

An Improved Neutrosophic Number Optimization Method for Optimal Design of Truss Structures

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To overcome the complex calculation and difficult solution problems in existing solution methods of neutrosophic number (NN) optimization models, this paper proposes an improved NN optimization method to solve NN optimization models by use of the Matlab built-in function “fmincon()” corresponding to the indeterminacy I and the indeterminate scale λ . Next, the proposed NN optimization method is applied to a three-bar planar truss structural design with indeterminate information to achieve the minimum weight objective under stress and deflection constraints as a NN nonlinear optimization design example. The optimal solutions of the truss structural design demonstrate the feasibility and flexibility of the proposed NN nonlinear optimization method under indeterminate environment. Finally, by taking some specified indeterminate scale we can also obtain a suitable optimal solution to satisfy some specified actual requirement under indeterminate environments.

Keywords: Neutrosophic number; neutrosophic number optimization method; neutrosophic number optimization model; indeterminate scale; truss structure design.

1. Introduction

In past decades, various optimization methods have been applied to optimal design problems of truss structures with some constraints. Usually, there are three types of truss structural design problems: size, layout, and topology optimizations. Because the truss structural optimization design is a critical and challenging activity in mechanical engineering and civil engineering fields, a lot of researchers mainly focus on the developments or improvements of optimal algorithms for optimal design problems of truss structures in determinate environments.^{1,2,4,5,7,8,10} However, in real engineering design problems, there exist some indeterminate data, such as Young’s modulus, stiffness, and yield stress of materials, which are generally given as interval ranges rather than the unique crisp values in design handbooks on materials due to the uncertainty/indeterminacy of material characteristics. Furthermore, the applied force/torque for truss structures may be indeterminate/changeable in real situations. Hence, the uncertain problems are inevitable and have to be taken into account in

engineering optimization problems; otherwise, the obtained optimal solution may become infeasible or its performance can degrade significantly. Thus, robust optimization approaches have been proposed for handling uncertain design problems.^{18,24} However, the robust optimal solutions obtained from the robust optimization approaches may be conservative and relatively insensitive to input uncertainty, and also the unique crisp optimal solution obtained from an indeterminate problem may be unreasonable due to the conservative/overestimated design. On the other hand, existing some uncertain optimization approaches are to provide uncertain objective programming models involving uncertain variables/parameters, and then these models were turned to crisp objective programming models to find unique crisp optimal solutions.^{9,11,16,17} From an indeterminate viewpoint, however, an indeterminate optimization problem should contain possible ranges of the optimal solutions corresponding to various indeterminate ranges to be suited to indeterminate design requirements rather than the unique crisp optimal solution.^{3,23}

Then, the indeterminacy of structural design is connected with lack of accurate data of design factors. In this case, how to express indeterminate information is an important problem. Hence, Smarandache¹²⁻¹⁴ first introduced a concept of indeterminacy, which is denoted by the symbol " I " as the indeterminate value/range, and then presented a concept of a neutrosophic number (NN) $z = d + uI$ for $d, u \in R$ (all real numbers), which consists of both the determinate part d and the indeterminate part uI . It is obvious that it can express determinate and/or indeterminate information in incomplete, uncertain, and indeterminate problems. After that, Ye^{19,22} first applied NNs to decision making problems. Then, Kong *et al.*⁶ and Ye²⁰ applied NNs to fault diagnosis problems under NN environment. Further, Smarandache¹⁵ introduced an interval function (so-called neutrosophic function $g(x) = [g_1(x), g_2(x)]$ for $x \in R$) to describe indeterminate problems by the form of interval functions. Based on the Smarandache's neutrosophic function (interval function), Ye *et al.*²¹ utilized the neutrosophic functions (interval functions) to express the joint roughness coefficient and the shear strength in rock mechanics under indeterminate environment. Jiang and Ye³ defines basic operations of NNs and NN functions for objective functions and constraints in optimization models, and proposed a general NN nonlinear optimization model for the optimal design of truss structures. They utilized the Lagrangian multipliers for the neutrosophic number optimization model and the Karush-Kuhn-Tucker (KKT) necessary conditions to obtain the NN/interval optimal solution. Ye²³ also presented a NN linear programming (NNLP) method for NNLP problems to obtain the possible ranges of the optimal solution by the simplex algorithm under indeterminate environments.

