Certain Single-Valued Neutrosophic Graphs with Application

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Abstract

In this research paper, we present certain types of single-valued neutrosophic graphs, including edge regular single-valued neutrosophic graphs and totally edge regular single-valued neutrosophic graphs. We investigate some of their related properties. We describe an application of single-valued neutrosophic graph in decision making process and present the procedure of our method that is used in our application in an algorithm.

Keywords: Edge regular single-valued neutrosophic graphs, Totally edge regular single-valued neutrosophic graphs.

Mathematics Subject Classification 2010: 03E72, 68R10, 68R05

1 Introduction

Fuzzy set theory is the generalized concept of classical set theory. In classical set theory, there are only two possibilities, that is, either the statement is true or false. However, many statements have variable values which can be handled more accurately using fuzzy set theory. In 1965, Zadeh [25] introduced the notion of fuzzy sets to handle the problems with uncertainties. Fuzzy set theory [25] plays a vital role in complex phenomena which is not easily characterized by classical set theory. In 1983, Atanassov [6] proposed the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. He added a new component which determines the falsity membership degree in the definition of fuzzy sets. The idea of intuitionistic fuzzy sets is more meaningful as well as intensive due to the presence of truth membership degree, indeterminacy membership degree and falsity membership degree, where the indeterminacy membership degree of intuitionistic fuzzy sets is its hesitation part by default. The truth membership degree and the falsity membership degree are more or less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Smarandache[9-10] introduced the idea of neutrosophic sets by combining the non-standard analysis. Neutrosophic set is a mathematical tool for dealing real life problems having imprecise, indeterminacy and inconsistent data. Neutrosophic set theory, as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory, is applied in a variety of fields, including control theory, decision making problems, topology, medicines and in many more real life problems. Wang et al.[20] presented the notion of single-valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. A single-valued neutrosophic set has three components: truth membership degree, indeterminacy membership degree and falsity membership degree. These three components of a single-valued neutrosophic set are not dependent and their values are contained in the standard unit interval [0, 1]. Single-valued neutrosophic sets are the generalization of intuitionistic fuzzy sets. Single-valued neutrosophic sets have been a new hot research topic and many researchers have addressed this issue. Majumdar and Samanta [11] studied similarity and entropy of single-valued neutrosophic sets. Ye[12-15] proposed correlation coefficients of single-valued neutrosophic sets, and applied it to single-valued neutrosophic decision making problems. To simplify neutrosophic sets, Ye [24] introduced a multicriteria decision making method using aggregation operators.

Graph theory has become a powerful conceptual framework for modeling and solution of combinatorial problems that arise in various areas, including computer sciences, engineering and mathematics. Single-valued neutrosophic graphs, as the generalization of graphs, have many properties which are the basis of different techniques that are used in modern mathematics. Dhavaseelan et al. [10] defined strong neutrosophic graphs. Broumi et al. [8, 9] portrayed single-valued neutrosophic graphs. Akram and Shahzadi [1] introduced the notion of neutrosophic soft graphs with applications. Akram [3] introduced the notion of single-valued neutrosophic planar graphs. Akram et al. [2] also introduced the single-valued neutrosophic hypergraphs. Representation of graphs using intuitionistic neutrosophic soft sets was discussed in [5]. In this research paper, we present certain types of single-valued neutrosophic graphs, including edge regular single-valued neutrosophic graphs and totally edge regular single-valued neutrosophic graphs. We describe an application of single-valued neutrosophic graph in decision making process and present the procedure of our method that is used in our application in an algorithm.

2 Single-valued neutrosophic graphs

Definition 2.1. [1] A single-valued neutrosophic graph $G = (X, Y)$ is a pair, where $X : N \to [0, 1]$ is a single-valued neutrosophic set on N and $Y : N \times N \to [0, 1]$ is a single-valued neutrosophic relation on N such that

$$\begin{aligned} t_Y(st) &\leq \min\{t_X(s), t_X(t)\}, \\ i_Y(st) &\leq \min\{i_X(s), i_X(t)\}, \\ f_Y(st) &\leq \max\{f_X(s), f_X(t)\}, \end{aligned}$$

for all $s, t \in N$, where $t_Y(st) = 0$, $i_Y(st) = 0$ and $f_Y(st) = 0$, for all $s, t \in N \times N - L$. X and Y are called the single-valued neutrosophic vertex set of G and the single-valued neutrosophic edge set of G, respectively. A single-valued neutrosophic relation Y is said to be symmetric if $t_Y(st) = t_Y(ts)$, $i_Y(st) = i_Y(ts)$ and $f_Y(st) = f_Y(ts)$, for all $s, t \in N$.

Example 2.1. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4, s_5\}$ and $L = \{s_1s_2, s_1s_4, s_1s_5, s_2s_3, s_2s_4, s_3s_4,$ The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 1.

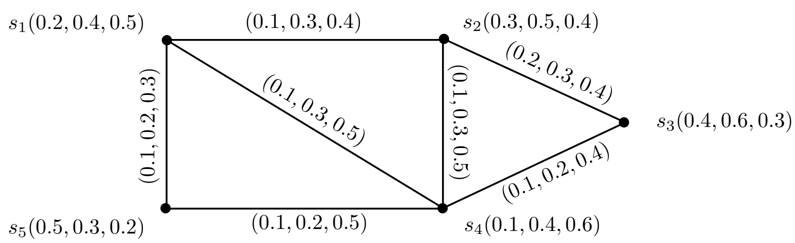

Figure 1: Single-valued neutrosophic graph G

Definition 2.2. A single-valued neutrosophic path $\mathcal{P}$ is a sequence of distinct vertices $s = s_1, s_2, s_3, \ldots, s_n = t$ such that, for all k, $t_Y(s_ks_{k+1}) > 0$, $i_Y(s_ks_{k+1}) > 0$ and $f_Y(s_ks_{k+1}) > 0$. A single-valued neutrosophic path is said to be a single-valued neutrosophic cycle if $s = t$.

Example 2.2. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4, s_5\}$ and $L = \{s_1s_5, s_2s_3, s_2s_4, s_3s_4, s_4s_5\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 2.

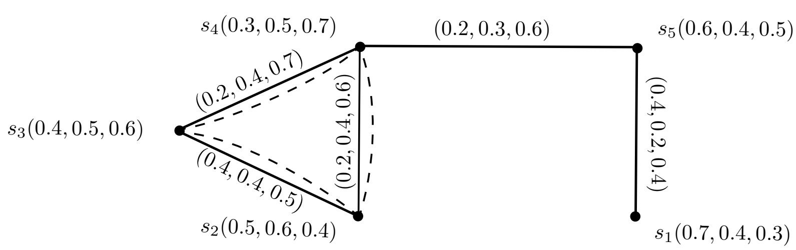

Figure 2: Single-valued neutrosophic graph G

The path $\mathcal{P}$ from s_2 to s_1 is shown with thick lines and the cycle $\mathcal{C}$ from s_2 to s_2 is shown with dashed lines in Fig. 2.

Definition 2.3. [9] The order and the size of a single-valued neutrosophic graph G are denoted by $\mathcal{O}(G)$ and $\mathcal{S}(G)$, respectively, and are defined as

$$\begin{aligned}\mathcal{O}(G) &= (\sum_{s\in N} t_X(s), \sum_{s\in N} i_X(s), \sum_{s\in N} f_X(s)),\\ \mathcal{S}(G) &= (\sum_{st\in L} t_Y(st), \sum_{st\in L} i_Y(st), \sum_{st\in L} f_Y(st)).\end{aligned}$$

Definition 2.4. [9] The degree and the total degree of a vertex s of a single-valued neutrosophic graph G are denoted by $\mathcal{D}_G(s) = (\mathcal{D}_t(s), \mathcal{D}_i(s), \mathcal{D}_f(s))$ and $\mathcal{TD}_G(s) = (\mathcal{TD}_t(s), \mathcal{TD}_i(s), \mathcal{TD}_f(s))$, respectively, and are defined as

$$\begin{aligned}\mathcal{D}_G(s) &= (\sum_{s\neq t} t_Y(st), \sum_{s\neq t} i_Y(st), \sum_{s\neq t} f_Y(st)),\\ \mathcal{TD}_G(s) &= (\sum_{s\neq t} t_Y(st) + t_X(s), \sum_{s\neq t} i_Y(st) + i_X(s), \sum_{s\neq t} f_Y(st) + f_X(s)),\end{aligned}$$

for $st \in L$, where $s \in N$.

