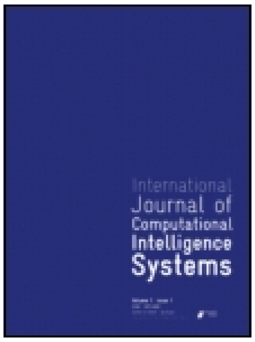

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An Improved Weighted Correlation Coefficient Based on Integrated Weight for Interval Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems

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Abstract

This paper presents a new correlation coefficient measure, which satisfies the requirement of this measure equaling one if and only if two interval neutrosophic sets (INSs) are the same. And an objective weight of INSs is presented to unearth and utilize deeper information that is uncertain. Using the proposed weighted correlation coefficient measure of INSs, a decision-making method is developed, which takes into account the influence of the evaluations' uncertainty and both the objective and subjective weights.

Keywords: Interval neutrosophic sets, objective weight, integrated weight, correlation coefficient, multi-criteria decision-making.

1. Introduction

The seminal theory of fuzzy sets (FSs) that was proposed by Zadeh in 1965 [1] is regarded as an important tool for solving multi-criteria decision-making (MCDM) problems [2, 3]. Since then, many new extensions that have resolved issues surrounding imprecise, incomplete and uncertain information have been suggested [4]. For example, Turksen [5] introduced the interval-valued fuzzy set (IVFS) using an interval

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number instead of one specific value to define the membership degree. Furthermore, in order to depict fuzzy information comprehensively, Atanassov and Gargov [6, 7] defined IFSs and interval-valued intuitionistic fuzzy sets (IVIFSs), which can handle incomplete and inconsistent information. Hesitant fuzzy sets (HFSs) were introduced by Torra and Narukawa [8] to deal with situations where people are hesitant in expressing their preference regarding objects in a decision-making process. Moreover, all these extensions of FSs have been developed by authors working in various fields [9-11] with further extensions still being proposed [12-17]. In particular, Florentin Smarandache [18, 19] introduced neutrosophic logic and neutrosophic sets (NSs) in 1995, with the latter being characterized by the functions of truth, indeterminacy and falsity. What's more, the three functions' values lie in $]0^{-}, 1^{+}[$, the non-standard unit interval, which is the extension of the standard interval [0, 1] of IFS. Additionally, the uncertainty shown here, i.e. the indeterminacy factor, is immune to truth and falsity values, while the incorporated uncertainty depends on the degrees of belongingness and non-belongingness in an IFS [20].

However, NSs are difficult to apply in actual decision-making problems. Therefore, the single-valued neutrosophic set (SVNS) was put forward, with a number of MCDM methods being proposed under a single-valued neutrosophic environment [21-27], and some other extensions of NSs have been introduced [28-30]. In consideration of the fact that using exact numbers to describe the degrees of truth, falsity and indeterminacy about a particular statement is sometimes infeasible in real situations, Wang et al. [31] proposed the concept of INSs and presented the set-theoretic operators of an INS. What's more, the operations of an INS were discussed in Ref. 32. To correct deficiencies in Ref. 31, Zhang et al. [33] refined the INS's operations, proposed a comparison approach between interval neutrosophic numbers (INNs) and developed the aggregation operators for INSs. In addition, kinds of MCDM methods utilizing INSs were put forward, including those using aggregation operators [33], a fuzzy cross-entropy [34], similarity measures [35] and outranking [36].

The correlation coefficient is an important tool for judging the relationship between two objects, and under fuzzy circumstances, the correlation coefficient is a principal vehicle for calculating the fuzziness of information in FS theory, which has been widely developed. For example, Chiang and Lin [37] introduced the correlation of FSs and in 1991 Gerstenkorn and Manko [38] defined the correlation of IFSs. However, Hong and Hwang [39] pointed out that the correlation coefficient in Ref. 38 did not satisfy the condition that $K(A,B)=1$ if and only if $A=B$, where $K(A,B)$ denotes the correlation coefficient between two FSs A and B. They also generalized the correlation coefficient of IFSs in a probability space [39] and proved that the method proposed overcame the shortcoming mentioned above in the case of finite spaces. Furthermore, Hung and Wu [40] defined the correlation coefficient of IFSs by utilizing the concept of centroids and introduced the concept of positive and negative correlation. Based on Ref. 40, Hanafy et al. [41] defined the correlation coefficient of generalized IFSs whose degrees of membership and non-membership lie between 0 and 0.5. Moreover, Bustince and Burillo [42] discussed the correlation coefficient under an interval-valued intuitionistic fuzzy environment and demonstrated their properties. Additionally, in an interval-valued intuitionistic fuzzy environment, the correlation coefficient can also be an effective vehicle. For example, based on the correlation coefficient method of IVIFSs proposed in Ref. 42, Ye [43] developed a weighted correlation coefficient measure to solve MCDM problems with incompletely known criterion weight information, where the weight is determined by the entropy measure. Furthermore, the correlation coefficient has been widely applied in various scientific fields, such as decision making [44-46], pattern recognition [47] and machine learning [48].

The correlation coefficient measure is also effective under neutrosophic environments. Hanafy et al. [49] defined the correlation and correlation coefficient of NSs, and Ye [21] introduced the correlation and correlation coefficient of SVNSs and utilized this measure to solve MCDM problems. Following the correlation coefficient in Ref. 49, Broumi and Smarandache [50] proposed the correlation coefficient measure and the weighted correlation coefficient measure of INSs. Nevertheless, there are some drawbacks in certain situations regarding the correlation coefficient measure defined in Ref. 21. In order to overcome these disadvantages, Ye [51] developed an improved correlation coefficient measure of SVNSs and extended it to INSs.

With regard to MCDM problems, alternatives are evaluated under various criteria. Therefore, criteria weights reflect the relative importance in ranking alternatives from a set of those available. With respect to multiple weights, they can be divided into two categories: subjective weights and objective weights [52]. Subjective weights are related to the preferences or judgments of decision makers, while objective weights usually refer to the relative importance of various criteria without any consideration of the decision maker's preferences. The subjective weight measure and objective weight measure have both been extensively studied.

Regarding the subjective weight measure, Saatty [53, 54] proposed an eigenvector method using pairwise weight ratios to obtain the weights of belonging of each member of the set. Subsequently, Keeney and Raiffa [55] discussed some direct assessing methods to determine the subjective weight. Based on Ref. 54, Cogger and Yu [56] introduced a new eigenweight vector whose computation is easier than Saatty's method. Moreover, Chu [57] proposed a weighted least-squares method, several examples of which were shown to compare favourably with the eigenvector method. In order to deal with mixed multiplicative and fuzzy preference relations, Wang et al. [58] presented a chi-squared method.

As for the objective weight, based on the notion of contrast intensity and the conflicting character of the evaluation criteria, Diakoulaki et al. [59] proposed the importance of criteria through an inter-criteria correlation method to obtain the objective weight. Wu [60] made use of the maximizing deviation method and constructed a non-liner programming model to obtain the objective weight. Moreover, Zou et al. [61] proposed a new weight evaluation process, which utilized the entropy measure, and applied it in a water quality assessment.

In general, the subjective method reflects the preference of the decision maker, while the objective method makes use of mathematical models to unearth the objective information. However, the subjective method may be influenced by the level of the decision maker's knowledge and the objective method neglects the decision maker's preference. The most common method of overcoming this shortage, and benefiting from not only the expertise of decision makers but also the relative importance of evaluation information, is to integrate the subjective and objective weights to explore a decision-making process that approaches, as closely as possible, the actual one. For instance, Ma et al. [62] set up a two-objective programming model by integrating the subjective and objective approaches to solve decision-making problems; moreover this two-objective programming problem can be solved by making use of the linear weighted summation method. Similarly, Wang and Parkan [63] utilized a linear programming technique to integrate the subjective fuzzy preference relation and the objective decision matrix information in three different ways.