However, in NNLP and NN nonlinear optimization problems, the optimization calculations cannot be carried by means of the MATLAB built-in routines, such as "linprog()" and "fmincon()," which result in the complex calculation and difficult solution problems by existing solution methods.^{3,23} To overcome these drawbacks,

this paper presents an improved NN optimization method to realize the implementation of the solution method for NN optimization models by use of the Matlab built-in function "fmincon()" corresponding to the indeterminacy $I \in [\inf I, \sup I]$ and the indeterminate scale $\lambda \in [0, 1]$. Next, the proposed NN optimization method is applied to the optimization problem of truss structural design with indeterminate information, where the obtained optimal solutions (possible optimal interval range) can satisfy the design requirements in indeterminate situations. In some specified case, designers can also get one specified optimal solution by specifying some suitable indeterminate scale $\lambda \in [0, 1]$ to satisfy the specified design requirement.

The remainder of this paper is structured as follows. Section 2 describes the indeterminate expression of NNs and some concepts of NN functions, which are used for the NN objective functions and constraints in indeterminate optimization problems. Section 3 proposes an improved NN optimization method to implement the solution method of NN optimization models by use of the Matlab built-in function "fmincon()". In Sec. 4, the proposed NN optimization method is applied to a three-bar truss structural design with indeterminate information, and then the optimal design solutions of the truss structure demonstrate the feasibility and flexibility of the proposed NN optimization design method. Section 5 contains some conclusions and future research direction.

2. NN and NN Optimization Model

2.1. Indeterminate expression based on NNs

In design problems of truss structures, there are some indeterminate design variables and/or parameters, such as allowable tensile and compressive stresses and external forces. For example, the allowable stress of some metal material is given in design handbooks by a possible range between 120 MPa and 140 MPa, denoted by $[\sigma_T] = [120, 140]$ MPa, which reveals the value of $[\sigma_T]$ is an indeterminate range within the interval $[120, 140]$. To effectively express the indeterminate information, a NN $z = d + uI$ for $d, u \in R$ (all real numbers), introduced by Smarandache (1998, 2013, 2014), can represent it as $z = 120 + 20I$ MPa for $I \in [0, 1]$, where its determinate part is $d = 120$ MPa, its indeterminate part is $uI = 20I$ MPa. Then, the indeterminacy I in NN usually belongs to the indeterminate interval $[\inf I, \sup I]$, i.e., $I \in [\inf I, \sup I]$. For another example, if some external force is within $[20, 23]$ kN, then it can be expressed as the NN $z = 20 + 3I$ kN for $I \in [0, 1]$ or $z = 20 + 0.3I$ kN for $I \in [0, 10]$ according to some represented needs.

It is clear that the NN $z = d + uI$ for $d, u \in R$ can express its determinate and/or indeterminate information, where I is usually specified as a possible interval range $[\inf I, \sup I]$ in actual applications. It is noteworthy that there are the crisp value $z = d$ if $uI = 0$ and the indeterminate value $z = uI$ if $d = 0$ as two special cases. Therefore, NNs are very suitable for the expression of determinate and/or indeterminate information under indeterminate environments.

For convenience, let Z be all NNs (Z domain), and then a NN is denoted by $z = d + uI = [d + u(\inf I), d + u(\sup I)]$ for $I \subseteq [\inf I, \sup I]$ and $z \in Z$.

2.2. NN functions and optimization model in Z domain

In optimal design problems of truss structures, a general optimization model is composed of the objective functions and constrained functions. Then in indeterminate optimization problems, objective functions and constrained functions may contain indeterminate information (variables and/or parameters). To express indeterminate optimization models in NN environment, we introduce NN functions to general NN optimization models in Z domain.^{3,23}

Definition 1 (Refs. 13 and 23). A NN function in n variables (unknowns) and Z domain is defined as follows:

$$f(\mathbf{x}, I) : Z^n \rightarrow Z, \quad (1)$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ for $\mathbf{x} \in Z^n$ is a n -dimensional vector, I is indeterminacy, and $f(\mathbf{x}, I)$ is either a NN linear function or a NN nonlinear function.

For example, $f_1(\mathbf{x}, I) = x_1^2 + (4 + I)x_2^2$ for $\mathbf{x} = [x_1, x_2]^T \in Z^2$ is a NN nonlinear function, while $f_2(\mathbf{x}, I) = (3 + 2I)x_1 + x_2 + 2 + 3I$ for $\mathbf{x} = [x_1, x_2]^T \in Z^2$ is a NN linear function.

