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APPROACH TO IMAGE SEGMENTATION BASED ON INTERVAL NEUTROSOPHIC SET

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Abstract. As a generalization of the fuzzy set and intuitionistic fuzzy set, the neutrosophic set (NS) have been developed to represent uncertain, imprecise, incomplete and inconsistent information existing in the real world. Now the interval neutrosophic set (INS) which is an expansion of the neutrosophic set have been proposed exactly to address issues with a set of numbers in the real unit interval, not just one specific number. After definition of concepts and operations, INS is applied to image segmentation. Images are converted to the INS domain, which is described using three membership interval sets: T, I and F. Then, in order to increase the contrast between membership and evaluate the indeterminacy, a fuzzy intensification for each element in the interval set is made and a score function in the INS is defined. Finally, the proposed method is employed to perform image segmentation using the traditional k-means clustering. The experimental results on a variety of images demonstrate that the proposed approach can segment different sorts of images. Especially, it can segment "clean" images and images with various levels of noise.

1. **Introduction.** In 1965, Zadeh L A [29] proposed a mathematical method of describing ambiguity – fuzzy set theory, dealing with the object and its fuzzy concept as a fuzzy set, and then building an appropriate membership function. Besides, this method analyzes the fuzzy object by utilizing operation and transformation of the fuzzy set, and can well express and deal with vagueness and uncertainty information.

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Since fuzzy theory is easier to be accepted by people than classical set theory, it is applied to handling uncertainty in various fields.

However, the traditional fuzzy set only uses one real number to represent the membership degree of the fuzzy set. As sometimes the membership is uncertain and hard to be defined by a crisp value [28], the concept of the interval-valued fuzzy set, which is used to express the uncertainty of the membership degree, was proposed [26]. In 1986, Atanassov K T [2] introduced the intuitionistic fuzzy set − an extension of the traditional fuzzy set, which takes the membership degree, non-membership and hesitancy degrees into consideration. Therefore, it is more flexible and practical to deal with vagueness and uncertainty compared with the traditional fuzzy set. Subsequently, the intuitionistic fuzzy set was extended to the interval-valued intuitionistic fuzzy set [3].

In [18], Liu X established the fuzzy theory using AFS algebras and AFS structure. Then he proposed the concept of fuzzy variables and a simple controlling model based on AFS theory. In 1999, Smarandache F put forward the concept of the neutrosophic set [25], which enhances measurement of independent uncertainty. And it is a generalization of the existing fuzzy set, the interval fuzzy set and the intuitionistic fuzzy set. The neutrosophic set describes fuzzy nature of the real world in a more detailed way with the true membership function, the indeterminate membership function and the false membership function. After that, Wang H et al. [28, 30] proposed the interval neutrosophic set to make the neutrosophic set more comprehensive.

The fact that image processing is a process from three-dimensional to two-dimensional images which increase the uncertainty of image information. Besides, many concepts in the image are also vague and imprecise. For example, there are no precise definitions for concepts like edge, smoothness and contrast, and they are fuzzy in nature. In addition, there is also fuzziness or uncertainty in the grey level of the image. First of all, the grey level is vague. If pixels are gradually brighten and there is a pixel with certain grey level in the intermediate zone, it's hard to judge whether it belongs to the "dark" or "bright" region. Secondly, due to the ambiguous boundary or the object contour, it is difficult to determine whether a pixel is an edge pixel or not. At the same time, grey levels are ambiguous when human vision processes images. Image segmentation is one of the most difficult tasks in image processing and pattern recognition, and it plays an important role in a variety of applications [9]. However, all of these uncertainties would cause the complexity of image segmentation.

Fuzzy theory is applied to the field of image segmentation in order to better express fuzzy information and concepts. Furthermore, the image segmentation with fuzzy theory retains more information from the original image than hard segmentation methods [27, 8]. Therefore, fuzzy image segmentation has become an important branch as well as a popular research topic in the field of image segmentation.

In recent years, fuzzy theory is added to the threshold method for image segmentation, such as the two-dimensional fuzzy entropy threshold method of s-function proposed by Cheng H D et al. [5]. Besides, image segmentation can be regarded as the classification of the feature space such as gray scale and texture. The clustering algorithms, such as the fuzzy clustering algorithm [4, 7] are also common, and can be used for image segmentation. According to a series of fuzzy C-means clustering algorithms, the IN and neutrosophic fuzzy clustering algorithm (NCM) was introduced into the traditional fuzzy C-means clustering algorithms in [14].