Example 2.3. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3\}$ and $L = \{s_1s_2,\ s_2s_3, s_3s_1\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 3.

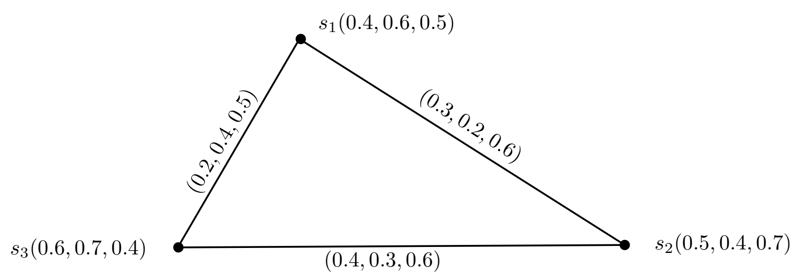

Figure 3: Single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{O}(G) = (1.5, 1.7, 1.6)$, $\mathcal{S}(G) = (0.9, 0.9, 1.7)$, $\mathcal{D}_G(s_1) = (0.5, 0.6, 1.1)$, $\mathcal{D}_G(s_2) = (0.7, 0.5, 1.2)$, $\mathcal{D}_G(s_3) = (0.6, 0.7, 1.1)$, $\mathcal{TD}_G(s_1) = (0.9, 1.2, 1.6)$, $\mathcal{TD}_G(s_2) = (1.2, 0.9, 1.9)$ and $\mathcal{TD}_G(s_3) = (1.2, 1.4, 1.5)$.

Definition 2.5. A single-valued neutrosophic graph $G = (X, Y)$ is called a regular single-valued neutrosophic graph of degree (m_1, m_2, m_3) if $\mathcal{D}_G(s) = (m_1, m_2, m_3)$, for all $s \in N$.

Example 2.4. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2,\ s_2s_3, s_3s_4, s_4s_1\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 4.

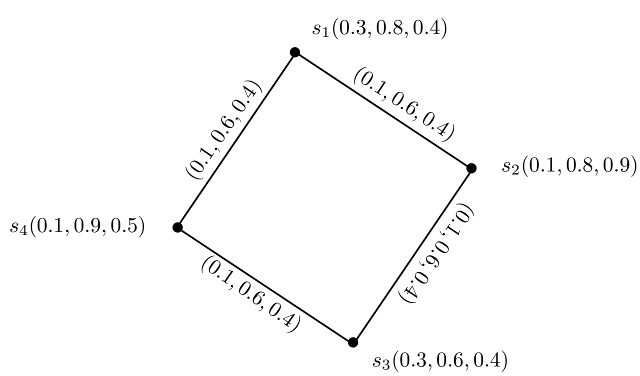

Figure 4: Regular single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.2, 1.2, 0.8) = \mathcal{D}_G(s_2) = \mathcal{D}_G(s_3) = \mathcal{D}_G(s_4)$. Hence G is a regular single-valued neutrosophic graph.

Definition 2.6. A single-valued neutrosophic graph $G = (X, Y)$ is called a totally regular single-valued neutrosophic graph of degree (n_1, n_2, n_3) if $\mathcal{TD}_G(s) = (n_1, n_2, n_3)$, for all $s \in N$.

Example 2.5. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ and $L = \{s_1s_2,\ s_2s_3, s_3s_4, s_4s_5, s_5s_6, s_6$ The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 5.

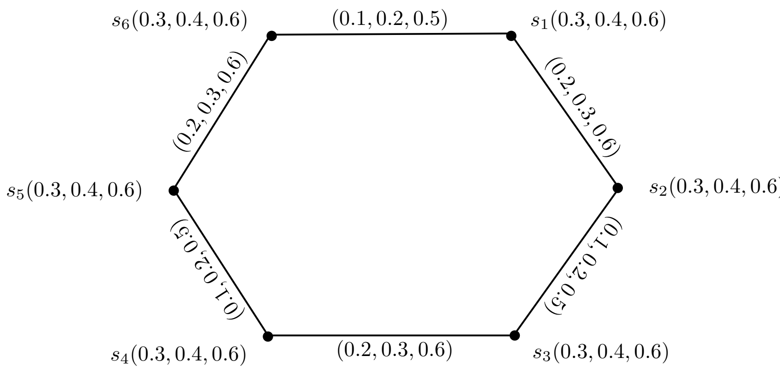

Figure 5: Totally regular single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.3, 0.5, 1.1) = \mathcal{D}_G(s_2) = \mathcal{D}_G(s_3) = \mathcal{D}_G(s_4) = \mathcal{D}_G(s_5) = \mathcal{D}_G(s_6)$ and $\mathcal{TD}_G(s_1) = (0.6, 0.9, 1.7) = \mathcal{TD}_G(s_2) = \mathcal{TD}_G(s_3) = \mathcal{TD}_G(s_4) = \mathcal{TD}_G(s_5) = \mathcal{TD}_G(s_6)$. Hence G is a totally regular single-valued neutrosophic graph.

Remark 2.1. The above two concepts are independent, that is, it is not necessary that totally regular single-valued neutrosophic graph is regular single-valued neutrosophic graph and vice versa.

Example 2.6. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2,\ s_2s_3, s_3s_4, s_4s_1\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 6.

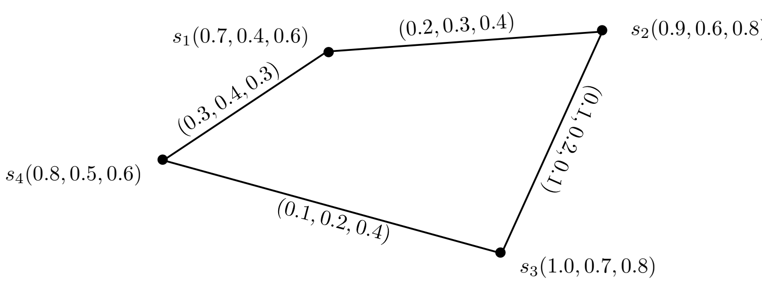

Figure 6: Totally regular single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.5, 0.7, 0.7)$, $\mathcal{D}_G(s_2) = (0.3, 0.5, 0.5)$, $\mathcal{D}_G(s) = (0.2, 0.4, 0.5)$, $\mathcal{D}_G(s_4) = (0.4, 0.6, 0.7)$ and $\mathcal{TD}_G(s_1) = (1.2, 1.1, 1.3) = \mathcal{TD}_G(s_2) = \mathcal{TD}_G(s_3) = \mathcal{TD}_G(s_4)$. Therefore G is a totally regular single-valued neutrosophic graph but not a regular single-valued neitrosophic graph.

Definition 2.7. The degree and the total degree of an edge st of a single-valued neutrosophic graph G are denoted by $\mathcal{D}_G(st) = (\mathcal{D}_t(st), \mathcal{D}_i(st), \mathcal{D}_f(st))$ and $\mathcal{TD}_G(st) = (\mathcal{TD}_t(st), \mathcal{TD}_i(st), \mathcal{TD}_f(st))$, respectively, and are defined as

$$\mathcal{D}_G(st) = \mathcal{D}_G(s) + \mathcal{D}_G(t) - 2(t_Y(st), i_Y(st), f_Y(st)),$$

$$\mathcal{TD}_G(st) = \mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)).$$

Example 2.7. Consider a crisp graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3\}$ and $L = \{s_1s_2, s_1s_3\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 7.

