International Journal of Engineering & Technology, 7 (4.10) (2018) 593-597

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Characterizations of Strong and Balanced Neutrosophic Complex Graphs

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Abstract

In this paper, the concepts of neutrosophic complex graph, complete neutrosophic complex graph, strong neutrosophic complex graph, balanced neutrosophic complex graph and strictly balanced neutrosophic complex graph are introduced. Some of the interesting properties and related examples are established.

Keywords: Neutrosophic complex graph, complement of a NCG, $compNCG$, density of a NCG, $balanced\,NCG$, $strt\text{-}balanced\,NCG$, total degree of a vertex, size and order of NCG.

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1. Introduction

The graph theory is an extremely useful tool for solving combinatorial problems in different areas such algebra, number theory, operation research, computer science, networks etc. as the result of rapid increasing in the size of networks the graph problems become uncertain and we deal these aspects with the method of fuzzy logic[8]. Al. Hawary [1] introduced the concept of balanced fuzzy graphs and some operations of fuzzy graphs. Ramot [5, 6], Zhang [9] introduced the idea of a complex fuzzy sets, complex fuzzy logic and its applications. R.Narmada Devi [7] discussed the concept of neutrosophic complex set and its operations. S. Broumi [2,3,4] were introduced the concepts of single valued neutrosophic graph, its types and their application in real life problem.

2. Preliminaries

Definition 2.1

[7] Let X be a nonempty set. A neutrosophic complex set $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in X\}$ is defined on the universe of discourse X which is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A that assigns a complex values grade of $T_A(x),I_A(x)$ and $F_A(x)$ in A for any $x\in X$. The values $T_A(x),I_A(x)$ and $F_A(x)$ and their sum may all within the unit circle in the complex plane and so is of the following form $T_A(x)=p_A(x).e^{j\mu_A(x)}$, $I_A(x)=q_A(x).e^{j\nu_A(x)}$ and $F_A(x)=r_A(x).e^{j\omega_A(x)}$ where $p_A(x),q_A(x),r_A(x)$ and $\mu_A(x),\nu_A(x),\omega_A(x)$ are respectively real valued and $p_A(x),q_A(x),r_A(x)\in[0,1]$ such that $0\le p_A(x)+q_A(x)+r_A(x)\le 3$.

Definition 2.2:

[7] Let $A=\langle x,T_A(x),I_A(x),F_A(x)\rangle$ and $B=\langle x,T_B(x),I_B(x),F_B(x)\rangle$ be any two neutrosophic complex sets in X. Then the union and intersection of A and B are denoted and defined as

(i) $A\cup B=\langle x,T_{A\cup B}(x),I_{A\cup B}(x),F_{A\cup B}(x)\rangle$ where

$$T_{A\cup B}(x)=[p_A(x)\vee p_B(x)].e^{j[\mu_A(x)\vee\mu_B(x)]},$$
$$I_{A\cup B}(x)=[q_A(x)\vee q_B(x)].e^{j[\nu_A(x)\vee\nu_B(x)]} \text{ and}$$
$$F_{A\cup B}(x)=[r_A(x)\wedge r_B(x)].e^{j[\omega_A(x)\wedge\omega_B(x)]}.$$

(ii) $A\cap B=\langle x,T_{A\cap B}(x),I_{A\cap B}(x),F_{A\cap B}(x)\rangle$ where

$T_{A\cap B}(x)=[p_A(x)\wedge p_B(x)].e^{j[\mu_A(x)\wedge\mu_B(x)]}$,
$I_{A\cap B}(x)=[q_A(x)\wedge q_B(x)].e^{j[\nu_A(x)\wedge\nu_B(x)]}$ and
$F_{A\cap B}(x)=[r_A(x)\vee r_B(x)].e^{j[\omega_A(x)\vee\omega_B(x)]}$.