Recently, Guo Y et al. [9, 14] proposed some new clustering algorithms and image segmentation methods based on the neutrosophic set.

Because the interval neutrosophic set could make the neutrosophic set more consummate and its the capacity of processing the images with noises, in this paper, the interval neutrosophic set is employed to process the images with noise, and then an approach to image segmentation is proposed based on the interval neutrosophic set. First, the images are converted into interval neutrosophic images. Then, in order to increase the contrast between memberships and evaluate the indeterminacy, the fuzzy intensification for each element in the interval set is completed, and a score function in INS is also defined. Finally, the images in the INS domain are segmented using the traditional k-means clustering method. The experimental results concerning a variety of images demonstrate that the proposed approach can segment different images well.

The remainder of this paper is organized as follows. In section 2, related work is described. Experimental results and analysis are discussed in Section 3. Finally, conclusions are presented in Section 4.

2. **Related Work.**

2.1. **Neutrosophic Set.** Neutrosophy is a new branch of philosophy with new philosophical arguments. In [23], Smarandache F first introduced the non-standard analysis, and the fuzzy logic was extended to the neutrosophic logic. And the fuzzy set is generalized to the neutrosophic set similarly. The neutrosophic set (NS) and its some properties are discussed in [23, 24].

Definition 2.1. (Neutrosophic set). Let X be a universe of discourse, a neutrosophic set A is defined on universe X. An element x in set A is noted as $x(t, i, f)$, A can be expressed as:

$$A = \{[x, (T_A(x), I_A(x), F_A(x))] | x \in X\}$$

where, T, I and F are real standard or non-standard sets of $]^{-}0, 1^{+}[$, with $sup\ T = t\_sup$, $inf\ T = t\_inf$, $sup\ I = i\_sup$, $inf\ I = i\_inf$, $sup\ F = f\_sup$, $inf\ F = f\_inf$ and $n\_sup = t\_sup + i\_sup + f\_sup$, $n\_inf = t\_inf + i\_inf + f\_inf$, so $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$. T, I and F are called neutrosophic components. T is the degree of the true membership function in set A, I is the degree of the indeterminate membership function in set A and F is the degree of the false membership function in set A ([23]). Element $x(t, i, f)$ belongs to A in the following way: it is t% true, i% indeterminacy, and f% false, where t varies in T, i varies in I, and f varies in F([24]).

2.2. **Neutrosophic Image.** In the field of neutrosophic, we describe an image as an neutrosophic image (NI) [8].

Definition 2.2. (Neutrosophic image)[11, 12]. Let U be a universe of the discourse in this paper. $W = w * w$ is a collection of image pixels, where $W \subseteq U$ and w is an arguement, it was determined by the concrete situation and personality. Then a neutrosophic image can be characterized by three membership sets T, I and F [8, 9].

For a given image, we convert each pixel $P(i, j)$ in the image domain into neutrosophic domain. Then each pixel $P(i, j)$ in the neutrosophic image can be described as $P_{NS}(i,j)$, and $P_{NS}(i,j) = \{T(i,j), I(i,j), F(i,j)\}$. As the membership degrees $T(i,j)$, $I(i,j)$ and $F(i,j)$ are defined in [6, 13, 14, 15]:

$$T(i,j) = \frac{\bar{g}(i,j) - \bar{g}_{min}}{\bar{g}_{max} - \bar{g}_{min}} \tag{1}$$

$$\bar{g}(i,j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} g(m,n) \tag{2}$$

$$I(i,j) = \frac{\delta(i,j) - \delta_{min}}{\delta_{max} - \delta_{min}} \tag{3}$$

where,

$$\delta(i,j) = abs(g(i,j) - \bar{g}(i,j)) \tag{4}$$

$$F(i,j) = 1 - T(i,j) \tag{5}$$

where, $g(i,j)$ is the gray value of the pixel $P(i,j)$, $\bar{g}(i,j)$ is the region mean value of $g(i,j)$, and $\delta(i,j)$ is the absolute value of the difference between intensity $g(i,j)$ and its local mean value at $g(i,j)$. The value of I measures the indeterminacy degree of P_{NS}. And according to various noises, the w value varies.

