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# Attribute Control Chart Using the Repetitive Sampling Under Neutrosophic System

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**ABSTRACT** In this paper, an attribute control chart using the repetitive sampling under the neutrosophic statistics system is discussed. The necessary measures of the proposed control chart under the neutrosophic statistics system are given. The control chart coefficients of the proposed control chart are determined using an algorithm under neutrosophic statistics system. The efficiency of the proposed control chart in terms of neutrosophic average run length is discussed over the existing control chart under neutrosophic statistics system. From the comparison studies, it is found that the proposed control chart under neutrosophic statistics system is more sensitive in detecting a shift in the process as compared with the existing control chart under neutrosophic statistics system. An industrial application of the proposed control chart under neutrosophic statistics system is also given.

**INDEX TERMS** Attribute chart, average run length, control chart, neutrosophic logic, shift.

## I. INTRODUCTION

The Shewhart control charts have been widely used in the industry to monitor the controlling the number of non-conforming items or defective items during the manufacturing process. The Shewhart control chart consists of three control limits are known as upper control limit (UCL), lower control limit (LCL) and the central limit (CL). A process is declared as out-of-control if the plotting statistic exceed the LCL or UCL. The control charts help to keep the process near the CL and minimize the non-conforming items. The Shewhart X-bar control chart is applied when the data is continuous and Shewhart np control chart is used when data is discrete. The X-bar control chart is more informative than the attribute control chart. However, the earlier control chart cannot be applied when the purpose is monitoring of non-conforming items. In addition, the np-control chart is quite simple and easy to apply in the industry. Further, as mentioned by [1], the attribute control chart can also be applied when the quality of interest is categorical. However, fuzzy approach [2] has the ability to monitor more than one quality characteristics simultaneously.

The fuzzy logic is applied when the parameters or the observations are imprecise or unclear. The fuzzy attribute control charts can be applied to monitor the categorical characteristics in indeterminacy environment. According to [3] “Due to the emphasis on the user’s feelings and psychological

factors, there are many fuzzy attributes of quality from the fitness-for-use viewpoint. As a result, there are not only two distinct judgments (applicable or inapplicable) when evaluating the quality from a fitness point of view. In this sense, considering the fuzzy property of fitness-for-use quality is more practical”. The fuzzy chart for monitoring attribute data was proposed by [4]. The economic aspects of fuzzy attribute chart were considered by [5]. The application of such fuzzy charts is given by [6] and [7]. Later on, [8], [9], and [10] designed charts to monitor imprecise data. The fuzzy charts using weighted average methods was considered by [2] and [11]. The range chart was proposed by [12]. A detailed discussion and applications can be seen in [2], [3], and [13]–[24].

Reference [25] proposed the repetitive sampling which is used when there is no decision on the information of the first sample. This sampling found to be more efficient than the single sampling scheme in average sample number (ASN) and average run length (ARL). A rich literature is available on the repetitive sampling in the area of acceptance sampling and the control charts. The control chart based on the process capability index (PCI) using this sampling was considered by [26]. Reference [27] worked on an attribute and variable charts using this sampling scheme. A detailed discussion and applications of charts using the repetitive sampling can be seen in [28]–[33].

According to [34] “A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are defined above, is called Neutrosophic Logic (NL)”. The NL is the extension of the traditional fuzzy approach. Therefore, the neutrosophic statistics (NS) is untiled the NL is the generalization of the classical statistics. The NS is applied when the data is ambiguous, vague and uncertain. Reference [35] and [36] introduced neutrosophic numbers in rock engineering. Reference [37]–[40] introduced the NS in acceptance sampling plans. Reference [41] introduced the NS in the area of control chart. The variance of proposed chart using the NS was proposed by [42]. Reference [43] proposed the chart to monitor failure censored reliability using the NS.

According to the best of our knowledge, there is no work on attribute control chart using repetitive sampling under the NS. In this paper, an attribute control chart using the repetitive sampling under the neutrosophic statistics system is discussed. The necessary measures of the proposed control chart under the neutrosophic statistics system are given. The control chart coefficients of the proposed control chart are determined using an algorithm under neutrosophic statistics system. The efficiency of the proposed control chart in terms of neutrosophic average run length (NARL) is discussed over the existing control chart under neutrosophic statistics system. From the comparison studies, it is found that the proposed control chart under neutrosophic statistics system is more sensitive in detecting a shift in the process as compared to the existing control chart under neutrosophic statistics system. An industrial application of the proposed control chart under neutrosophic statistics system is also given.