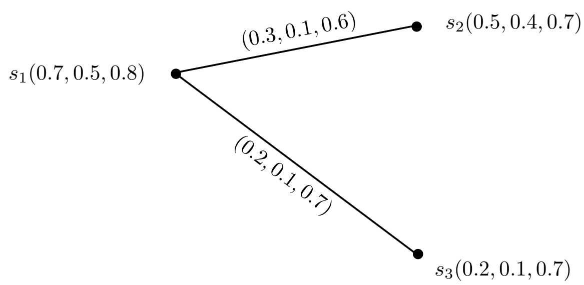

Figure 7: Single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.5, 0.2, 1.3)$, $\mathcal{D}_G(s_2) = (0.3, 0.1, 0.6)$, and $\mathcal{D}_G(s_3) = (0.2, 0.1, 0.7)$.

- The degree of each edge is given as:

$$\begin{aligned}
\mathcal{D}_G(s_1s_2) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (0.7, 0.5, 0.8) + (0.5, 0.4, 0.7) - 2(0.3, 0.1, 0.6),\\
&= (0.2, 0.1, 0.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_1s_3) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_3) - 2(t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\
&= (0.7, 0.5, 0.8) + (0.4, 0.2, 0.6) - 2(0.2, 0.1, 0.7),\\
&= (0.3, 0.1, 0.6).
\end{aligned}$$

- The total degree of each edge is given as:

$$\begin{aligned}
\mathcal{TD}_G(s_1s_2) &= \mathcal{D}_G(s_1s_2) + (t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (0.2, 0.1, 0.7) + (0.3, 0.1, 0.6),\\
&= (0.5, 0.2, 1.3).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(s_1s_3) &= \mathcal{D}_G(s_1s_3) + (t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\
&= (0.3, 0.1, 0.6) + (0.2, 0.1, 0.7),\\
&= (0.5, 0.2, 1.3).
\end{aligned}$$

Definition 2.8. The maximum degree of a single-valued neutrosophic graph G is defined as $\Delta(G) = (\Delta_t(G), \Delta_i(G), \Delta_f(G))$, where

$$\Delta_t(G) = \max\{\mathcal{D}_t(s) : s \in N\},$$

$$\Delta_i(G) = \max\{\mathcal{D}_i(s) : s \in N\},$$

$$\Delta_f(G) = \max\{\mathcal{D}_f(s) : s \in N\}.$$

Definition 2.9. The minimum degree of a single-valued neutrosophic graph G is defined as $\delta(G) = (\delta_t(G), \delta_i(G), \delta_f(G))$, where

$$\delta_t(G) = \min\{\mathcal{D}_t(s) : s \in N\},$$

$$\delta_i(G) = \min\{\mathcal{D}_i(s) : s \in N\},$$

$$\delta_f(G) = \min\{\mathcal{D}_f(s) : s \in N\}.$$

Example 2.8. Consider the single-valued neutrosophic graph $G = (X, Y)$ as shown in Fig. 7. By direct calculations, we have $\Delta(G) = (0.5, 0.2, 1.3)$ and $\delta(G) = (0.2, 0.1, 0.6)$.

Definition 2.10. Let N be a nonempty set. A single-valued neutrosophic graph $G = (X, Y)$ on N is said to be an edge regular single-valued neutrosophic graph if every edge in G has the same degree (q_1, q_2, q_3).

Remark 2.2. Let $G = (X, Y)$ be an edge regular single-valued neutrosophic graph. Then G is said to be an equally edge regular single-valued neutrosophic graph if $q_1 = q_2 = q_3$.

Example 2.9. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3\}$ and $L = \{s_1s_2, s_1s_3, s_2s_3\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 8.

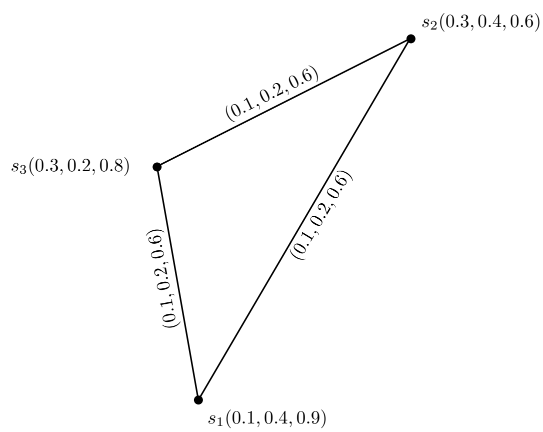

Figure 8: Edge regular single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.2, 0.4, 1.2)$, $\mathcal{D}_G(s_2) = (0.2, 0.4, 1.2)$ and $\mathcal{D}_G(s_3) = (0.2, 0.4, 1.2)$. The degree of each edge is given below:

$$\begin{aligned}\mathcal{D}_G(s_1s_2) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\ &= (0.2, 0.4, 1.2) + (0.2, 0.4, 1.2) - 2(0.1, 0.2, 0.6),\\ &= (0.2, 0.4, 1.2).\end{aligned}$$

$$\begin{aligned}\mathcal{D}_G(s_1s_3) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_3) - 2(t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\ &= (0.2, 0.4, 1.2) + (0.2, 0.4, 1.2) - 2(0.1, 0.2, 0.6),\\ &= (0.2, 0.4, 1.2).\end{aligned}$$

$$\begin{aligned}\mathcal{D}_G(s_2s_3) &= \mathcal{D}_G(s_2) + \mathcal{D}_G(s_3) - 2(t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\ &= (0.2, 0.4, 1.2) + (0.2, 0.4, 1.2) - 2(0.1, 0.2, 0.6),\\ &= (0.2, 0.4, 1.2).\end{aligned}$$

It is easy to see that each edge of single-valued neutrosophic graph G has the same degree. Hence G is an edge regular single-valued neutrosophic graph.

Definition 2.11. Let N be a nonempty set. A single-valued neutrosophic graph $G = (X, Y)$ on N is said to be a totally edge regular single-valued neutrosophic graph if every edge in G has the same total degree (p_1, p_2, p_3).

Example 2.10. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3\}$ and $L = \{s_1s_2, s_1s_3, s_2s_3\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 9.

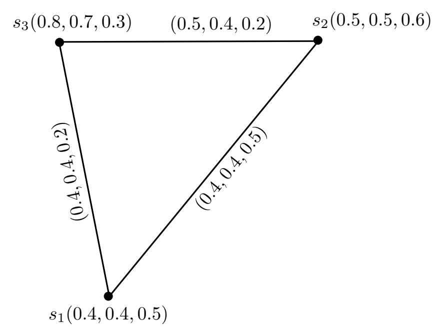

Figure 9: Totally edge regular single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.8, 0.8, 0.7)$, $\mathcal{D}_G(s_2) = (0.9, 0.8, 0.7)$ and $\mathcal{D}_G(s_3) = (0.9, 0.8, 0.4)$.

- The degree of each edge is given below:

$$\begin{aligned}
\mathcal{D}_G(s_1s_2) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (0.8, 0.8, 0.7) + (0.9, 0.8, 0.7) - 2(0.4, 0.4, 0.5),\\
&= (0.9, 0.8, 0.4).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_1s_3) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_3) - 2(t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\
&= (0.8, 0.8, 0.7) + (0.9, 0.8, 0.4) - 2(0.4, 0.4, 0.2),\\
&= (0.9, 0.8, 0.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_2s_3) &= \mathcal{D}_G(s_2) + \mathcal{D}_G(s_3) - 2(t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\
&= (0.9, 0.8, 0.7) + (0.9, 0.8, 0.4) - 2(0.5, 0.4, 0.2),\\
&= (0.8, 0.8, 0.7).
\end{aligned}$$

It is easy to see that $\mathcal{D}_G(s_1s_2) \neq \mathcal{D}_G(s_1s_3) \neq \mathcal{D}_G(s_2s_3)$. So G is not an edge regular single-valued neutrosophic graph.

