Certain Networks Models Using Single-valued Neutrosophic Directed Hypergraphs

Muhammad Akram and Anam Luqman
Department of Mathematics, University of the Punjab, Lahore, PAKISTAN
m.akram@pucit.edu.pk, anamluqman7@yahoo.com

Abstract

A directed hypergraph is powerful tool to solve the problems that arises in different fields, including computer networks, social networks and collaboration networks. In this research paper, we apply the concept of single-valued neutrosophic sets to directed hypergraphs. We introduce certain new concepts, including single-valued neutrosophic directed hypergraphs, single-valued neutrosophic line directed graphs and dual single-valued neutrosophic directed hypergraphs. We describe applications of single-valued neutrosophic directed hypergraphs in manufacturing and production networks, collaboration networks and social networks. We develop and implement algorithm for our certain networks models based on single-valued neutrosophic directed hypergraphs

Key-Words : Single-valued neutrosophic directed hypergraphs, Dual single-valued neutrosophic directed hypergraphs, Single-valued neutrosophic line directed graphs, Applications.

1 Introduction

Fuzzy set theory generalizes the concept of the classical set theory. In classical set theory, we may only conclude either the statement is true or false. However, many statements have variable answers which can be handled more accurately using fuzzy set theory. Zadeh gave the concept of fuzzy sets in 1965 to solve problems with uncertainties [32]. Fuzzy sets and fuzzy logic are playing a vital role in controlling and modeling uncertain systems in various fields, including society and nature, clustering, linguistics and decision-making. In 1986, Atanassov [6] illustrated the extended form of fuzzy set by adding a new component, called, intuitionistic fuzzy set(IFS). The notion of intuitionistic fuzzy set(IFS) is more meaningful as well as inventive due to the presence of degree of membership, degree of non-membership and the hesitation margin. In 1998, Smarandache [26] submitted the idea of neutrosophic set(NS). A NS has three constituents: truth-membership, indeterminacy-membership and falsity-membership, in which each membership value is a real standard or non-standard subset of the unit interval $]0^-, 1^+[$. In real-life problems, NSs can be applied more appropriately by using the single-valued neutrosophic sets(SVNSs), defined by Wang et al [28]. A SVNS generalizes the concept of IFS. In SVNS, three components are not dependent and their values are contained in the unit interval [0, 1]. Majumdar and Samanta [19] studied similarity and entropy of SVNSs. Ye [29] proposed correlation coefficients of SVNSs to solve single-valued neutrosophic decision-making problems. To simplify neutrosophic sets, he also [31] proposed a method of multicriteria decision-making using aggregation operators.
Graph theory has become a powerful conceptual framework for modeling and solution of combinatorial problems that arise in various areas, including Mathematics, Computer Science and Engineering. Hypergraphs [9], a generalization of graphs, have many properties which are the basis of different

techniques that are used in modern Mathematics. The applicability of graph theory has widened by the generalization of undirected graphs, called undirected hypergraphs, which have been proved to be more useful as mathematical modeling tools. In real World applications, hypergraph techniques appear very useful in many places, including declustering problems which are important to increase the performance of parallel databases. Hypergraphs can be demonstrated as a useful engine(or tool) to model concepts and systems in different fields of discrete mathematics. There are many complex phenomena and concepts in many areas, including rewriting systems, problem solving, databases and logic programming which can be represented using hypergraphs. Directed hypergraphs are used to solve and model certain problems arising in deductive databases and in model checking.
Fuzzy graphs were narrated by Rosenfeld [25] in 1975. After that in 1987, some remarks on fuzzy graphs were represented by Bhattacharya [10]. He showed that all the concepts of crisp graph theory do not have similarities in fuzzy graph theory. Several concepts on fuzzy graphs and fuzzy hypergraphs were discussed by Mordeson and Nair [20]. Parvathi at el. described some operations on intuitionistic fuzzy graphs in [21]. Kaufmann [14] gave the idea of fuzzy hypergraphs and Chen [12] defined the interval-valued fuzzy hypergraphs. Generalization and redefinition of fuzzy hypergraphs were discussed by Lee-Kwang and Keon-Myung [17]. Parvathi et al. [22] established the notion of IF hypergraph. Later, Akram and Dudek extended this idea and studied its various properties in [5]. They also represented various applications of intuitionistic fuzzy hypergraphs such as radio coverage network and clustering problem. Parvathi et al. [23] established the notion of IF directed hypergraphs. The minimum spanning of SVN tree and its clustering method were studied by Ye [30]. Broumi et al. [8] portrayed single-valued neutrosophic graphs. Akram and Shahzadi [2] introduced the notion of neutrosophic soft graphs with applications. Akram et al. [1] also introduced the single-valued neutrosophic hypergraphs.
This paper is classified as follows: In section 2, concepts of SVN directed hypergraphs are described. The concepts of simple, elementary, support simple and sectionally elementary SVN directed hypergraphs are introduced. Section 3 deals with concepts of SVN line directed graphs, 2−section of a SVN directed hypergraphs and dual SVN directed hypergraphs. We describe the construction of dual SVN directed hypergraphs. In section 4, we discuss how the concept of SVN directed hypergraphs and SVN line directed graphs can be helpful to understand and analyse the production and manufacturing networks, social networks and collaboration networks. In the last section, we conclude our results. Throughout the paper, following notations and terminologies are used:

Notations	Description
$D = (V, H)$	Single-valued neutrosophic directed hypergraph with vertices V and SVN directed hyperedges H.
$FS(D)$	Fundamental sequence of SVNDHG D.
χ	Strength of a SVN directed hyperpath.
$D^* = (V^*, H^*)$	Dual SVN directed hypergraph of D.
$L(D) = (V_L, H_L)$	SVN line directed graph of D.
$[D]_2 = (V, E)$	2−section graph of D.
$(0, 0, 0)$	**0**

2 Single-Valued Neutrosophic Directed Hypergraphs

Definition 2.1. [26] Let X be a set of points with a general element in X denoted by x. A *neutrosophic set*(NS) N in X is an object having the form $N = \{(x, \alpha_N(x), \beta_N(x), \gamma_N(x))|x \in X\}$, which is characterized by a truth-membership function α_N, an indeterminacy-membership function β_N and falsity-membership function γ_N, where the functions $\alpha_N : X \to]0^-, 1^+[$, $\beta_N : X \to]0^-, 1^+[$ and $\gamma_N : X \to]0^-, 1^+[$ are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on the sum of $\alpha_N(x)$, $\beta_N(x)$ and $\gamma_N(x)$, therefore $0^- \leq \alpha_N(x)+\beta_N(x)+\gamma_N(x) \leq 3^+$.

In real life applications, it is complicated to use neutrosophic set in scientific and engineering problems having the values from real standard or non-standard subset of $]0^-, 1^+[$. To apply neutrosophic sets in real life problems more conveniently, we use single-valued neutrosophic sets.

Definition 2.2. [28] Let X be a set of points with a general element in X denoted by x. A *single-valued neutrosophic set*(SVNS) S in X is an object having the form $S = \{(x, \alpha_S(x), \beta_S(x), \gamma_S(x))|x \in X\}$, which is characterized by a truth-membership function α_S, an indeterminacy-membership function β_S and falsity-membership function γ_S, where the functions $\alpha_S : X \to [0, 1]$, $\beta_S : X \to [0, 1]$ and $\gamma_S : X \to [0, 1]$ are subsets of $[0, 1]$. There is a restriction on the sum of $\alpha_S(x)$, $\beta_S(x)$ and $\gamma_S(x)$, such that $0 \leq \alpha_S(x)+\beta_S(x)+\gamma_S(x) \leq 3$.

Definition 2.3. [1] The *support* of a SVNS $S = \{(x, \alpha_S(x), \beta_S(x), \gamma_S(x)) : x \in X\}$ is defined as *supp*(S)=$\{x \in X|\alpha_S(x) \neq 0, \beta_S(x) \neq 0, \gamma_S(x) \neq 0\}$. *supp*$(S)$ is a crisp set.

We now define a single-valued neutrosophic directed hypergraph.

Definition 2.4. A *single-valued neutrosophic directed hypergraph* on a non-empty set X is defined as an ordered pair $D = (V, H)$, where $V = \{A_1, A_2, A_3, \ldots, A_k\}$ is a family of non-trivial single-valued neutrosophic subsets on X and H is a single-valued neutrosophic relation on SVNSs A_i such that

1. $\alpha_H(E_i) = \alpha_H(\{v_1, v_2, v_3, \cdots, v_r\}) \leq \min\{\alpha_{A_i}(v_1), \alpha_{A_i}(v_2), \alpha_{A_i}(v_3), \cdots, \alpha_{A_i}(v_r)\}$,
   $\beta_H(E_i) = \beta_H(\{v_1, v_2, v_3, \cdots, v_r\}) \leq \min\{\beta_{A_i}(v_1), \beta_{A_i}(v_2), \beta_{A_i}(v_3), \cdots, \beta_{A_i}(v_r)\}$,
   $\gamma_H(E_i) = \gamma_H(\{v_1, v_2, v_3, \cdots, v_r\}) \leq \max\{\gamma_{A_i}(v_1), \gamma_{A_i}(v_2), \gamma_{A_i}(v_3), \cdots, \gamma_{A_i}(v_r)\}$,
   for all $v_1, v_2, v_3, \cdots, v_r \in X$.
2. $X = \bigcup_k supp(A_k)$, for all $A_k \in V$.