**II. DESIGN OF CHART USING THE NS**

Let  $n_N \in \{n_L, n_U\}$  be a neutrosophic random sample is taken from the population having some uncertain observations,  $D_N \in \{D_L, D_U\}$  be neutrosophic defective values recorded from  $n_N \in \{n_L, n_U\}$  and  $p_N \in \{p_L, p_U\}$  be the neutrosophic probability of non-conforming items. Based on this information, [41] defined the following form of the neutrosophic binomial distribution

$$\sum_{d_N=|LCL_N|+1}^{|UCL_N|} \binom{n}{d_N} p_N^{d_N} (1-p_N)^{n-d_N}, \quad d_N = 0, 1, \dots, n; \quad d_N \in \{d_L, d_U\}, \quad p_N \in \{p_L, p_U\} \quad (1)$$

where  $|y|$  denotes the positive integer values.

We propose the following attribute control chart using the repetitive sampling under the neutrosophic statistical interval method (NSIM).

*Step-1:* Record the number of non-conforming items  $D_N \in \{D_L, D_U\}$  from  $n_N \in \{n_L, n_U\}$ .

*Step-2:* If  $LCL_{N2} \leq D_N \leq UCL_{N2}$ ; the process is an in-control state. Note here that  $LCL_{N2} \in \{LCL_{L2}, LCL_{U2}\}$  and  $UCL_{N2} \in \{UCL_{L2}, UCL_{U2}\}$  are inner neutrosophic lower limits and neutrosophic upper limits, respectively.

*Step-3:* If  $D_N \geq UCL_{N1}$  or  $D_N \leq LCL_{N1}$ ; the process is declared as out-of-control. Note here that  $LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\}$  and  $UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\}$  are outer neutrosophic lower limits and neutrosophic upper limits, respectively.

The proposed attribute control chart is the generalization of several control charts. For example, the proposed control chart reduces to [41] under the NS when no repetitive sampling is needed. The proposed control chart also reduces to [27] when no uncertain observations or parameters are noted in the data. The neutrosophic in-control control limits for the proposed chart are given by

$$UCL_{N1} = np_{N0} + k_{N1}\sqrt{np_{N0}(1-p_{N0})}; \quad p_{N0} \in \{p_{L0}, p_{U0}\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\} \quad (2)$$

$$UCL_{N2} = np_{N0} + k_{N2}\sqrt{np_{N0}(1-p_{N0})}; \quad p_{N0} \in \{p_{L0}, p_{U0}\}, \quad k_{N2} \in \{k_{L2}, k_{U2}\} \quad (3)$$

$$LCL_{N1} = np_{N0} - k_{N1}\sqrt{np_{N0}(1-p_{N0})}; \quad p_{N0} \in \{p_{L0}, p_{U0}\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\} \quad (4)$$

$$LCL_{N2} = np_{N0} - k_{N2}\sqrt{np_{N0}(1-p_{N0})}; \quad p_{N0} \in \{p_{L0}, p_{U0}\}, \quad k_{N2} \in \{k_{L2}, k_{U2}\} \quad (5)$$

Note here that  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  are the neutrosophic coefficients. Now, we derive the necessary measures to assess the performance of the proposed attribute control chart when the process is in-control state at  $p_{N0} \in \{p_{L0}, p_{U0}\}$ . According to the above-mentioned attribute control chart, the process will be out-of-control at the first sample when  $D_N \geq UCL_{N1}$  or  $D_N \leq LCL_{N1}$ . Therefore, the neutrosophic probability that the process is in-control, say  $P_{inN1}^0$  at first sample is given by

$$P_{inN1}^0 = P \{LCL_{N1} \leq D_N \leq UCL_{N1} | p_{N0}\} \quad (6)$$

Using Eq. (1), it can be written as follows

$$P_{inN1}^0 = \sum_{d_N=|LCL_{N1}|+1}^{|UCL_{N1}|} \binom{n}{d_N} p_{N0}^{d_N} (1-p_{N0})^{n-d_N}; \quad d_N = 0, 1, \dots, n; \quad d_N \in \{d_L, d_U\}, \quad p_{N0} \in \{p_{L0}, p_{U0}\}, \quad UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\}; \quad LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\} \quad (7)$$

