Air quality analysis using complex neutrosophic concept lattice

Prem Kumar Singh[a,*]

[a]*Amity Institute of Information Technology,*
Amity University,
Sector-125, Noida - 201313
Uttar Pradesh–INDIA.

Abstract

The precise measurement of uncertainty exists in the complex fuzzy attributes and their graphical analytics is a mathematically expensive tasks for knowledge processing researchers. To deal with this problem recently, the calculus of complex neutrosophic sets are introduced to characterize the uncertainty in data based on its truth, indeterminacy, and falsity membership–value, independently. In this process, a major problem is addressed while finding some of the interesting patterns in the given complex neutrosophic data set. To solve this problem, the current paper proposes a method for discovery of complex neutrosophic concepts and their graphical structure visualization using the properties of Lower Neighbors algorithm. One of the suitable examples of the proposed method is illustrated for precise measurement of uncertainty exists in Air Quality Index (AQI) and its pattern at given phase of time.

Keywords: Air Quality Index (AQI); Complex fuzzy sets; Complex neutrosophic set; Concept lattice; Formal Concept Analysis(FCA); Three–way fuzzy concept lattice

*Corresponding author

Email address: premsingh.csjm@gmail.com;premsingh.csjm@yahoo.com (Prem Kumar Singh)

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1. Introduction

Recently, the calculus of complex vague set concept lattice [20] and its properties [21–23] is introduced which has been given a new orientation to characteristic the complex data sets in more understandable manner when compared to approaches available in unipolar fuzzy space [7, 12, 19]. The reason is that the calculus of complex set [8–10] and concept lattice theory [11] provides a well established mathematical framework to measure the human cognitive thought [7, 12]. To measure the fluctuation in uncertainty the calculus of complex fuzzy sets [8–9] helps in its precise representation using amplitude and phase term in unipolar or bipolar space as discussed by Prem Kumar Singh [18–20]. In this process, an important problem was addressed while handling the three–way fuzzy attributes [21–23]. To achieve this goal, properties of complex neutrosophic sets [2] are introduced for handling multi–decision attributes [6, 28]. This extensive version of complex fuzzy set [23–24] and its properties in the neutrosophic or three–way polar space [26–28] given a new orientation to analyze the data sets based on applied abstract algebra [1, 11, 17, 32–34]. Towards this extension recently, Prem Kumar Singh [21–23] introduces neutrosophic and complex vague concept lattice [20] for precise approximation of computational linguistics [37–40]. In case the data set contains complex neutrosophy attributes [2] then charactering them based on their acceptation, rejection and uncertain regions is major concern. One of the examples for complex neutrosophic attributes exists which creates an issue for precise analysis of knowledge processing task. One of the suitable examples is 22° temperature used to consider as cool in summer season, warm in winter season whereas fair (or uncertain) in spring season. This interpretation of cognitive thought changes at each phase of time. In this case adequate measurement of human cognitive thought is major issue with its numerical representation and graphical analytics. This problem is dovetail in many cases of multi–dimensional criteria which affect the human life as Fire Danger Index (FDI) which turns into Bushfire [1]. Similarly, measuring the Air Quality Index (AQI) is another concern for researchers at the given phase of time to characterize its changes by acceptation, rejection and uncertain regions. The motivation is to provide the information contained in the data sets in an understandable manner based on its maximal acceptation regions, minimal rejection or uncertain regions. To solve this problem, the

[1] $https://en.wikipedia.org/wiki/Bushfires\_in\_Australia$

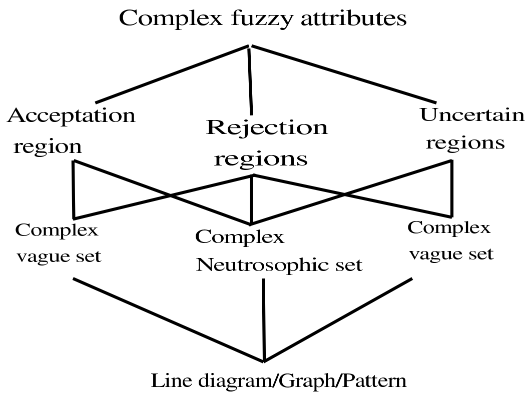

Figure 1: The motivation to introduce complex neutrosophic concept lattice

current paper focuses on depth analysis of complex neutrosophic context and its graphical structure visualization based on applied abstract algebra.

Recenlty, some of the researchers started analysis on neutrosophic attributes and its data sets based on lattice theory [13, 15–16] and graphical analytics [21–23] to approximate them in three–way decision space [35–37]. All of these approaches fails in precise measurement of periodic changes in three–way or neutrosophic fuzzy attributes. One of the suitable examples is Air Quality Index (AQI) [2] of any country changes at each interval of time. In this case, measuring the AQI based on its acceptation, rejection or uncertain regions is a computationally expensive tasks for the researchers. The reason is that the AQI values used to fluctuate several times in a day. The level of ozone used to become high from morning to afternoon to early evening. Similarly, the particle pollution is high at the day time which may increase subsequently in the busy or office time i.e. morning and evening. In all of these cases precise representation of uncertainty measuring its changes at given phase of time is mathematically expensive tasks. To conquer this problem recently, some of the approaches based on complex neutrosophic sets [2] and its lattice theory [9] is introduced for knowledge processing tasks [14, 20, 29–30] of multi–decision attributes [1, 23, 42–43] at δ–granulation [38–41]. Table 1 shows that, the calculus of complex neutrosophic set provides