Here $\{E_1, E_2, E_3, \cdots, E_r\}$ is the family of directed hyperedges.

Definition 2.5. A SVN *directed hyperedge*(or *hyperarc*) is defined as an ordered pair $A = (u, v)$, where u and v are disjoint subsets of nodes. u is taken as the *tail* of A and v is called its *head*. $t(A)$ and $h(A)$ are used to denote the *tail* and *head* of SVN directed hyperarc, respectively.
In SVNDHG $D = (V, H)$, any two vertices s and t are adjacent vertices if they both belong to the same directed hyperedge. A *source* vertex s is defined as a vertex in D if $h(x) \neq s$, for each $x \in H$. A *destination* vertex d is defined as a vertex if $t(x) \neq d$, for every $x \in H$.

Definition 2.6. A SVNDHG $D = (V, H)$ can be represented by an incidence matrix. The *incidence matrix* of a SVNDHG is defined by an $n \times m$ matrix $[b_{ij}]$ as:

$$b_{ij} = \begin{cases} (\alpha_{A_j}(a_i), \beta_{A_j}(a_i), \gamma_{A_j}(a_i)), & \text{if} \quad a_i \in A_j, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

We illustrate the concept of a single-valued neutrosophic directed hypergraph with an example.

Example 2.1. Consider a SVNDHG $D = (V, H)$, such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $H = \{H_1, H_2, H_3, H_4\}$, where

$$H_1 = \{(v_1, 0.2, 0.3, 0.4), (v_3, 0.2, 0.4, 0.2), (v_5, 0.5, 0.5, 0.3)\},$$

$$H_2 = \{(v_1, 0.2, 0.3, 0.4), (v_2, 0.1, 0.3, 0.5), (v_3, 0.2, 0.4, 0.2), (v_4, 0.1, 0.2, 0.3)\},$$
$$H_3 = \{(v_3, 0.2, 0.4, 0.2), (v_4, 0.1, 0.2, 0.3), (v_7, 0.1, 0.2, 0.3)\},$$
$$H_4 = \{(v_5, 0.5, 0.5, 0.3), (v_6, 0.4, 0.5, 0.6), (v_7, 0.1, 0.2, 0.3)\}.$$

The incidence matrix of $D = (V, H)$ is given in Table 1.

Table 1: Incidence matrix of SVNDHG

I_D	H_1	H_2	H_3	H_4
v_1	$(0.2, 0.3, 0.4)$	$(0.2, 0.3, 0.4)$	$\mathbf{0}$	$\mathbf{0}$
v_2	$\mathbf{0}$	$(0.1, 0.3, 0.5)$	$\mathbf{0}$	$\mathbf{0}$
v_3	$(0.2, 0.4, 0.2)$	$(0.2, 0.4, 0.2)$	$(0.2, 0.4, 0.2)$	$\mathbf{0}$
v_4	$\mathbf{0}$	$(0.1, 0.2, 0.3)$	$(0.1, 0.2, 0.3)$	$\mathbf{0}$
v_5	$(0.5, 0.5, 0.3)$	$\mathbf{0}$	$\mathbf{0}$	$(0.5, 0.5, 0.3)$
v_6	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$(0.4, 0.5, 0.6)$
v_7	$\mathbf{0}$	$\mathbf{0}$	$(0.1, 0.2, 0.3)$	$(0.1, 0.2, 0.3)$

The SVNDHG is shown in Figure 1.

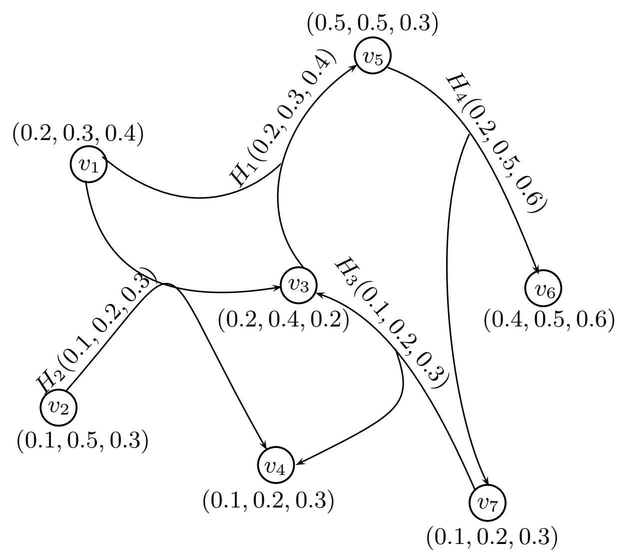

Figure 1: SVNDHG

Definition 2.7. The *height* of a SVNDHG $D = (V, H)$ is defined as $h(D) = \{\max(H_k), \max(H_l), \min(H_m) \mid H_k, H_l, H_m \in H\}$, where $H_k = \max(\alpha_{H_j}(v_i))$, $H_l = \max(\beta_{H_j}(v_i))$ and $H_m = \min(\gamma_{H_j}(v_i))$. Here, $\alpha_{H_j}(v_i)$, $\beta_{H_j}(v_i)$ and $\gamma_{H_j}(v_i)$ denote the truth-membership, indeterminacy and falsity-membership values of vertex v_i to directed hyperedge H_j, respectively.

Definition 2.8. A SVNS $S = \{(x, \alpha_S(x), \beta_S(x), \gamma_S(x)) : x \in X\}$ is called an *elementary* single-valued neutrosophic set if α_S, β_S and γ_S are single valued on the support of S.
A SVNDHG $D = (V, H)$ is an *elementary* SVNDHG if its all directed hyperedges are elementary.

Definition 2.9. The *strength* of a single-valued neutrosophic directed hyperedge is defined as $\eta(H_i) = \{\min_{v_j} \in H_i(\alpha_{H_i}(v_j) : \alpha_{H_i}(v_j) > 0), \min_{v_j} \in H_i(\beta_{H_i}(v_j) : \beta_{H_i}(v_j) > 0), \max_{v_j} \in H_i(\gamma_{H_i}(v_j) : \gamma_{H_i}(v_j) > 0)\}$.

The *strength* of directed hyperedge describes that the objects having the participation degree at least $\eta(H_i)$ are grouped in the hyperedge H_i.

Definition 2.10. A SVNDHG $D = (V, H)$ is *simple* if $A_j,\ A_k \in H$ and $A_j \leq A_k$ implies $A_j = A_k$.
A SVNDHG $D = (V, H)$ is called *support simple* if $A_j,\ A_k \in H$, $supp(A_j) = supp(A_k)$ and $A_j \leq A_k$, then $A_j = A_k$.
A SVNDHG $D = (V, H)$ is called *strongly support simple* if $A_j,\ A_k \in H$ and $supp(A_j) = supp(A_k)$ imply that $A_j = A_k$.

Theorem 2.1. *A SVNDHG $D = (V, H)$ is single-valued neutrosophic directed graph(possibly with loops) if and only if D is support simple, elementary and all the hyperedges have two(or one) element support.*

Theorem 2.2. *Let $D = (V, H)$ be an elementary SVNDHG. Then D is support simple if and only if D is strongly support simple.*
Proof. Suppose that $D = (V, H)$ is elementary, support simple and $supp(A_j) = supp(A_k)$ for $A_j, A_k \in H$. We assume that $h(A_j) \leq h(A_k)$. Since D is elementary we have $A_j \leq A_k$, and since D is support simple we have $A_j = A_k$. Hence D is strongly support simple.
The converse part of the theorem can be proved trivially by using the definitions.

Theorem 2.3. *Let $D = (V, H)$ be a strongly support simple SVNDHG of order n. Then $|H| \leq 2^n - 1$. The equality holds if and only if $\{supp(A_j)|A_j \in H\}$=$\mathcal{P}(V) \setminus \emptyset$.*
Proof. Since every non trivial subset of V can be the support set of at most one $A_j \in H$ so $|H| \leq 2^n - 1$. The second part is trivial.

Theorem 2.4. *Let $D = (V, H)$ be a simple, elementary SVNDHG of order n. Then $|H| \leq 2^n - 1$. The equality holds if and only if $\{supp(A_j)|A_j \in H\}$=$\mathcal{P}(V) \setminus \emptyset$.*
Proof. Since D is simple and elementary, each non trivial subset of V can be the support set of at most one $A_j \in H$. Hence $|H| \leq 2^n - 1$. We now prove that there exists an elementary, simple D having $|H| = 2^n - 1$. Let $A = \{(\alpha_B(v), \beta_B(v), \gamma_B(v))|B \subseteq V\}$ be the set of mappings such that

$$\alpha_B(v) = \begin{cases} \frac{1}{|B|}, & \text{if} v \in B, \\ 0, & \text{otherwise.} \end{cases}$$

$$\beta_B(v) = \begin{cases} \frac{1}{|B|}, & \text{if} v \in B, \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_B(v) = \begin{cases} \frac{1}{|B|}, & \text{if} v \in B, \\ 0, & \text{otherwise.} \end{cases}$$

Then every set containing single element has height $(1, 1, 1)$, height of every set containing two elements is $(0.5, 0.5, 0.5)$ and so on. Hence D is elementary, simple and $|H| = 2^n - 1$.