As mentioned above, many objective weight measures have been proposed with the entropy weight being one of the most widely used approaches for solving MCDM problems [61, 64, 65]. The entropy is also an important concept in the fuzzy environment. The fuzzy entropy was first introduced by Zadeh [1, 4] to measure uncertain information. In 1972, Luca and Termini [66] proposed the axiomatic definition of the entropy of FSs and defined the entropy using the non-probability concept. Moreover, Trillas and Riera [67] proposed general expressions for the entropy and in 1982 Yager [68] defined the fuzziness degree of an FS in terms of a lack of distinction between the FS and its complement. Fan and Xie [69] proposed the fuzzy entropy measure induced by distance, and similarly the entropy has been widely developed in an intuitionistic fuzzy environment. Bustince and Burillo [70] provided an axiom definition of an intuitionistic fuzzy entropy. Based on the axiomatic definition of the entropy of Luca et al. [66], Szmidt et al. [71] extended it into IFSs and proposed an entropy measure for IFSs as a result of a geometric interpretation of IFSs using a ratio of distances between them; furthermore, they also proposed some new entropy measures based on the similarity measures in Ref. 72. With regard to the neutrosophic environment, Majumdar et al. [73] introduced the entropy of SVNSs by providing an axiomatic definition based on the entropy's definition of an FS proposed by Luca et al. [66], and proposed a new entropy measure based on the notion that the uncertainty of a SVNS is due to the belongingness, non-belongingness and indeterminacy parts. Moreover, the relationships among the similarity measures, distance measures and entropy measures of FSs, IVFSs, IFSs and NSs have also been investigated [73-77]. The entropy is also effective in dealing with practical problems. For example, as mentioned above, the entropy can be used to obtain the objective weight in MCDM problems [43, 61, 65, 78].

However, most contributions on measuring the correlation coefficient and entropy concentrate on extensions of FSs and little effort has been made in this regard on INSs, which will restrict its potential scientific and engineering applications. Furthermore, the extant research about the correlation coefficient mostly only utilizes the objective measure under an environment where information about the criterion weight for alternatives is completely unknown or incompletely unknown [43, 78]. However, the influence caused by the uncertainty of an evaluations still exists, whereas the information about the criterion weight is known and the objective weight can avert the non-determinacy and arbitrariness caused by the subjective weight [79]. Therefore, a lot more work on this issue needs to be conducted. Consequently, the correlation coefficient measure, weighted correlation coefficient measure and entropy measure for INSs are extended in this paper, and an objective weight measure based on the entropy for INSs is also proposed. Additionally, the notion that the weighted correlation coefficient measure should make use of the integrated weight is proposed. Furthermore, a MCDM procedure based on the weighted correlation coefficient measure, which considers both the subjective and objective weights, is established, and an illustrative example is provided to demonstrate the applicability of the proposed measures.

The rest of this paper is organized as follows. Section 2 briefly introduces IVIFSs, NSs, SVNSs and INSs, as well as some operations for INSs, such as intersection and union. In Section 3, the correlation coefficient measure, weighted correlation coefficient measure, entropy measure, and their properties for INSs are developed. In addition, an objective weight measure that makes use of the entropy for INSs is explored. In Section 4, a decision-making procedure based on the weighted correlation coefficient measure using the integrated weight for MCDM problems is provided. In Section 5, an illustrative example is presented to illustrate the proposed method and a comparative analysis and discussion are also provided. Finally, conclusions are drawn in Section 6.

2. Preliminaries

In this section, some basic concepts and definitions related to INSs are introduced; these will be used in the rest of the paper.

Definition 1. Let X be a space of points (objects), with a generic element in X denoted by x. An IFS A in X is characterized by a membership function $\mu_A(x)$ and a non-membership function $\nu_A(x)$. For each point x in X, we have $\mu_A(x)$, $\nu_A(x) \subseteq [0,1]$, $x \in X$. Thus, the IFS A can be denoted by [7]:

$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$.

Definition 2. Let *A* and *B* be two IFSs in the universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$ and $A = \{< x_i, \mu_A(x_i), \nu_A(x_i) > | x_i \in X\}$, and $B = \{< x_i, \mu_B(x_i), \nu_B(x_i) > | x_i \in X\}$; then the correlation coefficient of *A* and *B* is defined by[80]:

$$C(A,B) = \frac{\sum_{i=1}^{n}(\mu_A(x_i)\cdot\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i))}{\max(\sum_{i=1}^{n}(\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)), \sum_{i=1}^{n}(\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)))}$$

where $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$ and $\pi_B(x_i) = 1 - \mu_B(x_i) - \nu_B(x_i)$ are called the degree of uncertainty (or hesitation).

Definition 3. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, therefore, $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$ [81].

Definition 4. An NS A is contained in the other NS B, denoted as $A \subseteq B$, if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \le \inf I_B(x)$, $\sup I_A(x) \le \sup I_B(x)$, $\inf F_A(x) \le \inf F_B(x)$ and $\sup F_A(x) \le \sup F_B(x)$ for $x \in X$ [81].

Since it is difficult to apply NSs to practical problems, Ye [22] reduced the NSs of nonstandard intervals into a type of SVNS of standard intervals that preserved the operations of NSs.

Definition 5. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, and $F_A(x): X \to [0,1]$. Then, a simplification of A is denoted by[22]:

$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$

which is called an SVNS and is a subclass of NSs.

Definition 6. Let X be a space of points (objects) with generic elements in X denoted by X. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0,1]$, and $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$, $x \in X$ [81].

Only the subunitary interval of [0, 1] is considered, which is a subclass of an NS. Therefore, all INSs are clearly NSs.

For any FS A, its complement A^c is defined by $m_{A^c}(x) = 1 - m_A(x)$ for all x in X. The complement of an INS A is also denoted by A^c.

Definition 7. Let A and B be two INSs, then[81, 82]:

(1) $A = B$, if and only if $A \subseteq B$ and $A \supseteq B$;

(2) $A^c = \{< x, [\inf F_A(x), \sup F_A(x)], [1 - \sup I_A(x), 1 - \inf I_A(x)], [\inf T_A(x), \sup T_A(x)] >\}$; and

(3) $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$ and $\sup F_A(x) \ge \sup F_B(x)$, for any $x \in X$.

A distance function or metric is a generalization of the concept of physical distance, and in FS theory, it describes how far one element is away from another. Ye [83] defined the Hamming distance measure between two INSs.

Definition 8. Let A and B be two INSs in the universe discourse $X = \{x_1, x_2, \cdots, x_n\}$, then the distance measure between them can be defined as follows[83]:

The Hamming distance:

$$d_H(A,B) = \frac{1}{6}\sum_{i=1}^{n}\big[|\inf T_A(x_i) - \inf T_B(x_i)| + |\sup T_A(x_i) - \sup T_B(x_i)| + |\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + |\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)| \big] \quad (1)$$

The normalized Hamming distance:

$$d_{nH}(A,B) = \frac{1}{6n}\sum_{i=1}^{n}\big[|\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + |\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + |\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)| \big] \quad (2)$$

Definition 9. Let A and B be two INSs in the universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$ and $A = \{< x_i, [\inf T_A(x_i), \sup T_A(x_i)], [\inf I_A(x_i), \sup I_A(x_i)], [\inf F_A(x_i), \sup F_A(x_i)] > | x_i \in X\}$ and $B = \{< x_i, [\inf T_B(x_i), \sup T_B(x_i)], [\inf I_B(x_i), \sup I_B(x_i)], [\inf F_B(x_i), \sup F_B(x_i)] > | x_i \in X\}$, then the correlation coefficient of A and B is defined by[50]:

$$K(A,B) = \frac{C(A,B)}{\sqrt{E(A) \cdot E(B)}}, \quad (3)$$

where the correlation of two INSs A and B is given by:

$$C(A,B) = \frac{1}{2}\sum_{i=1}^{n}\big[\inf T_A(x_i)\cdot \inf T_B(x_i) + \sup T_A(x_i)\cdot \sup T_B(x_i) + \inf I_A(x_i)\cdot \inf I_B(x_i) + \sup I_A(x_i)\cdot \sup I_B(x_i) + \inf F_A(x_i)\cdot \inf F_B(x_i) + \sup F_A(x_i)\cdot \sup F_B(x_i)\big]$$

and the informational intuitional energies of two IVIFSs A and B are defined as:

$$E(A) = \sum_{i=1}^{n}\frac{1}{2}\big[(\inf T_A(x_i))^2 + (\sup T_A(x_i))^2 + (\inf T_A(x_i))^2 + (\sup T_A(x_i))^2 + (\inf F_A(x_i))^2 + (\sup F_A(x_i))^2\big],$$

$$E(B) = \sum_{i=1}^{n}\frac{1}{2}\big[(\inf T_B(x_i))^2 + (\sup T_B(x_i))^2 + (\inf T_B(x_i))^2 + (\sup T_B(x_i))^2 + (\inf F_B(x_i))^2 + (\sup F_B(x_i))^2\big].$$

However, as Ye [51] mentioned, this correlation coefficient measure in Definition 9 cannot guarantee that the correlation coefficient of two INSs equals one if and only if two INSs are the same [51].