Definition 2.3:
[7] Let $A=\langle x,T_A(x),I_A(x),F_A(x)\rangle$ and $B=\langle x,T_B(x),I_B(x),F_B(x)\rangle$ be any two neutrosophic complex sets in X. Then (i) $A\subseteq B$ if $T_A(x)\le T_B(x),I_A(x)\le I_B(x)$ and $F_A(x)\ge F_B(x)$, for all $x\in X$. That is $p_A(x)\le p_B(x),\mu_A(x)\le\mu_B(x)$, $q_A(x)\le q_B(x),\nu_A(x)\le\nu_B(x)$, $r_A(x)\ge r_B(x)$ and $\omega_A(x)\ge\omega_B(x)$, (ii) The complement of A is defined as $A^C=\langle x,T_{A^C}(x),I_{A^C}(x),F_{A^C}(x)\rangle$ where $T_{A^C}(x)=[1-p_A(x)].e^{j[1-\mu_A(x)]}$, $I_{A^C}(x)=[1-q_A(x)].e^{j[1-\nu_A(x)]}$ and $F_{A^C}(x)=[1-r_A(x)].e^{j[1-\omega_A(x)]}$, for all $x\in X$.

3. View on Strong Neutrosophic Complex Graphs

Definition 3.1:
Let $G^*=(V,E)$ be a crisp graph. A pair $G=(A,B)$ is called a neutrosophic complex graph (in short. NCG) on a crisp graph $G^*=(V,E)$, where $A=\langle x,T_A(x),I_A(x),F_A(x)\rangle$ is a neutrosophic complex set on V, for every $x\in V$ and $B=\langle xy,T_B(xy),I_B(xy),F_B(xy)\rangle$ is a neutrosophic complex set on E such that $T_B(xy)\le\min[T_A(x),T_A(y)]$, $I_B(xy)\le\min[I_A(x),I_A(y)]$ and $F_B(xy)\ge\max[F_A(x),F_A(y)]$ for every $xy\in E\subseteq V\times V$. Then A and B are neutrosophic complex vertex set on V and neutrosophic complex edge set on E respectively.

Definition 3.2:.
A neutrosophic complex graph $\bar{G}=(\bar{A},\bar{B})$ is called a complement of a neutrosophic complex graph $G=(A,B)$ if
(i) $\bar{A}=A$,
(ii) $T_{\bar{B}}(xy)\le\min[T_A(x),T_A(y)-T_B(xy)]$,
(iii) $I_{\bar{B}}(xy)\le\min[I_A(x),I_A(y)-I_B(xy)]$,
(iv) $F_{\bar{B}}(xy)\ge\max[F_A(x),F_A(y)-F_B(xy)]$, for all $xy\in E$.

Definition 3.3:
. Let $G=(A,B)$ be any neutrosophic complex graph of a crisp graph $G^*=(V,E)$. Then $H=(A_1,B_1)$ is called a neutrosophic complex subgraph(inshort., $NCSubG$) if
(i) $T_{A_1}(x)=T_A(x),I_{A_1}(x)=I_A(x),F_{A_1}(x)=F_A(x)$, for all $x\in V_1\subseteq V$,
(ii) $T_{A_1}(x)=T_A(x)$, $I_{A_1}(x)=I_A(x),F_{A_1}(x)=F_A(x)$, for all $xy\in E_1\subseteq E$.

Definition 3.4:
A neutrosophic complex graph G is a complete neutrosophic complex graph (in short. $compNCG$) $T_B(xy)=\min[T_A(x),T_A(y)]$, $I_B(xy)=\min[I_A(x),I_A(y)]$ and $F_B(xy)=\max[F_A(x),F_A(y)]$ for every $x,y\in V$.

Definition 3.5:
A neutrosophic complex graph G is a strong neutrosophic complex graph (in short. $StNCG$) $T_B(xy)=\min[T_A(x),T_A(y)]$, $I_B(xy)=\min[I_A(x),I_A(y)]$ and $F_B(xy)=\max[F_A(x),F_A(y)]$ for every $xy\in E$.

Proposition 3.1:
Let $G=(A,B)$ be any strong neutrosophic complex fuzzy graph. Then
(i) Every strong neutrosophic complex graph is the generalization of strong complex fuzzy graph.
(ii) Every strong neutrosophic complex graph is the generalization of strong neutrosophic fuzzy graph.
(iii) Every strong neutrosophic complex graph is the generalization of strong intuitionistic fuzzy graph.
(iv) Every strong neutrosophic complex graph is the generalization of strong vague fuzzy graph.
(v) Every strong neutrosophic complex graph is the generalization of strong fuzzy graph.