Definition 2.11. Let $D = (V, H)$ be a SVNDHG. Consider $\lambda \in [0, 1]$, $\mu \in [0, 1]$ and $\nu \in [0, 1]$ such that $0 \leq \lambda + \mu + \nu \leq 3$. Then the $(\lambda, \mu, \nu)-$level directed hypergraph of D is defined as an ordered pair $D^{(\lambda,\mu,\nu)} = (V^{(\lambda,\mu,\nu)}, H^{(\lambda,\mu,\nu)})$, where $H^{(\lambda,\mu,\nu)} = \{H_i^{(\lambda,\mu,\nu)}|H_i \in H\}$, $V^{(\lambda,\mu,\nu)} = \bigcup\limits_{H_i \in H} H_i^{(\lambda,\mu,\nu)}$ and
$H_i^{(\lambda,\mu,\nu)} = \{v_j \in V|\alpha_{H_i}(v_j) \geq \lambda, \beta_{H_i}(v_j) \geq \mu, \gamma_{H_i}(v_j) \leq \nu\}$.

Definition 2.12. Let D=(V, H) be a SVNDHG such that $h(D)$=(u, v, w).
Let $D^{(u_i,v_i,w_i)} = (V^{(u_i,v_i,w_i)}, H^{(u_i,v_i,w_i)})$ be the (u_i, v_i, w_i)-level hypergraphs of D. The sequence of real numbers $(u_1, v_1, w_1), (u_2, v_2, w_2), ..., (u_n, v_n, w_n)$,
$0 < u_n < u_{n+1} <, ..., < u_1 = u, 0 < v_n < v_{n+1} <, ..., < v_1 = v$, and $w_n > w_{n+1} >, ..., > w_1 = w > 0$, which satisfies the properties:

1. if $u_{i+1} < u' < u_i, v_{i+1} < v' < v_i, w_{i+1} > w' > w_i (w_i < w' < w_{i+1})$, then $H^{(u',v',w')} = H^{(u_i,v_i,w_i)}$,
2. $H^{(u_i,v_i,w_i)} \sqsubseteq H^{(u_{i+1},v_{i+1},w_{i+1})}$,

is *fundamental sequence* of SVNDHG D, denoted by $FS(D)$. The set of (u_i, v_i, w_i)-level hypergraphs $\{D^{(u_1,v_1,w_1)}$
$, D^{(u_2,v_2,w_2)}, ..., D^{(u_n,v_n,w_n)}\}$ is known as *core hypergraphs* of SVNDHG D and is denoted by $c(D)$. The corresponding sequence of $(u_i, v_i, w_i)-$level directed hypergraphs $\{D^{(u_1,v_1,w_1)} \subseteq D^{(u_2,v_2,w_2)} \subseteq ... \subseteq D^{(u_n,v_n,w_n)}\}$ is called the D *induced fundamental sequence.*

Example 2.2. e Consider a SVNDHG D=(V, H), where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $H = \{H_1, H_2, H_3, H_4\}$. Incidence matrix of D is given in Table 2.

Table 2: Incidence matrix of D

	H_1	H_2	H_3	H_4
v_1	$(0.8, 0.7, 0.1)$	$(0.9, 0.8, 0.1)$	**0**	$(0.5, 0.4, 0.3)$
v_2	$(0.8, 0.7, 0.1)$	$(0.9, 0.8, 0.1)$	$(0.5, 0.4, 0.3)$	$(0.5, 0.4, 0.3)$
v_3	**0**	**0**	$(0.3, 0.3, 0.4)$	**0**
v_4	$(0.5, 0.4, 0.3)$	**0**	$(0.5, 0.4, 0.3)$	**0**
v_5	**0**	**0**	**0**	$(0.5, 0.4, 0.3)$

The corresponding graph is shown in Figure 2.

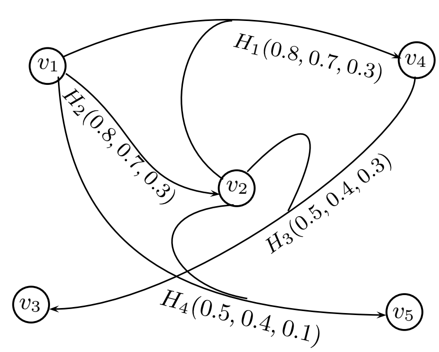

Figure 2: SVNDHG

By routine calculations, we have $h(D) = (0.9, 0.8, 0.1)$, $H^{(0.9,0.8,0.1)} = \{\{v_1, v_2\}\}$, $H^{(0.8,0.7,0.1)} = \{\{v_1, v_2\}\}$ and $H^{(0.5,0.4,0.3)} = \{\{v_1, v_2\}, \{v_1, v_2, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_4\}\}$. Therefore, the $FS(D)$ is $\{(0.9, 0.8, 0.1), (0.5, 0.4, 0.3)\}$. The *set of core hypergraphs* is $c(D) = \{D^{(0.9,0.8,0.1)} = (V_1, H_1), D^{(0.5,0.4,0.3)} = (V_2, H_2)\}$. Note that $H^{(0.9,0.8,0.1)} \subseteq H^{(0.5,0.4,0.3)}$ and $H^{(0.9,0.8,0.1)} \neq H^{(0.5,0.4,0.3)}$, $H_i \nsubseteq H_j$ for all $H_i, H_j \in H$, hence D is *simple*. Further, it can be seen that $supp(H_i) = supp(H_j)$ for all $H_i, H_j \in H$

implies $H_i = H_j$. Thus, D is *strongly support simple* and *support simple*. The *induced fundamental sequence* of D is given in Figure 3.

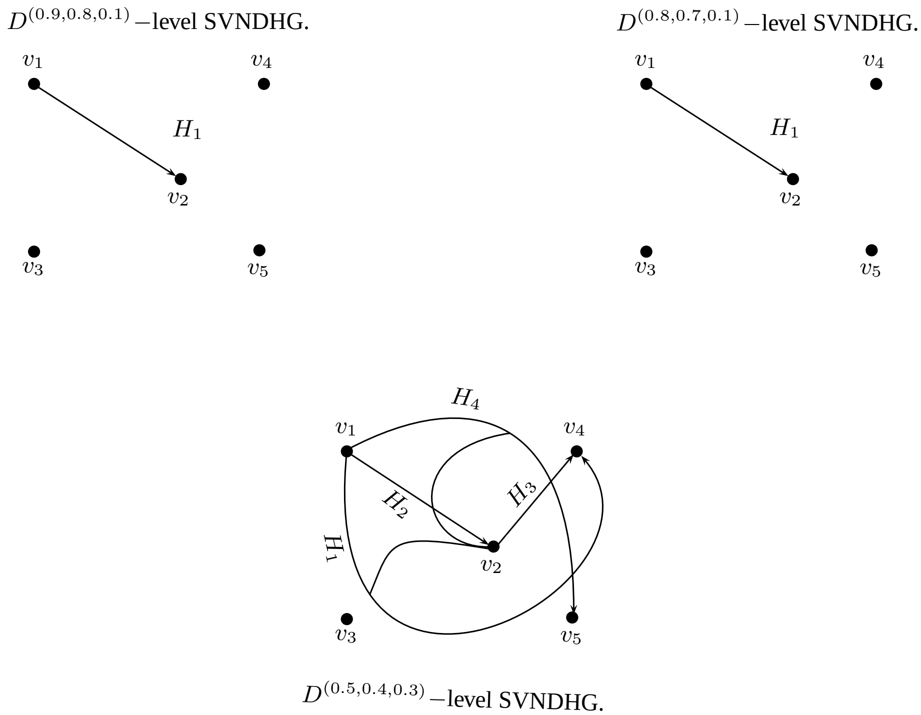

Figure 3: Induced fundamental sequence of D.

Definition 2.13. Let D=(V, H) be a SVNDHG and $FS(D) = \{(u_1, v_1, w_1), (u_2, v_2, w_2), ..., (u_n, v_n, w_n)\}$ be the fundamental sequence of D. If for each $H_i \in H$ and each $(l, m, n) \in ((u_{i+1}, v_{i+1}, w_{i+1}), (u_i, v_i, w_i)]$, we have $H_i^{(l,m,n)} = H_i^{(u_i,v_i,w_i)}$ for all $(u_i, v_i, w_i) \in FS(D)$, then D is called *sectionally elementary*.
It can be noted that D is *sectionally elementary* if and only if $\alpha_{H_i}(x), \beta_{H_i}(x), \gamma_{H_i}(x) \in FS(D)$ for all $H_i \in H$ and for every $x \in V$.

Definition 2.14. Let D=(V, H) be a SVNDHG. The *partial single-valued neutrosophic directed hypergraph* of D is defined as an ordered pair D'=(V', H'), where $H' \subseteq H$ and $V' = \bigcup_i \{supp(H_i')|H_i' \in H\}$. Then D' is called *partial SVNDHG* generated by H'.

Definition 2.15. A SVNDHG D=(V, H) is said to be *ordered* if $c(D)$ is ordered. That is, if $c(D) = \{D^{(u_1,v_1,w_1)}, D^{(u_2,v_2,w_2)}, \cdots, D^{(u_n,v_n,w_n)}\}$, then $D^{(u_1,v_1,w_1)} \subseteq D^{(u_2,v_2,w_2)} \subseteq \cdots \subseteq D^{(u_n,v_n,w_n)}$.
The sequence is called *simply ordered* if it is *ordered* and if whenever $H^* \in H_{j+1}^* \setminus H_j^*$, then $H^* \nsubseteq V_j$. Thus the SVNDHG is also *simply ordered*.

