13563) = 0.432,\ \mathrm{F}_{\widetilde{N}_2}(13563) = 0.361.$$

In this case, we must calculate the score degree of 13563 in both $\widetilde{\mathrm{N}}_1$ and $\widetilde{\mathrm{N}}_2$ and select the maximum value.

$$SC_{\widetilde{N}_1}(13563) = 2 + 0.262 - 0.739 - 0.739 = 0.783,$$
$$\text{and } SC_{\widetilde{N}_2}(13563) = 2 + 0.568 - 0.432 - 0.361 = 1.775.$$

Since the score degree of 13563 in $\widetilde{N}_2$ is greater than $\widetilde{N}_1$, then the neutrosophication value of 13563 is $\widetilde{N}_2$, as in Table 1.

We will apply the previous steps on the remaining data as follows:

The value 13867 locates in the range of $\widetilde{N}_2 = \langle 13350{,}13650{,}13950; 0.80{,}0.20{,}0.10\rangle$, and $\widetilde{N}_3 = \langle 13650{,}13950{,}14250; 0.90{,}0.20{,}0.10\rangle$.

Then

$$T_{\widetilde{N}_2}(13867) = 0.221, I_{\widetilde{N}_2}(13867) = 0.156, F_{\widetilde{N}_2}(13867) = 0.751.$$
$$T_{\widetilde{N}_3}(13867) = 0.651, I_{\widetilde{N}_3}(13867) = 0.421, F_{\widetilde{N}_3}(13867) = 0.349,$$
$$SC_{\widetilde{N}_2}(13867) = 2 + 0.221 - 0.156 - 0.751 = 1.314,$$
$$\text{and } SC_{\widetilde{N}_3}(13867) = 2 + 0.651 - 0.421 - 0.349 = 1.881.$$

So, the neutrosophication value of 13867 is $\widetilde{N}_3$.

Also, the value of 14,696 locates in the range of $\widetilde{N}_5 = \langle 14250{,}14550{,}14850; 0.75{,}0.10{,}0.30\rangle$, $\widetilde{N}_6 = \langle 14550{,}14850{,}15150; 0.90{,}0.10{,}0.10\rangle$, then

$$T_{\tilde{N}_5}(14696) = 0.385, I_{\tilde{N}_5}(14696) = 0.538, F_{\tilde{N}_5}(14696) = 0.641.$$

$$T_{\tilde{N}_6}(14696) = 0.438, I_{\tilde{N}_6}(14696) = 0.562, F_{\tilde{N}_6}(14696) = 0.562.$$

$$SC_{\tilde{N}_5}(14696) = 2 + 0.385 - 0.538 - 0.641 = 1.206,$$

$$\text{and } SC_{\tilde{N}_6}(14696) = 2 + 0.438 - 0.562 - 0.562 = 1.314.$$

So, the neutrosophication value of 14,696 is $\tilde{N}_6$.

The value 15,460 locates in the range of $\tilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, and $\tilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, then

$$T_{\tilde{N}_8}(15460) = 0.773, I_{\tilde{N}_8}(15460) = 0.226, F_{\tilde{N}_8}(15460) = 0.226.$$

$$T_{\tilde{N}_9}(15460) = 0.023, I_{\tilde{N}_9}(15460) = 0.973, F_{\tilde{N}_9}(15460) = 0.973.$$

$$SC_{\tilde{N}_8}(15460) = 2 + 0.773 - 0.226 - 0.226 = 2.321,$$

$$\text{and } SC_{\tilde{N}_9}(15460) = 2 + 0.023 - 0.973 - 0.976 = 0.074.$$

So, the neutrosophication value of 15,460 is $\tilde{N}_8$.

The value of 15,311 locates in the range of $\tilde{N}_7 = \langle 14850,15150,15450; 0.60,0.30,0.40\rangle$ and $\tilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, then

$$T_{\tilde{N}_7}(15311) = 0.278, I_{\tilde{N}_7}(15311) = 0.675, F_{\tilde{N}_7}(15311) = 0.722.$$

$$SC_{\tilde{N}_7}(15311) = 2 + 0.278 - 0.675 - 0.722 = 0.881.$$

$$T_{\tilde{N}_8}(15311) = 0.429, I_{\tilde{N}_8}(15311) = 0.570, F_{\tilde{N}_8}(15311) = 0.570.$$

$$SC_{\tilde{N}_8}(15311) = 2 + 0.429 - 0.570 - 0.570 = 1.289.$$

So, the neutrosophication value of 15,311 is $\tilde{N}_8$.

The value of 15,603 locates in the range of $\tilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$ and $\tilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, then

$$T_{\tilde{N}_8}(15603) = 0.392, I_{\tilde{N}_8}(15603) = 0.608, F_{\tilde{N}_8}(15603) = 0.608.$$

$$SC_{\tilde{N}_8}(15603) = 2 + 0.392 - 0.608 - 0.608 = 1.176.$$

$$T_{\tilde{N}_9}(15603) = 0.357, I_{\tilde{N}_9}(15603) = 0.592, F_{\tilde{N}_9}(15603) = 0.643.$$

$$SC_{\tilde{N}_9}(15603) = 2 + 0.357 - 0.592 - 0.643 = 1.122.$$

So, the neutrosophication value of 15,603 is $\tilde{N}_8$.

The value of 15,861 locates in the range of $\tilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, and $\tilde{N}_{10} = \langle 15750,16050,16350; 0.90,0.10,0.30\rangle$, then

$$T_{\tilde{N}_9}(15861) = 0.441, I_{\tilde{N}_9}(15861) = 0.496, F_{\tilde{N}_9}(15861) = 0.559.$$

$$SC_{\tilde{N}_9}(15861) = 2 + 0.441 - 0.496 - 0.559 = 1.386.$$

$$T_{\tilde{N}_{10}}(15861) = 0.333, I_{\tilde{N}_{10}}(15861) = 0.667, F_{\tilde{N}_{10}}(15861) = 0.741.$$

$$SC_{\tilde{N}_{10}}(15861) = 2 + 0.333 - 0.667 - 0.741 = 0.925.$$

So, the neutrosophication value of 15861 is $\tilde{N}_9$.

