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Control Chart for Variance Using Repetitive Sampling Under Neutrosophic Statistical Interval System

MUHAMMAD ASLAM
Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia
e-mail: aslam_ravian@hotmail.com

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ABSTRACT In this paper, a control chart for variance using the repetitive sampling under the neutrosophic statistics is proposed. The proposed control chart can be applied for monitoring the variability in the process when some observations/parameters are unclear and indeterminate. The operational procedure of the proposed control chart and the necessary measures are given for the practical use of the proposed control chart. The control chart parameters are determined through the algorithm under neutrosophic statistical interval method. The efficiency of the proposed control chart is compared with the existing control chart in neutrosophic average run length. From the simulation study and a real example, it is concluded that the proposed control chart is more efficient in detecting a shift in the process and more adequate, effective, informative, and flexible under the uncertainty environment.

INDEX TERMS Neutrosophic statistical interval method, neutrosophic average run length, variance, repetitive sampling, uncertainty, control chart.

I. INTRODUCTION

To meet the high standard of quality of the product, it is necessary to monitor the variation in the process. The presence of the variation in the process may shift the process from the set target. According to [1] "High quality in a product can be only achieved if the manufacturing process meets the given specifications limits. To the manufacturer to produce the defect-free product, the variation in the process should be controlled". A control chart is an important tool, which has been commonly used for the monitoring variation in the process. The control chart is a graph having lower control limit (LCL), central limit (CL) and the upper control limit (UCL). The process is said to be out-of-control if the plotting statistic is beyond LCL or UCL. The late indication about the shift in the process causes the production of non-conforming items. The control chart immediately indicates when the process is shifted from the specified target. Therefore, the control charts are designed to help the operators to fix the issue in the manufacturing process before the deeper problem in the process. The Shewhart S^2 control chart has been commonly applied for the monitoring of the variation in the process. The S^2 chart using the double sampling was proposed by [2]. Reference [3] proposed the ARL-unbiased control chart. Reference [4] designed the efficient control chart to monitor the variation in the process. Reference [5] worked on the variance control chart using combined sampling schemes. Reference [6] compared the efficiency of interquartile range chart with existing control charts. For more details, the reader may refer to [7]–[9].

The control charts are usually designed using the single sampling scheme. In this type control charts, a decision about the state of the control chart is decided based on sample information from the single sample selected from the process. The repetitive sampling (RS) scheme was originally proposed by [10] which is applied when the experimenter is indecisive about the state of the process at the first sample.

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The repetitive sampling based control charts are found to be more efficient in average run length (ARL) than the control chart using the single sampling. Reference [11] designed control chart for process capability using the RS. Reference [12] worked on t-chart using this sampling scheme. Reference [13] deigned sign chart using the RS. Reference [14] proposed a hybrid type control chart using the RS. Reference [15] proposed the S^2 control chart using the RS. More details on the RS and ranked sampling control charts can be read in [16]–[21].

The fuzzy control charts, which are more sensitive than the Shewhart control, are applied when parameters are uncertain [22]. Reference [23] "observations include human judgments, and evaluations and decisions, a continuous random variable of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. These variability causes create vagueness in the measurement system". Reference [23] proposed the dispersion control chart using the fuzzy logic. Reference [23] proposed a variance control chart using α-cut. [24] designed control chart under uncertainty. The applications of the control chart using the fuzzy approach can be seen in [25]. More details can be seen in [26]–[29].

Reference [30] mentioned that the neutrosophic logic which deals with indeterminacy is the generalization of the fuzzy logic. Reference [31] used the neutrosophic logic to develop the idea of the neutrosophic statistics (NS). The NS can be applied for the analysis of the data, which is drawn from pupation or sample having uncertain observations or the parameters. The NS is the extension of the classical statistics and found to be more effective to analyze the data under the uncertainty. The NS provides the analysis results in indeterminacy interval rather than the determined values. Therefore, the NS is more adequate and effective to a method to apply under the uncertainty environment. References [32] and [33] suggested that NS is a more effective method than the classical statistics in rock measuring issues. References [34] and [35] originally introduced the NS in the area of acceptance sampling plans. Reference [36] introduced the NS in the area of control chart first time. Reference [1] proposed the variance control chart using the NS.

By exploring the literature and best of our knowledge, there is no work on the S^2 control chart using the repetitive sampling under the NS. In this paper, we will design a variance control chart using the neutrosophic statistical interval method (NSIM). We expect that the proposed control chart will be more efficient in detecting a shift in the process and more adequate, effective, informative and flexible in the uncertainty environment.

