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Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components

R.Jansi[1], K.Mohana[2] and Florentin Smarandache[3]
[1] Research Scholar, [2]Assistant Professor,
[1, 2] Department of Mathematics, Nirmala College for Women, Coimbatore.
[3] Department of Mathematics, University of Mexico, USA.
Email ID: mathematicsgasc@gmail.com[1], riyaraju1116@gmail.com[2], fsmarandache@gmail.com[3].

Abstract: In this paper, we study the new concept of Pythagorean neutrosophic set with T and F as dependent neutrosophic components [PNS]. Pythagorean neutrosophic set with T and F as dependent neutrosophic components [PNS] is introduced as a generalization of neutrosophic set (In neutrosophic sets, there are three special cases, here we take one of the special cases. That is, membership and non-membership degrees are dependent components and indeterminacy is independent) and Pythagorean fuzzy set. In PNS sets, membership, non-membership and indeterminacy degrees are gratifying the condition $0 \leq \left(u_A(x)\right)^2 + \left(\zeta_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 2$ instead of $u_A(x) + \zeta_A(x) + v_A(x) > 2$ as in neutrosophic sets. We investigate the basic operations of PNS sets. Also, the correlation measure of PNS set is proposed and proves some of their basic properties. The concept of this correlation measures of PNS set is the extension of correlation measures of Pythagorean fuzzy set and neutrosophic set. Then, using correlation of PNS set measure, the application of medical diagnosis is given.

Keywords: Pythagorean fuzzy set, Pythagorean Neutrosophic set with T and F as dependent neutrosophic components [PNS], Correlation measure and Medical diagnosis.

Introduction

Fuzzy sets were firstly initiated by L.A.Zadeh [36] in 1965. Zadeh's idea of fuzzy set evolved as a new tool having the ability to deal with uncertainties in real-life problems and discussed only membership function. After the extensions of fuzzy set theory Atanassov [7] generalized this concept and introduced a new set called intuitionistic fuzzy set (IFS) in 1986, which can be describe the non-membership grade of an imprecise event along with its membership grade under a restriction that the sum of both membership and non-membership grades does not exceed 1. IFS has its greatest use in practical multiple attribute decision making problems.In some practical problems.In some practical problems, the sum of membership and non-membership degree to which an alternative satisfying attribute provided by decision maker(DM) may be bigger than 1.

Yager [30] was decided to introduce the new concept known as Pythagorean fuzzy sets. Pythagorean fuzzy sets has limitation that their square sum is less than or equal to 1. IFS was failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [22] in 1995 introduced new concept known as neutrosophic set(NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. A neutrosophic set includes truth membership, falsity membership and indeterminacy membership.

In 2006, F.Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [22]. In neutrosophic set [22], if truth membership and falsity membership are 100% dependent and indeterminacy is 100% independent, that is $0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 2$. Sometimes in real life, we face many problems which cannot be handled by using neutrosophic for example when $u_A(x) + \zeta_A(x) + v_A(x) > 2$. In such condition, a neutrosophic set has no ability to obtain any satisfactory result. To state this condition, we give an example: the truth membership, falsity membership and indeterminacy values are $\frac{8}{10}, \frac{5}{10}$ and $\frac{9}{10}$ respectively. This satisfies the condition that their sums exceeds 2 and are not presented to neutrosophic set. So, In Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] of condition is as their square sum does not exceeds 2. Here, T and F are dependent neutrosophic components and we make $u_A(x), v_A(x) as$ Pythagorean, then $(u_A(x))^2 + (v_A(x))^2 \leq 1$ with $u_A(x), v_A(x)$ in [0,1]. If $\zeta_A(x)$ is an Independent from them, then $0 \leq \zeta_A(x) \leq 1$. Then $0 \leq (u_A(x))^2 + (\zeta_A(x))^2 + (v_A(x))^2 \leq 2$, with $u_A(x), \zeta_A(x), v_A(x)$ in [0,1]. We consider in general the degree of dependence between $u_A(x), \zeta_A(x), v_A(x)$ is 1 , hence $u_A(x), \zeta_A(x), v_A(x) \leq 3 - 1 = 2$.

