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An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets

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ABSTRACT

In this paper, a new outranking approach for multi-criteria decision-making (MCDM) problems is developed in the context of a simplified neutrosophic environment, where the truth-membership degree, indeterminacy-membership degree and falsity-membership degree for each element are singleton subsets in [0,1]. Firstly, the novel operations of simplified neutrosophic sets (SNSs) and relational properties are developed. Then some outranking relations for simplified neutrosophic number (SNNs) are defined, based on ELECTRE, and the properties within the outranking relations are further discussed in detail. Additionally, based on the outranking relations of SNNs, a ranking approach is developed in order to solve MCDM problems. Finally, two practical examples are provided to illustrate the practicality and effectiveness of the proposed approach. Moreover, a comparison analysis based on the same example is also conducted.

1. Introduction

In many cases, it is difficult for decision-makers to precisely express a preference when solving MCDM problems with inaccurate, uncertain or incomplete information. Under these circumstances, Zadeh's fuzzy sets (FSs) [1], where the membership degree is represented by a real number between zero and one, are regarded as an important tool for solving MCDM problems [2,3], fuzzy logic and approximate reasoning [4], and pattern recognition [5].

However, FSs cannot handle certain cases where it is hard to define the membership degree using one specific value. In order to overcome the lack of knowledge of non-membership degrees, Atanassov [6] introduced intuitionistic fuzzy sets (IFSs), an extension of Zadeh's FSs. Furthermore, Gau and Buehrer [7] defined vague sets and subsequently, Bustince [8] pointed out those vague sets and IFSs are mathematically equivalent objects. To date, IFSs have been widely applied in solving MCDM problems [9–16], neural networks [17,18], medical diagnosis [19], color region extraction [20,21], and market prediction [22]. IFSs simultaneously take into account the membership degree, non-membership degree and hesitation degree. Therefore, they are more flexible and practical when addressing fuzziness and uncertainty than traditional FSs. Moreover, in some actual cases, the membership degree, non-membership degree and hesitation degree of an element in IFSs may not be a specific number; hence, they were extended to interval-valued intuitionistic fuzzy sets (IVIFSs) [23]. In order to handle situations where people are hesitant in expressing their preference regarding objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra and Narukawa [24,25]. Furthermore, Zhu et al. [26,27] proposed dual HFSs and outlined their operations and properties. Chen et al. [28] proposed interval-valued hesitant fuzzy sets (IVHFSs) and applied them to MCDM problems. Farhadinia [29] discussed the correlation for dual IVHFSs and Peng et al. [30] introduced a MCDM approach with hesitant interval-valued intuitionistic fuzzy sets (HIVIFSs), which is an extension of dual IVHFSs.

Although the theory of FSs has been developed and generalized, it cannot deal with all types of uncertainty in different real-world problems. Types of uncertainty, such as indeterminate and inconsistent information, cannot be managed. For example, when an expert is asked for their opinion about a certain statement, he or she may say the possibility that the statement is true is 0.5, the possibility that the

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statement is false is 0.6 and the degree that he or she is not sure is 0.2 [31]. This issue is beyond the scope of FSs and IFSs and therefore some new theories are required.

Smarandache [32,33] proposed neutrosophic logic and neutrosophic sets (NSs) and subsequently Rivieccio [34] pointed out that a NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity and it lies within $]0^-, 1^+[$, the non-standard unit interval. Clearly this is the extension of the standard interval [0, 1]. Furthermore, the uncertainty presented here, i.e. the indeterminacy factor, is dependent on the truth and falsity values, whereas the incorporated uncertainty is dependent on the degrees of belongingness and non-belongingness of IFSs [35]. Additionally, the aforementioned example of NSs can be expressed as $x(0.5, 0.2, 0.6)$. However, without a specific description, NSs are difficult to apply to real-life situations. Therefore, single-valued neutrosophic sets (SVNSs) were proposed, which are an extension of NSs [31,35]. Majumdar et al. [35] introduced a measure of entropy of SVNSs. Furthermore, the correlation coefficients of SVNSs as well as a decision-making method using SVNSs were introduced [36]. In addition, Ye [37] also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval [0,1], and proposed an MCDM method using the aggregation operators of SNSs. Wang et al. [38] and Lupiáñez [39] proposed the concept of interval neutrosophic sets (INSs) and provided the set-theoretic operators of INSs. Broumi and Smarandache [40] discussed the correlation coefficient of INSs and Zhang et al. [41] developed a MCDM method based on aggregation operators within an interval neutrosophic environment. Furthermore, Ye [42,43] proposed the similarity measures between SVNSs and INSs, which were based on the relationship between similarity measures and distances. However, in certain cases, the operations of SNSs provided by Ye [37] may be impractical. For example, the sum of any element and the maximum value should be equal to the maximum value, but this is not always the case during operations [37]. The similarity measures and distances of SVNSs that are based on those operations may also be unrealistic. Peng et al. [44] defined the novel operations and aggregation operators, which were based on the operations in Ye [37], and applied them to MCDM problems.

Furthermore, the aforementioned methods using SNSs always involve operations and measures whose influence on the final solution may be significant. However, there is another method that can overcome these shortcomings, namely the relation model. Relation models utilize outranking relations or priority functions for ranking the alternatives in terms of their priority according to the criteria. Recently relation models have been widely acknowledged as they are more related to the actual decision-making process. The ELECTRE methods, which were originally developed by Benayoun and Roy [45,46] are typical of this field. Subsequently, ELECTRE I, II, III, IV, IS, and TRI [45–48] were developed, which are extensions of ELECTRE. To date, the ELECTRE methods have been successfully and widely used in various fields, including biological engineering [49,50], energy sources [51,52], environmental studies [53,54], economics [55], value engineering [56], medical science [57], communication and transportation [58], and location selection problems [59,60]. For example, Devi and Yadav [59] developed an MCDM method based on ELECTRE in order to solve plant location problems where the evaluation values are IFSs. Moreover, Hatami-Marbini and Tavana [61] proposed an extension of the ELECTRE I method to solve group decision-making problems within a fuzzy environment. Vahdani et al. [62] proposed a novel ELECTRE method to solve MCDM problems with interval weights and data. Vahdani and Hadipour [63] developed a novel ELECTRE method to solve MCDM problems with interval-valued fuzzy information. Yang et al. [64] proposed an outranking method for MCDM problems with duplex linguistic information. Yang and Wang [65] developed a decision aiding technique based on incomplete preference information and applied it to solve MCDM problems. Wang et al. [66,67] proposed outranking approaches to solve MCDM problems with HFSs and hesitant fuzzy linguistic term sets. Chen [68] developed an ELECTRE outranking method to solve multiple criteria group decision-making problems with interval type-2 fuzzy sets.