II. PREMIERES

Let $X_{Ni} \in \{X_L, X_U\}$; $i = 1, 2, 3, \ldots, n_N$ be a neutrosophic random variable (NRV) consisting of lower value X_L and upper value X_U. Suppose also that $\mu_N = \sum_{i=1}^{N_N} X_{Ni}/N_N$; $\mu_N \in \{\mu_L, \mu_U\}$ and $\sigma_N^2 = \sum_{i=1}^{n_N} (X_{Ni} - \mu_N)^2 / (N_N - 1)$; $\sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}$ represent the neutrosophic population mean and variance corresponding to X_L and X_U, respectively. The neutrosophic estimators of the population mean is $\bar{X}_N = \sum_{i=1}^{n_N} X_{Ni}/n_N$; $\bar{X}_N \in \left\{\bar{X}_L, \bar{X}_U\right\}$ and population variance is $S_N^2 = \sum_{i=1}^{n_N} \left(X_{Ni} - \bar{X}_N\right)^2 / (n_N - 1)$; $S_N^2 \in \left\{S_L^2, S_U^2\right\}$. From [1] and [31], $S_N^2 \in \left\{S_L^2, S_U^2\right\}$ has approximate neutrosophic normal distribution with neutrosophic mean $\sigma_N^2 \in \left\{\sigma_L^2, \sigma_L^2\right\}$ and neutrosophic variance $2(\sigma_N^2)^2/(n_N - 1)$. Also, $(\mathrm{n_N} - 1)\, S_N^2/\sigma_N^2$; $S_N^2 \in \left\{S_L^2, S_L^2\right\}, \sigma_N^2 \in \left\{\sigma_L^2, \sigma_L^2\right\}$ is modelled by the neutrosophic Chi-Square distribution (χ_N^2; $\chi_N^2 \in \left\{\chi_L^2, \chi_U^2\right\}$) having a neutrosophic degree of freedom (ndf) $\nu_N = n_N - 1$; $n_N \in \{n_L, n_U\}$. Also, $G_N \in \{G_L, G_U\}$ presents the neutrosophic cumulative distribution function (ncdf), see for example, [1].

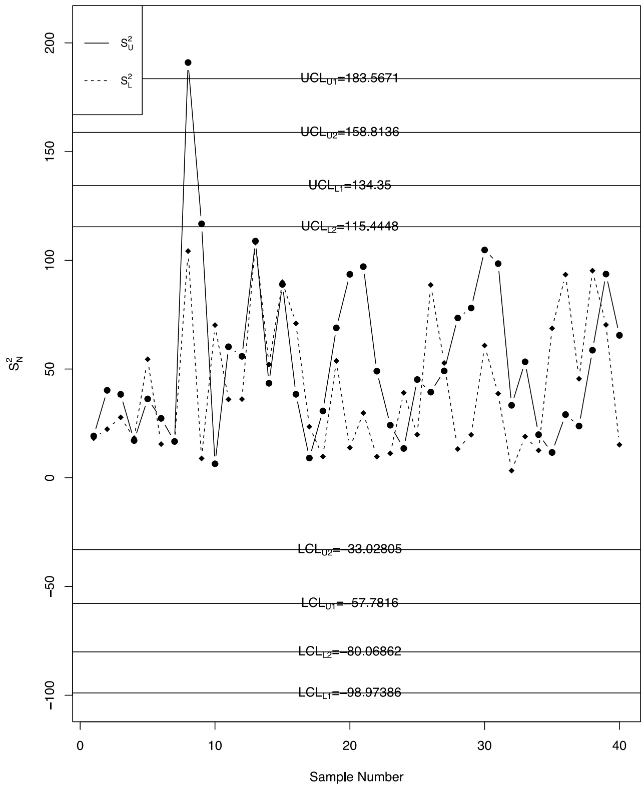

FIGURE 1. The proposed control chart for the simulation data.

III. DESIGN OF THE PROPOSED CHART

Based on the given information, we propose the following control chart for the repetitive sampling under the NSIM.

Step-1: Compute S_N^2 based on NRV of size n_N.

Step-2: Declare the process is an in-control state if $LCL_{N2} \leq S_N^2 \leq UCL_{N2}$; where $LCL_{N2} \in \{LCL_{L2}, LCL_{U2}\}$ and $UCL_{N2} \in \{UCL_{L2}, UCL_{U2}\}$ are neutrosophic upper control limits.

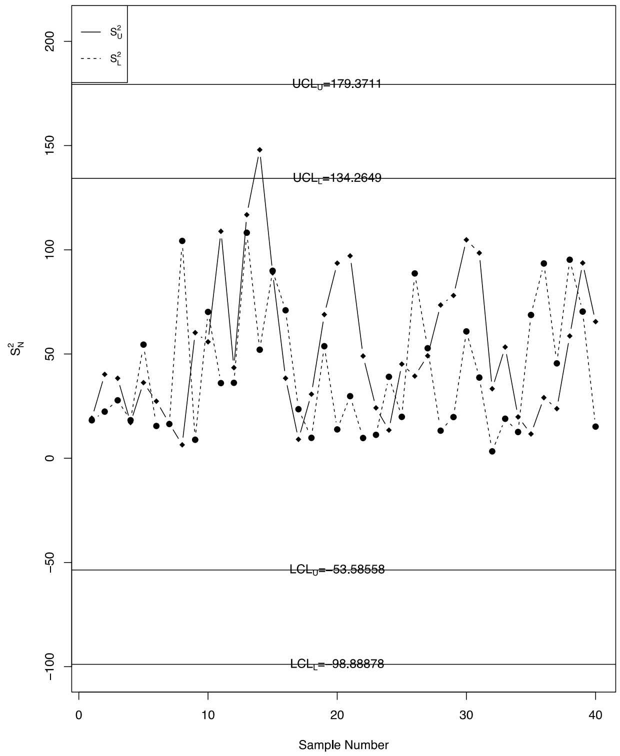

FIGURE 2. The existing chart for the simulated data.

Step-3: The process is said to be out-of-control if $S_N^2 \geq UCL_{N1}$ or $S_N^2 \leq UCL_{N1}$; where $LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\}$ and $UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\}$ are neutrosophic lower control limits, otherwise repeat the process.