Proposition 2.5. *Let D=(V, H) be a SVNDHG. If D is elementary, then it is ordered. Further, if D is an ordered SVNDHG with simple support, then D is elementary.*

Example 2.3. Consider a SVNDHG D=(V, H), where $V = \{v_1, v_2, v_3, v_4\}$ and $H = \{H_1, H_2, H_3\}$, which is represented by the incidence matrix given in Table 3.

Table 3: Incidence matrix of D.

	H_1	H_2	H_3
v_1	$(0.7, 0.5, 0.1)$	$\mathbf{0}$	$(0.5, 0.3, 0.1)$
v_2	$(0.7, 0.5, 0.1)$	$(0.5, 0.3, 0.1)$	$\mathbf{0}$
v_3	$\mathbf{0}$	$(0.5, 0.3, 0.1)$	$(0.5, 0.3, 0.1)$
v_4	$\mathbf{0}$	$\mathbf{0}$	$(0.5, 0.3, 0.1)$

Here, $FS(D) = \{(0.7, 0.5, 0.1), (0.5, 0.3, 0.1)\}$. The SVNDHG D is sectionally elementary. As for each $H_i \in H$ and for all $(l, m, n) \in ((0.5, 0.3, 0.1), (0.7, 0.5, 0.1)]$, we have $H_i^{(l,m,n)} = H_i^{(0.7,0.5,0.1)}$. It can be seen that $H_1^{(0.6,0.35,0.1)} = \{v_1, v_2\} = H_1^{(0.7,0.5,0.1)}$ and so on.
The corresponding SVNDHG is shown in Figure 4.

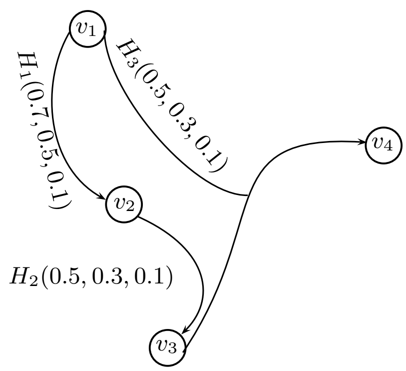

Figure 4: Sectionally elementary SVNDHG.

3 Single-valued neutrosophic line directed hypergraphs

Definition 3.1. A *SVN directed hyperpath* of length k in a SVNDHG $D = (V, H)$ is defined as an alternating sequence $v_1, H_1, v_2, H_2, \cdots, v_k, H_k, v_{k+1}$ of distinct points and directed hyperedges such that

(i) $\alpha_H(H_i) > 0$, $\beta_H(H_i) > 0$ and $\gamma_H(H_i) > 0$,

(ii) $v_i, v_{i+1} \in H_i$, $i = 1, 2, 3, \cdots, k$.

A *SVN directed hyperpath* is called a *SVN directed hypercycle* if $v_1 = v_{k+1}$.

Definition 3.2. A SVNDHG $D = (V, H)$ is *connected* if a SVN directed hyperpath exists between each pair of distinct nodes.

Definition 3.3. Let any two vertices, say s and t, be connected through a SVN directed hyperpath of length k in a SVNDHG. Then the *strength* of the SVN directed hyperpath is defined as,

$$\chi^k(s,t)=\{\alpha_H(H_1) \wedge \alpha_H(H_2) \wedge \cdots \wedge \alpha_H(H_k), \beta_H(H_1) \wedge \beta_H(H_2) \wedge \cdots \wedge \beta_H(H_k), \gamma_H(H_1) \vee \gamma_H(H_2) \vee \cdots \vee \gamma_H(H_k)\},$$

$s \in H_1$, $t \in H_k$. $H_1, H_2, \cdots, H_k$ are directed hyperedges.
The strength of connectedness between s and t is defined as,
$\chi^{\infty}(s,t)$=$\{\sup_k \alpha(\chi^k(s,t)), \sup_k \beta(\chi^k(s,t)), \inf_k \gamma(\chi^k(s,t))\}$.

Theorem 3.1. *A SVNDHG $D = (V, H)$ is connected if and only if $\chi^{\infty}(s,t) > 0$, for all $s, t \in V$.*
Proof. Suppose that $D = (V, H)$ is connected SVNDHG. Then between each pair of distinct vertices there exists a SVN directed hyperpath such that
$\chi^k(s,t) > 0$
$\Rightarrow \{\sup_k \alpha(\chi^k(s,t)), \sup_k\{\beta(\chi^k(s,t)), \inf_k\{\gamma(\chi^k(s,t))|k = 1, 2, \cdots\} > 0$
$\Rightarrow \chi^{\infty}(s,t) > 0$, for all $s, t \in V$.
Conversely, suppose that $\chi^{\infty}(s,t) > 0$
$\Rightarrow \{\sup_k \alpha(\chi^k(s,t)), \sup_k\{\beta(\chi^k(s,t)), \inf_k\{\gamma(\chi^k(s,t))|k = 1, 2, \cdots\} > 0$. This shows that there exists at least one directed hyperpath between each pair of vertices. Hence D is connected.

Definition 3.4. A SVNDHG $D = (V, H)$ is called *linear* if for every SVN directed hyperedge $H_i, H_j \in H$

(i) $supp(H_i) \subseteq supp(H_j)$ implies $i = j$,

(ii) $|supp(H_i) \bigcap supp(H_j)| \leq 1$.

We now define the dual SVN directed hypergraphs.

Definition 3.5. Let $D = (V, H)$ be a SVNDHG on a universal set V. The *dual single-valued neutrosophic directed hypergraph* of D is defined as an ordered pair $D^* = (V^*, H^*)$, where

1 $V^* = H$ is single-valued neutrosophic set of vertices of D^*.

2 If $|V| = n$, then H^* is the SVNS on the set of directed hyperedges $\{V_1, V_2, V_3, \cdots, V_n\}$ such that $V_i = \{H_j|v_i \in H_j, H_j$ is the directed hyperedge in $D\}$. This means that V_i is the set of those directed hyperedges which contain the vertex v_i as a common vertex.

The truth-membership, indeterminacy and falsity-membership values of V_i are defined as,

$$\alpha^*_H(V_i)=\inf\{\alpha_H(H_j) : v_i \in H_j\},\ \beta^*_H(V_i)=\inf\{\beta_H(H_j) : v_i \in H_j\},\ \gamma^*_H(V_i)=\sup\{\gamma_H(H_j) : v_i \in H_j\}.$$

We describe the method of construction of dual single-valued neutrosophic directed hypergraph D^* of a SVNDHG D as a simple procedure given below. We also describe an example.

Construction 1.

Let $D = (V, H)$ be a single-valued neutrosophic directed hypergraph. The procedure of constructing the dual single-valued neutrosophic directed hypergraph contains the following steps.

1. Make the single-valued neutrosophic set of vertices of D^* as V^*=H.

2. Define a one to one function $f : V \to H$ from the set of vertices to the set of directed hyperedges of D in the way that if the directed hyperedges $H_s, H_{s+1}, H_{s+2}, \cdots, H_j$ contain the vertex v_i, then v_i is mapped onto $H_s, H_{s+1}, H_{s+2}, \cdots, H_j$ as shown in Figure 5.

3. Draw the directed hyperedges $\{V_1, V_2, \cdots, V_n\}$ of D^* such that $V_i = \{H_j|f(v_i) = H_j\}$.

4. Make the directed hyperedges as the vertex H_j of D^* belongs to $h(V_i)$ if and only if $v_i \in t(H_j)$ in D
   and similarly H_j is in $t(V_i)$ if and only if $v_i \in h(H_j)$.

5. Calculate the truth-membership, indeterminacy and falsity-membership values of directed hyperedges in D^* as $\alpha_{H^*}(V_i)$=inf$\{\alpha_H(H_j) : v_i \in H_j\}$, $\beta_{H^*}(V_i)$=inf$\{\beta_H(H_j) : v_i \in H_j\}$, $\gamma_{H^*}(V_i)$=sup$\{\gamma_H(H_j) : v_i \in H_j\}$.

Example 3.1. Consider a SVNDHG $D = (V, H)$, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $H = \{H_1, H_2, H_3\}$ as shown in Figure 5. The *dual* SVNDHG $D^* = (V^*, H^*)$ is shown with dashed lines such that $V^* = \{h_1, h_2, h_3\}$, $H^* = \{V_1, V_2, V_3, V_4, V_5, V_6\}$. The incidence matrix of D^* is given in Table 4.

Table 4: Incidence matrix of dual single-valued neutrosophic directed hypergraph.

