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Application of Neutrosophic Soft Sets to K-Algebras

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Abstract: Neutrosophic sets and soft sets are two different mathematical tools for representing vagueness and uncertainty. We apply these models in combination to study vagueness and uncertainty in K-algebras. We introduce the notion of single-valued neutrosophic soft (SNS) K-algebras and investigate some of their properties. We establish the notion of $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras and describe some of their related properties. We also illustrate the concepts with numerical examples.

Keywords: K-algebras; single-valued neutrosophic soft K-algebras; $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras

1. Introduction

The notion of a K-algebra $(G, \cdot, \odot, e)$ was first introduced by Dar and Akram [1] in 2003 and published in 2005. A K-algebra is an algebra built on a group $(G, \cdot, e)$ by adjoining an induced binary operation $\odot$ on G, which is attached to an abstract K-algebra $(G, \cdot, \odot, e)$. This system is, in general, non-commutative and non-associative with a right identity e, if $(G, \cdot, e)$ is non-commutative. For a given group G, the K-algebra is proper if G is not an elementary abelian two-group. Thus, a K-algebra is abelian, and being non-abelian purely depends on the base group G. In 2004, Dar and Akram [2] further renamed a K-algebra on a group G as a $K(G)$-algebra due to its structural basis G. The $K(G)$-algebras have been characterized by their left and right mappings in [2] when the group is abelian. The K-algebras have also been characterized by their left and right mappings in [3] when the group is non-abelian. In 2007, Dar and Akram [4] also studied K-homomorphisms of K-algebras.

Logic is an essential tool for giving applications in mathematics and computer science, and it is also a technique for laying a foundation. Non-classical logic takes advantage of the classical logic to handle information with various facts of uncertainty, including the fuzziness and randomness. In particular, non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain in formation. Among all kinds of uncertainties, the incomparability is the most important one that is frequently encountered in our daily lives. Fuzzy set theory, a generalization of classical set theory introduced by Zadeh [5], has drawn the attention of many researchers who have extended the fuzzy sets to intuitionistic fuzzy sets [6], interval-valued intuitionistic fuzzy sets [6], and so on, which are also applied to some decision-making process. On the other hand, Molodtsov [7] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields, including game theory, operations research, Riemann integration and Perron integration. In 1998, Smarandache [8] proposed the idea of neutrosophic sets. He mingled tricomponent logic, non-standard analysis and philosophy. It is a branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. For convenient and advantageous usage of neutrosophic sets in science and engineering, Wang et al. [9] proposed the single-valued neutrosophic sets, whose three independent components have values in the standard unit interval.

Garg and Nancy [10–12] developed a hybrid aggregation operator by using the two instances of the neutrosophic sets, single-valued neutrosophic sets and interval-valued neutrosophic sets. They introduced the concept of some new linguistic prioritized aggregation operators to deal with uncertainty in linguistic terms. To aggregate single-valued neutrosophic information, they developed some new operators to resolve the multi-criteria decision-making problems such as the Muirhead mean, the single-valued neutrosophic prioritized Muirhead mean and the single-valued neutrosophic prioritized Muirhead dual. Maji in [13] initiated the concept of neutrosophic soft sets. Certain notions of fuzzy K-algebras have been studied in [14–18]. Recently, Akram et al. [19,20] introduced single-valued neutrosophic K-algebras and single-valued neutrosophic topological K-algebras. In this paper, we introduce the notion of single-valued neutrosophic soft K-algebras and investigate some of their properties. We establish the notion of $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras and describe some of their related properties. We also illustrate the concepts with numerical examples. The remaining research article is arranged as follows. Section 2 consists of some basic definitions related to K-algebras and single-valued neutrosophic soft sets. In Section 3, the notion of single-valued neutrosophic soft K-algebras is proposed. To have a generalized viewpoint of single-valued neutrosophic soft K-algebras, Section 4 poses the concept of $(\in, \in \vee q)$-single-valued neutrosophic K-algebras with some examples. Finally, some concluding remarks are given in Section 5.

2. Preliminaries

Definition 1 ([1])**.** *Let $(G, \cdot, e)$ be a group such that each non-identity element is not of order two. Let a binary operation $\odot$ be introduced on the group G and defined by $\odot(s,t) = s \odot t = st^{-1}$ for all $s, t \in G$. If e is the identity of the group G, then:*

(1) *e takes the shape of the right $\odot$-identity and not that of the left $\odot$-identity.*
(2) *Each non-identity element ($s \neq e$) is $\odot$-involutionary because $s \odot s = ss^{-1} = e$.*
(3) *G is $\odot$-nonassociative because $(s \odot t) \odot u = s \odot (u \odot t^{-1}) \neq s \odot (t \odot u)$ for all $s, y, u \in G$.*
(4) *G is $\odot$-noncommutative since $s \odot t \neq t \odot s$ for all $s, t \in G$.*
(5) *If G is an elementary Abelian two-group, then $s \odot t = s \cdot t$.*

Definition 2 ([1])**.** *A K-algebra is a structure $(G, \cdot, \odot, e)$ on a group G, where $\odot : G \times G \to G$ is defined by $\odot(s,t) = s \odot t = s.t^{-1}$, if it satisfies the following axioms:*

(i) $((s \odot t) \odot (s \odot u)) = (s \odot (u^{-1} \odot t^{-1}) \odot s)$,
(ii) $(s \odot (s \odot t)) = ((s \odot t^{-1}) \odot s)$,
(iii) $s \odot s = e$,
(iv) $s \odot e = s$,
(v) $e \odot s = s^{-1}$ *for all s, t, $u \in G$.*

Definition 3 ([1])**.** *Let $\mathcal{K}$ be a K-algebra, and let $\mathcal{H}$ be a nonempty subset of $\mathcal{K}$. Then, $\mathcal{H}$ is called a subalgebra of $\mathcal{K}$ if $u \odot v \in \mathcal{H}$ for all $u, v \in \mathcal{H}$.*

Definition 4 ([1])**.** *Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be two K-algebras. A mapping $f : \mathcal{K}_1 \to \mathcal{K}_2$ is called a homomorphism if it satisfies the following condition:*

- *$f(u \odot v) = f(u) \odot f(v)$ for all $u, v \in \mathcal{K}$.*

Definition 5 ([7])**.** *Let Z be an initial universe, and let $\mathbb{R}$ be a universe of parameters. Then, $(\zeta, \mathbb{M})$ is called a soft set (SS), where $\mathbb{M} \subset \mathbb{R}$, $P(Z)$ is the power set of Z and ζ is a set-valued function, which is defined as $\zeta : \mathbb{M} \to P(Z)$.*

Definition 6 ([7])**.** *Let for two soft sets $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ over the common universe Z; the pair $(\zeta, \mathbb{M})$ is a soft subset of $(\eta, \mathbb{N})$, denoted by $(\zeta, \mathbb{M}) \subseteq (\eta, \mathbb{N})$ if it satisfies the following conditions:*

(a) $\mathbb{M} \subseteq \mathbb{N}$,
(b) $\zeta(\theta) \subseteq \eta(\theta)$ for any $\theta \in \mathbb{M}$.

Definition 7 ([9])**.** *Let Z be a universal set of objects. A single-valued neutrosophic set (SNS) $\mathcal{A}$ in Z is characterized by three membership functions, i.e., the $(\mathcal{T}_{\mathcal{A}})$-truth membership function, $(\mathcal{I}_{\mathcal{A}})$-indeterminacy membership function and $(\mathcal{F}_{\mathcal{A}})$-falsity membership function, where $\mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(s) \in [0,1]$ for all $s \in Z$. There is no restriction on the sum of these three components. Therefore, $0 \leq \mathcal{T}_{\mathcal{A}}(s) + \mathcal{I}_{\mathcal{A}}(s) + \mathcal{F}_{\mathcal{A}}(s) \leq 3$.*

Definition 8. *A single-valued neutrosophic set $\mathcal{A}$ in a non-empty set Z is called a single-valued neutrosophic point if:*

$$\mathcal{T}_{\mathcal{A}}(v) = \begin{cases} \alpha \in (0,1], & \text{if } v = u \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(v) = \begin{cases} \beta \in (0,1], & \text{if } v = u \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathcal{F}_{\mathcal{A}}(v) = \begin{cases} \gamma \in [0,1), & \text{if } v = u \\ 0, & \text{otherwise,} \end{cases}$$

with support u and value (α, β, γ), denoted by $u(\alpha, \beta, \gamma)$. This single-valued neutrosophic point is said to "belong to" a single-valued neutrosophic set $\mathcal{A}$, written as $u(\alpha, \beta, \gamma) \in \mathcal{A}$ if $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma$ and said to be "quasicoincident with" a single-valued neutrosophic set $\mathcal{A}$, written as $u(\alpha, \beta, \gamma)\ q\ \mathcal{A}$ if $\mathcal{T}_{\mathcal{A}}(u) + \alpha > 1, \mathcal{I}_{\mathcal{A}}(u) + \beta > 1, \mathcal{F}_{\mathcal{A}}(u) + \gamma < 1$.

Definition 9 ([19])**.** *Let $\mathcal{K}$ be a K-algebra, and let $\mathcal{A}$ be a single-valued neutrosophic set in $\mathcal{K}$ such that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$. Then, $\mathcal{A}$ is called a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if the following conditions hold:*

(1) $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$ for all $s \neq e \in \mathcal{K}$.
(2) $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
$\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
$\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$ for all $s, t \in \mathcal{K}$.

Definition 10 ([19])**.** *A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in $\mathcal{K}$ is called an $(\tilde{a}, \tilde{b})$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following condition:*

- $u_{(\alpha_1,\beta_1,\gamma_1)}\ \tilde{a}\ \mathcal{A},\ v_{(\alpha_2,\beta_2,\gamma_2)}\ \tilde{a}\ \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}\ \tilde{b}\ \mathcal{A}$,

  for all $u, v \in G, \alpha_1, \alpha_2 \in (0,1], \beta_1, \beta_2 \in (0,1], \gamma_1, \gamma_2 \in [0,1)$ and $\tilde{a}, \tilde{b} \in \{\in, q, \in \vee q, \in \wedge q\}$.

Definition 11 ([19])**.** *A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following conditions:*

(a) $e_{(\alpha,\beta,\gamma)} \in \mathcal{A} \Rightarrow (u)_{(\alpha,\beta,\gamma)} \in \vee q\ \mathcal{A}$,
(b) $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}, v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2)} \in \vee q\ \mathcal{A}$,

for all $u, v \in G, \alpha, \alpha_1, \alpha_2 \in (0,1], \beta, \beta_1, \beta_2 \in (0,1], \gamma, \gamma_1, \gamma_2 \in [0,1)$.