In general, NN optimization problems in n design variables and Z domain can be defined as the following general form of a NN optimization model:

$$\begin{aligned} \min & f(\mathbf{x}, I), \\ \text{s.t. } & g_k(\mathbf{x}, I) \leq 0, \quad k = 1, 2, \dots, m, \\ & h_j(\mathbf{x}, I) = 0, \quad j = 1, 2, \dots, s, \\ & \mathbf{x} \in Z^n, \end{aligned} \quad (2)$$

where $f(\mathbf{x}, I)$ is a NN objective function and $g_1(\mathbf{x}, I), g_2(\mathbf{x}, I), \dots, g_m(\mathbf{x}, I), h_1(\mathbf{x}, I), h_2(\mathbf{x}, I), \dots, h_s(\mathbf{x}, I)$ are NN constrained functions for $\mathbf{x} \in Z^n$ and $I \in [\inf I, \sup I]$.

When the NN optimal solution of design variables satisfies all these constrained conditions in a NN optimization model, it is feasible and otherwise is unfeasible. Generally, the optimal solution of design variables and the value of the NN objective function are usually within some optimal indeterminate range for an indeterminate optimization problem.^{3,23}

However, it is usually difficult to solve NN linear/nonlinear optimization models in indeterminate linear/nonlinear optimization design problems by existing solution methods. To easily solve the NN linear/nonlinear optimization model, this paper uses a Matlab built-in function "fmincon()" for the implementation of the solution method in NN optimization problems.

3. Solution Method using the Matlab Built-in Function “fmincon()”

In this study, we propose an approach of de-neutrosophication, where the indeterminacy $I \in [\inf I, \sup I]$ can be represented as $I(\lambda) = (1-\lambda)(\inf I) + \lambda(\sup I)$ for $\lambda \in [0, 1]$. Thus, NN can be represented as $z(\lambda) = d + uI(\lambda) = d + u[(1-\lambda)(\inf I) + \lambda(\sup I)]$ for $\lambda \in [0, 1]$. Thus λ may be considered as an indeterminate scale in real engineering design problems. For example, assume that a NN is $z = 120 + 20I$ for $I \in [0, 1]$. There are $z(0) = 120$ for $\lambda = 0$ (determinacy), $z(1) = 140$ for $\lambda = 1$ (maximum indeterminacy), and $z(0.5) = 130$ for $\lambda = 0.5$ (moderate indeterminacy).

For the optimization approach of de-neutrosophication, it allows some specification of the indeterminacy $I \in [\inf I, \sup I]$ as $I(\lambda) = (1-\lambda)(\inf I) + \lambda(\sup I)$ by an indeterminate scale $\lambda \in [0,1]$. Especially when $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 1$ are considered as the minimum indeterminacy/determinacy, the moderate indeterminacy, and the maximum indeterminacy, respectively, in an NN optimization problem, we can obtain their corresponding optimal solutions. Usually, the NN optimization solutions reveal an optimal interval range (but not always) corresponding to the interval range $[0, 1]$ of the indeterminate scale λ . It is obvious that the optimal solutions are changed as the indeterminacy $I \in [\inf I, \sup I]$ and the indeterminate scale $\lambda \in [0, 1]$ are changed in NN optimization problems.

According to the proposed optimization technique of de-neutrosophication, the NN optimization model (2) with $I \in [\inf I, \sup I]$ can be turned into the following optimization model with the indeterminate scale $\lambda \in [0, 1]$:

$$\begin{aligned} &\min f(\mathbf{x}, \lambda) \\ &\text{s.t. } g_k(\mathbf{x}, \lambda) \leq 0, \quad k = 1, 2, \dots, m \\ &\quad h_j(\mathbf{x}, \lambda) = 0, \quad j = 1, 2, \dots, s \\ &\quad \mathbf{x} \in R^n. \end{aligned} \tag{3}$$

Thus, the optimization model (3) can be easily solved by use of the Matlab built-in function “fmincon()”. To illustrate the effectiveness of the proposed method, we consider the following numerical example to show the optimal solutions corresponding to the specified indeterminate scales of λ .