The proposed control consists of four control limits with two-neutrosophic control limits coefficient under the NSIM. The proposed chart shows the control limits in indeterminacy interval. Under this complex system, the operational procedure of the proposed control chart is the same as in double sampling and repetitive sampling under classical statistics. Also, the proposed control chart is the generalization of the control chart proposed by [1] under the NSIM and [15] when no certainty in the observations or in the parameters. The neutrosophic control limits $LCL_{N1} \in \{LCL_{L1}, LCL_{U1}\}$, $UCL_{N1} \in \{UCL_{L1}, UCL_{U1}\}$, $LCL_{N2} \in \{LCL_{L2}, LCL_{U2}\}$, $UCL_{N2} \in \{UCL_{L2}, UCL_{U2}\}$ of the proposed control chart are given as

$$LCL_{N1} = \sigma_N^2 - k_{N1}\sqrt{2\left(\sigma_N^2\right)^2/(n_N-1)}; \quad \sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\} \tag{1}$$

$$UCL_{N1} = \sigma_N^2 + k_{N1}\sqrt{2\left(\sigma_N^2\right)^2/(n_N-1)}; \quad \sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\} \tag{2}$$

$$LCL_{N2} = \sigma_N^2 - k_{N2}\sqrt{2\left(\sigma_N^2\right)^2/(n_N-1)}; \quad \sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N2} \in \{k_{L2}, k_{U2}\} \tag{3}$$

$$UCL_{N2} = \sigma_N^2 + k_{N2}\sqrt{2\left(\sigma_N^2\right)^2/(n_N-1)}; \quad \sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N2} \in \{k_{L2}, k_{U2}\} \tag{4}$$

Note here that $k_{N1} \in \{k_{L1}, k_{U1}\}$ and $k_{N2} \in \{k_{L2}, k_{U2}\}$ are the control limits coefficients. The probability when the process is out-of-control using the single sampling under the NSIM is given by

$$\begin{aligned} P(S_N^2 \geq UCL_{N1}) &= 1 - G_N\left(\frac{(n_N-1)\,UCL_{N1}}{\sigma_N^2}\right) \\ &= 1 - G_N\left((n_N-1)\left(1 + k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right); \\ &\sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\}, \quad n_N \in \{n_L, n_U\} \end{aligned} \tag{5}$$

or

$$\begin{aligned} P\left(S_N^2 \leq LCL_{N1}\right) &= G_N\left(\frac{(n_N-1)\,UCL_{N1}}{\sigma_N^2}\right) \\ &= G_N\left((n_N-1)\left(1 - k_{N1}\sqrt{2/(n_N-1)}\right)\right); \\ &\sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\}, \quad n_N \in \{n_L, n_U\} \end{aligned} \tag{6}$$

Suppose that the process is declared to be an in-control state at $\sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}$. Therefore, the probability of out-of-control for the single sampling under the NSIM can be written as follows

$$\begin{aligned} P_{outN1}^0 = 1 - G_N&\left((n_N-1)\left(1 + k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right) \\ &+ G_N\left((n_N-1)\left(1 - k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right); \\ &\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\}, \quad n_N \in \{n_L, n_U\} \end{aligned} \tag{7}$$

The neutrosophic probability of indecision at the first sample is given by

$$\begin{aligned} P_{rept}^{0N} &= P\left(UCL_{N2} < S_N^2 \leq UCL_{N1}|\sigma_N^2\right) \\ &\quad + P\left(LCL_{N1} < S_N^2 \leq LCL_{N2}|\sigma_N^2\right); \\ &\quad \sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\} \end{aligned} \tag{8}$$

The Eq. (8) can be written as follows

$$\begin{aligned} P_{reptN}^0 &= G_N\left((n_N-1)\left(1 + k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right) \\ &\quad - G_N\left((n_N-1)\left(1 + k_{N2}\sqrt{\frac{2}{(n_N-1)}}\right)\right) \end{aligned}$$

TABLE 1. The values of NARL when $n_N \in \{3, 4\}$.

	r_{0N}=300	r_{0N}=370
n_N	[3,4]	[3,4]
k_N	[(5.8737, 3.9607),(5.2171, 5.1036)]	[(6.0840, 5.6889),(5.4234, 4.5386)]
c		
1	[300.93, 300.15]	[371.99, 374.31]
1.1	[166.72, 159.31]	[203.55, 193.75]
1.2	[101.98, 94.51]	[123.48, 112.45]
1.3	[67.31, 61.04]	[81.06, 71.23]
1.4	[47.16, 42.13]	[56.61, 48.31]
1.5	[34.65, 30.66]	[41.53,34.59]
1.6	[26.47, 23.28]	[31.71, 25.89]
1.7	[20.88, 18.30]	[25.02, 20.08]
1.8	[16.92, 14.81]	[20.29, 16.05]
1.9	[14.02, 12.28]	[16.83, 13.16]
2	[11.85, 10.39]	[14.24, 11.02]
3	[4.18, 3.78]	[5.05, 3.76]
4	[2.58, 2.39]	[3.08, 2.32]

TABLE 2. The values of NARL when $n_N \in \{4, 6\}$.

	r_{0N}=300	r_{0N}=370
n_N	[4,6]	[4,6]
k_N	[(5.2314, 3.5838),(4.6238,4.5484)]	[(5.4133,4.5360),(4.7725, 4.6760)]
c		
1	[300.17, 309.25]	[370.21,374.86]
1.1	[157.81, 150.29]	[191.84,178.40]
1.2	[92.64, 83.35]	[111.45,97.23]
1.3	[59.16, 51.10]	[70.65,58.74]
1.4	[40.36, 33.87]	[47.95,38.45]
1.5	[29.02, 23.88]	[34.36,26.81]
1.6	[21.78, 17.69]	[25.72,19.68]
1.7	[16.94, 13.64]	[19.96,15.05]
1.8	[13.56, 10.88]	[15.97,11.91]
1.9	[11.13, 8.92]	[13.09,9.70]
2	[9.33, 7.48]	[10.97,8.09]
3	[3.24, 2.71]	[3.75,2.83]
4	[2.04, 1.78]	[2.32,1.83]

$$+ G_N\left((n_N-1)\left(1-k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right)$$
$$- G_N\left((n_N-1)\left(1-k_{N2}\sqrt{\frac{2}{(n_N-1)}}\right)\right);$$

$$\sigma_N^2 \in \left\{\sigma_L^2, \sigma_U^2\right\}, \quad k_{N1} \in \{k_{L1}, k_{U1}\}, \quad n_N \in \{n_L, n_U\} \tag{9}$$

