An Application of Neutrosophic Bipolar Vague On Multi-Criteria Decision Making Problems

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Abstract- In this paper, we studied the concept of neutrosophic bipolar vague set and some of its operations. Also, we propose score, certainty and accuracy functions to compare the neutrosophic bipolar vague sets. Then, we develop the neutrosophic bipolar vague weighted average operator (A_ω) and neutrosophic bipolar vague weighted geometric operator (G_ω) to aggregate the neutrosophic bipolar vague information. Furthermore, based on the (A_ω) and (G_ω) operators and the score, certainty and accuracy functions, we develop a neutrosophic bipolar vague multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of neutrosophic bipolar vague numbers to select the most desirable one(s). Finally, numerical examples of the method were given to demonstrate the application and effectiveness of the developed method.

Keywords- Neutrosophic set, neutrosophic bipolar vague set, average operator, geometric operator, score, certainty and accuracy functions, multi-criteria decision making.

1. INTRODUCTION:

To handle with imprecision and uncertainty, concept of fuzzy sets and intuitionistic fuzzy sets originally introduced by Zadeh [30] and Atanassov [1], respectively. Then, Smarandache [21] proposed concept of neutrosophic set which is generalization of fuzzy set theory and intuitionistic fuzzy sets. These sets models have been studied by many authors; on application [4,6,8,12-14,18,19], theory [22-24,25-29,31,32], and so on. Bosc and Pivert [2] said that "Bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative affects. Positive information states what is possible, satisfactory, permitted, desired, or considered as being acceptable. On the other hand, negative statements express what is impossible, rejected, or forbidden. Negative preferences correspond to constraints, since they specify which values or objects have to be rejected (i.e., those that do not satisfy the constraints), while positive preferences correspond to wishes, as they specify which objects

are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes." Therefore, Lee [10,11] introduced the concept of bipolar fuzzy sets which is an generalization of the fuzzy sets. Neutrosophic vague set is a combination of neutrosophic set and vague set which was defined by Shawkat Alkhazaleh[20].

In this paper, we introduced the concept of neutrosophic bipolar vague sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets and vague sets. Also, we give some operations and operators on the neutrosophic bipolar vague. The operations and operators generalizes the operations and operators of fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets and vague sets which has been previously proposed. Therefore, in section 2, we studied the concept of neutrosophic bipolar vague set and its some of its operations. In section 3, we introduced the concept of score, certainity and accuracy funtions of the neutrosophic bipolar vague sets for comparision. In the same section, we also develop the neutrosophic bipolar vague weighted average operator (A_ω) and neutrosophic bipolar vague weighted geometric operator (G_ω) operator to aggregate the neutrosophic bipolar vague information. In section 4, based on the (A_ω) and (G_ω) operators and the score, certainty and accuracy functions, we develop a neutrosophic bipolar vague multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of neutrosophic bipolar vague numbers to select the most desirable one(s) and give numerical examples to demonstrate the application and effectiveness of the developed method.

2. PRELIMINARIES

Definition 2. 1[10,11]: Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by

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positive membership function μ_A^+: X→[0,1] and a negative membership function μ_A^-: X→[-1,0]. For sake of simplicity, we shall use the symbol A={< x, μ_A^+(x), μ_A^-(x)>: x ∈ X}.

Definition 2. 2[10,11]: Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i)$\mu_{A\cup B}^+(x) = \max\{ \mu_A^+(x), \mu_B^+(x)\}$.

(ii) $\mu_{A\cup B}^-(x) = \min\{ \mu_A^-(x), \mu_B^-(x)\}$.

(iii) $\mu_{A\cap B}^+(x) = \min\{ \mu_A^+(x), \mu_B^+(x)\}$.

(iv) $\mu_{A\cap B}^-(x) = \max\{ \mu_A^-(x), \mu_B^-(x)\}$.

(v) $\mu_{\bar{A}}^+(x)= 1-\mu_A^+(x)$ and $\mu_{\bar{A}}^-(x)= -1-\mu_A^+(x)$ for all x∈ X.

Definition 2.3(5): A vague set A in the universe of discourse U is a pair (t_A ,f_A) where t_A : U→[0,1], f_A : U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all u∈ U .The function t_A and f_A are called true membership function and false membership function respectively. The interval [t_A ,1-f_A] is called the vague value of u in A, and denoted by v_A(u), i.e v_A(u)=[t_A ,1-f_A].

Definition 2.4[5]: Let A be a non-empty set and the vague set A and B in the form A= {<x, t_A ,1-f_A >:x∈ X}, B={<x, t_B ,1-f_B >:x∈ X}.

Then,

(i)A⊆ B if and only if t_A(x)≤t_A(x) and 1-f_B(x) ≤ 1-f_B(x).