The value of 16,807 locates in the range of $\tilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$, $\tilde{N}_{13} = \langle 16650,16950,17250; 0.90,0.10,0.30\rangle$ then,

$$T_{\tilde{N}_{12}}(16807) = 0.381, I_{\tilde{N}_{12}}(16807) = 0.618, F_{\tilde{N}_{12}}(16807) = 0.618.$$

$$SC_{\tilde{N}_{12}}(16807) = 2 + 0.381 - 0.618 - 0.618 = 1.145.$$

$$T_{\tilde{N}_{13}}(16807) = 0.471, I_{\tilde{N}_{13}}(16807) = 0.529, F_{\tilde{N}_{13}}(16807) = 0.634.$$

$$SC_{\tilde{N}_{13}}(16807) = 2 + 0.471 - 0.529 - 0.634 = 1.308.$$

So, the neutrosophication value of 16807 is $\widetilde{N}_{13}$.

The value of 16919 locates in the range of $\widetilde{N}_{12} = \langle 16350, 16650, 16950; 0.80, 0.20, 0.20\rangle$, $\widetilde{N}_{13} = \langle 16650, 16950, 17250; 0.90, 0.10, 0.30\rangle$, then

$$T_{\widetilde{N}_{12}}(16919) = 0.063, I_{\widetilde{N}_{12}}(16919) = 0.917, F_{\widetilde{N}_{12}}(16919) = 0.917.$$

$$SC_{\widetilde{N}_{12}}(16919) = 2 + 0.063 - 0.917 - 0.917 = 0.229.$$

$$T_{\widetilde{N}_{13}}(16919) = 0.807, I_{\widetilde{N}_{13}}(16919) = 0.193, F_{\widetilde{N}_{13}}(16919) = 0.372.$$

$$SC_{\widetilde{N}_{13}}(16919) = 2 + 0.807 - 0.193 - 0.372 = 2.24.$$

So, the neutrosophication value of 16919 is $\widetilde{N}_{13}$.

The value of 16388 locates in the range of $\widetilde{N}_{11} = \langle 16050, 16350, 16650; 0.85, 0.10, 0.15\rangle$, $\widetilde{N}_{12} = \langle 16350, 16650, 16950; 0.80, 0.20, 0.20\rangle$, then

$$T_{\widetilde{N}_{11}}(16388) = 0.742, I_{\widetilde{N}_{11}}(16388) = 0.214, F_{\widetilde{N}_{11}}(16388) = 0.257.$$

$$SC_{\widetilde{N}_{11}}(16388) = 2 + 0.742 - 0.214 - 0.257 = 2.271.$$

$$T_{\widetilde{N}_{12}}(16388) = 0.101, I_{\widetilde{N}_{12}}(16388) = 0.898, F_{\widetilde{N}_{12}}(16388) = 0.898.$$

$$SC_{\widetilde{N}_{12}}(16388) = 2 + 0.101 - 0.898 - 0.898 = 0.305.$$

So, the neutrosophication value of 16388 is $\widetilde{N}_{11}$.

The value of 15433 locates in the range of $\widetilde{N}_{7} = \langle 14850, 15150, 15450; 0.60, 0.30, 0.40\rangle$, and $\widetilde{N}_{8} = \langle 15150, 15450, 15750; 0.80, 0.20, 0.20\rangle$, then

$$T_{\widetilde{N}_{7}}(15433) = 0.034, I_{\widetilde{N}_{7}}(15433) = 0.960, F_{\widetilde{N}_{7}}(15433) = 0.966.$$

$$SC_{\widetilde{N}_{7}}(15433) = 2 + 0.034 - 0.960 - 0.966 = 0.108.$$

$$T_{\widetilde{N}_{8}}(15433) = 0.754, I_{\widetilde{N}_{8}}(15433) = 0.245, F_{\widetilde{N}_{8}}(15433) = 0.245.$$

$$SC_{\widetilde{N}_{8}}(15433) = 2 + 0.754 - 0.245 - 0.245 = 2.264.$$

So, the neutrosophication value of 15433 is $\widetilde{N}_{8}$.

The value of 15497 locates in the range of $\widetilde{N}_{8} = \langle 15150, 15450, 15750; 0.80, 0.20, 0.20\rangle$ and $\widetilde{N}_{9} = \langle 15450, 15750, 16050; 0.70, 0.20, 0.30\rangle$ then,

$$T_{\widetilde{N}_{8}}(15497) = 0.674, I_{\widetilde{N}_{8}}(15497) = 0.325, F_{\widetilde{N}_{8}}(15497) = 0.325.$$

$$SC_{\widetilde{N}_{8}}(15497) = 2.024.$$

Also,

$$T_{\widetilde{N}_{9}}(15497) = 0.109, I_{\widetilde{N}_{9}}(15497) = 0.874, F_{\widetilde{N}_{9}}(15497) = 0.890.$$

$$SC_{\widetilde{N}_{9}}(15497) = 0.345.$$

So, the neutrosophication value of 15,433 is $\widetilde{N}_{8}$.

The value of 15,145 locates in the range of $\widetilde{N}_{6} = \langle 14550, 14850, 15150; 0.90, 0.10, 0.10\rangle$, and $\widetilde{N}_{7} = \langle 14850, 15150, 15450; 0.60, 0.30, 0.40\rangle$, then

$$T_{\widetilde{N}_{6}}(15145) = 0.015, I_{\widetilde{N}_{6}}(15145) = 0.985, F_{\widetilde{N}_{6}}(15145) = 0.985.$$

$$SC_{\widetilde{N}_{6}}(15145) = 0.045.$$

Also,

$$T_{\widetilde{N}_{7}}(15145) = 0.59, I_{\widetilde{N}_{7}}(15145) = 0.311, F_{\widetilde{N}_{7}}(15145) = 0.41.$$

$$SC_{\widetilde{N}_{7}}(15145) = 1.869.$$

So, the neutrosophication value of 15,145 is $\widetilde{N}_{7}$.