- The total degree of each edge is calculated as:

$$\begin{aligned}
\mathcal{TD}_G(s_1s_2) &= \mathcal{D}_G(s_1s_2) + (t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (1.3, 1.2, 0.9).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(s_1s_3) &= \mathcal{D}_G(s_1s_3) + (t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\
&= (1.3, 1.2, 0.9).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(s_2s_3) &= \mathcal{D}_G(s_2s_3) + (t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\
&= (1.3, 1.2, 0.9).
\end{aligned}$$

It is easy to see that each edge of single-valued neutrosophic graph G has the same total degree. So G is a totally edge regular single-valued neutrosophic graph.

Remark 2.3. A single-valued neutrosophic graph G is an edge regular single-valued neutrosophic graph if and only if $\Delta_{\mathcal{D}}(G) = \delta_{\mathcal{D}}(G) = (q_1, q_2, q_3)$.

Example 2.11. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2, s_2s_3, s_3s_4\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 10.

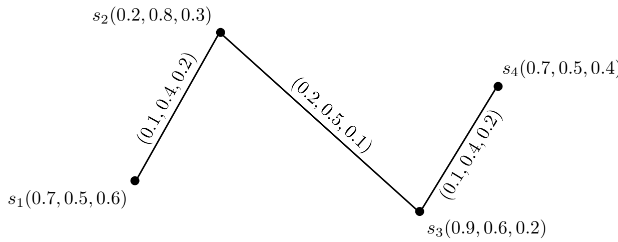

Figure 10: Single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.1, 0.4, 0.2)$, $\mathcal{D}_G(s_2) = (0.3, 0.9, 0.3)$, $\mathcal{D}_G(s_3) = (0.3, 0.9, 0.3)$ and $\mathcal{D}_G(s_4) = (0.1, 0.4, 0.2)$.

- The degree of each edge is given below:

$$\begin{aligned}
\mathcal{D}_G(s_1s_2) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (0.1, 0.4, 0.2) + (0.3, 0.9, 0.3) - 2(0.1, 0.4, 0.2),\\
&= (0.2, 0.5, 0.1).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_2s_3) &= \mathcal{D}_G(s_2) + \mathcal{D}_G(s_3) - 2(t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\
&= (0.3, 0.9, 0.3) + (0.3, 0.9, 0.3) - 2(0.2, 0.5, 0.1),\\
&= (0.2, 0.8, 0.4).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_3s_4) &= \mathcal{D}_G(s_3) + \mathcal{D}_G(s_4) - 2(t_Y(s_3s_4), i_Y(s_3s_4), f_Y(s_3s_4)),\\
&= (0.3, 0.9, 0.3) + (0.1, 0.4, 0.2) - 2(0.1, 0.4, 0.2),\\
&= (0.2, 0.5, 0.1).
\end{aligned}$$

It is easy to see that $\mathcal{D}_G(s_1s_2) \neq \mathcal{D}_G(s_2s_3)$. So G is not an edge regular single-valued neutrosophic graph.

- The total degree of each edge is calculated as:

$$\begin{aligned}
\mathcal{TD}_G(s_1s_2) &= \mathcal{D}_G(s_1s_2) + (t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (0.3, 0.9, 0.3).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(s_2s_3) &= \mathcal{D}_G(s_2s_3) + (t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\
&= (0.4, 1.3, 0.5).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(s_3s_4) &= \mathcal{D}_G(s_3s_4) + (t_Y(s_3s_4), i_Y(s_3s_4), f_Y(s_3s_4)),\\
&= (0.3, 0.9, 0.3).
\end{aligned}$$

It is easy to see that $\mathcal{TD}_G(s_1s_2) \neq \mathcal{TD}_G(s_2s_3)$. So G is not a totally edge regular single-valued neutrosophic graph.

Remark 2.4. A complete single-valued neutrosophic graph G may not be an edge regular single-valued neutrosophic graph.

Example 2.12. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2, s_1s_3, s_1s_4, s_2s_3, s_2s_4, s_3s_4\}$. The corresponding complete single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. **??**.

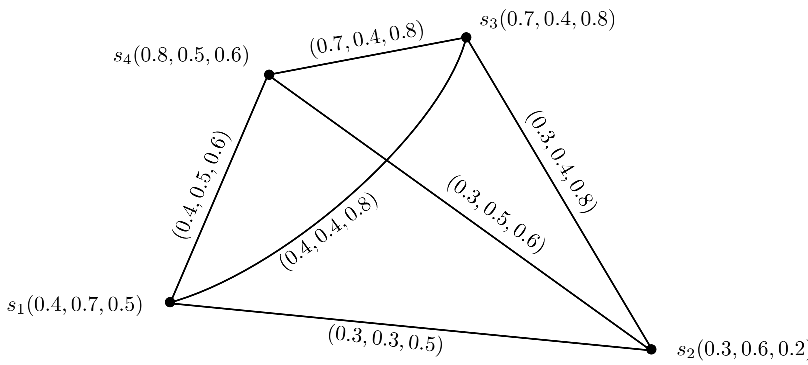

Figure 11: Complete single-valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (1.1, 1.2, 1.9)$, $\mathcal{D}_G(s_2) = (0.9, 1.2, 1.9)$, $\mathcal{D}_G(s_3) = (1.4, 1.2, 2.4)$ and $\mathcal{D}_G(s_4) = (1.4, 1.4, 2.0)$. The degree of each edge is given below:

$$\begin{aligned}
\mathcal{D}_G(s_1s_2) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1s_2), i_Y(s_1s_2), f_Y(s_1s_2)),\\
&= (1.1, 1.2, 1.9) + (0.9, 1.2, 1.9) - 2(0.3, 0.3, 0.5),\\
&= (1.4, 2.0, 2.8).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_1s_3) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_3) - 2(t_Y(s_1s_3), i_Y(s_1s_3), f_Y(s_1s_3)),\\
&= (1.1, 1.2, 1.9) + (1.4, 1.2, 2.4) - 2(0.4, 0.4, 0.8),\\
&= (1.7, 1.6, 2.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_1s_4) &= \mathcal{D}_G(s_1) + \mathcal{D}_G(s_4) - 2(t_Y(s_1s_4), i_Y(s_1s_4), f_Y(s_1s_4)),\\
&= (1.1, 1.2, 1.9) + (1.4, 1.4, 2.0) - 2(0.4, 0.5, 0.6),\\
&= (1.7, 1.6, 2.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_2s_3) &= \mathcal{D}_G(s_2) + \mathcal{D}_G(s_3) - 2(t_Y(s_2s_3), i_Y(s_2s_3), f_Y(s_2s_3)),\\
&= (0.9, 1.2, 1.9) + (1.4, 1.2, 2.4) - 2(0.3, 0.4, 0.8),\\
&= (1.7, 1.6, 2.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_2s_4) &= \mathcal{D}_G(s_2) + \mathcal{D}_G(s_4) - 2(t_Y(s_2s_4), i_Y(s_2s_4), f_Y(s_2s_4)),\\
&= (0.9, 1.2, 1.9) + (1.4, 1.4, 2.0) - 2(0.3, 0.5, 0.6),\\
&= (1.7, 1.6, 2.7).
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_G(s_3s_4) &= \mathcal{D}_G(s_3) + \mathcal{D}_G(s_4) - 2(t_Y(s_3s_4), i_Y(s_3s_4), f_Y(s_3s_4)),\\
&= (1.4, 1.2, 2.4) + (1.4, 1.4, 2.0) - 2(0.7, 0.4, 0.8),\\
&= (1.4, 1.8, 2.8).
\end{aligned}$$

It is easy to see that each edge of single-valued neutrosophic graph G has not the same degree. Therefore, G is a complete single-valued neutrosophic graph but not an edge regular single-valued neutrosophic graph.