(ii)A∪ B={<max(t_A(x),t_B(x)), max(1-f_A(x),1-f_B(x))>/x∈X}.

(iii) A∩ B={<min(t_A(x),t_B(x)), min(1-f_A(x),1-f_B(x))>/x∈X}.

(iv)$\bar{A}$={<x, f_A(x) ,1-t_A(x)>:x∈X}.

Definition 2.5[21]: Let X be a universe of discourse. Then a neutrosophic set is defined as:

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, which is characterized by a truth-membership function T_A:X →]0−,1+[, an indeterminacy membership function I_A:X →]0−,1+[and a falsity-membership function F_A:X →]0−,1+[. There is not restriction on the sum of T_A(x), I_A(x) and F_A(x), so $0^-\leq$ supT_A(x) $\leq$ supI_A(x) $\leq$ supF_A(x) $\leq 3+$.

Definition 2.6[17]: A neutrosophic bipolar vague set A in X is defined as an object of the form,

A={<
x,
$[T^-(x),T^+(x)]^+, [I^-(x),I^+(x)]^+, [F^-(x),F^+(x)]^+,$

$[T^-(x),T^+(x)]^-, [I^-(x),I^+(x)]^-, [F^-(x),F^+(x)]^-$
>: x∈X}, where

$T^+ = 1 - F^-, F^+ = 1 - T^-$, and

T^+, I^+, F^+: X→[2,0] and T^-, I^-, F^-:X→[-2,0].

The positive membership degree $[T^-(x),T^+(x)]^+, [I^-(x),I^+(x)]^+, [F^-(x),F^+(x)]^+$ denotes the truth membership, indeterminate membership and false membership of an element x∈X corresponding to a neutrosophic bipolar vague set A and the negative membership degree $[T^-(x),T^+(x)]^-, [I^-(x),I^+(x)]^-, [F^-(x),F^+(x)]^-$ denotes the truth membership, indeterminate membership and false membership of an element x∈X to some implicit counter-property corresponding to a neutrosophic bipolar vague set A.

Definition 2.7[17]: Let A={< x, $[T_A^-,T_A^+]^+, [I_A^-,I_A^+]^+, [F_A^-,F_A^+]^+, [T_A^-,T_A^+]^-, [I_A^-,I_A^+]^-, [F_A^-,F_A^+]^->$} and

B={< x, $[T_B^-,T_B^+]^+, [I_B^-,I_B^+]^+, [F_B^-,F_B^+]^+, [T_B^-,T_B^+]^-, [I_B^-,I_B^+]^-, [F_B^-,F_B^+]^->$} be two neutrosophic bipolar vague sets. Then A⊆B if and only if

$[T_A^- \leq T_B^-]^+, [T_A^+ \leq T_B^+]^+, [I_A^- \geq I_B^-]^+, [I_A^+ \geq I_B^+]^+, [F_A^- \geq F_B^-]^+, [F_A^+ \geq F_B^+]^+$, and
$[T_A^- \geq T_B^-]^-, [T_A^+ \geq T_B^+]^-, [I_A^- \leq I_B^-]^-, [I_A^+ \leq I_B^+]^-, [F_A^- \leq F_B^-]^-, [F_A^+ \leq F_B^+]^-$ for all x∈X.

Definition 2.8[17]: Let A={< x, $[T_A^-,T_A^+]^+, [I_A^-,I_A^+]^+, [F_A^-,F_A^+]^+, [T_A^-,T_A^+]^-, [I_A^-,I_A^+]^-, [F_A^-,F_A^+]^->$} and B={< x, $[T_B^-,T_B^+]^+, [I_B^-,I_B^+]^+, [F_B^-,F_B^+]^+, [T_B^-,T_B^+]^-,$

$[I_B^-,I_B^+]^-, [F_B^-,F_B^+]^->$} be two neutrosophic bipolar vague sets. Then A=B if and only if $[T_A^- = T_B^-]^+, [T_A^+ = T_B^+]^+, [I_A^- = I_B^-]^+, [I_A^+ = I_B^+]^+, [F_A^- = F_B^-]^+, [F_A^+ = F_B^+]^+$, and $[T_A^- = T_B^-]^-, [T_A^+ = T_B^+]^-, [I_A^- = I_B^-]^-, [I_A^+ = I_B^+]^-, [F_A^- = F_B^-]^-, [F_A^+ = F_B^+]^-$ for all x∈X.