[2] *https://en.wikipedia.org/wiki/Air_quality_index*

Table 1: Some necessary conditions for the uses of complex neutrosophic set

	Complex fuzzy set	Complex vague set	Complex neutrosophic set
Domain	Universe of Discourse	Universe of Discourse	Universe of Discourse
Co–domain	Unipolar–value in unit circle [0, 1]	Bipolar–valued in unit in circle [0, 1]	Three–valued in unit circle $[0, 1]^3$
Truth membership	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
False membership	No	Yes in [0, 1]	Yes in $[0, 1]^3$
Indeterminacy membership	No	1–True –false	Yes in $[0, 1]^3$
Amplitude term	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Phase term measurement	Yes $[0, 2\pi]$	Yes $[0, 2\pi]$	Yes in $[0, 2\pi]$
Uncertainty measurement	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Fluctuation measurement	Yes	Yes	Yes
Graph	Yes	Yes	Yes

more adequate measurement of uncertainty and its changes in the complex fuzzy attributes based on its truth, indeterminacy and uncertain regions, independently. This is one of the major advantages of complex neutrosophic set which motivated the current study to utilize its properties in knowledge processing tasks. The motivation is to provide a compressed graphical structure visualization of the complex neutrosophic data sets in the concept lattice based on their super and sub concept hierarchy. The objective is to find some of the useful pattern in the given complex neutrosophic context for multi–decision process as shown in Figure 1. To fulfil this need, calculus of applied lattice theory [11, 33] and its extensive properties [18–22] is utilized in this paper for generating the complex neutrosophic concepts and its hierarchical order visualization based on their Lower Neighbor. The reason is concept of Lower Neighbor provides a easier way to discover the concepts within limited time complexity [3–5]. In this way, the proposed method provides a basis of an algorithm for compressed graphical visualization of complex neutrosophic context in the concept lattice. To provide a more readable and easier way to understand the pattern in the given complex fuzzy context while extracting the information. It can be considered as one of the significant output of the proposed method in field of complex data set analysis.

Remaining part of the paper is organized as follows: Section 2 provides some basic preliminaries about complex neutrosophic sets. Section 3 provides a method for generating the complex neutrosophic concepts using their Lower Neighbor. Section 4 provides illustration of the proposed method with an example. Section 5 contains discussions followed by conclusions, and references.

2. Complex neutrosophic context and its graphical visualization

Recently, it seems that. handling complex neutrosophic data set like measuring the quality of AQI is mathematically rigorous tasks. To deal with these types of complex or seasonal data sets one solution is to represent them matrix format and try to visualize them in the graph. The current section shows some useful definitions in this section to acheive this goal adequately:

Definition 1. (Complex fuzzy set) [24–26] : A complex fuzzy set Z can be defined over a universe of discourse U having a single fuzzy membership–value at given phase of time. The complex–valued grade of membership of an element $z \in U$ can be characterized by $\mu_Z(z)$. The membership–values that $\mu_Z(z)$ may receive all values within the unit circle of a defined complex plane in the form $\mu_Z(z)$ $=r_z(x)e^{iw_z(x)}$, where i$=\sqrt{-1}$, both $r_Z(z)$ and $w_Z(z)$ are real–valued and $r_Z(z) \in [0, 1]$. The complex fuzzy set Z may be represented as the set of ordered pairs :

$$Z=\{(z, \mu_Z(z)) : z \in U\}= \left\{(z, r_Z(z)e^{iwZ(z)}) : z \in U\right\}$$

Example 1: Let us suppose, an expert wants to measure the level of AQI index of the given geographical regions (i.e. object–x_1) based on its saturation value of PM_{10} (i.e. attribute y_1). The user collected the data and saw that the saturation value of PM_{10} changes 50 percent in six to seven months. This complex fuzzy attributes can be written using the properties of complex fuzzy set as follows: $0.5e^{i1.2\pi}$. In case the user want to represent the indeterminacy and falsity regions then properties of neutrosophic set can be useul.

Definition 2. (Neutrosophic set) [27]: It provides a way to characterize the uncertainty and vagueness in attributes $y \in Y$ based on truth–membership function $T_N(y)$, a indeterminacy–membership function $I_N(y)$ and a falsity–membership function $F_N(y)$. The $T_N(y)$, $I_N(y)$ and $F_N(y)$ are real standard or non–standard subsets of $]0^-, 1^+[$as given below:

$$T_N : Y \rightarrow]0^-, 1^+[,$$

$$I_N : Y \rightarrow]0^-, 1^+[,$$

$$F_N : Y \rightarrow]0^-, 1^+[.$$

The neutrosophic set can be represented as follows:

$N = \{(x, T_N(y), I_N(x), F_N(y)) : y \in Y\}$ where $0^- \leq T_N(y) + I_N(y) + F_N(y) \leq 3^+$.

It is noted that $0^- = 0 - \epsilon$ where 0 is its standard part and ϵ is its non–standard part. Similarly, $1^+ = 1 + \epsilon$ ($3^+ = 3 + \epsilon$) where 1 (or 3) is standard part and ϵ is its non–standard part. The real standard (0, 1) or [0, 1] can be also used to represent the neutrosophic set. The union and intersection among neutrosophic sets N_1 and N_2 can be computed as follows:

$\bullet N_1 \bigcup N_2 = \{(x, T_{N_1}(x) \vee T_{N_2}(x), I_{N_1}(x) \wedge I_{N_2}(x), F_{N_1}(x) \wedge F_{N_2}(x)) : x \in X\}$
The intersection of N_1 and N_2 can be defined as follows:

$\bullet N_1 \bigcap N_2 = \{(x, T_{N_1}(x) \wedge T_{N_2}(x), I_{N_1}(x) \vee I_{N_2}(x), F_{N_1}(x) \vee F_{N_2}(x)) : x \in X\}$.