Example 1. *Consider a K-algebra* $\mathcal{K} = (G, \cdot, \odot, e)$, *where* $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$ *is the cyclic group of order eight and Caley's table for* $\odot$ *is given as:*

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7
e	e	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ *in* $\mathcal{K}$ *as follows:*

$$\mathcal{T}_{\mathcal{A}}(s) = \begin{cases} 0.9 & \text{if } s = e \\ 0.7 & \textit{for all } s \neq e \in G, \end{cases}$$
$$\mathcal{I}_{\mathcal{A}}(s) = \begin{cases} 0.8 & \text{if } s = e \\ 0.6 & \textit{for all } s \neq e \in G, \end{cases}$$
$$\mathcal{F}_{\mathcal{A}}(s) = \begin{cases} 0 & \text{if } s = e \\ 0.4 & \textit{for all } s \neq e \in G. \end{cases}$$

We take

$\alpha = 0.3, \alpha_1 = 0.6, \alpha_2 = 0.3,$
$\beta = 0.4, \beta_1 = 0.5, \beta_2 = 0.3,$
$\gamma = 0.5, \gamma_1 = 0.5, \gamma_2 = 0.6,$
where $\alpha, \alpha_1, \alpha_2 \in (0,1], \beta, \beta_1, \beta_2 \in (0,1], \gamma, \gamma_1, \gamma_2 \in [0,1)$.

By direct calculations, it is easy to see that $\mathcal{A}$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\mathcal{K}$.

Theorem 1. *Let* $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ *be a single-valued neutrosophic set in* $\mathcal{K}$. *Then,* $\mathcal{A}$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\mathcal{K}$ *if and only if:*

$$\begin{aligned} \text{(i) } & \mathcal{T}_{\mathcal{A}}(u) \geq \min(\mathcal{T}_{\mathcal{A}}(e), 0.5), \\ & \mathcal{I}_{\mathcal{A}}(u) \geq \min(\mathcal{I}_{\mathcal{A}}(e), 0.5), \\ & \mathcal{F}_{\mathcal{A}}(u) \leq \max(\mathcal{F}_{\mathcal{A}}(e), 0.5). \end{aligned}$$

$$\begin{aligned} \text{(ii) } & \mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5), \\ & \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5), \\ & \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5) \text{ for all } u, v \in G. \end{aligned}$$

Definition 12 **([13]).** *Suppose an initial universe* Z *and a universe of parameters* $\mathbb{R}$. *A single-valued neutrosophic soft set (SNSS) is a pair* $(\zeta, \mathbb{M})$, *where* $\mathbb{M} \subset \mathbb{R}$, ζ *is a set-valued function defined as* $\zeta : \mathbb{M} \to P(Z)$. $P(Z)$ *is a set containing all single-valued neutrosophic (SN) subsets of* Z. *Each parameter in* $\mathbb{R}$ *is considered as a neutrosophic word or a sentence containing a neutrosophic word.*

Definition 13 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two single-valued neutrosophic soft sets over a common universe Z, then the pair $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft subset of $(\eta, \mathbb{N})$, denoted by $(\zeta, \mathbb{M}) \subseteq (\eta, \mathbb{N})$, if it satisfies the following conditions:*

(a) *Parametric set $\mathbb{M}$ is a subset of parametric set $\mathbb{N}$,*
(b) *$\zeta(\theta)$ is a subset of $\eta(\theta)$, for any $\theta \in \mathbb{M}$.*

Definition 14 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z, then the extended intersection is denoted by $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$, where $\mathbb{Q} = \mathbb{M} \cup \mathbb{N}$ and defined as:*

$$\vartheta(\theta) = \begin{cases} \zeta(\theta) & \textit{if } \theta \in \mathbb{M} - \mathbb{N}, \\ \eta(\theta) & \textit{if } \theta \in \mathbb{N} - \mathbb{M}, \\ \zeta(\theta) \cap \eta(\theta) & \textit{if } \theta \in \mathbb{M} \cap \mathbb{N} \ \text{ for all } \theta \in \mathbb{Q}. \end{cases}$$

Definition 15 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z. We denote their extended union by $(\zeta, \mathbb{M}) \cup_{ex} (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$, where $\mathbb{Q} = \mathbb{M} \cup \mathbb{N}$ and defined as:*

$$\vartheta(\theta) = \begin{cases} \zeta(\theta) & \textit{if } \theta \in \mathbb{M} - \mathbb{N}, \\ \eta(\theta) & \textit{if } \theta \in \mathbb{N} - \mathbb{M}, \\ \zeta(\theta) \cup \eta(\theta) & \textit{if } \theta \in \mathbb{M} \cap \mathbb{N} \ \text{ for all } \theta \in \mathbb{Q}. \end{cases}$$

Definition 16 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z; the restricted intersection is an SNSS over Z and denoted by $(\vartheta, \mathbb{M} \cap \mathbb{N})$ with $\mathbb{M} \cap \mathbb{N} \neq \emptyset$, where $(\vartheta, \mathbb{M} \cap \mathbb{N}) = (\zeta, \mathbb{M}) \cap (\eta, \mathbb{N})$ and $\vartheta(\theta) = \zeta(\theta) \cap \eta(\theta)$ for all $\theta \in \mathbb{M} \cap \mathbb{N}$.*

Definition 17 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z, then their restricted union is an SNSS over Z and denoted by $(\vartheta, \mathbb{M} \cap \mathbb{N})$ with $\mathbb{M} \cap \mathbb{N} \neq \emptyset$, where $(\zeta, \mathbb{M}) \cup (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$ and $\vartheta(\theta) = \zeta(\theta) \cup \eta(\theta)$* for all $\theta \in \mathbb{M} \cap \mathbb{N}$.

Definition 18 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z; the "AND" operation is denoted by $(\zeta, \mathbb{M})$ AND $(\eta, \mathbb{N})$= $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N}) = (\vartheta, \mathbb{M} \times \mathbb{N})$, where for all $(l, m) \in \mathbb{M} \times \mathbb{N}$, $\vartheta(l, m) = \zeta(l) \cap \eta(m)$, and the truth-membership, indeterminacy-membership and falsity-membership of $\vartheta(l, m)$ are defined as $\mathcal{T}_{\vartheta(l,m)} = \min\{\mathcal{T}_{\vartheta(l)}, \mathcal{T}_{\vartheta(m)}\}, \mathcal{I}_{\vartheta(l,m)} = \min\{\mathcal{I}_{\vartheta(l)}, \mathcal{I}_{\vartheta(m)}\}, \mathcal{F}_{\vartheta(l,m)} = \max\{\mathcal{F}_{\vartheta(l)}, \mathcal{F}_{\vartheta(m)}\}$, for all $l \in \mathbb{M}$ and for all $m \in \mathbb{N}$.*

Definition 19 ([13])**.** *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two SNSSs over Z; the "OR" operation is denoted by $(\zeta, \mathbb{M})$ OR $(\eta, \mathbb{N})$= $(\zeta, \mathbb{M}) \vee (\eta, \mathbb{N}) = (\vartheta, \mathbb{M} \times \mathbb{N})$, where for all $(l, m) \in \mathbb{M} \times \mathbb{N}$, $\vartheta(l, m) = \zeta(l) \cup \eta(m)$, and the truth-membership, indeterminacy-membership and falsity-membership of $\vartheta(l, m)$ are defined as $\mathcal{T}_{\vartheta(l,m)} = \max\{\mathcal{T}_{\vartheta(l)}, \mathcal{T}_{\vartheta(m)}\}, \mathcal{I}_{\vartheta(l,m)} = \max\{\mathcal{I}_{\vartheta(l)}, \mathcal{I}_{\vartheta(m)}\}, \mathcal{F}_{\vartheta(l,m)} = \min\{\mathcal{F}_{\vartheta(l)}, \mathcal{F}_{\vartheta(m)}\}$, for all $l \in \mathbb{M}$ and for all $m \in \mathbb{N}$.*

3. Single-Valued Neutrosophic Soft K-Algebras

Definition 20. *Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft set (SNSS) over $\mathcal{K}$. The pair $(\zeta, \mathbb{M})$ is called a single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$ if the following conditions are satisfied:*

$$\begin{aligned} &\text{(i)} \quad \mathcal{T}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{T}_{\zeta_\theta}(s), \mathcal{T}_{\zeta_\theta}(t)\}, \\ &\text{(ii)} \quad \mathcal{I}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{I}_{\zeta_\theta}(s), \mathcal{I}_{\zeta_\theta}(t)\}, \\ &\text{(iii)} \ \mathcal{F}_{\zeta_\theta}(s \odot t) \leq \max\{\mathcal{F}_{\zeta_\theta}(s), \mathcal{F}_{\zeta_\theta}(t)\} \text{ for all } s, t \in G. \end{aligned}$$

A single-valued neutrosophic soft K-algebra also satisfies the following properties:

$$\mathcal{T}_{\zeta_\theta}(e) \geq \mathcal{T}_{\zeta_\theta}(s),$$
$$\mathcal{I}_{\zeta_\theta}(e) \geq \mathcal{I}_{\zeta_\theta}(s),$$
$$\mathcal{F}_{\zeta_\theta}(e) \leq \mathcal{F}_{\zeta_\theta}(s) \text{ for all } s \neq e \in G.$$

Example 2. *Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where G is the cyclic group of order nine given as $G = \{e, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8\}$. Consider the following Cayley's table:*

$\odot$	e	w	w^2	w^3	w^4	w^5	w^6	w^7	w^8
e	e	w^8	w^7	w^6	w^5	w^4	w^3	w^2	w
w	w	e	w^8	w^7	w^6	w^5	w^4	w^3	w^2
w^2	w^2	w	e	w^8	w^7	w^6	w^5	w^4	w^3
w^3	w^3	w^2	w	e	w^8	w^7	w^6	w^5	w^4
w^4	w^4	w^3	w^2	w	e	w^8	w^7	w^6	w^5
w^5	w^5	w^4	w^3	w^2	w	e	w^8	w^7	w^6
w^6	w^6	w^5	w^4	w^3	w^2	w	e	w^8	w^7
w^7	w^7	w^6	w^5	w^4	w^3	w^2	w	e	w^8
w^8	w^8	w^7	w^6	w^5	w^4	w^3	w^2	w	e