Definition 2.9[17]: Let A={< x, $[T_A^-,T_A^+]^+, [I_A^-,I_A^+]^+, [F_A^-,F_A^+]^+, [T_A^-,T_A^+]^-,$

$[I_A^-,I_A^+]^-, [F_A^-,F_A^+]^->$} and B={< x, $[T_B^-,T_B^+]^+, [I_B^-,I_B^+]^+, [F_B^-,F_B^+]^+, [T_B^-,T_B^+]^-,$

$[I_B^-,I_B^+]^-, [F_B^-,F_B^+]^->$} be two neutrosophic bipolar vague sets. Then their union is defined as:

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A∪B= {$\max[T_A^-, T_B^-]^+, \max[T_A^+, T_B^+]^+,$

$\min[I_A^-, I_B^-]^+, \min[I_A^+, I_B^+]^+,$

$\min[F_A^-, F_B^-]^+, \min[F_A^+, F_B^+]^+,$

$\min[T_A^-, T_B^-]^-, \min[T_A^+, T_B^+]^-,$

$\max[I_A^-, I_B^-]^-, \max[I_A^+, I_B^+]^-,$

$\max[F_A^-, F_B^-]^-, \max[F_A^+, F_B^+]^-$} for all x∈X.

Definition 2.10[17]**:** Let A={< x, $[T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^-$>} and

B={<x, $[T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^-$>} be two neutrosophic bipolar vague sets. Then their intersection is defined as:

A∩B={$\min[T_A^-, T_B^-]^+, \min[T_A^+, T_B^+]^+,$

$\max[I_A^-, I_B^-]^+, \max[I_A^+, I_B^+]^+,$

$\max[F_A^-, F_B^-]^+, \max[F_A^+, F_B^+]^+,$

$\max[T_A^-, T_B^-]^-, \max[T_A^+, T_B^+]^-,$

$\min[I_A^-, I_B^-]^-, \min[I_A^+, I_B^+]^-,$

$\min[F_A^-, F_B^-]^-, \min[F_A^+, F_B^+]^-$} for all x∈X.

Definition 2.11[17]**:** Let A={< x, $[T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^-$>:$x \in X$} be a neutrosophic bipolar vague set in X. Then the complement of A is denoted by $\bar{A}$ and is defined by, $\bar{A}$={<$[F_A^-, F_A^+]^+, [1 - I_A^-, 1 - I_A^+]^+, [T_A^-, T_A^+]^+, [F_A^-, F_A^+]^-, [1 - I_A^-, 1 - I_A^+]^-, [T_A^-, T_A^+]^-$>} for all x∈X.

3. NEUTROSOPHIC BIPOLAR VAGUE NUMBER

Definition 3.1: Let $\widetilde{a_1} = <[T_1^-, T_1^+]^+, [I_1^-, I_1^+]^+, [F_1^-, F_1^+]^+,$

$[T_1^-, T_1^+]^-, [I_1^-, I_1^+]^-, [F_1^-, F_1^+]^->$ and $\widetilde{a_2} = <[T_2^-, T_2^+]^+, [I_2^-, I_2^+]^+, [F_2^-, F_2^+]^+,$

$[T_2^-, T_2^+]^-, [I_2^-, I_2^+]^-, [F_2^-, F_2^+]^->$ be two neutrosophic bipolar vague numbers. Then the operations for NBVNs are defined as below:

i. $\lambda\widetilde{a_1} = <$
$1 - (1 - [T_1^-, T_1^+]^+)^\lambda, ([I_1^-, I_1^+]^+)^\lambda,$

$([F_1^-, F_1^+]^+)^\lambda, -(-[T_1^-, T_1^+]^-)^\lambda,$

$-(-[I_1^-, I_1^+]^-)^\lambda,$

$-\left(1 - \left(1 - (-[F_1^-, F_1^+]^-)\right)^\lambda\right) >$

ii. $\widetilde{a_1}^\lambda = <([T_1^-, T_1^+]^+)^\lambda, 1 - (1 - [I_1^-, I_1^+]^+)^\lambda, 1 - (1 - [F_1^-, F_1^+]^+)^\lambda, -\left(1 - \left(1 - (-[T_1^-, T_1^+]^-)\right)^\lambda\right), -(-[I_1^-, I_1^+]^-)^\lambda, -(-F^-, F^+]^-)^\lambda >$

iii. $\widetilde{a_1} + \widetilde{a_2} = <[T_1^-, T_1^+]^+ + [T_2^-, T_2^+]^+ - [T_1^-, T_1^+]^+.[T_2^-, T_2^+]^+,$

$[I_1^-, I_1^+]^+.[I_2^-, I_2^+]^+,$

$[F_1^-, F_1^+]^+.[F_2^-, F_2^+]^+,$

$-([T_1^-, T_1^+]^-.[T_2^-, T_2^+]^-),$

$-(-[I_1^-, I_1^+]^- - [I_2^-, I_2^+]^- - [I_1^-, I_1^+]^-.[I_2^-, I_2^+]^-), -(-[F_1^-, F_1^+]^- - [F_2^-, F_2^+]^- - [F_1^-, F_1^+]^-.[F_2^-, F_2^+]^-) >$

iv. $\widetilde{a_1}.\widetilde{a_2} = <[T_1^-, T_1^+]^+.[T_2^-, T_2^+]^+, [I_1^-, I_1^+]^+ + [I_2^-, I_2^+]^+ - [I_1^-, I_1^+]^+.[I_2^-, I_2^+]^+, [F_1^-, F_1^+]^+ + [F_2^-, F_2^+]^+ - [F_1^-, F_1^+]^+.[F_2^-, F_2^+]^+, -(-[T_1^-, T_1^+]^- - [T_2^-, T_2^+]^- - [T_1^-, T_1^+]^-.[T_2^-, T_2^+]^-),$

