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An Approach to Neutrosophic Subrings

Vildan Çetkin*¹, Halis Aygün²

Abstract

In this article we aim to construct some algebra on single valued neutrosophic sets. For this reason, we propose a new notion which is called a neutrosophic subring by combining the ring structure and neutrosophic sets. Then we establish some fundamental characteristics of the presented notion.

Keywords: neutrosophic set, single valued neutrosophic set, classical ring, homomorphism of rings.

1. INTRODUCTION

In human life situations, different types of uncertainties are encountered. Since the classical set is invalid to handle the described uncertainties, Zadeh [16] first gave the definition of a fuzzy set. According to this definition, a fuzzy set is a function described by a membership value which has taken degrees in a unit interval. But, later it has been seen that this definition is inadequate by consideration not only the membership degree but also the non-membership degree. So, Atanassov [2] described a new theory named as intuitionistic fuzzy set theory to handle mentioned ambiguity. Since this set has some problems in applications, Smarandache [14] introduced neutrosophy to solve the problems that involve indeterminate and inconsistent information. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as the interactions with different ideational spectra"[14]. Neutrosophic set is a generalization both of the fuzzy set and intuitionistic fuzzy set, where all of

the membership functions are represented independently in a different way of intuitionistic fuzzy set. Wang et al. [15] specified the definition of a neutrosophic set, named as a single valued neutrosophic set, to make more applicable the theory to real life problems. According to this definition, The single valued neutrosophic set (SVNS) is an extension of a classical set, (intuitionistic) fuzzy set, vague set and etc. Vasantha Kandasamy and Florentin Smarandache [8] discussed some algebraic structures on neutrosophic sets.

So far, the theory of SVNS is applied the direction on algebra and topology by some authors (see [1, 3, 4, 10, 12, 13]). Liu [9] defined the concept of a fuzzy ring. Later, Martinez [11] and Dixit et al.[5] studied on fuzzy ring and obtain certain ring theoretical analogue. Hur et al.[6] proposed the notion of an intuitionistic fuzzy subring. Vasantha Kandasamy and Florentin Smarandache [7] studied the neutrosophic rings.

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In this work, in a different direction from [7], we give an approach to a single valued neutrosophic subring of a classical ring as a continuation study of neutrosophic algebraic structures discussed in [4]. We define neutrosophic subring and also present some properties of this structure. Moreover, we examine homomorphic image and preimage of a neutrosophic subring. By this way, we obtain the generalized form of the fuzzy subring and intuitionistic fuzzy subring of a classical ring.

2. PRELIMINARIES

Throughout this section, X denotes the universal set which is non-empty.

Definition 2.1 [14] A neutrosophic set N on X is defined by:

$$N = \{ \langle x, t_N(x), i_N(x), f_N(x) \rangle : x \in X \}$$

where $t_N, i_N, f_N : X \rightarrow]-0, 1^+[$ are functions satisfy the inequality $-0 \leq t_N(x) + i_N(x) + f_N(x) \leq 3^+$

According to the original definition, the neutrosophic set takes the value from real standard or non-standard subsets of $] -0, 1^+[$. Since it is not appropriate to consider the degree which belongs to a real standard or a non-standard subset of $] -0, 1^+[$, in real life applications, especially in medical, engineering and statistical problems etc. For this reason, we prefer to deal with the following revised definition instead of the original definition of Smarandache.

Definition 2.2 [15] A single valued neutrosophic set (SVNS) N on X is characterized by the truth-membership function t_N ; the indeterminacy-membership function i_N and the falsity-membership function f_N . For each point x in X ; the values $t_N(x), i_N(x), f_N(x)$ take place in the real unit interval $[0,1]$.