In some cases, several different kinds of weight may be taken into account at the same time. In order to solve this problem, the integration measure of different kinds of weights is required.

Definition 10. Let $w = (w_1, w_2, \cdots, w_n)$ and $\theta = (\theta_1, \theta_2, \cdots, \theta_n)$ be two different types of weight vector. The final integrated weight vector $W = (W_1, W_2, \cdots, W_n)$ can be calculated as follows[79]:

$$W_i = \frac{w_i\theta_i}{\sum_{i=1}^{n} w_i\theta_i} \tag{4}$$

3. The Weighted Correlation Coefficient Measure for an INS

In this section, a new correlation coefficient measure, the weighted correlation coefficient measure for INSs and their properties are developed. Moreover, an objective weight measure for the INS that utilizes the entropy is also explored.

3.1. *The correlation coefficient measure for an INS*

In order to overcome the deficiency presented in Definition 9, a novel correlation coefficient measure is proposed that is motivated by the correlation coefficient measure of IFSs suggested by Xu [80].

Definition 11. A mapping K: $INS(X)\times INS(X)\to[0,1]$ is called the INSs correlation coefficient measure if K satisfies the following properties:

(KP1) $0\le K(A,B)\le 1$;

(KP2) $K(A,B)=K(B,A)$; and

(KP3) $K(A,B)=1$ if and only if $A=B$.

Definition 12. Let two INSs A and B in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$ be $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>|x_i\in X\}$ and $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>|x_i\in X\}$. Then a measure between A and B is defined by the following formula:

$$K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)} = \frac{\sum_{i=1}^{n} C(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^{n} T(A(x_i)),\sum_{i=1}^{n} T(B(x_i))\right)} \tag{5}$$

where $C(A,B)$ means the correlation between two INSs A and B; $T(A)$ and $T(B)$ refer to the information energies of the two INSs, respectively. They are provided by:

$$C(A(x_i),B(x_i))=\frac{1}{2}[\inf T_A(x_i)\cdot\inf T_B(x_i)+\sup T_A(x_i)\cdot\sup T_B(x_i) +\inf I_A(x_i)\cdot\inf I_B(x_i)+\sup I_A(x_i)\cdot\sup I_B(x_i) +\inf F_A(x_i)\cdot\inf F_B(x_i)+\sup F_A(x_i)\cdot\sup F_B(x_i)], \tag{6}$$

$$T(A(x_i))=\frac{1}{2}\Big[(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf I_A(x_i))^2 +(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2\Big], \tag{7}$$

$$T(B(x_i))=\frac{1}{2}\Big[(\inf T_B(x_i))^2+(\sup T_B(x_i))^2+(\inf I_B(x_i))^2 +(\sup I_B(x_i))^2+(\inf F_B(x_i))^2+(\sup F_B(x_i))^2\Big] \tag{8}$$

Theorem 1. *The proposed measure* $K(A,B)$ *satisfies all the axioms given in Definition 11.*

Proof.

(*KP*1) According to Definition 6, $[\inf T_A(x_i),\sup T_A(x_i)]$, $[\inf I_A(x_i),\sup I_A(x_i)]$, $[\inf F_A(x_i),\sup F_A(x_i)]$, $[\inf T_B(x_i),\sup T_B(x_i)]$, $[\inf I_B(x_i),\sup I_B(x_i)]$ and $[\inf F_B(x_i),\sup F_B(x_i)]\subseteq[0,1]$ exist for any $i\in\{1,2,\cdots,n\}$. Thus, it holds that $C(A,B)\ge 0$, $T(A)\ge 0$ and $T(B)\ge 0$. Therefore, $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}\ge 0$. According to the Cauchy–Schwarz inequality: $\left(a_1b_1+a_2b_2+\cdots+a_nb_n\right)^2 \le\left(a_1^2+a_2^2+\cdots+a_n^2\right)\cdot\left(b_1^2+b_2^2+\cdots+b_n^2\right)$ where $a_i,b_i\in R$, $i=1,2,\cdots,n$, $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}\le 1$. Therefore, $0\le K(A,B)\le 1$ holds.

(*KP*2) According to Eq. (6), it is known that $C(A,B)=C(B,A)$, and it's clear that $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=\frac{C(A,B)}{\max\left(T(B),T(A)\right)}= K(B,A)$.

(*KP*3) If $A=B$, $\inf T_A(x_i)=\inf T_B(x_i)$, $\sup T_A(x_i)=\sup T_B(x_i)$, $\inf I_A(x_i)=\inf I_B(x_i)$, $\sup I_A(x_i)=\sup I_B(x_i)$, $\inf F_A(x_i)=\inf F_B(x_i)$ and $\sup F_A(x_i)=\sup F_B(x_i)$. Thus, $C(A,B)=\frac{1}{2}\sum_{i=1}^{n}\Big[(\inf T_A(x))^2+(\sup T_A(x_i))^2+(\inf I_A(x_i))^2 +(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2\Big]$ and $T(A)=T(B)=\frac{1}{2}\sum_{i=1}^{n}\Big[(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+ (\inf I_A(x_i))^2+(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2\Big]$, i.e. $C(A,B)=T(A)=T(B)$. Thus, it is clear that $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=1$.

If $K(A,B)=\dfrac{C(A,B)}{\max\left(T(A),T(B)\right)}=1$, then $C(A,B)=\max(T(A),T(B))$. According to the Cauchy–Schwarz inequality, $C(A,B)\le\sqrt{T(A)\cdot T(B)}\le\max(T(A),T(B))$. Thus, $C(A,B)=\sqrt{T(A)\cdot T(B)}=\max(T(A),T(B))$. If $C(A,B)=\sqrt{T(A)\cdot T(B)}$, there exists a nonzero real number η such that $\inf T_A(x_i)=\eta\inf T_B(x_i)$, $\sup T_A(x_i)=\eta\sup T_B(x_i)$, $\inf I_A(x_i)=\eta\inf I_B(x_i)$, $\sup I_A(x_i)=\eta\sup I_B(x_i)$, $\inf F_A(x_i)=\eta\inf F_B(x_i)$ and $\sup F_A(x_i)=\eta\sup F_B(x_i)$ for any $x_i\in X$. Besides, if $\sqrt{T(A)\cdot T(B)}=\max(T(A),T(B))$, $T(A)=T(B)$. Based on these two conditions, it is obvious that $\eta=1$ (i.e. $A=B$).

Hence, Theorem 1 is true, which means the measure $K(A,B)$ defined in Definition 12 is a correlation coefficient measure. □

Property 1. $K(A,A)$ is the supremum of all $K(A,B)$; in other words, $K(A,A)\ge K(A,B)$, $\forall A,B\in INS$.

Proof.

Property 1 is easy to yield from Theorem 1, and according to this theorem, $0\le K(A,B)\le 1$ and $K(A,A)=1$. Thus, Property 1 is true. □

Property 1 implies that the correlation coefficient between an INS and itself is always greater than or equal to the correlation coefficient between the INS and any other INS defined in the same universe.