Consider a set of parameters $\mathbb{M} = \{l_1, l_2, l_3, \}$ and a set-valued function $\zeta : \mathbb{M} \to P(G)$, where the membership, indeterminacy-membership and non-membership values of the elements of G at parameters l_1, l_2, l_3 are given as:

$$\begin{aligned}
\text{(i) } \mathcal{T}_{\zeta_{l_1}}(e) &= 0.9,\ \ \mathcal{I}_{\zeta_{l_1}}(e) = 0.3,\ \ \mathcal{F}_{\zeta_{l_1}}(e) = 0.3,\\
\mathcal{T}_{\zeta_{l_1}}(s) &= 0.6,\ \ \mathcal{I}_{\zeta_{l_1}}(s) = 0.2\ ,\ \ \mathcal{F}_{\zeta_{l_1}}(s) = 0.4,\\
\text{(ii) } \mathcal{T}_{\zeta_{l_2}}(e) &= 0.8,\ \ \mathcal{I}_{\zeta_{l_2}}(e) = 0.7\ ,\ \ \mathcal{F}_{\zeta_{l_2}}(e) = 0.4,\\
\mathcal{T}_{\zeta_{l_2}}(s) &= 0.7,\ \ \mathcal{I}_{\zeta_{l_2}}(s) = 0.6\ ,\ \ \mathcal{F}_{\zeta_{l_2}}(s) = 0.5,\\
\text{(iii) } \mathcal{T}_{\zeta_{l_3}}(e) &= 0.9,\ \ \mathcal{I}_{\zeta_{l_3}}(e) = 0.6\ ,\ \ \mathcal{F}_{\zeta_{l_3}}(e) = 0.6,\\
\mathcal{T}_{\zeta_{l_3}}(s) &= 0.8,\ \ \mathcal{I}_{\zeta_{l_3}}(s) = 0.5\ ,\ \ \mathcal{F}_{\zeta_{l_3}}(s) = 0.7
\end{aligned}$$

for all $s \neq e \in G$. The function ζ is defined as:

$$\begin{aligned}
\zeta(l_1) = \{&(e, 0.9, 0.3, 0.3), (w, 0.6, 0.2, 0.4), (w^2, 0.6, 0.2, 0.4), (w^3, 0.6, 0.2, 0.4),\\
&(w^4, 0.6, 0.2, 0.4), (w^5, 0.6, 0.2, 0.4), (w^6, 0.6, 0.2, 0.4),\\
&(w^7, 0.6, 0.2, 0.4), (w^8, 0.6, 0.2, 0.4)\},
\end{aligned}$$

$$\begin{aligned}
\zeta(l_2) = \{&(e, 0.8, 0.7, 0.4), (w, 0.7, 0.6, 0.5), (w^2, 0.7, 0.6, 0.5), (w^3, 0.7, 0.6, 0.5),\\
&(w^4, 0.7, 0.6, 0.5), (w^5, 0.7, 0.6, 0.5), (w^6, 0.7, 0.6, 0.5),\\
&(w^7, 0.7, 0.6, 0.5), (w^8, 0.7, 0.6, 0.5)\},
\end{aligned}$$

$$\begin{aligned}
\zeta(l_3) = \{&(e, 0.9, 0.6, 0.6), (w, 0.8, 0.5, 0.7), (w^2, 0.8, 0.5, 0.7), (w^3, 0.8, 0.5, 0.7),\\
&(w^4, 0.8, 0.5, 0.7), (w^5, 0.8, 0.5, 0.7), (w^6, 0.8, 0.5, 0.7),\\
&(w^7, 0.8, 0.5, 0.7), (w^8, 0.8, 0.5, 0.7)\}.
\end{aligned}$$

Consider a set $\mathbb{N} = \{l_1, l_2\}$ of parameters and a set-valued function $\eta : \mathbb{N} \to P(G)$, where the membership, indeterminacy-membership and non-membership values of the elements of G at parameters l_1, l_2 are defined as:

$$\begin{aligned}
\text{(i) } & \mathcal{T}_{\eta_{l_1}}(e) = 0.9,\ \mathcal{I}_{\eta_{l_1}}(e) = 0.8,\ \mathcal{F}_{\eta_{l_1}}(e) = 0.2,\\
& \mathcal{T}_{\eta_{l_1}}(s) = 0.5,\ \mathcal{I}_{\eta_{l_1}}(s) = 0.2,\ \mathcal{F}_{\eta_{l_1}}(s) = 0.5,\\
\text{(ii) } & \mathcal{T}_{\eta_{l_2}}(e) = 0.3,\ \mathcal{I}_{\eta_{l_2}}(e) = 0.5,\ \mathcal{F}_{\eta_{l_2}}(e) = 0.6,\\
& \mathcal{T}_{\eta_{l_2}}(s) = 0.1,\ \mathcal{I}_{\eta_{l_2}}(s) = 0.4,\ \mathcal{F}_{\eta_{l_2}}(s) = 0.8
\end{aligned}$$

for all $s \neq e \in G$. The function η is defined as:

$$\begin{aligned}
\eta(l_1) = \{&(e, 0.9, 0.8, 0.2), (w, 0.5, 0.2, 0.5), (w^2, 0.5, 0.2, 0.5), (w^3, 0.5, 0.2, 0.5),\\
&(w^4, 0.5, 0.2, 0.5), (w^5, 0.5, 0.2, 0.5), (w^6, 0.5, 0.2, 0.5),\\
&(w^7, 0.5, 0.2, 0.5), (w^8, 0.5, 0.2, 0.5)\},\\
\eta(l_2) = \{&(e, 0.3, 0.5, 0.6), (w, 0.1, 0.4, 0.8), (w^2, 0.1, 0.4, 0.8), (w^3, 0.1, 0.4, 0.8),\\
&(w^4, 0.1, 0.4, 0.8), (w^5, 0.1, 0.4, 0.8), (w^6, 0.1, 0.4, 0.8),\\
&(w^7, 0.1, 0.4, 0.8), (w^8, 0.1, 0.4, 0.8)\}.
\end{aligned}$$

Evidently, the set $(\zeta, \mathbb{M})$ and the set $(\eta, \mathbb{N})$ comprises SNSSs. Since $\zeta(\theta)$, $\eta(\theta)$ are single-valued neutrosophic K-subalgebras for all $\theta \in \mathbb{M}$ and $\theta \in \mathbb{N}$. It is concluded that the pairs $(\zeta, \mathbb{M})$, $(\eta, \mathbb{N})$ are single-valued neutrosophic soft K-subalgebras.

Example 3. *Consider K-algebra on dihedral group D4 given as $G = \{e, a, b, c, w, x, y, z\}$, where $c = ab, w = a^2, x = a^3, y = a^2b, z = a^3b$, and Caley's table for $\odot$ is given as:*

$\odot$	e	a	b	c	w	x	y	z
e	e	x	b	c	w	a	y	z
a	a	e	c	y	x	w	z	b
b	b	c	e	x	y	z	w	a
c	c	y	a	e	z	b	x	w
w	w	a	y	z	e	x	b	c
x	x	w	z	b	a	e	c	y
y	y	z	w	a	b	c	e	x
z	z	b	x	w	c	y	a	e

Consider a set of parameters $\mathbb{M} = \{l_1, l_2, l_3, \}$ and a set-valued function $\zeta : \mathbb{M} \to P(G)$, where the membership, indeterminacy-membership and non-membership values of the elements of G at parameters l_1, l_2, l_3 are given as:

$$\begin{aligned}
(i)\ & \mathcal{T}_{\zeta_{l_1}}(e) = 0.7,\ \mathcal{I}_{\zeta_{l_1}}(e) = 0.7,\ \mathcal{F}_{\zeta_{l_1}}(e) = 0.3,\\
& \mathcal{T}_{\zeta_{l_1}}(s) = 0.5,\ \mathcal{I}_{\zeta_{l_1}}(s) = 0.2,\ \mathcal{F}_{\zeta_{l_1}}(s) = 0.7,\\
(ii)\ & \mathcal{T}_{\zeta_{l_2}}(e) = 0.9,\ \mathcal{I}_{\zeta_{l_2}}(e) = 0.8,\ \mathcal{F}_{\zeta_{l_2}}(e) = 0.4,\\
& \mathcal{T}_{\zeta_{l_2}}(s) = 0.2,\ \mathcal{I}_{\zeta_{l_2}}(s) = 0.2,\ \mathcal{F}_{\zeta_{l_2}}(s) = 0.9,\\
(iii)\ & \mathcal{T}_{\zeta_{l_3}}(e) = 0.5,\ \mathcal{I}_{\zeta_{l_3}}(e) = 0.5,\ \mathcal{F}_{\zeta_{l_3}}(e) = 0.3,\\
& \mathcal{T}_{\zeta_{l_3}}(s) = 0.1,\ \mathcal{I}_{\zeta_{l_3}}(s) = 0.3,\ \mathcal{F}_{\zeta_{l_3}}(s) = 0.8
\end{aligned}$$

for all $s \neq e \in G$. The function ζ is defined as:

$$\zeta(l_1) = \{(e, 0.7, 0.7, 0.3), (a, 0.5, 0.2, 0.7), (b, 0.5, 0.2, 0.7), (c, 0.5, 0.2, 0.7), \\ (w, 0.5, 0.2, 0.7), (x, 0.5, 0.2, 0.7), (y, 0.5, 0.2, 0.7), (z, 0.5, 0.2, 0.7)\},$$

$$\zeta(l_2) = \{(e, 0.9, 0.8, 0.4), (a, 0.2, 0.2, 0.9), (b, 0.2, 0.2, 0.9), (c, 0.2, 0.2, 0.9), \\ (w, 0.2, 0.2, 0.9), (x, 0.2, 0.2, 0.9), (y, 0.2, 0.2, 0.9), (z, 0.2, 0.2, 0.9)\},$$

$$\zeta(l_3) = \{(e, 0.5, 0.5, 0.3), (a, 0.1, 0.3, 0.8), (b, 0.1, 0.3, 0.8), (c, 0.1, 0.3, 0.8), \\ (w, 0.1, 0.3, 0.8), (x, 0.1, 0.3, 0.8), (y, 0.1, 0.3, 0.8), (z, 0.1, 0.3, 0.8)\}.$$

Consider a set $\mathbb{N} = \{l_1, l_2\}$ of parameters and a set-valued function $\eta : \mathbb{N} \to P(G)$, where the truth, indeterminacy and falsity membership values of the elements of G at parameters l_1, l_2 are defined as:

$$\begin{aligned} (i)\ & \mathcal{T}_{\eta_{l_1}}(e) = 0.8,\ \mathcal{I}_{\eta_{l_1}}(e) = 0.8,\ \mathcal{F}_{\eta_{l_1}}(e) = 0.2, \\ & \mathcal{T}_{\eta_{l_1}}(s) = 0.6,\ \mathcal{I}_{\eta_{l_1}}(s) = 0.3,\ \mathcal{F}_{\eta_{l_1}}(s) = 0.7, \\ (ii)\ & \mathcal{T}_{\eta_{l_2}}(e) = 0.6,\ \mathcal{I}_{\eta_{l_2}}(e) = 0.4,\ \mathcal{F}_{\eta_{l_2}}(e) = 0.3, \\ & \mathcal{T}_{\eta_{l_2}}(s) = 0.5,\ \mathcal{I}_{\eta_{l_2}}(s) = 0.4,\ \mathcal{F}_{\eta_{l_2}}(s) = 0.9 \end{aligned}$$

for all $s \neq e \in G$. The function η is defined as:

$$\eta(l_1) = \{(e, 0.8, 0.8, 0.2), (a, 0.6, 0.3, 0.7), (b, 0.6, 0.3, 0.7), (c, 0.6, 0.3, 0.7), \\ (w, 0.6, 0.3, 0.7), (x, 0.6, 0.3, 0.7), (y, 0.6, 0.3, 0.7), (z, 0.6, 0.3, 0.7)\},$$

$$\eta(l_2) = \{(e, 0.6, 0.4, 0.3), (a, 0.5, 0.4, 0.9), (b, 0.5, 0.4, 0.9), (c, 0.5, 0.4, 0.9), \\ (w, 0.5, 0.4, 0.9), (x, 0.5, 0.4, 0.9), (y, 0.5, 0.4, 0.9), (z, 0.5, 0.4, 0.9)\}.$$

Obviously, the set $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ comprises SNSSs. Since for $\theta \in \mathbb{M}$ and $\theta \in \mathbb{N}$, the sets $\zeta(\theta)$, $\eta(\theta)$ are single-valued neutrosophic K-subalgebras. This concludes that the pair $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are single-valued neutrosophic soft K-subalgebras.