Correlation coefficients are beneficial tools used to determine the degree of similarity between objects. The importance of correlation coefficients in fuzzy environments lies in the fact that these types of tools can feasibly be applied to problems of pattern recognition, MADM, medical diagnosis and clustering, etc. In other research, Ye[33] proposed three vector similarity measure for SNSs, an instance of SVNS and INS, includingthe Jaccard, Dice, and cosine similarity measures for SVNS and INSs, and applied them to multi-criteria decision-making problems with simplified neutrosophic information. Hanafy et al. [16] proposed the correlation coefficients of neutrosophic sets and studied some of their basic properties. Based on centroid method, Hanafy et al. [17], introduced and studied the concepts of correlation and correlation coefficient of neutrosophic sets and studied some of their properties.

Recently Bromi and Smarandache defined the Haudroff distance between neutrosophic sets and some similarity measures based on the distance such as; set theoretic approach and matching function to calculate the similarity degree between neutrosophic sets. In the same year, Broumi and Smarandache [11] also proposed the correlation coefficient between interval neutrosphic sets.

In this paper, we have to study the concept of Pythagorean neutrosophic set with T and F are neutrosophic components and also define the correlation measure of Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] and prove some of its properties. Then, using correlation of Pythagorean neutrosophic fuzzy set with T and F are dependent neutrosophic components [PNS] measure, the application of medical diagnosis is given.

Preliminaries

Definition 2.1 [1] Let E be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{< x, u_A(x), v_A(x) >: x \in E\}$$

Where $u_A: E \to [0,1]$ and $v_A: E \to [0,1]$ such that $0 \leq u_A(x) + v_A(x) \leq 1$ for any $x \in E$. Where, $u_A(x)$ and $v_A(x)$ is the degree of membership and degree of non-membership of the element x, respectively.

Definition 2.2 [18, 24]
Let X be a non-empty set and I the unit interval [0, 1]. A Pythagorean fuzzy set S is an object having the form $A = \{(x, u_A(x), v_A(x)) : x \in X\}$ where the functions $u_A : X \to [0,1]$ and $v_A : X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq \left(u_A(x)\right)^2 + (v_A(x))^2 \leq 1$ for each $x \in X$.

Definition 2.3[15] Let X be a non-empty set (universe). A neutrosophic set A on X is an object of the form: $A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$,

Where $u_A(x), \zeta_A(x), v_A(x) \in [0,1], 0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 2, for\ all\ x\ in\ X$. $u_A(x)$ is the degree of membership, $\zeta_A(x)$ is the degree of inderminancy and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ and $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent components.

Definition 2.4 Let X be a nonempty set and I the unit interval [0,1]. A neutrosophic set A and B of the form $A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$. Then

1) $A^C = \{(x, v_A(x), \zeta_A(x), u_A(x)) : x \in X\}$
2) $A \cup B = \{(x, \max(u_A(x), u_B(x)), \min(\zeta_A(x), \zeta_B(x)), \min(v_A(x), v_B(x))) : x \in X\}$
3) $A \cap B = \{(x, \min(u_A(x), u_B(x)), \max(\zeta_A(x), \zeta_B(x)), \max(v_A(x), v_B(x)) : x \in X\}$

3. Pythagorean Neutrosophic set with T and F are dependent neutrosophic components [PNS]:

Definition 3.1 Let X be a non-empty set (universe). A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A on X is an object of the form $A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$,

Where $u_A(x), \zeta_A(x), v_A(x) \in [0,1], 0 \leq \left(u_A(x)\right)^2 + \left(\zeta_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 2, for\ all\ x\ in\ X$. $u_A(x)$ is the degree of membership, $\zeta_A(x)$ is the degree of inderminancy and $v_A(x)$ is the degree of non-membership .Here $u_A(x)$ and $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent components.

Definition 3.2 Let X be a nonempty set and I the unit interval [0, 1]. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A and B of the form

$A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$. Then

1) $A^C = \{(x, v_A(x), \zeta_A(x), u_A(x)) : x \in X\}$
2) $A \cup B = \{(x, \max(u_A(x), u_B(x)), \max(\zeta_A(x), \zeta_B(x)), \min(v_A(x), v_B(x))) : x \in X\}$
3) $A \cap B = \{(x, \max(u_A(x), u_B(x)), \max(\zeta_A(x), \zeta_B(x)), \min(v_A(x), v_B(x)) : x \in X\}$

Definition 3.3 Let X be a nonempty set and I the unit interval [0, 1]. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A and B of the form
$A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$.
Then the correlation coefficient of A and B

$$\rho(A, B) = \frac{C(A, B)}{\sqrt{C(A, A). C(B, B)}} \quad (1)$$

$$C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

$$C(A,A)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$

$$C(B,B)=\sum_{i=1}^{n}\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

Preposition 3.4 The defined correlation measure between PNS A and PNS B satisfies the following properties

(i) $0\leq\ \rho(A,B)\leq 1$

(ii) $\rho(A,B)=1\ if\ and\ only\ if\ A=B$

(iii) $\rho(A,B)=\ \rho(B,A)$.