Example 2: Example 1 represents the acceptation part of PM_{10} in the given year. In case, the expert wants to measure the acceptation, rejection or indeterminacy part exists in AQI then the properties of neutrosophic set can be useful. To illustrate the problem, let us consider an expert founds that the level of PM_{10} in the given area is 60 percent accepted, 20 rejected and 10 percent uncertain for the health of citizens. This neutrosophic value can be written as (0.6, 0.2, 0.1) where 0.6 represents the truth–membership value, 0.2 indeterminacy–membership value, and 0.1 falsity–membership value. Now suppose the user want to measure the changes on the acceptation, rejection and uncertain regions of neutrosophic value at the given year. In this case, the properties of complex neutrosophic set can be useful.

Definition 3. (Complex neutrosophic set) [2] : A complex neutrosophic set Z can be defined over a universe of discourse U. The uncertainty in the attributes $z \in U$ can be characterized by true $^{-}0 < r_{T_z} < 1^{+}$, indeterminacy $^{-}0 \leq r_{I_z} < 1^{+}$ and falsity membership–value $^{-}0 \leq r_{F_z} < 1^{+}$, independently with a given phase of time (0, 2π). It can be observed that, the "amplitude" term in complex neutrosophic set satisfies the property $^{-}0 \leq r_{T_z} + r_{I_z} + r_{F_z} \leq 3^{+}$ whereas the "phase" term can be characterized by $w^r_{T_z}$, $w^r_{I_z}$ and $w^r_{F_z}$ in real–valued interval $[0, 2\pi]$. It can be represented as $Z = \Big\{(z, (r_{T_z} e^{w^r_{T_z}}, r_{I_z} e^{w^r_{I_z}}, r_{F_z} e^{w^r_{F_z}})) : z \in U\Big\}$.

Example 3: Let us extend the Example 2, that the expert agreed that quality of PM_{10} (i.e. y_1) is accepted 60 percent at the end of six to seven months, 20 percent rejected at the end of four to five months whereas the user is uncertain 10 percent at the end of nine to tenth month of a year. This complex query can be written using the complex neutrosophic set as given below:

$x_1 = (0.6e^{1.2\pi}, 0.2e^{0.7\pi}, 0.1e^{1.6\pi})/y_1$ where 2π is considered as phase term to represent the year.

Definition 4. (Complex neutrosophic graph) [20] : A complex neutrosophic fuzzy graph G=(V, μ_c, ρ_c) is a non–empty set in which the value of vartices μ_c: $V \rightarrow (r_c^T(v).e^{iarg_c^T(v)}, r_c^I(v).e^{iarg_c^I(v)}, r_c^F(v).e^{iarg_c^F(v)})$ and edges ρ_c: $V \times V \rightarrow (r_c^T(v \times v).e^{iarg_c^T(v \times v)}, r_c^I(v \times v).e^{iarg_c^I(v \times v)}, r_c^F(v \times v).e^{iarg_c^F(v \times v)})$. It means the membership–values can be characterized by the truth, indeterminate and falsity membership–values within the unit circle of a complex argand plane in the given period of time. It can be represented through amplitude and phase term of defined complex neutrosophic set as follows:

$$r_c^T(v_i \times v_j).e^{iarg_c^T(v_i \times v_j)} \leq min\left(r_c^T(v_i), r_c^T(v_i)\right).e^{imin\left(arg_c^T(v_i), arg_c^T(v_j)\right)}.$$

$$r_c^I(v_i \times v_j).e^{iarg_c^I(v_i \times v_j)} \geq max\left(r_c^I(v_i), r_c^I(v_i)\right).e^{imax\left(arg_c^I(v_i), arg_c^I(v_j)\right)}.$$

$$r_c^F(v_i \times v_j).e^{iarg_c^F(v_i \times v_j)} \geq max\left(r_c^F(v_i), r_c^F(v_i)\right).e^{imax\left(arg_c^F(v_i), arg_c^F(v_j)\right)}.$$

The given complex fuzzy graph is complete iff:

$r_c(v_i \times v_j).e^{iarg_c(v_i \times v_j)} = min\left(r_c(v_i), r_c(v_i)\right).e^{imin(arg_c(v_i), arg_c(v_j))}$ for the truth, indeterminacy and falsity membership function, independently.

Example 4: Let us suppose, the expert wants to analyze the four given areas x_1, x_2, x_3, x_4 based on the level of PM_{10} and its changes as shown in in Table 2. The corresponding relationship among them is shown in Table 3. The obtained complex neutrosophic contexts shown in Table 3 and 4 can be visualized in using the vertices V and edges E of a defined complex neutrosophic graph as shown in Figure 2.

Definition 5. (Lattice structure of neutrosophic set) [38–39] : Let N_1 and N_2 are neutrosophic sets in the universe of discourse X. Then $N_1 \subseteq N_2$ iff $T_{N_1}(x) \leq T_{N_2}(x)$, $I_{N_1}(x) \geq I_{N_2}(x)$, $F_{N_1}(x) \geq F_{N_2}(x)$ for any $x \in X$. $(N, \wedge, \vee)$ is bounded lattice. Also the structure $(N, \wedge, \vee, (1,0,0), (0,1,1), \neg)$ follow the D–Morgan algebra. Similarly, this lattice structure can be used to represent the three–way fuzzy concept lattice and their concept using Gödel logic .