Example 1. Assume $A=\{<x,[0.7,0.8],[0.0,0.1],[0.1,0.2]>\}$, and $B=\{<x,[0.4,0.5],[0.2,0.3],[0.3,0.4]>\}$, then $C(A,B)=0.41$, $T(A)=0.595$, and $T(B)=0.395$; thus, $K(A,B)=\dfrac{C(A,B)}{\max\left(T(A),T(B)\right)}=\dfrac{0.41}{\max(0.595,0.395)}=0.689$.

3.2. *The weighted correlation coefficient measure for an INS*

In Section 3.1, a correlation coefficient measure for INSs was proposed. However, this correlation coefficient measure does not take into consideration the relative importance of each INN in INSs. In many situations, such as MCDM [43, 78], different INNs may have different weights. In the following paragraphs, the weighted correlation coefficient between INSs, which is based on the correlation coefficient measure between INSs defined in Definition 12, will be introduced.

Definition 13. Let $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>|x_i\in X\}$ and $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>|x_i\in X\}$ be two INSs in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$. Let $w=\{w_1,w_2,\cdots,w_n\}$ be the weight vector of the elements x_i $(i=1,2,\cdots,n)$. Then a measure between A and B can be defined by the following formula:

$$K(A,B)=\frac{\sum_{i=1}^n w_iC(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^n w_iT(A(x_i)),\sum_{i=1}^n w_iT(B(x_i))\right)} \quad (9)$$

where $C(A(x_i),B(x_i))$, $T(A(x_i))$ and $T(B(x_i))$ satisfy Eqs. (6)-(8).

Theorem 2. *The proposed measure $K(A,B)$ in Definition 13 satisfies all the axioms given in Definition 11.*

Proof.

($P1$) According to Theorem 1, $C(A(x_i),B(x_i))\ge 0$, $T(A(x_i))\ge 0$ and $T(B(x_i))\ge 0$ $(i=1,2,\cdots,n)$. Besides, $w_i\ge 0$, thus, $K(A,B)=\dfrac{\sum_{i=1}^n w_iC(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^n w_iT(A(x_i)),\sum_{i=1}^n w_iT(B(x_i))\right)}>0$. According to the Cauchy–Schwarz inequality, $\sum_{i=1}^n w_iC(A(x_i),B(x_i))\le\sqrt{\sum_{i=1}^n w_iT(A(x_i))}\sqrt{\sum_{i=1}^n w_iT(B(x_i))}$ $\le\max\left(\sum_{i=1}^n w_iT(A(x_i)),\sum_{i=1}^n w_iT(B(x_i))\right)$. Therefore, $K(A,B)\le 1$.

($P2$) According to Theorem 1, it is known that $C(A(x_i),B(x_i))=C(B(x_i),A(x_i))$ exists for any $i\in\{i,2,\cdots,n\}$. Therefore, it's obvious that $\sum_{i=1}^n w_iC(A(x_i),B(x_i))=\sum_{i=1}^n w_iC(B(x_i),A(x_i))$. Thus, $K(A,B)=\dfrac{\sum_{i=1}^n w_iC(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^n w_iT(A(x_i)),\sum_{i=1}^n w_iT(B(x_i))\right)}=$ $\dfrac{\sum_{i=1}^n w_iC(B(x_i),A(x_i))}{\max\left(\sum_{i=1}^n w_iT(B(x_i)),\sum_{i=1}^n w_iT(A(x_i))\right)}=K(B,A)$.

($P3$) According to Theorem 1, $C(A(x_i),B(x_i))=\max(T(A(x_i)),T(B(x_i)))$ is true for any $i\in\{i,2,\cdots,n\}$ if $A=B$. Therefore,

$\sum_{i=1}^{n} w_i C(A(x_i), B(x_i)) = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$ is proved to be correct. Hence, if $A = B$, $K(A,B) = 1$.

If $K(A,B) = 1$, $\sum_{i=1}^{n} w_i C(A(x_i), B(x_i)) = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. According to the Cauchy–Schwarz inequality: $\sum_{i=1}^{n} w_i C(A(x_i), B(x_i)) \le \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))}$ $\le \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. Thus, $\sum_{i=1}^{n} w_i C(A(x_i), B(x_i)) = \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))}$ $= \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. If $\sum_{i=1}^{n} w_i C(A(x_i), B(x_i)) = \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))}$, there exists a nonzero real number η such that $\inf T_A(x_i) = \eta \inf T_B(x_i)$, $\sup T_A(x_i) = \eta \sup T_B(x_i)$, $\inf I_A(x_i) = \eta \inf I_B(x_i)$, $\sup I_A(x_i) = \eta \sup I_B(x_i)$, $\inf F_A(x_i) = \eta \inf F_B(x_i)$ and $\sup F_A(x_i) = \eta \sup F_B(x_i)$ for any $x_i \in X$. Besides, if $\sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))} =$ $\max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$, $T(A) = T(B)$. Based on these two conditions, it is obvious that $\eta = 1$ (i.e. $A = B$). □

Thus, Theorem 2 holds, which signifies that the measure $K(A,B)$ defined by Eq. (9) is a correlation coefficient measure. For convenience, it is called a weighted correlation coefficient measure.

Example 2. Assume $A = \{<x_1,[0.7,0.8],[0.0,0.1],[0.1,0.2]>, <x_2,[0.6,0.7],[0.1,0.2],[0.1,0.3]>\}$, $B = \{<x_1,[0.4,0.5],[0.2,0.3],[0.3,0.4]>, <x_2,[0.4,0.6],[0.1,0.3],[0.2,0.4]>\}$, and $w = \{0.4, 0.6\}$. Thus, $C(A(x_1), B(x_1)) = 0.41$, $C(A(x_2), B(x_2)) = 0.435$, $T(A(x_1)) = 0.595$, $T(A(x_2)) = 0.5$, $T(B(x_1)) = 0.395$, and $T(B(x_2)) = 0.41$; therefore, $\sum_{i=1}^{2} w_i C(A(x_i), B(x_i)) = 0.425$, $\sum_{i=1}^{2} w_i T(A(x_i)) = 0.538$, and $\sum_{i=1}^{2} w_i T(B(x_i)) = 0.404$. Thus, $K(A,B) = 0.790$.

As noted in Section 1, the integrated weight can not only benefit from the decision makers' expertise, but also from the relative importance of evaluation information. In order to assess the relative importance of weights accurately and comprehensively, it's better to utilize the integrated weight rather than only the subjective or objective weights in order to obtain the weighted correlation coefficient.

The subjective and objective weights should be calculated in order to compute the integrated weight. The subjective weight that mirrors the individual preference can be evaluated by the decision maker, while the objective weight that reflects the relative importance contained in the decision matrix should be calculated by mathematical methods. Certainly, many kinds of objective weight measures have been proposed and every measure has its own advantages. Due to the fact that the more equivocal the information is, the less important it will be [84], the entropy weight measure will be utilized to obtain the objective weight.

3.3. *The entropy weight measure for an INS*

In this section, the entropy measure and an objective weight measure based on the entropy for an INS are proposed.

The entropy is an important concept which is named after Claude Shannon who first introduced the concept. In information theory, the entropy is a measure for calculating the uncertainty associated with a random variable as it characterizes the uncertainty about the source of information. Thus the entropy is a measure of uncertainty. Based on the axiomatic definition of the entropy measure for SVNSs in Ref. 73, the entropy for INSs can be defined as follows.

