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Classification of the State of Manufacturing Process under Indeterminacy

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Abstract: In this paper, the diagnosis of the manufacturing process under the indeterminate environment is presented. The similarity measure index was used to find the probability of the in-control and the out-of-control of the process. The average run length (ARL) was also computed for various values of specified parameters. An example from the Juice Company is considered under the indeterminate environment. From this study, it is concluded that the proposed diagnosis scheme under the neutrosophic statistics is quite simple and effective for the current state of the manufacturing process under uncertainty. The use of the proposed method under the uncertainty environment in the Juice Company may eliminate the non-conforming items and alternatively increase the profit of the company.

Keywords: similarity index; diagnosis; process; indeterminacy; neutrosophic statistics

1. Introduction

To control the non-conforming products in the industry is an important task for industrial engineers. Their mission is to minimize the non-conforming product which can be achieved only if the problems in the manufacturing process can be tackled immediately. The control charts are essential tools in the industry to monitor the manufacturing process. These tools are used to indicate the state of the process. A timely indication about the state of the process leads to the high quality of the product. Epprecht et al. [1] and Chiu and Kuo [2] proposed a chart for monitoring one, and more than one, non-conforming product, respectively. Hsu [3] designed a variable chart using the improved sampling schemes. Ho and Quinino [4] proposed an attribute chart to control the variation in the process. Aslam et al. [5] and Aslam et al. [6] worked on a time-truncated chart for the Birnbaum-Saunders distribution and the Weibull distribution respectively. Jeyadurga et al. [7] worked on an attribute chart under truncated life tests.

To analyze the vague and fuzzy data, the fuzzy logic is applied. The fuzzy logic is applied to analyze the data when the experimenters are unsure about the exact values of the parameters. Therefore, the monitoring of the process having fuzzy data is done using the fuzzy-based control charts. Afshari and Gildeh [8] and Ercan Teksen and Anagun [9] worked on fuzzy attribute and variable charts, respectively. Fadaei and Pooya [10] worked on a fuzzy operating characteristic curve. For more details, the reader may refer to Jamkhaneh et al. [11] who discussed the rectifying fuzzy single sampling plan. Senturk and Erginel [12] studied variable control charts using fuzzy approach. Ercan Teksen and Anagun [9] worked on the fuzzy X-bar and R-charts. More details on fuzzy logic can be seen in Lee and Kim [13] and Grzegorzewski [14].

A fuzzy and imprecise data usually have indeterminate values. Fuzzy and vague data only considered the membership of the truth and false values. A neutrosophic logic deals with membership of truth, false and indeterminacy values. Therefore, the neutrosophic logic is useful to analyze the data having indeterminacy. Smarandache [15] introduced the neutrosophic statistics, which analyze the data when indeterminacy is presented. Aslam [16] and Aslam and Arif [17] introduced the neutrosophic statistics in the area of quality control. More details about the neutrosophic logic can be seen in references [18–23].

The similarity measure index (SMI) has been widely used in a variety of fields for classification purposes. In medical sciences, this index is used to classify the patients having a particular disease or not under indeterminacy, see De and Mishra [24]. By exploring the literature and to the best of the author's knowledge, there is no work on the process monitoring using SMI. In this paper, a method to classify the state of the process using SMI is introduced. The operational process of the proposed method is also given. The proposed classification method is simple in application compared to the existing method under classical statistics. It is expected that the proposed diagnosis method for the manufacturing process under the indeterminate environment will be effective, adequate and easy compared to the existing control charts under classical statistics. In Section 2, the SMI index is introduced in process control. A comparative study and application are given in Sections 3 and 4, respectively. Some concluded remarks are given in the last section.