Finally, the probability of out-of-control for the repetitive sampling under the NSIM is given by

TABLE 3. The values of NARL when $n_N \in \{9, 10\}$.

	r_{0N}=300	r_{0N}=370
n_N	[9,10]	[9,10]
k_N	[(4.1807, 3.4441),(4.0979, 3.4762)]	[(4.3148, 4.2484),(4.2244, 3.3250)]
c		
1	[301.71, 305.75]	[370.62,370.46]
1.1	[131.97, 129.72]	[159.14,153.15]
1.2	[67.48, 64.88]	[80.35,74.88]
1.3	[38.82, 36.73]	[45.85,41.54]
1.4	[24.46, 22.89]	[28.76,25.41]
1.5	[16.57, 15.38]	[19.43,16.79]
1.6	[11.89, 10.97]	[13.94,11.80]
1.7	[8.94, 8.23]	[10.48,8.73]
1.8	[6.99, 6.42]	[8.20,6.74]
1.9	[5.65, 5.19]	[6.63,5.38]
2	[4.69, 4.31]	[5.51,4.43]
3	[1.76, 1.65]	[2.01,1.64]
4	[1.28, 1.23]	[1.40,1.22]

$$P^0_{outN} = \frac{P^0_{outN1}}{1 - P^0_{reptN}}; \quad P^0_{outN} \in \left\{P^0_{outL}, P^0_{outU}\right\} \tag{10}$$

The neutrosophic average run length (NARL) provides the indeterminacy interval in which first out-of-control is expected. The NARL at $\sigma^2_N \in \left\{\sigma^2_L, \sigma^2_U\right\}$ is given by

$$\mathrm{NARL_{0N}} = \frac{1}{P^0_{outN}}; \quad \mathrm{ARL_{0N}} \in \{\mathrm{ARL_{0L}}, \mathrm{ARL_{0U}}\} \tag{11}$$

We suppose that due to some external variations, the process is shifted at new value $\sigma^2_{N1} = c\sigma^2_N$; $\sigma^2_{N1} \in \left\{\sigma^2_{L1}, \sigma^2_{U1}\right\}$, where c shows the shift constant. Therefore, the probability of out-of-control at $\sigma^2_{N1} = c\sigma^2_N$; $\sigma^2_{N1} \in \left\{\sigma^2_{L1}, \sigma^2_{U1}\right\}$ for the single sampling under the NSIM can be written as follows

$$P^1_{outN1} = P\left(S^2_N \geq UCL_{N1}|\sigma^2_{N1}\right) + P\left(S^2_N \leq LCL_{N1}|\sigma^2_{N1}\right); \quad \sigma^2_{N1} \in \left\{\sigma^2_{L1}, \sigma^2_{U1}\right\} \tag{12}$$

or

$$P^1_{outN1} = 1 - G_N\left(\frac{(n_N-1)}{c}\left(1 + k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right) + G_N\left(\frac{(n_N-1)}{c}\left(1 - k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right); \quad k_{N1} \in \{k_{L1}, k_{U1}\}, \quad n_N \in \{n_L, n_U\} \tag{13}$$

The neutrosophic probability of indecision at the first sample is given by

$$P^1_{reptN} = 2\left(G_N\left(\frac{(n_N-1)}{c}\left(1 + k_{N1}\sqrt{\frac{2}{(n_N-1)}}\right)\right) - G_N\left(\frac{(n_N-1)}{c}\left(1 + k_{N2}\sqrt{\frac{2}{(n_N-1)}}\right)\right)\right); \quad k_{N2} \in \{k_{L2}, k_{U2}\}, \quad n_N \in \{n_L, n_U\} \tag{14}$$

Finally, the probability of out-of-control for the repetitive sampling under the NSIM is given by

$$P^1_{outN} = \frac{P^1_{outN1}}{1 - P^1_{reptN}}; \quad P^1_{outN} \in \left\{P^1_{outL}, P^1_{outU}\right\} \tag{15}$$

The NARL at $\sigma^2_{N1} \in \left\{\sigma^2_{L1}, \sigma^2_{U1}\right\}$ is given by

$$\mathrm{NARL_{1N}} = \frac{1}{P^1_{outN}}; \quad \mathrm{ARL_{1N}} \in \{\mathrm{ARL_{1L}}, \mathrm{ARL_{1U}}\} \tag{16}$$

TABLE 4. The comparison in NARL when $n_N \in \{4, 6\}$ and $ARL_{0N} \in \{300, 300\}$.

