AN EXTENSION OF THE TOPSIS FOR MULTI-ATTRIBUTE GROUP DECISION MAKING UNDER NEUTROSOPHIC ENVIRONMENT

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AN EXTENSION OF THE TOPSIS FOR MULTI-ATTRIBUTE GROUP DECISION MAKING UNDER NEUTROSOPHIC ENVIRONMENT

AMIRHOSSEIN NAFEI, YONGYI GU, AND WENJUN YUAN

Abstract. Neutrosophic set as a generalization of the classical fuzzy sets and intuitionistic fuzzy sets is an effective tool to deal with inconsistent, imprecise and vague information. TOPSIS is a multiple attribute method to identify solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from a nadir point. In this paper, we first develop a new Hamming distance between single-valued neutrosophic numbers and then present an extension of the TOPSIS method for multi-attribute group decision-making (MAGDM) based on single-valued neutrosophic sets, where the information about attribute values and attribute weights are expressed by decision-makers based on neutrosophic numbers.

1. SECTION HEAD

Multi-attribute decision making (MADM) as a component of decision science is a substantial and essential part of daily life which can be applied in various areas, such as society, economics, management, military, and engineering technology. In most cases, it is intricate for decision-makers to accurately reveal a preference when solving MADM problems with imprecise, vague or incomplete information. Under these conditions, in the past few decades, various types of sets, such as fuzzy sets [27], interval-valued fuzzy sets [28], intuitionistic fuzzy sets [1, 3], interval-valued intuitionistic fuzzy sets [2], type 2 fuzzy sets [8, 11], type *n*fuzzy sets [8], hesitant fuzzy sets [24] and neutrosophic set theory [21], have been introduced and widely used in the solution of significant decision-making problems. The neutrosophic set theory which is an extension of the intuitionistic fuzzy set provides a practical tool to deal with indeterminate and inconsistent information that exist commonly in the

real conditions. A given neutrosophic set such as N has three independent components, namely the truth- membership $T_N(x)$, the indeterminacy membership $I_N(x)$ and falsity-membership $F_N(x)$.

The technique for order performance by similarity to ideal solution (TOPSIS) was first developed by Hwang and Yoon [11] for solving a MADM problem. It bases upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). In the process of TOPSIS, the performance ratings and the weights of the criteria are given as crisp values. In recent years a lot of MADM methods [4, 12–14, 25, 26] and multi-attribute group decision making (MAGDM) methods [6, 15, 17] based on the extension of the TOPSIS method have been proposed.

In order to evaluate human resources, Jin et al. [12] introduced an extended TOPSIS method for MADM based on intuitionistic fuzzy sets where the attribute values given by decision-makers are the intuitionistic fuzzy numbers. Wei and Liu [26] presented an extended TOPSIS method based on uncertain linguistic variables to manage high technological risks. In order to resolve MADM problems, Liu in [14] presents an extension of the TOPSIS method where the weights and decision values of the alternatives are considered as interval vague values. Liu and Su [13] proposed an extended TOPSIS based on trapezoid fuzzy linguistic numbers and present a method for determining attribute weights. Rădulescu. C. and Rădulescu. I. [19] by modifying the variable ρ in the Minkowski distance measure proposed an extended TOPSIS method for ranking cloud service providers. Verma et al. [25] proposed an interval-valued intuitionistic fuzzy TOPSIS method for solving a facility location problem. Balin [4] proposed an extension of TOPSIS based on interval-valued spherical fuzzy sets to select the most effective stabilizing system for naval ships. In [6] Chen proposed a symmetric approach to extend the TOPSIS to the fuzzy environment for MAGDM problems in which the weights of various attributes and ratings of alternatives in regard to the different attributes indicated by linguistic variables. By defining a distance formula of generalized interval-valued fuzzy numbers in [15] Liu proposed an extended TOPSIS method for MAGDM problems where the attribute values and weights given by different decision-makers are all generalized interval-valued fuzzy numbers. In this respect, to choose adequate security mechanisms in e-business processes, Mohammadi et al. [17] proposed a fuzzy TOPSIS method based on group recommendation.