$-([I_1^-, I_1^+]^-.[I_2^-, I_2^+]^-), -([F_1^-, F_1^+]^-.[F_2^-, F_2^+]^-) >$

Where $\lambda > 0$.

Definition 3.2: Let $\widetilde{a_1} = <[T_1^-, T_1^+]^+, [I_1^-, I_1^+]^+, [F_1^-, F_1^+]^+,$

$[T_1^-, T_1^+]^-, [I_1^-, I_1^+]^-, [F_1^-, F_1^+]^- >$ be a neutrosophic bipolar vague number. Then, the score function $s(\widetilde{a_1})$, accuracy function $a(\widetilde{a_1})$, and certainty function $c(\widetilde{a_1})$ of an NBVN are defined as follows:

i. $\tilde{s}(\widetilde{a_1}) = \left\{ \begin{array}{c} [T_1^-, T_1^+]^+ + (1 - [I_1^-, I_1^+]^+) + \\ (1 - [F_1^-, F_1^+]^+) + (1 + [T_1^-, T_1^+]^-) - \\ [I_1^-, I_1^+]^- - [F_1^-, F_1^+]^- \end{array} \right\} / 6$

ii. $\tilde{a}(\widetilde{a_1}) = [T_1^-, T_1^+]^+ - [F_1^-, F_1^+]^+ + [T_1^-, T_1^+]^- - [F_1^-, F_1^+]^-$

iii. $\tilde{c}(\widetilde{a_1}) = [T_1^-, T_1^+]^+ - [F_1^-, F_1^+]^-$

Definition 3.3: Let $\widetilde{a_1} = <[T_1^-, T_1^+]^+, [I_1^-, I_1^+]^+, [F_1^-, F_1^+]^+,$

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$[T_1{}^-, T_1{}^+]^-, [I_1{}^-, I_1{}^+]^-, [F_1{}^-, F_1{}^+]^- >$ and

$\widetilde{a_2} = < [T_2{}^-, T_2{}^+]^+, [I_2{}^-, I_2{}^+]^+, [F_2{}^-, F_2{}^+]^+,$

$[T_2{}^-, T_2{}^+]^-, [I_2{}^-, I_2{}^+]^-, [F_2{}^-, F_2{}^+]^- >$ be two neutrosophic bipolar vague number. The comparison method can be defined as follows:

i. If $\tilde{s}(\widetilde{a_1}) > \tilde{s}(\widetilde{a_2})$, then $\widetilde{a_1}$ is greater than $\widetilde{a_2}$, that is, $\widetilde{a_1}$ is superior to $\widetilde{a_2}$, denoted by $\widetilde{a_1} > \widetilde{a_2}$.

ii. If $\tilde{s}(\widetilde{a_1}) = \tilde{s}(\widetilde{a_2})$ and $\tilde{a}(\widetilde{a_1}) > \tilde{a}(\widetilde{a_2})$ then $\widetilde{a_1}$ is greater than $\widetilde{a_2}$, that is, $\widetilde{a_1}$ is superior to $\widetilde{a_2}$, denoted by $\widetilde{a_1} > \widetilde{a_2}$.

iii. If $\tilde{s}(\widetilde{a_1}) = \tilde{s}(\widetilde{a_2})$, $\tilde{a}(\widetilde{a_1}) = \tilde{a}(\widetilde{a_2})$ and $\tilde{c}(\widetilde{a_1}) > \tilde{c}(\widetilde{a_2})$, then $\widetilde{a_1}$ is greater than $\widetilde{a_2}$, that is, $\widetilde{a_1}$ is superior to $\widetilde{a_2}$, denoted by $\widetilde{a_1} > \widetilde{a_2}$.

iv. If $\tilde{s}(\widetilde{a_1}) = \tilde{s}(\widetilde{a_2})$, $\tilde{a}(\widetilde{a_1}) = \tilde{a}(\widetilde{a_2})$ and $\tilde{c}(\widetilde{a_1}) = \tilde{c}(\widetilde{a_2})$, then $\widetilde{a_1}$ is equal to $\widetilde{a_2}$, that is, $\widetilde{a_1}$ is indifferent to $\widetilde{a_2}$, denoted by $\widetilde{a_1} = \widetilde{a_2}$.