Proof:

(i) $0\leq\ \rho(A,B)\leq 1$

As the membership, inderminate and non-membership functions of the PNS lies between 0 and 1, $\rho(A,B)$ also lies between 0 and 1.

We will prove $C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$

$$=\left(\left(u_A(x_1)\right)^2.\left(u_B(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_B(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_B(x_1)\right)^2\right)+$$
$$\left(\left(u_A(x_2)\right)^2.\left(u_B(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_B(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\left(\left(u_A(x_n)\right)^2.\left(u_B(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_B(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_B(x_n)\right)^2\right)$$

By Cauchy-Schwarz inequality, $(x_1y_1+x_2y_2+\cdots+x_ny_n)^2\leq({x_1}^2+{x_2}^2+\cdots+{x_n}^2).({y_1}^2+{y_2}^2+\cdots+{y_n}^2)$, where $(x_1+x_2+\cdots+x_n)\in R^n\ and\ (y_1+y_2+\cdots+y_n)\in R^n$, we get

$$\left(C(A,B)\right)^2\ =\ \left(\left(u_A(x_1)\right)^4+\left(\zeta_A(x_1)\right)^4+\left(v_A(x_1)\right)^4\right)+\left(\left(u_A(x_2)\right)^4+\left(\zeta_A(x_2)\right)^4+\ \left(v_A(x_2)\right)^4\right)+$$
$$...+\left(\left(u_A(x_n)\right)^4+\left(\zeta_A(x_n)\right)^4+\left(v_A(x_n)\right)^4\right)$$
$$\times\left(\left(u_B(x_1)\right)^4+\ \left(\zeta_B(x_1)\right)^4+\left(v_B(x_1)\right)^4\right)+\ \left(\left(u_B(x_2)\right)^4+\ \left(\zeta_B(x_2)\right)^4+$$
$$\left(v_B(x_2)\right)^4\right)+\cdots+\ \left(\left(u_B(x_n)\right)^4+\ \left(\zeta_B(x_n)\right)^4+\left(v_B(x_n)\right)^4\right)$$

$$=\left(\left(u_A(x_1)\right)^2.\left(u_A(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_A(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_A(x_1)\right)^2\right)$$
$$+\ \left(\left(u_A(x_2)\right)^2.\left(u_A(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_A(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_A(x_2)\right)^2\right)+\cdots+$$

$$\left(\left(u_A(x_n)\right)^2.\left(u_A(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_A(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_A(x_n)\right)^2\right)\times$$
$$\left(\left(u_B(x_1)\right)^2\left(u_B(x_1)\right)^2+\left(\zeta_B(x_1)\right)^2\left(\zeta_B(x_1)\right)^2+\left(v_B(x_1)\right)^2\left(v_B(x_1)\right)^2\right)+$$
$$\left(\left(u_B(x_2)\right)^2\left(u_B(x_2)\right)^2+\left(\zeta_B(x_2)\right)^2\left(\zeta_B(x_2)\right)^2+\left(v_B(x_2)\right)^2\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\left(\left(u_B(x_n)\right)^2\left(u_B(x_n)\right)^2+\left(\zeta_B(x_n)\right)^2+\left(v_B(x_n)\right)^2\left(v_B(x_n)\right)^2\right)$$

$$= C(A,A)\times C(B,B).$$

Therefore, $\left(C(A,B)\right)^2 \le C(A,A)\times C(B,B)$ and thus $\rho(A,B)\le 1$.