I_{D^*}	V_1	V_2	V_3	V_4	V_5	V_6
h_1	$(0.2, 0.1, 0.1)$	$(0.2, 0.1, 0.1)$	**0**	**0**	$(0.2, 0.1, 0.1)$	**0**
h_2	**0**	$(0.2, 0.1, 0.1)$	$(0.4, 0.3, 0.3)$	$(0.4, 0.3, 0.3)$	**0**	**0**
h_3	**0**	$(0.2, 0.1, 0.1)$	**0**	$(0.4, 0.3, 0.3)$	**0**	$(0.4, 0.3, 0.3)$

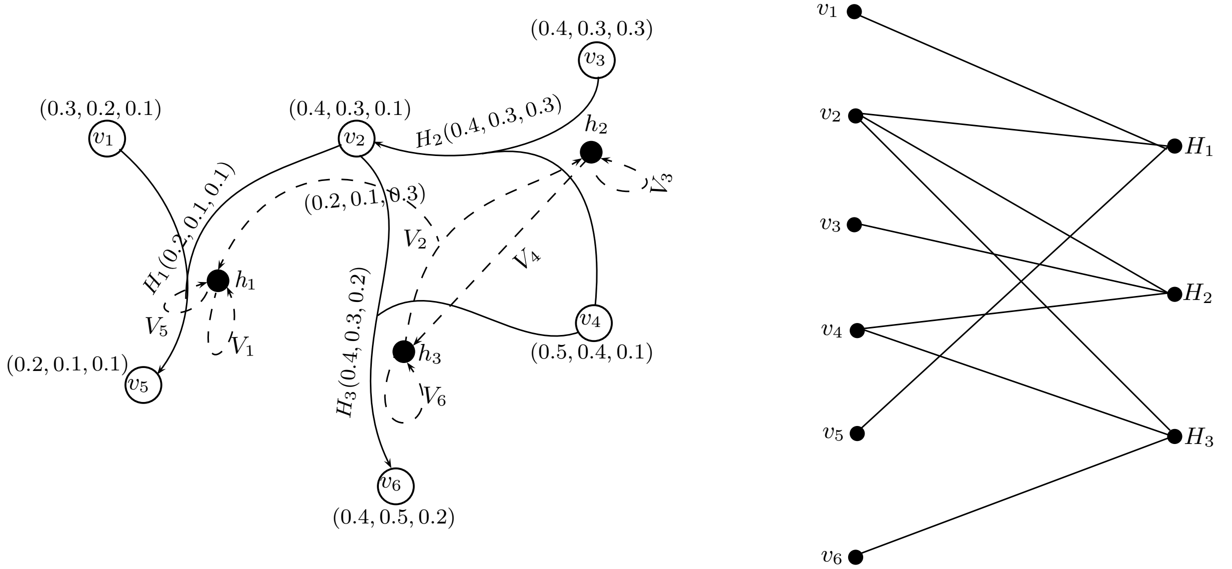

Figure 5: SVNDHG and its dual directed hypergraph D^*.

Theorem 3.2. *Let D be a single-valued neutrosophic directed hypergraph. Then $D^{**} = D$.*

Theorem 3.3. *The dual SVNDHG of a linear SVN hypergraph is also linear, that is, if D is linear then D^* is also linear.*

Proof. Let $D = (V, H)$ be a linear single -valued neutrosophic directed hypergraph and $D^* = (V^*, H^*)$. Suppose on contrary that D^* is not linear then there exists V_i and V_j such that $|supp(V_i) \bigcap supp(V_j)| = 2$. Let $|supp(V_i) \bigcap supp(V_j)| = \{H_l, H_m\}$. Then the duality of D^* implies that $v_i, v_j \in H_l$ and $v_i, v_j \in H_m$, which is a contradiction to the statement that D is linear. Hence D^* is linear.

Definition 3.6. Let $D = (V, H)$ be a SVNDHG. The *single-valued line directed graph* of D is the directed graph $L(D) = (V_L, H_L)$, such that

1. $V_L = H$,
2. $\{A_i, A_j\} \in H_L$ if and only if $|supp(A_i) \bigcap supp(A_j)| \neq \emptyset$ for $i \neq j$.

The truth-membership, indeterminacy and falsity-membership values of vertices and hyperedges of $L(D)$ are defined as,

(i) $V_L(A_i) = H(A_i)$,

(ii) $\alpha_{H_L}\{A_i, A_j\} = \min\{\alpha_H(A_i), \alpha_H(A_j) | A_i, A_j \in H\}$, $\beta_{H_L}\{A_i, A_j\} = \min\{\beta_H(A_i), \beta_H(A_j) | A_i, A_j \in H\}$ and $\gamma_{H_L}\{A_i, A_j\} = \max\{\gamma_H(A_i), \gamma_H(A_j) | A_i, A_j \in H\}$,

respectively.

Example 3.2. Consider a SVNDHG $D = (V, H)$ as given in Figure 6. The SVN line directed hypergraph $L(D) = (V_L, H_L)$ of D is shown with dashed hyperedges.

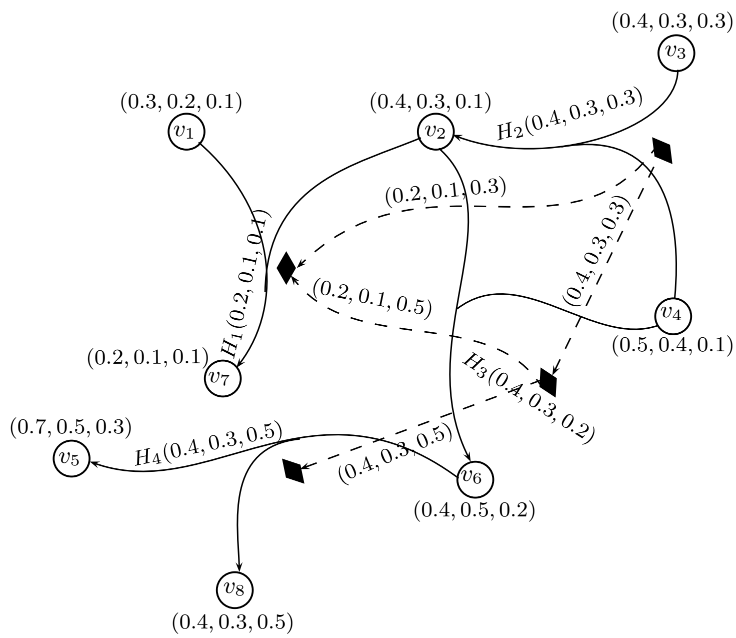

Figure 6: SVNDHG and its line directed graph $L(D)$.

Theorem 3.4. *Let $G = (U, W)$ be a simple SVN digraph. Then G is the SVN line graph of a linear SVN directed hypergraph.*

Proof. Let $G = (U, W)$ be a simple SVN digraph on a set of universe Z. With no loss of generality, suppose that G is connected. A SVNDHG $D = (V, H)$ can be formed from G as,

1. Take the set of edges of G as the vertices of D. Let $W = \{w_1, w_2, w_3, \cdots, w_n\}$ be the directed edges of G and Z^D be the set of vertices of D, then $Z^D = W$. Let $V = \{\rho_1, \rho_2, \rho_3, \cdots, \rho_r\}$ be the collection of non-trivial SVNSs on Z, such that $\rho_k(w_i) = 1$, $i = 1, 2, 3, \cdots, n$.
2. Let $Z = \{z_1, z_2, z_3, \cdots, z_m\}$, then the set of directed hyperedges of D is $H^D = \{H_1, H_2, H_3, \cdots, H_n\}$, where H_j are those edges of G in which z_i is the incidence vertex, that is, $H_i = \{w_j | z_i \in w_j, j = 1, 2, 3, \cdots, n.\}$. Further, $H(H_i) = U(z_i)$, $i = 1, 2, 3, \cdots, n$.

We claim that D is a linear SVNDHG. Consider the directed hyperedge $H_j = \{w_1, w_2, w_3, \cdots, w_k\}$. From the definition of SVNDG, we have
$\alpha_{H(H_i)} = \inf\{\wedge_j \alpha_{\rho_j}(w_1), \wedge_j \alpha_{\rho_j}(w_2), \cdots, \wedge_j \alpha_{\rho_j}(w_k)\} = \alpha_U(z_i) \leq 1$,
$\beta_{H(H_i)} = \inf\{\wedge_j \beta_{\rho_j}(w_1), \wedge_j \beta_{\rho_j}(w_2), \cdots, \wedge_j \beta_{\rho_j}(w_k)\}$
$= \beta_U(z_i) \leq 1$ and $\gamma_{H(H_i)} = \sup\{\vee_j \gamma_{\rho_j}(w_1), \vee_j \gamma_{\rho_j}(w_2), \cdots, \vee_j \gamma_{\rho_j}(w_k)\} = \gamma_U(z_i) \leq 1$, $1 \leq i \leq n$, and $\bigcup_r (\rho_r) = Z^D$, for all $\rho_r \in V$.
Thus D is SVNDHG. We now prove that D is linear.

1. Since the truth-membership, indeterminacy and falsity-membership values of all the vertices of D are same. Therefore, $supp(\rho_i) \subseteq supp(\rho_j)$ implies $i = j$, for each $1 \leq i, j \leq r$.
2. On contrary, suppose that $supp(\rho_i) \bigcap supp(\rho_j) = \{w_l, w_m\}$, that is, the both edges w_l, w_m have two incident vertices in G, which is a contradiction to the statement that G is simple. Hence $|supp(\rho_i) \bigcap supp(\rho_j)| \leq 1$, $1 \leq i, j \leq r$.