The value of 15,163 locates in the range of $\widetilde{N}_{7} = \langle 14850, 15150, 15450; 0.60, 0.30, 0.40\rangle$, $\widetilde{N}_{8} = \langle 15150, 15450, 15750; 0.80, 0.20, 0.20\rangle$, then

$$T_{\widetilde{N}_7}(15163) = 0.6,\ I_{\widetilde{N}_7}(15163) = 0.330,\ F_{\widetilde{N}_7}(15163) = 0.426.$$

$$SC_{\widetilde{N}_7}(15163) = 1.844.$$

Also

$$T_{\widetilde{N}_8}(15163) = 0.034, I_{\widetilde{N}_8}(15163) = 0.965, F_{\widetilde{N}_8}(15163) = 0.965.$$

$$SC_{\widetilde{N}_8}(15163) = 0.104.$$

So, the neutrosophication value of 15,163 is $\widetilde{N}_7$.

The value of 15,984 locates in the range of $\widetilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle, \widetilde{N}_{10} = \langle 15750,16050,16350; 0.90,0.10,0.30\rangle$, then

$$T_{\widetilde{N}_9}(15984) = 0.154,\quad I_{\widetilde{N}_9}(15984) = 0.824,\quad F_{\widetilde{N}_9}(15984) = 0.846.$$

$$SC_{\widetilde{N}_9}(15984) = 0.484.$$

Also,

$$T_{\widetilde{N}_{10}}(15984) = 0.702,\ I_{\widetilde{N}_{10}}(15984) = 0.298,\ F_{\widetilde{N}_{10}}(15984) = 0.454,$$
$$SC_{\widetilde{N}_{10}}(15984) = 1.95.$$

So, the neutrosophication value of 15984 is $\widetilde{N}_{10}$.

The value of 16859 locates in the range of $\widetilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$, $\widetilde{N}_{13} = \langle 16650,16950,17250; 0.90,0.10,0.30\rangle$, then

$$T_{\widetilde{N}_{12}}(16859) = 0.242,\ I_{\widetilde{N}_{12}}(16859) = 0.757,\quad F_{\widetilde{N}_{12}}(16859) = 0.757,$$

$$SC_{\widetilde{N}_{12}}(16859) = 0.728.$$

Also,

$$T_{\widetilde{N}_{13}}(16859) = 0.627, I_{\widetilde{N}_{13}}(16859) = 0.373,\quad F_{\widetilde{N}_{13}}(16859) = 0.512,$$

$$SC_{\widetilde{N}_{13}}(16859) = 1.442.$$

So, the neutrosophication value of 16859 is $\widetilde{N}_{13}$.

The value of 18150 locates in the range of $\widetilde{N}_{16} = \langle 17550,17850,18150; 0.65,0.20,0.35\rangle, \widetilde{N}_{17} = \langle 17850,18150,18450; 0.90,0.10,0.10\rangle$, then

$$T_{\widetilde{N}_{16}}(18150) = 0, I_{\widetilde{N}_{16}}(18150) = 1,\quad F_{\widetilde{N}_{16}}(18150) = 1,$$

$$SC_{\widetilde{N}_{16}}(18150) = 0.$$

Also,

$$T_{\widetilde{N}_{17}}(18150) = 0.90, I_{\widetilde{N}_{17}}(18150) = 0.1,\ F_{\widetilde{N}_{17}}(18150) = 0.1,$$

$$SC_{\widetilde{N}_{17}}(18150) = 2.7.$$

So, the neutrosophication value of 18150 is $\widetilde{N}_{17}$.

The value of 18970 locates in the range of $\widetilde{N}_{19} = \langle 18450,18750,19050; 0.60,0.20,0.30\rangle$, $\widetilde{N}_{20} = \langle 18750,19050,19350; 0.90,0.10,0.10\rangle$, then

$$T_{\widetilde{N}_{19}}(18970) = 0.16, I_{\widetilde{N}_{19}}(18970) = 0.786,\quad F_{\widetilde{N}_{19}}(18970) = 0.813.$$

$$SC_{\widetilde{N}_{19}}(18970) = 0.561.$$

Also,

$$T_{\widetilde{N}_{20}}(18970) = 0.66, I_{\widetilde{N}_{20}}(18970) = 0.34,\quad F_{\widetilde{N}_{20}}(18970) = 0.34.$$

$$SC_{\widetilde{N}_{20}}(18970) = 1.98.$$

So, the neutrosophication value of 18,970 is $\widetilde{N}_{20}$.

The value of 19,328 locates in the range of $\widetilde{N}_{20} = \langle 18750{,}19050{,}19350; 0.90{,}0.10{,}0.10\rangle$, $\widetilde{N}_{21} = \langle 19050{,}19350{,}19; 0.90{,}0.10{,}0.10\rangle$, then

$$T_{\widetilde{N}_{20}}(19328) = 0.066,\ I_{\widetilde{N}_{20}}(19328) = 0.992,\quad F_{\widetilde{N}_{20}}(19328) = 0.992.$$

$$SC_{\widetilde{N}_{20}}(19328) = 0.082.$$

Also,

$$T_{\widetilde{N}_{21}}(19328) = 0.834, I_{\widetilde{N}_{21}}(19328) = 0.166,\quad F_{\widetilde{N}_{21}}(19328) = 0.166.$$

$$SC_{\widetilde{N}_{21}}(19328) = 2.502.$$

So, the neutrosophication value of 19,328 is $\widetilde{N}_{21}$.

The value of 19,337 locates in the range of $\widetilde{N}_{20} = \langle 18750{,}19050{,}19350; 0.90{,}0.10{,}0.10\rangle$,

$\widetilde{N}_{21} = \langle 19050{,}19350{,}19; 0.90{,}0.10{,}0.10\rangle$, then

$$T_{\widetilde{N}_{20}}(19337) = 0.039, I_{\widetilde{N}_{20}}(19337) = 0.961,\quad F_{\widetilde{N}_{20}}(19337) = 0.961.$$

$$SC_{\widetilde{N}_{20}}(19337) = 0.117.$$

Also,

$$T_{\widetilde{N}_{21}}(19337) = 0.861, I_{\widetilde{N}_{21}}(19337) = 0.139,\quad F_{\widetilde{N}_{21}}(19337) = 0.139.$$

$$SC_{\widetilde{N}_{21}}(19337) = 2.583.$$

So, the neutrosophication value of 19,337 is $\widetilde{N}_{21}$.