Definition 6. (Neutrosophic fuzzy concepts) [21] [32–33]: Let us suppose, a set of attribute i.e. $(B) = \{y_j, (T_B(y_j), I_B(y_j), F_N(y_j)) \in [0,1]^3 : \forall y_j \in Y\}$

Table 2: A representation of PM_{10} and its fluctuation at the given areas using complex neutrosophic set

Vertex	y_1
x_1	$(0.5e^{i0.7\pi}, 0.3e^{i1.2\pi}, 0.2e^{i1.8\pi})$
x_2	$(0.7e^{i0.2\pi}, 0.6e^{i1.6\pi}, 0.1e^{i0.4\pi})$
x_3	$(0.4e^{i0.4\pi}, 0.5e^{i0.8\pi}, 0.6e^{i2\pi})$
x_4	$(0.8e^{i0.3\pi}, 0.7e^{i1.7\pi}, 0.3e^{i0.7\pi})$

Table 3: A complex neutrosophic relation among the given areas using their PM_{10}

Edges	y_1
(x_1, x_2)	$(0.5e^{i0.2\pi}, 0.3e^{i1.2\pi}, 0.1e^{i0.4\pi})$
(x_1, x_3)	$(0.4e^{i0.4\pi}, 0.3e^{i0.8\pi}, 0.2e^{i1.8\pi})$
(x_2, x_4)	$(0.7e^{i0.2\pi}, 0.6e^{i1.6\pi}, 0.1e^{i0.4\pi})$
(x_3, x_4)	$(0.4e^{i0.3\pi}, 0.5e^{i0.8\pi}, 0.3e^{i0.7\pi})$

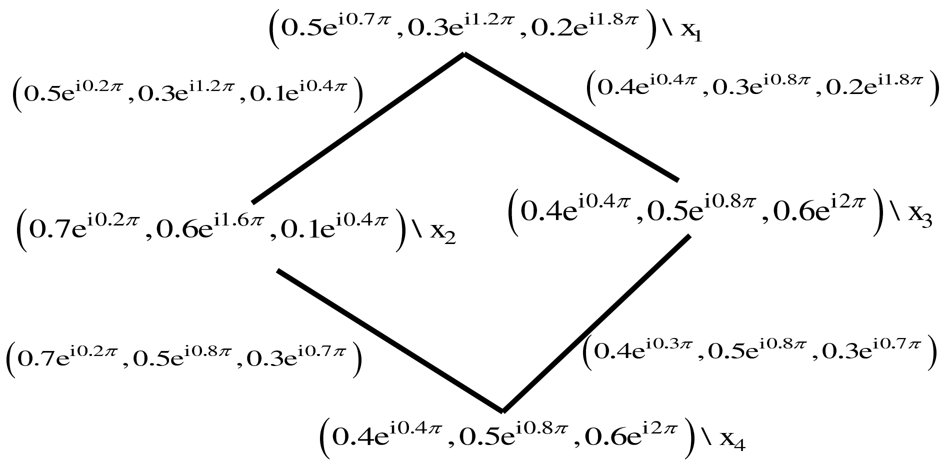

Figure 2: A three–way complex neutrosophic graph visualization of Table 2 and 3

where $j \leq m$. For the selected three–polar attribute set find their covering objects set in the given fuzzy context i.e.

$(A)= \{x_i, (T_A(x_i), I_A(x_i), F_A(x_i)) \in [0,1]^3 : \forall x_i \in X\}$ where $i \leq n$.

The obtain pair (A, B) is called as a neutrosophic fuzzy concept iff: $A^{\uparrow}= B$ and $B^{\downarrow}= A$. It can be interpreted as neutrosophic set of objects having maximal truth membership value, minimum indeterminacy and minimum falsity membership value with respect to integrating the information from the common set of fuzzy attributes in a defined space $[0,1]^3$ using component–wise Gödel residuated lattice. After that, none of the fuzzy set of objects (or attributes) can be found which can make the membership value of the obtained fuzzy set of attributes (or objects) bigger. Then pair of neutrosophic set (A, B) is called as a formal concepts, where A is called as extent, and B is called as intent. In this process, a problem arises when the truth, falsity and indeterminacy value of a neutrosophic attributes changes at each given phase of time. To overcome from this issue, a method is proposed in the next section for generating the complex neutrosophic concepts based on their Lower Neighbors as it is considered as one of the easier and cost effective method [3].

3. A proposed method for generating the complex neutrosophic concept

Generating the complex neutrosophic concepts is addressed as one of the major issues for precise analysis of complex data sets based on its acceptation, rejection, and uncertain regions. To deal with this problem recently subset based algorithms are introduced to handle the neutrosophic context [20–23]. This paper focuses on generating the complex neutrosophic concepts based on their Lower neighbor algorithm. One of the most suitable reason behind this method is that it provides an easier way to understand the concept generation when compared to other algorithms. The steps of the proposed method is as follows:

Step (1) The first complex neutrosophic concepts can be investigated by exploring all the objects set $\uparrow$ i.e.

$$\Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{I_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\Big)^{\uparrow} = \Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\Big).$$

The membership–value for the complex neutrosophic set of attributes can be computed as follows:

Amplitude:

min $(y_j, r_{T_{y_j}})$ for true membership,

max $(y_j, r_{I_{y_j}})$ for indeterminacy membership,

max $(y_j, r_{F_{y_j}})$ for false membership,

Phase term:

min $(y_j, e^{w^r_{T_{xj}}})$ for true phase term,

max $(y_j, e^{w^r_{I_{yj}}})$ for indeterminacy phase term.

max $(y_j, e^{w^r_{F_{yj}}})$ for false phase term.

Step (2) The Lower Neighbor of the complex fuzzy concepts generated at Step 1 can be investigated using uncovered attributes i.e. :y_k=$Y - y_j$ where $j \leq m$ and $k \leq m|$.

Step (3) The obtained complex neutrosophic set of attributes set can be explored using the Galois connection ($\downarrow$) on Amplitude = (1.0, 0.0, 0.0) and Phase=$(0, 2\pi)$ term. The covering objects set can be found by ($\downarrow$) as follows:

$$\Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{yj}}}, r_{I_{y_j}} e^{w^r_{I_{yj}}}, r_{F_{y_j}} e^{w^r_{F_{yj}}})\Big)^{\downarrow} = \Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{xi}}}, r_{I_{x_i}} e^{w^r_{x_{xi}}}, r_{F_{x_i}} e^{w^r_{F_{xi}}})\Big).$$

Compute the membership–values for the obtained objects:

Amplitude:

min $(x_i, r_{T_{x_i}})$ for true membership,

max$(x_i, r_{I_{x_i}})$ for Indeterminacy membership,

max$(x_i, r_{F_{x_i}})$ for false membership, Phase term:

min $(x_i, e^{w^r_{T x_i}})$ for true phase term,

max $(x_i, e^{w^r_{I x_i}})$ for indeterminacy phase term.

max $(x_i, e^{w^r_{F x_i}})$ for false phase term.