In this research, we first develop a distance measure to calculate the distance between single-valued neutrosophic numbers and then present an extended TOPSIS method for MAGDM under the neutrosophic environment where the attribute values and weights given by decision-makers (DMs) are represented by single-valued neutrosophic numbers (SVNNs). The key of our proposed method is that the different neutrosophic decision matrices presented by different decision-makers are converted into a single matrix and create an aggregated group decision matrix. The remaining

of this research is marshaled as follows: in the next section, we will briefly review the basic concepts of neutrosophic sets, the operation rules of single-valued neutrosophic sets, and the distance between them. Section 3 presents a distance measure to calculate the distance between SVNNs and describes the steps of the proposed method to rank the alternatives. Section 4 gives a numerical example to explain the validity of the proposed method. The study is concluded in Section 5.

2. Preliminaries

This section provides a brief review of particular preliminaries regarding neutrosophic sets, the distance between neutrosophic sets (NSs) and some other important concepts.

Definition 1. [22]: A neutrosophic set (NS) N in a domain X(finite universe of objectives) can be represented by $T_N : X \to]0^-, 1^+[$, $I_N : X \to]0^-, 1^+[$ and $F_N : X \to]0^-, 1^+[$ that satisfy the condition $0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+ \forall x \in X$. Where $T_N(x)$, $I_N(x)$ and $F_N(x)$ denote the truth, indeterminacy and falsity membership functions, respectively.

Definition 2. [18]: A neutrosophic set N is contained in another neutrosophic set M, if and only if :

$$\inf T_N(x) \leq \inf T_M(x),$$
$$Sup\, T_N(x) \leq Sup\, T_M(x),$$
$$\inf I_N(x) \geq \inf I_M(x),$$
$$Sup\, I_N(x) \geq Sup\, I_M(x),$$
$$\inf F_N(x) \geq \inf F_M(x),$$
$$Sup\, F_N(x) \geq Sup\, F_M(x),$$

for all $x \in X$.

Definition 3. [20]: The complement of a neutrosophic set N is denoted by N^c and can be defined as $T_N^c(x) = \{1^+\} \ominus T_N(x)$, $I_N^c(x) = \{1^+\} \ominus I_N(x)$ and $F_N^c(x) = \{1^+\} \ominus F_N(x)$ for all $x \in X$.

Definition 4. [16, 23]: Let X be a domain**.** A single-valued neutrosophic set (SVNS) N in the domain X can be denoted as $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$, where $T_N : X \to [0,1]$, $I_N : X \to [0,1]$ and $F_N : X \to [0,1]$ are three maps in X that satisfy the condition $0 \leq T_N(x) + F_N(x) + I_N(x) \leq 3 \,\forall x \in X$. The numbers $T_N(x)$, $F_N(x)$ and $I_N(x)$ are the degree of truth, falsity and indeterminacy membership of element x to N, respectively.

Remark 1. For a SVNS N, the trinary $(T_N(x), I_N(x), F_N(x))$ is called a single-valued neutrosophic number(SVNN). For convenience, the trinary $(T_N(x), I_N(x), F_N(x))$ is often denoted by (T, I, F).

Definition 5. [9, 16]: Let $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ be two SVNNs. The mathematical operations between x and y are defined as follows:

$$I. x \oplus y = (T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2), \tag{2.1}$$

$$II. x \otimes y = (T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2), \tag{2.2}$$

$$III. \lambda x = \left(1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda\right), \lambda > 0, \tag{2.3}$$

$$IV. x^\lambda = \left(T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda\right), \lambda > 0. \tag{2.4}$$