In case, when there is no decision about the state of the process at the first sample, the operational process of the control chart will be repeated. The probability of the repetition, say  $P_{repN}^0$  is given by

$$P_{repN}^0 = \sum_{d_N=|UCL_{N2}|+1}^{UCL_{N1}} \binom{n}{d_N} p_{N0}^{d_N} (1-p_{N0})^{n-d_N} + \sum_{d_N=|LCL_{N1}|+1}^{LCL_{N2}} \binom{n}{d_N} p_{N0}^{d_N} (1-p_{N0})^{n-d_N}; \quad UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\}, \quad LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\}, \quad UCL_{N2} \in \{UCL_{L2}, UCL_{U2}\}, \quad LCL_{N2} \in \{LCL_{L2}, LCL_{U2}\} \quad (8)$$

TABLE 1. The NARL when  $n_N \in [50, 50]$ .

	NARL <sub>N0</sub> =300	NARL <sub>N0</sub> =370
$k_N$	[3.80810797, 0.10773763),( 2.8508100, 0.02145566)]	[(3.4805580, 0.05542655), (3.4521577, 0.09645442)]
$p_{N0}$	[0.0292,0.142]	[0.0379,0.028]
$c$	ARL <sub>N1</sub>	ARL <sub>N1</sub>
0	[304.41, 319.69]	[372.15, 374.49]
0.01	[289.89, 290.00]	[352.07, 356.48]
0.02	[276.23, 263.46]	[333.31, 339.53]
0.05	[239.78, 199.26]	[283.92, 294.37]
0.08	[209.16, 152.59]	[243.22, 256.45]
0.1	[191.43, 128.56]	[220.04, 234.53]
0.2	[126.43, 58.58]	[137.77, 154.27]
0.3	[86.98, 29.56]	[90.47, 105.73]
0.4	[61.94, 16.22]	[61.85, 75.01]
0.5	[45.43, 9.51]	[43.76, 54.82]
0.8	[20.59, 2.53]	[18.20, 24.61]
0.9	[16.40, 1.73]	[14.17, 19.56]
0.95	[14.73, 1.44]	[12.58, 17.54]
1.0	[13.27, 1.20]	[11.22, 15.78]
1.5	[5.43, 1]	[4.21, 6.40]
2.0	[2.68, 1]	[1.94, 3.15]

The neutrosophic probability that the process is in-control state using the repetitive sampling is given by

$$P_{inN}^0 = \frac{P_{inN1}^0}{1 - P_{reptN}^0}; \quad P_{inN}^0 \in \{P_{inL}^0, P_{inU}^0\} \quad (9)$$

The neutrosophic average run length (NARL) when the process is at  $p_{N0} \in \{p_{L0}, p_{U0}\}$  is defined by

$$NARL_{N0} = \frac{1}{1 - \left\{ \frac{P_{inN1}^0}{1 - P_{reptN}^0} \right\}}; \quad NARL_{N0} \in \{NARL_{NL}, NARL_{NU}\} \quad (10)$$

We now derive the necessary measures when the process is shifted from in-control state to out-of-control state due to some uncountable factors. Suppose  $p_{N1} = (1 + c)p_{N0}$   $p_{N1} \in \{p_{L1}, p_{U1}\}$  denote the neutrosophic percent defective at the shifted process, where  $c$  is shift constant. Therefore, the neutrosophic probability that the process is in-control at  $p_{N1} \in \{p_{L1}, p_{U1}\}$ , say  $P_{inN1}^1$  at first sample is given by

$$P_{inN1}^0 = P\{LCL_{N1} \leq D_N \leq LCL_{N1} | p_{N0}\} \quad (11)$$

Using Eq. (1), it can be written as follows

$$P_{inN1}^1 = \sum_{d_N=|LCL_{N1}|+1}^{|UCL_{N1}|} \binom{n}{d_N} p_{N1}^{d_N} (1 - p_{N1})^{n-d_N};$$