2.3. **Interval Neutrosophic Set And Interval Neutrosophic Image.** When we apply the theory of the neutrosophic set to image processing [10], T, I and F are belonging to $]^{-}0, 1^{+}[$. However, due to the fact that the membership degree, non-membership degree and hesitation degree are represented by a single value in neutrosophic set, ambiguity of information cannot be well expressed. In order to define the neutrosophic image more precisely, the interval neutrosophic set is introduced and the interval neutrosophic image will be defined in this paper.

Definition 2.3. (Interval neutrosophic set) [28]. Let X be a universe of discourse, and A be a interval neutrosophic set defined on universe X. An element x in set A is noted as $x(t,i,f)$, A can be expressed as:

$$A = \{[x, (T_A(x), I_A(x), F_A(x))] | x \in X\}$$

For each point x in set A, the $T_A(x)$, $I_A(x)$ and $F_A(x) \subset [0,1]$, $T_A(x)$ denotes the membership degree of point x in set A, $I_A(x)$ denotes the hesitancy degree of point x in set A, and $F_A(x)$ denotes the non-membership degree of point x in set A. And $0 \leq \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \leq 3$.

When X is continuous, an interval neutrosophic set A can be expressed as $A = \int_X < T(x), I(x), F(x) >| x, x \in X$. When X is discrete, an interval neutrosophic set A can be expressed as $A = \sum_{i=1}^{n} < T(x_i), I(x_i), F(x_i) >| x_i, x_i \in X$.

An element x, called interval wise number, can be expressed as $x = ([T_1, T_2]$, $[I_1, I_2]$, $[F_1, F_2])$ in the interval neutrosophic set.

According to the advantages of the interval neutrosophic set, we define the interval neutrosophic image (INI) analogously as follows.

Definition 2.4. (Interval neutrosophic image). For a given image, we convert each pixel $P(i,j)$ in the image domain into interval neutrosophic domain. A pixel $P(i,j)$ in the image is described as P_{INS}, and $P_{INS}(i,j) = T(i,j), I(i,j), F(i,j)$. Here $P_{INS}(i,j)$ is an interval number set, it can be expressed as:

$$P_{INS}(i,j) = \{[T_1(i,j), T_2(i,j)], [I_1(i,j), I_2(i,j)], [F_1(i,j), F_2(i,j)]\} \tag{6}$$

In the neutrosophic set, the range of pixels is variational, and is affected by the surrounding noise. At the same time, taking singular points and edge points into account, we select different w to get new pixel $P_{INS}(i,j)$. There, T, I, F respectively denote the membership degree, hesitancy degree and non-membership degree.

2.4. **Intensification Operation.** In [19, 20], an intensification operation for fuzzy set was given, and above membership values were applied to modified intensifier operator , which used the square root of the maximum/ minimum to stretch contrast value using the formula. After image conversion, some areas will become blurred. Therefore, we use the intensification operation for images process, which can make images become clearer. The images segmentation effect will be better.

Definition 2.5. (Intensification operation). In fuzzy set[19], the intensification operation is defined as :

$$\mu(i,j) = \begin{cases} M \cdot \mu^2(i,j), & 0 \leq \mu(i,j) \leq t \\ 1 - 2 \cdot (1 - \mu(i,j))^2, & t \leq \mu(i,j) \leq 1 \end{cases}$$

where $'\mu(i,j)'$ is the membership value, M and t are constants. Here we let $M = 0.3$ and $t = 1$ in this paper.

2.5. **Score Function.** Score function has been extensive applicated in many areas of mathematics, both pure and applied, and is a key part of likelihood theory. For better image segmentation, a new score function is introduced to express the pixel-level features based on the uncertainty interval number in the interval neutrosophic image.

Definition 2.6. (Scoer function). Let $A = \{T_A, I_A, F_A\}$ be an interval neutrosophic set, the score function $S(A)$ can be defined as follows:

$$S(A) = \frac{1}{2} \sum_{t_A \in T_A, i_A \in I_A, f_A \in F_A} (t_A - i_A - f_A)$$

where, $t_A \in T_A, i_A \in I_A$ and $f_A \in F_A$.