Definition 6. [10, 20]: The complement of a SVNS N is denoted by N^c and is defined as $T_N^c(x) = F_N(x), I_N^c(x) = 1 - I(x)$ and $F_N^c(x) = T_N(x)$ for all $x \in X$. Therefore we have :

$$N^c = \{x, F_N(x), 1 - I_N(x), T_N(x); x \in X\}.$$

Definition 7. [7]: Let $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$ and $M = \{x, T_M(x), I_M(x), F_M(x); x \in X\}$ be two single-valued neutrosophic sets, the Hamming distance between N and M is defined as follow:

$$d_H(x,y) = \frac{1}{6}\left(|T_N(x) - T_M(x)| + |I_N(x) - I_M(x)| + |F_N(x) - F_M(x)|\right), \tag{2.5}$$

also, the Euclidian distance between N and M is defined as follow:

$$d_E(N,M) = \sqrt{\frac{1}{6}\left((T_N(x) - T_M(x))^2 + (I_N(x) - I_M(x))^2 + (F_N(x) - F_M(x))^2\right)}. \tag{2.6}$$

Definition 8. [5] $d(N,M)$ is said a distance measure between neutrosophic sets if satisfies the following properties:
P1: $d(N,M) \geq 0$.
P2: $d(N,M) = 0$ if and only if $N = M$ for all $N, M \in$ NSs.
P3: $d(N,M) = d(M,N)$.
P4: If $N \subseteq M \subseteq O$ where $O \in$ NSs in X then: $d(N,O) \geq d(N,M)$ and $d(N,O) \geq d(M,O)$.

3. The proposed method

In this section, we first propose a new Hamming distance based on the Hausdorff metric between single-valued neutrosophic numbers. Then will use this distance to present a new multi-attribute group decision-making method (MAGDM) based on the combination of neutrosophic sets and extended TOPSIS method.

3.1. *Extended Hausdorff distance*

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universe of objectives. Consider two neutrosophic sets N and M in X where $N = \{x_i, T_N(x_i), I_N(x_i), F_N(x_i); x_i \in X\}$ and $M = \{x_i, T_M(x_i), I_M(x_i), F_M(x_i); x_i \in X\}$. Then denote

$$d(N,M) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{(|T_N(x_i) - T_M(x_i)| + |I_N(x_i) - I_M(x_i)| + |F_N(x_i) - F_M(x_i)|)}{6} \right.$$
$$\left. + \frac{\max(|T_N(x_i) - T_M(x_i)|, |I_N(x_i) - I_M(x_i)|, |F_N(x_i) - F_M(x_i)|)}{3} \right] \quad (3.1)$$

Theorem 1. $d(N,M)$ *is a distance between two neutrosophic sets N and M in X.*

Proof. It is obvious $d(N,M)$ satisfies $P1 - P3$ of Definition (8). Therefore we only need to prove $d(N,M)$ satisfies P4. To this aim let $O = \{x, T_O(x), I_O(x), F_O(x); x \in X\}$ be another neutrosophic set, if $N \subseteq M \subseteq O$ then we have:

$$d(N,M) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{(|T_N(x_i) - T_M(x_i)| + |I_N(x_i) - I_M(x_i)| + |F_N(x_i) - F_M(x_i)|)}{6} \right.$$
$$\left. + \frac{\max(|T_N(x_i) - T_M(x_i)|, |I_N(x_i) - I_M(x_i)|, |F_N(x_i) - F_M(x_i)|)}{3} \right]$$

and

$$d(N,O) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{(|T_N(x_i) - T_O(x_i)| + |I_N(x_i) - I_O(x_i)| + |F_N(x_i) - F_O(x_i)|)}{6} \right.$$
$$\left. + \frac{\max(|T_N(x_i) - T_O(x_i)|, |I_N(x_i) - I_O(x_i)|, |F_N(x_i) - F_O(x_i)|)}{3} \right]$$