In this paper, the novel operations of SNSs are developed, some outranking relations of simplified neutrosophic numbers (SNNs) that are based on ELECTRE are proposed, and the rational properties of SNNs are discussed. Furthermore, an outranking approach for MCDM problems with SNNs is developed, which could overcome the drawbacks that were discussed earlier in the paper.

The paper is structured as follows. In Section 2, the definition and novel operations of SNSs are provided. In Section 3, some outranking relations of SNNs that are based on ELECTRE are defined and some properties are discussed. In Section 4, an outranking approach for MCDM problems with SNNs is developed. Two worked examples appear in Section 5 and conclusions are drawn in Section 6.

2. Preliminaries

2.1. NSs and SNSs

In this section, the definitions of NSs and SNSs are introduced in order to assist with the latter analysis.

Definition 1 *([32])*. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, therefore $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2 *([32])*. An NS A is contained in another NS B, denoted by $A \subseteq B$, if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$ and $\sup F_A(x) \geq \sup F_B(x)$ for any $x \in X$.

Since it is difficult to apply NSs to practical problems, Ye [37] reduced NSs of nonstandard intervals into the SNSs of standard intervals, which preserves the operations of NSs.

Definition 3 *([37])*. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0,1], that is $T_A(x) : X \to [0, 1]$, $I_A(x) : X \to [0, 1]$, and $F_A(x) : X \to [0, 1]$. Then, a simplification of A is denoted by:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\} \tag{1}$$

which is called an SNS and is a subclass of NSs. The set of all SNSs in X is represented as $SNSS(X)$. If, in particular, X has only one element, A is called a simplified neutrosophic number (SNN), which can be denoted by $A = < T_A, I_A, F_A >$. The set of all SNNs is represented as SNNS.

The operations of SNSs are also defined by Ye [37].

Definition 4 *([37])*. Let A and B be two SNSs, the following operations can be true.

(1) $A+B=\{<x, T_A(x)+T_B(x)-T_A(x)\cdot T_B(x), I_A(x)+I_B(x)-I_A(x)\cdot I_B(x), F_A(x)+F_B(x)-F_A(x)\cdot F_B(x)>|x\in X\}$;
(2) $A\cdot B=\{<x, T_A(x)\cdot T_B(x), I_A(x)\cdot I_B(x), F_A(x)\cdot F_B(x)>|x\in X\}$;
(3) $\lambda\cdot A=\{<x, 1-(1-T_A(x))^\lambda, 1-(1-I_A(x))^\lambda, 1-(1-F_A(x))^\lambda>|x\in X\}, \quad \lambda>0$;
(4) $A^\lambda=\{<x, T_A^\lambda(x), I_A^\lambda(x), F_A^\lambda(x)>|x\in X\}, \quad \lambda>0$.

Definition 4 has certain limitations and these are now outlined.

(1) In some situations, operations such as $A+B$ and $A\cdot B$ might be impractical. This is demonstrated in Example 1.

Example 1. Let $A=\{<x, 0.5, 0.5, 0.5>\}$ and $B=\{<x, 1, 0, 0>\}$ be two SNSs. Clearly, $B=\{<x, 1, 0, 0>\}$ is the larger of these SNSs. Theoretically, the sum of any number and the maximum number should be equal to the maximum value. However, according to Definition 4, $A+B=\{<x, 1, 0.5, 0.5>\}\neq B$, therefore the operation "+" cannot be accepted. Similar contradictions exist in other operations of Definition 4, and thus those defined above are incorrect.

(2) The correlation coefficient of SNSs [36], which is based on the operations of Definition 4, cannot be accepted in some specific cases.

Example 2. Let $A_1=\{<x, 0.8, 0, 0>\}$ and $A_2=\{<x, 0.7, 0, 0>\}$ be two SNSs, and $B=\{<x, 1, 0, 0>\}$ be the largest SNS. According to the correlation coefficient of SNSs [36], $W_1(A_1, B)=W_2(A_2, B)=1$ can be obtained, but this does not indicate which one is the best. However, it is clear that A_1 is superior to A_2.

(3) In addition, the cross-entropy measure for SNSs [42], which is based on the operations of Definition 4, cannot be accepted in specific cases.

Example 3. Let $A_1=\{<x, 0.1, 0, 0>\}$ and $A_2=\{<x, 0.9, 0, 0>\}$ be two SNSs, and $B=\{<x, 1, 0, 0>\}$ be the largest SNS According to the cross-entropy measure for SNSs [42], $S_1(A_1, B)=S_2(A_2, B)=1$ can be obtained, which indicates that A_1 is equal to A_2. However, it is not possible to discern which one is the best, but as $T_{A_2}(x)>T_{A_1}(x)$, $I_{A_2}(x)>I_{A_1}(x)$ and $F_{A_2}(x)>F_{A_1}(x)$ for any x in X, it is clear that A_2 is superior to A_1.

(4) If $I_A(x)=I_B(x)$ for any x in X, then A and B are both reduced to two IFSs. However, the operations presented in Definition 4 are not in accordance with the operations of two IFSs [9–22].

Definition 5 *([37])*. Let $X=\{x_1, x_2, \ldots, x_n\}$ and A and B be two SNSs, then A is contained in B, i.e. $A\subseteq B$ if and only if $T_A(x)\leq T_B(x)$, $I_A(x)\geq I_B(x)$ and $F_A(x)\geq F_B(x)$ for any $x\in X$.