Hence we obtain the following propertity $0 \le \rho(A,B) \le 1$

(ii) $\rho(A,B) = 1 \; if \; and \; only \; if \; A = B$

Let the two PNS A and B be equal (i.e A = B). Hence for any

$u_A(x_i) = u_B(x_i)$, $\zeta_A(x_i) = \zeta_B(x_i)$ and $v_A(x_i) = v_B(x_i)$,

Then $C(A,A) = C(B,B) = \sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$

And $C(A,B) = \sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$

$$= \sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right) = C(A,A)$$

Hence

$$\rho(A,B) = \frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}}$$

$$= \frac{C(A,A)}{\sqrt{C(A,A).C(A,A)}} = 1$$

Let the $\rho(A,B) = 1$.Then, the unite measure is possible only if

$$\frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}} = 1$$

This refer that $u_A(x_i) = u_B(x_i)$, $\zeta_A(x_i) = \zeta_B(x_i)$ and $v_A(x_i) = v_B(x_i)$,

for all i. Hence A = B.

(iii) If $\rho(A,B) = \rho(B,A)$, it obvious that

$$\frac{C(A,B)}{\sqrt{C(A,A).C_{NPFS}(B,B)}} = \frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}} = \rho(B,A)$$

as

$$C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$
$$=\sum_{i=1}^{n}\left(\left(u_B(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$
$$C(B,A)$$

Hence the proof.

Definition 3.5

Let A and B be two PNSs, then the correlation coefficient is defined as

$$\rho'(A,B)=\frac{C(A,B)}{max\{C(A,A).C(B,B)\}} \quad (2)$$

Theorem 3.6

The defined correlation measure between PNS A and PNS B satisfies the following properties

(i) $0\leq\ \rho'(A,B)\leq 1$

(ii) $\rho'(A,B)=1\ if\ and\ only\ if\ A=B$

(iii) $\rho'(A,B)=\ \rho'(B,A)$.

Proof: The property (i) and (ii) is straight forward, so omit here. Also $\rho'(A,B)\geq 0$ is evident. We now prove only $\rho'(A,B)\leq 1$.

Since Theorem 3.4, we have $(C(A,B))^2\leq C(A,A).C(B,B)$. Therefore, $C(A,B)\leq max\{C(A,A),C(B,B)\}$ and thus $\rho'(A,B)\leq 1$.

However, in many practical situations, the different set may have taken different weights, and thus, weight ω_i of the element $x_i\in X\ (i=1,2,\dots,n)$ should be taken into account. In the following, we develop a weighted correlation coefficient between PNSs. Let $\omega=\{\omega_1,\omega_2,\dots,\omega_n\}$ be the weight vector of the elements $x_i(i=1,2,\dots.,n)$ with $\omega_i\geq 0\ and\ \sum_{i=1}^{n}\omega_i=1$, then we have extended the above correlation coefficient $\rho(A,B)\ and\ \rho'(A,B)$ to weighted correlation coefficient as follows:

$$\rho''=\frac{C_\omega(A,B)}{\sqrt{C_\omega(A,A).C_\omega(B,B)}} \quad (3)$$

$$C_\omega(A,B)=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

$$C_\omega(A,A)=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$

$$C_\omega(B,B)=\sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

And

$$\rho''' = \frac{C_\omega(A,B)}{max\{C_\omega(A,A).C_\omega(B,B)\}} \quad (4)$$

$$= \frac{\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2 + \left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2 + \left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)}{max\begin{Bmatrix}\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2 + \left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2 + \left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right), \\ \sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2 + \left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2 + \left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)\end{Bmatrix}}$$

It can be easy to verify that if $\omega = \left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$, then Equation (3) and (4) reduce that (1) and (2), respectively.

Theorem 3.7

Let $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T$ be the weight vector of $x_i(i = 1,2,\ldots.,n)$ with $\omega_i \geq 0$ *and* $\sum_{i=1}^{n}\omega_i = 1$, then the weighted correlation coefficient between the PNSs A and B defined by Equation (3) satisfies:

(i) $0 \leq \rho''(A,B) \leq 1$

(ii) $\rho''(A,B) = 1$ *if and only if* $A = B$

(iii) $\rho''(A,B) = \rho''(B,A)$.

Proof:

The property (i) and (ii) are straight forward so omit here. Also $\rho''(A,B) \geq 0$ is evident so we need to show only $\rho''(A,B) \leq 1$.