It's easy to see

$$|T_N(x_i) - T_O(x_i)| \geq |T_N(x_i) - T_M(x_i)|,$$
$$|I_N(x_i) - I_O(x_i)| \geq |I_N(x_i) - I_M(x_i)|,$$
$$|F_N(x_i) - F_O(x_i)| \geq |F_N(x_i) - F_M(x_i)|,$$

so we have:

$$\frac{(|T_N(x_i) - T_O(x_i)| + |I_N(x_i) - I_O(x_i)| + |F_N(x_i) - F_O(x_i)|)}{6}$$
$$+ \frac{\max(|T_N(x_i) - T_O(x_i)|, |I_N(x_i) - I_O(x_i)|, |F_N(x_i) - F_O(x_i)|)}{3} \geq$$
$$\frac{(|T_N(x_i) - T_M(x_i)| + |I_N(x_i) - I_M(x_i)| + |F_N(x_i) - F_M(x_i)|)}{6}$$
$$+ \frac{\max(|T_N(x_i) - T_M(x_i)|, |I_N(x_i) - I_M(x_i)|, |F_N(x_i) - F_M(x_i)|)}{3}$$

Therefore we can get the inequality $d(N,O) \geq d(N,M)$. By the same reason we can get $d(N,O) \geq d(M,O)$. So $d(N,M)$ satisfies $P4$ of Definition 8. That is to say, $d(N,M)$ is a distance measure between neutrosophic sets N and M. $\qquad\square$

3.2. *The extended TOPSIS method for multi-attribute group decision-making based on NSs*

Suppose that $A = \{A_1, A_2, \ldots, A_n\}$ be a set of alternatives, $B = \{C_1, C_2, \ldots, C_m\}$ be a set of attributes and $D = \{D_1, D_2, \ldots D_k\}$ be a set of decision-makers (DMs). Let $\bar{w}_p = \left[\bar{w}_1^p, \bar{w}_2^p, \ldots, \bar{w}_m^p\right]$ be a vector of weights for attributes determined by DM D_p where $\bar{w}_j^p$ is a single-valued neutrosophic number denoting the weight of attribute C_j given by decision-maker D_p. $1 \leq j \leq m$ and $1 \leq p \leq k$.

Assume that W_p represents the weight of DM D_p. If a decision group has k members then $W_P = \frac{1}{k}$, where $W_p \in [0,1]$ and $\sum_{p=1}^{k} W_p = 1$.

Let $X_p = [x_{ij}]_{m \times n}$ be a decision matrix of the n alternatives in regard to the m attributes characterized by decision-maker D_p, shown as follows:

$$
X_p = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array}
\begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array}
\left[\begin{array}{cccc}
x_{11}^p & x_{12}^p & \ldots & x_{1m}^p \\
x_{21}^p & x_{22}^p & \ldots & x_{2m}^p \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1}^p & x_{n2}^p & \ldots & x_{nm}^p
\end{array} \right]. \qquad (3.2)
$$

Where $x_{ij} = (T_{ij}, I_{ij}, F_{ij})$ is a single value neutrosophic number for the alternative A_i in regard to the attribute C_j.

The procedure of our proposed method can be summarized as follows:

Step 1. According to the weighting vector $\bar{w}_p$, the decision matrix X_P and the multiplication operator of SVNSs presented in Eq. (2.2) calculate the weighted decision matrix (WDM) EV_P as follows:

$$
EV_p = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array}
\begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array}
\left[\begin{array}{cccc}
x_{11}^p \otimes \bar{w}_1^P & x_{12}^p \otimes \bar{w}_2^P & \ldots & x_{1m}^p \otimes \bar{w}_m^P \\
x_{21}^p \otimes \bar{w}_1^P & x_{22}^p \otimes \bar{w}_2^P & \ldots & x_{2m}^p \otimes \bar{w}_m^P \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1}^p \otimes \bar{w}_1^P & x_{n2}^p \otimes \bar{w}_2^P & \ldots & x_{nm}^p \otimes \bar{w}_m^P
\end{array} \right]
=
\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array}
\begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array}
\left[\begin{array}{cccc}
y_{11}^p & y_{12}^p & \ldots & y_{1m}^p \\
y_{21}^p & y_{22}^p & \ldots & y_{2m}^p \\
\vdots & \vdots & \ddots & \vdots \\
y_{n1}^p & y_{n2}^p & \ldots & y_{nm}^p
\end{array} \right].
$$
$$(3.3)$$