Clearly, if the equalities are not accepted, then $A\subset B$ is obtained.

2.2. The novel operations of SNNs

Some novel operations of SNNs are now defined.

Definition 6. Let A and B be two SNNs, the operations of SNNs can be defined as follows:

(1) $\lambda A=<1-(1-T_A)^\lambda, (I_A)^\lambda, (F_A)^\lambda>, \quad \lambda>0$;
(2) $A^\lambda=<(T_A)^\lambda, 1-(1-I_A)^\lambda, 1-(1-F_A)^\lambda>, \quad \lambda>0$;
(3) $A+B=<T_A+T_B-T_A\cdot T_B, I_A\cdot I_B, F_A\cdot F_B>$;
(4) $A\cdot B=<T_A\cdot T_B, I_A+I_B-I_A\cdot I_B, F_A+F_B-F_A\cdot F_B>$.

Theorem 1. Let A, B and C be three SNNs, then the following equations can be true.

(1) $A+B=B+A$;
(2) $A\cdot B=B\cdot A$;
(3) $\lambda(A+B)=\lambda A+\lambda B, \quad \lambda>0$;
(4) $(A\cdot B)^\lambda=A^\lambda+B^\lambda, \quad \lambda>0$;
(5) $\lambda_1 A+\lambda_2 A=(\lambda_1+\lambda_2)A, \quad \lambda_1>0, \lambda_2>0$;
(6) $A^{\lambda_1}\cdot A^{\lambda_2}=A^{(\lambda_1+\lambda_2)}, \quad \lambda_1>0, \lambda_2>0$;
(7) $(A+B)+C=A+(B+C)$;
(8) $(A\cdot B)\cdot C=A\cdot(B\cdot C)$.

Example 4. Let $A=<0.6, 0.1, 0.2>$ and $B=<0.5, 0.3, 0.4>$ be two SNNs, and $\lambda=2$, then the following results can be achieved:

(1) $2\cdot A=<1-(1-0.6)^2, 0.1^2, 0.2^2>=<0.84, 0.01, 0.04>$;
(2) $A^2=<0.6^2, 1-(1-0.1)^2, 1-(1-0.2)^2>=<0.36, 0.19, 0.36>$;
(3) $A+B=<0.6+0.5-0.6\times 0.5, 0.1\times 0.3, 0.2\times 0.4>=<0.80, 0.03, 0.08>$;
(4) $A\cdot B=<0.6\times 0.5, 0.1+0.3-0.1\times 0.3, 0.2+0.4-0.2\times 0.4>=<0.30, 0.37, 0.52>$.

Definition 7. Let $A\in SNNS$, then the complement of an SNN A is denoted by A^C, which is defined by $A^C=<1-T_A, 1-I_A, 1-F_A>$.

Definition 8. Let A and B be two SNNs, then $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$.

Definition 9 *([35])*. Let A and B be any two SNNs, then the normalized Euclidean distance between A and B can be defined as follows:

$$d(A, B)=\sqrt{\frac{1}{3n}(|\tilde{T}_A-\tilde{T}_B|^2+|\tilde{I}_A-\tilde{I}_B|^2+|\tilde{F}_A-\tilde{F}_B|^2)}. \tag{2}$$

3. The outranking relations of SNNs

The binary relations between two SNNs that are based on ELECTRE are now defined.

Definition 10. Let A and B be two SNNs, then the strong dominance relation, weak dominance relation, and indifference relation of SNNs can be defined as follows.

(1) If $T_A \geq T_B$, $I_A < I_B$ and $F_A < F_B$ or $T_A > T_B$, $I_A = I_B$ and $F_A = F_B$, then A strongly dominates B (B is strongly dominated by A), denoted by $A \succ_S B$.
(2) If $T_A \geq T_B$, $I_A \geq I_B$ and $F_A < F_B$ or $T_A \geq T_B$, $I_A < I_B$ and $F_A \geq F_B$, then A weakly dominates B (B is weakly dominated by A), denoted by $A \succ_W B$.
(3) If $T_A = T_B$, $I_A = I_B$ and $F_A = F_B$, then A is indifferent to B, denoted by $A \sim_I B$.
(4) If none of the relations mentioned above exist between A and B for any $x \in X$, then A and B are incomparable, denoted by $A \perp B$.

Property 1. Let $A, B \in SNNS$, then the following properties can be obtained:

(1) if $B \subset A$, then $A \succ_S B$;
(2) if $A \succ_S B$, then $B \subseteq A$;
(3) $A \sim_I B$ if and only if $A = B$.

Proof. (1) If $B \subset A$, then $T_B < T_A$, $I_B > I_A$ and $F_B > F_A$. $A \succ_S B$ is definitely validated according to the strong dominance relation in Definition 10.

(2) If $A \succ_S B$, then based on Definition 10, $T_A \geq T_B$, $I_A < I_B$ and $F_A < F_B$ or $T_A > T_B$, $I_A = I_B$ and $F_A = F_B$ are realized. From Definition 5, $B \subseteq A$ is obtained.

(3) Necessity: $A \sim_I B \Rightarrow A = B$.

According to the indifference relation in Definition 10, it is known that $T_A = T_B$, $I_A = I_B$ and $F_A = F_B$. Clearly, $A \subseteq B$ and $B \subseteq A$ are achieved, then $A = B$.

Sufficiency: $A = B \Rightarrow A \sim_I B$.

If $A = B$, then it is known that $A \subseteq B$ and $B \subseteq A$, which means $T_A \leq T_B$, $I_A \geq I_B$, $F_A \geq F_B$ and $T_A \geq T_B$, $I_A \leq I_B$ and $F_A \leq F_B$; then $T_A = T_B$, $I_A = I_B$ and $F_A = F_B$ are obtained. Due to the indifference relation in Definition 10, $A \sim_I B$ is definitely validated. □

Property 2. Let A, B and C be three SNNs, if $A \succ_S B$ and $B \succ_S C$, then $A \succ_S C$.