$$d_N = 0, 1, \dots, n; \quad d_N \in \{d_L, d_U\},$$

$$p_{N1} \in \{p_{L1}, p_{U1}\}, \quad UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\};$$

$$LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\} \quad (12)$$

In case, when there is no decision about the state of the process at the first sample, the operational process of the control

chart will be repeated. The probability of the repetition, say  $P_{reptN}^1$  is given by

$$P_{reptN}^1 = \sum_{d_N=|UCL_{N2}|+1}^{UCL_{N1}} \binom{n}{d_N} p_{N1}^{d_N} (1 - p_{N1})^{n-d_N}$$

$$+ \sum_{d_N=|LCL_{N1}|+1}^{LCL_{N2}} \binom{n}{d_N} p_{N1}^{d_N} (1 - p_{N1})^{n-d_N};$$

$$UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\};$$

$$LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\} \quad (13)$$

The neutrosophic probability that the process is in-control state using the repetitive sampling is given by

$$P_{inN}^1 = \frac{P_{inN1}^1}{1 - P_{reptN}^1}; \quad P_{inN}^1 \in \{P_{inL}^1, P_{inU}^1\} \quad (14)$$

The neutrosophic average run length (NARL) when the process is at  $p_{N1} \in \{p_{L1}, p_{U1}\}$  is defined by

$$NARL_{N1} = \frac{1}{1 - \left\{ \frac{P_{inN1}^1}{1 - P_{reptN}^1} \right\}};$$

$$NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\} \quad (15)$$

Suppose that  $r_{0N} \in \{r_{0L}, r_{0U}\}$  denotes the pre-fixed values of  $NARL_{N0} \in \{NARL_{NL}, NARL_{NU}\}$ . The values of neutrosophic control chart coefficients  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  and  $NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\}$  are determined for various combinations of  $r_{0N} \in \{r_{0L}, r_{0U}\}$ ,  $c$  and  $n_N \in \{n_L, n_U\}$  placed in Tables 1-3. From Tables 1-3, we note that values of indeterminacy interval of  $NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\}$  decrease as  $n_N \in \{n_L, n_U\}$  increases from  $n_N \in \{50, 50\}$  to  $n_N \in \{150, 150\}$ . For an example, when  $NARL_{N0} \in \{300, 300\}$ ,

TABLE 2. The NARL when  $n_N \in [150, 50]$ .

	ARL <sub>N0</sub> =300	ARL <sub>N0</sub> =370
$k_N$	[(3.2059705, 0.23039384), 3.2300457, 0.07855410]	[(3.03770802, 0.03943507), 3.0061115, 0.01933103]
$p_{N0}$	[0.0162,0.0361]	[0.1174, 0.2145]
$c$	ARL <sub>N1</sub>	ARL <sub>N1</sub>
0	[304.24, 306.77]	[371.97, 388.0445]
0.01	[286.61, 282.13]	[327.69,342.437]
0.02	[270.21, 259.80]	[287.91,301.2283]
0.05	[227.43, 204.33]	[194.38,203.6801]
0.08	[192.66, 162.37]	[132.17,138.4758]
0.1	[173.06, 140.06]	[103.07,107.9297]
0.2	[105.02, 70.96]	[33.59,35.03192]
0.3	[67.24, 39.15]	[13.19,13.70168]
0.4	[45.03, 23.17]	[5.97,6.190185]
0.5	[31.33, 14.52]	[2.99,3.100434]
0.8	[12.58, 4.61]	[1,1.66]
0.9	[9.72, 3.34]	[1,1]
0.95	[8.60, 2.86]	[1,1]
1.0	[7.64, 2.47]	[1,1]
1.5	[2.79, 0.66]	[1,1]
2.0	[1.24, 1]	[1,1]

TABLE 3. The NARL when  $NARL_{N0} = 300$  and  $370$ .