Step 2. Based on the obtained WDMs and the weight of decision-makers we can get the aggregated group decision matrix AG of all decision-makers $D_1, D_2, \ldots, D_k$ as follows:

$$AG = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{cccc} D_1 & D_2 & \cdots & D_k \\ \left[\begin{array}{cccc} G_{11} & G_{12} & \cdots & G_{1k} \\ G_{21} & G_{22} & \cdots & G_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nk} \end{array} \right] \end{array}. \tag{3.4}$$

Where G_{ip} is a neutrosophic value, representing the sum of alternatives in regard to DM D_p, can be calculated as follows:

$G_{ip} = W_P[y_{i1}^p \oplus y_{i2}^p \oplus \cdots \oplus y_{im}^p]$ where W_P is the weight of decision-maker D_P and $\oplus$ is the addition operator presented in Eq. (2.1).

Step 3. Based on the obtained aggregated group decision matrix we know that the elements G_{ip} are SVNNs. The absolute neutrosophic positive ideal solution (NPIS) P^+ and the neutrosophic negative ideal solution (NNIS) P^- can be defined as follows:

$$\begin{aligned} P^+ &= (G_1^+, G_2^+, ..., G_k^+), \\ P^- &= (G_1^-, G_2^-, ..., G_k^-). \end{aligned} \tag{3.5}$$

Where $G_j^+ = (1,0,0)$ and $G_j^- = (0,1,1)$, $j = 1,2,..,k$. Also we can select the virtual positive ideal solution and negative ideal solution by selecting the best values for each attribute from all alternatives as follows:

$$\begin{cases} G_j^+ = (\max_i T_{ij}, \min_i I_{ij}, \min_i F_{ij}) = (T_j^+, I_j^+, F_j^+), \\ G_j^- = (\min_i T_{ij}, \max_i I_{ij}, \max_i F_{ij}) = (T_j^-, I_j^-, F_j^-), \end{cases} \quad 1 \leq j \leq k. \tag{3.6}$$

Step 4. Based on the proposed distance measure in Eq. (3.1). calculate the distance between alternative A_i and the elements in the obtained positive ideal solution P^+ as follows:

$$d_i^+ = \sum_{j=1}^n G_{ij} - G_j^+ = \frac{1}{n} \sum_{j=1}^n \Big[\frac{\left(\left|T_{ij} - T_j^+\right| + \left|I_{ij} - I_j^+\right| + \left|F_{ij} - F_j^+\right| \right)}{6}$$
$$+ \frac{\max\left(\left|T_{ij} - T_j^+\right|, \left|I_{ij} - I_j^+\right|, \left|F_{ij} - F_j^+\right| \right)}{3} \Big] \tag{3.7}$$

also, the degree of distance between the alternative A_i and the elements in the obtained negative ideal solution P^- can be calculated as follows:

$$d_i^- = \sum_{j=1}^n G_{ij} - G_j^- = \frac{1}{n} \sum_{j=1}^n \Big[\frac{\left(\left|T_{ij} - T_j^-\right| + \left|I_{ij} - I_j^-\right| + \left|F_{ij} - F_j^-\right| \right)}{6}$$
$$+ \frac{\max\left(\left|T_{ij} - T_j^-\right|, \left|I_{ij} - I_j^-\right|, \left|F_{ij} - F_j^-\right| \right)}{3} \Big]. \tag{3.8}$$

Where $1 \leq i \leq n, 1 \leq j \leq k$.