Proof. According to the strong dominance relation in Definition 10, if $A \succ_S B$, then $T_A \geq T_B$, $I_A < I_B$ and $F_A < F_B$ or $T_A > T_B$, $I_A = I_B$ and $F_A = F_B$.

If $B \succ_S C$, then $T_B \geq T_C$, $I_B < I_C$ and $F_B < F_C$ or $T_B > T_C$, $I_B = I_C$ and $F_B = F_C$.

Therefore, the further derivations are:

$$\text{If}\quad \left.\begin{array}{l} T_A \geq T_B, I_A < I_B \text{ and } F_A < F_B \\ T_B \geq T_C, I_B < I_C \text{ and } F_B < F_C \end{array}\right\} \Rightarrow T_A \geq T_C, I_A < I_C \text{ and } F_A < F_C, \text{ then based on Definition 10, } A \succ_S C \text{ is realized.}$$

$$\text{If}\quad \left.\begin{array}{l} T_A \geq T_B, I_A < I_B \text{ and } F_A < F_B \\ T_B > T_C, I_B = I_C \text{ and } F_B = F_C \end{array}\right\} \Rightarrow T_A > T_C, I_A < I_C \text{ and } F_A < F_C, \text{ then based on Definition 10, } A \succ_S C \text{ is achieved.}$$

$$\text{If}\quad \left.\begin{array}{l} T_A > T_B, I_A = I_B \text{ and } F_A = F_B \\ T_B \geq T_C, I_B < I_C \text{ and } F_B < F_C \end{array}\right\} \Rightarrow T_A > T_C, I_A < I_C \text{ and } F_A < F_C, \text{ then based on Definition 10, } A \succ_S C \text{ is obtained.}$$

$$\text{If}\quad \left.\begin{array}{l} T_A > T_B, I_A = I_B \text{ and } F_A = F_B \\ T_B \geq T_C, I_B = I_C \text{ and } F_B = F_C \end{array}\right\} \Rightarrow T_A > T_C, I_A = I_C \text{ and } F_A = F_C, \text{ then based on Definition 10, } A \succ_S C \text{ is realized.}$$

Therefore, if $A \succ_S B$ and $B \succ_S C$, then $A \succ_S C$. □

Property 3. Let A, B and C be three SNNs, if $A \sim_I B$ and $B \sim_I C$, then $A \sim_I C$.

Proof. Clearly, if $A \sim_I B$ and $B \sim_I C$, then $A \sim_I C$ is surely validated. □

Property 4. Let A, B and C be three SNNs, then the following results can be achieved.

(1) The strong dominance relations are categorized into:
① irreflexivity: $\forall A \in SNNS$, $A \not\succ_S A$;
② asymmetry: $\forall A, B \in SNNS$, $A \succ_S B \Rightarrow B \not\succ_S A$;
③ transitivity: $\forall A, B, C \in SNNS$, $A \succ_S B$, $B \succ_S C \Rightarrow A \succ_S C$.
(2) The weak dominance relations are categorized into:
④ irreflexivity: $\forall A \in SNNS$, $A \not\succ_W A$;
⑤ asymmetry: $\forall A, B \in SNNS$, $A \succ_W B \Rightarrow B \not\succ_W A$;
⑥ non-transitivity: $\exists A, B, C \in SNNS$, $A \succ_W B$, $B \succ_w C A \succ_W C$.

(3) The indifference relations are categorized into:

⑦ reflexivity: $\forall A \in SNNS, A \sim_I A$;

⑧ symmetry: $\forall A, B \in SNNS, A \sim_I B \Rightarrow B \sim_I A$;

⑨ transitivity: $\forall A, B, C \in SNNS, A \sim_I B, B \sim_I C \Rightarrow A \sim_I C$.

According to Definition 10, it is clear that ③, ⑦, ⑧ and ⑨ are true, and ①, ②, ④, ⑤ and ⑥ need to be proven.

Example 5. ①, ②, ④, ⑤ and ⑥ are exemplified as follows.

(1) If $A = < 0.6, 0.3, 0.2 >$ is a SNN, then $A \not\succ_S A$ can be obtained.
(2) If $A = < 0.7, 0.2, 0.4 >$ and $B = < 0.5, 0.3, 0.5 >$ are two SNNs, then $A \succ_S B$, but $B \not\succ_S A$ is achieved.
(3) If $A = < 0.5, 0.2, 0.3 >$ is a SNS, then $A \not\succ_W A$ is realized.
(4) If $A = < 0.4, 0.3, 0.3 >$ and $B = < 0.3, 0.3, 0.4 >$ are two SNNs, then $A \succ_W B$ is obtained, however $B \not\succ_W A$.
(5) If $A = < 0.7, 0.3, 0.4 >$, $B = < 0.6, 0.2, 0.5 >$ and $C = < 0.6, 0.3, 0.3 >$ are three SNNs, then $A \succ_W B$ and $B \succ_W C$ are achieved, however $A \perp C$.

Property 5. Let a_1 and a_2 be two actions, the performances for actions a_1 and a_2 be in the form of SNNs, and $P = S \cup W \cup I$ mean that "a_1 is at least as good as a_2", then four situations may arise:

(1) $a_1 P a_2$ and not $a_2 P a_1$, that is $a_1 \succ_S a_2$ or $a_1 \succ_W a_2$;
(2) $a_2 P a_1$ and not $a_1 P a_2$, that is $a_2 \succ_S a_1$ or $a_2 \succ_W a_1$;
(3) $a_1 P a_2$ and $a_2 P a_1$, that is $a_1 \sim_I a_2$;
(4) not $a_1 P a_2$ and not $a_2 P a_1$, that is $a_1 \perp a_2$.

