

# AN INTRODUCTION TO THE NEUTROSOPHIC FUZZY IN QUEUE

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**Abstract**—In this paper one generalizes the classical fuzzy and imprecise fuzzy to the notion of “neutrosophic fuzzy” in order to be able to queue model. In this paper, we focus on developing an neutrosophic probability which is a new problem solving queue operation in the real standard domain by implementing a novel term “intuitionistic fuzzy neutrosophic logic.” In order to attain this, we define a with operation standard and non-standard real subsets of queue output is obtained, based on the fuzzy queue. Uncertainty Principle of a queue behavior, and derived supremum shape function and infimum shape function, total shape function in parametric non-linear programme state of Zadeh’s Exclusion Principle (in fuzzy). Neutrosophic fuzzy is close related to neutrosophic logic and neutrosophic set, and etymologically derived from “neutrosophy” [24, 25].

**Index Terms**—Imprecise fuzzy probability, neutrosophic probability, neutrosophic logic, neutrosophic set, non-standard interval queue, Uncertainty Principle, Zadeh’s Exclusion Principle.

## I. INTRODUCTION

One consequence of the Uncertainty Principle says that it is impossible to fully predict the behavior of a queue, also the queue principle cannot apply at the actual level. For example the bulk queue, queue state of an arrival can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an arrival (queue state) belongs and does not belong to a set (a place) in the same time; or an arrival (queue state) belongs to two different sets (two different places) in the same time. It is a question of “alternative worlds” theory very well represented by the neutrosophic set theory. In parametric Equation on the behavior of queue fuzzy and “crisp queue” in Queue theory, the queue function  $Z$  which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the

domain of definition (the left and right waiting line test fails, intersecting the graph in more lines). How to describe a queue shape  $f$  in the infinite system that belongs to two distinct places  $P_1$  and  $P_2$  in the same time?  $f \in P_1$  and  $f \notin P_1$  as a true contradiction, or how to describe two distinct bulk queue  $b_1$  and  $b_2$ , they belong to the same queue or queue state in the same time? Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?

In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines. How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint? We better describe them, using the attribute “neutrosophic” than “fuzzy” or any other, Queue arrival that neither exists nor non-exists.

## II. DEFINITION: NEUTROSOPHIC PROBABILITY QUEUE

Is a generalization queue of the classical probability in which the chance that an event  $A$  occurs is  $t\%$  true - where  $t$  varies in the subset  $T$ ,  $i\%$  indeterminate - where  $i$  varies in the subset  $I$ , and  $f\%$  false - where  $f$  varies in the subset  $F$ . One notes  $NP(q) = (T, I, F)$ . It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

The membership function of the performance measure of Neutrosophic Probability queue  $P(\tilde{A}, \tilde{S})$  that is,

$$\delta_{P(q)}(Z) = \sup_{a \in X, s \in X} \{ \min\{a, \mu_{\tilde{A}}(a), \mu_{\tilde{S}}(a)\} / z = P(q) \}$$

for every  $a \in X, s \in X$  and  $X$  in real line  $a \in [0, 1]$

The non-membership function of the performance measure of Neutrosophic Probability queue  $P(q)$ , that

$$\delta_{P(q)}(Z) = \inf_{a \in X, s \in X} \{ \max\{ a, \nu_\lambda(a), \nu_\mu(a) \} / z = P(q) \}$$

for every  $a \in X, s \in X$  and  $X$  in real line  $a \in [0, 1]$ .

The membership function of the performance measure of Neutrosophic Probability queue  $P(q)$  that is,

$$\delta_{P(q)}(Z) = \sup_{a \in X, s \in X} \{ \min\{ s, \mu_{\bar{A}}(s), \mu_{\bar{S}}(s) \} / z = P(q) \}$$

for every  $a \in X, s \in X$  and  $X$  in real line  $a \in [0, 1]$

The non-membership function of the performance measure of Neutrosophic Probability queue  $P(q)$ , that is

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for every  $a \in X, s \in X$  and  $X$  in real line  $a \in [0, 1]$ . Then the right shape function of a triangular fuzzy number  $P(q)$ .