In other words, N may be shown as

$$N = \sum_{i=1}^n \langle t_N(x), i_N(x), f_N(x) \rangle / x_i, x_i \in X.$$

Since the membership functions t_N, i_N, f_N are defined from the universal set X into the unit interval $[0,1]$ as $t_N, i_N, f_N : X \rightarrow [0,1]$, a (single-valued) neutrosophic set N will be denoted by a mapping described by $N : X \rightarrow [0,1]^3$ and write

$N(x) = (t_N(x), i_N(x), f_N(x))$, for simplicity. The family of all single-valued neutrosophic sets on X is denoted by $SNS(X)$.

Definition 2.3 [12, 15] Let $N_1, N_2 \in SNS(X)$. Then

(1) N_1 is contained in N_2 ; denoted as $N_1 \subseteq N_2$, if and only if $N_1(x) \leq N_2(x)$. This means that $t_{N_1}(x) \leq t_{N_2}(x)$, $i_{N_1}(x) \leq i_{N_2}(x)$ and $f_{N_1}(x) \geq f_{N_2}(x)$. Two sets N_1, N_2 are called equal, i.e., $N_1 = N_2$ iff $N_1 \subseteq N_2$ and $N_2 \subseteq N_1$.

(2) the union of N_1 and N_2 is defined as $N(x) = N_1(x) \vee N_2(x)$, where $t_N(x) = t_{N_1}(x) \vee t_{N_2}(x)$, $i_N(x) = i_{N_1}(x) \vee i_{N_2}(x)$, $f_N(x) = f_{N_1}(x) \wedge f_{N_2}(x)$, for each $x \in X$.

(3) the intersection of N_1 and N_2 is defined as $N(x) = N_1(x) \wedge N_2(x)$, where $t_N(x) = t_{N_1}(x) \wedge t_{N_2}(x)$, $i_N(x) = i_{N_1}(x) \wedge i_{N_2}(x)$, $f_N(x) = f_{N_1}(x) \vee f_{N_2}(x)$, for each $x \in X$.

(4) N^c denotes the complement of the SVNS N and it is defined by $N^c(x) = (f_N(x), 1 - i_N(x), t_N(x))$, for each $x \in X$. Hence $(N^c)^c = N$.

The details of the set theoretical operations can be found in [12, 15].

Definition 2.4 [4] Let $g : X_1 \rightarrow X_2$ be a mapping between classical sets, $N_1 \in SNS(X_1)$ and $N_2 \in SNS(X_2)$. Then the image $g(N_1) \in SNS(X_2)$ and it is defined as follows.

$$\begin{aligned} g(N_1)(x_1) &= (t_{g(N_1)}(x_2), i_{g(N_1)}(x_2), f_{g(N_1)}(x_2)) \\ &= (g(t_{N_1})(x_2), g(i_{N_1})(x_2), g(f_{N_1})(x_2)), \end{aligned}$$

$\forall x_2 \in X_2$, where

$$g(t_{N_1})(x_2) = \begin{cases} \vee t_{N_1}(x_1), & \text{if } x_1 \in g^{-1}(x_2) \\ 0, & \text{otherwise} \end{cases}$$

$$g(i_{N_1})(x_2) = \begin{cases} \vee i_{N_1}(x_1), & \text{if } x_1 \in g^{-1}(x_2) \\ 0, & \text{otherwise} \end{cases}$$

$$g(f_{N_1})(x_2) = \begin{cases} \wedge f_{N_1}(x_1), & \text{if } x_1 \in g^{-1}(x_2) \\ 0, & \text{otherwise} \end{cases}$$

And the preimage $g^{-1}(N_2) \in SNS(X_1)$ and it is defined as:

$$\begin{aligned} &g^{-1}(N_2)(x_1) \\ &= (t_{g^{-1}(N_2)}(x_1), i_{g^{-1}(N_2)}(x_1), f_{g^{-1}(N_2)}(x_1)) \\ &= (t_{N_2}(g(x_1)), i_{N_2}(g(x_1)), f_{N_2}(g(x_1))) \\ &= N_2(g(x_1)), \forall x_1 \in X_1. \end{aligned}$$

3. NEUTROSOPHIC SUBRINGS

Now, we introduce the notion of neutrosophic subrings of a (classical) ring in a similar way of fuzzy case. We show that being a neutrosophic subring is preserved under a classical ring homomorphism. Also, we study some fundamental properties of a neutrosophic subring.