	ARL <sub>N0</sub> =300	ARL <sub>N0</sub> =370
$n_N$	[50,150]	[20,200]
$k_N$	[(3.41163588, 0.01374361), 3.1956378, 0.03093748]	[(3.4033867, 0.08835755), 3.47013388, 0.17708702]
$p_{N0}$	[0.0393,0.036]	[0.1324,0.0172]
$c$		
0	[304.24,314.04]	[370.82, 374.81]
0.01	[287.97, 288.78]	[347.78, 349.30]
0.02	[272.76, 265.89]	[326.43, 325.84]
0.05	[232.69, 209.05]	[271.18, 265.99]
0.08	[199.73, 166.06]	[226.76, 218.87]
0.1	[180.77, 143.21]	[201.97, 193.00]
0.2	[113.72, 72.47]	[117.60, 107.72]
0.3	[75.02, 39.94]	[72.38, 64.27]
0.4	[51.51, 23.61]	[46.69, 40.54]
0.5	[36.59, 14.79]	[31.34, 26.80]
0.8	[15.38, 4.68]	[11.40, 9.58]
0.9	[12.01, 3.39]	[8.53, 7.18]
0.95	[10.68, 2.91]	[7.43, 6.27]
1.0	[9.53, 2.51]	[6.50, 5.49]
1.5	[3.61, 2.31]	[2.02, 1.78]
2.0	[1.66, 1]	[1, 1]

$n_N \in \{50, 50\}$  and  $c = 0.01$ , the indeterminacy interval of  $NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\}$  is [286.615, 282.1398] and it is [289.8948, 290.0023] when  $n_N \in \{150, 150\}$ .

The following neutrosophic algorithm is applied to find the values of  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  and  $NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\}$ .

Step-1: Pre-fix the values of  $n_N \in \{n_L, n_U\}$  and  $c$ .

Step-2: Determine the values of  $p_{N0} \in \{p_{L0}, p_{U0}\}$ ,  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  such that

$NARL_{N0} \in \{NARL_{NL}, NARL_{NU}\} \geq r_{0N} \in \{r_{0L}, r_{0U}\}$ . Several combinations of  $p_{N0} \in \{p_{L0}, p_{U0}\}$ ,  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  will exist where  $NARL_{N0} \geq r_{0N}$ .

Step-3: Select those values of  $p_{N0} \in \{p_{L0}, p_{U0}\}$ ,  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  where  $NARL_{N0}$  is same or very close to  $r_{0N} \in \{r_{0L}, r_{0U}\}$ .

Step-4: Use the values of  $p_{N0} \in \{p_{L0}, p_{U0}\}$ ,  $k_{N1} \in \{k_{L1}, k_{U1}\}$  and  $k_{N2} \in \{k_{L2}, k_{U2}\}$  to find  $NARL_{N1} \in \{NARL_{NL}, NARL_{NU}\}$  for various shifts.

TABLE 4. The comparison in NARL when  $ARL_{N0} = 370$  when  $n_N \in [50, 50]$ .

c	Existing Chart	Proposed Chart
0	[371.80, 384.87]	[372.15, 374.49]
0.01	[333.48, 339.55]	[352.07, 356.48]
0.02	[298.49, 299.67]	[333.31, 339.53]
0.05	[213.04, 207.24]	[283.92, 294.37]
0.08	[152.57, 145.26]	[243.22, 256.45]
0.1	[122.81, 115.64]	[220.04, 234.53]
0.2	[45.54, 41.31]	[137.77, 154.27]
0.3	[19.77, 17.54]	[90.47, 105.73]
0.4	[9.88, 8.66]	[61.85, 75.01]
0.5	[5.59, 4.88]	[43.76, 54.82]
0.8	[1.84, 1.66]	[18.20, 24.61]
0.9	[1.49, 1.36]	[14.17, 19.56]
0.95	[1.37, 1.27]	[12.58, 17.54]
1.0	[1.28, 1.20]	[11.22, 15.78]
1.5	[1.00, 1.00]	[4.21, 6.40]
2.0	[1.00, 1.00]	[1.94, 3.15]

III. SIMULATION STUDY

A control chart under the NS is said to be more efficient than the other if provides the smaller values of NARL at same parameters. The smaller the values of NARL means the quick indication about the shift in the process. Here, we will discuss the advantages of the proposed control chart over the neutrosophic attribute control chart proposed by [41].

A. COMPARISON IN THE NARL VALUES

We first present the theoretical comparison between the proposed control chart with the attribute control chart proposed by [41]. For fair comparison, we set the same values of all specified neutrosophic parameters. The values of NARL when  $NARL_{N0} \in \{370, 370\}$  and  $n_N \in \{50, 50\}$  are given in Table 4.