Definition 3.6:.
Let $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be any two neutrosophic complex graphs on the crisp graphs $G^*=(V_1,E_1)$ and $G^{**}=(V_2,E_2)$ respectively. Then the Cartesian product $G_1\times G_2=(A_1\times A_2,B_1\times B_2)$ is a neutrosophic complex graph which is defined as follows:

(i) $T_{A_1\times A_2}(x_1,x_2)=\min\{T_{A_1}(x_1),T_{A_2}(x_2)\}$,
$I_{A_1\times A_2}(x_1,x_2)=\min\{I_{A_1}(x_1),I_{A_2}(x_2)\}$ and
$F_{A_1\times A_2}(x_1,x_2)=\max\{F_{A_1}(x_1),F_{A_2}(x_2)\}$,
for all $(x_1,x_2)\in V=V_1\times V_2$

(ii) $T_{B_1\times B_2}((x,x_2),(x,y_2))=\min\{T_{A_1}(x),T_{B_2}(x_2,y_2)\}$,
$I_{B_1\times B_2}((x,x_2),(x,y_2))=\min\{I_{A_1}(x),I_{B_2}(x_2,y_2)\}$ and
$F_{B_1\times B_2}((x,x_2),(x,y_2))=\max\{F_{A_1}(x),F_{B_2}(x_2,y_2)\}$,
for every $x\in V$, $(x_2,y_2)\in E_2$

(iii) $T_{B_1\times B_2}((x_1,z),(y_1,z))=\min\{T_{B_1}(x_1,y_1),T_{A_2}(z)\}$,
$I_{B_1\times B_2}((x_1,z),(y_1,z))=\min\{I_{B_1}(x_1,y_1),I_{A_2}(z)\}$ and
$F_{B_1\times B_2}((x_1,z),(y_1,z))=\max\{F_{B_1}(x_1,y_1),F_{A_2}(z)\}$,
for every $z\in V_2$, $(x_1,y_1)\in E_1$

Proposition 3.2:.
If $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be any two strong neutrosophic complex graphs on the crisp graphs $G^*=(V_1,E_1)$ and $G^{**}=(V_2,E_2)$ respectively. Then the Cartesian product $G_1\times G_2=(A_1\times A_2,B_1\times B_2)$ is a neutrosophic complex graph on the crisp graph $G=G^*\times G^{**}=(V,E)$ where $V=V_1\times V_2$ and $E=E_1\times E_2$.

Proof:

Let G_1 and G_2 be strong neutrosophic complex graphs. Then there exists $x_j, x_j \in E_j, j = 1,2$ such that

$T_{B_j}\left(x_j, x_j\right) = \min\left\{T_{A_j}\left(x_j\right), T_{A_j}\left(y_j\right)\right\}$,

$I_{B_j}\left(x_j, x_j\right) = \min\left\{I_{A_j}\left(x_j\right), I_{A_j}\left(y_j\right)\right\}$ and

$F_{B_j}\left(x_j, x_j\right) = \max\left\{F_{A_j}\left(x_j\right), F_{A_j}\left(y_j\right)\right\}$ for $j = 1,2$.

Let $E = \left\{(x,x_2)(x,y_2) / x \in V_1, (x_2,y_2) \in E\right\} \mathrm{U} \left\{(x_1,z)(y_1,z) / z \in V_2, (x_1,y_1) \in E_1\right\}$.

Suppose that $(x,x_2)(x,y_2) \in E$, we have

(i)
$$T_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \min\left\{T_{A_1}(x), T_{B_2}(x_2y_2)\right\}$$
$$= \min\left\{T_{A_1}(x), T_{A_2}(x_2), T_{A_2}(y_2)\right\}. \quad (3.1)$$

(ii)
$$I_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \min\left\{I_{A_1}(x), I_{B_2}(x_2y_2)\right\}$$
$$= \min\left\{I_{A_1}(x), I_{A_2}(x_2), I_{A_2}(y_2)\right\}. \quad (3.2)$$

(iii)
$$F_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \max\left\{F_{A_1}(x), F_{B_2}(x_2y_2)\right\}$$
$$= \max\left\{F_{A_1}(x), F_{A_2}(x_2), F_{A_2}(y_2)\right\}. \quad (3.3)$$