### III. NON-STANDARD REAL QUEUE AND NON-STANDARD REAL SETS

Let  $T, I, F$  be standard or non-standard real subsets  $\in K$   $-0, 1+ M,$

with  $\sup T = t_{\sup}, \inf T = t_{\inf},$

$\sup I = i_{\sup}, \inf I = i_{\inf},$

$\sup F = f_{\sup}, \inf F = f_{\inf},$

and  $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup},$

$n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}.$

Obviously:  $t_{\sup}, i_{\sup}, f_{\sup} [ 1+,$  and  $t_{\inf}, i_{\inf}, f_{\inf} m -0,$

whereas  $n_{\sup} \leq 3+$  and  $n_{\inf} \geq -0.$

The subsets  $T, I, F$  are not necessarily intervals, but may be any real subsets of queue: discrete or continuous; single-element, finite, or (either countably or uncountably) infinite; union or intersection of various subsets queue; etc. They may also overlap queue. These real subsets of queue could represent the relative errors in determining  $t, i, f$  (in the case when the subsets  $T, I, F$  are reduced to points).

This representation is closer to the human mind reasoning. It characterizes/catches the *imprecision* of knowledge or linguistic inexactitude received by various observers (that's why  $T, I, F$  are subsets - not necessarily single-elements), *uncertainty* due to incomplete knowledge or acquisition errors or stochasticity (that's why the subset  $I$  exists), and *vagueness* due to lack of clear contours or limits (that's why  $T, I, F$  are subsets and  $I$  exists; in particular for the appurtenance to the neutrosophic queue sets). One

has to specify the superior ( $x_{\sup}$ ) and inferior ( $x_{\inf}$ ) limits of the queue subsets because in many problems arises the necessity to compute them.

The real number  $x$  is said to be infinitesimal if and only if for all positive integers  $n$  one has  $|x| < 1/n$ . Let  $n > 0$  be a such infinitesimal number. The *hyper-real number set* is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers  $1+ = 1+n$ , where "1" is its standard part and "n" its non-standard part, and  $-0 = 0-n$ , where "0" is its standard part and "n" its nonstandard part.

Then, we call  $n -0, 1+ n$  non-standard unit interval. Obviously,  $0$  and  $1$ , and analogously non-standard numbers infinitely small but less than  $0$  or infinitely small but greater than  $1$ , belong to the non-standard unit interval. Actually, by "a" one signifies a monad, i.e. a set of hyper-real numbers in non-standard analysis:

$\alpha(-a) = \{a-x: \mu x. *, x \text{ is infinitesimal}\}$ , and similarly "b+" is a monad:

$\alpha(b+) = \{b+x: \mu x. *, x \text{ is infinitesimal}\}.$

Generally, the left and right borders of a non-standard interval  $\alpha -a, b+ \alpha$  are vague, imprecise, themselves being non-standard (sub)sets  $.(-a)$  and  $.(b+)$  as defined above.

Combining the two before mentioned definitions one gets, what we would call, a binad of

"-c+":

$\alpha(-c+) = \{c-x: x\%. *, x \text{ is infinitesimal}\} \cup \{c+x: x\%. *, x \text{ is infinitesimal}\}$ , which is a collection of open punctured neighborhoods (balls) of  $c$ .

Of course,  $-a < a$  and  $b+ > b$ . No order between  $-c+$  and  $c$ .

Addition of non-standard finite numbers with themselves or with real numbers:

$-a + b = -(a + b)$

$a + b+ = (a + b)+$

$-a + b+ = -(a + b)+$

$-a + -b = -(a + b)$  (the left monads absorb themselves)

$a+ + b+ = (a + b)+$  (analogously, the right monads absorb themselves)

Similarly for subtraction, multiplication, division, roots, and powers of non-standard finite numbers with themselves or with real numbers.

By extension let  $\inf \alpha -a, b+ \alpha = -a$  and  $\sup \alpha -a, b+ \alpha = b+$ .