Proposition 1. *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two single-valued neutrosophic soft K-subalgebras. Then, the extended intersection of $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ is a single-valued neutrosophic soft K-subalgebra.*

Proof. For any $\theta \in \mathbb{Q}$, the following three cases arise.

First case: If $\theta \in \mathbb{M} - \mathbb{N}$, then $\vartheta(\theta) = \zeta(\theta)$ and $\zeta(\theta)$ being single-valued neutrosophic K-subalgebra implies that $\vartheta(\theta)$ is also a single-valued neutrosophic K-subalgebra since $(\zeta, \mathbb{M})$ is an SNS K-subalgebra.

Second case: If $\theta \in \mathbb{N} - \mathbb{M}$, then $\vartheta(\theta) = \eta(\theta)$ and $\eta(\theta)$ being single-valued neutrosophic K-subalgebra implies that $\vartheta(\theta)$ is a single-valued neutrosophic K-subalgebra since $(\eta, \mathbb{N})$ is an SNS K-subalgebra.

Third case: Now, if $\theta \in \mathbb{M} \cap \mathbb{N}$, then $\vartheta(\theta) = \zeta(\theta) \cap \eta(\theta)$, which is again a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Thus, in any case, $\vartheta(\theta)$ is a single-valued neutrosophic K-subalgebra. Consequently, $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N})$ is a K-subalgebraover $\mathcal{K}$.

□

Proposition 2. *If $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are two SNS K-subalgebras over $\mathcal{K}$, then $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N})$ is an SNS K-subalgebra.*

Proof. Let $(l,m) \in \mathbb{Q}, \zeta(l), \zeta(m)$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$, where $\mathbb{Q} = \mathbb{M} \times \mathbb{N}$, which implies that $\vartheta(l,m) = \zeta(l) \cap \eta(m)$ is also a single-valued neutrosophic K-subalgebra over $\mathcal{K}$. Hence, $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N})$ is an SNS K-subalgebra of $\mathcal{K}$. □

Proposition 3. *If $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are two SNS K-subalgebras and $\zeta(l) \subseteq \eta(l)$ for all $l \in \mathbb{M}$, then $(\zeta, \mathbb{M})$ is an SNS K-subalgebra of $(\eta, \mathbb{N})$.*

Proof. Since $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are SNS K-subalgebras and $\zeta(l), \eta(l)$ are two single-valued neutrosophic K-subalgebras, also $\zeta(l) \subseteq \eta(l)$. Therefore, $(\zeta, \mathbb{M})$ is an SNS K-subalgebra of $(\eta, \mathbb{N})$. □

Proposition 4. *Let $(\zeta, \mathbb{M})$, $(\eta, \mathbb{N})$ be two SNS K-subalgebras. If $\mathbb{M} \cap \mathbb{N} = \emptyset$, then $(\zeta, \mathbb{M}) \cup_{ex} (\eta, \mathbb{N})$ is an SNS K-subalgebra over $\mathcal{K}$.*

Proof. The proof follows from Definition 15. □

Theorem 2. *If $(\zeta, \mathbb{M})$ is an SNS K-subalgebra, then for a non-empty collection $\{(\vartheta_i, \mathbb{N}_i) \mid i \in \Omega\}$ of SNS K-subalgebras of $(\zeta, \mathbb{M})$, the following results hold:*

(i) $\bigcap\limits_{i\in\Omega}{}_{ex}(\vartheta_i, \mathbb{N}_i)$ *is an SNS K-subalgebra of* $(\zeta, \mathbb{M})$.
(ii) $\bigwedge_{i\in\Omega}$ *is an SNS K-subalgebra of* $\bigwedge_{i\in\Omega}(\zeta, \mathbb{M})$.
(iii) *For the disjoint intersection of two parametric sets $\mathbb{N}_i, \mathbb{N}_j$, $\forall i, j \in \Omega$, $\bigvee\limits_{i\in\Omega}{}_{ex}(\vartheta_i, \mathbb{N}_i)$ is an SNS K-subalgebra of $\bigvee\limits_{i\in\Omega}(\zeta, \mathbb{M})$.*

Definition 21 ([13]). *Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft set over Z. Then, for each $\alpha, \beta, \gamma \in [0,1]$, the set $(\zeta, \mathbb{M})^{(\alpha,\beta,\gamma)} = (\zeta^{(\alpha,\beta,\gamma)}, \mathbb{M})$ is called an (α, β, γ)-level soft set of $(\zeta, \mathbb{M})$ and defined as:*
$\zeta_\theta^{(\alpha,\beta,\gamma)} = \{\mathcal{T}_{\zeta_\theta} \geq \alpha, \mathcal{I}_{\zeta_\theta} \geq \beta, \mathcal{F}_{\zeta_\theta} \leq \gamma\}$, *for all* $\theta \in \mathbb{M}$.

Theorem 3. *If $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft set over $\mathcal{K}$, then $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft K-subalgebra if and only if $(\zeta, \mathbb{M})^{(\alpha,\beta,\gamma)}$ is a soft K-subalgebra for all $\alpha, \beta, \gamma \in [0,1]$.*

Proof. Consider that $(\zeta, \mathbb{M})$ is an SNS K-subalgebra. Then, for all $\alpha, \beta, \gamma \in [0,1], \theta \in \mathbb{M}$ and $u_1, u_2 \in \zeta_\theta{}^{(\alpha,\beta,\gamma)}$, $\mathcal{T}_{\zeta_\theta}(u_1) \geq \alpha$, $\mathcal{T}_{\zeta_\theta}(u_2) \geq \alpha, \mathcal{I}_{\zeta_\theta}(u_1) \geq \beta, \mathcal{I}_{\zeta_\theta}(u_2) \geq \beta, \mathcal{F}_{\zeta_\theta}(u_1) \leq \gamma, \mathcal{F}_{\zeta_\theta}(u_2) \leq \gamma$. It follows that $\mathcal{T}_{\zeta_\theta}(u_1 \odot u_2) \geq \min(\mathcal{T}_{\zeta_\theta}(u_1), \mathcal{T}_{\zeta_\theta}(u_2)) \geq \alpha, \mathcal{I}_{\zeta_\theta}(u_1 \odot (u_2)) \geq \min(\mathcal{I}_{\zeta_\theta}(u_1), \mathcal{I}_{\zeta_\theta}(u_2)) \geq \beta, \mathcal{F}_{\zeta_\theta}(u_1 \odot (u_2)) \leq \max(\mathcal{F}_{\zeta\theta}(u_1), \mathcal{F}_{\zeta_\theta}(u_2)) \leq \gamma$; which implies that $u_1 \odot u_2 \in \zeta_\theta{}^{(\alpha,\beta,\gamma)}$. Hence, $\zeta_\theta{}^{(\alpha,\beta,\gamma)}$ is a soft K-subalgebra for all $\alpha, \beta, \gamma \in [0,1]$. The converse part is obvious. □

Definition 22. *Let φ and ρ be two functions, where $\varphi: S_1 \to S_2$ and $\rho: \mathbb{M} \to \mathbb{N}$ and $\mathbb{M}$ and $\mathbb{N}$ are subsets of the universe of parameters $\mathbb{R}$ from S_1 and S_2, respectively. The pair (φ, ρ) is said to be a single-valued neutrosophic soft function from S_1 to S_2.*

Definition 23. *Let the pair (φ, ρ) be a single-valued neutrosophic soft function from $\mathcal{K}_1$ into $\mathcal{K}_2$, then the pair (φ, ρ) is called a single-valued neutrosophic soft homomorphism if φ is a homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and is said to be a single-valued neutrosophic soft bijective homomorphism if φ is an isomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and ρ is an injective map from $\mathbb{M}$ to $\mathbb{N}$.*

Definition 24 ([13]). *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two single-valued neutrosophic soft sets over G_1 and G_2, respectively, and let (φ, ρ) be an SNS function from G_1 into G_2. Then, under the single-valued neutrosophic soft function (φ, ρ), the image of $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft set on $\mathcal{K}_2$, denoted by $(\varphi, \rho)(\zeta, \mathbb{M})$ and defined as: for all $l \in \rho(\mathbb{M})$ and $v \in G_2$, $(\varphi, \rho)(\zeta, \mathbb{M}) = (\varphi(\zeta), \rho(\mathbb{M}))$, where:*

$$\mathcal{T}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigvee_{\varphi(u)=v} \ \bigvee_{\rho(a)=l} \zeta_a(u) & \text{if } u \in \rho^{-1}(v), \\ 1, & \text{otherwise,} \end{cases}$$

$$\mathcal{I}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigvee_{\varphi(u)=v} \ \bigvee_{\rho(a)=l} \zeta_a(u) & \text{if } u \in \rho^{-1}(v), \\ 1, & \text{otherwise,} \end{cases}$$
$$\mathcal{F}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigwedge_{\varphi(u)=v} \ \bigwedge_{\rho(a)=l} \zeta_a(u) & \text{if } u \in \rho^{-1}(v), \\ 0, & \text{otherwise.} \end{cases}$$

Under the single-valued neutrosophic soft function (φ, ρ), the preimage of $(\eta, \mathbb{N})$ is denoted as $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ and defined as for all $a \in \rho^{-1}(\mathbb{N})$ and for all $u \in G_1$, $(\varphi, \rho)^{-1}(\eta, \mathbb{N}) = (\varphi^{-1}(\eta), \rho^{-1}(\mathbb{N}))$, where:

$$\mathcal{T}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{T}_{\eta_{\rho(a)}}(\varphi(u)),$$
$$\mathcal{I}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{I}_{\eta_{\rho(a)}}(\varphi(u)),$$
$$\mathcal{F}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{F}_{\eta_{\rho(a)}}(\varphi(u)).$$

Theorem 4. *Let (φ, ρ) be a single-valued neutrosophic soft homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and $(\eta, \mathbb{N})$ be a single-valued neutrosophic soft K-subalgebra on $\mathcal{K}_2$. Then, $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ is an SNS K-subalgebra on $\mathcal{K}_1$.*