We know that

$$T_{A_1\times A_2}\left((x_1,x_2)\right) = \min\left\{T_{A_1}(x_1), T_{A_2}(x_2)\right\},$$
$$T_{A_1\times A_2}\left((x_1,y_2)\right) = \min\left\{T_{A_1}(x_1), T_{A_2}(y_2)\right\},$$
$$I_{A_1\times A_2}\left((x_1,x_2)\right) = \min\left\{I_{A_1}(x_1), I_{A_2}(x_2)\right\},$$
$$I_{A_1\times A_2}\left((x_1,y_2)\right) = \min\left\{I_{A_1}(x_1), I_{A_2}(y_2)\right\},$$
$$F_{A_1\times A_2}\left((x_1,x_2)\right) = \min\left\{F_{A_1}(x_1), F_{A_2}(x_2)\right\} \text{ and}$$
$$F_{A_1\times A_2}\left((x_1,y_2)\right) = \max\left\{F_{A_1}(x_1), F_{A_2}(y_2)\right\},$$

But

(i)
$$\min\left\{T_{A_1\times A_2}(x,x_2), T_{A_1\times A_2}(x,y_2)\right\}$$
$$= \min\left\{\min\left\{T_{A_1}(x), T_{A_2}(x_2)\right\}, \min\left\{T_{A_1}(x), T_{A_2}(y_2)\right\}\right\}$$

(ii)
$$\min\left\{I_{A_1\times A_2}(x,x_2), I_{A_1\times A_2}(x,y_2)\right\}$$
$$= \min\left\{\min\left\{I_{A_1}(x), I_{A_2}(x_2)\right\}, \min\left\{I_{A_1}(x), I_{A_2}(y_2)\right\}\right\}$$

(iii)
$$\max\left\{F_{A_1\times A_2}(x,x_2), F_{A_1\times A_2}(x,y_2)\right\}$$
$$= \max\left\{\max\left\{F_{A_1}(x), F_{A_2}(x_2)\right\}, \max\left\{F_{A_1}(x), F_{A_2}(y_2)\right\}\right\}$$

This implies that

a) $\min\left\{T_{A_1\times A_2}(x,x_2), T_{A_1\times A_2}(x,y_2)\right\} = \min\left\{T_{A_1}(x), T_{A_2}(x_2), T_{A_2}(y_2)\right\}$.

b) $\min\left\{I_{A_1\times A_2}(x,x_2), I_{A_1\times A_2}(x,y_2)\right\} = \min\left\{I_{A_1}(x), I_{A_2}(x_2), I_{A_2}(y_2)\right\}$.

c) $\max\left\{F_{A_1\times A_2}(x,x_2), F_{A_1\times A_2}(x,y_2)\right\} = \max\left\{F_{A_1}(x), F_{A_2}(x_2), F_{A_2}(y_2)\right\}$

Hence the equations (3.1), (3.2) and (3.3) becomes

$T_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \min\left\{T_{A_1\times A_2}(x,x_2), T_{A_1\times A_2}(x,y_2)\right\}$,

$I_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \min\left\{I_{A_1\times A_2}(x,x_2), I_{A_1\times A_2}(x,y_2)\right\}$

and $F_{B_1\times B_2}\left((x,x_2),(x,y_2)\right) = \max\left\{F_{A_1\times A_2}(x,x_2), F_{A_1\times A_2}(x,y_2)\right\}$.

Therefore $G = G_1 \times G_2$ is a strong neutrosophic complex graph.

4. Balanced and Strictly Balanced Neutrosophic Complex Graphs

Definition 4.1:.

Let $G = (A,B)$ be any neutrosophic complex graph of a crisp graph $G^* = (V,E)$. Then the density of a neutrosophic complex graph G is defined by $D(G) = \left\langle T_D(G), I_D(G), F_D(G)\right\rangle$ where

$$T_D(G) = \frac{2\sum_{x,y\in V} T_B(xy)}{\sum_{x,y\in E}\left[T_A(x)\wedge T_A(y)\right]} \qquad I_D(G) = \frac{2\sum_{x,y\in V} I_B(xy)}{\sum_{x,y\in E}\left[I_A(x)\wedge I_A(y)\right]} \quad \text{and}$$

$$F_D(G) = \frac{2\sum_{x,y\in V} F_B(xy)}{\sum_{x,y\in E}\left[F_A(x)\vee F_A(y)\right]}.$$

Definition 4.2:.

A neutrosophic complex graph $G = (A,B)$ is balanced (in short., *balanced NCG*) if $D(H) \subseteq D(G)$ where $H = (A_1, B_1)$ is a neutrosophic complex subgraph of $G = (A,B)$. That is, $T_D(H) \le T_D(G)$, $I_D(H) \le I_D(G)$ and $F_D(H) \ge F_D(G)$.