c	Proposed Chart	Existing Chart
	NARL	NARL
1	[300.17, 309.25]	[300.03,490.72]
1.1	[157.81, 150.29]	[167.75,236.62]
1.2	[92.64, 83.35]	[103.63,130.01]
1.3	[59.16, 51.10]	[69.11,78.91]
1.4	[40.36, 33.87]	[48.93,51.76]
1.5	[29.02, 23.88]	[36.34,36.10]
1.6	[21.78, 17.69]	[28.05,26.46]
1.7	[16.94, 13.64]	[22.35,20.20]
1.8	[13.56, 10.88]	[18.28,15.94]
1.9	[11.13, 8.92]	[15.29,12.93]
2	[9.33, 7.48]	[13.03,10.74]
3	[3.24, 2.71]	[4.84,3.58]
4	[2.049, 1.78]	[3.02,2.21]

Suppose that $\mathrm{NARL_{0N}}$ be specified by r_{0N}. The values of $\mathrm{NARL_{1N}}$ for various combinations of $n_N \in \{n_L, n_U\}$, $\mathrm{ARL_{0N}} \in \{\mathrm{ARL_{0L}}, \mathrm{ARL_{0U}}\}$ and c are presented in Tables 1-3. From Tables 1-3, we note the decreasing trend in the indeterminacy interval of $\mathrm{NARL_{1N}}$ when c changes from 1 to 4. The indeterminacy interval of $\mathrm{NARL_{1N}}$ is increasing when $\mathrm{ARL_{0N}}$ changes from 300 to 370. The indeterminacy interval of $\mathrm{NARL_{1N}}$ is decreasing when $n_N \in \{n_L, n_U\}$ increases.

The following algorithm is used under the NSIM to determine $k_{N1} \in \{k_{L1}, k_{U1}\}, k_{N2} \in \{k_{L2}, k_{U2}\}$ and $\mathrm{ARL_{1N}} \in \{\mathrm{ARL_{1L}}, \mathrm{ARL_{1U}}\}$.

Step-1: Pre-fix the indeterminacy interval of $n_N \in \{n_L, n_U\}$ and shift value c.

Step-2: Specify the suitable ranges for $k_{N1} \in \{k_{L1}, k_{U1}\}$, $k_{N2} \in \{k_{L2}, k_{U2}\}$, where $k_{L1} < k_{U1}$ and $k_{L2} < k_{U2}$. Determine the indeterminacy interval of $k_{N1} \in \{k_{L1}, k_{U1}\}, k_{N2} \in \{k_{L2}, k_{U2}\}$ such that $\mathrm{NARL_{0N}} \geq r_{0N}$.

Step-3: During the simulation, we note that several combinations of $k_{N1} \in \{k_{L1}, k_{U1}\}, k_{N2} \in \{k_{L2}, k_{U2}\}$ are available that satisfy the condition $\mathrm{NARL_{0N}} \geq r_{0N}$.

Step-4: choose that values of $k_{N1} \in \{k_{L1}, k_{U1}\}, k_{N2} \in \{k_{L2}, k_{U2}\}$ where $\mathrm{NARL_{0N}}$ is same or very close to r_{0N}.

Step-5: Determine $\mathrm{ARL_{1N}} \in \{\mathrm{ARL_{1L}}, \mathrm{ARL_{1U}}\}$ for various values of c.

IV. ADVANTAGES OF THE PROPOSED CHART

In this section, we will discuss the advantages of the proposed control chart over the control charts proposed by [1] under the NSIM. We will compare three-control chart in terms of NARL and using the simulation data.

A. COMPARISON IN NARL

We will now compare the proposed chart under the NSIM with chart proposed by [1] under the NSIM in NARL. A control chart having the smaller values of NARL at the same parameters is said to be a more efficient control chart. Let $R_n = \mathrm{ARL_{1U}} - \mathrm{ARL_{1L}}$ be the neutrosophic range of indeterminacy interval of NARL. The smaller the values of R_n better the control chart. The values of NARL of both control charts when $n_N \in \{4, 6\}$, $\mathrm{ARL_{0N}} \in \{300, 300\}$ and $\mathrm{ARL_{0N}} \in \{370, 370\}$ are shown in Tables 4-5. We compare both control chart in the rage of indeterminacy interval of NARL. From Tables 4-5, we note that the proposed control chart under NSIM has smaller values of NARL as compared to [1] control chart. For example, when $n_N \in \{4, 6\}$, $\mathrm{ARL_{0N}} \in \{300, 300\}$ and $c = 1.2$. The values of indeterminacy interval of NARL

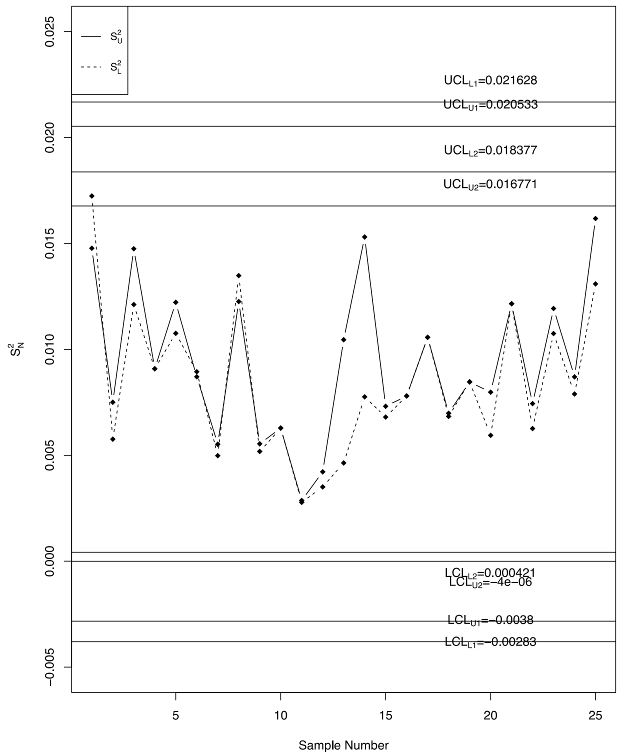

FIGURE 3. The proposed control chart for the real data.