4. An outranking approach for MCDM with simplified neutrosophic information

The MCDM ranking/selection problems with simplified neutrosophic information consist of a group of alternatives, denoted by $A = \{a_1, a_2, \ldots, a_n\}$. Each alternative is evaluated based on criteria, denoted by $C = \{c_1, c_2, \ldots, c_m\}$. a_{ij} is the value of alternative a_i for criterion c_j, and a_{ij} $(i = 1, \ldots, n; j = 1, \ldots, m)$ are in the form of SNNs. The corresponding weight is w_j, where $w_j \in [0, 1]$ and $\sum_{j=1}^{m} w_j = 1$. This method is suitable if the amount of decision-makers is small and they could evaluate these criteria in the form of SNNs. For convenience, the value of a_{ij} is denoted by $a_{ij} = < T_{ij}, I_{ij}, F_{ij} >$ $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$.

This method is an integration of SNNs and the outranking method to manage the MCDM problems mentioned above. In general, there are benefit criteria and cost criteria in MCDM problems and the cost-type criterion values can be transformed into benefit-type criterion values as follows:

$$\beta_{ij} = \begin{cases} a_{ij}, & \text{for benefit criterion } c_j \\ (a_{ij})^c, & \text{for cost criterion } c_j \end{cases}, \quad (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m). \tag{3}$$

here $(a_{ij})^c$ is the complement of a_{ij} as defined in Definition 7.

It is now feasible to develop a new approach for the MCDM problems mentioned above.

Step 1. Transform the decision matrix.

The SNN decision matrix $R = (a_{ij})_{n \times m}$ can be transformed into a normalized SNN decision matrix $R = (\beta_{ij})_{n \times m}$ based on Eq. (3).

Step 2. Determine the weighted normalized matrix.

According to the weight vector for the criteria, the weighted normalized decision matrix can be constructed using the following formula:

$$\gamma_{ij} = \beta_{ij} w_j, \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, m. \tag{4}$$

where w_j is the weight of the jth criterion with $\sum_{j=1}^{m} w_j = 1$.

Step 3. Determine the concordance and discordance set of subscripts.

The concordance set of subscripts, which should satisfy the constraint $a_{ij} P a_{kj}$, is represented as:

$$O_{ik} = \{j | a_{ij} P a_{kj}\} \quad (i, k = 1, 2, \ldots, n). \tag{5}$$

$a_{ij} P a_{kj}$ represents $a_{ij} >_S a_{kj}$ or $a_{ij} >_W a_{kj}$ or $a_{ij} \sim a_{kj}$.

The discordance set of subscripts for criteria is the complementary subset, therefore:

$$D_{ik} = J - O_{ik}. \tag{6}$$

Step 4. Determine the concordance and discordance matrix.

By using the weight vector w that is associated with the criteria, the concordance index $C(a_i, a_k)$ is represented as:

$$C(a_i, a_k) = \sum_{j \in O_{ik}} w_j. \tag{7}$$

Thus, the concordance matrix C is:

$$C = \begin{pmatrix} - & c_{12} & \cdots & c_{1n} \\ c_{21} & - & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & - \end{pmatrix}. \quad (8)$$

The discordance index $D(a_i, a_k)$ is represented as:

$$D_{ik} = \frac{\max\limits_{j \in D_{ik}} \left\{ d(a_{ij}, a_{kj}) \right\}}{\max\limits_{j \in J} \left\{ d(a_{ij}, a_{kj}) \right\}}. \quad (9)$$

here $d(a_{ij}, a_{kj})$ denotes the normalized Euclidean distance between a_{ij} and a_{kj} as defined in Definition 9.

Thus, the discordance matrix D is:

$$D = \begin{pmatrix} - & d_{12} & \cdots & d_{1n} \\ d_{21} & - & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ d_{n1} & d_{n2} & \cdots & - \end{pmatrix}. \quad (10)$$

Step 5. Determine the effective concordance and discordance matrix.

Given a preference threshold value $\tilde{p}$, which could judge the possible superiority of a_i with respect to a_k, then the effective concordance matrix is represented as:

$$\sigma(a_i, a_s) = \begin{cases} 1: & \text{If } C(a_i, a_k) \geq \tilde{p} \\ 0: & \text{If } C(a_i, a_k) < \tilde{p} \end{cases}, \quad (11)$$

where $\tilde{p}$ can be determined according to the decision-makers' preference, and $\tilde{p} = \sum_{i=1}^{n}\sum_{k=1}^{n} C(a_i, a_k)/n(n-1)$.

Similarly, given a veto threshold value $\tilde{v}$, which could judge the possible inferiority of a_i with respect to a_k, then the effective discordance matrix is represented as:

$$\delta(a_i, a_k) = \begin{cases} 1: & \text{If } D(a_i, a_k) \geq \tilde{v} \\ 0: & \text{If } D(a_i, a_k) < \tilde{v} \end{cases}, \quad (12)$$

where $\tilde{v}$ can be determined by the decision-makers, and $\tilde{v} = \sum_{i=1}^{n}\sum_{k=1}^{n} D(a_i, a_k)/n(n-1)$.

Step 6. Determine the outranking matrix.

Subsequently the outranking matrix is defined with the following formula.

$$\pi(a_i, a_k) = \sigma(a_i, a_k) \cdot \delta(a_i, a_k). \quad (13)$$

Step 7. Eliminate the less attractiveness alternatives and rank the alternatives.

The outranking matrix π indicates the partial preference ordering of the alternatives. If $\pi_{ik} = 1$, then a_i is preferred to A_k for both concordance and discordance indexes. However, a_i still has a chance of being dominated by other alternatives. Therefore, the condition which makes a_i an effective alternative is as follows:

$$\begin{cases} \pi_{ik} = 1 \text{ for at least one unit element} & \text{for } k = 1, 2, \ldots, n; i \neq k \\ \pi_{hi} = 0 \text{ for all } h & \text{for } h = 1, 2, \ldots, n; h \neq i; h \neq k \end{cases}. \quad (14)$$

If any column of the matrix $\pi = (\pi_{ij})$ has at least one element of 1, then this column is dominated by the corresponding row. Thus, any column which has an element of 1 can be eliminated. The threshold values $\tilde{p}$ and $\tilde{v}$, which were given in Step 5, are approximate and influence the final ranking. Thus the best alternative can be obtained by changing the threshold value.