2.6. **Clustering Analysis.**

Definition 2.7. (Clustering Algorithms). In fact, the clustering algorithm is to classify similar points into the same group [9, 1]. Let $X = X_i,\ \{i = 1,\ 2,\ ...,\ n\}$ be a data set, the clustering causes a division $C = C_1,\ C_2,\ ...,\ C_m$, which satisfies:

$$X = \bigcup_{i=1}^{m} C_i$$

where,

$$C_i \neq \emptyset, for \ \ i = 1,\ 2,\ ...,\ m$$

$$C_i \cap C_j = \emptyset,\ for\ i,\ j = 1,\ 2,\ ...,\ m,\ i \neq j$$

We regard the image as a set of features. To achieve the image segmentation by clustering the feature set. Among clustering methods, the k-means algorithm is widely used. It is an algorithm to combine the objects into k groups based on the features. In the present study, the k-means algorithm is used for image segmentation. As each cluster should be as compact as possible, it is important to define an objective function for a clustering analysis method. In [9] and [17], the objective function of the k-means is defined as follows:

$$J = \sum_{j=1}^{x} \sum_{i=1}^{k} \parallel X_i^j - C_j \parallel$$

where the $\parallel X_i^j - C_j \parallel$ is a distance measure between a data point and its cluster center.

2.7. **Interval Neutrosophic Image Segmentation.** All interval neutrosophic image segmentation[22, 21] steps can be summarized as follows:
Step 1: Input target image, read the pixels.
Step 2: Transform the image into interval neutrosophic image.
Step 3: Intensification operation for the interval neutrosophic image.
Step 4: Calculate the scoer function for each pixel.
Step 5: Cluster the score function values.
Step 6: Image segmentation.

3. **Experimental Results And Comparative Analysis.**

3.1. **Evaluation Standard.** In this section, we utilize an objective criterion $PSNR$ [16] to evaluate the image segmentation results. It is an abbreviation of "Peak Signal to Noise Ratio", which means the vertex signal reaches the Noise Ratio has certain limitations. $PSNR$ is generally used as an engineering project for maximum signal and background noise. $PSNR$ is a most common and widely used objective measure evaluation quality. Although many experimental results showed that $PSNR's$ score is not consistent with the visual quality seen by human eyes, it is possible that image with higher $PSNR$ looks worse than those with lower $PSNR$. This is because visual perception of human eyes is not absolute, and its perceptual results can be influenced by many factors.

There is no standard to assess an images' quality objectively that can be accepted by all currently. So here we rely on the $PSNR$ to evaluate results. The output image will be different from the original image after image processing. To measure the quality of a processed image, we usually measure level of satisfactory accordinng to the $PSNR$. It is the relative value of the mean square error between the original image and the processed image.

Definition 3.1. (Peak Signal to Noise Ratio). Function of $PSNR$ is defined as follows:

$$PSNR = 10 \times lg(\frac{MAX_I^2}{MSE}) = 20 \times lg(\frac{MAX_I}{\sqrt{MSE}})$$

$$PSNR = 10 \times log_{10}(\frac{255 \times 255}{MSE})$$

TABLE 1. The value of PSNR in different ranges for the three images ($W = w * w$ is a collection of image pixels)

range	$w_1 = 3$ $w_2 = 3$	$w_1 = 3$ $w_2 = 4$	$w_1 = 3$ $w_2 = 5$	$w_1 = 3$ $w_2 = 6$	$w_1 = 3$ $w_2 = 7$	$w_1 = 3$ $w_2 = 8$
Lena1	21.9715	22.1662	21.5974	21.9076	21.1507	21.6422
Lena2	22.8407	23.1224	22.5366	22.8503	22.3337	22.6582
Pepper	20.9797	21.4961	20.4562	20.8171	20.1147	20.4185

$$MSE = \frac{1}{m * n} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|I(i,j) - K(i,j)\|^2$$

where the MAX_I represents the maximum value of the image color, in this paper we read the image as gray image, so the $MAX_I = 255$. MSE represents the mean square error, which is the mean square error between the original image and the processing image. $I(i,j)$ is the pixel value of the original image and $K(i,j)$ is the pixel value of the processing image. $m * n$ is the size of the image. Unit of $PSNR$ is dB. The higher $PSNR$ value is, the less distortion there is.

3.2. **Experimental Results.** In the experiment, we divide an image into three categories, and we apply the proposed approach to a mass of images. We only take three examples here. Figure 1 shows the three noiseless original gray images. In the first part of the experiment, we select the optimal range for regions. We have defined W in Definition 2.2. And $W = w * w$ is a collection of image pixels, where w is a variable. And according to various noises, the w value varies. We also need to get two different w values to acquire an interval value. From Table 1, we can see that the region value of $w_1 = 3, w_2 = 4$ are optimal. The larger w_2 is, the smaller $PSNR$ is. Here we are only listing some of the results. After that, there is no need to consider the influence of range selection for the image segmentation in the following experiments.