Example 1. A NN minimization problem with $I \in [0, 2]$ and two variables is considered subject to several NN nonlinear inequality constraints, which is constructed as the following optimization model:

$$\begin{aligned} \min f(\mathbf{x}, I) &= \{[x_1 + (1 + I)]^2 + 3(x_2 - 1.3)^2\} \{[x_1 - (1.2 + I)]^2 \\ &\quad + 0.5(x_2 - 0.5)^2\}, \\ \text{s.t. } g_1(\mathbf{x}, I) &= 3x_1 - x_1x_2 + 4x_2 - (6 + I) \leq 0, \\ g_2(\mathbf{x}, I) &= 2x_1 + x_2 - (2 + 2I) \leq 0, \\ g_3(\mathbf{x}, I) &= (2 + I)x_1 - 4x_2^2 - 4x_2 \leq 0, \\ g_4(\mathbf{x}) &= -x_1 \leq 0, \\ g_5(\mathbf{x}) &= -x_2 \leq 0. \end{aligned} \tag{4}$$

According to the introduced optimization technique of de-neutrosophication, the NN nonlinear optimization model (4) with $I \in [0, 2]$ can be turned into the following optimization model with the indeterminate scale $\lambda \in [0, 1]$:

$$\begin{aligned} \min f(\mathbf{x}, \lambda) &= \{[x_1 + (1 + 2\lambda)]^2 + 3(x_2 - 1.3)^2\} \{[x_1 - (1.2 + 2\lambda)]^2 \\ &\quad + 0.5(x_2 - 0.5)^2\}, \\ \text{s.t. } g_1(\mathbf{x}, \lambda) &= 3x_1 - x_1x_2 + 4x_2 - (6 + 2\lambda) \leq 0, \\ g_2(\mathbf{x}, \lambda) &= 2x_1 + x_2 - (2 + 4\lambda) \leq 0, \\ g_3(\mathbf{x}, \lambda) &= (2 + 2\lambda)x_1 - 4x_1^2 - 4x_2 \leq 0, \\ g_4(\mathbf{x}) &= -x_1 \leq 0, \\ g_5(\mathbf{x}) &= -x_2 \leq 0. \end{aligned} \quad (5)$$

Then, we discuss the implementation of the solution method by the Matlab built-in function "fmincon()". When we consider $\lambda = 0$ as the minimum indeterminacy, $\lambda = 0.5$ as the moderate indeterminacy, and $\lambda = 1$ as the maximum indeterminacy in the NN nonlinear optimization problem, we can obtain the following three specified optimal solutions:

- (1) $x_1^* = 0.8406$, $x_2^* = 0.3187$, and $f(\mathbf{x}^*, I) = 1.1026$ for $\lambda = 0$;
- (2) $x_1^* = 1.6415$, $x_2^* = 0.7170$, and $f(\mathbf{x}^*, I) = 5.0656$ for $\lambda = 0.5$;
- (3) $x_1^* = 2$, $x_2^* = 1$, and $f(\mathbf{x}^*, I) = 40.2987$ for $\lambda = 1$.

Obviously, the optimization problem with the indeterminacy $I \in [0, 2]$ reveals different optimal results corresponding to the three specified indeterminate scales of λ . However, its general optimal solutions are $x_1^* = [0.8406, 2]$ and $x_2^* = [0.3187, 1]$ for $f(\mathbf{x}^*, I) = [1.1026, 40.2987]$, which usually are interval optimal ranges.

4. Optimal Design of Truss Structures under NN Environment

To illustrate the application of the proposed NN optimization method, this section will research on the NN optimization design problem of a three-bar planar truss structure with indeterminate information to realize the primal investigation of the truss structural optimization design in NN environment.

A well-known three-bar planar truss structure, which is shown in Fig. 1, is presented to minimize the weight of the truss structure subject to stress constraints on each of the truss members and the vertical and horizontal displacement constraints at the loading point of a loaded three-bar planar truss with some indeterminate information for $I \in [0, 1]$. According to the design requirements, the optimization problem of the truss structure can be constructed by the following NN optimization model:

$$\begin{aligned} \min f(\mathbf{x}, I) &= \rho L(2\sqrt{2}x_1 + x_2), \\ \text{s.t. } \sigma_1(\mathbf{x}, I) &= \frac{F(2x_1 + \sqrt{2}x_2)}{2(x_1^2 + \sqrt{2}x_1x_2)} - [\sigma_{T1}] \leq 0, \end{aligned}$$