Definition 3.1 Let $H = (H, +, \cdot)$ be a classical ring and $N \in SNS(H)$. Then N is called a neutrosophic subring of H if the following properties are satisfied for each $x, h \in H$.

(H1) $N(x + h) \geq N(x) \wedge N(h)$.

(H2) $N(-h) \geq N(h)$.

(H3) $N(x \cdot h) \geq N(x) \wedge N(h)$.

$NSR(H)$ denotes the collection of all neutrosophic subrings of H .

Throughout this study, H denotes a classical ring, unless otherwise specified.

Example 3.2 Let us consider $H = Z_4 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$ as the classical ring with the operations \oplus and \odot defined by $\bar{r} \oplus \bar{s} = \overline{r + s}$ and $\bar{r} \odot \bar{s} = \overline{r \cdot s}$ for all $r, s \in Z_4$, respectively. Define the neutrosophic set N on H as follows.

$$N = \{ \langle 0.8, 0.4, 0.1 \rangle / \bar{0} + \langle 0.5, 0.3, 0.5 \rangle / \bar{1} + \langle 0.7, 0.4, 0.3 \rangle / \bar{2} + \langle 0.5, 0.3, 0.5 \rangle / \bar{3} \}$$

It is easy to verify that the neutrosophic set defined above is a neutrosophic subring of H .

Theorem 3.3. Let H be a fixed classical ring and $N \in SNS(H)$. Then $N \in NSR(H)$ iff the conditions given below hold.

(1) $N(h_1 - h_2) \geq N(h_1) \wedge N(h_2)$, for all $h_1, h_2 \in H$.

(2) $N(h_1 \cdot h_2) \geq N(h_1) \wedge N(h_2)$, for all $h_1, h_2 \in H$.

Proof. Let N be a neutrosophic subring of H . Then the following inequality is valid from the conditions (H1) and (H2) as follows:

$$\begin{aligned} N(h_1 - h_2) &= N(h_1 + (-h_2)) \geq N(h_1) \wedge \\ &N(-h_2) \geq N(h_1) \wedge N(h_2). \end{aligned}$$

Conversely, suppose that the conditions (1) and (2) are satisfied. Then the following is clearly obtained.

$$N(0) = N(h_1 - h_1) \geq N(h_1) \wedge N(h_1) = N(h_1),$$

for each $h_1 \in H$ (where 0 is the unit of the sum operation of H).

$$N(-h_1) = N(0 - h_1) \geq N(0) \wedge N(h_1) \geq N(h_1) \wedge N(h_1) = N(h_1),$$

for each $h_1 \in H$. By using these inequalities, we now obtain that

$$\begin{aligned} N(h_1 + h_2) &= N(h_1 - (-h_2)) \geq N(h_1) \wedge \\ &N(-h_2) \geq N(h_1) \wedge N(h_2). \end{aligned}$$

Theorem 3.4. If $N_1, N_2 \in SNS(H)$ are neutrosophic subrings of H , then so the intersection $N_1 \cap N_2$ is.

Proof. Let $h_1, h_2 \in H$ be arbitrary. By Theorem 3.3, we need to show that

$$(N_1 \cap N_2)(h_1 - h_2) \geq (N_1 \cap N_2)(h_1) \wedge (N_1 \cap N_2)(h_2)$$

And

$$(N_1 \cap N_2)(h_1 \cdot h_2) \geq (N_1 \cap N_2)(h_1) \wedge (N_1 \cap N_2)(h_2)$$