2. The Proposed Chart Based on SMI

Suppose that $Z_N = s_N + u_N I; Z_N \in [Z_L, Z_U]$ is a neutrosophic number having a determined part s_N and an indeterminate part $u_N I$, $I \in [\inf I, \inf U]$ denotes the indeterminacy. Note here that $Z_N \in [Z_L, Z_U]$ is reduced to the determined number $Z_N = s_N$ when no indeterminacy is found. The practitioners cannot record observations of the variable of interest in the precise and determined form in the presence of indeterminacy. The monitoring of the data having neutrosophic numbers using classical statistics as discussed in reference [25] may mislead decision-makers regarding the state of the process. For example, the practitioners decide the process is in the control state using classical statistics, but in fact, some observations are in the indeterminacy interval. More details on this issue can be seen in reference [26]. Suppose that t_U, f_U and I_U presents the probabilities of the non-defective, defective and indeterminate. For the classification of the state of the process, let $t = 1$ and $f = 0$ show that the process is in control. Therefore, the value of SMI close to 1 indicates that the process is in control and the values away from the SMI show the process is out-of-control. The SMI from De and Mishra [24] is given by:

$$SMI = \sqrt{\left(1 - \frac{|(t_L - t_U) - (I_L - I_U) - (f_L - f_U)|}{3}\right)(1 - |(t_L - t_U) + (I_L - I_U) + (f_L - f_U)|)} \tag{1}$$

Note here $0 \le t_L, I_L, f_L \le 1$, $0 \le t_U, I_U, f_U \le 1$, $0 \le t_L + f_L \le 1$, $0 \le t_U + f_U \le 1$, $t_L + I_L + f_L \le 2$, $t_U + I_U + f_U \le 2$.

Based on SMI, the following classification procedure is proposed to diagnose the state of the manufacturing process.

Step 1: Select a random sample of size n and determine t_U, f_U and I_U.

Step 2: Compute the values of SMI. Classify the process in-control if SMI $\ge$ 0.95, otherwise, the out-of-control.

The operational process of the proposed method is also given with the help of Figure 1.

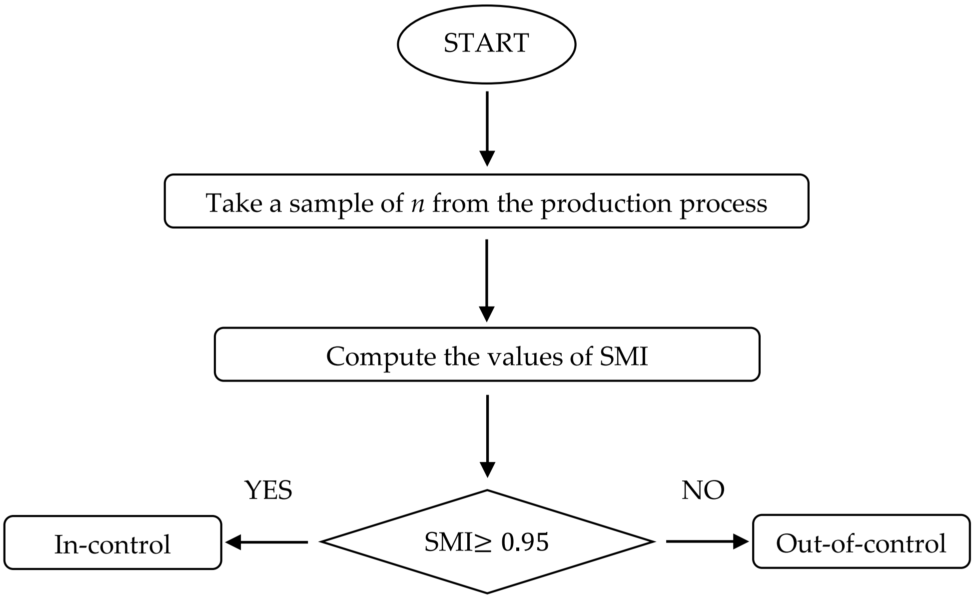

Figure 1. The operational process of the proposed method.