Based on the study given in [15,21] we define some weighted aggregation operators related to neutrosophic bipolar vague sets as follows;

Definition 3.4: Let $\tilde{a}_j = < [T_j{}^-, T_j{}^+]^+, [I_j{}^-, I_j{}^+]^+, [F_j{}^-, F_j{}^+]^+,$

$[T_j{}^-, T_j{}^+]^-, [I_j{}^-, I_j{}^+]^-, [F_j{}^-, F_j{}^+]^- >$
(j=1,2,…,n) be a family of neutrosophic bipolar vague numbers. A mapping $A_\omega: Q_n \to Q$ is called neutrosophic bipolar vague weighted average operator if it satisfies,

$A_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) = \sum_{j=1}^{n} \omega_j \tilde{a}_j$

$$= < 1 - \prod_{j=1}^{n}\left(1 - [T_j{}^-, T_j{}^+]^+\right)^{\omega_j}, \prod_{j=1}^{n}\left([I_j{}^-, I_j{}^+]^+\right)^{\omega_j},$$

$$\prod_{j=1}^{n}\left([F_j{}^-, F_j{}^+]^+\right)^{\omega_j},$$
$$- \prod_{j=1}^{n}\left(-[T_j{}^-, T_j{}^+]^-\right)^{\omega_j},$$

$$-(1 - \prod_{j=1}^{n}(1 - (-[I_j{}^-, I_j{}^+]^-))^{\omega_j},$$

$$-(1 - \prod_{j=1}^{n}(1 - (-[F_j{}^-, F_j{}^+]^-))^{\omega_j} >$$

where ω_j is the weight of $\tilde{a}_j$(j=1,2…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Based on the study given in [ ] we give the theorem related to neutrosophic bipolar vague sets as follows;

Theorem 3.5: Let $\tilde{a}_j = < [T_j{}^-, T_j{}^+]^+, [I_j{}^-, I_j{}^+]^+, [F_j{}^-, F_j{}^+]^+,$

$[T_j{}^-, T_j{}^+]^-, [I_j{}^-, I_j{}^+]^-, [F_j{}^-, F_j{}^+]^- >$
(j=1,2…,n) be a family of neutrosophic bipolar vague numbers. Then,

i. If $\tilde{a}_j = \tilde{a}$ for all j=1,2,…,n then, $A_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) = \tilde{a}$.

ii. $min_{j=1,2,\dots,n}\{\tilde{a}_j\} \leq A_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) \leq max_{j=1,2,\dots,n}\{\tilde{a}_j\}$

iii. If $\tilde{a}_j \leq \tilde{a}_j{}^*$ for all j=1,2,…,n then, $A_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) \leq A_\omega(\tilde{a}_1{}^*, \tilde{a}_2{}^*, \tilde{a}_3{}^*, \dots, \tilde{a}_n{}^*)$

Based on the study given in [15,21] we define some weighted aggregation operators related to neutrosophic bipolar vague sets as follows;

Definition 3.6: Let $\tilde{a}_j = < [T_j{}^-, T_j{}^+]^+, [I_j{}^-, I_j{}^+]^+, [F_j{}^-, F_j{}^+]^+,$

$[T_j{}^-, T_j{}^+]^-, [I_j{}^-, I_j{}^+]^-, [F_j{}^-, F_j{}^+]^- >$
(j=1,2,…,n) be a family of neutrosophic bipolar vague numbers. A mapping $G_\omega: Q_n \to Q$ is called neutrosophic bipolar vague weighted geometric operator if it satisfies,

$G_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) = \sum_{j=1}^{n} \omega_j \tilde{a}_j$

$$= < \prod_{j=1}^{n}\left([T_j{}^-, T_j{}^+]^+\right)^{\omega_j},$$
$$1 - \prod_{j=1}^{n}\left(1 - [I_j{}^-, I_j{}^+]^+\right)^{\omega_j}, \ 1 - \prod_{j=1}^{n}\left(1 - [F_j{}^-, F_j{}^+]^+\right)^{\omega_j}, -(1 - \prod_{j=1}^{n}\left(1 - (-[T_j{}^-, T_j{}^+]^-)\right)^{\omega_j},$$
$$-\prod_{j=1}^{n}\left(-[I_j{}^-, I_j{}^+]^-\right)^{\omega_j}, -\prod_{j=1}^{n}\left(-[F_j{}^-, F_j{}^+]^-\right)^{\omega_j} >$$

where ω_j is the weight of $\tilde{a}_j$(j=1,2…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Based on the study given in [15,21] we give the theorem related to neutrosophic bipolar vague sets as follows;

Theorem 3.7: Let $\tilde{a}_j = < [T_j{}^-, T_j{}^+]^+, [I_j{}^-, I_j{}^+]^+, [F_j{}^-, F_j{}^+]^+,$