Proof. Assume that $u_1, u_2 \in \mathcal{K}_1$, then we have:

$$\varphi^{-1}(\mathcal{T}_{\eta_\theta})(u_1 \odot u_2) = \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1))$$
$$\varphi^{-1}(\mathcal{T}_{\eta_\theta})(u_1 \odot u_2) \geq \min\{\mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_2))\}$$
$$\varphi^{-1}(\mathcal{T}_{\eta_\theta})(u_1 \odot u_2) \geq \min\{\varphi^{-1}(\mathcal{T}_{\eta_\theta})(u_1), \varphi^{-1}(\mathcal{T}_{\eta_\theta})(u_2)\},$$

$$\varphi^{-1}(\mathcal{I}_{\eta_\theta})(u_1 \odot u_2) = \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1))$$
$$\varphi^{-1}(\mathcal{I}_{\eta_\theta})(u_1 \odot u_2) \geq \min\{\mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_2))\}$$
$$\varphi^{-1}(\mathcal{I}_{\eta_\theta})(u_1 \odot u_2) \geq \min\{\varphi^{-1}(\mathcal{I}_{\eta_\theta})(u_1), \varphi^{-1}(\mathcal{I}_{\eta_\theta})(u_2)\},$$

$$\varphi^{-1}(\mathcal{F}_{\eta_\theta})(u_1 \odot u_2) = \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1))$$
$$\varphi^{-1}(\mathcal{F}_{\eta_\theta})(u_1 \odot u_2) \leq \max\{\mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_2))\}$$
$$\varphi^{-1}(\mathcal{F}_{\eta_\theta})(u_1 \odot u_2) \leq \max\{\varphi^{-1}(\mathcal{F}_{\eta_\theta})(u_1), \varphi^{-1}(\mathcal{F}_{\eta_\theta})(u_2)\}.$$

Therefore, $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ is an SNS K-subalgebra over $\mathcal{K}_1$. □

Remark 1. *Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft K-subalgebra, and let (φ, ρ) be a single-valued neutrosophic soft homomorphism from $\mathcal{K}_1$ into $\mathcal{K}_2$. Then, $(\varphi, \rho)(\zeta, \mathbb{M})$ may not be a single-valued neutrosophic soft K-subalgebra over $\mathcal{K}_2$.*

4. $(\in, \in \vee q)$-Single-Valued Neutrosophic Soft *K*-Algebras

Definition 25. *Suppose $\mathcal{K}$ is a K-algebra. Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft set. The pair $(\zeta, \mathbb{M})$ is called an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra if $\zeta(\theta)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ for all $\theta \in \mathbb{M}$.*

Example 4. *Consider two cyclic groups $G_1 = \{< u >: u^6 = e\}$ and $G_2 = \{< v >: v^2 = e\}$, where $G = G_1 \times G_2 = \{(e,e), (e,v), (u,e), (u,v), (u^2,e), (u^2,v), (u^3,e), (u^3,v), (u^4,e), (u^4,v), (u^5,e), (u^5,v)\}$ is a group. Consider a K-algebra $\mathcal{K}$ on $G = \{e, \acute{x}_1, \acute{x}_2, \acute{x}_3, \acute{x}_4, \acute{x}_5, \acute{x}_6, \acute{x}_7, \acute{x}_8, \acute{x}_9, \acute{x}_{10}, \acute{x}_{11}\}$, where $e = (e,e)$, $\acute{x}_1 = (e,v)$, $\acute{x}_2 = (u,e)$, $\acute{x}_3 = (u,v)$, $\acute{x}_4 = (u^2,e)$, $\acute{x}_5 = (u^2,v)$, $\acute{x}_6 = (u^3,e)$, $\acute{x}_7 = (u^3,v)$, $\acute{x}_8 = (u^4,e)$, $\acute{x}_9 = (u^4,v)$, $\acute{x}_{10} = (u^5,e)$, $\acute{x}_{11} = (u^5,v)$ and $\odot$ is defined by Caley's table as:*

$\odot$	e	$\acute{x}_1$	$\acute{x}_2$	$\acute{x}_3$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_{10}$	$\acute{x}_{11}$
e	e	$\acute{x}_1$	$\acute{x}_{10}$	$\acute{x}_{11}$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_2$	$\acute{x}_3$
$\acute{x}_1$	$\acute{x}_1$	e	$\acute{x}_{11}$	$\acute{x}_{10}$	$\acute{x}_9$	$\acute{x}_8$	$\acute{x}_7$	$\acute{x}_6$	$\acute{x}_5$	$\acute{x}_4$	$\acute{x}_3$	$\acute{x}_2$
$\acute{x}_2$	$\acute{x}_2$	$\acute{x}_3$	e	$\acute{x}_1$	$\acute{x}_{10}$	$\acute{x}_{11}$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_4$	$\acute{x}_5$
$\acute{x}_3$	$\acute{x}_3$	$\acute{x}_2$	$\acute{x}_1$	e	$\acute{x}_{11}$	$\acute{x}_{10}$	$\acute{x}_9$	$\acute{x}_8$	$\acute{x}_7$	$\acute{x}_6$	$\acute{x}_5$	$\acute{x}_4$
$\acute{x}_4$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_2$	$\acute{x}_3$	e	$\acute{x}_1$	$\acute{x}_{10}$	$\acute{x}_{11}$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_6$	$\acute{x}_7$
$\acute{x}_5$	$\acute{x}_5$	$\acute{x}_4$	$\acute{x}_3$	$\acute{x}_2$	$\acute{x}_1$	e	$\acute{x}_{11}$	$\acute{x}_{10}$	$\acute{x}_9$	$\acute{x}_8$	$\acute{x}_7$	$\acute{x}_6$
$\acute{x}_6$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_2$	$\acute{x}_3$	e	$\acute{x}_1$	$\acute{x}_{10}$	$\acute{x}_{11}$	$\acute{x}_8$	$\acute{x}_9$
$\acute{x}_7$	$\acute{x}_7$	$\acute{x}_6$	$\acute{x}_5$	$\acute{x}_4$	$\acute{x}_3$	$\acute{x}_2$	$\acute{x}_1$	e	$\acute{x}_{11}$	$\acute{x}_{10}$	$\acute{x}_9$	$\acute{x}_8$
$\acute{x}_8$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_2$	$\acute{x}_3$	e	$\acute{x}_1$	$\acute{x}_{10}$	$\acute{x}_{11}$
$\acute{x}_9$	$\acute{x}_9$	$\acute{x}_8$	$\acute{x}_7$	$\acute{x}_6$	$\acute{x}_5$	$\acute{x}_4$	$\acute{x}_3$	$\acute{x}_2$	$\acute{x}_1$	e	$\acute{x}_{11}$	$\acute{x}_{10}$
$\acute{x}_{10}$	$\acute{x}_{10}$	$\acute{x}_{11}$	$\acute{x}_8$	$\acute{x}_9$	$\acute{x}_6$	$\acute{x}_7$	$\acute{x}_4$	$\acute{x}_5$	$\acute{x}_2$	$\acute{x}_3$	e	$\acute{x}_1$
$\acute{x}_{11}$	$\acute{x}_{11}$	$\acute{x}_{10}$	$\acute{x}_9$	$\acute{x}_8$	$\acute{x}_7$	$\acute{x}_6$	$\acute{x}_5$	$\acute{x}_4$	$\acute{x}_3$	$\acute{x}_2$	$\acute{x}_1$	e

Let $\mathbb{M} = \{l_1, l_2\}$ be a set of parameters and $\zeta : \mathbb{M} \to P(G)$ be a set-valued function defined as follows:

$$\begin{aligned}\zeta(l_1) = \{&(e, 0.9, 0.8, 0.5), (\acute{x}_1, 0.5, 0.8, 0.5), (\acute{x}_2, 0.5, 0.8, 0.5), (\acute{x}_3, 0.5, 0.8, 0.5),\\ &(\acute{x}_4, 0.5, 0.8, 0.5), (\acute{x}_5, 0.5, 0.8, 0.5), (\acute{x}_6, 0.5, 0.8, 0.5), (\acute{x}_7, 0.5, 0.8, 0.5),\\ &(\acute{x}_8, 0.5, 0.8, 0.5), (\acute{x}_9, 0.5, 0.8, 0.5), (\acute{x}_{10}, 0.5, 0.8, 0.5), (\acute{x}_{11}, 0.5, 0.8, 0.5)\},\end{aligned}$$

$$\begin{aligned}\zeta(l_2) = \{&(e, 0.7, 0.8, 0.4), (\acute{x}_1, 0.6, 0.5, 0.5), (\acute{x}_2, 0.6, 0.5, 0.5), (\acute{x}_3, 0.6, 0.5, 0.5),\\ &(\acute{x}_4, 0.6, 0.5, 0.5), (\acute{x}_5, 0.6, 0.5, 0.5), (\acute{x}_6, 0.6, 0.5, 0.5), (\acute{x}_7, 0.6, 0.5, 0.5),\\ &(\acute{x}_8, 0.6, 0.5, 0.5), (\acute{x}_9, 0.6, 0.5, 0.5), (\acute{x}_{10}, 0.6, 0.5, 0.5), (\acute{x}_{11}, 0.6, 0.5, 0.5)\}.\end{aligned}$$

We can see that $(\zeta, \mathbb{M})$ is an SNSS over $\mathcal{K}$. By Theorem 1, it is evident that $\zeta(\theta)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra for all $\theta \in \mathbb{M}$. Since $\mathcal{T}_{\mathcal{A}}(u) \geq \min(\mathcal{T}_{\mathcal{A}}(e), 0.5)$, $\mathcal{I}_{\mathcal{A}}(u) \geq \min(\mathcal{I}_{\mathcal{A}}(e), 0.5)$, $\mathcal{F}_{\mathcal{A}}(u) \leq \max(\mathcal{F}_{\mathcal{A}}(e), 0.5)$ and $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5)$, $\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5)$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5)$, for all $u, v \in G$. This implies that $(\zeta, \mathbb{M})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$.

Theorem 5. *If the pair $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are two $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebras, then $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N})$ is also an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$.*

Proof. Consider a K-algebra $\mathcal{K}$. Let for any $(l, m) \in \mathbb{Q}$, $\zeta(l)$ and $\eta(m)$ be two $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebras, where $\mathbb{Q} = \mathbb{M} \times \mathbb{N}$. This implies that $\vartheta(l, m) = \zeta(l) \cap \eta(m)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Hence, $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N})$ is an $(\in, \in, \vee q)$-single-valued neutrosophic soft K-subalgebra over $\mathcal{K}$. □

Example 5. *Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where G is the cyclic group of order nine given as $G = \{e, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8\}$, and Cayley's table for $\odot$ is given in Example 2.*
Consider a set of parameters $\mathbb{M} = \{l_1, l_2\}$ and a set-valued function $\zeta : \mathbb{M} \to P(G)$ defined as:

$$\begin{aligned}\zeta(l_1) = \{&(e, 0.9, 0.3, 0.3), (w, 0.6, 0.3, 0.4), (w^2, 0.6, 0.3, 0.4), (w^3, 0.6, 0.3, 0.4),\\ &(w^4, 0.6, 0.3, 0.4), (w^5, 0.6, 0.3, 0.4), (w^6, 0.6, 0.3, 0.4),\\ &(w^7, 0.6, 0.3, 0.4), (w^8, 0.6, 0.3, 0.4)\},\end{aligned}$$

$$\begin{aligned}\zeta(l_2) = \{&(e, 0.9, 0.8, 0.4), (w, 0.8, 0.5, 0.5), (w^2, 0.8, 0.5, 0.5), (w^3, 0.8, 0.5, 0.5),\\ &(w^4, 0.8, 0.5, 0.5), (w^5, 0.8, 0.5, 0.5), (w^6, 0.8, 0.5, 0.5),\\ &(w^7, 0.8, 0.5, 0.5), (w^8, 0.8, 0.5, 0.5)\},\end{aligned}$$