#### IV. OPERATIONS WITH STANDARD AND NON-STANDARD REAL SUBSETS OF QUEUE

Let  $S1$  and  $S2$  be two (unidimensional) standard or non-standard real subsets, then one defines:

Addition of sets:

$S1+S2 = \{x/x=s1+s2, \text{ where } s1 \in S1 \text{ and } s2 \in S2\}$ ,  
with  $\inf S1+S2 = \inf S1 + \inf S2, \sup S1+S2 = \sup S1 + \sup S2$ ;

and, as some particular cases, we have

$\{a\}+S2 = \{x/x=a+s2, \text{ where } s2 \in S2\}$   
with  $\inf \{a\}+S2 = a + \inf S2, \sup \{a\}+S2 = a + \sup S2$ ;  
also  $\{1\}+S2 = \{x/x=1+s2, \text{ where } s2 \in S2\}$   
with  $\inf \{1\}+S2 = 1 + \inf S2, \sup \{1\}+S2 = 1 + \sup S2$ .

Subtraction of sets:

$S1-S2 = \{x/x=s1-s2, \text{ where } s1 \in S1 \text{ and } s2 \in S2\}$ .  
For real positive subsets (most of the cases will fall in this range) one gets  
 $\inf S1-S2 = \inf S1 - \sup S2, \sup S1-S2 = \sup S1 - \inf S2$ ;

and, as some particular cases, we have

$\{a\}-S2 = \{x/x=a-s2, \text{ where } s2 \in S2\}$ ,  
with  $\inf \{a\}-S2 = a - \sup S2, \sup \{a\}-S2 = a - \inf S2$ ;  
also  $\{1\}-S2 = \{x/x=1-s2, \text{ where } s2 \in S2\}$ ,  
with  $\inf \{1\}-S2 = 1 - \sup S2, \sup \{1\}-S2 = 100 - \inf S2$ .

Multiplication of sets:

$S1.S2 = \{x/x=s1.s2, \text{ where } s1 \in S1 \text{ and } s2 \in S2\}$ .  
For real positive subsets (most of the cases will fall in this range) one gets  
 $\inf S1.S2 = \inf S1 . \inf S2, \sup S1.S2 = \sup S1 . \sup S2$ ;  
and, as some particular cases, we have  
 $\{a\}.S2 = \{x/x=a.s2, \text{ where } s2 \in S2\}$ ,  
with  $\inf \{a\}.S2 = a * \inf S2, \sup \{a\}.S2 = a . \sup S2$ ;  
also  $\{1\}.S2 = \{x/x=1.s2, \text{ where } s2 \in S2\}$ ,  
with  $\inf \{1\}.S2 = 1 . \inf S2, \sup \{1\}.S2 = 1 . \sup S2$ .

Division of a set by a number:

Let  $k \in \mathbb{R}^*$ , then  $S1 \div k = \{x/x=s1/k, \text{ where } s1 \in S1\}$ ,  
Let  $(T1, I1, F1)$  and  $(T2, I2, F2)$  be standard or non-standard triplets of real subsets which  
 $\in P(K -0, 1+ M)^3$ , where  $P(\alpha -0, 1+ \alpha)$  is the set of all

subsets of non-standard unit interval

$\alpha -0, 1+ \alpha$ , then we define:

$(T1, I1, F1) + (T2, I2, F2) = (T1+T2, I1+I2, F1+F2)$ ,  
 $(T1, I1, F1) - (T2, I2, F2) = (T1-T2, I1-I2, F1-F2)$ ,  
 $(T1, I1, F1) . (T2, I2, F2) = (T1.T2, I1.I2, F1 . F2)$ .

#### V. NEUTROSOPHIC STATISTICS

Is the analysis of the events described by the neutrosophic probability. This is also a generalization of the classical statistics and imprecise statistics.