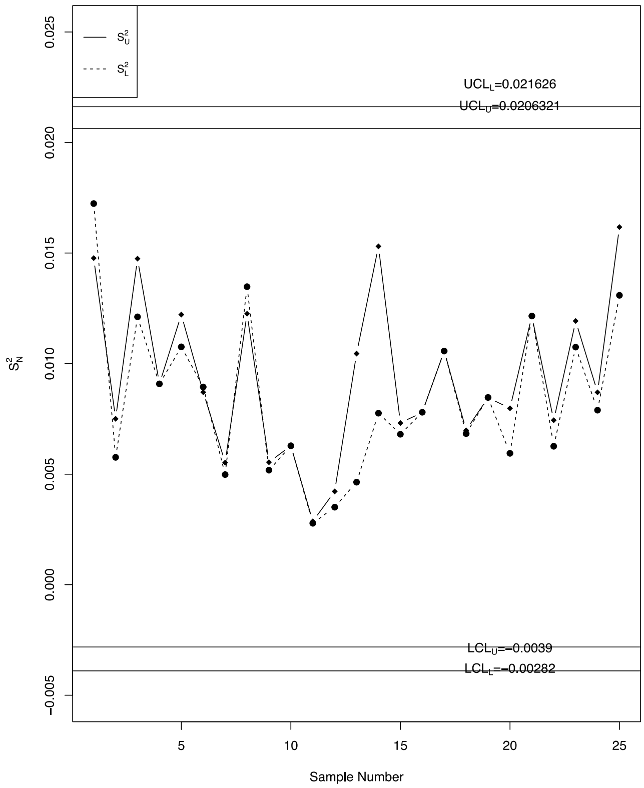

FIGURE 4. the existing control chart for the real data.

and R_n of proposed chart are $\text{ARL}_{1\text{N}} \in \{92.64, 83.35\}$ and $R_n = 9$. On the other hand, values of indeterminacy interval of NARL and R_n of [1] chart are $\text{ARL}_{1\text{N}} \in \{103.63, 130.01\}$ and $R_n = 27$. From these values, we note that the proposed control chart will give the indication in the shift between 83th and 92$^{\text{nd}}$ samples while the existing control chart will provide the single between 103th and 130$^{\text{th}}$ samples. The By comparing the proposed control chart with [1] control chart, we conclude that the proposed control chart is more efficient than the existing control chart in detecting a shift in the process. Therefore, the proposed control chart is more effective than the existing control chart under the NSIM.

B. COMPARISON IN NARL

Now, we compare the efficiency of the proposed control chart under the NSIM over [1] using the simulation data generated from the neutrosophic normal distribution with neutrosophic mean and variance $\mu_N \in \{0, 0\}$ and $\sigma_N^2 \in \{4, 6.25\}$. First 20 observations are generated assuming the process is in-control and next 20 observations are generated for the shifted process when $c = 1.9$. Let $n_N \in \{5, 5\}$ and $\text{ARL}_{0\text{N}} \in \{370, 370\}$ for this study. The values of statistic $S_N^2 \in \left\{S_L^2, S_L^2\right\}$, are computed and plotted on Figure 1. From Figure 1, it can be seen in that the proposed control chart detects shift at the 8$^{\text{th}}$ sample. On Figure 2, the values of statistic $S_N^2 \in \left\{S_L^2, S_L^2\right\}$ are plotted on the existing control chart. From Figure 2, it is noted that the existing control chart detects the first shift in the process at the 14$^{\text{th}}$ sample. By comparing Figure 1 with Figure 2, it is concluded that the proposed chart has the ability to detect an early shift in the process as compared to [1].

V. CASE STUDY

A well-known automobile company in Jeddah, Saudi Arabia is interested to apply the proposed control chart for the monitoring of engine piston rings. The diameter of engine piston rings is the quality of interest which is an important variable, see [37]. A small increase or decrease in the diameter may cause the defective product. The instrument does the measurement of diameter, which causes the uncertainty in the observations. As mentioned above, measurements done by the devices or by human subjectivity may cause to have some uncertain values in the data, see [23]. The data having uncertain observations cannot be monitored effectively using [15] control chart under classical statistics. We borrowed the piston rings from [1] and reported in Table 6 for the easy reference.

The values of $S_N^2 \in \left\{S_L^2, S_L^2\right\}$ are calculated and shown in Table 6. The neutrosophic control limits when $n_N \in \{5, 5\}$, $\text{ARL}_{0\text{N}} \in \{370, 370\}$, $k_{N1} \in \{4.8582, 4.9535\}$, $k_{N1} \in \{3.5665, 3.9374\}$ and $\sigma_N^2 \in \{0.0035, 0.0034\}$ are computed as follows

$$LCL_{L1} = \sigma_L^2 - k_{L1}\sqrt{2\left(\sigma_L^2\right)^2 / (n_N - 1)} = -0.0028$$

TABLE 5. The comparison in NARL when $n_N \in \{4, 6\}$ and $ARL_{0N} \in \{370, 370\}$.