Now, consider a set $\mathbb{N} = \{t_1, t_2\}$ of parameters and a set-valued function $\eta : \mathbb{N} \to P(G)$ defined as:

$$\begin{aligned}\eta(t_1) = \{&(e, 0.9, 0.8, 0.3), (w, 0.6, 0.7, 0.4), (w^2, 0.6, 0.7, 0.4), (w^3, 0.6, 0.7, 0.4),\\ &(w^4, 0.6, 0.7, 0.4), (w^5, 0.6, 0.7, 0.4), (w^6, 0.6, 0.7, 0.4),\\ &(w^7, 0.6, 0.7, 0.4), (w^8, 0.6, 0.7, 0.4)\},\\ \eta(t_2) = \{&(e, 0.7, 0.7, 0.5), (w, 0.5, 0.6, 0.3), (w^2, 0.5, 0.6, 0.3), (w^3, 0.5, 0.6, 0.3),\\ &(w^4, 0.5, 0.6, 0.3), (w^5, 0.5, 0.6, 0.3), (w^6, 0.5, 0.6, 0.3),\\ &(w^7, 0.5, 0.6, 0.3), (w^8, 0.5, 0.6, 0.3)\}.\end{aligned}$$

Clearly, the set $(\zeta, \mathbb{M})$ and the set $(\eta, \mathbb{N})$ comprises $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras. By Theorem 1, the sets $\zeta(l)$, $\eta(t)$ are $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebras for all $l \in \mathbb{M}$ and for all $t \in \mathbb{N}$. For all $(l, t) \in \mathbb{M} \times \mathbb{N}$, $(\zeta, \mathbb{M})$ AND $(\eta, \mathbb{N})$= $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N}) = (\vartheta, \mathbb{M} \times \mathbb{N})$, where a set-valued function $\vartheta : \mathbb{M} \times \mathbb{N} \to P(G)$ is defined as:

$$\begin{aligned}\vartheta(l_1, t_1) = \{&(e, 0.9, 0.3, 0.3), (w, 0.6, 0.3, 0.4), (w^2, 0.6, 0.3, 0.4), (w^3, 0.6, 0.3, 0.4),\\ &(w^4, 0.6, 0.3, 0.4), (w^5, 0.6, 0.3, 0.4), (w^6, 0.6, 0.3, 0.4),\\ &(w^7, 0.6, 0.3, 0.4), (w^8, 0.6, 0.3, 0.4)\},\end{aligned}$$

$$\begin{aligned}\vartheta(l_2, t_2) = \{&(e, 0.7, 0.7, 0.5), (w, 0.5, 0.5, 0.5), (w^2, 0.5, 0.5, 0.5), (w^3, 0.5, 0.5, 0.5),\\ &(w^4, 0.5, 0.5, 0.5), (w^5, 0.5, 0.5, 0.5), (w^6, 0.5, 0.5, 0.5),\\ &(w^7, 0.5, 0.5, 0.5), (w^8, 0.5, 0.5, 0.5)\},\end{aligned}$$

$$\begin{aligned}\vartheta(l_1, t_2) = \{&(e, 0.7, 0.3, 0.5), (w, 0.5, 0.3, 0.4), (w^2, 0.5, 0.3, 0.4), (w^3, 0.5, 0.3, 0.4),\\ &(w^4, 0.5, 0.3, 0.4), (w^5, 0.5, 0.3, 0.4), (w^6, 0.5, 0.3, 0.4),\\ &(w^7, 0.5, 0.3, 0.4), (w^8, 0.5, 0.3, 0.4)\},\end{aligned}$$

$$\begin{aligned}\vartheta(l_2, t_1) = \{&(e, 0.9, 0.8, 0.4), (w, 0.6, 0.5, 0.5), (w^2, 0.6, 0.5, 0.5), (w^3, 0.6, 0.5, 0.5),\\ &(w^4, 0.6, 0.5, 0.5), (w^5, 0.6, 0.5, 0.5), (w^6, 0.6, 0.5, 0.5),\\ &(w^7, 0.6, 0.5, 0.5), (w^8, 0.6, 0.5, 0.5)\}.\end{aligned}$$

Clearly, $(\vartheta, \mathbb{M} \times \mathbb{N}) = \vartheta(l, t) = \zeta(l) \cap \eta(t)$for all $(l, t) \in \mathbb{M} \times \mathbb{N}$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras.

Theorem 6. *If $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are two $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebras of $\mathcal{K}$ with $\mathbb{M} \cap \mathbb{N} \neq \emptyset$, then $(\zeta, \mathbb{M}) \cap (\zeta, \mathbb{N})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebras over $\mathcal{K}$.*

Proof. By Definition 16, for any $l \in \mathbb{Q}$, both $\zeta(l)$ and $\eta(l)$ are $(\in, \in \vee q)$-single-valued neutrosophic *K*-subalgebras since $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are $(\in, \in \vee q)$-single-valued neutrosophic soft *K*-subalgebras. Therefore, $\vartheta(l) = \zeta(l) \cap \eta(l)$ is an $(\in, \in \vee q)$-single-valued neutrosophic *K*-subalgebra. Consequently, $(\zeta, \mathbb{M}) \cap (\eta, \mathbb{N})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft *K*-subalgebra of $\mathcal{K}$. □

Example 6. *Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where G is the cyclic group of order nine given as $G = \{e, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8\}$, and Cayley's table for $\odot$ is given in Example 2. Consider a set of parameters $\mathbb{M} = \{l_1, l_2\}$ and set-valued function $\zeta : \mathbb{M} \to P(G)$ and a set of parameters $\mathbb{N} = \{t_1, t_2\}$ with set-valued functions $\eta : \mathbb{N} \to P(G)$, which are defined in Example 5.*

We show that if $\mathbb{M} \cap \mathbb{N} \neq \emptyset$, then $(\zeta, \mathbb{M}) \cap (\zeta, \mathbb{N})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra over $\mathcal{K}$. Now, if $\mathbb{M} = \{l_1, l_2\}$, $\mathbb{N} = \{t_1, t_2\}$, $\mathbb{M} \cap \mathbb{N} \neq \emptyset$, $(\zeta, \mathbb{M}) \cap (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$ and $\vartheta : \mathbb{Q} \to P(Z)$ is a set-valued function, where $\mathbb{Q} = \mathbb{M} \cap \mathbb{N}$, then the following cases can be considered.

(i) $\mathbb{Q} = \mathbb{M} \cap \mathbb{N} = \{l_1\}$, *whenever* $t_1 = l_1$ *or* $t_2 = l_1$.
(ii) $\mathbb{Q} = \mathbb{M} \cap \mathbb{N} = \{l_2\}$, *whenever* $t_1 = l_2$ *or* $t_2 = l_2$.
(iii) $\mathbb{Q} = \mathbb{M} \cap \mathbb{N} = \{t_1\}$, *whenever* $l_1 = t_1$ *or* $l_2 = t_1$.
(iv) $\mathbb{Q} = \mathbb{M} \cap \mathbb{N} = \{t_2\}$, *whenever* $l_1 = t_2$ *or* $l_2 = t_2$.

Now, in each case, set-valued function ϑ *is defined as:*

$\vartheta(l_1) = \zeta(l_1) \cap \eta(t_1) = \vartheta(t_1) = \{(e, 0.9, 0.3, 0.3), (w, 0.6, 0.3, 0.4), (w^2, 0.6, 0.3, 0.4), (w^3, 0.6, 0.3, 0.4),$
$(w^4, 0.6, 0.3, 0.4), (w^5, 0.6, 0.3, 0.4), (w^6, 0.6, 0.3, 0.4), (w^7, 0.6, 0.3, 0.4), (w^8, 0.6, 0.3, 0.4)\}$,
where $t_1 = l_1$ *or* $l_1 = t_1$,

$\vartheta(l_2) = \zeta(l_2) \cap \eta(t_1) = \{(e, 0.9, 0.8, 0.4), (w, 0.6, 0.5, 0.5), (w^2, 0.6, 0.5, 0.5), (w^3, 0.6, 0.5, 0.5),$
$(w^4, 0.6, 0.5, 0.5), (w^5, 0.6, 0.5, 0.5), (w^6, 0.6, 0.5, 0.5), (w^7, 0.6, 0.5, 0.5), (w^8, 0.6, 0.5, 0.5)\}$, *where* $t_1 = l_2$,

$\vartheta(t_2) = \zeta(l_1) \cap \eta(t_2) = \{(e, 0.7, 0.3, 0.5), (w, 0.5, 0.3, 0.4), (w^2, 0.5, 0.3, 0.4), (w^3, 0.5, 0.3, 0.4),$
$(w^4, 0.5, 0.3, 0.4), (w^5, 0.5, 0.3, 0.4), (w^6, 0.5, 0.3, 0.4), (w^7, 0.5, 0.3, 0.4), (w^8, 0.5, 0.3, 0.4)\}$, *where* $l_1 = t_2$.

Clearly $\vartheta(\theta)$, *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra for all* $\theta \in \mathbb{Q}$, *which implies that* $(\vartheta, \mathbb{Q})$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\mathcal{K}$, *where* $\mathbb{Q} = \mathbb{M} \cap \mathbb{N}$.

Theorem 7. *If* $(\zeta, \mathbb{M})$ *is an* $(\in, \in \vee q)$ *K-subalgebra of* $\mathcal{K}$, *then for a non-empty collection* $\{(\vartheta_i, \mathbb{N}_i) \mid i \in \Omega\}$ *of* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebras of* $(\zeta, \mathbb{M})$, *the following results hold:*

(i) $\bigcap_{i \in \Omega} {}_{ex} (\vartheta_i, \mathbb{N}_i)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $(\zeta, \mathbb{M})$.
(ii) $\bigwedge_{i \in \Omega} (\vartheta_i, \mathbb{N}_i)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\bigwedge_{i \in \Omega} (\zeta, \mathbb{M})$.
(iii) *For the disjoint intersection of two parametric sets* $\mathbb{N}_i, \mathbb{N}_j$, $\forall i, j \in \Omega$, $\bigvee_{i \in \Omega} {}_{ex} (\vartheta_i, \mathbb{N}_i)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\bigvee_{i \in \Omega} (\zeta, \mathbb{M})$.

Proof. The proof follows from Definitions 14, 18 and 19. □

Theorem 8. *Let* $(\zeta, \mathbb{M})$ *and* $(\eta, \mathbb{N})$ *be two* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebras, then* $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N})$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\mathcal{K}$.