Finally, the value of 18,876 locates in the range of $\widetilde{N}_{19} = \langle 18450{,}18750{,}19050; 0.60{,}0.20{,}0.30\rangle$, $\widetilde{N}_{20} = \langle 18750{,}19050{,}19350; 0.90{,}0.10{,}0.10\rangle$, then

$$T_{\widetilde{N}_{19}}(18876) = 0.348, I_{\widetilde{N}_{19}}(18876) = 0.536,\quad F_{\widetilde{N}_{19}}(18876) = 0.594.$$

$$SC_{\widetilde{N}_{19}}(18876) = 1.218.$$

Also,

$$T_{\widetilde{N}_{20}}(18876) = 0.378, I_{\widetilde{N}_{20}}(18876) = 0.622,\quad F_{\widetilde{N}_{20}}(18876) = 0.622.$$

$$SC_{\widetilde{N}_{20}}(18876) = 1.134.$$

So, the neutrosophication value of 18,876 is $\widetilde{N}_{19}$.

Table 1. Actual and neutrosophication values of student enrollments.

Years	Actual Enrollments	Neutrosophication Values of Enrollments $\widetilde{N}$
1971	13,055	$\widetilde{N}_1$
1972	13,563	$\widetilde{N}_2$
1973	13,867	$\widetilde{N}_3$
1974	14,696	$\widetilde{N}_6$
1975	15,460	$\widetilde{N}_8$
1976	15,311	$\widetilde{N}_8$
1977	15,603	$\widetilde{N}_8$
1978	15,861	$\widetilde{N}_9$
1979	16,807	$\widetilde{N}_{13}$
1980	16,919	$\widetilde{N}_{13}$
1981	16,388	$\widetilde{N}_{11}$
1982	15,433	$\widetilde{N}_8$
1983	15,497	$\widetilde{N}_8$
1984	15,145	$\widetilde{N}_7$
1985	15,163	$\widetilde{N}_7$
1986	15,984	$\widetilde{N}_{10}$

1987	16,859	$\widetilde{N}_{13}$
1988	18,150	$\widetilde{N}_{17}$
1989	18,970	$\widetilde{N}_{20}$
1990	19,328	$\widetilde{N}_{21}$
1991	19,337	$\widetilde{N}_{21}$
1992	18,876	$\widetilde{N}_{19}$

Step 5: Construct the neutrosophic logical relationships (NLRs) as in Table 2:

Table 2. Neutrosophic logical relationships.

$\widetilde{N}_1 \rightarrow \widetilde{N}_2$	$\widetilde{N}_2 \rightarrow \widetilde{N}_3$	$\widetilde{N}_3 \rightarrow \widetilde{N}_6$	$\widetilde{N}_6 \rightarrow \widetilde{N}_8$	$\widetilde{N}_8 \rightarrow \widetilde{N}_8$
$\widetilde{N}_8 \rightarrow \widetilde{N}_9$	$\widetilde{N}_9 \rightarrow \widetilde{N}_{13}$	$\widetilde{N}_{13} \rightarrow \widetilde{N}_{13}$	$\widetilde{N}_{13} \rightarrow \widetilde{N}_{11}$	$\widetilde{N}_{11} \rightarrow \widetilde{N}_8$
$\widetilde{N}_8 \rightarrow \widetilde{N}_7$	$\widetilde{N}_7 \rightarrow \widetilde{N}_7$	$\widetilde{N}_7 \rightarrow \widetilde{N}_{10}$	$\widetilde{N}_{10} \rightarrow \widetilde{N}_{13}$	$\widetilde{N}_{13} \rightarrow \widetilde{N}_{17}$
$\widetilde{N}_{17} \rightarrow \widetilde{N}_{20}$	$\widetilde{N}_{20} \rightarrow \widetilde{N}_{21}$	$\widetilde{N}_{21} \rightarrow \widetilde{N}_{21}$	$\widetilde{N}_{21} \rightarrow \widetilde{N}_{19}$	

Step 6: Based on NLR, begin to establish the neutrosophic logical relationship groups (NLRGs) as in Table 3.

Table 3. Neutrosophic logical relationship groups (NLRGs) of enrollments.

$\widetilde{N}_1 \rightarrow \widetilde{N}_2$		
$\widetilde{N}_2 \rightarrow \widetilde{N}_3$		
$\widetilde{N}_3 \rightarrow \widetilde{N}_6$		
$\widetilde{N}_6 \rightarrow \widetilde{N}_8$		
$\widetilde{N}_7 \rightarrow \widetilde{N}_7$	$\widetilde{N}_7 \rightarrow \widetilde{N}_{10}$	
$\widetilde{N}_8 \rightarrow \widetilde{N}_7$	$\widetilde{N}_8 \rightarrow \widetilde{N}_8$	$\widetilde{N}_8 \rightarrow \widetilde{N}_9$
$\widetilde{N}_9 \rightarrow \widetilde{N}_{13}$		
$\widetilde{N}_{10} \rightarrow \widetilde{N}_{13}$		
$\widetilde{N}_{11} \rightarrow \widetilde{N}_8$		
$\widetilde{N}_{13} \rightarrow \widetilde{N}_{11}$	$\widetilde{N}_{13} \rightarrow \widetilde{N}_{13}$	$\widetilde{N}_{13} \rightarrow \widetilde{N}_{17}$
$\widetilde{N}_{17} \rightarrow \widetilde{N}_{20}$		
$\widetilde{N}_{20} \rightarrow \widetilde{N}_{21}$		
$\widetilde{N}_{21} \rightarrow \widetilde{N}_{19}$	$\widetilde{N}_{21} \rightarrow \widetilde{N}_{21}$	

Step 7: Calculate the forecasted values as in Table 4:

To calculate the forecasted value of 13,055 in year 1971, do the following:

- Look at the neutrosophication value of 13055 in year 1971 which is $\widetilde{N}_1$ as it appears in Table 1.
- Go to NLRG which is presented in Table 3, and because $\widetilde{N}_1$is the first neutrosophication value of data, then it is not caused by any other value (i.e., $\neq\rightarrow \widetilde{N}_1$) as in Table 3.

Therefore, the forecasted value of 13,055 is—Which means leaving it empty, as we illustrated in Step 7, Rule 1 of the proposed algorithm.