Example 4.1:.

Consider neutrosophic complex graph $G = (A,B)$ for a crisp graph $G^* = (V,E)$ where $V = \{u,v,z\}$ and $E = \{uv, uz, vz\}$. Let neutrosophic complex subgraphs of G are $H_1 = (A_1, B_1)$, $H_2 = (A_2, B_2)$, $H_3 = (A_3, B_3)$, $H_4 = (A_4, B_4)$, $H_5 = (A_5, B_5)$ and $H_6 = (A_6, B_6)$ whose vertex sets and edge sets are given by $V_1 = \{u,v\}, E_1 = \{uv\}, V_2 = \{u,z\}, E_2 = \{uz\}$, $V_3 = \{v,z\}, E_3 = \{vz\}$, $V_4 = \{u,v,z\}, E_4 = \{uv, vz\}$, $V_5 = \{u,v,z\}, E_5 = \{uv, uz\}, V_6 = \{u,v,z\}$ and $E_6 = \{uz, vz\}$ respectively. Therefore the density of a neutrosophic complex graph $D(G) = \langle 0.27, 0.27, 2.71\rangle$.

But

$D(H_1) = \langle 0.18, 0.18, 2.76\rangle \subseteq D(G)$,

$D(H_2) = \langle 0.18, 0.18, 2.76\rangle \subseteq D(G)$,

$D(H_3) = \langle 0.18, 0.18, 3.68\rangle \subseteq D(G)$,

$D(H_4) = \langle 0.16, 0.16, 3.05\rangle \subseteq D(G)$,

$D(H_5) = \langle 0.16, 0.16, 2.97\rangle \subseteq D(G)$

and $D(H_6) = \langle 0.18, 0.18, 2.97\rangle \subseteq D(G)$. This implies that the given graph $G = (A,B)$ is a balanced neutrosophic complex graph.

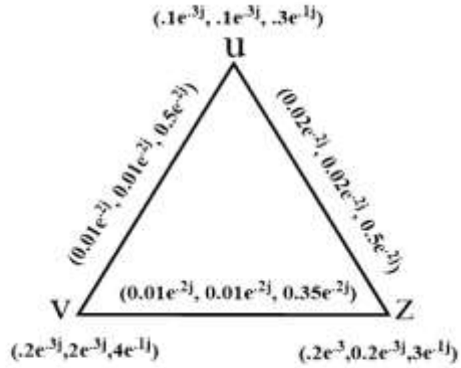

Figure 1: Balanced neutrosophic complex graph

Proposition 4.1:
Every complete neutrosophic complex graph is a balanced neutrosophic complex graph.

Proof:
Let $G=(A,B)$ be any complete neutrosophic complex graph. By the definition, we have

(i) $T_B(xy)=\min[T_A(x),T_A(y)]$

(ii) $I_B(xy)=\min[I_A(x),I_A(y)]$

(iii) $F_B(xy)=\max[F_A(x),F_A(y)]$ for every $x,y\in V$.

This implies that

(i) $\sum_{x,y\in V} T_B(xy)=\sum_{xy\in E}[T_A(x)\wedge T_A(y)]$

(ii) $\sum_{x,y\in V} I_B(xy)=\sum_{xy\in E}[I_A(x)\wedge I_A(y)]$

(iii) $\sum_{x,y\in V} F_B(xy)=\sum_{xy\in E}[F_A(x)\vee F_A(y)]$

Now $T_D(G)=\dfrac{2\sum_{x,y\in V} T_B(xy)}{\sum_{xy\in E}[T_A(x)\wedge T_A(y)]}$, $I_D(G)=\dfrac{2\sum_{x,y\in V} I_B(xy)}{\sum_{xy\in E}[I_A(x)\wedge I_A(y)]}$ and $F_D(G)=\dfrac{2\sum_{x,y\in V} F_B(xy)}{\sum_{xy\in E}[F_A(x)\vee F_A(y)]}$. This implies that $D(G)=(2,2,2)$. Let $H=(A_1,B_1)$ be a be a nonempty neutrosophic complex subgraph of G. Then $D(H)=(2,2,2)$ for every $H\subseteq G$. Hence G is a balanced neutrosophic complex graph.