Note here that unlike the traditional control charts under classical statistics, the proposed chart using SMI is independent of the control limits and the control limits coefficients. The proposed chart reduces to the traditional control charts under classical statistics if no indeterminacy is found. Suppose that the probability of in-control of the process is determined from SMI. Let $SMI = P_{in}$, the P_{in} for the process is given by

$$P_{in} = \sqrt{\left(1 - \frac{|(t_L - t_U) - (I_L - I_U) - (f_L - f_U)|}{3}\right)}(1 - |(t_L - t_U) + (I_L - I_U) + (f_L - f_U)|) \tag{2}$$

The average run length (ARL) is used to see when on the average the process is expected to be out-of-control. The ARL under indeterminacy is given by:

$$ARL = \frac{1}{\left[\sqrt{\left(1 - \frac{|(t_L - t_U) - (I_L - I_U) - (f_L - f_U)|}{3}\right)}(1 - |(t_L - t_U) + (I_L - I_U) + (f_L - f_U)|)\right]} \tag{3}$$

The values of t_U, f_U and I_U for various values of n are given in Tables 1–3. Tables 1 and 2 are given when $n = 25$ and $n = 50$, respectively. Table 3 is presented for a variable sample size. In Table 4, the values of P_{in} and ARL are given for the parameters given in Tables 1–3. The classification of the state of the process based on SMI is also presented in Table 4. The process is said to be the in-control (IN) state if SMI $\geq$ 0.95 and the out-of-control (OOC) state if SMI < 0.95. It is noted no specific trend in ARL values. The following algorithm is used to classify the state of the process using the proposed method.

Step 1: Specify n and determine the values of t_U, f_U and I_U.
Step 2: Use the SMI to find the probability of in-control.
Step 3: Classify the process IN if SMI $\geq$ 0.95 and OOC if SMI < 0.95.

Table 1. Neutrosophic data when n = 25.

Sample No.	Sample Size	Number of Defective Units D	f_U	Number of Non-Defective Units ND	t_U	Number of Indeterminate Units I	I_U
1	25	3	0.12	21	0.84	1	0.04
2	25	4	0.16	19	0.76	2	0.08
3	25	2	0.08	23	0.92	0	0
4	25	5	0.2	20	0.8	4	0.16
5	25	2	0.08	22	0.88	1	0.04
6	25	1	0.04	22	0.88	2	0.08
7	25	0	0	20	0.8	5	0.2
8	25	4	0.16	21	0.84	0	0
9	25	6	0.24	17	0.68	2	0.08
10	25	1	0.04	23	0.92	1	0.04
11	25	2	0.08	20	0.8	3	0.12
12	25	5	0.2	18	0.72	2	0.08
13	25	4	0.16	19	0.76	2	0.08
14	25	8	0.32	16	0.64	1	0.04
15	25	3	0.12	21	0.84	1	0.04
16	25	2	0.08	21	0.84	2	0.08
17	25	5	0.2	17	0.68	3	0.12
18	25	3	0.12	19	0.76	3	0.12
19	25	7	0.28	17	0.68	1	0.04
20	25	1	0.04	23	0.92	1	0.04
21	25	0	0	23	0.92	2	0.08
22	25	2	0.08	19	0.76	4	0.16
23	25	5	0.2	17	0.68	3	0.12
24	25	7	0.28	17	0.68	1	0.04
25	25	8	0.32	17	0.68	0	0

Table 2. Neutrosophic data when n = 50.

Sample No.	Sample Size	Number of Defective Units D	f_U	Number of Non-Defective Units ND	t_U	Number of Indeterminate Units I	I_U
1	50	1	0.02	48	0.96	1	0.02
2	50	2	0.04	47	0.94	1	0.02
3	50	3	0.06	45	0.9	2	0.04
4	50	5	0.1	43	0.86	2	0.04
5	50	2	0.04	43	0.86	5	0.1
6	50	6	0.12	41	0.82	3	0.06
7	50	1	0.02	46	0.92	3	0.06
8	50	2	0.04	44	0.88	4	0.08
9	50	7	0.14	37	0.74	6	0.12
10	50	8	0.16	34	0.68	6	0.12
11	50	1	0.02	47	0.94	2	0.04
12	50	6	0.12	43	0.86	1	0.02
13	50	1	0.02	41	0.82	8	0.16
14	50	3	0.06	39	0.78	8	0.16
15	50	6	0.12	41	0.82	3	0.06
16	50	3	0.06	45	0.9	2	0.04
17	50	9	0.18	40	0.8	1	0.02
18	50	2	0.04	41	0.82	7	0.14
19	50	4	0.08	46	0.92	0	0
20	50	6	0.12	43	0.86	1	0.02
21	50	1	0.02	47	0.94	2	0.04
22	50	7	0.14	43	0.86	0	0
23	50	2	0.04	45	0.9	3	0.06
24	50	0	0	48	0.96	2	0.04
25	50	1	0.02	48	0.96	1	0.02