Also, to calculate the forecasted value of 13,563 in year 1972, do the following:

- Look at the neutrosophication value of 13,563 in year 1972 which is $\widetilde{N}_2$ as it appears in Table 1, and because $\widetilde{N}_2$ is caused by $\widetilde{N}_1$ (i.e., $\widetilde{N}_1 \rightarrow \widetilde{N}_2$), then
- Go to Table 3, and look at the NLRG which starts with $\widetilde{N}_1$, and we noted that it is $\widetilde{N}_1 \rightarrow \widetilde{N}_2$. Then the forecasted value of 13,563 is the middle value of $\widetilde{N}_2$.

Another illustrating example for calculating the forecasted value of 18,876 in year 1992:

- Look at the neutrosophication value of 18,876 in year 1992 which is $\widetilde{N}_{19}$ as it appears in Table 1. Since $\widetilde{N}_{19}$ is caused by $\widetilde{N}_{21}$, then
- Go to Table 3, and look at the NLRG which starts with $\widetilde{N}_{21}$ (i.e., $\widetilde{N}_{21} \rightarrow \widetilde{N}_{19}, \widetilde{N}_{21} \rightarrow \widetilde{N}_{21}$). Then the forecasted value of 18876 is the average of the middle values of $\widetilde{N}_{19}$, $\widetilde{N}_{21}$, and it will equal 19,050.

The other forecasted values are calculated in the same manner.

Table 4. Actual and forecasted values of enrollments.

Years	Actual Enrollments	Forecasted Values of Enrollments
1971	13,055	–
1972	13,563	13,650
1973	13,867	13,950
1974	14,696	14,850
1975	15,460	15,450
1976	15,311	15,450
1977	15,603	15,450
1978	15,861	15,450
1979	16,807	16,950
1980	16,919	17,150
1981	16,388	17,150
1982	15,433	15,450
1983	15,497	15,450
1984	15,145	15,450
1985	15,163	15,600
1986	15,984	15,600
1987	16,859	16,950
1988	18,150	17,150
1989	18,970	19,050
1990	19,328	19,350
1991	19,337	19,050
1992	18,876	19,050

The actual and forecasted values of enrollments appear in Figure 1.

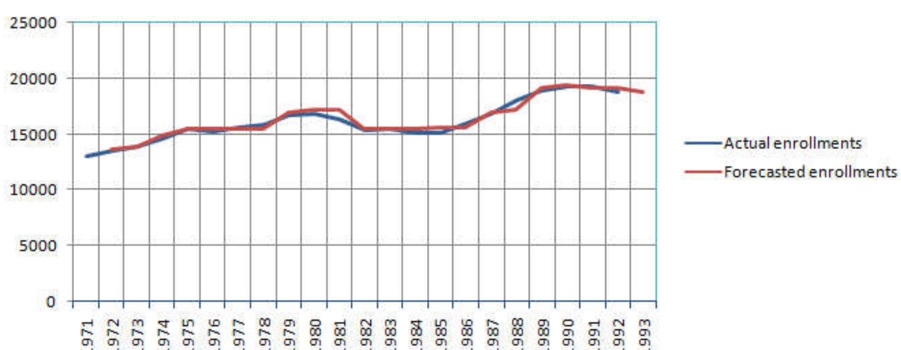

Figure 1. Forecasted and actual enrollments.

The forecasted enrollment data obtained with the suggested method, along with the forecasted data obtained with the models in [14,17,43–46], are presented in Table 5.

Table 5. Forecasted values by suggested method and other methods.

Years	Actual Values	Forecasted Values						
		Proposed	[43]	[44]	[45]	[46]	[14]	[17]
1971	13,055	–	–	–	–	–	–	–
1972	13,563	13,650	14,242.0	14,025	13,250	14,031.35	14,586	13,693
1973	13,867	13,950	14,242.0	14,568	13,750	14,795.36	14,586	13,693
1974	14,696	14,850	14,242.0	14,568	13,750	14,795.36	15,363	14,867
1975	15,460	15,450	15,774.3	15,654	14,500	14,795.36	15,363	15,287
1976	15,311	15,450	15,774.3	15,654	15,375	16,406.57	15,442	15,376
1977	15,603	15,450	15,774.3	15,654	15,375	16,406.57	15,442	15,376
1978	15,861	15,450	15,774.3	15,654	15,625	16,406.57	15,442	15,376
1979	16,807	16,950	16,146.5	16,197	15,875	16,406.57	15,442	16,523
1980	16,919	17,150	16,988.3	17,283	16,833	17,315.29	17,064	16,606
1981	16,388	17,150	16,988.3	17,283	16,833	17,315.29	17,064	17,519
1982	15,433	15,450	16,146.5	16,197	16,500	17,315.29	15,438	16,606
1983	15,497	15,450	15,474.3	15,654	15,500	16,406.57	15,442	15,376
1984	15,145	15,450	15,474.3	15,654	15,500	16,406.57	15,442	15,376
1985	15,163	15,600	15,474.3	15,654	15,125	16,406.57	15,363	15,287
1986	15,984	15,600	15,474.3	15,654	15,125	16,406.57	15,363	15,287
1987	16,859	16,950	16,146.5	15,654	16,833	16,406.57	15,438	16,523
1988	18,150	17,150	16,988.3	16,197	16,667	17,315.29	17,064	17,519
1989	18,970	19,050	19,144.0	17,283	18,125	19,132.79	19,356	19,500
1990	19,328	19,350	19,144.0	18,369	18,750	19,132.79	19,356	19,000
1991	19,337	19,050	19,144.0	19,454	19,500	19,132.79	19,356	19,500
1992	18,876	19,050	19,144.0	19,454	19,500	19,132.79	19,356	19,500

By comparing the proposed method with other existing methods in Table 5, the RMSE and AFE tools confirm that the suggested method is better than others, as shown in Table 6.

Table 6. Error measures.

Tool	Proposed	[43]	[44]	[45]	[46]	[14]	[17]
RMSE	342.68	478.45	781.47	646.67	805.17	642.68	493.56
AFE (%)	1.44	2.39	3.61	2.98	4.28	2.96	2.33

We combined forecasted values with respect to all methods in Figure 2.