5. Illustrative examples

In this section, an example for a MCDM problem with simplified neutrosophic information and an example for a MCDM problem with intuitionistic fuzzy information are used to demonstrate the application of the proposed decision-making method, as well as for the comparison analysis.

5.1. Two examples for MCDM problems with simplified neutrosophic information and intuitionistic fuzzy information

Example 6 *([37])*. There is an investment company, which wants to invest a sum of money in the best option. This company has set up a panel which has to choose between four possible alternatives for investing the money: (1) a_1 is a car company; (2) a_2 is a food company; (3) a_3 is a computer company; and (4) a_4 is an arms company. The investment company must make a decision using the following three criteria: (1) c_1 is the risk; (2) c_2 is the growth; and (3) c_3 is the customer satisfaction; these are all benefit type criteria. The weight vector of the criteria is represented by $w = (0.35, 0.25, 0.4)$. The four possible alternatives are to be evaluated under the above three criteria in the form of SNNs for each decision-maker, as shown in the following simplified neutrosophic decision matrix R:

$$R = \begin{pmatrix} <0.4, 0.2, 0.3> & <0.4, 0.2, 0.3> & <0.8, 0.8, 0.5> \\ <0.6, 0.1, 0.2> & <0.6, 0.1, 0.2> & <0.5, 0.8, 0.8> \\ <0.3, 0.2, 0.3> & <0.5, 0.2, 0.3> & <0.5, 0.7, 0.8> \\ <0.7, 0.0, 0.1> & <0.6, 0.1, 0.2> & <0.6, 0.7, 0.8> \end{pmatrix}.$$

The procedures for obtaining the best alternative are now outlined.

Step1. Transform the decision matrix.

Since all the criteria are of the benefit type, $R' = R$ can be obtained.

Step 2. Determine the weighted normalized matrix.

Based on Eq. (4), the weighted normalized matrix R'' can be obtained.

$$R'' = \begin{pmatrix} <0.1637, 0.6687, 0.6178> & <0.1637, 0.6687, 0.6178> & <0.4307, 0.9457, 0.7579> \\ <0.2744, 0.5623, 0.5253> & <0.2744, 0.5623, 0.5253> & <0.2154, 0.9457, 0.9146> \\ <0.1174, 0.6687, 0.6178> & <0.2154, 0.6687, 0.6178> & <0.2154, 0.9147, 0.9146> \\ <0.3439, 0.0000, 0.3981> & <0.2744, 0.5623, 0.5253> & <0.2744, 0.9147, 0.9146> \end{pmatrix}.$$

Step 3. Determine the concordance and discordance set of subscripts.

According to Eqs. (5) and (6), the concordance set of subscripts is obtained as follows:

$$\begin{array}{l} O_{12} = \{3\};\ O_{21} = \{1,2\};\ O_{31} = \{2\};\ O_{41} = \{1,2\};\ O_{13} = \{1,3\};\ O_{23} = \{1,2\}; \\ O_{32} = \{3\};\ O_{42} = \{1,2,3\};\ O_{14} = \{3\};\ O_{24} = \{2\};\ O_{34} = \varnothing;\ O_{43} = \{1,2,3\} \end{array}.$$

The discordance set of subscripts is obtained as follows:

$$\begin{array}{l} D_{12} = \{1,2\};\ D_{21} = \{3\};\ D_{31} = \{1,3\};\ D_{41} = \{3\};\ D_{13} = \{2\};\ D_{23} = \{3\}; \\ D_{32} = \{1,2\};\ D_{42} = \varnothing;\ D_{14} = \{1,2\};\ D_{24} = \{1,3\};\ D_{34} = \{1,2,3\};\ D_{43} = \varnothing \end{array}.$$

where $\varnothing$ denotes "empty".

Step 4. Determine the concordance and discordance matrix.

With regard to the weight vector w associated with the criteria, the concordance index is represented as follows:

$$C = \begin{pmatrix} - & 0.40 & 0.75 & 0.40 \\ 0.60 & - & 0.60 & 0.25 \\ 0.25 & 0.40 & - & 0 \\ 0.60 & 1 & 1 & - \end{pmatrix}.$$

The discordance index can be calculated as follows. For example, $D_{2,3} = \frac{\max\{d(a_{23}, a_{33})\}}{\max\{d(a_{21}, a_{31}), d(a_{22}, a_{32}), d(a_{23}, a_{33})\}} = \frac{\max\{0.0179\}}{\max\{0.1218, 0.0882, 0.0179\}} = \frac{0.0179}{0.01218} = 0.1472$.

Here:

$$d(a_{21}, a_{31}) = \left(\frac{1}{3}((0.2744 - 0.1174)^2 + (0.5623 - 0.6687)^2 + (0.5253 - 0.6178)^2)\right)^{1/2} = 0.1218;$$

$$d(a_{22}, a_{32}) = \left(\frac{1}{3}((0.2744 - 0.2154)^2 + (0.5623 - 0.6687)^2 + (0.5253 - 0.6178)^2)\right)^{1/2} = 0.0882;$$

and

$$d(a_{21}, a_{31}) = \left(\frac{1}{3}((0.2154 - 0.2154)^2 + (0.9457 - 0.9147)^2 + (0.9146 - 0.9146)^2)\right)^{1/2} = 0.0179.$$

Therefore, the discordance index matrix is as follows:

$$D = \begin{pmatrix} - & 0.6730 & 0.1928 & 1 \\ 1 & - & 0.1472 & 1 \\ 1 & 1 & - & 1 \\ 0.3077 & 0 & 0 & - \end{pmatrix}.$$

Step 5. Determine the effective concordance and discordance matrix.