Theorem 2.1. *Let $G=(X,Y)$ be a single-valued neutrosophic graph. Then $\sum_{st\in L}\mathcal{D}_G(st)=\sum_{st\in L}\mathcal{D}_{G^*}(st)(t_Y(st),i_Y(st),f_Y$ where $\mathcal{D}_{G^*}(st)=\mathcal{D}_{G^*}(s)+\mathcal{D}_{G^*}(t)-2$ for all $s,t\in N$.*

Theorem 2.2. *Let $G=(X,Y)$ be a single-valued neutrosophic graph. Then $\sum_{st\in L}\mathcal{TD}_G(st)=\sum_{st\in L}\mathcal{D}_{G^*}(st)(t_Y(st),i_Y(st),$ $\mathcal{S}(G)$, where $\mathcal{D}_{G^*}(st)=\mathcal{D}_{G^*}(s)+\mathcal{D}_{G^*}(t)-2$ for all $s,t\in N$.*

Proof. Since the total degree of each edge in a single-valued neutrosophic graph G is $\mathcal{TD}_G(st)=\mathcal{D}_G(st)+(t_Y(st),i_Y(st),f_Y(st)$
Therefore,

$$\begin{aligned}
\sum_{st\in L}\mathcal{TD}_G(st) &= \sum_{st\in L}(\mathcal{D}_G(st)+(t_Y(st),i_Y(st),f_Y(st))),\\
\sum_{st\in L}\mathcal{TD}_G(st) &= \sum_{st\in L}\mathcal{D}_G(st)+\sum_{st\in L}(t_Y(st),i_Y(st),f_Y(st)),\\
\sum_{st\in L}\mathcal{TD}_G(st) &= \sum_{st\in L}\mathcal{D}_{G^*}(st)(t_Y(st),i_Y(st),f_Y(st))+\mathcal{S}(G).
\end{aligned}$$

This completes the proof. □

Theorem 2.3. *Let $G^*=(N,L)$ be an edge regular crisp graph of degree q and $G=(X,Y)$ be an edge regular single-valued neutrosophic graph of degree (q_1,q_2,q_3) of G^*. Then the size of G is $(\frac{mq_1}{q},\frac{mq_2}{q},\frac{mq_3}{q})$, where $|L|=m$.*

Proof. Let $G=(X,Y)$ be an edge regular single-valued neutrosophic graph of an edge regular crisp graph $G^*=(N,L)$. Therefore, $\mathcal{D}_G(st)=(q_1,q_2,q_3)$ and $\mathcal{D}_{G^*}(st)=q$ for each edge $st\in L$. Since,

$$\begin{aligned}
\sum_{st\in L}\mathcal{D}_G(st) &= \sum_{st\in L}\mathcal{D}_{G^*}(st)(t_Y(st),i_Y(st),f_Y(st)),\\
\sum_{st\in L}(q_1,q_2,q_3) &= q\sum_{st\in L}(t_Y(st),i_Y(st),f_Y(st)),\\
m(q_1,q_2,q_3) &= q\mathcal{S}(G),\\
(mq_1,mq_2,mq_3) &= q\mathcal{S}(G),\\
\mathcal{S}(G) &= (\frac{mq_1}{q},\frac{mq_2}{q},\frac{mq_3}{q}).
\end{aligned}$$

This completes the proof. □

Theorem 2.4. *Let $G^*=(N,L)$ be an edge regular crisp graph of degree q and $G=(X,Y)$ be a totally edge regular single-valued neutrosophic graph of degree (p_1,p_2,p_3) of G^*. Then the size of G is $(\frac{mp_1}{q+1},\frac{mp_2}{q+1},\frac{mp_3}{q+1})$, where $|L|=m$.*

Proof. Let $G=(X,Y)$ be a totally edge regular single-valued neutrosophic graph of an edge regular crisp graph $G^*=(N,L)$. Therefore, $\mathcal{D}_G(st)=(p_1,p_2,p_3)$ and $\mathcal{D}_{G^*}(st)=q$ for each edge $st\in L$. Since,

$$\begin{aligned}
\sum_{st\in L}\mathcal{TD}_G(st) &= \sum_{st\in L}\mathcal{D}_{G^*}(st)(t_Y(st),i_Y(st),f_Y(st))+\mathcal{S}(G),\\
\sum_{st\in L}(p_1,p_2,p_3) &= q\sum_{st\in L}(t_Y(st),i_Y(st),f_Y(st))+\mathcal{S}(G),\\
m(p_1,p_2,p_3) &= q\mathcal{S}(G)+\mathcal{S}(G),\\
(mp_1,mp_2,mp_3) &= (q+1)\mathcal{S}(G),\\
\mathcal{S}(G) &= (\frac{mp_1}{q+1},\frac{mp_2}{q+1},\frac{mp_3}{q+1}).
\end{aligned}$$

This completes the proof. □

Theorem 2.5. *Let $G^*=(N,L)$ be a crisp graph. Suppose that $G=(X,Y)$ be an edge regular single-valued neutrosophic graph of degree (q_1,q_2,q_3) and a totally edge regular single-valued neutrosophic graph of degree (p_1,p_2,p_3) of G^*. Then the size of G is $m(p_1-q_1,p_2-q_2,p_3-q_3)$, where $|L|=m$.*

Proof. Let $G = (X, Y)$ be an edge regular single-valued neutrosophic graph and a totally edge regular single-valued neutrosophic graph of a crisp graph $G^* = (N, L)$. Therefore, $\mathcal{D}_G(st) = (q_1, q_2, q_3)$ and $\mathcal{TD}_G(st) = (p_1, p_2, p_3)$ for each edge $st \in L$. Since,

$$\begin{aligned}
\mathcal{TD}_G(st) &= \mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)),\\
\sum_{st\in L} \mathcal{TD}_G(st) &= \sum_{st\in L} \mathcal{D}_G(st) + \sum_{st\in L} (t_Y(st), i_Y(st), f_Y(st)),\\
m(p_1, p_2, p_3) &= m(q_1, q_2, q_3) + \mathcal{S}(G),\\
\mathcal{S}(G) &= m(p_1 - q_1, p_2 - q_2, p_3 - q_3).
\end{aligned}$$

This completes the proof. □

Theorem 2.6. *Let $G^* = (N, L)$ be a crisp graph, which is a cycle on m vertices. Suppose that $G = (X, Y)$ be a single-valued neutrosophic graph of G^*. Then $\sum_{s_k\in N} \mathcal{D}_G(s_k) = \sum_{s_k s_l\in L} \mathcal{D}_G(s_k s_l)$.*

Proof. Let $G = (X, Y)$ be a single-valued neutrosophic graph of G^*. Suppose that G^* be a cycle $s_1, s_2, s_3, \ldots, s_m, s_1$ on m vertices. Then

$$\begin{aligned}
\sum_{s_k s_l\in L} \mathcal{D}_G(s_k s_l) &= \mathcal{D}_G(s_1 s_2) + \mathcal{D}_G(s_2 s_3) + \ldots + \mathcal{D}_G(s_m s_1),\\
&= [\mathcal{D}_G(s_1) + \mathcal{D}_G(s_2) - 2(t_Y(s_1 s_2), i_Y(s_1 s_2), f_Y(s_1 s_2))][\mathcal{D}_G(s_2)\\
&+ \mathcal{D}_G(s_3) - 2(t_Y(s_2 s_3), i_Y(s_2 s_3), f_Y(s_2 s_3))] + \ldots + [\mathcal{D}_G(s_m)\\
&+ \mathcal{D}_G(s_1) - 2(t_Y(s_m s_1), i_Y(s_m s_1), f_Y(s_m s_1))],\\
&= 2\mathcal{D}_G(s_1) + 2\mathcal{D}_G(s_2) + \ldots + 2\mathcal{D}_G(s_m) - 2(t_Y(s_1 s_2), i_Y(s_1 s_2), f_Y(s_1 s_2)),\\
&- 2(t_Y(s_2 s_3), i_Y(s_2 s_3), f_Y(s_2 s_3)) - \ldots - 2(t_Y(s_m s_1), i_Y(s_m s_1), f_Y(s_m s_1)),\\
&= 2\sum_{s_k\in N} \mathcal{D}_G(s_k) - 2\sum_{s_k s_l\in L} (t_Y(s_k s_l), i_Y(s_k s_l), f_Y(s_k s_l)),\\
&= \sum_{s_k\in N} \mathcal{D}_G(s_k) + \sum_{s_k\in N} \mathcal{D}_G(s_k) - 2\sum_{s_k s_l\in L} (t_Y(s_k s_l), i_Y(s_k s_l), f_Y(s_k s_l)),\\
&= \sum_{s_k\in N} \mathcal{D}_G(s_k) + 2\sum_{s_k s_l\in L} (t_Y(s_k s_l), i_Y(s_k s_l), f_Y(s_k s_l))\\
&- 2\sum_{s_k s_l\in L} (t_Y(s_k s_l), i_Y(s_k s_l), f_Y(s_k s_l)),\\
&= \sum_{s_k\in N} \mathcal{D}_G(s_k).
\end{aligned}$$