c	Proposed chart	Existing Chart
	NARL	NARL
1	[370.21,374.86]	[370.33, 379.28]
1.1	[191.84,178.40]	[192.50, 180.33]
1.2	[111.45,97.23]	[112.25, 98.22]
1.3	[70.65,58.74]	[71.47, 59.32]
1.4	[47.95,38.45]	[48.73, 38.83]
1.5	[34.36,26.81]	[35.09, 27.08]
1.6	[25.72,19.68]	[26.40, 19.87]
1.7	[19.96,15.05]	[20.60, 15.20]
1.8	[15.97,11.91]	[16.55, 12.04]
1.9	[13.09,9.70]	[13.64, 9.81]
2	[10.97,8.09]	[11.48, 8.18]
3	[3.75,2.83]	[4.04, 2.86]
4	[2.32,1.838]	[2.51, 1.85]

$$UCL_{L1} = \sigma_L^2 + k_{l1}\sqrt{2\left(\sigma_L^2\right)^2 / (n_N - 1)} = 0.0216$$

$$LCL_{L2} = \sigma_L^2 - k_{l2}\sqrt{2\left(\sigma_L^2\right)^2 / (n_N - 1)} = 0.0004$$

$$UCL_{L2} = \sigma_L^2 + k_{l2}\sqrt{2\left(\sigma_L^2\right)^2 / (n_N - 1)} = 0.0183$$

$$LCL_{U1} = \sigma_U^2 - k_{u1}\sqrt{2\left(\sigma_U^2\right)^2 / (n_N - 1)} = -0.0038$$

$$UCL_{U1} = \sigma_U^2 + k_{u1}\sqrt{2\left(\sigma_U^2\right)^2 / (n_N - 1)} = 0.0205$$

$$LCL_{U2} = \sigma_U^2 - k_{u2}\sqrt{2\left(\sigma_U^2\right)^2 / (n_N - 1)} = -0.00004$$

$$UCL_{U2} = \sigma_U^2 + k_{u2}\sqrt{2\left(\sigma_U^2\right)^2 / (n_N - 1)} = 0.0167$$

The values of statistic $S_N^2 \in \{S_L^2, S_L^2\}$ are plotted in Figure 3. From Figure 3, we can note that several points are near the control limits. We also note that the 2nd sample is in indeterminacy interval. The existing chart by [1] in Figure 4 indicate the process is in control. The proposed chart for this real data is implemented as follows

Step-1: Compute $S_N^2 \in [0.014772, 0.017242], \ldots S_N^2 \in [0.016177, 0.013088]$ as in Table 6 based on NRV of size $n_N \in \{5, 5\}$.

Step-2: Declare the process is in-control state if $[0.0004, 0.0028] \leq S_N^2 \leq [0.0183, 0.0216]$.

By comparing Figure 3 with Figure 4, we conclude that the proposed control chart indicates some shifting issues in the process although the process is in control state. We can conclude that the proposed control chart has the ability to detect an early shift in the process as compared to the existing control chart. As the proposed control chart provides results in indeterminacy interval rather than the determined values, therefore, the proposed control chart is more adequate under uncertainty environment.

VI. CONCLUDING REMARKS

In this paper, a control chart for variance using the repetitive sampling under the neutrosophic statistics (NS) is proposed. The proposed control chart is the generalization of several control charts. The proposed control chart using the repetitive sampling is more flexible and allow to repeat the process if the operations are in-decision at first sample information. The proposed control chart provides the indeterminacy interval for the NARL values which is more desirable under the uncertainty. From the comparison, we conclude that the proposed is more sensitive in detecting the shift in the process. The proposed control chart can be used for the monitoring of process in the automobile industry, aerospace industry and food industry. The proposed control chart for the monitoring of big data in marine science can be extended for future research. The proposed chart using the cost model can also be considered as future work. The joint monitoring of mean and variance under NSIM can be considered as future research.

TABLE 6. Real example data.

	Sample Observation					S_N^2
1	[74.03,74.03]	[74.002,73.991]	[74.019,74.019]	[73.992,73.992]	[74.008,74.001]	[0.014772,0.017242]
2	[73.995,73.995]	[73.992,74.003]	[74.001,74.001]	[74.011,74.011]	[74.004,74.004]	[0.007503,0.005762]
3	[73.988,74.017]	[74.024,74.024]	[74.021,74.021]	[74.005,74.005]	[74.002,73.995]	[0.014748,0.012116]
4	[74.002,74.002]	[73.996,73.996]	[73.993,73.993]	[74.015,74.015]	[74.009,74.009]	[0.009083,0.009083]
5	[73.992,73.992]	[74.007,74.007]	[74.015,74.015]	[73.989,73.989]	[74.014,73.998]	[0.012219,0.010756]
6	[74.009,74.009]	[73.994,74.001]	[73.997,73.997]	[73.985,73.985]	[73.993,73.993]	[0.008706,0.008944]
7	[73.995,73.998]	[74.006,74.006]	[73.994,73.994]	[74,74]	[74.005,74.005]	[0.005523,0.00498]
8	[73.985,73.985]	[74.003,74.01]	[73.993,73.993]	[74.015,74.015]	[73.988,73.988]	[0.012256,0.01348]
9	[74.008,74.005]	[73.995,73.995]	[74.009,74.009]	[74.005,74.005]	[74.004,74.004]	[0.005541,0.005177]
10	[73.998,73.998]	[74,74]	[73.99,73.99]	[74.007,74.007]	[73.995,73.995]	[0.006285,0.006285]
11	[73.994,73.998]	[73.998,73.998]	[73.994,73.994]	[73.995,73.995]	[73.99,74.001]	[0.002864,0.002775]
12	[74.004,74.004]	[74,74.002]	[74.007,74.005]	[74,74.001]	[73.996,73.996]	[0.004219,0.003507]
13	[73.983,73.993]	[74.002,74.002]	[73.998,73.998]	[73.997,73.997]	[74.012,74.005]	[0.010455,0.004637]
14	[74.006,74.006]	[73.967,73.985]	[73.994,73.994]	[74,74]	[73.984,73.996]	[0.015304,0.007759]
15	[74.012,74.012]	[74.014,74.012]	[73.998,73.998]	[73.999,73.999]	[74.007,74.007]	[0.007314,0.006804]
16	[74,74]	[73.984,73.984]	[74.005,74.005]	[73.998,73.998]	[73.996,73.996]	[0.007797,0.007797]
17	[73.994,73.994]	[74.012,74.012]	[73.986,73.986]	[74.005,74.005]	[74.007,74.007]	[0.010569,0.010569]
18	[74.006,74.006]	[74.01,74.011]	[74.018,74.018]	[74.003,74.003]	[74,74.001]	[0.006986,0.006834]
19	[73.984,73.984]	[74.002,74.002]	[74.003,74.003]	[74.005,74.005]	[73.997,73.997]	[0.008468,0.008468]
20	[74,74]	[74.01,74.01]	[74.013,74.009]	[74.02,74.015]	[74.003,74.003]	[0.007981,0.005941]
21	[73.982,73.982]	[74.001,74.001]	[74.015,74.015]	[74.005,74.005]	[73.996,73.996]	[0.012153,0.012153]
22	[74.004,74.004]	[73.999,73.999]	[73.99,73.99]	[74.006,74.006]	[74.009,74.002]	[0.007436,0.006261]
23	[74.01,74.01]	[73.989,73.989]	[73.99,73.99]	[74.009,74.005]	[74.014,74.011]	[0.011929,0.010747]
24	[74.015,74.011]	[74.008,74.008]	[73.993,73.993]	[74,74]	[74.01,74.011]	[0.008701,0.007893]
25	[73.982,73.982]	[73.984,73.989]	[73.995,73.995]	[74.017,74.012]	[74.013,74.01]	[0.016177,0.013088]

