Paper:

Correlation Coefficients of Refined-Single Valued Neutrosophic Sets and Their Applications in Multiple Attribute Decision-Making

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The paper presents the correlation coefficient of refined-single valued neutrosophic sets (Refined-SVNSs) based on the extension of the correlation of single valued neutrosophic sets (SVNSs), and then a decision making method is proposed by the use of the weighted correlation coefficient of Refined-SVNSs. Through the weighted correlation coefficient between the ideal alternative and each alternative, we can rank all alternatives and the best one of all alternatives can be easily identified as well. Finally, to prove this decision making method proposed in this paper is useful to deal with the actual application, we use an example to illustrate it.

Keywords: refined-SVNSs, correlation coefficients, decision making, SVNSs

1. Introduction

Neutrosophic set (NS) proposed by Smarandache [1] is an important tool to solve multi-criteria decision making problems. Since then, many new extensions about incomplete, uncertain and imprecise information have been presented. For examples, in 2005, Wang et al. [2] introduced the concept of an interval neutrosophic set (INS). The single valued neutrosophic set (SVNS) was introduced for the first time by Smarandache in 1998 in his book [3]; reviewed in [4], which is also mentioned by Wang et al. [5] in 2010. In 2013, Smarandache refined the neutrosophic set: truth value T is refined into types of sub-truths such as T_1, T_2, etc., similarly indeterminacy I is split/refined into types of sub-indeterminacies I_1, I_2, etc., and the sub-falsehood F is split into F_1, F_2, etc. Therefore, Smarandache [6] introduced the concept of a Refined-SVNS. In a decision making problem, if the given criteria have many sub-criteria, we will subdivide these criteria. In 2014, Ye [7] presented a concept of a simplified neutrosophic set (SNS). Now INS, SNS, and SVNS have been developed by many researchers in various fields [8–22]. But there are few studies and researches on the Refined-SVNS. So we propose a decision making method on correlation coefficients of Refined-SVNSs and use a decision making example to prove this method in this paper.

The rest organizations of this paper are as follows. Section 2: briefly introduces NS, SVNS and Refined-SVNS. Section 3: introduces the Correlation Coefficients and the Correlation Coefficients of Refined-SVNS. Section 4: gives a decision making method based on the weighted correlation coefficient measures of Refined-SVNSs. Section 5: presents an example with Refined-SVNS to illustrate the proposed methods and gives a conclusion. Finally, Section 6: concludes.

2. Some Concepts of NS, SVNS, and Refined-SVNS

Definition 1 [1]. Set X be a universe of discourse, with a generic element denoted x in X. Then a NS is defined as:

$$A=\left\{\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle \middle| x\in X\right\},$$

in which $T_A(x): X \to]^-0,1^+[$ means truth membership function, $I_A(x): X \to]^-0,1^+[$ means indeterminacy membership and $F_A(x): X \to]^-0,1^+[$ means falsity membership function. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$ and there is no relation on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $^-0 \le \sup T_A(x)+\sup I_A(x)+\sup F_A(x) \le 3^+$.

Obviously, just using this Definition 1, we cannot apply the neutrosophic set to deal with the practical problems. Therefore, Smarandache [3] introduced the concept of a SVNS, which is an extension of NS.

Definition 2 [3]. Set X be a universe of discourse, with a generic element denoted x in X. Then a SVNS is defined as:

$$A=\left\{\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle \middle| x\in X\right\},$$

in which $T_A(x),I_A(x),F_A(x)\in[0,1]$, $0\le T_A(x)+I_A(x)+F_A(x)\le 3$.

When we deal the practical problem, the given criteria maybe have many sub-criteria, we should subdivide these criteria. Therefore, Smarandache [6] introduced the concept of a Refined-SVNS.