Remark 4.4:
Every strong neutrosophic complex graph is a balanced neutrosophic complex graph.

Definition 4.3:
A neutrosophic complex graph G is strictly balanced (inshort., $Strt\text{-}balanced$NCG) if for every $x,y\in V$ and for all nonempty neutrosophic complex subgraph $H=(A_j,B_j), j\in J, D(H)=D(G)$.

Example 4.2:
Consider NCG G for a crisp graph G^* where $V=\{a,b,c\}$ and $E=\{ab,bc,ac\}$. Let $H_1=(A_1,B_1)$, $H_2=(A_2,B_2)$, $H_3=(A_3,B_3)$, $H_4=(A_4,B_4)$, $H_5=(A_5,B_5)$ and $H_6=(A_6,B_6)$ be six $NCSubG$ s of G whose vertex sets and edge sets are given by $V_1=\{a,b\},E_1=\{ab\}$, $V_2=\{b,c\},E_2=\{bc\},V_3=\{a,c\}$, $E_3=\{ac\}$, , $V_4=\{a,b,c\},E_4=\{ac,ab\}$, $V_5=\{a,b,c\},E_5=\{ab,bc\}$, $V_6=\{a,b,c\}$ and $E_6=\{bc,ac\}$ respectively. Therefore the density of a neutrosophic complex graph G is equal to the density of all the above neutrosophic complex subgraphs of G, that is., $D(G)=(2,2,2)=D(H_j)$ where $j=\{1,2,3,4,5,6\}$. Hence G is a strictly balanced neutrosophic complex graph.

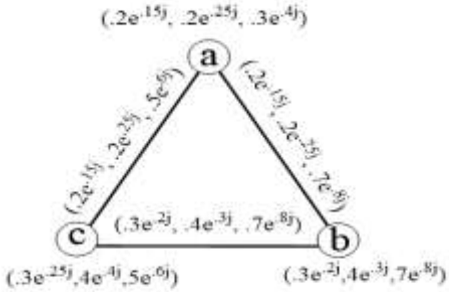

Figure 2: Strictly balanced neutrosophic complex graph

Proposition 4.2:
The complement of a strictly balanced neutrosophic complex graph is a strictly balanced neutrosophic complex graph.

Definition 4.4:.
Let $G=(A,B)$ be any neutrosophic complex graph of a crisp graph $G^*=(V,E)$. Then the total degree of a vertex $x\in V$. Then

(i) T-degree of a vertex $u\in V$ is defined as $T_{\deg_G}(x)=\sum_{x\neq y} T_B(xy)$,

(ii) T-degree of a vertex $u\in V$ is defined as $I_{\deg_G}(x)=\sum_{x\neq y} I_B(xy)$,

(iii) F-degree of a vertex $u\in V$ is defined as $F_{\deg_G}(x)=\sum_{x\neq y} F_B(xy), xy\in E$.

Definition 4.5:.
Let $G=(A,B)$ be any neutrosophic complex graph of a crisp graph $G^*=(V,E)$. Then the total degree of a vertex $x\in V$ is defined by $d_G(x)=\langle T_{d_G}(x),I_{d_G}(x),F_{d_G}(x)\rangle$ where $T_{d_G}(x)=T_{\deg_G}(x)+T_A(x)$, $I_{d_G}(x)=I_{\deg_G}(x)+I_A(x)$ and $F_{d_G}(x)=F_{\deg_G}(x)+F_A(x)$.

Definition 4.6:.
Let $G=(A,B)$ be any neutrosophic complex graph of a crisp graph $G^*=(V,E)$. Then the total degree of a vertex $x\in V$. Then the size of neutrosophic complex graph G is defined by $S(G)=\langle T_S(G),I_S(G),F_S(G)\rangle$ where $T_S(G)=\sum_{x,y\in V} T_B(xy)$, $I_S(G)=\sum_{x,y\in V} I_B(xy)$ and $F_S(G)=\sum_{x,y\in V} F_B(xy)$.

Definition 4.7:.
Let $G=(A,B)$ be any neutrosophic complex graph of a crisp graph $G^*=(V,E)$. Then the total degree of a vertex $x\in V$ is defined by $Ord(G)=\langle T_{Ord}(G),I_{Ord}(G),F_{Ord}(G)\rangle$ where $T_{Ord}(G)=\sum_{x_i\in V} T_A(x_i)$, $I_{Ord}(G)=\sum_{x_i\in V} I_A(x_i)$ and $F_{Ord}(G)=\sum_{x_i\in V} F_A(x_i)$.