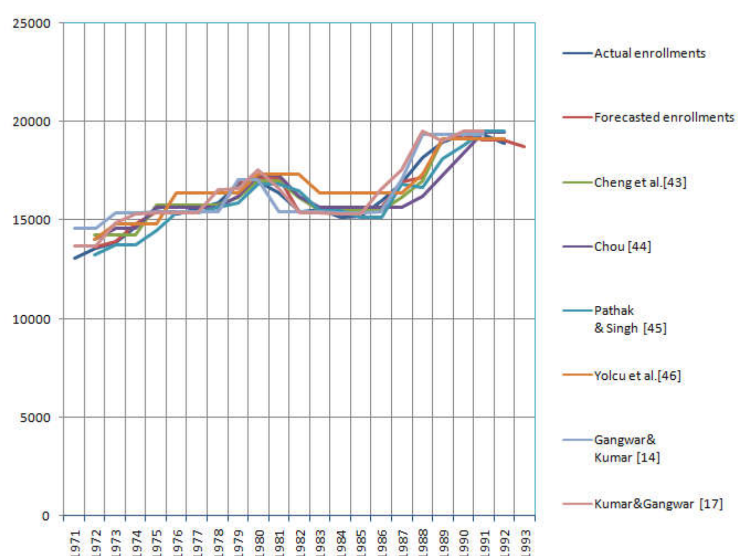

Figure 2. Comparison figures between all forecasted values.

If we plan to find the second-order neutrosophic logical relationships of the previous example by applying the proposed method of forecasting based on the second-order NTS, they are as shown in Table 7.

Table 7. Second-order NLR.

$\tilde{N}_1, \tilde{N}_2 \rightarrow \tilde{N}_3$
$\tilde{N}_2, \tilde{N}_3 \rightarrow \tilde{N}_6$
$\tilde{N}_3, \tilde{N}_6 \rightarrow \tilde{N}_8$
$\tilde{N}_6, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_9$
$\tilde{N}_8, \tilde{N}_9 \rightarrow \tilde{N}_{13}$
$\tilde{N}_9, \tilde{N}_{13} \rightarrow \tilde{N}_{13}$
$\tilde{N}_{13}, \tilde{N}_{13} \rightarrow \tilde{N}_{11}$ $\tilde{N}_{13}, \tilde{N}_{11} \rightarrow \tilde{N}_8$
$\tilde{N}_{11}, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_7$
$\tilde{N}_8, \tilde{N}_7 \rightarrow \tilde{N}_7$
$\tilde{N}_7, \tilde{N}_7 \rightarrow \tilde{N}_{10}$
$\tilde{N}_7, \tilde{N}_{10} \rightarrow \tilde{N}_{13}$
$\tilde{N}_{10}, \tilde{N}_{13} \rightarrow \tilde{N}_{17}$
$\tilde{N}_{13}, \tilde{N}_{17} \rightarrow \tilde{N}_{20}$
$\tilde{N}_{17}, \tilde{N}_{20} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{20}, \tilde{N}_{21} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{21}, \tilde{N}_{21} \rightarrow \tilde{N}_{19}$

The second-order neutrosophic logical relationship groups of the previous example are as shown in Table 8.

Table 8. Second-order NLRGs.

$\tilde{N}_1, \tilde{N}_2 \rightarrow \tilde{N}_3$		
$\tilde{N}_2, \tilde{N}_3 \rightarrow \tilde{N}_6$		
$\tilde{N}_3, \tilde{N}_6 \rightarrow \tilde{N}_8$		
$\tilde{N}_6, \tilde{N}_8 \rightarrow \tilde{N}_8$		
$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_8$	$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_9$	$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_7$
$\tilde{N}_8, \tilde{N}_9 \rightarrow \tilde{N}_{13}$		
$\tilde{N}_9, \tilde{N}_{13} \rightarrow \tilde{N}_{13}$		
$\tilde{N}_{13}, \tilde{N}_{13} \rightarrow \tilde{N}_{11}$		
$\tilde{N}_{13}, \tilde{N}_{11} \rightarrow \tilde{N}_8$		
$\tilde{N}_{11}, \tilde{N}_8 \rightarrow \tilde{N}_8$		
$\tilde{N}_8, \tilde{N}_7 \rightarrow \tilde{N}_7$		
$\tilde{N}_7, \tilde{N}_7 \rightarrow \tilde{N}_{10}$		
$\tilde{N}_7, \tilde{N}_{10} \rightarrow \tilde{N}_{13}$		
$\tilde{N}_{10}, \tilde{N}_{13} \rightarrow \tilde{N}_{17}$		
$\tilde{N}_{13}, \tilde{N}_{17} \rightarrow \tilde{N}_{20}$		
$\tilde{N}_{17}, \tilde{N}_{20} \rightarrow \tilde{N}_{21}$		
$\tilde{N}_{20}, \tilde{N}_{21} \rightarrow \tilde{N}_{21}$		
$\tilde{N}_{21}, \tilde{N}_{21} \rightarrow \tilde{N}_{19}$		

We compared forecasted values of enrollments based on the second order of neutrosophic logical relationship groups of the proposed method with the method of second order presented by Gautam and Singh [47]. The results are shown in Table 9.

Table 9. Actual and forecasted values of enrollments based on the second order of the proposed method vs. the Gautam and Singh [47] method.

Years	Actual Enrollments	Second-Order Forecasted Values of the Proposed Method	Forecasted Values in [47]
1971	13,055	–	–
1972	13,563	–	–
1973	13,867	13,950	13,800
1974	14,696	14,850	14,400
1975	15,460	15,450	15,300
1976	15,311	15,450	15,300
1977	15,603	15,450	15,600
1978	15,861	15,450	15,600
1979	16,807	16,950	16,800
1980	16,919	16,950	16,800
1981	16,388	16,350	16,200
1982	15,433	15,450	15,300
1983	15,497	15,450	15,300
1984	15,145	15,450	15,000
1985	15,163	15,150	15,000
1986	15,984	16,050	15,900
1987	16,859	16,950	16,800
1988	18,150	18,150	18,000
1989	18,970	19,050	18,900
1990	19,328	19,350	19,200
1991	19,337	19,350	19,200
1992	18,876	18,750	18,600

The MSE and AFE of the two methods are presented in Table 10.