Definition 14. A real function $E: INS(X) \to [0,1]$ is called the entropy on *INS*(*X*), if *E* satisfies the following properties:

(EP1) $E(A) = 0$ (minimum) if A is a crisp set ($\forall A \in P(X)$);

(EP2) $E(A) = 1$ (maximum) if $T_A(x) = I_A(x) = F_A(x)$ (i.e. $\inf T_A(x) = \inf I_A(x) = \inf F_A(x)$ and $\sup T_A(x) = \sup I_A(x) = \sup F_A(x)$) for any $x \in X$; and

(EP3) $E(A) \le E(B)$ if A is less fuzzy than B or B is more uncertain than A, i.e. (1) $\inf T_A(x) - \inf F_A(x) \le \inf I_B(x) - \inf F_B(x)$ and $\sup T_A(x) - \sup F_A(x) \le \sup I_B(x) - \sup F_B(x)$ for $\inf T_A(x) \ge \inf F_A(x)$ and $\sup T_A(x) \le \sup F_A(x)$ or $\inf T_A(x) - \inf F_A(x) \ge \inf I_B(x) - \inf F_B(x)$ and

$\sup T_A(x) - \sup F_A(x) \geq \sup I_B(x) - \sup F_B(x)$ for $\inf T_A(x) \leq \inf F_A(x)$, $\sup T_A(x) \leq \sup F_A(x)$ and $\sup T_A(x) \geq \sup T_B(x)$; and (2) $\inf I_A(x) \leq \inf I_B(x)$ and $\sup I_A(x) \leq \sup I_B(x)$ for $\inf I_B(x) + \sup I_B(x) \leq 1$ or $\inf I_A(x) \geq \inf I_B(x)$ and $\sup I_A(x) \geq \sup I_B(x)$ for $\inf I_B(x) + \sup I_B(x) \geq 1$;

(EP4) $E(A) = E(A^c)$.

A great deal of research has demonstrated the connection among the distance measure, the similarity measure and the entropy measure of FSs [73-77]. Having taken these studies into account, the entropy measure of INSs based on the distance measure defined in Definition 8 is now proposed.

Definition 15. Let A be an INS in the universe discourse $X = \{x_1, x_2, \cdots, x_n\}$, and assume that $E(A): N(X) \to [0,1]$. $E(A)$ is a measure such that:

$$E(A) = 1 - d(A, A^c). \quad (10)$$

where $d(A, A^c)$ refers to the distance measure between INS A and its complementary set A^c utilizing Eq. (2).

Theorem 3. *The proposed measure $E(A)$ satisfies all the axioms given in Definition 14.*

Proof. Let $A = \{< x_i, [\inf T_A(x_i), \sup T_A(x_i)], [\inf I_A(x_i), \sup I_A(x_i)], [\inf F_A(x_i), \sup F_A(x_i)] > | x_i \in X\}$ and $B = \{< x_i, [\inf T_B(x_i), \sup T_B(x_i)], [\inf I_B(x_i), \sup I_B(x_i)], [\inf F_B(x_i), \sup F_B(x_i)] > | x_i \in X\}$ be two INSs.

($EP1$) If an INS A is a crisp set, i.e. $\inf T_A(x_i) = \sup T_A(x_i) = 1$, $\inf I_A(x_i) = \sup I_A(x_i) = 0$, and $\inf F_A(x_i) = \sup F_A(x_i) = 0$ or $\inf T_A(x_i) = \sup T_A(x_i) = 0$, $\inf I_A(x_i) = \sup I_A(x_i) = 1$, and $\inf F_A(x_i) = \sup F_A(x_i) = 1$. By using Definition 7, the complementary set of A can be calculated, i.e. $\inf T_{A^c}(x_i) = \sup T_{A^c}(x_i) = 0$, $\inf I_{A^c}(x_i) = \sup I_{A^c}(x_i) = 1$, and $\inf F_{A^c}(x_i) = \sup F_{A^c}(x_i) = 1$ or $\inf T_{A^c}(x_i) = \sup T_{A^c}(x_i) = 1$, $\inf I_{A^c}(x_i) = \sup I_{A^c}(x_i) = 0$, and $\inf F_{A^c}(x_i) = \sup F_{A^c}(x_i) = 0$ respectively. Therefore, it's obvious that $E(A) = 0$.

($EP2$) If $T_A(x_i) = I_A(x_i) = F_A(x_i)$ and $\inf T_A(x_i) + \sup T_A(x_i) = 1$, by using Eq. (10), the entropy can be calculated: $E(A) = 1 - \frac{1}{6n}\sum_{i=1}^{n}[|\inf T_A(x_i) - \inf F_A(x_i)| + |\sup T_A(x_i) - \sup F_A(x_i)| + |\inf I_A(x_i) + \sup I_A(x_i) - 1| + |\sup I_A(x_i) + \inf I_A(x_i) - 1| + |\inf F_A(x_i) - \inf T_A(x_i)| + |\sup F_A(x_i) - \sup T_A(x_i)|] = 1$.

($EP3$) $\inf T_A(x_i) - \inf F_A(x_i) \leq \inf I_B(x_i) - \inf F_B(x_i)$ and $\sup T_A(x_i) - \sup F_A(x_i) \leq \sup I_B(x_i) - \sup F_B(x_i)$ for $\inf T_A(x_i) \geq \inf F_A(x_i)$ and $\sup T_A(x_i) \leq \sup F_A(x_i)$ or $\inf T_A(x_i) - \inf F_A(x_i) \geq \inf I_B(x_i) - \inf F_B(x_i)$ and $\sup T_A(x_i) - \sup F_A(x_i) \geq \sup I_B(x_i) - \sup F_B(x_i)$ for $\inf T_A(x_i) \leq \inf F_A(x_i)$, $\sup T_A(x_i) \leq \sup F_A(x_i)$ and $\sup T_A(x_i) \geq \sup T_B(x_i)$. Thus, it is quite obvious that $|\inf T_A(x) - \inf F_A(x)| + |\sup T_A(x_i) - \sup F_A(x_i)| \geq |\inf T_B(x_i) - \inf F_B(x_i)| + |\sup T_B(x_i) - \sup F_B(x_i)|$. $\inf I_A(x_i) \leq \inf I_B(x_i)$ and $\sup I_A(x_i) \leq \sup I_B(x_i)$ for $\inf I_B(x_i) + \sup I_B(x_i) \leq 1$ or $\inf I_A(x_i) \geq \inf I_B(x_i)$ and $\sup I_A(x_i) \geq \sup I_B(x_i)$ for $\inf I_B(x_i) + \sup I_B(x_i) \geq 1$, thus $|(\inf F_A(x_i) + \sup F_A(x_i)) - 1| \geq |(\inf F_B(x_i) + \sup F_B(x_i)) - 1|$. Therefore, $2[|\inf T_A(x_i) - \inf F_A(x_i)| + |\sup T_A(x_i) - \sup F_A(x_i)| + |\inf I_A(x_i) + \sup I_A(x_i) - 1|] \geq 2[|\inf T_B(x_i) - \inf F_B(x_i)| + |\sup T_B(x_i) - \sup F_B(x_i)| + |\inf I_B(x_i) + \sup I_B(x_i) - 1|]$ $(i = 1, 2, \cdots, n)$. Thus, $E(A) \leq E(B)$.

($EP4$) By using Eq. (10), $E(A)$ and $E(A^c)$ can be respectively calculated:

$E(A) = 1 - \frac{1}{6n} \times 2 \times \sum_{i=1}^{n}[|\inf T_A(x_i) - \inf F_A(x_i)| + |\sup T_A(x_i) - \sup F_A(x_i)| + |\inf I_A(x_i) + \sup I_A(x_i) - 1|]$;

and $E(A^c) = 1 - \frac{1}{6n} \times 2 \times \sum_{i=1}^{n}[|\inf F_A(x_i) - \inf T_A(x_i)| + |\sup F_A(x_i) - \sup T_A(x_i)| + |1 - (\inf I_A(x_i) + \sup I_A(x_i))|]$. Therefore, $E(A) = E(A^c)$. □

Thus, Theorem 3 holds which indicates the measure proposed in Definition 15 is an entropy measure.

Example 3. Assume $A = \{< x, [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] >\}$, then $A^c = \{< x, [0.1, 0.2], [0.9, 1.0], [0.7, 0.8] >\}$, and $E(A) = 1 - \frac{1}{6} \times 2 \times [|0.7 - 0.1| + |0.8 - 0.2| + |0 + 0.1 - 1|] = 0.3$.