Definition 3 [6]. Set X be a universe of discourse, with a generic element denoted x in X. Then a Refined-SVNS is defined as:

$$A=\Big\{\langle x,\big(T_{1A}(x),T_{2A}(x),\ldots,T_{kA}(x)\big),\\ \big(I_{1A}(x),I_{2A}(x),\ldots,I_{kA}(x)\big),\\ \big(F_{1A}(x),F_{2A}(x),\ldots,F_{kA}(x)\big)\rangle\big|x\in X\Big\}$$

here k is a positive integer, $T_{1A}(x),T_{2A}(x),\ldots,T_{kA}(x)\in[0,1]$, $I_{1A}(x),I_{2A}(x),\ldots,I_{kA}(x)\in[0,1]$, $F_{1A}(x),F_{2A}(x),\ldots,F_{kA}(x)\in[0,1]$, and $0\le T_{iA}(x)+I_{iA}(x)+F_{iA}(x)\le 3$ for $i=1,2,\ldots,k$.

Definition 4 [14]. Set X be a universe of discourse, and L and M be two SVNSs, $L=\{\langle x,T_L(x_i),I_L(x_i),F_L(x_i)\rangle\mid x_i\in X\}$ and $M=\{\langle x,T_M(x_i),I_M(x_i),F_M(x_i)\rangle\mid x_i\in X\}$. The correlation coefficients measure of two SVNSs L and M is:

$$N(L,M)=\frac{C(L,M)}{\max\{C(L,L),C(M,M)\}}\\ =\sum_{k=1}^{n}\Big[T_L(x_i)\cdot T_M(x_i)+I_L(x_i)\cdot I_M(x_i)+F_L(x_i)\cdot F_M(x_i)\Big]\Big/\\ \max\Big\{\sum_{k=1}^{n}\Big[T_L^2(x_i)+I_L^2(x_i)+F_L^2(x_i)\Big],\\ \sum_{k=1}^{n}\Big[T_M^2(x_i)+I_M^2(x_i)+F_M^2(x_i)\Big]\Big\}.\quad\ldots\ldots(1)$$

Theorem 1. The correlation coefficients measure $N(L,M)$ satisfies the following properties (1)–(3) [14]:

(1) $0\le N(L,M)\le 1$;

(2) $N(L,M)=1$ if and only if $L=M$;

(3) $N(L,M)=N(M,L)$;

Their proofs can be consulted in [14].

3. Correlation Coefficients Measure Methods of Refined-SVNSs

Definition 5. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a universe of discourse, and L and M be two Refined-SVNSs,

$$L=\{\langle x_i,(T_{1L}(x_i),T_{2L}(x_i),\ldots,T_{k_iL}(x_i)),\\ (I_{1L}(x_i),I_{2L}(x_i),\ldots,I_{k_iL}(x_i)),\\ (F_{1L}(x_i),F_{2L}(x_i),\ldots,F_{k_iL}(x_i))\rangle|x_i\in X\},\\ M=\{\langle x_i,(T_{1M}(x_i),T_{2M}(x_i),\ldots,T_{k_iM}(x_i)),\\ (I_{1M}(x_i),I_{2M}(x_i),\ldots,I_{k_iM}(x_i)),\\ (F_{1M}(x_i),F_{2M}(x_i),\ldots,F_{k_iM}(x_i))\rangle|x_i\in X\}.$$

Here k_i is a positive integer, and all $T_{jL}(x_i),I_{jL}(x_i),F_{jL}(x_i)$ and $T_{jM}(x_i)$, $I_{jM}(x_i)$, $F_{jM}(x_i)\in[0,1](i=1,2,\ldots,n;j=1,2,\ldots,k_i)$.

As an extension of Definition 4, we present a correlation coefficients measure between two Refined-SVNSs L and M as follows:

$$N(L,M)=\frac{C(L,M)}{\max\{C(L,L),C(M,M)\}}\\ =\sum_{i=1}^{n}\sum_{j=1}^{k_i}\big[T_{jL}(x_i)\cdot T_{jM}(x_i)+I_{jL}(x_i)\cdot I_{jM}(x_i)\\ +F_{jL}(x_i)\cdot T_{jM}(x_i)\big]/k_i\Big/\\ \max\Big\{\sum_{i=1}^{n}\sum_{j=1}^{k_i}\big[T_{jL}{}^2(x_i)+I_{jL}{}^2(x_i)+F_{jL}{}^2(x_i)\big]/k_i,\\ \sum_{i=1}^{n}\sum_{j=1}^{k_i}\big[T_{jM}{}^2(x_i)+I_{jM}{}^2(x_i)+F_{jM}{}^2(x_i)\big]/k_i\Big\}\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Theorem 1. The correlation coefficients measure $N(L,M)$ between two Refined-SVNSs L and M satisfies the following properties:

(1) $N(L,M)=N(M,L)$;

(2) $0\le N(L,M)\le 1$;

(3) $N(L,M)=1$ if and only if $L=M$;

Proof.
(1) For $T_{jL}(x_i)\cdot T_{jM}(x_i)+I_{jL}(x_i)\cdot I_{jM}(x_i)+F_{jL}(x_i)\cdot F_{jM}(x_i)=T_{jM}(x_i)\cdot T_{jL}(x_i)+I_{jM}(x_i)\cdot I_{jL}(x_i)+F_{jM}(x_i)\cdot F_{jL}(x_i)$, so we can get $N(L,M)=N(M,L)$.

(2) For $0\le T_{jM}(x_i)\le 1$ then $0\le T_{jL}(x_i)\cdot T_{jM}(x_i)\le 1$, $0\le I_{jM}(x_i)\le 1$ then $0\le I_{jL}(x_i)\cdot I_{jM}(x_i)\le 1$ and $0\le F_{jM}(x_i)\le 1$ then $0\le F_{jL}(x_i)\cdot F_{jM}(x_i)\le 1$, so we can get $N(L,M)\ge 0$. Next, we prove $N(L,M)\le 1$;

$$C(L,M)\\ =\sum_{i=1}^{n}\sum_{j=1}^{k_i}\big[T_{jL}(x_i)\cdot T_{jM}(x_i)+I_{jL}(x_i)\cdot I_{jM}(x_i)\\ +F_{jL}(x_i)\cdot F_{jM}(x_i)\big]/k_i\\ =\frac{1}{k_1}\big[(T_{1L}(x_1)\cdot T_{1M}(x_1)+I_{1L}(x_1)\cdot I_{1M}(x_1)\\ +F_{1L}(x_1)\cdot F_{1M}(x_1))+\cdots+(T_{k_1L}(x_1)\cdot T_{k_1M}(x_1)\\ +I_{k_1L}(x_1)\cdot I_{k_1M}(x_1)+F_{k_1L}(x_1)\cdot F_{k_1M}(x_1))\big]\\ +\cdots+\frac{1}{k_n}\big[(T_{1L}(x_n)\cdot T_{1M}(x_n)+I_{1L}(x_n)\cdot I_{1M}(x_n)\\ +F_{1L}(x_n)\cdot F_{1M}(x_n))+\cdots+(T_{k_nL}(x_n)\cdot T_{k_nM}(x_n)\\ +I_{k_nL}(x_n)\cdot I_{k_nM}(x_n)+F_{k_nL}(x_n)\cdot F_{k_nM}(x_n))\big]$$

According to the Cauchy-Schwarz inequality:

$$(\alpha_1\beta_1+\alpha_2\beta_2+\cdots+\alpha_n\beta_n)^2\\ \le(\alpha_1{}^2+\alpha_2{}^2+\cdots+\alpha_n{}^2)(\beta_1{}^2+\beta_2{}^2+\cdots+\beta_n{}^2)$$

Where $\alpha_1,\alpha_2,\ldots,\alpha_n\in R^n$ and $\beta_1,\beta_2,\ldots,\beta_n\in R^n$, we can

Table 1. The Refined-SVNS decision matrix D.