Step (4). Apply the up operator $\uparrow$ on the constituted objects set:

$$\Big(x_i, (r_{R_{x_i}} e^{w^r_{T x_i}}, r_{I_{x_i}} e^{w^r_{x x_i}}, r_{F_{x_i}} e^{w^r_{F x_i}})\Big)^{\uparrow} = \Big(y_j, (r_{R_{y_j}} e^{w^r_{T y_j}}, r_{I_{y_j}} e^{w^r_{I y_j}}, r_{F_{y_j}} e^{w^r_{F y_j}})\Big).$$

Compute the neutrosophic membership–value for the obtained attributes:

Amplitude:

min $(y_j, r_{T_{y_j}})$ for true membership,

max $(y_j, r_{I_{y_j}})$ for indeterminacy membership,

max $(y_j, r_{F_{y_j}})$ for false membership,

Phase term:

min $(y_j, e^{w^r_{T x_j}})$ for true phase term,

max $(y_j, e^{w^r_{I y_j}})$ for indeterminacy phase term,

max $(y_j, e^{w^r_{F y_j}})$ for false phase term.

Step (5) The obtained pair of complex neutrosophic set of objects and attributes (A, B) represents the Lower Neighbor of given concept. The distinct Lower Neighbors having maximal acceptance of complex neutrosophic membership value while integrating the information among objects and attributes set can be considered as Next Neighbor.

Step (6) Similarly, all the complex neutrosophic concepts can be discovered using the uncovered attributes.

Step (7) The complex neutrosophic concepts lattice can be build using their Next Neighbor.

Step (8) Extract some of the meaningful information from the obtained lattice. The pseudo code for the proposed algorithm is shown in Table 4.

Complexity: Let us suppose, the number of objects and the number of attributes in the given three–way complex fuzzy context is n and m, respectively. To discover the Lower Neighbor of three–way complex fuzzy attributes takes O($n^3.m$) time complexity for the amplitude and phase term, respectively. The removal of similar Lower Neighbor takes at most O($n^3 * m^3$) time complexity for the amplitude and phase term, independently. This computation gives the proposed method takes O($|C|.n^6.m^6$) where, C is Lower Neighbor. In this way the proposed method shown in Table 4 takes less computation when compared to any of the available approaches for processing the three–way complex fuzzy data set.

4. Air quality measurement using complex neutrosophic concept lattice

Recently, Prem Kumar Singh [20–23] has paid attention towards analysis of uncertainty in data beyond the unipolar or bipolar fuzzy space. In this process, a major problem was addressed while handling the complex fuzzy attributes in which the uncertainty and its fluctuation changes at each given phase of time. One of the most suitable example of these type of data sets is Air Quality Index as it is considered one of the major issues in country like India [3]. To deal with this type of data several approaches [1, 14, 17, 27–32] based on properties of complex fuzzy sets are introduced, recently. The most interesting is one of the researcher tried to measure the uncertainty based on acceptation, rejection and uncertain part of the given attribute [2]. This method gives a way to characterize the complex data set based on its truth, falsity and indeterminacy–membership–values, independently with their periodic phase of time. Recently, some of the researchers paid the attention

[3] http://indianexpress.com/article/india/india-news-india/delhi-air-pollution-smog-health-effects-3739848/

Table 4: A proposed algorithm for building the complex neutrosophic concept lattice

Input: A three–way complex fuzzy context $\mathbf{K}$=$(X,\ Y,\ \tilde{R})$
where $|X|$=n, $|Y|$=m.
Output: The set of three–way complex fuzzy concepts
1. Find the maximal covering attributes for the objects set (X) using $(\uparrow)$:
(i) $\Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{I_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\Big)^{\uparrow} = \Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\Big)$.
(ii) Compute the neutrosophic membership–value for the obtained attributes:
min $(y_j, r_{T_{y_j}} e^{w^r_{T_{x_j}}})$ for true membership,
max $(y_j, r_{I_{y_j}} e^{w^r_{I_{y_j}}})$ for indeterminacy membership,
max $(y_j, r_{F_{y_j}} e^{w^r_{F_{y_j}}})$ for false membership,
(iii) Apply the operator $(\downarrow)$ on the obtained attribute set:
$$\Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\Big)^{\downarrow} = \Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\Big)$$
(iv) This gives first complex neutrosophic concept (A, B).
2. Find its Lower Neighbor:
3. **for** (k=0 to m)
Y_k=$Y - y_j$ where $j, k \leq m$
(i). New attribute set: y_k= $\{y_j, y_k\}$
(ii). Set maximal acceptance for the complex neutrosophic attributes
i.e. Amplitude = (1.0, 0.0, 0.0) and Phase=$(0, 2\pi)$
(iii). Apply the operator $(\downarrow)$ on the attributes
$$\Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\Big)^{\downarrow} = \Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\Big)$$
(iv). Compute the membership of the obtained objects using Step 1 (ii):
(v). Apply the operator $(\uparrow)$ on the constituted set of objects:
$$\Big(x_i, (r_{R_{x_i}} e^{w^r_{T_{x_i}}}, r_{I_{x_i}} e^{w^r_{x_{x_i}}}, r_{F_{x_i}} e^{w^r_{F_{x_i}}})\Big)^{\uparrow} = \Big(y_j, (r_{R_{y_j}} e^{w^r_{T_{y_j}}}, r_{I_{y_j}} e^{w^r_{I_{y_j}}}, r_{F_{y_j}} e^{w^r_{F_{y_j}}})\Big).$$
(vi). Compute the membership of obtained attributes as per Step 1 (ii).
End **for**.
4. Distinct Lower Neighbor is considered as Next Neighbor.
5. Similarly, generate all the Next Neighbor using uncovered attributes.
6. Build the complex neutrosophic concept lattice for knowledge extraction.