$[T_j{}^-, T_j{}^+]^-, [I_j{}^-, I_j{}^+]^-, [F_j{}^-, F_j{}^+]^- >$
(j=1,2…,n) be a family of neutrosophic bipolar vague numbers. Then,

i. If $\tilde{a}_j = \tilde{a}$ for all j=1,2,…,n then, $G_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) = \tilde{a}$.

ii. $min_{j=1,2,\dots,n}\{\tilde{a}_j\} \leq G_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_n) \leq max_{j=1,2,\dots,n}\{\tilde{a}_j\}$

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iii.   If $\tilde{a}_j \leq \tilde{a}_j^*$ for all j=1,2,…,n then, $G_\omega(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, …, \tilde{a}_n) \leq G_\omega(\tilde{a}_1^*, \tilde{a}_2^*, \tilde{a}_3^*, …, \tilde{a}_n^*)$.

Note that the aggregation results are still NBVNs.

4. NBVN- DECISION MAKING METHOD

In this section, we develop an approach based on the A_ω (or G_ω) operator and the above ranking method to deal with multiple criteria decision making problems with neutrosophic bipolar vague information.

Suppose that $A = \{A_1, A_2, …, A_m\}$ and $C = \{C_1, C_2, …, C_n\}$ is the set of alternatives and criterions or attributes, respectively. Let $\omega = (\omega_1, \omega_2, …, \omega_n)^T$ be the weight vector of attributes, such that $\sum_{j=1}^n \omega_j = 1$, $\omega_j \geq 0$ (j=1,2,…,n) and ω_j refers to the weight of attribute C_j. An alternative on criterions is evaluated by the decision maker, and the evaluation values are represented by the form of neutrosophic bipolar vague numbers. Assume that,

$(\widetilde{a_{ij}})_{m \times n} =$
$(< [T_{ij}^-, T_{ij}^+]^+, [I_{ij}^-, I_{ij}^+]^+, [F_{ij}^-, F_{ij}^+]^+,$
$[T_{ij}^-, T_{ij}^+]^-, [I_{ij}^-, I_{ij}^+]^-, [F_{ij}^-, F_{ij}^+]^- >)_{m \times n}$
is the decision matrix provided by the decision maker; $\widetilde{a_{ij}}$ is a neutrosophic bipolar vague number for alternative A_i associated with the criterions C_j. We have the conditions $[T_{ij}^-, T_{ij}^+]^+, [I_{ij}^-, I_{ij}^+]^+, [F_{ij}^-, F_{ij}^+]^+,$

$[T_{ij}^-, T_{ij}^+]^-, [I_{ij}^-, I_{ij}^+]^-, [F_{ij}^-, F_{ij}^+]^- \in [0,1]$
such that

$0 \leq [T_{ij}^-, T_{ij}^+]^+ + [I_{ij}^-, I_{ij}^+]^+ + [F_{ij}^-, F_{ij}^+]^+ - [T_{ij}^-, T_{ij}^+]^- - [I_{ij}^-, I_{ij}^+]^- - [F_{ij}^-, F_{ij}^+]^- \leq 6$ for i=1,2,…,m and j=1,2,…,n.

Now, we develop an algorithm as follows:

Algorithm

Step 1: Construct the decision matrix provided by the decision maker as,

$(\widetilde{a_{ij}})_{m \times n}$
$= (< [T_{ij}^-, T_{ij}^+]^+, [I_{ij}^-, I_{ij}^+]^+, [F_{ij}^-, F_{ij}^+]^+,$
$[T_{ij}^-, T_{ij}^+]^-, [I_{ij}^-, I_{ij}^+]^-, [F_{ij}^-, F_{ij}^+]^- >)_{m \times n}$

Step 2: Compute $\tilde{a}_i = A_\omega(\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, …, \tilde{a}_{in})$ (or $G_\omega(\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, …, \tilde{a}_{in})$) for each i=1,2…,m.

Step 3: Calculate the score values of $\tilde{s}(\widetilde{a_i})$ (i=1,2,…m) for the collective overall neutrosophic bipolar vague number of $\tilde{a}_i$(i=1,2,…,m)

Step 4: Rank all the software systems of $\tilde{a}_i$(i=1,2,…,m) according to the score values.