Step 5. Compute the relative closeness coefficient to choose the most appropriate and efficient decision by ranking the alternatives as follows:

$$R_i^* = \frac{d_i^-}{d_i^+ + d_i^-}, 1, ..., n \tag{3.9}$$

Step 6. Utilize the relative closeness coefficients to sort the alternatives. The bigger R_i^* is, the better alternative A_i is.

4. Illustrative example

In this section, an example based on TOPSIS method for MAGDM under the neutrosophic environment is used as a demonstration of the applications and the effectiveness of the proposed decision-making method.

Suppose that there is a panel to compare four car companies A_1, A_2, A_3 and A_4 as the alternatives. Also assume that three attributes such as "Quality (C_1)", "Design (C_2)" and "Price (C_3)". A committee of three decision-makers D_1, D_2 and D_3 has been formed to rank the alternatives and choose the best company. Assume that the decision values of company alternatives A_1, A_2, A_3 and A_4 in regard to the attributes "Quality", "Design" and "Price" given by the decision-makers D_1, D_2 and D_3 are represented by single-valued neutrosophic numbers, as shown in Table (1), Table (2) and Table (3), respectively.

TABLE 1. The decision values given by D_1

	Quality	Design	Price
Company1	(0.1771, 0.5573, 0.5013)	(0.1079, 0.3390, 0.4857)	(0.1932, 0.6289, 0.9274)
Company2	(0.8296, 0.7725, 0.4317)	(0.1822, 0.2101, 0.8944)	(0.8959, 0.1015, 0.9175)
Company3	(0.7669, 0.3119, 0.9976)	(0.0991, 0.5102, 0.1375)	(0.0991, 0.3909, 0.7136)
Company4	(0. 9345, 0.1790, 0.8116)	(0.4898, 0.9064, 0.3900)	(0.0442, 0.0546 , 0.6183)

TABLE 2. The decision values given by D_2

	Quality	Design	Price
Company1	(0.3433, 0.5493, 0.9542)	(0.6465, 0.7565, 0.2815)	(0.8352, 0.9727, 0.5906)
Company2	(0.9360, 0.3304, 0. 0319)	(0.8332, 0.4139, 0.2304)	(0.3225, 0.3278, 0.6604)
Company3	(0.1248, 0.6195, 0.3369)	(0.3983, 0.4923, 0.7111)	(0.5523, 0.8378, 0.0476)
Company4	(0.7306, 0.3606, 0.6627)	(0.7498, 0.6947, 0.6246)	(0.9791, 0.7391, 0.3488)

Therefore the corresponding decision matrices X_1, X_2 and X_3 can be shown as follows, respectively:

TABLE 3. The decision values given by D_3

	Quality	Design	Price
Company1	(0.4513, 0.5038, 0.3610)	(0.2815, 0.4494, 0.0839)	(0.1386, 0.1892, 0.4035)
Company2	(0.2409, 0.4896, 0.6203)	(0.7311, 0.9635, 0.9748)	(0.5882, 0.6671, 0.1220)
Company3	(0.2409, 0.8770, 0.8112)	(0.1378, 0.0423, 0.6513)	(0.3662, 0.5864, 0.2684)
Company4	(0.8562, 0.3531, 0.0193)	(0.8367, 0.9730, 0.2312)	(0.8068, 0.6751, 0.2578)

$$
X_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
C_1 & C_2 & C_3 \\
(0.1771, 0.5573, 0.5013) & (0.1079, 0.3390, 0.4857) & (0.1932, 0.6289, 0.9274) \\
(0.8296, 0.7725, 0.4317) & (0.1822, 0.2101, 0.8944) & (0.8959, 0.1015, 0.9175) \\
(0.7669, 0.3119, 0.9976) & (0.0991, 0.5102, 0.1375) & (0.0991, 0.3909, 0.7136) \\
(0.9345, 0.1790, 0.8116) & (0.4898, 0.9064, 0.3900) & (0.0442, 0.0546, 0.6183)
\end{bmatrix},
$$