Theorem 3.5. *A SVNDHG $D = (V, H)$ is connected if and only if its line directed graph $L(D)$ is connected.*
Proof. Suppose that $D = (V, H)$ is connected SVNDHG. Let $L(D) = (A, B)$ be the SVN line directed graph of D and H_i, H_j be any two distinct vertices of $L(D)$. Consider $v_i \in H_i$ and $v_j \in H_j$. Since D is connected, there exists a SVN directed hyperpath $v_i, H_i, v_{i+1}, H_{i+1}, \cdots, v_j, H_j$ between v_i and v_j. By definition of strength of connectedness, we have

$$\begin{aligned}
\chi^\infty(H_i, H_j) &= \sup_k\{\alpha(\chi^k(H_i, H_j))\}, \sup_k\{\beta(\chi^k(H_i, H_j))\}, \inf_k\{\gamma(\chi^k(H_i, H_j))\}, k = 1, 2, \cdots \\
&= \sup_k\{\alpha_B(H_i, H_{i+1}) \wedge \alpha_B(H_{i+1}, H_{i+2}) \wedge \cdots \wedge \alpha_B(H_{j-1}, H_j)\}, \sup_k\{\beta_B(H_i, H_{i+1}) \\
&\wedge \quad \beta_B(H_{i+1}, H_{i+2}) \wedge \cdots \wedge \beta_B(H_{j-1}, H_j)\}, \inf_k\{\gamma_B(H_i, H_{i+1}) \vee \gamma_B(H_{i+1}, H_{i+2}) \vee \cdots \vee \beta_B(H_{j-1}, H_j)\} \\
&= \sup\{\alpha_H(H_i) \wedge \alpha_H(H_{i+1}) \wedge \cdots \wedge \alpha_H(H_{j-1}) \wedge \alpha_H(H_j)\}, \sup\{\beta_H(H_i) \wedge \beta_H(H_{i+1}) \wedge \cdots \\
&\wedge \quad \alpha_H(H_{j-1}) \wedge \beta_H(H_j)\}, \inf\{\gamma_H(H_i) \vee \gamma_H(H_{i+1}) \vee \cdots \vee \gamma_H(H_{j-1}) \vee \gamma_H(H_j)\}, k = 1, 2, \cdots \\
&= \sup\{\alpha(\chi^k(v_i, v_j))\}, \sup\{\beta(\chi^k(v_i, v_j))\}, \inf\{\gamma(\chi^k(v_i, v_j))\}, k = 1, 2, \cdots \\
&= \chi^\infty(v_i, v_j) > 0.
\end{aligned}$$

Since H_i and H_j were chosen arbitrarily. Hence $L(D)$ is connected. The converse part of the theorem can be proved on the same lines.

Definition 3.7. The 2-*section* $[D]_2$ of a single-valued neutrosophic directed hypergraph $D = (V, H)$ is the SVN graph (V, E), where

(i) $V = V$, i.e., $[D]_2$ has the same set of vertices as D.

(ii) $E = \{h = v_i v_j | v_i \neq v_j, v_i v_j \in H_k, k = 1, 2, 3, \cdots\}$, i.e., two vertices v_i and v_j are adjacent in $[D]_2$ if they belong to the same directed hyperedge H_k in D and $\alpha_E(v_i v_j) = \inf\{\wedge_k \alpha_{H_k}(v_i), \wedge_k \alpha_{H_k}(v_j)\}$, $\beta_E(v_i v_j) = \inf\{\wedge_k \beta_{H_k}(v_i), \wedge_k \beta_{H_k}(v_j)\}$, $\gamma_E(v_i v_j) = \sup\{\vee_k \gamma_{H_k}(v_i), \vee_k \gamma_{H_k}(v_j)\}$.

Example 3.3. A SVNDHG $D = (V, H)$ and its 2-section $[D]_2$ is shown in Figure 7.

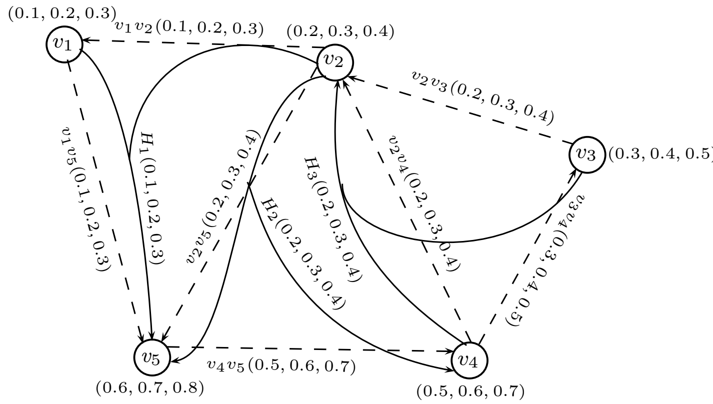

Figure 7: A SVNDHG and its 2−section.

4 Applications

Graphs and hypergraphs can be used to describe the complex network systems. The complex network systems, including social networks, World Wide Web, neural networks are investigated by means of simple graphs and digraphs. The graphs take the nodes as a set of objects or people and the edges define the relations between them. In many cases, it is not possible to give full description of real World systems using the simple graphs or digraphs. For example, if a collaboration network is represented through a simple graph. We only know that whether the two researchers are working together or not. We can not know if three or more researchers, which are connected in the network, are coauthors of the same article or not. Further, in various situations, the given data contains the information of existence, indeterminacy and non-existance. To overcome such type of difficulties in complex networks, we use single-valued neutrosophic directed hypergraphs to describe the relationships between three or more elements and the networks are then called the *hyper-networks* .

1. **Production and manufacturing networks**: In a production system, there is a set of goods which can be produced using different technologies or devices. A SVNDHG can be utilized more precisely to illustrate a production and manufacturing system. Consider a production system as given in Figure 4, where the set of square vertices represents the products which are taken as input to produce the other products as given in elliptical vertices.

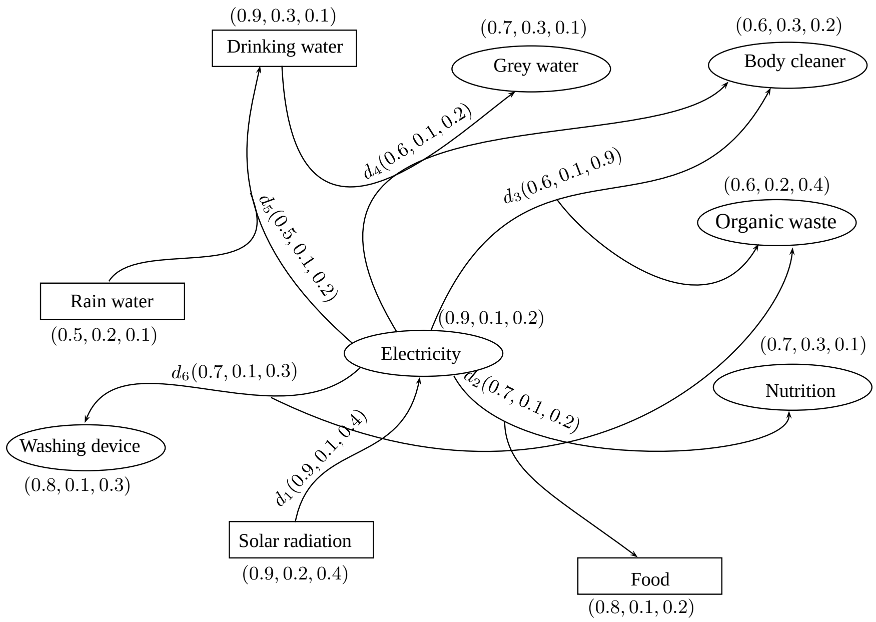

Figure 8: production system using a SVNDHG.

The set of directed hyperedges $\{d_1, d_2, d_3, d_4, d_5, d_6\}$ contains the devices or technologies which are used in our production system to design new products. Here we use the devices {Silicon photovoltaic system, Electric hob, Ultrasonic shower, Electric heater shower, Harvesting system, Washing machine}, which are represented by directed hyperedges. A directed hyperarc $(t(d), h(d))$ represents that the goods in set $t(d)$ are required to manufacture the products in the set $h(d)$. The product nodes are taken as storage. The truth-membership and falsity-membership values of each product node interpret that how much of the product is available to supply and unavailable to fulfill the demand, respectively. The indeterminacy value contains the imprecise or inexact information about the product. The truth-membership degree of each directed hyperedge(or device) describes that how much this technology is appropriate to manufacture the product. For example, the directed hyperedge d_2=({Electricity, Food}, {Nutrition}) interprets that the electric hob uses electricity and food to produce nutrition. It is noted that more than one technologies can be adopted to manufacture the same product using different or same inputs. The truth-membership degrees of each hyperedge evaluates the suitability of that device. For example, electric heater shower having membership degrees $(0.6, 0.1, 0.2)$ is more useful device than an electric shower $(0.6, 0.1, 0.4)$ to a body cleaner, as its falsity-membership value is less than an electric shower.

2.Collaboration network using a SVNDHG: We use a SVNDHG as a directed hyper-network to discuss the teamwork or joint efforts of researchers from different fields. Consider a SVNDHG $D = (V, H)$ as a collaboration network. The vertices or nodes of the hypergraph are taken as the researchers. The set of vertices V is $\{M_1, M_2, M_3, P_1, P_2, P_3, C_1, C_2, C_3, C_4, B_1, B_2, B_3, B_4\}$, where the subset of vertices $\{M_1, M_2, M_3\}$ represents the group of researchers in field of Mathematics, $\{P_1, P_2, P_3\}$ represents the group of researchers in field of Physics, $\{C_1, C_2, C_3, C_4\}$ represents the group of researchers in field of Chemistry and $\{B_1, B_2, B_3, B_4\}$ represents the group of researchers in field of Biology. The directed hyperedges of SVNDHG interpret the group of members which are working together at the same project. The corresponding SVNDHG is given in Figure 9.