Table 10. Error measures of the proposed method and the Gautam and Singh method [47].

Tool	Proposed	[47]
MSE	19,823.4	24,443.4
AFE (%)	0.60	0.81

From Table 10, it appears that our proposed method of second order is also better than the proposed method of second order presented by Gautam and Singh [47].

In addition, the third-order neutrosophic logical relationship groups of the previous example are constructed and shown in Table 11.

Table 11. Third-order NLRGs.

$\tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \rightarrow \tilde{N}_6$
$\tilde{N}_2, \tilde{N}_3, \tilde{N}_6 \rightarrow \tilde{N}_8$
$\tilde{N}_3, \tilde{N}_6, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_6, \tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_8, \tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_9$
$\tilde{N}_8, \tilde{N}_8, \tilde{N}_9 \rightarrow \tilde{N}_{13}$
$\tilde{N}_8, \tilde{N}_9, \tilde{N}_{13} \rightarrow \tilde{N}_{13}$
$\tilde{N}_9, \tilde{N}_{13}, \tilde{N}_{13} \rightarrow \tilde{N}_{11}$
$\tilde{N}_{13}, \tilde{N}_{13}, \tilde{N}_{11} \rightarrow \tilde{N}_8$
$\tilde{N}_{13}, \tilde{N}_{11}, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_{11}, \tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_7$
$\tilde{N}_8, \tilde{N}_8, \tilde{N}_7 \rightarrow \tilde{N}_7$
$\tilde{N}_8, \tilde{N}_7, \tilde{N}_7 \rightarrow \tilde{N}_{10}$
$\tilde{N}_7, \tilde{N}_7, \tilde{N}_{10} \rightarrow \tilde{N}_{13}$
$\tilde{N}_7, \tilde{N}_{10}, \tilde{N}_{13} \rightarrow \tilde{N}_{17}$
$\tilde{N}_{10}, \tilde{N}_{13}, \tilde{N}_{17} \rightarrow \tilde{N}_{20}$
$\tilde{N}_{13}, \tilde{N}_{17}, \tilde{N}_{20} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{17}, \tilde{N}_{20}, \tilde{N}_{21} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{20}, \tilde{N}_{21}, \tilde{N}_{21} \rightarrow \tilde{N}_{19}$

We also compared the forecasted values of enrollments based on the third order of neutrosophic logical relationship groups of the proposed method with the proposed methods of third order presented by [8,9,47], and the results are shown in Table 12.

Table 12. Actual and forecasted values of enrollments based on the third order of the proposed method vs. the methods presented by [8,9,47].

Years	Actual Enrollments	Third-Order Forecasted Values of the Proposed Method	Forecasted Values in [47]	Forecasted Values in [8]	Forecasted Values in [9]
1971	13,055	–	–	–	–
1972	13,563	–	–	–	–
1973	13,867	–	–	–	–
1974	14,696	14,850	14,400	14,500	14,750
1975	15,460	15,450	15,300	15,500	15,750
1976	15,311	15,450	15,300	15,500	15,500
1977	15,603	15,450	15,600	15,500	15,500
1978	15,861	15,750	15,600	15,500	15,500
1979	16,807	16,950	16,800	16,500	16,500
1980	16,919	16,950	16,800	16,500	16,500
1981	16,388	16,350	16,200	16,500	16,500
1982	15,433	15,450	15,300	15,500	15,500
1983	15,497	15,450	15,300	15,500	15,500
1984	15,145	15,150	15,000	15,500	15,250
1985	15,163	15,150	15,000	15,500	15,500
1986	15,984	16,050	15,900	15,500	15,500
1987	16,859	16,950	16,800	16,500	16,500
1988	18,150	18,150	18,000	18,500	18,500
1989	18,970	19,050	18,900	18,500	18,500
1990	19,328	19,350	19,200	19,500	19,500
1991	19,337	19,350	19,200	19,500	19,500
1992	18,876	18,750	18,600	18,500	18,750

The MSE and AFE of the methods are presented in Table 13.

Table 13. Error measures of the proposed method and the [8,9,47] methods.

Tool	Proposed	[47]	[8]	[9]
MSE	7367.316	25,493.6	86,694	76,509
AFE (%)	0.40	0.82	1.52	1.40

4.2. Numerical example 2

We verified the proposed method by solving the TAIEX2004 example [40], and by putting D_1 and D_2 equal 56 and 61, respectively, then U = $[5600.17, 6200.69]$. Also we calculated the suitable length as we illustrated previously and found that it is equal to 40. Therefore, the number of triangular neutrosophic numbers is equal to 12. For these neutrosophic numbers, the decision makers determined the truth, indeterminacy, and falsity degrees equal to 0.9,0.1,0.1, respectively. The actual and forecasted values of the TAIEX2004 example are presented in Table 14 and Figure 3.

Table 14. Actual and forecasted values of TAIEX2004.

Dates	Actual Values	Forecasted Values of the Proposed Method
01/11/2004	5656.17	–
02/11/2004	5759.61	5760.17
03/11/2004	5862.85	5813.5
04/11/2004	5860.73	5900.17
05/11/2004	5931.31	5900.17
08/11/2004	5937.46	5903.02
09/11/2004	5945.2	5903.02
10/11/2004	5948.49	5940.17

11/11/2004	5874.52	5940.17
12/11/2004	5917.16	5903.02
15/11/2004	5906.69	5903.02
16/11/2004	5910.85	5903.02
17/11/2004	6028.68	5940.17
18/11/2004	6049.49	5940.17
19/11/2004	6026.55	5940.17
22/11/2004	5838.42	5830.17
23/11/2004	5851.1	5830.17
24/11/2004	5911.31	5903.02
25/11/2004	5855.24	5830.17
26/11/2004	5778.65	5813.5
29/11/2004	5785.26	5813.5
30/11/2004	5844.76	5860.17
1/12/2004	5798.62	5830.17
02/12/2004	5867.95	5860.17
03/12/2004	5893.27	5900.17
06/12/2004	5919.17	5900.17
07/12/2004	5925.28	5903.02
08/12/2004	5892.51	5903.02
09/12/2004	5913.97	5900.17
10/12/2004	5911.63	5903.02
13/12/2004	5878.89	5903.02
14/12/2004	5909.65	5900.17
15/12/2004	6002.58	5903.02
16/12/2004	6019.23	6040.17
17/12/2004	6009.32	6040.17
20/12/2004	5985.94	6040.17
21/12/2004	5987.85	6040.17
22/12/2004	6001.52	6040.17
23/12/2004	5997.67	6040.17
24/12/2004	6019.42	6040.17
27/12/2004	5985.94	6040.17
28/12/2004	6000.57	6040.17
29/12/2004	6088.49	6040.17
30/12/2004	6100.86	6080.17
31/12/2004	6139.69	6080.17