The Table 4 clearly indicates that the values of indeterminacy interval of NARL from the proposed control are smaller than the control chart proposed by [41] at all values of  $c$ . For an example, when  $c = 0.01$ , the values of indeterminacy interval in NARL are  $NARL_{N1} \in \{333.48, 339.55\}$  from the proposed control chart and indeterminacy interval in NARL are  $NARL_{N1} \in \{352.07, 356.48\}$  from [41] control chart. By comparing both control charts, it can be seen that for the proposed control chart a shift in the process can be expected in between 333-339<sup>th</sup> samples while at the same parameters, the existing attribute control chart gives indication between 352-356<sup>th</sup> samples. From this comparison, we conclude that the proposed control chart is more efficient than chart proposed by [41] in detecting the shift in the process.

B. COMPARISON USING SIMULATION

We also show the advantage of the proposed control chart over the existing neutrosophic attribute chart using the simulated data. The data is generated from the neutrosophic binomial distribution with neutrosophic parameters  $n_N \in$

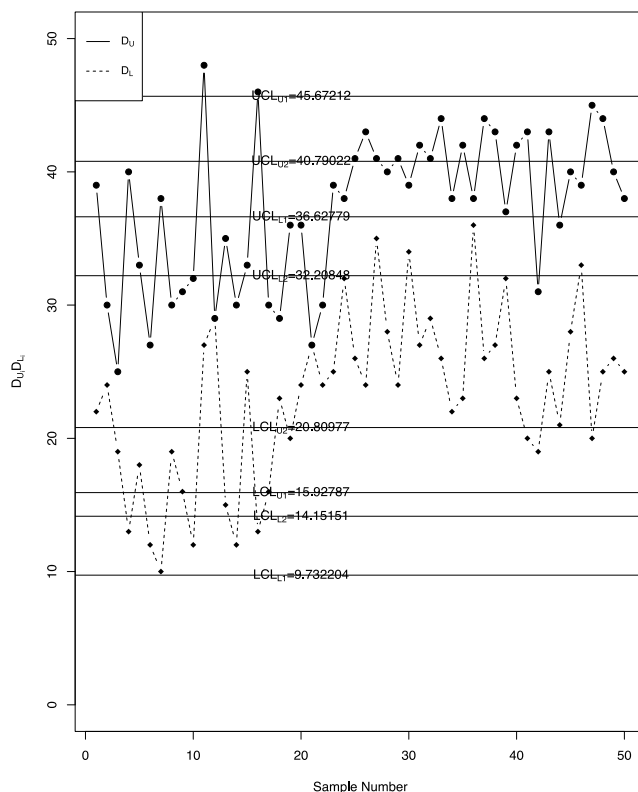


FIGURE 1. The proposed chart for the simulated data.

$\{150, 150\}$  and  $p_{N0} \in \{0.1174, 0.2145\}$ . We generated 50 sample from the neutrosophic binomial distribution. The first 25 values are generated from the in-control process and next 25 are generated at shifted process when  $c = 0.3$ . For this simulation study, we have  $NARL_{N0} \in \{370, 370\}$ ,  $n_N \in \{150, 150\}$ ,  $k_{N1} \in \{3.0377, 0.0394\}$  and  $k_{N2} \in \{3.0061, 0.0193\}$ . The calculated neutrosophic control limits are shown in Figure 1. The values of statistic  $D_N \in \{D_L, D_U\}$