First consider the following

$$t_{N_1 \cap N_2}(h_1 - h_2) = t_{N_1}(h_1 - h_2) \wedge t_{N_2}(h_1 - h_2)$$

$$\begin{aligned} &\geq (t_{N_1}(h_1) \wedge t_{N_1}(h_2)) \wedge (t_{N_2}(h_1) \wedge t_{N_2}(h_2)) \\ &= (t_{N_1}(h_1) \wedge t_{N_2}(h_1)) \wedge (t_{N_1}(h_2) \wedge t_{N_2}(h_2)) \\ &= t_{N_1 \cap N_2}(h_1) \wedge t_{N_1 \cap N_2}(h_2). \end{aligned}$$

The other inequalities $i_{N_1 \cap N_2}(h_1 - h_2) \geq i_{N_1 \cap N_2}(h_1) \wedge i_{N_1 \cap N_2}(h_2)$ and $f_{N_1 \cap N_2}(h_1 - h_2) \leq f_{N_1 \cap N_2}(h_1) \vee f_{N_1 \cap N_2}(h_2)$ are similarly proved for each $h_1, h_2 \in H$. For the second condition, let us consider the falsity-degree of the intersection,

$$\begin{aligned} f_{N_1 \cap N_2}(h_1 \cdot h_2) &= f_{N_1}(h_1 \cdot h_2) \vee f_{N_2}(h_1 \cdot h_2) \\ &\leq (f_{N_1}(h_1) \vee f_{N_1}(h_2)) \vee (f_{N_2}(h_1) \vee f_{N_2}(h_2)) \\ &= (f_{N_1}(h_1) \vee f_{N_2}(h_1)) \vee (f_{N_1}(h_2) \vee f_{N_2}(h_2)) \\ &= f_{N_1 \cap N_2}(h_1) \vee f_{N_1 \cap N_2}(h_2). \end{aligned}$$

The other inequalities $t_{N_1 \cap N_2}(h_1 \cdot h_2) \geq t_{N_1 \cap N_2}(h_1) \wedge t_{N_1 \cap N_2}(h_2)$ and $i_{N_1 \cap N_2}(h_1 \cdot h_2) \geq i_{N_1 \cap N_2}(h_1) \wedge i_{N_1 \cap N_2}(h_2)$ are similarly proved for each $h_1, h_2 \in H$.

Consequently, $N_1 \cap N_2$ is a neutrosophic subring of H , as desired.

Definition 3.5. [4] Let $N \in SNS(X)$ and $\beta \in [0,1]$ be given. Then the level sets, which are classical sets on X , of N are defined in a following way.

$$(t_N)_\beta = \{x \in X \mid t_N(x) \geq \beta\},$$

$$(i_N)_\beta = \{x \in X \mid i_N(x) \geq \beta\} \text{ and}$$

$$(f_N)^\beta = \{x \in X \mid f_N(x) \leq \beta\}.$$

Following results are easily proved by using Definition 3.5.

(1) If $N \subseteq M$ and $\beta \in [0,1]$, then $(t_N)_\beta \subseteq (t_M)_\beta$, $(i_N)_\beta \subseteq (i_M)_\beta$ and $(f_N)^\beta \supseteq (f_M)^\beta$.

(2) $\beta \leq \gamma$ implies $(t_N)_\gamma \subseteq (t_N)_\beta$, $(i_N)_\gamma \subseteq (i_N)_\beta$ and $(f_N)^\beta \subseteq (f_N)^\gamma$.

Proposition 3.6. $N \in NSR(H)$ iff for each $\beta \in [0,1]$, β -level sets of N , $(t_N)_\beta$, $(i_N)_\beta$ and $(f_N)^\beta$ are classical subrings of H .