$$
X_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
C_1 & C_2 & C_3 \\
(0.3433, 0.5493, 0.9542) & (0.6465, 0.7565, 0.2815) & (0.8352, 0.9727, 0.5906) \\
(0.9360, 0.3304, 0.0319) & (0.8332, 0.4139, 0.2304) & (0.3225, 0.3278, 0.6604) \\
(0.1248, 0.6195, 0.3369) & (0.3983, 0.4923, 0.7111) & (0.5523, 0.8378, 0.0476) \\
(0.7306, 0.3606, 0.6627) & (0.7498, 0.6947, 0.6246) & (0.9791, 0.7391, 0.3488)
\end{bmatrix},
$$

$$
X_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{bmatrix}
C_1 & C_2 & C_3 \\
(0.4513, 0.5038, 0.3610) & (0.2815, 0.4494, 0.0839) & (0.1386, 0.1892, 0.4035) \\
(0.2409, 0.4896, 0.6203) & (0.7311, 0.9635, 0.9748) & (0.5882, 0.6671, 0.1220) \\
(0.2409, 0.8770, 0.8112) & (0.1378, 0.0423, 0.6513) & (0.3662, 0.5864, 0.2684) \\
(0.8562, 0.3531, 0.0193) & (0.8367, 0.9730, 0.2312) & (0.8068, 0.6751, 0.2578)
\end{bmatrix}.
$$

Suppose that the attribute weights given by three DMs D_1, D_2 and D_3 are shown as follows:

$$
\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}
$$
$$W_1 = [(0.2834, 0.3900, 0.8344), (0.8962, 0.4979, 0.6096), (0.8266, 0.6948, 0.5747)],$$

$$
\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}
$$
$$W_2 = [(0.3260, 0.8844, 0.6748), (0.4564, 0.7209, 0.4385), (0.7138, 0.0186, 0.4378)],$$

$$
\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}
$$
$$W_3 = [(0.1170, 0.2462, 0.5466), (0.8147, 0.3427, 0.5619), (0.3249, 0.3757, 0.3958)].$$

Furthermore, because the decision group in this example has three members we can consider $W_1 = W_2 = W_3 = \frac{1}{3}$.

The proposed method is currently applied to solve this problem and the computational procedure is summarized as follows:

Step 1. Construct the weighted decision matrices EV_1, EV_2 and EV_3 as follows:

$$EV_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \left[\begin{array}{ccc} (0.0502, 0.7300, 0.9174) & (0.0967, 0.6681, 0.7992) & (0.1597, 0.8867, 0.9691) \\ (0.2351, 0.8612, 0.9059) & (0.1633, 0.6034, 0.9588) & (0.7405, 0.7258, 0.9649) \\ (0.2173, 0.5803, 0.9996) & (0.0888, 0.7540, 0.6633) & (0.0819, 0.8141, 0.8782) \\ (0.2648, 0.4992, 0.9688) & (0.4389, 0.9530, 0.7619) & (0.0365, 0.7115, 0.8377) \end{array}\right] \end{array},$$

$$EV_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \left[\begin{array}{ccc} (0.1119, 0.9479, 0.9851) & (0.2951, 0.9320, 0.5966) & (0.5962, 0.9732, 0.7698) \\ (0.3052, 0.9226, 0.6852) & (0.3803, 0.8364, 0.5679) & (0.2302, 0.3403, 0.8091) \\ (0.0407, 0.9650, 0.7908) & (0.1818, 0.8583, 0.8378) & (0.3942, 0.8408, 0.4646) \\ (0.2382, 0.9261, 0.8903) & (0.3422, 0.9148, 0.7892) & (0.6989, 0.7439, 0.6339) \end{array}\right] \end{array},$$

$$EV_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \left[\begin{array}{ccc} (0.0528, 0.6260, 0.7103) & (0.2293, 0.6381, 0.5987) & (0.0450, 0.4938, 0.6396) \\ (0.0282, 0.6153, 0.8278) & (0.5956, 0.9760, 0.9890) & (0.1911, 0.7922, 0.4695) \\ (0.0837, 0.9073, 0.9144) & (0.1122, 0.3705, 0.8473) & (0.1189, 0.7418, 0.5580) \\ (0.1002, 0.5124, 0.5553) & (0.6817, 0.9822, 0.6632) & (0.2621, 0.7972, 0.5516) \end{array}\right] \end{array}.$$