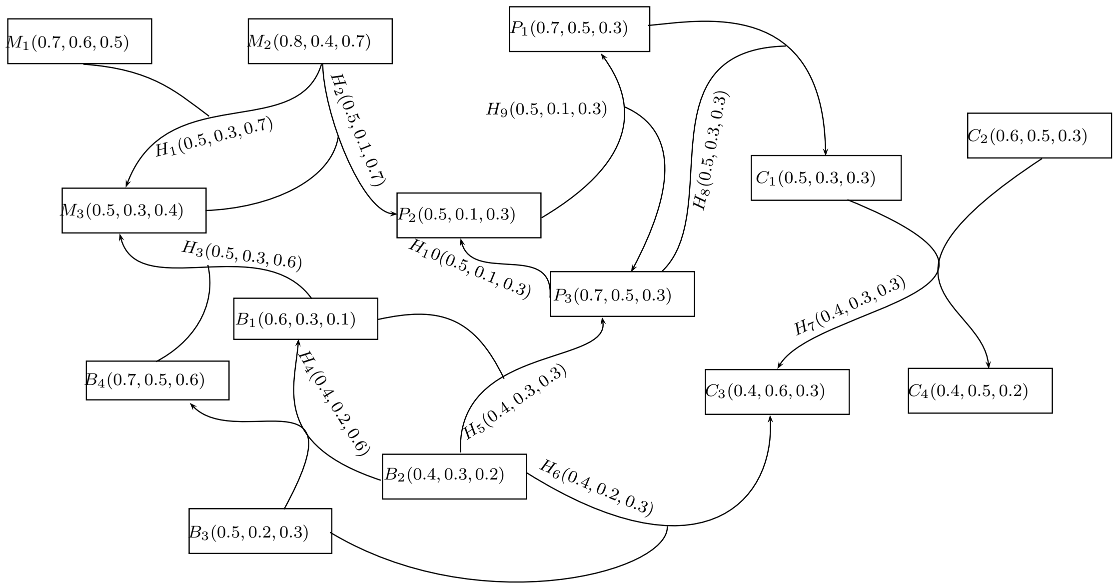

Figure 9: SVNDHG representing the collaboration network.

The truth-membership value of each researcher represents their published articles, indeterminacy shows their submitted articles that may be accepted or rejected and the falsity-membership value describes the rejected articles. For example, $(0.7, 0.6, 0.5)$ shows that the researcher M_1 has 70% publications, 60% submitted papers and 50% of his research work is rejected. The value of a SVN directed hyperedge depicts the joint work of the researchers which are connected through the hyperedge. For example, truth-membership, indeterminacy and falsity-membership values $(0.5, 0.1, 0.3)$ of H_2 describe that the researchers M_2, M_3, P_2 from the field of Mathematics and Physics have 50% publications, 10% submitted papers and 30% rejected papers, respectively, while working together. By calculating the strength of each SVN directed hyperedge, we can conclude that which group of researchers has better work done as compared to others. By routine calculations, we have
$\eta(H_1) = (0.5, 0.3, 0.7)$, $\eta(H_2) = (0.5, 0.1, 0.7)$,
$\eta(H_3) = (0.5, 0.3, 0.6)$, $\eta(H_4) = (0.4, 0.2, 0.6)$,
$\eta(H_5) = (0.4, 0.3, 0.3)$, $\eta(H_6) = (0.4, 0.2, 0.3)$,
$\eta(H_7) = (0.4, 0.3, 0.3)$, $\eta(H_8) = (0.5, 0.3, 0.3)$,
$\eta(H_9) = (0.5, 0.1, 0.3)$, $\eta(H_{10}) = (0.5, 0.1, 0.3)$.
Thus, we have H_8 is the strongest edge among the all. So we conclude that the researchers P_1, P_3 from the field of Physics and C_1 from the field of Chemistry have done more joint work as compared to others, i.e., they have 50% publications, 30% of their research work is submitted and 30% papers are rejected. The method adopted in our example can be explained by a simple algorithm given in Table 5.

Table 5: Algorithm for collaboration network

Algorithm 1
1. Input the degree of membership of all nodes(researchers) $v_1, v_2, \ldots, v_n$.
2. Input the number of directed hyperedges r.
3. Calculate the strength of SVN directed hyperedge $H_i = \{v_k, v_{k+1}, \ldots, v_l\}$, $1 \leq k \leq n-1$, $2 \leq l \leq n$ as, $S_i = \{\min_{v_j \in H_i} \alpha_{H_i}(v_j) \mid \alpha_{H_i}(v_j) > 0, \min_{v_j \in H_i} \beta_{H_i}(v_j) \mid \beta_{H_i}(v_j) > 0, \max_{v_j \in H_i} \gamma_{H_i}(v_j) \mid \gamma_{H_i}(v_j) > 0\}$, $1 \leq i \leq r$.
4. Find the strongest directed hyperedge using steps $5 - 14$.
5. **do** p from $1 \to r$
6. $J = (0, 0, 1)$
7. **do** q from $1 \to r$
8. **if**$(p \neq q)$ **then**
8. $J = (\max\{\alpha(J), \alpha(S_q)\}, \max\{\beta(J), \beta(S_q)\}, \min\{\gamma(J), \gamma(S_q)\})$
9. **end if**
10. **end do**
11. **if**$(\alpha(J) = \alpha(S_p), \beta(J) = \beta(S_p)$ and $\gamma(J) = \gamma(S_p))$ **then**
12. **print***, H_p is a strongest SVN directed hyperedge.
13. **end if**
14. **end do**

3. **Social networking**: A SVNDHG can also be used to study and understand the social networks, using people as nodes (or vertices) and relationships between two or more than two people as single-valued neutrosophic directed hyperedges. Consider the representation of social clubs and its members as a SVNDHG $D = (V, C)$, where $V = \{$Alen, Alex, Andy, Ben, Ava, Anna, Amy, Alice, Chris, Clay, Dave, Pal$\}$ interpret the members of different social clubs and the set of SVN directed hyperedges $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$ represents the social clubs. Each directed hyperedge(or social club) connects the people having some common characteristics to each other. The social hypernetwork is shown in Figure 10.

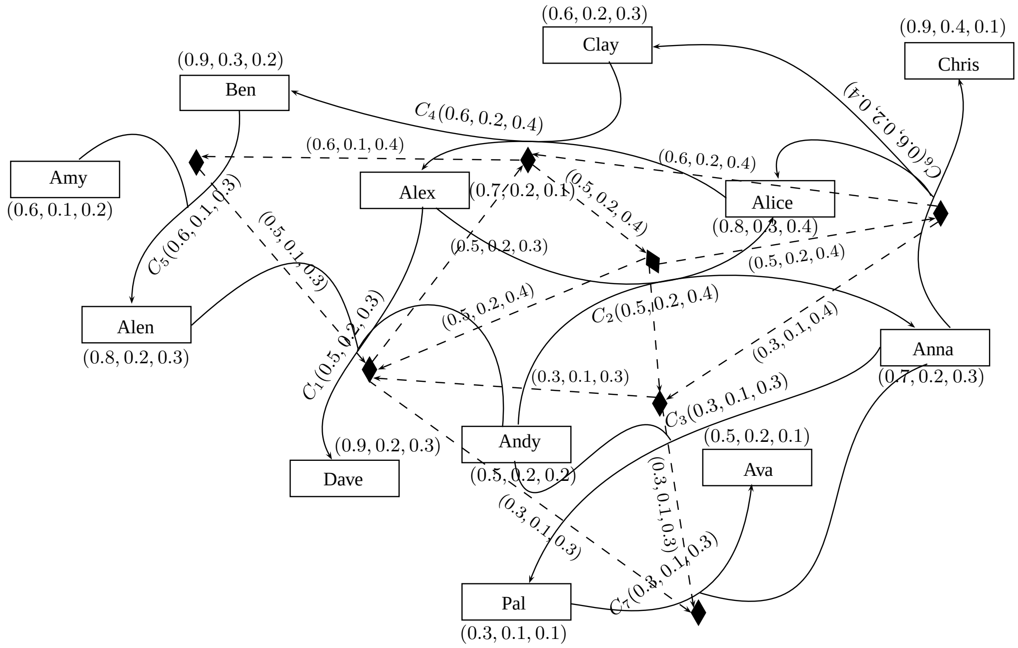

Figure 10: Representation of a social network using SVNDHG.