Proof. Let N be a neutrosophic subring of H , $\beta \in [0,1]$ and $h_1, h_2 \in (t_N)_\beta$ (similarly, $h_1, h_2 \in (i_N)_\beta, (f_N)^\beta$). By the assumption,

$t_N(h_1 - h_2) \geq t_N(h_1) \wedge t_N(h_2) \geq \beta \wedge \beta = \beta$ (and similarly, $i_N(h_1 - h_2) \geq \beta$, $f_N(h_1 - h_2) \leq \beta$). Hence $h_1 - h_2 \in (t_N)_\beta$ (and similarly, $h_1 - h_2 \in (i_N)_\beta, h_1 - h_2 \in (f_N)^\beta$), for each $\beta \in [0,1]$. In a similar way, we obtain $h_1 \cdot h_2 \in (t_N)_\beta$ (respectively, $h_1 \cdot h_2 \in (i_N)_\beta$ and $h_1 \cdot h_2 \in (f_N)^\beta$). This means that $(t_N)_\beta$ (and similarly, $(i_N)_\beta, (f_N)^\beta$) is a classical subring of H , for each $\beta \in [0,1]$.

Conversely, suppose that $(t_N)_\beta, (i_N)_\beta$ and $(f_N)^\beta$ are classical subrings of H . Let $h_1, h_2 \in H$ and $\beta = t_N(h_1) \wedge t_N(h_2)$, then $h_1, h_2 \in (t_N)_\beta$. Since $(t_N)_\beta$ is a subring of H , then $h_1 - h_2 \in (t_N)_\beta$ and $h_1 \cdot h_2 \in (t_N)_\beta$. This means that

$$t_N(h_1 - h_2) \geq \beta = t_N(h_1) \wedge t_N(h_2) \quad \text{and} \\ t_N(h_1 \cdot h_2) \geq \beta = t_N(h_1) \wedge t_N(h_2).$$

In similar computations, we obtain the desired inequalities as follows.

$$i_N(h_1 - h_2) \geq i_N(h_1) \wedge i_N(h_2), \quad i_N(h_1 \cdot h_2) \geq i_N(h_1) \wedge i_N(h_2), \\ f_N(h_1 - h_2) \leq f_N(h_1) \vee f_N(h_2) \quad \text{and}$$

$$f_N(h_1 \cdot h_2) \leq f_N(h_1) \vee f_N(h_2).$$

So this completes the proof.

Theorem 3.7. Let H_1 and H_2 be two classical rings and $g: H_1 \rightarrow H_2$ be a ring homomorphism. If N is a neutrosophic subring of H_1 , then $g(N)$, the image of N , is a neutrosophic subring of H_2 .

Proof. Suppose that N is a neutrosophic subring of H_1 and $k_1, k_2 \in H_2$. If $g^{-1}(k_1) = \emptyset$ or $g^{-1}(k_2) = \emptyset$, then it is obvious that $g(N)$ is a neutrosophic subring of H_2 . Assume that there exist $h_1, h_2 \in H_1$ such that $g(h_1) = k_1$ and $g(h_2) = k_2$. Since g is a homomorphism of rings,

$$g(h_1 - h_2) = g(h_1) - g(h_2) = k_1 - k_2 \quad \text{and} \\ g(h_1 \cdot h_2) = g(h_1) \cdot g(h_2) = k_1 \cdot k_2.$$

So, the followings become valid:

$$\begin{aligned} g(t_N)(k_1 - k_2) &= \bigvee_{k_1-k_2=g(h)} t_N(h) \\ &\geq t_N(h_1 - h_2). \\ g(i_N)(k_1 - k_2) &= \bigvee_{k_1-k_2=g(h)} i_N(h) \\ &\geq i_N(h_1 - h_2). \\ g(f_N)(k_1 - k_2) &= \bigwedge_{k_1-k_2=g(h)} f_N(h) \\ &\leq f_N(h_1 - h_2). \end{aligned}$$

By using the above inequalities, we now prove that $g(N)(k_1 - k_2) \geq g(N)(k_1) \wedge g(N)(k_2)$.