Now, we give a numerical example as follows:

Example 4.1: Let us consider the decision making problem adapted from Xu and Cia[21]. A customer who intends to buy a car. Four types of cars (alternatives) A_i(i=1,2,3,4) are available. The customer takes into ccount four attributes to evaluate the alternatives; C_1= Fuel Economy; C_2= Aerod; C_3= Comfort; C_4=Safety and the use of neutrosophic bipolar vague values to evaluate the four possible alternatives A_i(i=1,2,3,4) under the above four attributes. Also, the weight vector of the attributes C_j(j=1,2,3,4) is $\omega = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})^T$. Then,

Algorithm

Step 1: Construct the decision matrix provided by the customer as;

	C_1	C_2	C_3	C_4
A_1	[0.4,0.5][0.5,0.5] [0.5,0.6][-0.6,-0.4] [-0.3,-0.3][1.4,1.6]	[0.4,0.6][0.2,0.2] [0.4,0.6][-0.5,-0.3] [-0.6,-0.6][1.3,1.5]	[0.2,0.5][0.5,0.5] [0.5,0.8][-0.4,-0.1] [-0.5,-0.6][1.1,1.4]	[0.4,0.5][0.5,0.5] [0.5,0.6][-0.3,-0.2] [-0.2,-0.2][1.2,1.3]
A_2	[0.7,0.9][0.7,0.7] [0.1,0.3][-0.3,-0.2] [-0.7,-0.7][1.2,1.3]	[0.4,0.7][0.8,0.4] [0.3,0.6][-0.4,-0.3] [-0.4,-0.4][1.3,1.4]	[0.6,0.7][0.5,0.8][0.3,0.4][-0.1,-0.1] [-0.6,-0.6][1.1,1.1]	[0.2,0.5][0.2,0.2] [0.5,0.8][-0.5,-0.4] [-0.2,-0.1][1.4,1.5]
A_3	[0.5,0.7][0.5,0.5] [0.3,0.5][-0.4,-0.1] [-0.5,-0.6][1.1,1.4]	[0.3,0.6][0.4,0.4] [0.4,0.7][-0.2,-0.2] [-0.8,-0.6][1.2,1.2]	[0.7,0.8][0.6,0.3] [0.2,0.3][-0.5,-0.3] [-0.7,-0.7][1.3,1.5]	[0.6,0.7][0.5,0.5] [0.3,0.4][-0.1,-0.2] [-0.6,-0.6][1.2,1.1]
A_4	[0.5,0.9][0.3,0.3] [0.1,0.5][-0.4,-0.3] [-0.4,-0.4][1.3,1.4]	[0.4,0.6][0.2,0.2] [0.4,0.6][-0.5,-0.3] [-0.5,-0.5][1.3,1.5]	[0.3,0.5][0.7,0.8] [0.5,0.7][-0.5,-0.1] [-0.3,-0.6][1.1,1.5]	[0.4,0.7][0.6,0.8] [0.3,0.6][-0.4,-0.2] [-0.3,-0.5][1.2,1.4]

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Step 2: Compute $\tilde{a}_i = A_\omega(\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, \tilde{a}_{i4})$ for each i=1,2,3,4 as;

$\tilde{a}_1$=[0.3782,0.5271],[0.3866,0.3866],

[0.4728,0.6217], [-0.4748,-0.3018],

[-0.4067,-0.4231], [1.3223,1.4968]

$\tilde{a}_2$=[0.5820,0.8145],[0.6290,0.4988],

[0.1845,0.4179], [-0.2990,-0.2212],

[-0.5758,-0.5820], [1.2353,1.3218]

$\tilde{a}_3$=[0.5039,0.6936],[0.4826,0.4435],

[0.3063,0.4960], [-0.2907,-0.1487],

 [-0.5687,-0.6756], [1.1609,1.6024]

$\tilde{a}_4$=[0.4417,0.8016],[0.3285,0.3463],

[0.1983,0.5582], [-0.3260,-0.3314],

[-0.4044,-0.4675], [1.3359,1.3581]

Step 3: Calculate the score values $\tilde{s}(\widetilde{a_\iota})$ (i=1,2,3,4) for the collective overall neutrosophic bipolar vague number of $\widetilde{a_\iota}$ (i=1,2,3,4) as;

$\tilde{s}(\widetilde{a_1})$= 0.1895

$\tilde{s}(\widetilde{a_2})$= 0.3123

$\tilde{s}(\widetilde{a_3})$= 0.2925

$\tilde{s}(\widetilde{a_4})$= 0.2777

Step 4: Rank all the software systems of A_i (i=1,2,3,4) according to the score values as;

$A_2 > A_3 > A_4 > A_1$

And thus A_2 is the most desirable alternative.

Let us discuss another numerical example for easy understanding:

Example 4.2:

People who are affected by cancer, have a combination of treatments, such as surgery with chemotherapy and/or radiation therapy, hormone therapy, and immunotherapy. Our main objective is to find the best treatment from the above mentioned therapies. However, all the treatments can cause side effects. Our goal is to find the best treatment which cause the least side effects, extend the patient's life, cure the cancer and control its growth using an Neutrosophic Bipolar Vague set. Also the weight vector of the attributes C_j (j=1,2,3,4, is $\omega = (0.08, 0.62, 0.05, 0.25)^T$.