Table 3. Neutrosophic data with variable sample size.

Sample No.	Sample Size	Number of Defective units D	f_U	Number of Non-Defective Units ND	t_U	Number of Indeterminate Units I	I_U
1	100	12	0.120	78	0.780	10	0.10
2	80	8	0.100	67	0.838	5	0.06
3	80	6	0.075	69	0.863	5	0.06
4	100	9	0.090	89	0.890	2	0.02
5	110	10	0.091	99	0.900	1	0.01
6	110	12	0.109	98	0.891	0	0.00
7	100	11	0.110	85	0.850	4	0.04
8	100	16	0.160	79	0.790	5	0.05
9	90	10	0.111	66	0.733	14	0.16
10	90	6	0.067	72	0.800	12	0.13
11	110	20	0.182	89	0.809	1	0.01
12	120	15	0.125	99	0.825	6	0.05
13	120	9	0.075	108	0.900	3	0.03
14	120	8	0.067	107	0.892	5	0.04
15	110	6	0.055	95	0.864	9	0.08
16	80	8	0.100	72	0.900	0	0.00
17	80	10	0.125	69	0.863	1	0.01
18	80	7	0.088	68	0.850	5	0.06
19	90	5	0.056	78	0.867	7	0.08
20	100	8	0.080	88	0.880	4	0.04
21	100	5	0.050	88	0.880	7	0.07
22	100	8	0.080	91	0.910	1	0.01
23	100	10	0.100	88	0.880	2	0.02
24	90	6	0.067	80	0.889	4	0.04
25	90	9	0.100	80	0.889	1	0.01

Table 4. Classification of the process.

n = 25			n = 50			Variable Sample Size		
P_{in}	Classification	ARL	P_{in}	Classification	ARL	P_{in}	Classification	ARL
0.9451	OOC	18	0.9865	IN	74	0.9223	OOC	13
0.9165	OOC	12	0.9797	IN	49	0.9438	OOC	18
0.9729	IN	37	0.9660	IN	29	0.9526	IN	21
0.8265	OOC	6	0.9521	IN	21	0.9626	IN	27
0.9591	IN	24	0.9521	IN	21	0.9654	IN	29
0.9591	IN	24	0.9380	OOC	16	0.9629	IN	27
0.9309	OOC	14	0.9729	IN	37	0.9486	OOC	19
0.9451	OOC	18	0.9591	IN	24	0.9273	OOC	14
0.8869	OOC	9	0.9092	OOC	11	0.9040	OOC	10
0.9729	IN	37	0.8763	OOC	8	0.9300	OOC	14
0.9309	OOC	14	0.9797	IN	49	0.9335	OOC	15
0.9018	OOC	10	0.9521	IN	21	0.9398	OOC	17
0.9165	OOC	12	0.9380	OOC	16	0.9628	IN	27
0.8717	OOC	8	0.9237	OOC	13	0.9630	IN	27
0.9451	OOC	18	0.9380	OOC	16	0.9532	IN	21
0.9451	OOC	18	0.9660	IN	29	0.9660	IN	29
0.8869	OOC	9	0.9309	OOC	14	0.9526	IN	21
0.9165	OOC	12	0.9380	OOC	16	0.9480	OOC	19
0.8869	OOC	9	0.9729	IN	37	0.9526	IN	21
0.9729	IN	37	0.9521	IN	21	0.9591	IN	24
0.9729	IN	37	0.9797	IN	49	0.9591	IN	24
0.9165	OOC	12	0.9521	IN	21	0.9695	IN	33
0.8869	OOC	9	0.9660	IN	29	0.9591	IN	24
0.8869	OOC	9	0.9865	IN	74	0.9610	IN	26
0.8869	OOC	9	0.9865	IN	74	0.9619	IN	26

Note: IN = in-control and OOC = out-of-control.