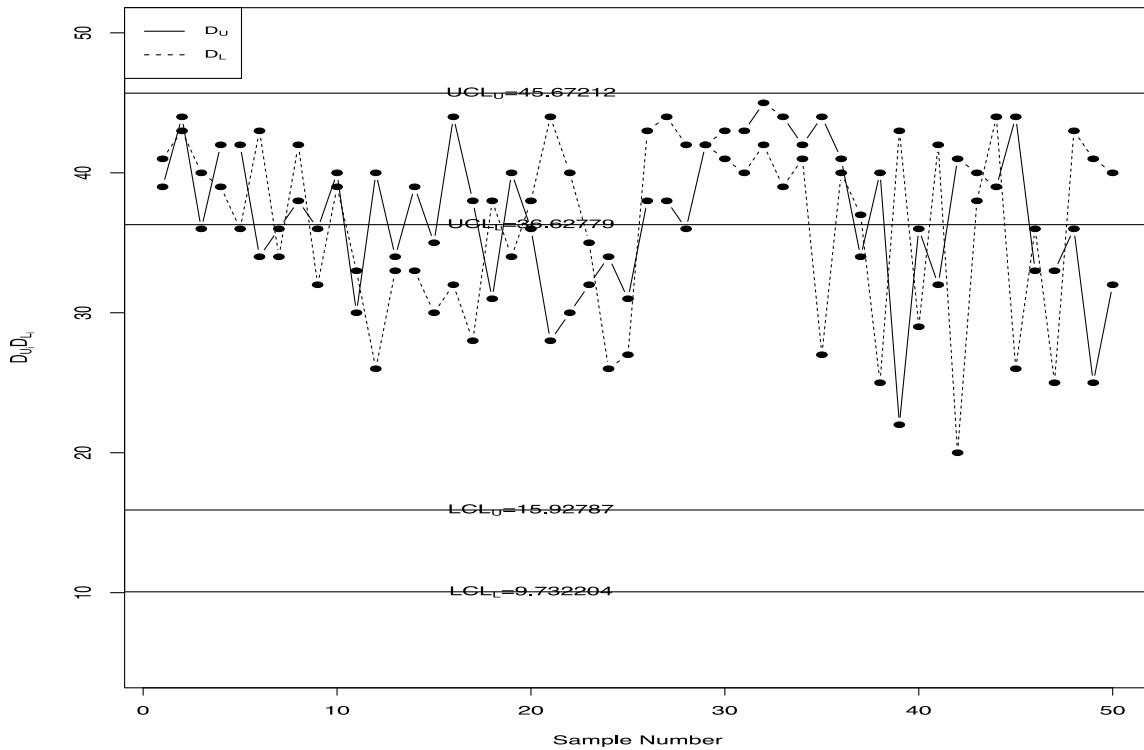


FIGURE 2. The existing chart for the simulated data.

are recorded and plotted in Figure 1. From Figure 1, it is clear that the proposed control chart detects shift at the 11<sup>th</sup> sample. The values of  $D_N \in \{D_L, D_U\}$  are also recorded for the existing control and plotted on Figure 2. From Figure 2, we note that the existing control chart does not provide any signal about the shift in the process. By comparing both figures, we reach the conclusion that the proposed chart is more sensitive in the detecting the shift as compared to [41] control chart.

IV. CASE STUDY

A juice company situated in the Lahore, Pakistan is interested to use the proposed control chart for monitoring of non-conforming product. As mentioned by [44] “Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can have an improper seal on either the side seam or the bottom panel”. During the inspection, the industrial experimenter is uncertain about the classification of some items either conforming or non-conforming. Due to the uncertainty, the industrial engineers can expect the percent non-conforming product from 0.028 to 0.0379. In this situation, when some observations are unclear and uncertain, the attribute control chart using the classical statistics cannot be applied for the monitoring of non-conforming

TABLE 5. Neutrosophic data of juice company [41].

Sample number	No.of nonconforming units $D_i$	Sample number	No.of nonconforming units
1	[12,13]	16	[8,8]
2	[15,15]	17	[10,10]
3	[8,10]	18	[5,8]
4	[10,10]	19	[13,13]
5	[4,4]	20	[11,13]
6	[7,7]	21	[20,20]
7	[16,16]	22	[18,20]
8	[9,11]	23	[24,24]
9	[14,14]	24	[15,15]
10	[10,10]	25	[9,12]
11	[5,8]	26	[12,12]
12	[6,8]	27	[7,10]
13	[17,17]	28	[13,15]
14	[12,15]	29	[9,9]
15	[22,22]	30	[6,9]

items. For this real example, let  $n_N \in \{50, 50\}$  and  $NARL_{N0} \in \{370, 370\}$ . The neutrosophic data is taken from [41] and reported in Table 5. The values of  $D_N \in \{D_L, D_U\}$  are plotted on Figure 3 and Figure 4 for the proposed control chart and [41] control chart, respectively. From Figures 3 and 4, it can be noted that some several samples are in the repetition and indeterminacy areas in Figure 3. We also note that some points are near the control limits, which needs