Proof. By Definition 14, let for any $l \in \mathbb{Q}$ the following three conditions arise:

(1) If $l \in \mathbb{M} - \mathbb{N}$, then $\vartheta(l) = \zeta(l)$ is an $(\in, \in, \vee q)$-single-valued neutrosophic K-subalgebra since $(\zeta, \mathbb{M})$ is an $(\in, \in, \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$.
(2) If $l \in \mathbb{N} - \mathbb{M}$, then we have $\vartheta(l) = \eta(l)$, which is an $(\in, \in, \vee q)$-single-valued neutrosophic K-subalgebra since $(\eta, \mathbb{N})$ is an $(\in, \in, \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$.
(3) Now, if $l \in \mathbb{M} \cap \mathbb{N}$, then $\vartheta(l) = \zeta(l) \cap \eta(l)$, which is also an $(\in, \in, \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Therefore, in each case, $\vartheta(l)$ is an $(\in, \in, \vee q)$-single-valued neutrosophic K-subalgebra. Consequently, $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N})$ is an $(\in, \in, \vee q)$-single-valued neutrosophic soft K-subalgebras of $\mathcal{K}$.

□

Example 7. *Consider a K-algebra* $\mathcal{K} = (G, \cdot, \odot, e)$, *where G is the cyclic group of order nine given as* $G = \{e, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8\}$, *and Cayley's table for* $\odot$ *is given in Example 2. Consider a set of* *parameters* $\mathbb{M} = \{l_1, l_2\}$, *set-valued function* $\zeta : \mathbb{M} \to P(G)$ *and a set of parameters* $\mathbb{N} = \{t_1, t_2\}$ *with set-valued functions* $\eta : \mathbb{N} \to P(G)$, *where* ζ *at parameters* l_1, l_2 *and* η *at parameters* t_1, t_2 *are defined as:*

$$\begin{aligned}\zeta(l_1) = \{&(e, 0.9, 0.3, 0.3), (w, 0.6, 0.3, 0.4), (w^2, 0.6, 0.3, 0.4), (w^3, 0.6, 0.3, 0.4),\\ &(w^4, 0.6, 0.3, 0.4), (w^5, 0.6, 0.3, 0.4), (w^6, 0.6, 0.3, 0.4),\\ &(w^7, 0.6, 0.3, 0.4), (w^8, 0.6, 0.3, 0.4)\},\end{aligned}$$

$$\begin{aligned}\zeta(l_2) = \{&(e, 0.9, 0.8, 0.4), (w, 0.8, 0.5, 0.5), (w^2, 0.8, 0.5, 0.5), (w^3, 0.8, 0.5, 0.5),\\ &(w^4, 0.8, 0.5, 0.5), (w^5, 0.8, 0.5, 0.5), (w^6, 0.8, 0.5, 0.5),\\ &(w^7, 0.8, 0.5, 0.5), (w^8, 0.8, 0.5, 0.5)\},\end{aligned}$$

and

$$\begin{aligned}\eta(t_1) = \{&(e, 0.9, 0.8, 0.3), (w, 0.6, 0.7, 0.4), (w^2, 0.6, 0.7, 0.4), (w^3, 0.6, 0.7, 0.4),\\ &(w^4, 0.6, 0.7, 0.4), (w^5, 0.6, 0.7, 0.4), (w^6, 0.6, 0.7, 0.4),\\ &(w^7, 0.6, 0.7, 0.4), (w^8, 0.6, 0.7, 0.4)\},\\ \eta(t_2) = \{&(e, 0.7, 0.7, 0.5), (w, 0.5, 0.6, 0.3), (w^2, 0.5, 0.6, 0.3), (w^3, 0.5, 0.6, 0.3),\\ &(w^4, 0.5, 0.6, 0.3), (w^5, 0.5, 0.6, 0.3), (w^6, 0.5, 0.6, 0.3),\\ &(w^7, 0.5, 0.6, 0.3), (w^8, 0.5, 0.6, 0.3)\}.\end{aligned}$$

Clearly, by Example 5, $(\zeta, \mathbb{M})$ *and* $(\eta, \mathbb{N})$ *are* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebras. Now, to show that* $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\mathcal{K}$*, where* $\mathbb{Q} = \mathbb{M} \cup \mathbb{N} = \{l_1, l_2, t_1, t_2\}$*, then by Definition 14, the following conditions can be considered:*

(i) *If* $\theta \in \mathbb{M} - \mathbb{N}$*, then* $\theta = \{l_1, l_2\}$ *and set-valued function* ϑ *at parameters* l_1, l_2 *is defined as:*
$\vartheta(l_1) = \zeta(l_1) = \{(e, 0.9, 0.3, 0.3), (w, 0.6, 0.3, 0.4), (w^2, 0.6, 0.3, 0.4), (w^3, 0.6, 0.3, 0.4), (w^4, 0.6, 0.3, 0.4), (w^5, 0.6, 0.3, 0.4), (w^6, 0.6, 0.3, 0.4), (w^7, 0.6, 0.3, 0.4), (w^8, 0.6, 0.3, 0.4)\}$,
$\vartheta(l_2) = \zeta(l_2) = \{(e, 0.9, 0.8, 0.4), (w, 0.8, 0.5, 0.5), (w^2, 0.8, 0.5, 0.5), (w^3, 0.8, 0.5, 0.5), (w^4, 0.8, 0.5, 0.5), (w^5, 0.8, 0.5, 0.5), (w^6, 0.8, 0.5, 0.5), (w^7, 0.8, 0.5, 0.5), (w^8, 0.8, 0.5, 0.5)\}$.
Since $\zeta(\theta)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra, therefore* $\vartheta(\theta)$ *is also an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\mathcal{K}$*, for all* $\theta \in \mathbb{M} - \mathbb{N}$.

(ii) *If* $\theta \in \mathbb{N} - \mathbb{M}$*, then* $\theta = \{t_1, t_2\}$ *and set-valued function* ϑ *at parameters* t_1, t_2 *is defined as:*
$\vartheta(t_1) = \eta(t_1) = \{(e, 0.9, 0.8, 0.3), (w, 0.6, 0.7, 0.4), (w^2, 0.6, 0.7, 0.4), (w^3, 0.6, 0.7, 0.4), (w^4, 0.6, 0.7, 0.4), (w^5, 0.6, 0.7, 0.4), (w^6, 0.6, 0.7, 0.4), (w^7, 0.6, 0.7, 0.4), (w^8, 0.6, 0.7, 0.4)\}$,
$\vartheta(t_2) = \eta(t_2) = \{(e, 0.7, 0.7, 0.5), (w, 0.5, 0.6, 0.3), (w^2, 0.5, 0.6, 0.3), (w^3, 0.5, 0.6, 0.3), (w^4, 0.5, 0.6, 0.3), (w^5, 0.5, 0.6, 0.3), (w^6, 0.5, 0.6, 0.3), (w^7, 0.5, 0.6, 0.3), (w^8, 0.5, 0.6, 0.3)\}$.
Since $\eta(\theta)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra, therefore* $\vartheta(\theta)$ *is also an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\mathcal{K}$*, for all* $\theta \in \mathbb{N} - \mathbb{M}$.

(iii) *Now, if* $\theta \in \mathbb{M} \cap \mathbb{N}$*, then* $\vartheta(\theta) = \zeta(\theta) \cap \eta(\theta)$*. By Example 6, it follows that* $\vartheta(\theta)$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic K-subalgebra of* $\mathcal{K}$*, for all* $\theta \in \mathbb{M} \cap \mathbb{N}$*. Therefore,* $(\zeta, \mathbb{M}) \cap_{ex} (\eta, \mathbb{N}) = (\vartheta, \mathbb{Q})$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\mathcal{K}$.

Theorem 9. *Let* $(\zeta, \mathbb{M})$*,* $(\eta, \mathbb{N})$ *be two* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebras with* $\mathbb{M} \cap \mathbb{N} = \emptyset$*, then* $(\zeta, \mathbb{M}) \cup_{ex} (\eta, \mathbb{N})$ *is an* $(\in, \in \vee q)$*-single-valued neutrosophic soft K-subalgebra of* $\mathcal{K}$.

Proof. The proof follows from Definition 15. □

We denote the set of all $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras of $\mathcal{K}$ by $\mathbb{N}(G, \mathbb{R})$.

Theorem 10. *Under the ordering relation* $\subset$, $(\mathbb{SN}(G,\mathbb{R}),\cup_{ex},\cap)$ *is a complete distributive lattice.*

Proof. Suppose that $(\zeta,\mathbb{M}),(\eta,\mathbb{N}) \in \mathbb{SN}(G,\mathbb{R})$, $(\zeta,\mathbb{M}) \cup_{ex} (\eta,\mathbb{N}) \in \mathbb{SN}(G,\mathbb{R})$ and $(\zeta,\mathbb{M}) \cap (\eta,\mathbb{N}) \in \mathbb{SN}(G,\mathbb{R})$. Consider $\{(\zeta,\mathbb{M}),(\eta,\mathbb{N})\}$ are an arbitrary collection of $(\mathbb{SN}(G,\mathbb{R}),\cup_{ex},\cap)$, since $(\zeta,\mathbb{M}) \cup_{ex} (\eta,\mathbb{N})$ is the supremum of $(\zeta,\mathbb{M})$ and $(\zeta,\mathbb{M}) \cap (\eta,\mathbb{N})$ the infimum of $(\eta,\mathbb{N})$, which shows that $(\mathbb{SN}(G,\mathbb{R}),\cup_{ex},\cap)$ is a complete lattice.

In order to show that it is a complete distributive lattice, i.e., for all $(\zeta,\mathbb{M}),(\eta,\mathbb{N}),(\vartheta,\mathbb{Q}) \in \mathbb{SN}(G,\mathbb{R})$, $(\zeta,\mathbb{M}) \cap ((\eta,\mathbb{N}) \cup_{ex} (\vartheta,\mathbb{Q})) = ((\zeta,\mathbb{M}) \cap (\eta,\mathbb{N})) \cup_{ex} ((\zeta,\mathbb{M}) \cap (\vartheta,\mathbb{Q}))$; let us suppose that $(\zeta,\mathbb{M}) \cap ((\eta,\mathbb{N}) \cup_{ex} (\vartheta,\mathbb{Q})) = (\mathbb{I},\mathbb{M} \cap (\mathbb{N} \cup \mathbb{Q}))$, $((\zeta,\mathbb{M}) \cap (\eta,\mathbb{N})) \cup_{ex} ((\zeta,\mathbb{M}) \cap (\vartheta,\mathbb{Q})) = (\mathbb{J},(\mathbb{M} \cap \mathbb{N}) \cup (\mathbb{M} \cap \mathbb{Q})) = (\mathbb{J},\mathbb{M} \cap (\mathbb{N} \cup \mathbb{Q}))$. For any $\theta \in \mathbb{M} \cap (\mathbb{N} \cup \mathbb{Q})$, $\theta \in \mathbb{M}$ and $\theta \in \mathbb{N} \cup \mathbb{Q}$, the following cases arise:

(i) $\theta \in \mathbb{M}$, $\theta \notin \mathbb{N}$ and $\theta \in \mathbb{Q}$. Then, $\mathbb{K}(\theta) = \zeta(\theta) \cap \vartheta(\theta) = \mathbb{J}(\theta)$,
(ii) $\theta \in \mathbb{M}$, $\theta \in \mathbb{N}$ and $\theta \notin \mathbb{Q}$. Then, $\mathbb{K}(\theta) = \zeta(\theta) \cap \eta(\theta) = \mathbb{J}(\theta)$,
(iii) $\theta \in \mathbb{M}$, $\theta \in \mathbb{N}$ and $\theta \in \mathbb{Q}$. Then, $\mathbb{K}(\theta) = \zeta(\theta) \cap (\eta(\theta) \cup \vartheta(\theta))$
$= (\zeta(\theta) \cap \eta(\theta)) \cup (\zeta(\theta) \cap \vartheta(\theta)) = \mathbb{J}(\theta)$.