	$\boldsymbol{C_1(C_{11}, C_{12}, \ldots, C_{1k_1})}$	…	$\boldsymbol{C_n(C_{n1}, C_{n2}, \ldots, C_{nk_n})}$
A_1	$\langle(T_{1A_1}, T_{2A_1}, \ldots, T_{k_1A_1}), (I_{1A_1}, I_{2A_1}, \ldots, I_{k_1A_1}), (F_{1A_1}, F_{2A_1}, \ldots, F_{k_1A_1})\rangle$	…	$\langle(T_{1A_1}, T_{2A_1}, \ldots, T_{k_nA_1}), (I_{1A_1}, I_{2A_1}, \ldots, I_{k_nA_1}), (F_{1A_1}, F_{2A_1}, \ldots, F_{k_nA_1})\rangle$
A_2	$\langle(T_{1A_2}, T_{2A_2}, \ldots, T_{k_1A_2}), (I_{1A_2}, I_{2A_2}, \ldots, I_{k_1A_2}), (F_{1A_2}, F_{2A_2}, \ldots, F_{k_1A_2})\rangle$	…	$\langle(T_{1A_2}, T_{2A_2}, \ldots, T_{k_nA_2}), (I_{1A_2}, I_{2A_2}, \ldots, I_{k_nA_2}), (F_{1A_2}, F_{2A_2}, \ldots, F_{k_nA_2})\rangle$
…	…	…	…
A_m	$\langle(T_{1A_m}, T_{2A_m}, \ldots, T_{k_1A_m}), (I_{1A_m}, I_{2A_m}, \ldots, I_{k_1A_m}), (F_{1A_m}, F_{2A_m}, \ldots, F_{k_1A_m})$	…	$\langle(T_{1A_1}, T_{2A_1}, \ldots, T_{k_nA_m}), (I_{1A_1}, I_{2A_1}, \ldots, I_{k_nA_m}), (F_{1A_1}, F_{2A_m}, \ldots, F_{k_nA_m})\rangle$

get

$$
\begin{aligned}
C(L,M)^2 \le \frac{1}{k_1{}^2}\Big[&\big(T_{1L}{}^2(x_1)+I_{1L}{}^2(x_1)+F_{1L}{}^2(x_1)\big)\cdot\\
&\big(T_{1M}{}^2(x_1)+I_{1M}{}^2(x_1)+F_{1M}{}^2(x_1)\big)+\cdots\\
&+\big(T_{k_1L}{}^2(x_1)+I_{k_1L}{}^2(x_1)+F_{k_1L}{}^2(x_1)\big)\cdot\\
&\big(T_{k_1M}{}^2(x_1)+I_{k_1M}{}^2(x_1)+F_{k_1M}{}^2(x_1)\big)\Big]+\cdots\\
&+\frac{1}{k_n{}^2}\Big[\big(T_{1L}{}^2(x_1)+I_{1L}{}^2(x_1)+F_{1L}{}^2(x_1)\big)\cdot\\
&\big(T_{1M}{}^2(x_1)+I_{1M}{}^2(x_1)+F_{1M}{}^2(x_1)\big)+\cdots\\
&+\big(T_{k_nL}{}^2(x_1)+I_{k_nL}{}^2(x_1)+F_{k_nL}{}^2(x_1)\big)\cdot\\
&\big(T_{k_nM}{}^2(x_1)+I_{k_nM}{}^2(x_1)+F_{k_nM}{}^2(x_1)\big)\Big]\\
&=C(L,L)C(M,M)
\end{aligned}
$$

For $C(L,M)^2 \le C(L,L)C(M,M)$, just $C(L,M) \le C(L,L)^{1/2}C(M,M)^{1/2}$.
Then, $C(L,M) \le \max\{C(L,L), C(M,M)\}$.
So we can get $N(L,M) = C(L,M) / \max\{C(L,L), C(M,M)\} \le 1$.

(3) If $L = M$ then $T_{jL}(x_i) = T_{jM}(x_i) I_{jL}(x_i) = I_{jM}(x_i)$, and $F_{jL}(x_i) = F_{jM}(x_i)$ for any $x_i \in X$ and $i = 1, 2, \ldots, n$, so we can get $N(L,M) = 1$, if and only if $L = M$.

Usually, all attributes have weights, maybe these weights are at the same values or different, but they are all belong 0 to 1 and the total value of them is 1. Now, we assume that the weight of each attribute C_i $(i = 1, 2, \ldots, n)$ is w_i. Then, we can introduce the weighted correlation coefficients measure between two Refined-SVNSs L and M as follow:

$$
\begin{aligned}
&W(L,M)\\
&=\sum_{i=1}^{n} w_i \sum_{j=1}^{k_i}\Big[\big(T_{jL}(x_i)\cdot T_{jM}(x_i)+I_{jL}(x_i)\cdot I_{jM}(x_i)\\
&\qquad +F_{jL}(x_i)\cdot F_{jM}(x_i)\big)\Big]/k_i \Big/\\
&\max\left\{\sum_{i=1}^{n} w_i \sum_{j=1}^{k_i}\left[T_{jL}{}^2(x_i)+I_{jL}{}^2(x_i)+F_{jL}{}^2(x_i)\right]/k_i,\right.\\
&\left.\sum_{i=1}^{n} w_i \sum_{j=1}^{k_i}\left[T_{jM}{}^2(x_i)+I_{jM}{}^2(x_i)+F_{jM}{}^2(x_i)\right]/k_i\right\}
\end{aligned} \tag{3}
$$

4. Building a Decision-Making Model Using the Correlation Coefficients

In a decision-making problem, there are a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$ and a set of attributes $C = \{C_1, C_2, \ldots, C_n\}$. Sometimes C_i $(i = 1, 2, \ldots, n)$ may be subdivided into some sub-attribute C_{ij} $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, k_i)$, then we can use a Refined-SVNS to express it:

$$
\begin{aligned}
A_r = \Big\{\big\langle C_i, &(T_{1A_r}(C_i), T_{2A_r}(C_i), \ldots, T_{k_iA_r}(C_i)),\\
&(I_{1A_r}(C_i), I_{2A_r}(C_i), \ldots, I_{k_iA_r}(C_i)),\\
&(F_{1A_r}(C_i), F_{2A_r}(C_i), \ldots, F_{k_iA_r}(C_i))\big\rangle \big| C_i \in C\Big\},\\
&r = 1, 2, \ldots, m \text{ and } i = 1, 2, \ldots, n.
\end{aligned} \tag{4}
$$

We could use a Refined-SVNS to denote the values of the three functions $T_{k_iS_r}(G_i)$, $I_{k_iS_r}(G_i)$, $F_{k_iS_r}(G_i)$ for convenience, so we establish the Refined-SVNS decision matrix D, which is shown in **Table 1**.

Step 1: Based on the Refined-SVNS decision matrix D, we can get the ideal solution (ideal Refined-SVNS) A_i^*.

When the attributes are benefit, A_i^* is shown as follows:

$$
\begin{aligned}
A_i^* = \Big\langle &\big(T_{1A_m}{}^{max}, T_{2A_m}{}^{max}, \ldots, T_{k_iA_m}{}^{max}\big),\\
&\big(I_{1A_m}{}^{max}, I_{2A_m}{}^{max}, \ldots, I_{k_iA_m}{}^{max}\big),\\
&\big(F_{1A_m}{}^{max}, F_{2A_m}{}^{max}, \ldots, F_{k_iA_m}{}^{max}\big)\Big\rangle,\\
&\text{for } i = 1, 2, \ldots, n.
\end{aligned} \tag{5}
$$

Table 2. The Refined-SVNS decision matrix D for four alternatives on three attributes/seven sub-attributes.

	$C_1(C_{11},C_{12})$	$C_2(C_{21},C_{22},C_{23})$	$C_3(C_{31},C_{32})$
A_1	$\langle(0.6,0.7),(0.2,0.1),(0.2,0.3)\rangle$	$\langle(0.9,0.7,0.8),(0.1,0.3,0.2),(0.2,0.2,0.1)\rangle$	$\langle(0.6,0.8),(0.3,0.2),(0.3,0.4)\rangle$
A_2	$\langle(0.8,0.7),(0.1,0.2),(0.3,0.2)\rangle$	$\langle(0.7,0.8,0.7),(0.2,0.4,0.3),(0.1,0.2,0.1)\rangle$	$\langle(0.8,0.8),(0.1,0.2),(0.1,0.2)\rangle$
A_3	$\langle(0.6,0.8),(0.1,0.3),(0.3,0.4)\rangle$	$\langle(0.8,0.6,0.7),(0.3,0.1,0.1),(0.2,0.1,0.2)\rangle$	$\langle(0.8,0.7),(0.4,0.3),(0.2,0.1)\rangle$
A_4	$\langle(0.7,0.6),(0.1,0.2),(0.2,0.3)\rangle$	$\langle(0.7,0.8,0.7),(0.2,0.2,0.1),(0.1,0.2,0.2)\rangle$	$\langle(0.7,0.7),(0.2,0.3),(0.2,0.3)\rangle$