According to Eq. (11), the preference threshold value $\tilde{p}$ can be obtained.

$$\tilde{p} = \frac{\sum_{i=1}^{4}\sum_{k=1}^{4} C(a_i, a_k)}{4 \times (4-1)} = 0.5208.$$

Thus, the effective concordance matrix can be obtained as follows.

$$\sigma = (\sigma(a_i, a_k)) = \begin{pmatrix} - & 0 & 1 & 0 \\ 1 & - & 1 & 0 \\ 0 & 0 & - & 0 \\ 1 & 1 & 1 & - \end{pmatrix}.$$

Similarly, the veto threshold value $\tilde{v}$ and the effective discordance matrix are calculated as follows.

$$\tilde{v} = \frac{\sum_{i=1}^{4}\sum_{k=1}^{4} D(a_i, a_k)}{4 \times (4-1)} = 0.6101, \quad \delta = (\delta(a_i, a_k)) = \begin{pmatrix} - & 1 & 0 & 1 \\ 1 & - & 0 & 1 \\ 1 & 1 & - & 1 \\ 0 & 0 & 0 & - \end{pmatrix}.$$

Step 6. Determine the outranking matrix.
Based on Step 5 and Eq. (13), the outranking matrix is calculated as follows.

$$\pi = (\pi(a_i, a_k)) = (\sigma(a_i, a_k)) \cdot (\delta(a_i, a_k)) = \begin{pmatrix} - & 0 & 1 & 0 \\ 1 & - & 1 & 0 \\ 0 & 0 & - & 0 \\ 1 & 1 & 1 & - \end{pmatrix} \cdot \begin{pmatrix} - & 1 & 0 & 1 \\ 1 & - & 0 & 1 \\ 1 & 1 & - & 1 \\ 0 & 0 & 0 & - \end{pmatrix} = \begin{pmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Step 7. Eliminate the less favorable alternatives and rank the alternatives.

In matrix π, the first column has a unit element and can be eliminated. Subsequently alternative a_2 is placed in the first order and a_1 is placed in the fourth order. Furthermore, if the threshold values $\tilde{p}$ and $\tilde{v}$ are changed in Step 5, then it can be seen that a_4 is placed in the second order and a_3 is placed in the third order. Therefore the final ranking is $a_2 \succ a_4 \succ a_3 \succ a_1$. Thus, the best alternative is a_2 whereas the worst alternative is a_1.

It should be noted that an SNS is a special case of an NS, and an NS is a set where each element of the universe has the degrees of truth, indeterminacy and falsity, which lie within $]0^-, 1^+[$, the non-standard unit interval. This is an extension of the standard interval [0, 1] as discussed by Ye [36,37,42]. In particular, the uncertainty presented here, i.e. the indeterminacy factor, is independent of truth and falsity values, whereas the incorporated uncertainty is dependent on the degree of belongingness and non-belongingness of IFSs [35]. Therefore, this leads to the theory that IFSs are a special case of SNSs. Moreover, SNSs can solve some problems that are beyond the scope of FSs and IFSs, as was discussed earlier. The proposed MCDM approach with simplified neutrosophic information can also solve MCDM problems with intuitionistic information. Therefore an intuitionistic fuzzy problem is presented, which can be solved by utilizing the proposed approach. Furthermore a comparison analysis is made with other IFS methods.

Example 7 *([15,69])*. The arrival of the mobile phone and its rapid and widespread growth may well be seen as one of the most significant developments in the fields of communication and information technology over the past two decades. A decision-maker wants to buy the mobile phone that is the best option. There are five possible mobile phones $a_j (j = 1, 2, 3)$ to be evaluated. The decision-maker must make a decision according to the following six criteria: (1) c_1 (basic requirements): reasonable price and a standard process applied; (2) c_2 (physical characteristics): design standards, weight, dimension, shape, water resistance, and raw material properties; (3) c_3 (technical features): talk time, standby time, and safety standards; c_4 (functionality): ease of use; c_5 (brand choice): market vision and technical support; and c_6 (customer excitement): games, diversity of ring tones, and business life facilitating services. The weight vector of the criteria is given by $w = (0.2, 0.1, 0.2, 0.2, 0.2, 0.1)$. The five possible alternatives are to be evaluated under the above six criteria in the form of intuitionistic fuzzy numbers, as shown in the following intuitionistic fuzzy decision matrix R:

$$R = \begin{pmatrix} <0.4, 0.2, 0.4> & <0.5, 0.3, 0.2> & <0.6, 0.1, 0.3> & <0.8, 0.1, 0.1> & <0.3, 0.4, 0.3> & <0.4, 0.6, 0.0> \\ <0.9, 0.1, 0.0> & <0.2, 0.4, 0.4> & <0.5, 0.4, 0.1> & <0.3, 0.5, 0.5> & <0.6, 0.3, 0.1> & <0.4, 0.4, 0.2> \\ <0.6, 0.2, 0.2> & <0.7, 0.2, 0.1> & <0.2, 0.5, 0.3> & <0.4, 0.5, 0.1> & <0.8, 0.2, 0.0> & <0.3, 0.5, 0.2> \end{pmatrix}.$$

In the following overview, the proposed decision-making approach is used to deal with intuitionsitic fuzzy information but the computation process is omitted. If the developed outranking decision-making method in Section 4 is used, the final outranking matrix is calculated as follows:

$$\pi = (\pi(a_i, a_k)) = (\sigma(a_i, a_k)) \cdot (\delta(a_i, a_k)) = \begin{pmatrix} - & 1 & 1 \\ 0 & - & 1 \\ 1 & 1 & - \end{pmatrix} \cdot \begin{pmatrix} - & 0 & 1 \\ 1 & - & 1 \\ 1 & 0 & - \end{pmatrix} = \begin{pmatrix} - & 0 & 1 \\ 0 & - & 1 \\ 1 & 0 & - \end{pmatrix}.$$

Then the final ranking is $a_2 \succ a_1 \succ a_3$. Therefore the best alternative is a_2, whereas the worst alternative is a_3.

5.2. A comparison analysis

In order to validate the feasibility of the proposed decision-making method, a comparative study was conducted with other methods. The comparison analysis includes two cases. One is the other methods that were outlined in Ye [36,37,42], which are compared to the proposed method using simplified neutrosophic information. In the other, some of the methods that were introduced in Liu and Wang [9], Pei and Zheng [10], Zeng and Su [12], Xu [13] and Ye [36,37,42] are compared with the proposed approach using intuitionistic fuzzy information.