FIGURE 1. Lena1, Lena2, Pepper

In Figure 2, we show the image segmentation results by the traditional k-means algorithm (in the middle of the figure) and interval neutrosophic set method (in the end of the figure). Figure 3-5 are three images with Gaussian noise (mean is 0 and variance is 0.01), and in these images, the first part is the original image with gaussian noise, the middle part is the segmentation by the traditional k-means algorithms, and the third part is the segmentation by interval neutrosophic set method. It can be concluded that the segmentation by interval neutrosophic set method has less noise than by the traditional k-means algorithm.

FIGURE 2. The first part is the original image, the middle part is the segmentation by the traditional k-means algorithms, and the third part is the segmentation by interval neutrosophic set method

FIGURE 3. For Lena1 image, the first part is the original image with gaussian noise (mean is 0 and variance is 0.01), the middle part is the segmentation by the traditional k-means algorithms, and the third part is the segmentation by interval neutrosophic set method

After that, we add different noises to the images, and want to obtain different results by calculating $PSNR$. First of all, we add gaussian noise, whose mean is 0 and the variance is 0.01 and 0.05 respectively. Then the salt noise with 0.01 mean value and speckle noise with 0.03 mean value are added. Compared to the traditional k-means algorithms, we can see from the Table 2, that for the image with gaussian noise and speckle noise, our method can obtain better results.

We compare the result of image segmentation with the approach in [9], which was proposed a new neutrosophic approach to image segmentation. In Figure 6, the first part is the original Lena image, the middle part is the segment result by the neutrosophic approach, and the third part is the segment result of the proposed method. From the experimental results, we can easily see that our method makes the contour of the segmented image clearer and the effect on the edge pixels more prominent. And compared with the method proposed in this paper, the neutrosophic approach to image segmentation only consider membership degree in the cluster calculation, not count non-membership degree and hesitation.

FIGURE 4. For Lena2 image, the first part is the original image with gaussian noise (mean is 0 and variance is 0.01), the middle part is the segmentation by the traditional k-means algorithms, and the third part is the segmentation by interval neutrosophic set method

FIGURE 5. For Pepper image, the first part is the original image with gaussian noise (mean is 0 and variance is 0.01), the middle part is the segmentation by the traditional k-means algorithms, and the third part is the segmentation by interval neutrosophic set method

TABLE 2. The value of PSNR for the three images with different noises

noise	gaussian noise(1)	gaussian noise(2)	salt noise	speckle noise
k-means	19.1443	13.8363	22.6033	19.5381
INI	21.2599	18.4293	21.1605	21.6448
noise	gaussian noise(1)	gaussian noise(2)	salt noise	speckle noise
k-means	19.1969	13.9415	23.3081	20.9689
INI	22.1729	18.6085	22.4192	22.6730
noise	gaussian noise(1)	gaussian noise(2)	salt noise	speckle noise
k-means	18.6936	13.8596	21.4768	19.4912
INI	20.6764	18.0361	20.7433	20.6611

The contribution of this study is that the proposed approach can better handle the indeterminacy and uncertainty of the images. It achieves better results on the images with noises.

FIGURE 6. For Lena image, the first part is the original image, the middle part is the segmentation by the approach in [9], and the third part is the segmentation by interval neutrosophic set method

4. **Conclusion and Outlook.** This paper proposes a new image segmentation method based on interval neutrosophic set. And the experimental results show that this methodology not only obtain the higher $PSNR$, but also achieves better segmentation results than the k-means algorithm. Furthermore, the method has no sufficient effect on suppression of all noise, and its extensive application will be further studied.