Since,

$$C_\omega(A,B) = \sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2 + \left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2 + \left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

$$= \omega_1\left(\left(u_A(x_1)\right)^2.\left(u_B(x_1)\right)^2 + \left(\zeta_A(x_1)\right)^2.\left(\zeta_B(x_1)\right)^2 + \left(v_A(x_1)\right)^2.\left(v_B(x_1)\right)^2\right) + \omega_2\left(\left(u_A(x_2)\right)^2.\left(u_B(x_2)\right)^2 + \left(\zeta_A(x_2)\right)^2.\left(\zeta_B(x_2)\right)^2 + \left(v_A(x_2)\right)^2.\left(v_B(x_2)\right)^2\right) + \cdots + \omega_n\left(\left(u_A(x_n)\right)^2.\left(u_B(x_n)\right)^2 + \left(\zeta_A(x_n)\right)^2.\left(\zeta_B(x_n)\right)^2 + \left(v_A(x_n)\right)^2.\left(v_B(x_n)\right)^2\right)$$

$$= \left(\sqrt{\omega_1}\left(u_A(x_1)\right)^2.\sqrt{\omega_1}\left(u_B(x_1)\right)^2 + \sqrt{\omega_1}\left(\zeta_A(x_1)\right)^2.\sqrt{\omega_1}\left(\zeta_B(x_1)\right)^2 + \sqrt{\omega_1}\left(v_A(x_1)\right)^2.\sqrt{\omega_1}\left(v_B(x_1)\right)^2\right) + \left(\sqrt{\omega_2}\left(u_A(x_2)\right)^2.\sqrt{\omega_2}\left(u_B(x_2)\right)^2 + \sqrt{\omega_2}\left(\zeta_A(x_2)\right)^2.\sqrt{\omega_2}\left(\zeta_B(x_2)\right)^2 + \sqrt{\omega_2}\left(v_A(x_2)\right)^2.\sqrt{\omega_2}\left(v_B(x_2)\right)^2\right) + \cdots + \left(\sqrt{\omega_n}\left(u_A(x_n)\right)^2.\sqrt{\omega_n}\left(u_B(x_n)\right)^2 + \sqrt{\omega_n}\left(\zeta_A(x_n)\right)^2.\sqrt{\omega_n}\left(\zeta_B(x_n)\right)^2 + \sqrt{\omega_n}\left(v_A(x_n)\right)^2.\sqrt{\omega_n}\left(v_B(x_n)\right)^2\right)$$

By using Cauchy-Schwarz inequality, we get

$$\left(C_\omega(A,B)\right)^2 \le \left(\omega_1\left(u_A(x_1)\right)^2.\left(u_A(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_A(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_A(x_1)\right)^2\right)+$$
$$\left(\omega_2\left(u_A(x_2)\right)^2.\left(u_A(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_A(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_A(x_2)\right)^2\right)+$$
$$\dots+\left(\omega_n\left(u_A(x_n)\right)^2.\left(u_A(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_A(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_A(x_n)\right)^2\right)\times$$
$$\left(\omega_1\left(u_B(x_1)\right)^2\left(u_B(x_1)\right)^2+\left(\zeta_B(x_1)\right)^2\left(\zeta_B(x_1)\right)^2+\left(v_B(x_1)\right)^2\left(v_B(x_1)\right)^2\right)+$$
$$\left(\omega_2\left(u_B(x_2)\right)^2\left(u_B(x_2)\right)^2+\left(\zeta_B(x_2)\right)^2\left(\zeta_B(x_2)\right)^2+\left(v_B(x_2)\right)^2\left(v_B(x_2)\right)^2\right)$$
$$+\dots+\left(\omega_n\left(u_B(x_n)\right)^2\left(u_B(x_n)\right)^2+\left(\zeta_B(x_n)\right)^2\left(\zeta_B(x_n)\right)^2+\left(v_B(x_n)\right)^2\left(v_B(x_n)\right)^2\right)$$
$$=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)\times$$
$$\sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$
$$=C_\omega(A,A)\times C_\omega(B,B)$$

Therefore, $C_\omega(A,B) \le \sqrt{C_\omega(A,A)\times C_\omega(B,B)}$ and hence $0 \le \rho''(A,B) \le 1$.

Theorem 3.8

The correlation coefficient of two PNSs A and B as defined in Equation (4), that is, $\rho'''(A,B)$ satisfies the same properties as those in Theorem 3.7

Proof: The proof of this theorem is similar to that of Theorem 3.6.