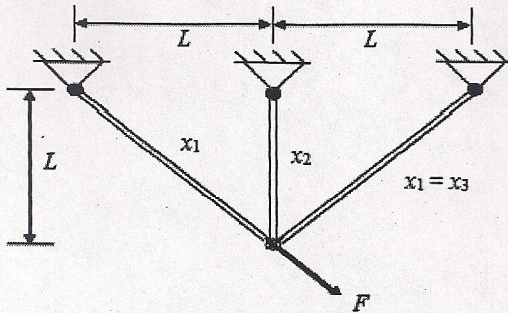


Fig. 1. Three-bar planar truss structure.

$$\begin{aligned} \sigma_2(\mathbf{x}, I) &= \frac{F_v}{x_1 + \sqrt{2}x_2} - [\sigma_{T2}] \leq 0, \\ \sigma_3(\mathbf{x}, I) &= \frac{F\sqrt{2}x_2}{2(x_1^2 + \sqrt{2}x_1x_2)} - [\sigma_{C3}] \leq 0, \\ \delta_v(\mathbf{x}, I) &= \frac{FL}{E(x_1 + \sqrt{2}x_2)} - \delta_v \leq 0, \\ \delta_h(\mathbf{x}, I) &= \frac{FL}{Ex_1} - \delta_h \leq 0, \\ x_{i\min} &\leq x_i \leq x_{i\max} \quad \text{for } i = 1, 2, \end{aligned} \tag{6}$$

where F is the applied load at the loading point; ρ is the material density; L is the length; E is Young's modulus; the design vector $\mathbf{x} = [x_1, x_2]^T$ is composed of two design variables x_1 and x_2 , which are denoted as the cross sections of bars 1 and 2; δ_v and δ_h are the allowable vertical and horizontal displacements of the loaded point, respectively; $[\sigma_T]$ is the allowable stress for bars 1, 2, and 3. The given data of the optimization model are shown in Table 1.

Thus, we use the Matlab built-in function "fmincon()" for the implementation of the solution method for the NN nonlinear optimization model of the truss structural design.

Firstly, according to the optimization technique of de-neutrosophication, the NN nonlinear optimization model (6) with the indeterminacy $I \in [0, 1]$ can be formulated

Table 1. The given data of the optimization model for $I \in [0, 1]$.

Applied load F (N)	Volume density ρ (kg/m ³)	Length L (mm)	Allowable stress $[\sigma_T]$ (MPa)	Young's modulus E (MPa)	Allowable vertical and horizontal displacements of loaded joint δ_v and δ_h (mm)	Cross section of bars 1, 2 (mm ²)
$(1 + 0.2I) \times 10^4$	7800	1000	$120 + 20I$	$(2 + 0.1I) \times 10^5$	$\delta_v = 0.01,$ $\delta_h = 0.1$	$x_{1\min} = x_{2\min} = 50,$ $x_{1\max} = x_{2\max} = 4000$

as follows:

$$\begin{aligned}
 \min f(\mathbf{x}, \lambda) &= 0.0078(2\sqrt{2}x_1 + x_2) \\
 \text{s.t. } \sigma_1(\mathbf{x}, \lambda) &= \frac{(1 + 0.2\lambda) \times 10^4(2x_1 + \sqrt{2}x_2)}{2(x_1^2 + \sqrt{2}x_1x_2)} - (120 + 20\lambda) \leq 0 \\
 \sigma_2(\mathbf{x}, \lambda) &= \frac{(1 + 0.2\lambda) \times 10^4}{x_1 + \sqrt{2}x_2} - (120 + 20\lambda) \leq 0 \\
 \sigma_3(\mathbf{x}, \lambda) &= \frac{(1 + 0.2\lambda) \times 10^4\sqrt{2}x_2}{2(x_1^2 + \sqrt{2}x_1x_2)} - (120 + 20\lambda) \leq 0 \\
 \delta_v(\mathbf{x}, \lambda) &= \frac{(1 + 0.2\lambda) \times 10^7}{(2 + 0.1\lambda) \times 10^5(x_1 + \sqrt{2}x_2)} - 0.01 \leq 0 \\
 \delta_h(\mathbf{x}, \lambda) &= \frac{(1 + 0.2\lambda) \times 10^7}{2 + 0.1\lambda \times 10^5x_1} - 0.1 \leq 0 \\
 50 \leq x_i &\leq 4000 \quad \text{for } i = 1, 2.
 \end{aligned} \tag{7}$$

Selecting different indeterminate scales of $\lambda \in [0, 1]$ in the NN nonlinear optimization problem, we can obtain the different specified optimal solutions of the three-bar planar truss structural design, which are shown in Table 2.