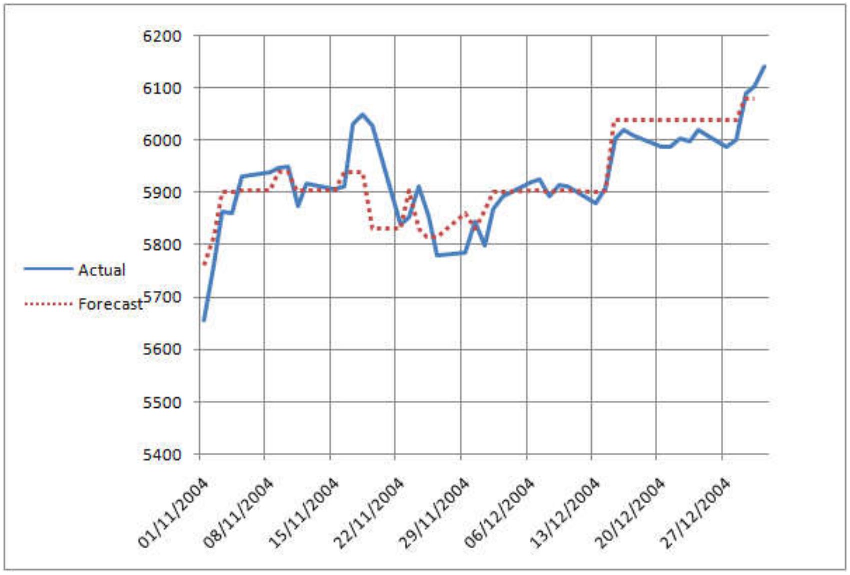

Figure 3. Actual and forecasted values of TAIEX2004.

The RMSE and AFE of the proposed method are presented in Table 15.

Table 15. Error measures of the proposed method.

Tool	Proposed
RMSE	42.05
AFE (%)	0.005

To confirm the performance of the suggested method, we compared it with other existing methods and the results are shown in Table 16 and Figure 4.

Table 16. Error measures of the proposed method and other existing methods which solved the TAIEX2004 example.

Methods	RMSE
Guan et al.'s method [40]	53.01
Huarng et al.'s method [48]	73.57
Chen and Kao's method [49]	58.17
Cheng et al.'s method [50]	54.24
Chen et al.'s method [51]	56.16
Chen and Chang's method [52]	60.48
Chen and Chen's method [53]	61.94
Yu and Huarng's method [54]	55.91
Proposed method	42.05

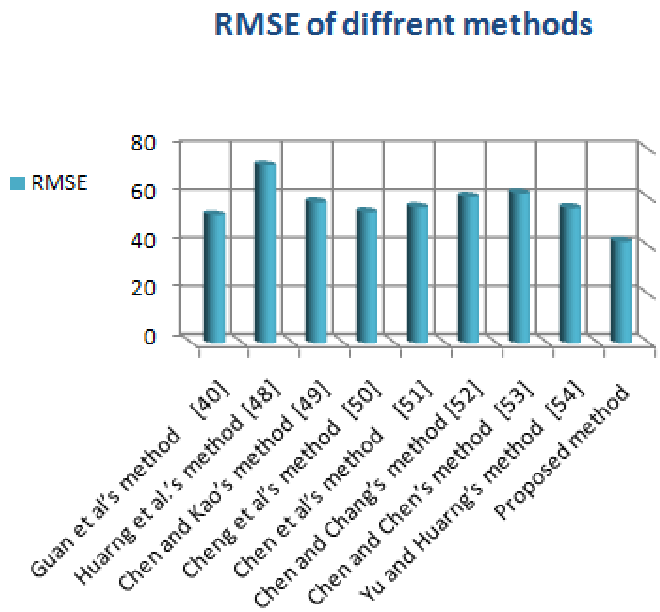

Figure 4. The RMSE of different methods that solved the TAIEX2004 example.

TAIEX2004 is used as a baseline to compare our method with other competitive methods, to compare and identify how all the methods can manage error reduction. The RMSE is a common approach used in financial analysis [55]. Compared with the existing methods as shown in Table 16, our proposed method can offer the least presence of errors since it has the most minimized RMSE. In other words, our method appears to be performing the best in reducing errors and ensuring all our analyses are accurate with insights. This may provide a new insight for business intelligence with artificial intelligence, cloud computing, and neutrosophic research.

5. Conclusion and future directions

The objective of this research was to enhance the accuracy rates of forecasting, since the forecasting accuracy rates in the existing approaches of fuzzy and intuitionistic fuzzy time series were not accurate enough. Thus, in this research we introduced the notion of first-and high-order neutrosophic time series data by defining the fitting length of intervals and proposing a novel method for calculating forecasted values. In order to obtain truth, indeterminacy, and falsity membership degrees of historical data, we defined triangular neutrosophic numbers. The neutrosophication process of historical time series data depends on the biggest score function of the triangular neutrosophic numbers. For the deneutrosophication process of first- and high-order NTS, we used simple arithmetic computations. The suggested approach of first- and high-order neutrosophic time series proved its superiority against other existing methods in the field of fuzzy, intuitionistic fuzzy, and neutrosophic time series. In the future, we plan to apply meta-heuristic optimization techniques for improving the accuracy of the suggested method. We will apply this model for predicting other time series, such as demand forecasting, electricity consumption, etc. Furthermore**,** we may consider using other approaches for comparing similarities of historical data, like information entropy.

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