Algorithm

Step 1: Construct the decision matrix provided by the customer as;

	C₁	C₂	C₃	C₄
A₁	[0.2,0.8][0.1,0.1] [0.2,0.8][-0.6,-0.4] [-0.2,-0.2][1.4,1.6]	[0.4,0.6][0.3,0.3] [0.4,0.6][-0.3,-0.2] [-0.3,-0.3][1.2,1.3]	[0.4,0.4][0.1,0.2] [0.6,0.6][-0.7,-0.3] [-0.6,-0.2][1.3,1.7]	[0.4,0.5][0.3,0.7] [0.5,0.6][-0.7,-0.4] [-0.5,-0.1][1.4,1.7]
A₂	[0.1,0.2][0.4,0.4] [0.8,0.9][-0.4,-0.3] [-0.7,-0.7][1.3,1.4]	[0.3,0.4][0.6,0.6] [0.6,0.7][-0.5,-0.4] [-0.1,-0.1][1.4,1.5]	[0.5,0.7][0.2,0.3] [0.3,0.5][-0.5,-0.3] [-0.3,-0.2][1.3,1.5]	[0.3,0.6][0.4,0.5] [0.4,0.7][-0.4,-0.2] [-0.6,-0.3][1.2,1.4]
A₃	[0.4,0.5][0.5,0.5] [0.5,0.6][-0.4,-0.3] [-0.6,-0.6][1.3,1.4]	[0.5,0.6][0.7,0.7] [0.4,0.5][-0.6,-0.3] [-0.4,-0.3][1.3,1.6]	[0.2,0.7][0.1,0.4] [0.3,0.8][-0.6,-0.4] [-0.5,-0.2][1.4,1.6]	[0.3,0.7][0.2,0.6] [0.3,0.7][-0.5,-0.5] [-0.6,-0.4][1.5,1.5]
A₄	[0.2,0.3][0.2,0.2] [0.7,0.8][-0.3,-0.2] [-0.4,-0.4][1.2,1.3]	[0.2,0.4][0.3,0.4] [0.6,0.8][-0.5,-0.4] [-0.1,-0.2]][1.4,1.5]	[0.6,0.8][0.3,0.4] [0.2,0.4][-0.4,-0.1] [-0.5,-0.3][1.1,1.4]	[0.3,0.6][0.5,0.6] [0.4,0.7][-0.5,-0.4] [-0.3,-0.1][1.4,1.5]

Step 2: Compute $\tilde{a}_i = A_\omega(\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, \tilde{a}_{i4})$ for each i=1,2,3,4 as;

$\tilde{a}_1$=[0.3861,.05917][0.2599,0.3326]

[0.4082,0.6138][-0.4087,-0.2564]

[-0.3676,-0.2415][1.2688.1.4367]

$\tilde{a}_2$=[0.2978,0.4642][0.4966,0.5359]

[0.5357,0.7021][-0.4643,-0.3216]

[-0.3355,0.2306][1.3351,1.4661]

$\tilde{a}_3$=[0.4351,0.6265][0.4519,0.6374]

[0.3734.0.5648][-0.5548,-0.3456]

[-0.4800,-0.3516][1.3531,1.5578]

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$\tilde{a}_4$=[0.2527,0.4806][0.3298,0.4187]

[0.5193,0.7472][-0.4744,-0.3529]

[-0.2056,-0.2003][1.3671,1.4781]

Step 3: Calculate the score values $\tilde{s}(\widetilde{a_i})$ (i=1,2,3,4) for the collective overall neutrosophic bipolar vague number of $(\widetilde{a_i})$ (i=1,2,3,4) as;

$\tilde{s}(\widetilde{a_1})$= 0.2168

$\tilde{s}(\widetilde{a_2})$=0.1225

$\tilde{s}(\widetilde{a_3})$=0.1712

$\tilde{s}(\widetilde{a_4})$=0.1209

Step 4: Rank all the software systems of A_i (i=1,2,3,4) according to the score values as;

$A_1 > A_3 > A_2 > A_4$

Thus A_1 is the most desirable alternative. (i.e.,) the best treatment for cancer patients is chemotherapy or radiation therapy.

5. CONCLUSIONS

This paper presented a neutrosophic bipolar vague set and its score, certainty and accuracy functions. Then, the A_ω and G_ω operators were proposed to aggregate the neutrosophic bipolar vague information. Furthermore, based on the A_ω and G_ω operators and the score, certainty and accuracy functions, we have developed a neutrosophic bipolar vague multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of neutrosophic bipolar vague numbers. The A_ω and G_ω operators are utilized to aggregate the neutrosophic bipolar vague information corresponding to each alternative to obtain the collective overall values of the alternatives, and then the alternatives are ranked according to the values of the score, certainty and accuracy functions to select the most desirable one(s). Finally, a numerical example of the method was given to demonstrate the application and effectiveness of the developed method.

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