From Table 2, we can obtain that its general optimal solutions are $x_1^* = [500, 571.4]$ and $x_2^* = [3182, 3636.5]$, and then the minimum value of the objective function is $f(\mathbf{x}^*, I) = [35.8503, 40.9718]$ for $I \in [0, 1]$.

Obviously, the optimal results are changed as the indeterminate scale of λ are changed under NN environment. If $\lambda = 0$, it is clear that the NN optimization problem is degenerated to the determinate optimization problem without the indeterminate part uI (i.e., conventional certain optimization problem). However, for a specified interval range of the indeterminacy $I \in [\inf I, \sup I]$ in actual applications, designers will take some indeterminate scale of $\lambda \in [0, 1]$ depending on some indeterminate condition to obtain the corresponding optimal solution, which can satisfy the actual requirement of the truss structural design. For example, if we take the specified indeterminate scale $\lambda = 0.5$ (the moderate indeterminacy) for $I \in [0, 1]$ in Table 2, then the specified optimal solution is $x_1^* = 536.6 \text{ mm}^2$ and $x_2^* = 3414.8 \text{ mm}^2$ to be suitable for the design requirement of the moderate indeterminacy.

Table 2. Optimal solutions of the truss structural design for different indeterminate scales of λ .

λ	x_1^* (mm ²)	x_2^* (mm ²)	$f(\mathbf{x}^*, \lambda)$ (kg)
$\lambda = 0$	500	3182	35.8503
$\lambda = 0.1$	507.5	3229.5	36.3854
$\lambda = 0.3$	522.2	3323.1	37.4397
$\lambda = 0.5$	536.6	3414.8	38.4735
$\lambda = 0.7$	550.7	3504.8	39.4873
$\lambda = 0.9$	564.6	3593.0	40.4817
$\lambda = 1$	571.4	3636.5	40.9718

Compared with existing NN optimization design methods (Jiang and Ye, 2016; Ye, 2017), the improved NN (indeterminate) optimization design method indicates the following main advantages:

- (i) The improved NN optimization design method shows its convenience and flexibility since the Matlab built-in function “fmincon()” can realize the optimal solution by a specified selection of indeterminate scale under an indeterminate (NN) condition.
- (ii) Existing NN optimization methods difficultly handle the complex NN optimization design problems under indeterminate environment, while the improved NN optimization method can overcome this drawback.
- (iii) The improved NN optimization method demonstrates its feasibility and rationality under indeterminate optimization design problems.

5. Conclusion

To overcome the complex calculation and difficult solution problems in existing solution methods of NN optimization models,^{3,23} this paper proposed an improved NN optimization method with constrained optimizations by using the Matlab built-in function “fmincon()” to reach the solution method corresponding to the indeterminacy $I \in [\inf I, \sup I]$ and the indeterminate scale $\lambda \in [0, 1]$. Next, the improved NN optimization method was applied to a three-bar planar truss structural design with indeterminate information. The obtained optimal solutions demonstrated the flexibility and rationality of the proposed NN optimization design method. However, the NN optimization problems may contain indeterminate optimal solutions, which can indicate possible optimal ranges of the design variables and objective function when the indeterminacy $I \in [\inf I, \sup I]$. Then, when the indeterminate scale $\lambda \in [0, 1]$ are specified as possible values in actual applications, we can also determinate the specified optimal design values of design variables to satisfy the specified indeterminate requirements, such as the minimum indeterminacy/determinacy, the moderate indeterminacy, and the maximum indeterminacy.

It is obvious that the improved NN optimization design method is easier and more feasible than existing NN optimization design methods of truss structures since the existing NN optimization design methods contain some drawbacks, such as complex calculation/difficult solution problems, under indeterminate environment. Then, the improved NN optimization design method can overcome the mentioned drawbacks and provide a new way for the linear/nonlinear optimal design under indeterminate environment.

The results of this study may lead to the development of effective NN optimization technique for solving other models of linear/nonlinear optimization problems in different fields. In the future, therefore, we shall further study solving methods for more complex NN linear/nonlinear optimization problems and apply them

to mechanical design and civil engineering areas under indeterminate (NN) environment.

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