Both $\mathbb{J}$ and $\mathbb{K}$ being the same operators implies that $(\zeta,\mathbb{M}) \cap ((\eta,\mathbb{N}) \cup_{ex} (\vartheta,\mathbb{Q})) = ((\zeta,\mathbb{M}) \cap (\eta,\mathbb{N})) \cup_{ex} ((\zeta,\mathbb{M}) \cap (\vartheta,\mathbb{Q}))$. This completes the proof. $\square$

Definition 26. *The extended product of two single-valued neutrosophic soft sets is denoted by* $(\zeta,\mathbb{M}) \underset{\cdot}{\odot} (\eta,\mathbb{N}) = (\zeta \circ \eta,\mathbb{Q})$, *where* $\mathbb{Q} = \mathbb{M} \cup \mathbb{N}$ *and* $(\zeta,\mathbb{M})$ *and* $(\eta,\mathbb{N})$ *are two single-valued neutrosophic soft sets over* Z, *defined as for all* $\theta \in \mathbb{Q}$.

$$(\zeta \circ \eta)(\theta) = \begin{cases} \zeta(\theta) & \text{if } \theta \in \mathbb{M} - \mathbb{N}, \\ \eta(\theta) & \text{if } \theta \in \mathbb{N} - \mathbb{M}, \\ \zeta(\theta) \circ \eta(\theta) & \text{if } \theta \in \mathbb{M} \cap \mathbb{N}. \end{cases}$$

Here, $\zeta(\theta) \circ \eta(\theta)$ *is the product of two single-valued neutrosophic sets.*

Lemma 1. $(\zeta_1,\mathbb{M}),(\zeta_2,\mathbb{M}),(\eta_1,\mathbb{N}),(\eta_2,\mathbb{N})$ *are two SNSSs over* $\mathcal{K}$ *such that* $(\zeta_1,\mathbb{M}) \subset (\zeta_2,\mathbb{M})$ *and* $(\eta_1,\mathbb{N}) \subset (\eta_2,\mathbb{N})$. *Then:*

$$\begin{aligned} &(a)\ (\zeta_1,\mathbb{M})\underset{\cdot}{\odot}(\eta_1,\mathbb{N}) &&\subset(\zeta_2,\mathbb{M})\underset{\cdot}{\odot}(\eta_2,\mathbb{N}), \\ &(b)\ (\zeta_1,\mathbb{M}) \cap (\eta_1,\mathbb{N}) &&\subset(\zeta_2,\mathbb{M}) \cap (\eta_2,\mathbb{N}), \\ &(c)\ (\zeta_1,\mathbb{M}) \cap_{ex} (\eta_1,\mathbb{N}) &&\subset(\zeta_2,\mathbb{M}) \cap_{ex} (\eta_2,\mathbb{N}), \\ &(d)\ (\zeta_1,\mathbb{M}) \cup (\eta_1,\mathbb{N}) &&\subset(\zeta_2,\mathbb{M}) \cup (\eta_2,\mathbb{N}), \\ &(e)\ (\zeta_1,\mathbb{M}) \cup_{ex} (\eta_1,\mathbb{N}) &&\subset(\zeta_2,\mathbb{M}) \cup_{ex} (\eta_2,\mathbb{N}). \end{aligned}$$

Lemma 2. *Let* $(\zeta,\mathbb{M}),(\eta,\mathbb{N})$ *and* $(\vartheta,\mathbb{Q})$ *be SNSSs over* $\mathcal{K}$. *Then,* $(\zeta,\mathbb{M})\underset{\cdot}{\odot}((\eta,\mathbb{N})\underset{\cdot}{\odot}(\vartheta,\mathbb{Q})) = ((\zeta,\mathbb{M})\underset{\cdot}{\odot}(\eta,\mathbb{N}))\underset{\cdot}{\odot}(\vartheta,\mathbb{Q})$, *where* $\underset{\cdot}{\odot}$ *is the operation of the product of SNSSs over* $\mathcal{K}$.

Theorem 11. *xxx Let* $\mathcal{K}$ *be a* K-*algebra. If* $(\zeta,\mathbb{M})$ *and* $(\eta,\mathbb{N})$ *are* $(\in,\in,\vee q)$*-single-valued neutrosophic soft* K-*subalgebras, then* $(\zeta,\mathbb{M}) \underset{\cdot}{\odot} (\eta,\mathbb{N})$ *is an* $(\in,\in\vee q)$*-single-valued neutrosophic soft* K-*subalgebras of* $\mathcal{K}$.

Proof. The proof follows from Definition 26. $\square$

Example 8. *Consider a K-algebra* $\mathcal{K} = (G,\cdot,\odot,e)$, *where G is the cyclic group of order nine given as* $G = \{e,w,w^2,w^3,w^4,w^5,w^6,w^7,w^8\}$, *and Cayley's table for* $\odot$ *is given in Example 2. Consider a set of parameters* $\mathbb{M} = \{l_1,l_2\}$ *and a set-valued function* $\zeta : \mathbb{M} \to P(G)$ *defined as:*

$$\zeta(l_1) = \{(e,0.9,0.3,0.3),(s,0.6,0.3,0.4)\}, \text{ for all } s \neq e \in G.$$

$$\zeta(l_2) = \{(e, 0.9, 0.8, 0.4), (s', 0.8, 0.5, 0.5)\}, \text{ for all } s' \neq e \in G.$$

Now, we consider a set $\mathbb{N} = \{t_1, t_2\}$ of parameters and a set-valued function $\eta : \mathbb{N} \to P(G)$, which is defined as:

$$\eta(t_1) = \{(e, 0.9, 0.8, 0.3), (w, 0.6, 0.7, 0.4)\}, \text{ for all } w \neq e \in G.$$

$$\eta(t_2) = \{(e, 0.7, 0.7, 0.5), (w', 0.5, 0.6, 0.3)\}, \text{ for all } w' \neq e \in G.$$

Clearly, the set $(\zeta, \mathbb{M})$ and the set $(\eta, \mathbb{N})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra for all $l_1, l_2 \in \mathbb{M}$ and $t_1, t_2 \in \mathbb{N}$. Now, we show that $(\zeta, \mathbb{M}) \odot (\eta, \mathbb{N}) = (\zeta \circ \eta, \mathbb{Q})$, where $\mathbb{Q} = \mathbb{M} \cup \mathbb{N}$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$. By Definition 26, the following conditions can be considered:

- *(i)* *If $\theta \in \mathbb{M} - \mathbb{N}$, then $\theta = \{l_1, l_2\}$ and $(\zeta \circ \eta)(\theta) = \zeta(\theta)$. Since $\zeta(\theta)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra, therefore $(\zeta \circ \eta)$ is also an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, for all $\theta \in \mathbb{M} - \mathbb{N}$.*
- *(ii)* *If $\theta \in \mathbb{N} - \mathbb{M}$, then $\theta = \{t_1, t_2\}$ and $(\zeta \circ \eta)(\theta) = \eta(\theta)$. Therefore, $(\zeta \circ \eta)$ is also an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ since $\eta(\theta)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra for all $\theta \in \mathbb{N} - \mathbb{M}$.*
- *(iii)* *If $\theta \in \mathbb{M} \cap \mathbb{N}$, then $(\zeta \circ \eta)(\theta) = \zeta(\theta) \circ \eta(\theta)$, where $\zeta(\theta) \circ \eta(\theta)$ is the product of two single-valued neutrosophic sets at parameter θ. Then, by Example 6, four conditions can be considered since $\theta \in \mathbb{M} \cap \mathbb{N}$ and corresponding to each condition product can be calculated as:*
  $(\zeta \circ \eta)(l_1) = \zeta(l_1) \circ \eta(t_1) = \{\langle (e, e), 0.9, 0.3, 0.3\rangle, \langle (e, w), 0.6, 0.3, 0.4\rangle, \langle (s, e), 0.6, 0.3, 0.4\rangle, \langle (s, w), 0.6, 0.3, 0.4\rangle\}$,
  $(\zeta \circ \eta)(l_2) = \zeta(l_2) \circ \eta(t_1) = \{\langle (e, e), 0.9, 0.8, 0.4\rangle, \langle (e, w), 0.6, 0.7, 0.4\rangle, \langle (s', e), 0.8, 0.5, 0.5\rangle, \langle (s', w), 0.6, 0.5, 0.5\rangle\}$,
  $(\zeta \circ \eta)(t_2) = \zeta(l_1) \circ \eta(t_2) = \{\langle (e, e), 0.7, 0.3, 0.5\rangle, \langle (e, w'), 0.5, 0.3, 0.3\rangle, \langle (s, e), 0.6, 0.3, 0.5\rangle, \langle (s, w'), 0.5, 0.3, 0.4\rangle\}$.

Clearly, $(\zeta \circ \eta)(\theta)$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, for all $\theta \in \mathbb{M} \cap \mathbb{N}$, which shows that $(\zeta, \mathbb{M}) \odot (\eta, \mathbb{N})$ is an $(\in, \in \vee q)$-single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$.

Theorem 12. *Let $\mathcal{K}$ be a K-algebra. Then, under the ordering relation $\subset$, $(\mathbb{SN}(G, \mathbb{R}), \odot, \cap)$ is a complete lattice.*

Proof. The proof is straightforward. □

5. Conclusions

The world of science and its related fields have accomplished such complicated processes for which consistent and complete information is not always conceivable. For the last few decades, a number of theories and postulates have been introduced by many researchers to handle indeterminate constituents in science and technologies. These theories include the theory of probability, interval mathematics, fuzzy set theory, intuitionistic fuzzy set theory, neutrosophic set theory, etc. Among all these theories, a powerful mathematical tool to deal with indeterminate and inconsistent data is the neutrosophic set theory introduced by Smarandache in 1998. This theory provides a mathematical model to cope up with executions having complex phenomena towards uncertainty. In 1999, Molodtsov introduced the concept of soft set theory to deal with the problems involving indeterminacy without setting the membership function. This theory provides a parameterized consideration to uncertainties. We have applied these mathematical representations in collaboration to scrutinize the factor of uncertainty in *K*-algebras. A *K*-algebra is a new kind of non-classic logical algebra. We have introduced the notion of single-valued neutrosophic soft *K*-algebras and studied related properties. To give a generalized point of view of single-valued neutrosophic soft

K-algebras, we have proposed the concept of $(\in, \in \vee q)$-single-valued neutrosophic soft K-algebras and investigated various conclusive results with some numerical examples. In our opinion, the future study of K-algebras can be connected with: (1) K-modules and single-valued neutrosophic K-modules; (2) rough K-algebras and single-valued neutrosophic rough K-algebras; (3) hyper-K-algebras and single-valued neutrosophic K-algebras.

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