towards three–way fuzzy concept lattice and its concept learning [15–16, 21–23] without measuring the periodic changes in the uncertainty. To fill this backdrop, current paper introduces a method based on three–way complex neutrosophic sets and its graph for precise measurement of uncertainty and its changes at given phase of time in section 3. The proposed algorithm shown in Table 4 is based on Next Neighbor algorithm to generate the complex neutrosophic concepts. To illustrate the proposed method one of the real–life examples for measuring the changes in AQI and its pattern is illustrated below:

Example 5: Let us suppose, an expert wants to analyze the Air Quality Index (AQI) of four geographical regions (x_1, x_2, x_3, x_4) based on periodic changes in several parameters like PM_{10}, $\mathrm{PM}_{2.5}$, NO_2, Carbon monoxide (Co), Lead (Pb), Ozone (O_3), Sulpher dioxide(So_2), Ammonia (NH_3) etc [4]. To illustrate the proposed method first three parameters PM_{10} (y_1), $\mathrm{PM}_{2.5}$ (y_2), NO_2 (y_3) is considered in this paper. The expert can write the changes in the level of these parameters at the given year based on acceptation, rejection and indeterminacy regions as shown in Table 5, 6 and 7, respectively. Table 8 represents the compact form of these contexts using the properties of complex neutrosophic sets. It can be called as three–way complex fuzzy context which is central notion of this paper. To understand the entries in Table 8 let us suppose : $\tilde{R}_{(x_1,y_1)}$ =$(0.5e^{i0.7\pi}, 0.3e^{i1.6\pi}, 0.3e^{i1.4\pi})$. This entry shows that the saturation values of PM_{10} is 50 percent acceptable in third to fourth months, 30 percent unacceptable in ninth to tenth months whereas it is 30 percent unpredictable in sixth to the seventh month of the given year. Similarly, other entries of three–way complex fuzzy matrix can be interpreted.

Step 1. The proposed algorithm shown in Section 3.1 starts the investigation for three–way complex fuzzy concepts using those attributes which covers the objects set maximally. The attribute which cover objects set maximally i.e. $\{(1.0, 1.0)/x_1 + (1.0, 1.0)/x_2 + (1.0, 1.0)/x_3 + (1.0, 1.0)/x_4\}$ can be found using the operator $(\uparrow)$ as shown below:
$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}^{\uparrow}=$
$\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 +$

[4] https://www.dpcc.delhigovt.nic.in/indexdup.php

Table 5: A truth complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2

	y_1	y_2	y_3
x_1	$0.5e^{i0.7\pi}$	$0.8e^{i1.7\pi}$	$0.4e^{i0.4\pi}$
x_2	$0.3e^{i0.5\pi}$	$0.4e^{i0.3\pi}$	$0.5e^{i0.4\pi}$
x_3	$0.4e^{i1.5\pi}$	$0.6e^{i1.6\pi}$	$0.3e^{i0.5\pi}$
x_4	$0.4e^{i0.2\pi}$	$0.2e^{i0.9\pi}$	$0.7e^{i1.2\pi}$

Table 6: A indeterminacy complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2

	y_1	y_2	y_3
x_1	$0.3e^{i1.6\pi}$	$0.7e^{i1.1\pi}$	$0.5e^{i0.2\pi}$
x_2	$0.5e^{i1.3\pi}$	$0.1e^{i0.8\pi}$	$0.4e^{i1.4\pi}$
x_3	$0.6e^{i1.9\pi}$	$0.6e^{i1.2\pi}$	$0.3e^{i1.4\pi}$
x_4	$0.7e^{i0.2\pi}$	$0.1e^{i0.5\pi}$	$0.2e^{i0.7\pi}$

Table 7: A falsity complex membership value for the PM_{10}, $PM_{2.5}$ and NO_2

	y_1	y_2	y_3
x_1	$0.3e^{i1.4\pi}$	$0.2e^{i0.5\pi}$	$0.4e^{i0.7\pi}$
x_2	$0.4e^{i1.3\pi}$	$0.4e^{i1.7\pi}$	$0.3e^{i0.5\pi}$
x_3	$0.5e^{i0.2\pi}$	$0.8e^{i0.9\pi}$	$0.4e^{i1.5\pi}$
x_4	$0.2e^{i0.5\pi}$	$0.9e^{i1.9\pi}$	$0.4e^{i0.2\pi}$

Table 8: A three–way complex neutrosophic context representation of Table 5 to 7

	y_1	y_2	y_3
x_1	$(0.5e^{i0.7\pi}, 0.3e^{i1.6\pi}, 0.3e^{i1.4\pi})$	$(0.8e^{i1.7\pi}, 0.7e^{i1.1\pi}, 0.2e^{i0.5\pi})$	$(0.4e^{i0.4\pi}, 0.5^{i0.2\pi}, 0.4e^{i0.7\pi})$
x_2	$(0.3e^{i0.5\pi}, 0.5e^{i0.4\pi}, 0.4e^{i1.3\pi})$	$(0.4e^{i0.3\pi}, 0.1e^{i0.8\pi}, 0.4e^{i1.7\pi})$	$(0.5e^{i0.4\pi}, 0.4^{i1.4\pi}, 0.3e^{i0.5\pi})$
x_3	$(0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.5e^{i0.2\pi})$	$(0.6e^{i1.6\pi}, 0.6e^{i1.2\pi}, 0.8e^{i0.9\pi})$	$(0.3e^{i0.5\pi}, 0.3^{i1.4\pi}, 0.4e^{i1.5\pi})$
x_4	$(0.4e^{i0.2\pi}, 0.7e^{i0.2\pi}, 0.2e^{i0.5\pi})$	$(0.2e^{i0.9\pi}, 0.1e^{i0.5\pi}, 0.9e^{i1.9\pi})$	$(0.7e^{i1.2\pi}, 0.2^{i0.7\pi}, 0.4e^{i0.2\pi})$

$(0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$.