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REFERENCES

[1] M. R. Anderberg, Cluster analysis for applications, *Probability & Mathematical Statistics New York Academic Press*, **1** (1973), 347–353.
[2] K. T. Atanassov, Intuitionistic fuzzy sets, *Physica-Verlag HD*, **20** (1986), 87–96.
[3] K. T. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, *Physica-Verlag HD*, (1999), 343–349.
[4] C. Bai, D. Dhavale and J. Sarkis, Complex investment decisions using rough set and fuzzy C-means: An example of investment in green supply chains, *European Journal of Operational Research*, **248** (2016), 507–521.
[5] H. D. Cheng, Y. H. Chen and X. H. Jiang, Thresholding using two-dimensionalh histogram and fuzzy entropy principle, *IEEE Transactions on Image Processing,* **9** (2000), 732–735.
[6] H. D. Cheng and Y. Guo, A new neutrosophic approach to image thresholding, *New Mathematics & Natural Computation*, **4** (2008), 291–308.
[7] C. Feng, D. Zhao and M. Huang, Image segmentation using CUDA accelerated non-local means denoising and bias correction embedded fuzzy C-means (BCEFCM), *Signal Processing*, **122** (2016), 164–189.
[8] Y. Guo, H. D. Cheng and W. Zhao et al., A novel image segmentation algorithm based on fuzzy C-means algorithm and neutrosophic set, *JCIS-2008 Proceedings*, 2008.
[9] Y. Guo and H. D. Cheng, New neutrosophic approach to image segmentation, *Pattern Recognition*, **42** (2009), 587–595.
[10] Y. Guo, A novel image edge detection algorithm based on neutrosophic set, *Pergamon Press, Inc*, **40** (2014), 3–25.

[11] Y. Guo and A. Sengür, A novel image segmentation algorithm based on neutrosophic similarity clustering, *Applied Soft Computing*, **25** (2014), 391–398.
[12] Y. Guo, A. Sengür and J. Ye, A novel image thresholding algorithm based on neutrosophic similarity score, *Measurement*, **58** (2014), 175–186.
[13] Y. Guo and A. Sengür, NECM: Neutrosophic evidential C-means clustering algorithm, *Neural Computing & Applications*, **26** (2014), 1–11.
[14] Y. Guo and A. Sengür, NCM: Neutrosophic C-means clustering algorithm, *Pattern Recognition*, **48** (2015), 2710–2724.
[15] Y. Guo and Y. Akbulut et al., An efficient image segmentation algorithm using neutrosophic graph cut, *Symmetry*, **9** (2017), 185.
[16] K. Hanbay and M. F. Talu, Segmentation of SAR images using improved artificial bee colony algorithm and neutrosophic set, *Applied Soft Computing Journal*, **21** (2014), 433–443.
[17] T. Kanungo, D. M. Mount and N. S. Netanyahu et al., An efficient k-means clustering algorithm: Analysis and implementation, *IEEE Transactions on Pattern Analysis & Machine Intelligence*, **24** (2002), 881–892.
[18] X. Liu, The fuzzy theory based on AFS algebras and AFS structure, *Journal of Mathematical Analysis & Applications*, **217** (1998), 459–478.
[19] S. K. Pal and R. A. King, Image enhancement using smoothing with fuzzy sets, *IEEE Transactions on Systems Man & Cybernetics*, **11** (1981), 494–501.
[20] S. K. Pal and R. A. King, Image enhancement using fuzzy sets, *Electronics Letters*, **16** (1980), 376–378.
[21] A. Sengur and Y. Guo, Color texture image segmentation based on neutrosophic set and wavelet transformation, *Computer Vision & Image Understanding*, **115** (2011), 1134–1144.
[22] J. Shan, H. D. Cheng and Y. Wang, A novel segmentation method for breast ultrasound images based on neutrosophic k-means clustering, *Medical Physics*, **39** (2012), 5669.
[23] F. Smarandache, A unifying field in logics: Neutrosophic logic, *Multiple-Valued Logic*, **8** (1999), 489–503.
[24] F. Smarandache, A unifying field in logics: Neutrsophic logic, neutrosophy, neutrosophic set, neutrosophic probability (Fourth edition), *University of New Mexico*, **332** (2002), 5–21.
[25] F. Smarandache, Neutrosophic Topologies, 1999.
[26] Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy Sets & Systems*, **20** (1986), 191–210.
[27] J. K. Udupa and S. Samarasekera, Fuzzy connectedness and object definition: Theory, algorithms, and applications in image segmentation, *Academic Press Inc.*, **58** (1996), 246–261.
[28] H. Wang, P. Madiraju and Y. Zhang et al, Interval neutrosophic sets, *Mathematics*, **1** (2004), 274–277.
[29] L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338–353.
[30] H. Zhang, J. Wang and X. Chen, Interval neutrosophic sets and their application in multicriteria decision making problems, *The Scientific World Journal*, **2014** (2014), 645953.
[31] X. Zhao and S. T. Wang, Neutrosophic image segmentation approach based on similarity, *Application Research of Computers*, **29** (2012), 2371–2374.

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