In the following paragraphs, based on the above entropy measure, an objective weight measure for an INS is proposed called the entropy weight measure.

The entropy can be regarded as a measure of the uncertainty degree involved in an FS, and it reflects the objective information contained in the decision values. Thus, utilizing the entropy as a vehicle to obtain the objective weight is a reasonable action. According to entropy theory [21, 78], if an FS provides less uncertainty than other ones, it should be paid more attention. Therefore, the bigger weight should be assigned to the less uncertain fuzzy information in MCDM problems, otherwise the fuzzy information will be considered unimportant, which means its weight will be smaller.

According to these theories, an entropy weight measure is established to determine the objective weight under an interval-valued neutrosophic environment:

$$H_j(A) = \frac{1 - E(A(x_j))}{n - \sum_{j=1}^{n} E(A(x_j))} \quad (11)$$

where A is an INS in the universe discourse $X = \{x_1, x_2, \cdots, x_n\}$, $A(x_j) = \{\langle [\inf T_A(x_i), \sup T_A(x_i)], [\inf I_A(x_i), \sup I_A(x_i)], [\inf F_A(x_i), \sup F_A(x_i)] \rangle\}$ and $E(A(x_j))$ is calculated by Eq. (10).

Property 2. The proposed weight measure satisfies the following properties:

($W1$) $H_j(A) \in [0,1]$; and

($W2$) $\sum_{j=1}^{n} H_j(A) = 1$.

Proof.

($W1$) Let $H = (H_1(A), H_2(A), \cdots, H_n(A))$ be an entropy weight vector calculated according to Equation (11). According to Theorem 3, it is known that the entropy value of INSs lies between 0 and 1, i.e., $E(A(x_j)) \in [0,1]$; thus, it's obvious that $1 - E(A(x_j)) \in [0,1]$ and $n - \sum_{j=1}^{n} E(A(x_j)) \in [0,1]$. Besides, $\left(1 - E(A(x_j))\right) + \left(n - 1 + \sum_{i=1, i\neq j}^{n} E(A(x_j))\right) = n - \sum_{j=1}^{n} E(A(x_j)) \geq 0$ and $\left(n - 1 + \sum_{i=1, i\neq j}^{n} E(A(x_j))\right) \geq 0$ hold, which means $n - \sum_{j=1}^{n} E(A(x_j)) \geq \left(1 - E(A(x_j))\right)$ is true. Based on these conclusions, it is possible to obtain $H_j(A) = \frac{1 - E(A(x_j))}{n - \sum_{j=1}^{n} E(A(x_j))} \in [0,1]$.

($W2$) Obviously, $\sum_{j=1}^{n} H_j(A) = \sum_{j=1}^{n} \frac{1 - E(A(x_j))}{n - \sum_{j=1}^{n} E(A(x_j))} = \frac{\sum_{j=1}^{n} 1 - E(A(x_j))}{n - \sum_{j=1}^{n} E(A(x_j))} = \frac{n - \sum_{j=1}^{n} E(A(x_j))}{n - \sum_{j=1}^{n} E(A(x_j))} = 1$.

Therefore, Property 2 holds. □

Example 4. Assume $A = \{< x_1, [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] >, < x_2, [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] >, < x_3, [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] >\}$. By using Eq. (10), it can be calculated that $E(A(x_1)) = 0.3$, $E(A(x_2)) = 0.767$ and $E(A(x_3)) = 0.467$. Moreover, according to Eq. (11), $H_1(A) = 0.477$, $H_1(B) = 0.159$ and $H_1(C) = 0.364$.

Example 5. Assume that there are three INSs $A = \{< x_1, [0.4, 0.5], [0.0, 0.1], [0.3, 0.4] >, < x_2, [0.6, 0.7], [0.4, 0.5], [0.1, 0.3] >\}$, $B = \{< x_1, [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] >, < x_2, [0.2, 0.4], [0.5, 0.6], [0.2, 0.4] >\}$ and $C = \{< x_1, [1,1], [0,0], [0,0] >, < x_2, [1,1], [0,0], [0,0] >\}$, and that $w = (0.5, 0.5)$ is the subjective weight vector. According to Eq. (9), the weighted correlation coefficient based on the subjective weight can be calculated: $K(A,C) = 0.55$ and $K(B,C) = 0.525$. Therefore, $K(A,C) > K(B,C)$ is true, which means that the relative similarity degree between A and C is more than that between B and C. Furthermore, by using Eq. (11), the objective weight matrix can be obtained: $H_A = (0.52, 0.48)$, and $H_B = (0.95, 0.05)$, and according to Eq. (4), the integrated weight matrix is $W = \begin{bmatrix} 0.52 & 0.48 \\ 0.95 & 0.05 \end{bmatrix}$. By using Eq. (9), the weighted correlation coefficient based on the subjective weight can be calculated: $K(A,C) = 0.546$ and $K(B,C) = 0.728$. Thus, $K(A,C) < K(B,C)$ is true, which means that the relative similarity degree between A and C is less than that between B and C.

The above example shows that the relative similarity degree may be different when using two different kinds of weight. The reason for this lies in the fact that the subjective weight only reflects the preference of decision maker and ignores the objective information included in the decision matrix; in contrast, the integrated weight can benefit from not only the decision makers' expertise but also the relative importance of evaluation information.

4. The Weighted Correlation Coefficient's Application to MCDM Problems

In this section, a model for MCDM problems that applies the weighted correlation coefficient measure for INSs and takes into account the integration of the objective and subjective weights is presented.

Assume there are *m* alternatives $A=\{A_1,\ A_2,\ \cdots,\ A_m\}$ and *n* criteria $C=\{C_1,\ C_2,\ \cdots,\ C_n\}$, whose subjective weight vector provided by the decision maker is $w=(w_1,w_2,\cdots,w_n)$, where $w_j\ge 0$ ($j=1,2,\cdots,n$), and $\sum_{j=1}^{n} w_j=1$. Let $R=(a_{ij})_{m\times n}$ be the interval neutrosophic decision matrix, where $a_{ij}=\langle T_{a_{ij}},I_{a_{ij}},F_{a_{ij}}\rangle$ is an evaluation value, denoted by INN, where $T_{a_{ij}}=[\inf T_{a_{ij}},\sup T_{a_{ij}}]$ indicates the truth-membership function that the alternative A_i satisfies the criterion C_j, $I_{a_{ij}}=[\inf I_{a_{ij}},\sup I_{a_{ij}}]$ indicates the indeterminacy-membership function that the alternative A_i satisfies the criterion C_j and $F_{a_{ij}}=[\inf F_{a_{ij}},\sup F_{a_{ij}}]$ indicates the falsity-membership function that the alternative A_i satisfies the criterion C_j.

In MCDM environments, the concept of an ideal point has been used to help identify the best alternative in the decision set [43]. An ideal alternative can be identified by using a maximum operator to determine the best value of each criterion among all alternatives [83]. Thus, an ideal INN in the ideal alternative A^* can be defined as:

$$\alpha_j^*=\left\langle [\mathrm{a}_j^*,b_j^*],[c_j^*,d_j^*],[e_j^*,f_j^*]\right\rangle=\left\langle [\max_i(\mathrm{a}_{ij}),\max_i(b_{ij})],\ [\min_i(\mathrm{a}_{ij}),\min_i(b_{ij})],[\min_i(\mathrm{a}_{ij}),\min_i(b_{ij})]\right\rangle, \tag{12}$$

where $i\in\{1,2,\cdots,m\}$ and $j=1,2,\cdots,n$.