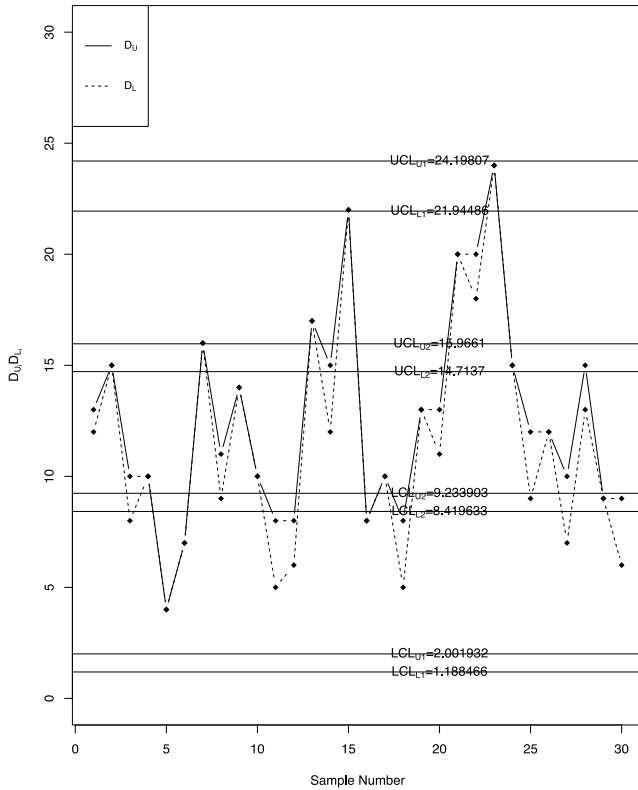


FIGURE 3. The proposed chart for the real data.

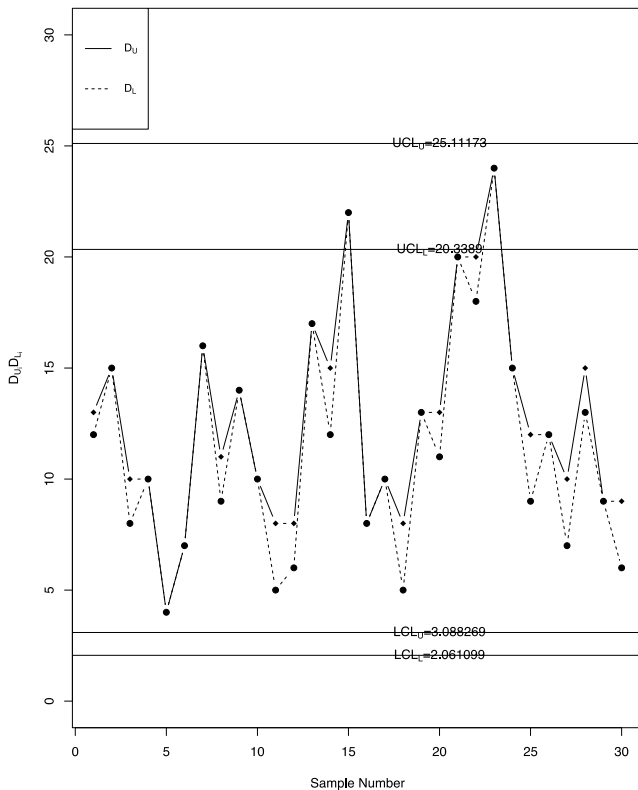


FIGURE 4. The existing chart for the real data.

industrial engineers attention. Figure 4 shows that the process is in-control state and indicate no point near the control limits.

### V. CONCLUDING REMARKS

In this manuscript, an attribute control chart using the repetitive sampling under the neurosophic statistics system is discussed. The necessary measures of the proposed control chart under the neurosophic statistics system are given. The proposed control chart is more sensitive and adequate to be used in uncertainty environment than the existing attribute control chart under classical statistics. From the simulation study and real example, it is concluded that the proposed control chart is more efficient in detecting a shift in the process. Therefore, the use of the proposed control chart in the industry will be helpful in minimize the non-conforming items in uncertainty. The proposed chart using other sampling scheme or for the big data can be considered for future research.

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