This completes the proof. □

Theorem 2.7. *Let $G = (X, Y)$ be a single-valued neutrosophic graph. Then Y is a constant function if and only if the following statements are equivalent:*

(a) *G is an edge regular single-valued neutrosophic graph,*

(b) *G is a totally edge regular single-valued neutrosophic graph.*

Proof. Let $G = (X, Y)$ be a single-valued neutrosophic graph. Suppose that Y is a constant function, then $t_Y(st) = l_1, i_Y(st) = l_2, f_Y(st) = l_3$ for all $st \in L$.
(a) ⇒ (b): Assume that G is an edge regular single-valued neutrosophic graph, i.e., $\mathcal{D}_G(st) = (q_1, q_2, q_3)$, for each edge $st \in L$. This implies that $\mathcal{TD}_G(st) = (l_1 + q_1, l_2 + q_2, l_3 + q_3)$ for each edge $st \in L$. This shows that G is an edge regular single-valued neutrosophic graph of degree $(l_1 + q_1, l_2 + q_2, l_3 + q_3)$.
(b) ⇒ (a): Suppose that G is a totally edge regular single-valued neutrosophic graph, i.e., $\mathcal{TD}_G(st) = (p_1, p_2, p_3)$ for all $st \in L$. This implies that $\mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)) = (p_1, p_2, p_3)$. This implies that $\mathcal{D}_G(st) = (p_1, p_2, p_3) - (t_Y(st), i_Y(st), f_Y(st))$. This implies that $\mathcal{D}_G(st) = (p_1 - l_1, p_2 - l_2, p_3 - l_3)$ for each edge $st \in L$.

Thus G is an edge regular single-valued neutrosophic graph of degree $(p_1-l_1, p_2-l_2, p_3-l_3)$. Hence the statements (a) and (b) are equivalent.
Conversely, suppose that (a) and (b) are equivalent. Assume that Y is not a constant function. This implies that $(t_Y(st), i_Y(st), f_Y(st)) \neq (t_Y(uv), i_Y(uv), f_Y(uv))$ for at least one pair of edges $st, uv \in L$. Assume that G is an edge regular single-valued neutrosophic graph. This implies that $\mathcal{D}_G(st) = \mathcal{D}_G(uv) = (q_1, q_2, q_3)$. This implies that $\mathcal{TD}_G(st) = \mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)) = (q_1, q_2, q_3) + (t_Y(st), i_Y(st), f_Y(st))$ and $\mathcal{TD}_G(uv) = \mathcal{D}_G(uv) + (t_Y(uv), i_Y(uv), f_Y(uv)) = (q_1, q_2, q_3) + (t_Y(uv), i_Y(uv), f_Y(uv))$. Since $(t_Y(st), i_Y(st), f_Y(st)) \neq (t_Y(uv), i_Y(uv), f_Y(uv))$. This implies that $\mathcal{TD}_G(st) \neq \mathcal{TD}_G(uv)$. This shows that G is not a totally edge regular single-valued neutrosophic graph, which contradicts our supposition. Now, suppose that G is a totally edge regular single-valued neutrosophic graph, i.e., $\mathcal{TD}_G(st) = \mathcal{TD}_G(uv) = (p_1, p_2, p_3)$. This implies that $\mathcal{TD}_G(st) = \mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)) = \mathcal{D}_G(uv) + (t_Y(uv), i_Y(uv), f_Y(uv))$. This implies that $\mathcal{D}_G(st) - \mathcal{D}_G(uv) = (t_Y(st), i_Y(st), f_Y(st)) - (t_Y(uv), i_Y(uv), f_Y(uv))$. Since $(t_Y(st), i_Y(st), f_Y(st)) \neq (t_Y(uv), i_Y(uv), f_Y(uv))$. This implies that $\mathcal{D}_G(st) - \mathcal{D}_G(uv) \neq 0$. This implies that $\mathcal{D}_G(st) \neq \mathcal{D}_G(uv)$. This shows that G is not an edge regular single-valued neutrosophic graph, which contradicts our supposition. Hence Y is a constant function. □

Theorem 2.8. *Let $G = (X, Y)$ be a single-valued neutrosophic graph. Assume that G is both edge regular single-valued neutrosophic of degree (q_1, q_2, q_3) and totally edge regular single-valued neutrosophic graph of degree (p_1, p_2, p_3). Then Y is a constant function.*

Proof. The proof is obvious. □

Remark 2.5. The converse of theorem 2.8 may not be true in general, that is, a single-valued neutrosophic graph $G = (X, Y)$, where Y is a constant function, may or may not be edge regular and totally edge regular single-valued neutrosophic graph.

Example 2.13. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2, s_2s_3, s_3s_4\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 12.

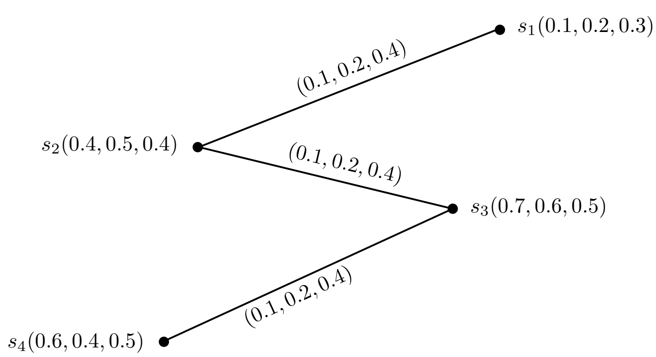

Figure 12: Single valued neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(s_1) = (0.1, 0.2, 0.4)$, $\mathcal{D}_G(s_2) = (0.2, 0.4, 0.8)$, $\mathcal{D}_G(s_3) = (0.2, 0.4, 0.8)$ and $\mathcal{D}_G(s_4) = (0.1, 0.2, 0.4)$. The degree of each edge is $\mathcal{D}_G(s_1s_2) = (0.1, 0.2, 0.4)$, $\mathcal{D}_G(s_2s_3) = (0.2, 0.4, 0.8)$ and $\mathcal{D}_G(s_3s_4) = (0.1, 0.2, 0.4)$. The total degree of each edge is $\mathcal{TD}_G(s_1s_2) = (0.2, 0.4, 0.8)$, $\mathcal{TD}_G(s_2s_3) = (0.3, 0.6, 1.2)$. It is clear from above calculations that G is neither an edge regular nor a totally edge regular single-valued neutrosophic graph.