Step2. Based on the obtained WDMs EV_1, EV_2, EV_3 and addition operator of SVNN shown in Eq. (2.1). construct the aggregated group decision matrix of all decision-makers as follows:

$$AG = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{ccc} D_1 & D_2 & D_3 \\ \left[\begin{array}{ccc} (0.0930, 0.1442, 0.2368) & (0.2491, 0.2866, 0.1508) & (0.1010, 0.0658, 0.0907) \\ (0.2780, 0.1257, 0.2794) & (0.2228, 0.0875, 0.1049) & (0.2274, 0.1586, 0.1281) \\ (0.1151, 0.1187, 0.1941) & (0.1748, 0.2300, 0.1026) & (0.0944, 0.0831, 0.1441) \\ (0.2009, 0.1128, 0.2061) & (0., 2830 \ 0.2101, 0.1485) & (0.2629, 0.1337, 0.0677) \end{array}\right] \end{array}.$$

Step 3. Determine the virtual NPIS and NNIS as:

$$G_j^+ = \left[\ (0.2780, 0.1128, 0.1941) \quad (0.2830, 0.0875, 0.1026) \quad (0.2629, 0.0658, 0.0677)\ \right],$$

$$G_j^- = \left[\ (0.0930, 0.1442, 0.2794) \quad (0.1748, 0.2866, 0.1508) \quad (0.0944, 0.1586, 0.1441)\ \right].$$

Step 4. Calculate the distance of each alternative from NPIS and NNIS, respectively, as follows:

$$d_1^+ = 0.3028,\ d_2^+ = 0.1377,\ d_3^+ = 0.2715,\ d_4^+ = 0.1435,$$
$$d_1^- = 0.1148,\ d_2^- = 0.2799,\ d_3^- = 0.1246,\ d_4^- = 0.2397.$$

Step5. Based on Eq. (3.9). the relative closeness coefficient of each candidate can be calculated as follows:

$$R_1^* = 0.2748, \ R_2^* = 0.6703, \ R_3^* = 0.3146, \ R_4^* = 0.6255.$$

Therefore, the ranking order of the four alternatives is A_2, A_4, A_3 and A_1. Obviously, the best selection is A_2.

Remark 2. In recent years a lot of extended TOPSIS methods to deal with MAGDM problems have been presented that only consider crisp or incomplete information on their calculation. But until now there hasn't been any TOPSIS method to consider and handle indeterminate and inconsistent information that exists commonly in real decision-making problems. In order to overcome this drawback, this paper for the first time presents an extended TOPSIS method for MAGDM problems based on a single-valued neutrosophic set. Although by using the neutrosophic sets we are faced with a large class of problems the proposed method has less calculation and is more flexible for decision making in the real world.

5. CONCLUSION

In general, decision-making problems are included uncertain and imprecise information, and neutrosophic sets can depict this kind of information easier and better. Because TOPSIS is an important decision-making method, and the neutrosophic sets can handle the incomplete, indeterminate and inconsistent data, it is important to establish an extended TOPSIS method based on NSs. In this paper, we first develop a distance measures which is an effective and simple tool to measure the distance between two single-valued neutrosophic numbers and then present an extended TOPSIS method to deal with multi-attribute group decision-making (MAGDM) under neutrosophic environment, where decision-makers express the attribute weights and attribute values for alternatives by using neutrosophic numbers. Although the proposed method presented in this paper is illustrated by a personal selection problem, however, it can also be applied to problems such as information project selection, material selection and many other areas of management decision problems.

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