When the attributes are cost, A_i^* is shown as follows:

$$A_i^* = \Big\langle \big(T_{1A_m}{}^{min}, T_{2A_m}{}^{min}, \dots, T_{k_iA_m}{}^{min}\big), \big(I_{1A_m}{}^{min}, I_{2A_m}{}^{min}, \dots, I_{k_iA_m}{}^{min}\big), \big(F_{1A_m}{}^{min}, F_{2A_m}{}^{min}, \dots, F_{k_iA_m}{}^{min}\big)\Big\rangle, \text{ for } i = 1,2,\dots,n. \quad (6)$$

So we can get the ideal alternative $A^* = \{A_1^*, A_2^*, \dots, A_n^*\}$.

Step 2: When the weights of attributes are given by $w = (w_1, w_2, \dots, w_n)$ with $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$. The correlation coefficients measure between the ideal alternative A^* and each alternative A_r $(r = 1,2,\dots,m)$ can be calculated according to Eqs. (4) and (5). Then we can obtain the values of $W(A_r, A^*)$ for $r = 1,2,\dots,m$.

Step 3: According the values of $W(A_r, A^*)$ for $r = 1,2,\dots,m$, all alternatives can be ranked in a descending order and the alternative of biggest $W(A_r, A^*)$ value just is the best choice.

Step 4: End.

5. Illustrative Examples

In this section, we give two examples with multiple attribute to demonstrate the application of the proposed method in this paper.

5.1. Example 1

Now, we discuss the decision-making problem adapted from [22]. A construction company wants to determine the selecting problem of construction projects. Now four construction projects are provided by decision makers, then we can get a set of four alternatives $A = A_1, A_2, A_3, A_4$. Then, in these construction projects, which one can be selected dependent on three main attributes. These attributes are financial state (C_1), environmental protection (C_2) and technology (C_3), and at the same time these attributes can be divided into seven sub-attributes: budget control (C_{11}) and risk/return ratio (C_{12}); public relation (C_{21}), geographical location (C_{22}), and health and safety (C_{23}); technical knowhow (C_{31}) and technological capability (C_{32}). Then, decision makers evaluate the value of the four possible alternatives under the above attributes by suitability judgments. With these values we can construct the Refined-SVNS decision matrix D, which is shown in **Table 2**.

For these attributes are benefit, so we can obtain the ideal alternative A^* by using Eq. (4) from the Refined-SVNS decision matrix D.

$$A^* = \{\langle(0.8,0.8),(0.1,0.1),(0.2,0.2)\rangle, \langle(0.9,0.8,0.8),(0.1,0.1,0.1),(0.1,0.1,0.1)\rangle, \langle(0.8,0.8),(0.1,0.2),(0.1,0.1)\rangle\}.$$

With the weight vector of the three attributes by $w = (0.4, 0.3, 0.3)$ on the opinion of the experts and Eq. (3), we can obtain the weighted correlation coefficients measure values between the ideal alternation A^* and each alternative A_r $(r = 1,2,3,4)$, the measure values are listed as follows:

$$W(A_1,A^*) = 0.9156,\ W(A_2,A^*) = 0.9603,$$
$$W(A_3,A^*) = 0.9308, \text{ and } W(A_4,A^*) = 0.8861.$$

Because of the measure values are $W(A_2,A^*) > W(A_3,A^*) > W(A_1,A^*) > W(A_4,A^*)$, the ranking order just is $A_2 \succ A_3 \succ A_1 \succ A_4$. Therefore, we can get the alternative A_2 as the best choice among all alternatives.

Comparing with the method of [22], the correlation coefficients measure between two Refined-SVNSs proposed in this paper is relatively simpler and easier, and we can obtain the same choice as in [22] through the weighted correlation coefficients measure values between the ideal alternation A^* and each alternative A_r $(r = 1,2,3,4)$.