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MUHAMMAD ASLAM received the M.Sc. degree in statistics from GC University Lahore, in 2004, with the Chief Minister of the Punjab Merit Scholarship, the M.Phil. degree in statistics from GC University Lahore, in 2006, with the Governor of the Punjab Merit Scholarship, and the Ph.D. degree in statistics from the National College of Business Administration and Economics Lahore, in 2010, under the supervision of Prof. Dr. M. Ahmad. He was a Lecturer of Statistics with the Edge College System International, from 2003 to 2006. He was a Research Assistant with the Department of Statistics, GC University Lahore, from 2006 to 2008. Then, he joined the Forman Christian College University, as a Lecturer, in 2009, where he was an Assistant Professor, from 2010 to 2012, and then as an Associate Professor, from 2012 to 2014. He was an Associate Professor of statistics with the Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia, from 2014 to 2017, where he is currently a Full Professor of statistics with the Department of Statistics. He taught summer course as a Visiting Faculty of statistics with Beijing Jiaotong University, China, in 2016. He has published more than 310 research papers in national and international well reputed journals including for example, the IEEE ACCESS, the *Journal of Applied Statistics*, the *European Journal of Operation Research*, *Information Sciences*, the *Journal of Process Control*, the *Journal of the Operational Research Society*, *Applied Mathematical Modeling*, the *The International Journal of Advanced Manufacturing Technology*, *Communications in Statistics*, the *Journal of Testing and Evaluation*, and the *Pakistan Journal of Statistics*. His papers have been cited more than 2400 times with h-index 25 and i-10 index 68 (Google Coalitions). His papers have also been cited more than 1200 times with h-index 20 (Web of Science Coalitions). He has authored one book published in Germany. He has also been HEC approved Ph.D. Supervisor, since 2011. He has supervised five Ph.D. theses, more than 25 M.Phil. theses and three M.Sc. theses. He is currently supervising one Ph.D. thesis and more than five M.Phil. theses in statistics. He is a Reviewer of more than 50 well reputed international journals. He has reviewed more than 140 research papers for various well reputed international journals. His name listed at second position among Statistician in the Directory of Productivity Scientists of Pakistan, in 2013, and also listed at first position, in 2014. He got 371 positions in the list of top 2210 profiles of Scientist of Saudi Institutions, in 2016. He is appointed as an External Examiner for 2016/2017-2018/2019 triennium with The University of Dodoma, Tanzania. His research interests include reliability, decision trees, industrial statistics, acceptance sampling, rank set sampling, neutrosophic statistics, and applied statistics. He is a member of the Islamic Countries Society of Statistical Sciences. He received Meritorious Services Award in Research from the National College of Business Administration and Economics, Lahore, in 2011, and the Research Productivity Award for the year 2012 by the Pakistan Council for Science and Technology. He is selected for the Innovative Academic Research and Dedicated Faculty Award 2017 by SPE, Malaysia. He received the King Abdulaziz University Excellence Awards in Scientific Research for the paper *Aslam, M., Azam, M., Khan, N. and Jun, C.-H. (2015)*. A New Mixed Control Chart to Monitor the Process, *International Journal of Production Research*, 53 (15), 4684–4693. He received the King Abdulaziz University Citation Award for the paper *Azam, M., Aslam, M. and Jun, C.-H. (2015)*. Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling, *International Journal of Advanced Manufacturing Technology*, 77:1927–1933, in 2018. He is a Member of the Editorial Board of the *Electronic Journal of Applied Statistical Analysis*, the *Asian Journal of Applied Science and Technology*, and the *Pakistan Journal of Commence and Social Sciences*.