Theorem 2.9. *Let $G = (X, Y)$ be a single-valued neutrosophic graph of $G^* = (N, L)$, where Y is a constant function. If G is a regular single-valued neutrosophic graph. Then G is an edge regular single-valued neutrosophic graph.*

Proof. Assume that Y is a constant function, that is, $t_Y(st) = l_1$, $i_Y(st) = l_2$ and $f_Y(st) = l_3$ for all $st \in L$. Suppose that G is a regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(s) = (m_1, m_2, m_3)$ for all $s \in N$. Now

$$\begin{aligned} \mathcal{D}_G(st) &= \mathcal{D}_G(s) + \mathcal{D}_G(t) - 2(t_Y(st), i_Y(st), f_Y(st)), \\ &= (m_1, m_2, m_3) + (m_1, m_2, m_3) - 2(l_1, l_2, l_3), \\ &= 2(m_1 - l_1, m_2 - l_2, m_3 - l_3), \end{aligned}$$

for all $st \in L$. Hence G is an edge regular single-valued neutrosophic graph. □

Theorem 2.10. *Let $G = (X, Y)$ be a single-valued neutrosophic graph of $G^* = (N, L)$, where Y is a constant function. If G is a regular single-valued neutrosophic graph. Then G is a totally edge regular single-valued neutrosophic graph.*

Proof. Let Y be a constant function, that is, $t_Y(st) = l_1$, $i_Y(st) = l_2$ and $f_Y(st) = l_3$ for all $st \in L$. Assume that G is a regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(s) = (m_1, m_2, m_3)$ for all $s \in N$. Then G is an edge regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(st) = (q_1, q_2, q_3)$. Now

$$\begin{aligned} \mathcal{TD}_G(st) &= \mathcal{D}_G(st) + (t_Y(st), i_Y(st), f_Y(st)), \\ &= (q_1, q_2, q_3) + (l_1, l_2, l_3), \\ &= 2(q_1 + l_1, q_2 + l_2, q_3 + l_3), \end{aligned}$$

for all $st \in L$. Hence G is a totally edge regular single-valued neutrosophic graph. □

Theorem 2.11. *Let $G^* = (N, L)$ be a regular crisp graph. Suppose that $G = (X, Y)$ is a single-valued neutrosophic graph of G^*. Then G is both regular and totally edge regular single-valued neutrosophic graph if and only if Y is a constant function.*

Proof. Let $G^* = (N, L)$ be a regular crisp graph. Suppose that $G = (X, Y)$ is a single-valued neutrosophic graph of G^*. Suppose that G is both regular and totally edge regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(s) = (m_1, m_2, m_3)$ for all $s \in N$ and $\mathcal{TD}_G(st) = (p_1, p_2, p_3)$ for all $st \in L$. Now

$$\begin{aligned} \mathcal{TD}_G(st) &= \mathcal{D}_G(s) + \mathcal{D}_G(t) - (t_Y(st), i_Y(st), f_Y(st)), \quad \forall st \in L, \\ (p_1, p_2, p_3) &= (m_1, m_2, m_3) + (m_1, m_2, m_3) - (t_Y(st), i_Y(st), f_Y(st)), \\ (t_Y(st), i_Y(st), f_Y(st)) &= (2m_1 - p_1, 2m_2 - p_2, 2m_3 - p_3), \end{aligned}$$

for all $st \in L$. Hence Y is a constant function. Conversely, let Y be a constant function, that is, $t_Y(st) = l_1$, $i_Y(st) = l_2$ and $f_Y(st) = l_3$ for all $st \in L$. So

$$\begin{aligned} \mathcal{D}_G(s) &= \sum_{st \in L} (t_Y(st), i_Y(st), f_Y(st)), \quad \forall s \in N, \\ &= \sum_{st \in L} (m_1, m_2, m_3), \\ &= (m_1, m_2, m_3)\mathcal{D}_{G^*}(s), \\ &= (m_1, m_2, m_3)m. \end{aligned}$$

This implies that $\mathcal{D}_G(s) = (mm_1, mm_2, mm_3)$ for all $s \in L$. Thus G is a regular single-valued neutrosophic graph. Now

$$\begin{aligned} \mathcal{TD}_G(st) &= \sum_{sa \in L, s \neq a} (t_Y(sa), i_Y(sa), f_Y(sa)) + \sum_{at \in L, a \neq t} (t_Y(at), i_Y(at), f_Y(at)), \\ &+ \; (t_Y(st), i_Y(st), f_Y(st)) \quad \forall st \in L, \\ &= \sum_{sa \in L, s \neq a} (l_1, l_2, l_3) + \sum_{at \in L, a \neq t} (l_1, l_2, l_3) + (l_1, l_2, l_3), \\ &= (l_1, l_2, l_3)(\mathcal{D}_{G^*}(s) - 1) + (l_1, l_2, l_3)(\mathcal{D}_{G^*}(t) - 1) + (l_1, l_2, l_3), \\ &= (l_1, l_2, l_3)(s - 1) + (l_1, l_2, l_3)(t - 1) + (l_1, l_2, l_3), \\ &= (2l_1, 2l_2, 2l_3)(s - 1) + (l_1, l_2, l_3), \end{aligned}$$

for all $st \in L$. Hence G is a totally edge regular single-valued neutrosophic graph. □

Theorem 2.12. *Let $G^* = (N, L)$ be a crisp graph. Suppose that $G = (X, Y)$ is a single-valued neutrosophic graph of G^*. Then Y is a constant function if and only if G is an edge regular single-valued neutrosophic graph.*

Proof. Let G be a regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(s) = (m_1, m_2, m_3)$, for all $s \in N$. Suppose that Y is a constant function, that is, $t_Y(st) = l_1$, $i_Y(st) = l_2$ and $f_Y(st) = l_3$, for all $st \in L$. Now

$$\begin{aligned}\mathcal{D}_G(st) &= \mathcal{D}_G(s) + \mathcal{D}_G(t) - 2(t_Y(st), i_Y(st), f_Y(st)), \quad \forall st \in L.\\ &= (m_1, m_2, m_3) + (m_1, m_2, m_3) - 2(l_1, l_2, l_3),\end{aligned}$$

this implies that $\mathcal{D}_G(st) = 2(m_1, m_2, m_3) - 2(l_1, l_2, l_3)$, for all $st \in L$. Hence G is an edge regular single-valued neutrosophic graph.
Conversely, assume that G is an edge regular single-valued neutrosophic graph, that is, $\mathcal{D}_G(st) = (q_1, q_2, q_3)$ for each edge $st \in L$. Now

$$\begin{aligned}\mathcal{D}_G(st) &= \mathcal{D}_G(s) + \mathcal{D}_G(t) - 2(t_Y(st), i_Y(st), f_Y(st)), \quad \forall st \in L,\\ (q_1, q_2, q_3) &= (m_1, m_2, m_3) + (m_1, m_2, m_3) - 2(t_Y(st), i_Y(st), f_Y(st)),\end{aligned}$$

this implies that $(t_Y(st), i_Y(st), f_Y(st)) = \frac{(q_1,q_2,q_3)-(2m_1,2m_2,2m_3)}{2}$, for all $st \in L$. Thus Y is a constant function. □

Definition 2.12. Let G^* be an edge regular crisp graph. Then a single-valued neutrosophic graph G of G^* is said to be a partially edge regular single-valued neutrosophic graph.

Example 2.14. It can be seen in example 2.12 that G^* is an edge regular crisp graph. Therefore, G is a partially edge regular single-valued neutrosophic graph.

Definition 2.13. Let G^* be an edge regular crisp graph. A single-valued neutrosophic graph G of G^* is said to be a full edge regular single-valued neutrosophic graph if it is both edge regular and partially edge regular.

Example 2.15. Consider a graph $G^* = (N, L)$ such that $N = \{s_1, s_2, s_3, s_4\}$ and $L = \{s_1s_2, s_2s_3, s_3s_4, s_1s_4\}$. The corresponding single-valued neutrosophic graph $G = (X, Y)$ is shown in Fig. 14.
By direct calculations, we have $\mathcal{D}_G(s_1) = (0.4, 0.8, 0.8)$, $\mathcal{D}_G(s_2) = (0.4, 0.8, 0.8)$, $\mathcal{D}_G(s_3) = (0.4, 0.8, 0.8)$ and $\mathcal{D}_G(s_4) = (0.4, 0.8, 0.8)$. The degree of each edge is $\mathcal{D}_G(s_1s_2) = (0.4, 0.8, 0.8)$, $\mathcal{D}_G(s_2s_3) = (0.4, 0.8, 0.8)$ $\mathcal{D}_G(s_3s_4) = (0.4, 0.8, 0.8)$ and $\mathcal{D}_G(s_1s_4) = (0.4, 0.8, 0.8)$. It is clear from calculations that G is full edge regular single-valued neutrosophic graph.