Proposition 4.3:.
If $G=(A,B)$ is neutrosophic complex graph of a crisp graph $G^*=(V,E)$, then for every $xy\in E$,

(i) $T_S(G)+T_S(\bar{G})\leq 2\sum_{x,y\in V} T_B(xy)$,

(ii) $I_S(G)+I_S(\bar{G})\leq 2\sum_{x,y\in V} I_B(xy)$,

(iii) $F_S(G)+F_S(\bar{G})\geq 2\sum_{x,y\in V} F_B(xy)$

Proof:
(i) $T_B(xy)\leq \min\left[T_A(x),T_A(y)\right]$,

(ii) $I_B(xy) \le \min\left[I_A(x), I_A(y)\right]$,

(iii) $F_B(xy) \ge \min\left[F_A(x), F_A(y)\right]$

(iv) $T_{\bar{B}}(xy) = \min\left[T_A(x), T_A(y)\right] - T_B(xy)$,

(v) $I_{\bar{B}}(xy) = \min\left[I_A(x), I_A(y)\right] - I_B(xy)$ and

(vi) $F_{\bar{B}}(xy) = \max\left[F_A(x), F_A(y)\right] - F_B(xy)$

We know that order of a neutrosophic complex graph G is equal to neutrosophic complex graph $\bar{G}$.

This implies that $Ord(G) = Ord(\bar{G})$ and we have $T_{\bar{B}}(xy) \le \min\left[T_A(x), T_A(y)\right], I_{\bar{B}}(xy) \le \min\left[I_A(x), I_A(y)\right]$ and $F_{\bar{B}}(xy) \ge \max\left[F_A(x), F_A(y)\right]$. But

$$T_{\bar{B}}(xy) + T_B(xy) = \min\left[T_A(x), T_A(y)\right],$$
$$I_{\bar{B}}(xy) + I_B(xy) = \min\left[I_A(x), I_A(y)\right] \text{ and}$$
$$F_{\bar{B}}(xy) + F_B(xy) = \max\left[F_A(x), F_A(y)\right]$$

This implies that

$$T_{\bar{B}}(xy) + T_B(xy) \le 2\min\left[T_A(x), T_A(y)\right],$$
$$I_{\bar{B}}(xy) + I_B(xy) \le 2\min\left[I_A(x), I_A(y)\right] \text{ and}$$
$$F_{\bar{B}}(xy) + F_B(xy) \ge 2\max\left[F_A(x), F_A(y)\right]$$

Taking summation on both side, we have

$$\sum_{x,y\in V} T_{\bar{B}}(xy) + \sum_{x,y\in V} T_B(xy) \le 2\sum_{x,y\in V} \min\left[T_A(x), T_A(y)\right],$$
$$\sum_{x,y\in V} I_{\bar{B}}(xy) + \sum_{x,y\in V} I_B(xy) \le 2\sum_{x,y\in V} \min\left[I_A(x), I_A(y)\right] \text{ and}$$
$$\sum_{x,y\in V} F_{\bar{B}}(xy) + \sum_{x,y\in V} F_B(xy) \ge 2\sum_{x,y\in V} \max\left[F_A(x), F_A(y)\right].$$

This implies that

$$T_S(G) + T_S(\bar{G}) \le 2\sum_{x,y\in V} \min\left[T_A(x), T_A(y)\right] = 2\sum_{x,y\in V} T_B(xy),$$
$$I_S(G) + I_S(\bar{G}) \le 2\sum_{x,y\in V} \min\left[I_A(x), I_A(y)\right] = 2\sum_{x,y\in V} I_B(xy)$$
$$\text{and } F_S(G) + F_S(\bar{G}) \ge 2\sum_{x,y\in V} \max\left[F_A(x), F_A(y)\right] = 2\sum_{x,y\in V} F_B(xy).$$

5. Conclusion

Nowadays, medical diagnosis comprises of uncertainties and increased volume of information available to physicians from new medical technologies. So, all collected information may be in neutrosophic complex form. The three components of a neutrosophic complex set are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. So, to deal more indeterminacy situations in medical diagnosis neutrosophic complex environment is more acceptable.

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