#### VI. NEUTROSOPHIC PROBABILITY SPACE

The universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

Let  $A$  and  $B$  be two neutrosophic events, and  $NP(A) = (T1, I1, F1), NP(B) = (T2, I2, F2)$  their neutrosophic probabilities. Then we define:

$NP(A \cap B) = NP(A) . NP(B)$ .

$NP(-A) = \{1\} - NP(A)$ .

$NP(A \cup B) = NP(A) + NP(B) - NP(A) . NP(B)$ .

1.  $NP(\text{impossible event}) = (T_{\text{imp}}, I_{\text{imp}}, F_{\text{imp}})$ ,  
where  $\sup T_{\text{imp}} \leq 0, \inf F_{\text{imp}} \geq 1$ ; no restriction on  $I_{\text{imp}}$ .

$NP(\text{sure event}) = (T_{\text{sur}}, I_{\text{sur}}, F_{\text{sur}})$ ,  
where  $\inf T_{\text{sur}} \geq 1, \sup F_{\text{sur}} \leq 0$ ; no restriction on  $I_{\text{sur}}$ .

$NP(\text{totally indeterminate event}) = (T_{\text{ind}}, I_{\text{ind}}, F_{\text{ind}})$ ;  
where  $\inf I_{\text{ind}} \geq 1$ ; no restrictions on  $T_{\text{ind}}$  or  $F_{\text{ind}}$ .

2.  $NP(A) \in \{(T, I, F), \text{ where } T, I, F \text{ are real subsets which may overlap}\}$ .

3.  $NP(A \cup B) = NP(A) + NP(B) - NP(A \cap B)$ .

4.  $NP(A) = \{1\} - NP(-A)$ .

#### VII. APPLICATIONS

#1. From a pool of refugees, waiting in a political refugee camp in Tamil Nadu to get the Indian visa,  $a\%$  have the chance to be accepted - where  $a$  varies in the set  $A$ ,  $r\%$  to be rejected - where  $r$  varies in the set  $R$ , and  $p\%$  to be in pending (not yet decided) - where  $p$  varies in  $P$ .

Say, for example, that the chance of someone Popescu in the pool to emigrate to India is (between)20-80% (considering different criteria of emigration one gets different percentages, we have to take care of all of them), the chance of being rejected is 10-15% or20-25%, and the chance of being in pending is 5% or 10%

or 15% . Then the neutrosophic probability that pescu emigrates to the India is  $NP(\text{Popescu}) = ( (20-80), (10-15)U(20-25), \{5,10,15\} )$ , closer to the life's thinking. This is a better approach than the classical probability, where  $20 \leq P(\text{Popescu}) \leq 80$ , because from the pending chance - which will be converted to acceptance or rejection - Popescu might get extra percentage in his will to emigration, and also the superior limit of the subsets sum  $80+20+25 > 100$  and in other cases one may have the inferior sum  $\geq 0$ , while in the classical fuzzy set theory the superior sum should be 100 and the inferior sum  $< 0$ .

#1. In a similar way, we could say about the element Popescu that Popescu  $( (20-80), (10-15)U(20-25), \{5,10,15\} )$  belongs to the set of accepted refugees.

#2. The probability that candidate C will win an election is say 15-20% true (percent of people voting for him), 25% false (percent of people voting against him), and 20% or 21% indeterminate (percent of people not coming to the ballot box, or giving a blank vote – not selecting anyone, or giving a negative vote - cutting all candidates on the list). Dialectic and dualism don't work in this case anymore.

#3. Another example, the probability that tomorrow it will rain is say 40-44% true according to meteorologists who have investigated the past years' weather, 20 or 24-25% false according to today's very sunny and droughty summer, and 5 or 10% undecided (indeterminate).

#4. The probability that Yankees will win tomorrow versus Cowboys is 80% true (according to their confrontation's history giving Yankees' satisfaction), 40-42% false (supposing Cowboys are actually up to the mark, while Yankees are declining), and 5 or 6 or 7% indeterminate (left to the hazard: sickness of players, referee's mistakes, atmospheric conditions during the game). These parameters act on players' psychology.