All the members of a social club connected through a SVN directed hyperedge share some common characteristics, including emotional intelligence, good behavior, communication skills and social sensitivity. For example, if the hyperedge C_1 describes the relation of *social sensitivity*(the capability to realize the emotions and thoughts of others) among the members of this club. Then the truth-membership, indeterminacy and falsity-membership values of each member indicate their sensitivity, unpredictable behavior and insensitivity towards the other members of the club. The truth-membership, indeterminacy and falsity-membership values $(0.6, 0.1, 0.3)$ of a SVN directed hyperedge C_6 interpret that 60% members have same chracteristics, 30% have different and 10% members of this club have unpredictable behavior. We use the concept of line directed graph to find out the common chracteristics of different members of distinct clubs. By routine calculations, we have

$$
\begin{aligned}
|supp(C_1)\bigcap supp(C_2)| &= \{Alex, Andy\}, & |supp(C_3)\bigcap supp(C_4)| &= \{\emptyset\},\\
|supp(C_1)\bigcap supp(C_3)| &= \{Andy\}, & |supp(C_3)\bigcap supp(C_5)| &= \{\emptyset\},\\
|supp(C_1)\bigcap supp(C_4)| &= \{Alex\}, & |supp(C_3)\bigcap supp(C_6)| &= \{Anna\},\\
|supp(C_1)\bigcap supp(C_5)| &= \{Alen\}, & |supp(C_3)\bigcap supp(C_7)| &= \{Pal\},\\
|supp(C_1)\bigcap supp(C_6)| &= \{\emptyset\}, & |supp(C_4)\bigcap supp(C_5)| &= \{Ben\},\\
|supp(C_1)\bigcap supp(C_7)| &= \{Dave\}, & |supp(C_4)\bigcap supp(C_6)| &= \{Alice, Clay\},\\
|supp(C_2)\bigcap supp(C_3)| &= \{Andy\}, & |supp(C_4)\bigcap supp(C_7)| &= \{\emptyset\},\\
|supp(C_2)\bigcap supp(C_4)| &= \{Alex, Alice\}, & |supp(C_5)\bigcap supp(C_6)| &= \{\emptyset\},\\
|supp(C_2)\bigcap supp(C_5)| &= \{\emptyset\}, & |supp(C_5)\bigcap supp(C_7)| &= \{\emptyset\},\\
|supp(C_2)\bigcap supp(C_6)| &= \{Alice, Anna\}, & |supp(C_6)\bigcap supp(C_7)| &= \{\emptyset\}.\\
|supp(C_2)\bigcap supp(C_7)| &= \{\emptyset\},
\end{aligned}
$$

The line directed graph of social network SVNDHG is given in Figure 10 with dashed lines. Each common edge between two social clubs describes the common characteristics of members of different clubs. For example, the edge C_1C_2 shows that the members of C_1 and C_2 have 50% common characteristics, 40% different to each other and 20% they have unpredictable behavior. The procedure followed in our example can be explained by means of simple algorithm given as follows.

Algorithm 2

1. Input the number of directed hyperedges m of SVNDHG $D = (V, H)$.

2. Input the degree of membership of all directed hyperedges $C_1, C_2, \ldots, C_m$.

3. Construct the SVN line directed graph $L(D) = (V_L, H_L)$ by taking $\{C_1, C_2, C_3, \ldots, C_m\}$ as set of vertices such that $V_L(C_i) = D(C_i)$, $1 \leq i \leq m$.

4. Draw an edge between C_i and C_j if $|C_i \cap C_j| \neq \emptyset$ and

$$H_L(C_iC_j) = (\min\{\alpha_H(C_i), \alpha_H(C_j)\}, \min\{\beta_H(C_i), \beta_H(C_j)\}, \max\{\gamma_H(C_i), \gamma_H(C_j)\}).$$

5. The edge C_iC_j describes the common characteristics of members of various clubs.

5 Conclusions

A single-valued neutrosophic set is an extension of fuzzy set as well as intuitionistic fuzzy set. The models based on single-valued neutrosophic sets are more precise, compatible and flexible in comparison to other traditional models. In this research paper, we have applied the notion of SVNS to the theory of directed hypergraphs. We have described the novel concepts, including single-valued neutrosophic directed hypergraphs, line directed graphs, dual directed hypergraphs and 2-section graphs. We have described some applications of single-valued neutrosophic directed hypergraphs in production system, social networking and collaboration networking to explain the flexibility of the model when the given data contains the part of indeterminacy. We plan to widen our research to (1) Bipolar fuzzy soft neutrosophic hypergraphs, (2) Interval valued neutrosophic hypergraphs, (3) Fuzzy rough neutrosophic hypergraphs and (4) Bipolar fuzzy rough directed hypergraphs.

Conflict of interest: The authors declare that they have no conflict of interest.

References

[1] M. Akram, S. Shahzadi, and A. Borumand saeid, (2016). *Single-valued neutrosophic hypergraphs*, TWMS Journal of Applied and Engineering Mathematics, 2016.

[2] M. Akram, and S. Shahzadi, (2016). *Neutrosophic soft graphs with application*, Journal of Intelligent & Fuzzy Systems, (2016) DOI: 10.3233/JIFS-16090.

[3] M. Akram and B. Davvaz, Strong intuitionistic fuzzy graphs, *Filomat* **26**(1) (2012) 177-196.

[4] M. Akram and G. Shahzadi, *Operations on single-valued neutrosophic graphs*, Journal of Uncertain System, **11**(1) (2017)1-22.

[5] M. Akram, and W. A. Dudek, (2013). *Intuitionistic fuzzy hypergraphs with applications*, Information Sciences, **218**, 182-193.

[6] K. Atanassov, (1986). *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems,**20**(1) , 87-96.

[7] K. Atanassov, (2002). *On index matrix representations of intuitionistic fuzzy graphs*, Notes on Intuitionistic Fuzzy Sets, **8**(4), 73-78.

[8] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, (2016). *Single-valued neutrosophic graphs*, Journal of New Theory, **10** , 86-101.

[9] C. Berge, (1973). *Graphs and hypergraphs*, Amsterdam: North-Holland Publishing Company.

[10] P. Bhattacharya, (1987). *Some remarks on fuzzy graphs*, Pattern Recognition Letters, **6**(5), 297-302.

[11] I. Bloch, (2009). *Geometry of spatial bipolar fuzzy sets based on bipolar fuzzy numbers and mathematical morphology* , Fuzzy Logic and Applications, Lecture Notes in Computer Science, **5571**, 237-245.

[12] S. M. Chen, (1997). *Interval-valued fuzzy hypergraph and fuzzy partition*, IEEE Transactions on Systems, Man and Cybernetics: Cybernetics, **27**(4), 725-733.

[13] G. Gallo, G. Longo, and S. Pallottino, (1993). *Directed hypergraphs and applications*, Discrete Applied Mathematics, **42**, 177-201.

[14] A. Kaufmann, (1977). *Introduction a la Thiorie des Sous-Ensemble Flous*, **1**, Masson: Paris.

[15] K.-M. Lee, (2000). *Bipolar-valued fuzzy sets and their basic operations*, Proc. Int. Conf., Bangkok, Thailand, 307-317.

[16] K.-M. Lee, (2004). *Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolar-valued fuzzy sets*, J. Fuzzy Logic Intell. Sys, **14**(2), 125-129.

[17] H. Lee-kwang, and K.-M. Lee, (1995). *Fuzzy hypergraph and fuzzy partition*, IEEE Transactions on Systems, Man and Cybernetics, **25**(1), 196-201.

[18] K. K. Myithili, R. Parvathi, and M. Akram, (2016). *Certain types of intuitionistic fuzzy directed hypergraphs*, Int. J. Mach. Learn. and Cyber, **7**(2), 287-295.

[19] P. Majumdar, and S. K. Samanta, (2014). *On similarity and entropy of neutrosophic sets*, Journal of Intelligent & Fuzzy Systems, **26**(3), 1245-1252.

[20] J. N. Mordeson, and P. S. Nair, (1998). *Fuzzy graphs and fuzzy hypergraphs*, Physica Verlag, Heidelberg, Second Edition 2001.

[21] R. Parvathi, M. G. Karunambigai, and K. T. Atanassov, (2009). *Operations on intuitionistic fuzzy graphs*, IEEE International Conference on Fuzzy Systems, 1396-1401.

[22] R. Parvathi, S. Thilagavathi, and M. G. Karunambigai, (2009). *Intuitionistic fuzzy hypergraphs*, Cybernetics and Information Technologies, **9**(2), 46-53.

[23] R. Parvathi, and S. Thilagavathi, (2013). *Intuitionistic fuzzy directed hypergraphs*, Advances in fuzzy sets and systems, **14**(1), 39-52.

[24] J. J. Peng, J. Q. Wang, H. Y. Zhang, and X. H. Chen, (2014). *An outrankingapproach for multi-criteria decision-making problems with simplified neutrosophic sets*, Applied Soft Computing, **25**, 336-346.

[25] A. Rosenfeld, (1975). *Fuzzy graphs*, in: L.A. Zadeh, K.S. Fu and M. Shimura(Eds), Fuzzy Sets and Their Applications, Academic Press, New York, 77-95.

[26] F. Smarandache, (1998). *Neutrosophy Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, USA.

[27] F. Smarandache, (1999). *A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic*, American Research Press, Rehoboth.

[28] H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman, (2010). *Single valued neutrosophic sets*, Multispace and Multistructure, **4**, 410-413.

[29] J. Ye, (2013). *Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment*, International Journal of General Systems, **42**(4), 386-394.

[30] J. Ye, (2014). *Single-valued neutrosophic minimum spanning tree and its clustering method*, Journal of Intelligent Systems , **23**(3), 311-324.

[31] J. Ye, (2014). *A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets*, Journal of Intelligent & Fuzzy Systems, **26**(5), 2459-2466.

[32] L.A. Zadeh, (1965). *Fuzzy sets*, Information and Control, **8**(3), 338-353.

[33] L.A. Zadeh, (1971). *Similarity relations and fuzzy orderings*. Information Sciences, **3**(2), 177-200.

[34] K. Zhu and J. Zhan, *Fuzzy parameterized fuzzy soft sets and decision making*, Int. J. Machine Learning & Cybernetics, **7**(6)(2016), 1207-1212.

[35] J. Zhan, Q. Liu and B. Davvaz, *A new rough set theory: rough soft hemirings*, J. Intell. Fuzzy Systems, **28**(2015), 1687-1697.

[36] J. Zhan and K. Zhu, *Reviews on decision making methods based on (fuzzy) soft sets and rough soft sets*, Journal of Intelligent and Fuzzy Systems, **29**(3)(2015), 1169-1176.