$$\begin{aligned} g(N)(k_1 - k_2) &= \\ &(g(t_N)(k_1 - k_2), g(i_N)(k_1 - k_2), g(f_N)(k_1 - k_2)) \\ &= \left(\bigvee_{k_1-k_2=g(h)} t_N(h), \bigvee_{k_1-k_2=g(h)} i_N(h), \bigwedge_{k_1-k_2=g(h)} f_N(h) \right) \\ &\geq (t_N(h_1 - h_2), i_N(h_1 - h_2), f_N(h_1 - h_2)) \\ &\geq (t_N(h_1) \wedge t_N(h_2), i_N(h_1) \wedge i_N(h_2), f_N(h_1) \vee f_N(h_2)) \\ &= (t_N(h_1), i_N(h_1), f_N(h_1)) \\ &\quad \wedge (t_N(h_2), i_N(h_2), f_N(h_2)) \end{aligned}$$

This is satisfied for each $h_1, h_2 \in H_1$ with $g(h_1) = k_1$ and $g(h_2) = k_2$. Then it is obvious that

$$\begin{aligned} g(N)(k_1 - k_2) &\geq \left(\bigvee_{k_1=g(h_1)} t_N(h_1), \bigvee_{k_1=g(h_1)} i_N(h_1), \bigwedge_{k_1=g(h_1)} f_N(h_1) \right) \\ &\wedge \left(\bigvee_{k_2=g(h_2)} t_N(h_2), \bigvee_{k_2=g(h_2)} i_N(h_2), \bigwedge_{k_2=g(h_2)} f_N(h_2) \right) \\ &= (g(t_N)(k_1), g(i_N)(k_1), g(f_N)(k_1)) \\ &\wedge (g(t_N)(k_2), g(i_N)(k_2), g(f_N)(k_2)) \\ &= g(N)(k_1) \wedge g(N)(k_2) \end{aligned}$$

In similar computations, it is seen that

$$g(N)(k_1 \cdot k_2) \geq g(N)(k_1) \wedge g(N)(k_2).$$

Hence being a neutrosophic subring is preserved under a homomorphism of rings.

Theorem 3.8. Let H_1 and H_2 be two classical rings and $g: H_1 \rightarrow H_2$ be a homomorphism of rings. If $M \in NSR(H_2)$, then the preimage $g^{-1}(M) \in NSR(H_1)$.

Proof. Suppose that M is a neutrosophic subring of H_2 and $h_1, h_2 \in H_1$. Since g is a homomorphism of rings, then the following inequality is obtained.

$$\begin{aligned} g^{-1}(M)(h_1 - h_2) &= (t_M(g(h_1 - h_2)), i_M(g(h_1 - h_2)), f_M(g(h_1 - h_2))) \\ &= (t_M(g(h_1) - g(h_2)), i_M(g(h_1) - g(h_2)), f_M(g(h_1) - g(h_2))) \\ &\geq (t_M(g(h_1)) \wedge t_M(g(h_2)), i_M(g(h_1)) \wedge i_M(g(h_2)), f_M(g(h_1)) \vee f_M(g(h_2))) \\ &= (t_M(g(h_1)), i_M(g(h_1)), f_M(g(h_1))) \\ &\quad \wedge (t_M(g(h_2)), i_M(g(h_2)), f_M(g(h_2))) \\ &= g^{-1}(M)(h_1) \wedge g^{-1}(M)(h_2) \end{aligned}$$

In similar computations, it is clear that $g^{-1}(M)(h_1 \cdot h_2) \geq g^{-1}(M)(h_1) \wedge g^{-1}(M)(h_2)$.

Therefore, $g^{-1}(M)$ is a neutrosophic subring of the classical ring H_1 .

4. CONCLUSION

The concept of a ring is one of the fundamental structures in algebra. The problem of classifying all rings (in a given class) up to homomorphism is far more complicated than the corresponding problem for groups. A single-valued neutrosophic set is a kind of neutrosophic set which is suitable to use in real world applications. So, we decided to combine these concepts and to propose the definition of a neutrosophic subring of a given crisp ring, in the direction of [4], and observe its fundamental properties. For further research, one can handle cyclic (respectively, symmetric, abelian) neutrosophic group structure.

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