#### VIII. REMARKS

Neutrosophic probability queue is useful to those arrivals events which involve some degree of indeterminacy (unknown) and more criteria of evaluation – as queue . This kind of probability is necessary because it provides a better representation

than classical probability to uncertain events.

#### IX. GENERALIZATIONS OF OTHER PROBABILITIES

In the case when the truth- and falsity-components are complementary, i.e. no indeterminacy and their sum is 1, one falls to the classical probability. As, for example, tossing dice or coins, or drawing cards from a well-shuffled deck, or drawing balls from an urn. An interesting particular case is for  $n=1$ , with  $0 \leq t, i, f \leq 1$ , which is closer to the classical probability.

For  $n=1$  and  $i=0$ , with  $0 \leq t, f \leq 1$ , one obtains the classical probability.

From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxism, pseudoparadoxism, and tautologism we transfer the "adjectives" to probabilities, i.e. we define the intuitionistic probability (when the probability space is incomplete), paraconsistent probability, faillibilist probability, dialetheist probability, paradoxist probability, pseudoparadoxist probability, and tautologic probability respectively.

Hence, the neutrosophic probability generalizes: The intuitionistic probability, which supports incomplete (not completely known/determined) probability spaces (for  $0 < n < 1$  and  $i=0, 0 \leq t, f \leq 1$ ) or incomplete events whose probability we need to calculate;

- The classical probability (for  $n=1$  and  $i=0$ , and  $0 \leq t, f \leq 1$ );
- The paraconsistent probability (for  $n > 1$  and  $i=0$ , with both  $t, f < 1$ );
- The dialetheist probability, which says that intersection of some disjoint probability spaces not empty (for  $t=f=1$  and  $i=0$ ; some paradoxist probabilities can be denoted this way);
- The faillibilist probability (for  $i > 0$ );
- The pseudoparadoxism (for  $n_{sup} > 1$  or  $n_{inf} < 0$ );
- The tautologism (for  $t_{sup} > 1$ ).

Compared with all other types of classical probabilities, the neutrosophic probability introduces a percentage of "indeterminacy" - due to unexpected parameters hidden in some probability spaces, and let each component  $t, i, f$  be even boiling *over* 1 to 1+ (overflowed) or freezing *under* 0 (underdried) to -0. For example: an element in some tautological probability space may have  $t > 1$ , called "overprobable" (i.e.  $t = 1+$ ). Similarly, an element in some paradoxist probability space may be "overindeterminate" (for

$i > 1$ ), or "overunprobable" (for  $f > 1$ , in some unconditionally false appurtenances); or "underprobable" (for  $t < 0$ , i.e.  $t = -0$ , in some unconditionally false appurtenances), "underindeterminate" (for  $i < 0$ , in some unconditionally true or false appurtenances), "underunprobable" (for  $f < 0$ , in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true ( $t > 1$ , and  $f < 0$  or  $i < 0$ ) and conditionally true appurtenances ( $t \leq 1$ , and  $f \leq 1$  or  $i \leq 1$ ).

#### X. OTHER EXAMPLES

Let's consider a neutrosophic set a collection of possible locations (positions) of particle x.

And let A and B be two neutrosophic sets.

One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between -0 and 1+.

For example:  $x(0.7, 0.1, 0.2)$  belongs to A (which means, with a probability of 70% particle x is in a position of A, with a probability of 20% x is not in A, and the rest is undecidable); or  $y(0, 0, 1)$  belongs to A (which normally means y is not for sure in A); or  $z(0, 1, 0)$  belongs to A (which means one does know absolutely nothing about z's affiliation with A).

More general,  $x(0.1-0.2), (0.30-0.35) \cup [0.70-0.71], \{0.1, 0.14, 0.18\}$  belongs to the set A, which means:

- With a probability in between 10-20% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- With a probability of 10% or 14% or 18% x is not in A;
- The indeterminacy related to the appurtenance of x to A is in between 30-35% or between 70-71% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and  $n_{sup} = 20\% + 71\% + 18\% > 100\%$  in this case.

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