Table 1
The results utilizing the different methods of Example 6.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Ye [36]	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
Ye [37]	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
Ye [42]	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1
The proposed method	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1

Table 2
The results obtained by utilizing the different methods for Example 7.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Liu and Wang [9]	$a_2 \succ a_3 \succ a_1$	a_2	a_1
Pei and Zheng [10]	$a_2 \succ a_3 \succ a_1$	a_2	a_1
Zheng and Su [12]	$a_3 \succ a_2 \succ a_1$	a_3	a_1
Xu [13]	$a_2 \succ a_3 \succ a_1$	a_2	a_1
Ye [36,37]	$a_1 \succ a_2 \succ a_3$	a_1	a_3
Ye [42]	$a_1 \succ a_3 \succ a_2$	a_1	a_2
The proposed method	$a_2 \succ a_1 \succ a_3$	a_2	a_3

Case 1. The proposed approach is compared with some methods using simplified neutrosophic information.

With regard to the three methods in Ye [36,37,42], some aggregation operators were developed in order to aggregate the simplified neutrosophic information first; then the cosine similarity measure, the correlation coefficient and the weighted cross-entropy between each alternative and the ideal alternative were calculated and used to determine the final ranking order of all the alternatives. If the methods in Ye [36,37,42] and the proposed method are utilized to solve the MCDM problem in Example 6, then the results can be obtained and are shown in Table 1.

According to the results presented in Table 1, if the methods of [36,37] are used, then the best alternative is a_4 while the worst alternative is a_1. If the proposed approach and the method of Ye [42] are used, then the best alternative is a_2 while the worst alternative is a_1. It can be seen that the result of the proposed approach is different from those that use the earlier methods of Ye [36,37], but is the same as that which uses Ye's later method [42]. There are a number of reasons why differences exist in the final rankings of all the compared methods and the proposed approach. Firstly, the measures and aggregation operators that are involved in those methods are related to some impractical operations as was discussed in Examples 1–3. Secondly, these different measures and aggregation operators also lead to different rankings and it is very difficult for decision-makers to confirm their judgments when using operators and measures that have similar characteristics and which always need a large amount of computation. Finally, the aggregation values, correlation coefficients and cross-entropy measures of SNSs were obtained firstly in Ye [36,37,42] and the differences were amplified in the final results due to the use of criteria weights.

Case 2. The proposed approach is compared with some methods using intuitionistic fuzzy information.

For the two methods used in Liu and Wang [9] and Pei and Zheng [10], the novel score function and the accuracy function of IFSs were developed and the best alternatives were obtained based on the values of these two functions. For the other two methods used in Zeng and Su [12] and Xu [13], the different distances between each alternative and the ideal solution were obtained and determined the final ranking order of all alternatives. If the four methods presented above and the methods in Ye [36,37,42] are all utilized to solve the MCDM problem in Example 7, then they will produce different results as seen in Table 2.

According to the results presented in Table 2, if the methods of Liu and Wang [9], Pei and Zheng [10] and Xu [13] and the proposed approach are utilized to solve MCDM problems with intuitionistic fuzzy information, then the best alternative is always a_2 whereas the worst alternative is a_1 or a_3. If the method of Zeng and Su [12] is used, then the final ranking is $a_3 \succ a_2 \succ a_1$ and the best alternative is a_3. However, the best alternative is always a_2 if the methods of Ye [36,37,42] are used. When compared with the methods that use intuitionistic fuzzy information, it can be seen that the result of the proposed approach is the same as those using the methods of Liu and Wang [9], Pei and Zheng [10] and Xu [13], but different from that using the method of Zeng and Su [12]. Moreover, the results of Ye [36,37,42] are different from those using all of the compared methods and the proposed approach. Three reasons can be put forward in order to explain these differences. Firstly, the results using the MCDM method with simplified neutrosophic information and those utilizing the MCDM method with intuitionistic fuzzy information are not the same. The main reason for this is that the operations of SNSs are different from those of the IFSs, which cannot satisfy the operations of the indeterminacy-membership degree of SNSs. Secondly, although those MCDM methods that use intuitionistic information were constructed using reliable theories and may be robust to a certain degree, they have a number of defects as those methods utilize a different score function, accuracy function, aggregation operators and distance, which also lead to different rankings. Finally, the impractical operations of Ye's [36,37,42] methods that were discussed earlier, and form part of those methods can, to a great extent, explain the differences that exist in the final rankings.

From the analysis presented above, it can be concluded that the main advantages of the approach developed in this paper over the other methods are not only due to its ability to effectively overcome the shortcomings of the compared methods, but also due to its ability to manage the preference information that is expressed by SNSs and IFSs. This means that it can avoid losing and distorting the preference information provided which makes the final results better correspond with real life decision-making problems.

6. Conclusion

SNSs can be applied in solving problems with uncertain, imprecise, incomplete and inconsistent information that exist in scientific and engineering situations. Based on the related research achievements of IFSs, the novel operations of SNSs were defined in this paper, and some outranking relations of SNNs were proposed that were derived from the ELECTRE method; furthermore their desirable properties were discussed in detail. Thus, a MCDM method was established that was based on outranking relations. By using the outranking method, the ranking of all alternatives can be determined and the best one can easily be identified. Two illustrative examples demonstrated the applicability of the proposed decision-making method. The contribution of this study is that an outranking approach for MCDM problems with SNNs could overcome the shortcomings of the existing methods as was discussed earlier. The proposed approach can deal with both simplified neutrosophic information and intuitionistic fuzzy information. Moreover, the comparison analysis showed that the final result produced by the proposed method is more precise and reliable than the results produced by the existing methods. In future research, the authors will continue to study the approach of obtaining the optimal values of $\tilde{p}$ and $\tilde{v}$ in ELECTRE by using a specific model.

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