Now, apply the operator $\downarrow$ to find maximal vague set of objects while integrating the information from these constitutes attributes as given below:

$\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}^{\downarrow}=\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}$.

It provides following three–way complex neutrosophic concepts:
1. Extent :

$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_3 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/x_4\}$

Intent:

$\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

Step 2. The Lower Neighbors of concepts shown in Step 1 can be found as follows:

(i) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1\}$.

It provides $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1+(0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2+ (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$.

(ii) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2\}$.

It provides: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1+(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2+ (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$.

(iii) $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\} \cup \{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$.

It provides : $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

Now following Lower Neighbor can be generated from above obtained complex neutrosophic attribute using the Galois connection (as illustrated in Step 1):

2. Extent :

$\{(0.5e^{i0.7\pi}, 0.3e^{i1.6\pi}, 0.3e^{i1.4\pi})/x_1 + (0.3e^{i0.5\pi}, 0.5e^{i0.4\pi}, 0.4e^{i1.3\pi})/x_2 + (0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.5e^{i0.2\pi})/x_3 + (0.4e^{i0.2\pi}, 0.7e^{i0.2\pi}, 0.2e^{i0.5\pi})/x_4\}$

Intent:

$\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.9e^{i1.4\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

3. Extent : $\{(0.8e^{i1.7\pi}, 0.7e^{i1.1\pi}, 0.2e^{i0.5\pi})/x_1 + (0.4e^{i0.3\pi}, 0.1e^{i0.8\pi}, 0.4e^{i1.7\pi})/x_2 + (0.6e^{i1.6\pi}, 0.6e^{i1.2\pi}, 0.8e^{i0.9\pi})/x_3 + (0.2e^{i0.9\pi}, 0.1e^{i0.5\pi}, 0.9e^{i1.9\pi})/x_4\}$
Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i1.9\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

4. Extent : $\{(0.4e^{i0.4\pi}, 0.5e^{i0.2\pi}, 0.4e^{i0.7\pi})/x_1 + (0.5e^{i0.4\pi}, 0.4e^{i1.4\pi}, 0.3e^{i0.5\pi})/x_2 + (0.3e^{i0.5\pi}, 0.3e^{i1.4\pi}, 0.4e^{i1.5\pi})/x_3 + (0.7e^{i1.2\pi}, 0.2e^{i0.7\pi}, 0.4e^{i0.2\pi})/x_4\}$
Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

It can be observed that each of the obtained Lower Neighbors are distinct. In this case, each of them can be considered as Next Neighbor as shown in Figure 4.

Step 3 Similarly, following concepts can be generated using the Next Neighbor of concept generated at Step 2:

5. Extent : $\{(0.5e^{i0.7\pi}, 0.7e^{i1.6\pi}, 0.3e^{i1.4\pi})/x_1 + (0.3e^{i0.3\pi}, 0.5e^{i0.8\pi}, 0.4e^{i1.7\pi})/x_2 + (0.4e^{i1.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i0.9\pi})/x_3 + (0.2e^{i0.2\pi}, 0.7e^{i0.5\pi}, 0.9e^{i1.4\pi})/x_4\}$
Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.5\pi})/y_3\}$

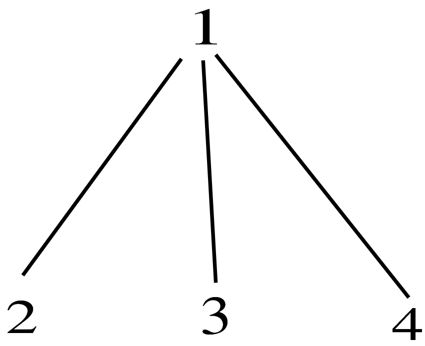

Figure 3: The three–way complex neutrosophic line diagram build at Step 2 using the proposed algorithm

6. Extent : $\{(0.4e^{i0.4\pi}, 0.5e^{i1.6\pi}, 0.4e^{i1.4\pi})/x_1+(0.3e^{i0.4\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.3\pi})/x_2+ (0.3e^{i0.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i1.5\pi})/x_3 + (0.4e^{i0.2\pi}, 0.7e^{i0.7\pi}, 0.9e^{i1.9\pi})/x_4\}$
Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (0.2e^{i0.3\pi}, 0.7e^{i1.2\pi}, 0.7e^{i1.9\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

7. Extent : $\{(0.3e^{i0.4\pi}, 0.7e^{i1.1\pi}, 0.5e^{i0.7\pi})/x_1+(0.4e^{i0.3\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.7\pi})/x_2+ (0.3e^{i0.5\pi}, 0.6e^{i1.2\pi}, 0.8e^{i1.5\pi})/x_3 + (0.2e^{i0.9\pi}, 0.2e^{i0.7\pi}, 0.9e^{i1.4\pi})/x_4\}$
Intent: $\{(0.3e^{i0.4\pi}, 0.7e^{i1.9\pi}, 0.5e^{i1.3\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