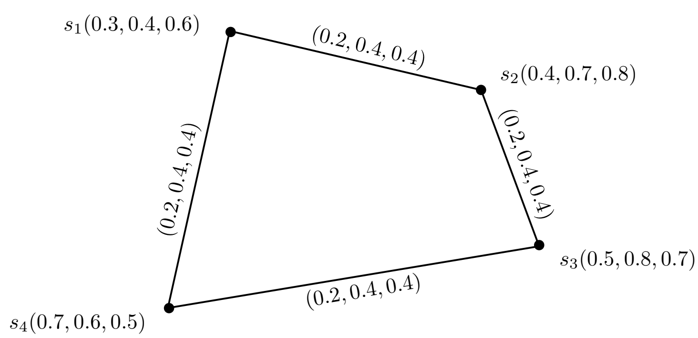

Figure 13: Full edge regular single-valued neutrosophic graph

Theorem 2.13. *Let $G = (X, Y)$ be a single-valued neutrosophic graph of a crisp graph $G^* = (N, L)$, where Y is a constant function. Then G is full edge regular single-valued neutrosophic graph if it is full regular single-valued neutrosophic graph.*

Proof. Let $G = (X, Y)$ be a single-valued neutrosophic graph of a crisp graph $G^* = (N, L)$. Suppose that Y is a constant function, that is, $(t_Y(st), i_Y(st), f_Y(st)) = (l_1, l_2, l_3)$ for each edge $st \in L$. Assume that G is full regular single-valued neutrosophic graph. Then G is both regular and partially regular. Therefore, $\mathcal{D}_G(s) = (m_1, m_2, m_3)$ and

$\mathcal{D}_{G^*}(s) = m$ for all $s \in N$. Since $\mathcal{D}_{G^*}(st) = \mathcal{D}_{G^*}(s) + \mathcal{D}_{G^*}(t) - 2$ for all $st \in L$. This shows that $\mathcal{D}_{G^*}(st) = 2m - 2$. Therefore, G^* is an edge regular single-valued neutrosophic graph. Now

$$\begin{aligned} \mathcal{D}_G(st) &= \mathcal{D}_G(s) + \mathcal{D}_G(t) - 2(t_Y(st), i_Y(st), f_Y(st)), \quad \forall st \in L. \\ &= (m_1, m_2, m_3) + (m_1, m_2, m_3) - 2(l_1, l_2, l_3), \end{aligned}$$

this implies that $\mathcal{D}_G(st) = 2(m_1 - l_1, m_2 - l_2, m_3 - l_3)$. This shows that G is an edge regular single-valued neutrosophic graph. Hence G is a full edge regular single-valued neutrosophic graph. □

3 Application

Detection of a Safe Root for an Airline Journey: Single valued neutrosophic graphs have now become a very important part of applied mathematics and is widely used tool in different areas with a number of applications such as biology, physics, transportation networks and social networks. Consider an example of single valued neutrosophic graphs in flight networks. Suppose we want to travel between different countries through an airline network. The airline companies aim to facilitate their passengers with high quality of services. Air traffic controllers have to make sure that company planes must arrive and depart at right time. This task is possible by planning efficient routes for the planes. Consider an airline network as given in Fig. 14, in which vertices and edges represent the countries and flights, respectively.

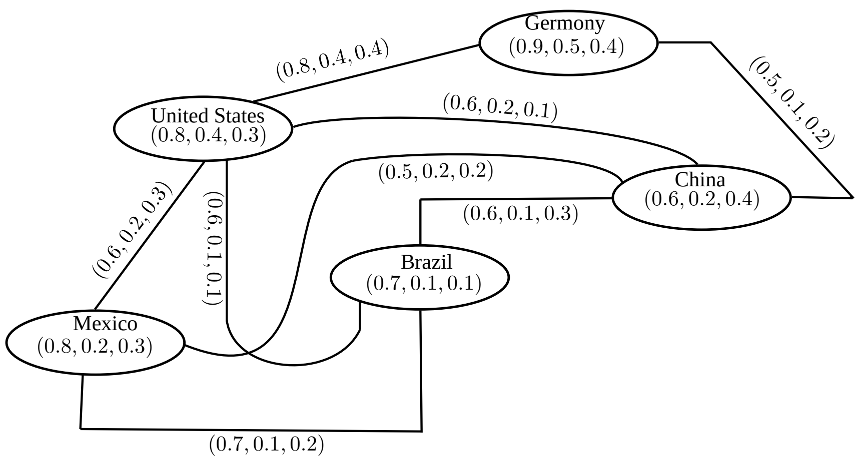

Figure 14: Single valued neutrosophic graph $G = (X, Y)$ of an airline network

The truth membership degree of each edge interprets that how much the flight is save. The indeterminacy membership degree of each edge shows the uncertain situations during a flight such as weather conditions, mechanical error and sabotage, etc. The falsity membership degree of each edge indicates the flaws of that flight. For example the edge between Germany and China indicates that the flight choosen for this travel is 50% safe, 10% depends on uncertain systems and 20% unsafe. The truth membership degree, the indeterminacy membership degree and the falsity membership degree of each edge is calculated by using the following relations

$$\begin{aligned} t_Y(st) &\leq \min\{t_X(s), t_X(t)\}, \\ i_Y(st) &\leq \min\{i_X(s), i_X(t)\}, \\ f_Y(st) &\leq \max\{f_X(s), f_X(t)\}, \end{aligned}$$

respectively, where $s, t \in N$. We want to find the protected route with maximum truth membership degree between the countries. It can be seen from Fig. 14 that there exist five routs between United States and germany. These are

$\mathbf{P_1}$: Germany to United States.

$\mathbf{P_2}$: Germany to China then China to United States.

P_3 : Germany to China, China to Mexico then Mexico to United States.

P_4 : Germany to China, China to Brazil, then Brazil to United States.

P_5 : Germany to China, China to Brazil, Brazil to Mexico then Mexico to United States.

After calculating the lengths of all the routs we get, $L(P_1) = (0.8, 0.4, 0.4)$, $L(P_2) = (1.1, 0.3, 0.3)$, $L(P_3) = (1.6, 0.5, 0.7)$, $L(P_4) = (1.7, 0.3, 0.6)$ and $L(P_5) = (2.4, 0.5, 1.0)$. From Fig. 14, it looks like traveling through Germany to United States is the most protected route but after calculating the lengths we find that the protected route is P_2 because of uncertain conditions. Similarly, one can find the protected route between other countries.
We now present the general procedure of our method which is used in our application from the following algorithm.

Algorithm

Step 1. Input the degrees of truth membership, indeterminacy membership and falsity membership of all m vertices(countries).

Step 2. Calculate the degrees of truth membership, indeterminacy membership and falsity membership of all edges using the following relations

$$t_Y(rs) \leq \min\{t_X(r), t_X(s)\},$$

$$i_Y(rs) \leq \min\{i_X(r), i_X(s)\},$$

$$f_Y(rs) \leq \max\{f_X(r), f_X(s)\},$$

respectively, where $r, s \in N$.

Step 3. Calculate all the possible routes P_k between the countries.

Step 4. Calculate the lengths of all the routs P_k using the following formula

$$L(P_k) = (\sum_{i=1}^{m-1} t_Y(s_i s_{i+1}), \sum_{i=1}^{m-1} i_Y(s_i s_{i+1}), \sum_{i=1}^{m-1} f_Y(s_i s_{i+1})), \quad k = 1, 2, \ldots, n.$$

Step 6. Find the protected route with maximum truth membership degree, minimum indeterminacy membership degree and minimum falsity membership degree.

4 Conclusion

Neutrosophic sets are the generalization of the concept of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic models give more flexibility, precisions and compatibility to the system as compared to the classical, fuzzy and intuitionistic fuzzy models. In this research paper, we have discussed certain types of edge-regular single-valued neutrosophic graphs. Here, we have established some theorems on single valued neutrosophic graphs. It is known that the single-valued neutrosophic graphs are successfully used to analysis the uncertain situation in the real World. Thus we aim to widen our research of fuzzification to (1) single-valued neutrosophic soft graphs, (2) single-valued neutrosophic rough fuzzy graphs, (3) Roughness in neutrosophic graphs and (4) Application of intuitionistic neutrosophic graphs in decision support systems.

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