3. Comparative Study

In this section, a comparison of the effectiveness of the proposed method is given over the control charts under classical statistics reported in reference [25]. According to Aslam et al. [26], a method which deals with indeterminacy is said to be more effective than the method which provides the determined values. The proposed method reduces to the traditional method under classical statistics if no indeterminacy is recorded. From reference [25], it is noted that the control chart under classical statistics does not consider the measure of indeterminacy which makes it limited to be used in an uncertainty environment. The performance of the existing control chart depends on the control limit coefficient which is determined through the complicated simulation process. On the other hand, the current method considered the measure of indeterminacy to evaluate the performance of the control chart. In addition, the proposed method is independent of the control limit coefficient. The proposed process can be applied easily to classify the state of the process. Note here that, the proposed method reduces to the method under classical statistics if no indeterminacy is found in the production data. The values of ARL from the proposed method and method under classical statistics discussed by Montgomery [25] are shown in Table 5 when $n = 25$ and $D = 2$. It is well-known theory that the smaller the values of ARL means more efficient the control chart process [25]. From Table 5, it can be seen that the proposed method provides the smaller values of ARL than the existing method. It means the proposed control chart has the ability to detect a shift in the process earlier than the method under classical statistics. For example, when $n = 25$ and $d = 2$, the value of ARL of the existing method from Table 5 is 37. On the other hand, the proposed method provides smaller values of ARL which are 24, 14, 18 and 12. From this comparison, it is concluded that the process is classified as IN. The industrial engineers can expect the process to be out-of-control at the 37th sample by using the existing method and on the 12th sample for sample number 22 using the proposed method. Therefore, the proposed method is efficient in detecting shifts earlier than the existing method. From this comparison, is the authors concluded that the proposed method is more effective than the existing charts as it considered the measure of indeterminacy and indicated when the process was OCC.

Table 5. The comparison of the proposed method with existing method when $n = 25$ and $D = 2$.

Sample No	ARL	Control Chart
3	37	Under classical statistics
5	24	Under neutrosophic statistics
11	14	Under neutrosophic statistics
16	18	Under neutrosophic statistics
22	12	Under neutrosophic statistics

4. Application

In this section, a discussion of the application of the proposed method in an orange juice company is given. According to Montgomery [25], "Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel". By inspection, it was found that a sample of 50 juice cans was formed. Some cans were found to be leaking and some were labeled as good. For some cans, the industrial engineer is indeterminate about whether the juice product is labeled as either conforming and non-conforming. Therefore, classical statistics cannot be applied to monitor the process in the presence of indeterminacy. The data for $n = 50$ is shown in Table 2. The classification of the state of the process for the juice cans is shown in Table 4. From Table 4, it is noted that the first five subgroups show that the process is the IN control state. The 5th subgroup shows that the process is OOC and industrial engineer should take action to bring back the process in the IN state. It is noted that overall eight samples are in OOC state. From this study, it is concluded that the use of the proposed method to classify the state of the process is quite easy, effective and adequate to be applied under an uncertainty environment.

5. Conclusions and Remarks

In this paper, the diagnosis of the manufacturing process under the indeterminate environment was presented. The similarity measure index was used to find the probability of the in-control and the out-of-control of the process. The average run length (ARL) was also computed for various values of specified parameters. An industrial example was given to explain the state of the process. An industrial example under the indeterminate environment was presented. From this study, it is concluded that the proposed diagnosis scheme under the neutrosophic statistics is quite simple and effective for the current state of the manufacturing process under uncertainty. The practitioners can apply the proposed method to save time and efforts in the industry. The proposed method using non-normal measures can be considered as future research.

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