8. Extent : $\{(0.4e^{i0.4\pi}, 0.7e^{i1.6\pi}, 0.4e^{i1.4\pi})/x_1+(0.3e^{i0.3\pi}, 0.5e^{i1.4\pi}, 0.4e^{i1.7\pi})/x_2+ (0.3e^{i0.5\pi}, 0.6e^{i1.9\pi}, 0.8e^{i1.5\pi})/x_3 + (0.2e^{i0.2\pi}, 0.7e^{i0.7\pi}, 0.9e^{i1.9\pi})/x_4\}$
Intent: $\{(1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_1 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_2 + (1.0e^{i2\pi}, 0.0e^{i2\pi}, 0.0e^{i2\pi})/y_3\}$

It can be observed that the above generated three–way complex fuzzy concepts and their compact visualization is shown in Figure 4 using the properties of complex neutrosophic graph. This graph shows that concept 1 is most generalized concept whereas concept number 8 is most specialized concepts. The concept number 1 represents that each of the chosen regions has 20 to 30 percent acceptable saturation value for the PM_{10} in month of second to four months, 50 to 70 percent un–acceptable level in month of seventh to eighth whereas 40 to 70 percent uncertain from ninth to eleven months. In this case, the expert can refer to authorized government body

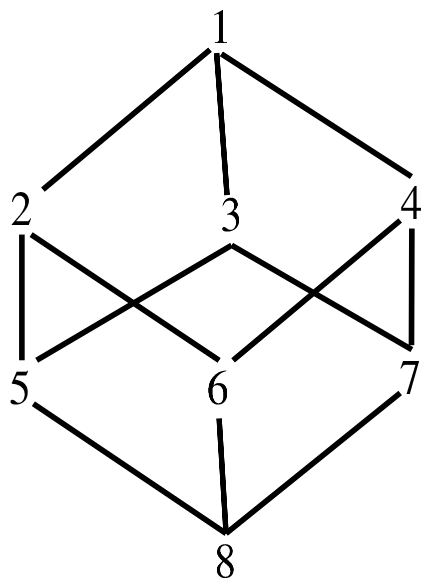

Figure 4: A three–way complex neutrosophic concept lattice generated from Table 8

for extra preparation in those months to reduce their health effects on the citizen. In a more precise way the expert may interpret the concept numbers 8. It represents that, the region x_1 has 40 percent acceptance level of each parameter in the month of second, 70 percent un–acceptable level in the month of ninth whereas 40 percent unpredictable in the month of seventh. The region x_2 has 30 percent acceptance level of each parameter in the month of first to second, 50 percent un–acceptable level in the month of eighth to ninth whereas 40 percent un–predictable in the month of ninth to tenth. The region x_3 has 30 percent acceptance level of each parameter in the month of second to third, 60 percent un–acceptable level in the month of tenth to eleven, whereas 80 percent un–predictable in the month of ninth to tenth. The region x_4 has 20 percent acceptance level of each parameter in the month of first, 70 percent un–acceptable level in the month of third to fourth whereas 90 percent un–predictable in the month of tenth to eleven. It can be observed that, these extracted patterns are more helpful in controlling or measuring the effect of AQI on the health of citizens in those areas. This will help in reducing the level of AQI and its fluctuation in those particular months to the certain levels using following methods:

1. Controlling emission from coal based power station,
2. Controlling hospitals waste,
3. Controlling diesel vehicles,
4. Controlling road or building construction dust,
5. Controlling the old and private vehicles etc.

It is one of the major and significant advantages of the proposed method towards measuring the pattern of AQI and its reduction which will help to the society.

Table 9 shows that, the proposed method has several advanatges to deal with complex neutrosophic context when compared to recently introduced methods. One of the most significant output of the proposed method provides a compressed line diagram and graphical analytics of the given complex neutrosophic context O($|C|.n^6.m^6$) time complexity. However, the proposed method unable to provide some of the complex neutrosophic concepts based on user required information granules. To overcome from this problem the author will focus on introducing connection of granular computing [41–42] to refine the neutrosophic [28] context for multi–polar attributes [23, 42].

5. Conclusions and Future research

This paper aimed at measuring changes in complex fuzzy attributes and its pattern based on truth, false and indeterminacy membership–values at given phase of time using the properties of complex neutrosophic concept lattice. To achieve this goal, a method is proposed in this paper for graphical analytics of complex neutrosophic data set using the calculus of Lower Neighbors algorithm which takes O($|C|.n^6.m^6$) time complexity. One of the suitable applications of the proposed method is also demonstrated for precise analysis of AQI and its interested pattern in the given yearm. However, the proposed method unable to provide an adequate analysis based on user required complex granules exists beyond the bipolar space. To deal with this problem author will focus on refined neutrosophic set [28] or complex multi–fuzzy sets [23, 42].

Table 9: Significant distinction of the proposed method when compared to other approaches

	Complex fuzzy set [24–25]	Complex vague set [20] [29–30]	Complex neutrosophic set [2, 6]	Proposed method
Domain	Universe of Discourse	Universe of Discourse	Universe of Discourse	Universe of Discourse
Co–domain	Three–polar Single–valued	Three–polar Interval–valued	Unit circle	Three–polar circle
True–region	[0, 1]	[0, 1]	[0, 1]	[0, 1]
False–region	[0, 1]	[0, 1]	[-1, 0)	[0, 1]
Uncertain regions	[0, 1]	[0, 1]	1-true -false	[0, 1]
Amplitude term	Yes	Yes	Yes	**Yes**
Phase term	No	No	Yes	**Yes**
Pattern	Yes	Yes	Yes	**Yes**
Graph	Yes	Yes	No	**Yes**
Lattice	Yes	Yes	No	**Yes**
Methodology	Subset	Subset	δ–equal	**Lower Neighbor**
Time complexity	O($2^m.n$)	O($2^m.n$)	O($2^m.n^2$)	O($n.m^3$)

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