5.2. Example 2

A university wants to rank the academy with some main attributes. Now there are five academies will be ranked, then we can get a set of five academics $A = \{A_1, A_2, A_3, A_4, A_5\}$. The rank of these academics depends on three main attributes and seven sub-attributes: (1) Teaching (C_1): teaching conditions (C_{11}), teachers troop (C_{12}) and teaching level (C_{13}); (2) The scientific research (C_2): teachers' scientific research (C_{21}), students' scientific research (C_{22}); (3) Server (C_3): social reputation (C_{31}), the employment situation (C_{32}).

Experts evaluate the value of the five academics under the above attributes by some data. With these values we can construct the Refined-SVNS decision matrix, which is shown in **Table 3**.

For these attributes are benefit, so we can obtain the ideal alternative $A*$ by using formula (4) from the

Table 3. The Refined-SVNS decision matrix D for five academics on three attributes/seven sub-attributes.

	$C_1(C_{11}, C_{12}, C_{13})$	$C_2(C_{21}, C_{22})$	$C_3(C_{31}, C_{32})$
A_1	$\langle(0.8, 0.9, 0.7), (0.2, 0.3, 0.2), (0.1, 0.1, 0.2)\rangle$	$\langle(0.9, 0.7), (0.2, 0.1), (0.2, 0.3)\rangle$	$\langle(0.8, 0.9), (0.1, 0.2), (0.2, 0.1)\rangle$
A_2	$\langle(0.7, 0.8, 0.7), (0.2, 0.4, 0.3), (0.3, 0.2, 0.2)\rangle$	$\langle(0.8, 0.5), (0.1, 0.2), (0.3, 0.4)\rangle$	$\langle(0.8, 0.8), (0.1, 0.2), (0.1, 0.2)\rangle$
A_3	$\langle(0.7, 0.6, 0.8), (0.2, 0.2, 0.1), (0.2, 0.3, 0.2)\rangle$	$\langle(0.6, 0.4), (0.1, 0.3), (0.4, 0.5)\rangle$	$\langle(0.8, 0.9), (0.3, 0.2), (0.2, 0.1)\rangle$
A_4	$\langle(0.8, 0.8, 0.7), (0.2, 0.2, 0.1), (0.1, 0.2, 0.2)\rangle$	$\langle(0.7, 0.6), (0.1, 0.2), (0.2, 0.3)\rangle$	$\langle(0.8, 0.7), (0.2, 0.3), (0.2, 0.3)\rangle$
A_5	$\langle(0.8, 0.7, 0.8), (0.2, 0.2, 0.1), (0.2, 0.2, 0.2)\rangle$	$\langle(0.6, 0.5), (0.2, 0.2), (0.3, 0.5)\rangle$	$\langle(0.8, 0.8), (0.2, 0.2), (0.1, 0.2)\rangle$

Refined-SVNS decision matrix D.

$$A^* = \{\langle(0.8,0.9,0.8),(0.2,0.2,0.1),(0.1,0.1,0.2)\rangle, \langle(0.9,0.7),(0.1,0.1),(0.2,0.3)\rangle, \langle(0.8,0.9),(0.1,0.2),(0.1,0.1)\rangle\}.$$

$$W(A_1,A^*) = 0.9950,\ W(A_2,A^*) = 0.9207,$$
$$W(A_3,A^*) = 0.8779,\ W(A_4,A^*) = 0.8987,$$
$$W(A_5,A^*) = 0.8947.$$

So, the ranking of five academics is $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$. Therefore, the academic named A_1 is the best one of all evaluated academics.

From above two examples, we can see that the correlation coefficients of refined-neutrosophic set can be used in actual engineering and scientific applications to help people to do some decision problems.

6. Conclusions

We presented the correlation coefficients measure of Refined-SVNSs in this paper and we use this method to deal with two actual decision-making applications. Through the correlation coefficients measure between the ideal alternative and each alternative, the ranking order of all alternatives can be got and the best alternative can be selected as well. Finally, the ranking order in the first example with correlation coefficients measure is probably agree with the ranking results of [22], the second example with correlation coefficients measure can get the rank of five evaluated academics, so the method proposed in this paper is suitable for actual applications in decision-making problems with Refined-SVNS. In the future, we shall go on studying the correlation coefficients measure between Refined-SVNSs and extending the proposed decision-making method to many other fields.

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