5. Application

In this section, we give some application of PNS in medical diagnosis problem using correlation measure.

Medical Diagnosis Problem

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult.In some practical problems, there is the possibility of each element having different truth membership , inderminate and false membership functions.The proposed correlation measure among the patients Vs. symptoms and symptoms Vs. diseases gives the proper medical diagnosis. Now, an example of a medical diagnosis will be presented

Example

Let P= $\{P_1, P_2, P_3\}$ be a set of patients, D= $\{Viral\ Fever, Malaria, Typhoid, Dengu\}$ be a set of diseases and S= $\{Temperature$, Headache, Cough, Joint pain$\}$ be a set of symptoms.

Table 1: M (the relation between Patient and Symptoms)

M	$Temperature$	Headache	Cough	Joint pain
P_1	(0.8,0.7,0.6)	(0.5,0.3,0.8)	(0.6,0.9,0.4)	(0.3,0.5,0.2)
P_2	(0.2,0.7,0.9)	(0.5,0.9,0.8)	(0.4,0.6,0.3)	(0.1,0.2,0.9)
P_3	(0.3,0.1,0.5)	(0.8,0.5,0.6)	(0.4,0.8,0.9)	(0.5,0.7,0.2)

Table 2: N (the relation between Symptoms and Diseases)

N	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
$Temperature$	(0.9,0.5,0.4)	(0.5,0.3,0.6)	(0.8,0.9,0.4)	(0.2,0.8,0.5)
Headache	(0.1,0.5,0.3)	(0.5,0.6,0.7)	(0.4,0.5,0.9)	(0.9,0.8,0.3)
Cough	(0.3,0.7,0.8)	(0.9,0.7,0.4)	(0.1,0.3,0.9)	(0.5,0.3,0.8)
Joint pain	(0.7,0.3,0.5)	(0.8,0.9,0.6)	(0.5,0.7,0.6)	(0.1,0.5,0.8)

Using Equations (1), we get the value of $\rho(A, B)$

Table 3: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.7670**	0.5363	0.5965	0.5446
P_2	0.4638	**0.6253**	0.4873	0.5434
P_3	0.4596	0.6606	0.6072	**0.7401**

Using Equations (2), we get the value of $\rho'(A, B)$

Table 4: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.6997**	0.5223	0.5786	0.5357
P_2	0.3670	**0.5292**	0.4358	0.5095
P_3	0.4269	0.6562	0.5784	**0.6729**

On the other hand, if we assign weights 0.10, 0.20, 0.30 and 0.40 respectively, then by applying correlation coefficient given in Equations (3) and (4), we can give the following values of the correlation coefficient:

Using Equations (3), we get the value of $\rho''(A, B)$

Table 5: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.7233**	0.6496	0.4527	0.4623
P_2	0.4390	**0.5469**	0.4758	0.4194
P_3	0.5123	0.6606	0.7229	**0.7638**

Using Equations (4), we get the value of $\rho'''(A, B)$

Table 6: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.6936**	0.5324	0.4280	0.4039
P_2	0.2812	**0.5316**	0.4245	0.4084
P_3	0.4321	0.6154	0.6727	**0.7518**

The highest correlation measure from the Tables 3,4,5,6 gives the proper medical diagnosis. Therefore, patient P_1 suffers from Viral Fever, patient P_2 suffers from Malaria and patient P_3 suffers from Dengu. Hence, we can see from the above four kinds of correlation coefficient indices that the results are same.

Conclusion

In this paper, we found the correlation measure of Pythagorean neutrosophic set with T and F are neutrosophic components (PNS) and proved some of their basic properties. Based on that the present paper have extended the theory of correlation coefficient from and neutrosophic sets (NS) to the Pythagorean neutrosophic set with T and F are neutrosophic components in which the constraint condition of sum of membership, non-membership and indeterminacy be less than two has been relaxed. Illustrate examples have handle the situation where the existing correlation coefficient in NS environment fails. Also to deal with the situations where the elements in a set are correlative, a weighted correlation coefficients has been defined. We studied an application of correlation measure of Pythagorean neutrosophic set with T and F are neutrosophic components in medical diagnosis.

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Conflicts of Interest

The authors declare no conflict of interest.

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