Based on Eq. (10) and the integrated weight matrix $W=\begin{pmatrix} W_{11} & W_{12} & \cdots & W_{1n}\\ W_{21} & W_{22} & \cdots & W_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ W_{m1} & W_{m2} & \cdots & W_{mn}\end{pmatrix}$, where W_{ij} is the integrated weight of alternative A_i under criterion C_j, the weighted correlation coefficient measure between the alternative A_i and the ideal alternative A^* can be denoted as:

$$K(A_i,A^*)=\frac{\sum_{j=1}^{n}W_{ij}[\mathrm{a}_j^*\cdot(\inf T_{a_{ij}})+b_j^*\cdot(\sup T_{a_{ij}})+\mathrm{c}_j^*\cdot(\inf I_{a_{ij}})+d_j^*\cdot(\sup I_{a_{ij}})+e_j^*\cdot(\inf F_{a_{ij}})+f_j^*\cdot(\sup F_{a_{ij}})]}{\max\left(\sum_{j=1}^{n}W_{ij}T\left(A_i\left(x_j\right)\right),\sum_{j=1}^{n}W_{ij}[(\mathrm{a}_j^*)^2+(\mathrm{b}_j^*)^2+(c_j^*)^2+(d_j^*)^2+(e_j^*)^2+(f_j^*)^2]\right)} \tag{13}$$

where $T\left(A_i\left(x_j\right)\right)$ can be obtained based on Eq. (7).

The larger the value of the weighted correlation coefficient $K(A_i,A^*)$ is, the closer the alternative A_i is to the ideal alternative A^*. Therefore, all the alternatives can be ranked according to the value of the weighted correlation coefficients so that the best alternative can be selected. In the following paragraphs, a procedure that considers the integrated weight to rank and select the most desirable alternative(s) is proposed based upon the weighted correlation coefficient measure.

Step 1. Calculate the distance between the set $A_{ij}=\{a_{ij}\}$ formed by the rating value a_{ij} and its complementary set A_{ij}^c.

By using Eq. (2), the distance matrix

$$D=\begin{bmatrix} d_{nh}(A_{11},A_{11}^c) & d_{nh}(A_{12},A_{12}^c) & \cdots & d_{nh}(A_{1n},A_{1n}^c)\\ d_{nh}(A_{21},A_{21}^c) & d_{nh}(A_{22},A_{22}^c) & \cdots & d_{nh}(A_{2n},A_{2n}^c)\\ \vdots & \vdots & \ddots & \vdots\\ d_{nh}(A_{m1},A_{m1}^c) & d_{nh}(A_{m2},A_{m2}^c) & \cdots & d_{nh}(A_{mn},A_{mn}^c)\end{bmatrix}$$

can be obtained.

Step 2. Calculate the entropy value of the set $A_{ij}=\{a_{ij}\}$.

By using Eq. (10) and the distance matrix D, the entropy value matrix

$$E=\begin{bmatrix} E(A_{11}) & E(A_{12}) & \cdots & E(A_{1n})\\ E(A_{21}) & E(A_{22}) & \cdots & E(A_{2n})\\ \vdots & \vdots & \ddots & \vdots\\ E(A_{m1}) & E(A_{m2}) & \cdots & E(A_{mn})\end{bmatrix}=$$

$$\begin{bmatrix} 1-d_{nh}(A_{11},A_{11}^c) & 1-d_{nh}(A_{12},A_{12}^c) & \cdots & 1-d_{nh}(A_{1n},A_{1n}^c)\\ 1-d_{nh}(A_{21},A_{21}^c) & 1-d_{nh}(A_{22},A_{22}^c) & \cdots & 1-d_{nh}(A_{2n},A_{2n}^c)\\ \vdots & \vdots & \ddots & \vdots\\ 1-d_{nh}(A_{m1},A_{m1}^c) & 1-d_{nh}(A_{m2},A_{m2}^c) & \cdots & 1-d_{nh}(A_{mn},A_{mn}^c)\end{bmatrix}$$

can be calculated.

Step 3. Calculate the objective weight matrix H.

By using Eq. (11) and the entropy value matrix E, it's easy to calculate the objective weight matrix:

$$H=\begin{bmatrix} H(A_{11}) & H(A_{12}) & \cdots & H(A_{1n}) \\ H(A_{21}) & H(A_{22}) & \cdots & H(A_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ H(A_{m1}) & H(A_{m2}) & \cdots & H(A_{mn}) \end{bmatrix}=$$

$$\begin{bmatrix} \frac{1-E(A_{11})}{\sum_{j=1}^{n}(1-E(A_{1j}))} & \frac{1-E(A_{12})}{\sum_{j=1}^{n}(1-E(A_{1j}))} & \cdots & \frac{1-E(A_{1n})}{\sum_{j=1}^{n}(1-E(A_{1j}))} \\ \frac{1-E(A_{21})}{\sum_{j=1}^{n}(1-E(A_{2j}))} & \frac{1-E(A_{22})}{\sum_{j=1}^{n}(1-E(A_{2j}))} & \cdots & \frac{1-E(A_{2n})}{\sum_{j=1}^{n}(1-E(A_{2j}))} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1-E(A_{m1})}{\sum_{j=1}^{n}(1-E(A_{mj}))} & \frac{1-E(A_{m2})}{\sum_{j=1}^{n}(1-E(A_{mj}))} & \cdots & \frac{1-E(A_{mn})}{\sum_{j=1}^{n}(1-E(A_{mj}))} \end{bmatrix}$$

.

Step 4. Calculate the integrated weight matrix W.

By using Eq. (4), the subjective weight $w=(w_1,w_2,\cdots,w_n)$ provided by the decision maker and the objective weight can be integrated, and the integrated weight matrix is:

$$W=\begin{bmatrix} W(A_{11}) & W(A_{12}) & \cdots & W(A_{1n}) \\ W(A_{21}) & W(A_{22}) & \cdots & W(A_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ W(A_{m1}) & W(A_{m2}) & \cdots & W(A_{mn}) \end{bmatrix}=$$

$$\begin{bmatrix} \frac{w_1H_{11}}{\sum_{j=1}^{n}w_jH_{1j}} & \frac{w_2H_{12}}{\sum_{j=1}^{n}w_jH_{1j}} & \cdots & \frac{w_nH_{1n}}{\sum_{j=1}^{n}w_jH_{1j}} \\ \frac{w_1H_{21}}{\sum_{j=1}^{n}w_jH_{2j}} & \frac{w_2H_{22}}{\sum_{j=1}^{n}w_jH_{2j}} & \cdots & \frac{w_nH_{2n}}{\sum_{j=1}^{n}w_jH_{2j}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_1H_{m1}}{\sum_{j=1}^{n}w_jH_{mj}} & \frac{w_2H_{m2}}{\sum_{j=1}^{n}w_jH_{mj}} & \cdots & \frac{w_nH_{mn}}{\sum_{j=1}^{n}w_jH_{mj}} \end{bmatrix}.$$

Step 5. Calculate the ideal alternative A^*.

By using Eq. (12), the ideal alternative A^* can be calculated.

Step 6. Calculate the weighted correlation coefficient between the alternative A_i and the ideal alternative A^*.

By using Eq. (13) and the integrated weight matrix, the weighted correlation coefficient value between A_i and A^* can be obtained.

Step 7. Rank the alternatives depending on the weighted correlation coefficient value.

5. Illustrative example

5.1. *An example of the weighted correlation coefficient measure for MCDM problems with INSs*

In this section, an example of an MCDM problem of alternatives is used to demonstrate the applicability and effectiveness of the proposed decision-making method.

Example 6. The decision-making problem adapted from Ref. 33 is to be considered. There is a panel with four possible alternatives: A_1, A_2, A_3, A_4. The decision must be taken according to the following three criteria: C_1, C_2 and C_3. The weight vector of the criteria is given by $w=(0.35,0.25,0.4)$. The four possible alternatives are evaluated by a decision maker under the above three criteria. In order to reflect reality more accurately and obtain more uncertainty information, the evaluation values are transformed into INNs, as shown in the following interval neutrosophic decision matrix *D*:

$$D=\begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \end{bmatrix}.$$

Let the ideal alternative be $A^*=<[1,1],[0,0],[0,0]>$. The decision-making procedure based on INSs is as follows.

Step 1. Calculate the distance between the set $A_{ij}=\{a_{ij}\}$ formed by the rating value a_{ij} and its complementary set A_{ij}^c.

By using Eq. (2), the distance matrix is

$$D=\begin{bmatrix} 0.23 & 0.33 & 0.40 \\ 0.50 & 0.50 & 0.33 \\ 0.23 & 0.30 & 0.37 \\ 0.70 & 0.53 & 0.23 \end{bmatrix}.$$

Step 2. Calculate the entropy value of the set $A_{ij}=\{a_{ij}\}$.

By using Eq. (10) and the distance matrix D, the entropy value matrix is $E=\begin{bmatrix}0.77 & 0.67 & 0.60\\ 0.50 & 0.50 & 0.67\\ 0.77 & 0.70 & 0.63\\ 0.30 & 0.47 & 0.77\end{bmatrix}$.

Step 3. Calculate the objective weight matrix H.

By using Eq. (11) and the entropy value matrix E, it's easy to calculate the objective weight matrix $H=\begin{bmatrix}0.24 & 0.34 & 0.42\\ 0.376 & 0.376 & 0.248\\ 0.26 & 0.33 & 0.41\\ 0.48 & 0.36 & 0.16\end{bmatrix}$.

Step 4. Calculate the integrated weight matrix W.

By using Eq. (4), the subjective weight $w=(0.35,0.25,0.4)$ provided by the decision maker and the objective weight can be integrated and the integrated weight matrix is $W=\begin{bmatrix}0.25 & 0.25 & 0.50\\ 0.405 & 0.29 & 0.305\\ 0.27 & 0.24 & 0.49\\ 0.52 & 0.28 & 0.20\end{bmatrix}$.

Step 5. Calculate the ideal alternative A^*.

By using Eq. (12), the following ideal alternative can be obtained: $A^*=\{\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle,$ $\langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle,\langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle\}$.

Step 6. Calculate the weighted correlation coefficient between the alternative A_i and the ideal alternative A^*. By using Eq. (12) and the integrated weight matrix, the weighted correlation coefficient value between A_i and A^* can be obtained, and $K(A_1,A^*)=0.9148$, $K(A_2,A^*)=0.899$, $K(A_3,A^*)=0.8517$, and $K(A_4,A^*)=0.9219$.

Step 7. Rank the alternatives depending on the weighted correlation coefficient value.

Based on the steps above, the final order $A_4 \succ A_1 \succ A_2 \succ A_3$ is obtained. Clearly, A_4 is the best alternative in this example.

5.2. *Comparison analysis and discussion*

In order to validate the feasibility of the proposed method, a comparative study with other methods was conducted, which includes two cases. In the first case, the proposed method is compared to the methods that were outlined in Refs. 33 and 35 using interval value neutrosophic information. In the second one, it is compared to the methods using single valued neutrosophic information introduced in Refs. 21, 85 and 51.

Case 1. The proposed method is compared with some methods that use interval neutrosophic information.

With regard to the method in Ref. 35, the similarity measures were firstly calculated and used to determine the final ranking order of all the alternatives, and then two aggregation operators were developed in order to aggregate the interval neutrosophic information [33]. The results from the different methods used to resolve the MCDM problem in Example 6 are shown in Table 1.

Table 1. The results of different methods using INSs.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 1 [35]	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Method 2 [35]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 3 [33]	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3
Method 4 [33]	$A_1 \succ A_4 \succ A_2 \succ A_3$	A_1	A_3
The proposed method	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3

From the results presented in Table 1, the best alternatives in Ref. 35 are A_4 and A_2 respectively, whilst the worst one is A_1. In contrast, by using the methods in Ref. 33, the best ones are A_4 and A_1 respectively, whilst the worst one is A_3. With regard to the proposed method in this paper, the best one is A_4, whilst the worst one is A_3. There are a number of reasons why differences exist between the final rankings of all the compared methods and the proposed method. Firstly, these different measures and aggregation operators also lead to different rankings, and it is very difficult for decision makers to confirm their judgments when using operators and measures that have similar characteristics. Secondly, the proposed method in this paper pays more attention to the impact that uncertainty has on the decision and also takes into consideration the integrated weight. Moreover, different aggregation operators lead to different rankings because the operators emphasize the decision makers' judgments differently. Method 3 in Ref. 33 uses the interval neutrosophic number weighted averaging (INNWA) operator, whilst method 4 in Ref. 33 utilizes the interval neutrosophic number weighted geometric (INNWG) operator. The INNWA operator is based on an arithmetic average and emphasizes the group's major points, while the INNWG operator emphasizes personal major points. That is the reason why results emanating from method 3 and method 4 in

Ref. 33 are different. By comparison, the proposed method in this paper focuses on the weighted correlation coefficient measure, which takes both the subjective and objective weights into consideration. Notwithstanding, the ranking of the proposed method is the same as that of the INNWA operator, which emphasizes the group's major points. Therefore, the proposed method is effective.

Case 2. The proposed method is compared with some methods that use simplified neutrosophic information. The comparison results are listed in Table 2.

Table 2. The results of different methods using SVNSs.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 5 [85]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 6 [21]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 7 [51]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
The proposed method	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1

From the results presented in Table 2, the worst alternatives of Refs. 85, 21, 51 and the proposed method are same, i.e., A_1. The best alternatives of Refs. 85, 21 and 51 are also the same one, i.e., A_2, but the best one of the proposed method is A_4. The reason why differences exist in the final rankings of the three compared methods and the proposed method is now provided. As mentioned in Case 1, the proposed weighted correlation coefficient method not only considers the subjective weight, which reflects the decision maker's subjective preference, but also refers to the objective weight, which mirrors the objective information in the decision matrix. This shows that the proposed method can also be used for MCDM problems with single valued neutrosophic information.

From the comparison analysis presented above, it can be concluded that the proposed method is more flexible and reliable in managing MCDM problems than the compared methods in an interval neutrosophic environment, which means that the method developed in this paper has certain advantages. Firstly, it can also be used to solve problems with preference information that is expressed by INSs as well as SVNSs. Secondly, it unearths the deeper information that is uncertain and utilizes it to make a precise decision. Furthermore, it is also capable of managing MCDM problems with a completely unknown criteria weight.

6. Conclusion

An NS has been applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information that exist in actual scientific and engineering applications. Moreover, the correlation coefficient measure is important in NS theory and the entropy measure captures the uncertainty of NSs. In this paper, a new correlation coefficient measure for INSs that satisfies the condition that the value equals one if and only if two INSs are the same was proposed, which was motivated by the correlation coefficient of IFSs. Additionally, the weighted correlation coefficient measure was extended and its property was developed. Furthermore, the entropy measure of INSs was defined based on the relationship between distance and the entropy. In order to obtain the integrated weight, an objective weight measure that utilizes the entropy for INSs was also discussed and the decision-making procedure for MCDM problems was established. Finally, an illustrative example demonstrated the applicability of the proposed decision-making method and a comparative analysis showed that the proposed methods were appropriate and effective for dealing with MCDM problems.

This study makes several contributions. Firstly, the method proposed is simple and convenient to compute and contributes to decreasing the loss of evaluation information. The feasibility and validity of the proposed method have been verified through the illustrative example and comparison analysis. Therefore, this method has a great deal of potential for dealing with issues regarding interval neutrosophic information in a number of environments, including cluster analysis and artificial intelligence. Secondly, the new correlation coefficient measure overcomes the shortcoming that the equivalent measure in Ref 50 does not satisfy the conditions that the value equals one if and only if two INSs are the same. In addition, this paper elaborates and demonstrates the viewpoint that the uncertainty of evaluation is related to its importance, and combining the subjective and objective weights can avoid the non-determinacy and arbitrariness that results from subjective opinions. Subsequently, based on these viewpoints, the paper makes further use of uncertainty information and proposes a weighted correlation coefficient decision-making method that takes both the subjective and objective weights into account, which can be helpful in making better decisions.

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