Data Envelopment Analysis Based on Triangular Neutrosophic Numbers

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Abstract: Data envelopment analysis (DEA) is one of the best mathematical techniques to compute the overall performance of units with some inputs and outputs. The original DEA methods are developed to tackle the information based on the crisp number but no ability to handle the indeterminacy, impreciseness, vagueness, inconsistent, and incompleteness information such as triangular neutrosophic numbers (TNNs). This study attempts to establish a new model of DEA, where the information on decision-making units is TNNs. Initially, the concept and features of a conventional DEA model and the comparative TNNs are discussed. Besides, some new ranking functions of TNNs are presented. Furthermore, based on the mentioned ranking functions, an algorithm for solving the new model has been established. A comparison of the new model with an existing method and other kinds of uncertainty tools has been provided. In comparison with the existing methods, the significant characteristic of the new model is that it can handle the triangular neutrosophic information simply and effectively. Finally, the implementation of this strategy for an example has been applied for various models of DEA.

1 Introduction

As a robust analytical tool for the benchmarking and efficiency evaluation, data envelopment analysis (DEA) is a technique for evaluating the relation efficiency of decision-making units (DMUs), developed initially by Charens *et al.* [1] within a printed-paper named CCR. They extended the non-parametric method introduced by Farrell [2] to gauge DMUs with multiple inputs and outputs. The Banker-Charnes-Cooper model (BCC) model is an extension of the previous model under the assumption of variable returns to scale [3]. In addition to CCR and BCC, several models discuss DEA from several perspectives: range adjusted measure (RAM) by Cooper *et al.* [4], slack-adjusted DEA by Sueyoshi [5], additive model by Seiford and Thrall [6], slacks based measure (SBM) model by Tone [7], and free disposal hull (FDH) model by Deprins *et al.* [8], all of which are DEA basic models.

At present, the DEA technique is developing rapidly, and it is used for the assessment of various industries and organisations [9–19]. In classical DEA models, DMUs are evaluated by considering input and output values to measure rational efficiency as compared to different DMUs; eventually, the measure to which rational efficiency belongs is obtained (0, 1). The first DEA models need precise and crisp data. However, in numerous cases such as when producing a system or in the preparation process, the banking system, insurance industry, and financial service system, data are unstable, uncertain, and complicated; therefore, cannot be accurately measured. Consequently, many analysts attempted to model DEA with completely different questioning hypotheses.

Fuzzy set (FS) is one of the essential tools to deal with the uncertainty phenomena [20–23]. The first attempt at using FSs in DEA can be found in the work of Sengupta [24]. Generally, using the fuzzy theory in DEA models can be classified as shown in Fig. 1 [25]. Several works investigated on fuzzy DEA models; some study information may be found in [26–40]. However, FSs can consider only the membership function and cannot deal with other parameters of vagueness.

For this reason, Atanassov [41] established the intuitionistic FS (IFS), which can define the membership and non-membership functions. There are also various research studies addressing the use of IFSs in DEA; see [42–45].

Although the theory of IFSs can handle incomplete information in various real-world issues, it cannot address all types of uncertainty, such as indeterminate information. Therefore, Smarandache [46–47] presented the neutrosophic set (NS) as a robust tool that generalises the classical and all kinds of FSs (FSs and IFSs). NSs can accommodate ambiguous, indeterminate, and conflicting information where the indeterminacy is clearly quantified and define three kinds of membership functions independently. In the past few years, the field of NS, logic, measure, probability, and statistics, pre-calculus, and calculus, and their applications in multiple areas have been extended and applied in various fields. For more information, see [48–59].

While NS theory has been widely used in various problems, there are not many studies that have implemented NS to tackle ambiguity or indeterminacy in DEA.

The utilisation of neutrosophic logic in DEA can be traced to Edalatpanah [60]. Kahraman *et al.* [61] presented a hybrid algorithm based on neutrosophic AHP and DEA for bringing a solution to the efficiency of private universities. Edalatpanah and Smarandache [62], based on some operators and natural logarithm, proposed an input-oriented DEA model with simplified neutrosophic numbers.

Recently, Abdelfattah [63] by converting a DEA with NS data into an interval DEA developed a new DEA model under triangular neutrosophic numbers (TNNs). He solved this model based on the approach of Despotis and Smirlis [64]. Although this approach is interesting, however, there are some limitations. One of them is that this method has high running times, mainly when we have many inputs and outputs. Furthermore, as stated in [65], the main flaw of [64] is the existence of several production frontiers in the steps of efficiency measure, and this leads to the lack of comparability between efficiencies. So, in this study, we design a new model of DEA with TNNs and establish a simple approach to solve it.

This paper is organised as follows: some basic knowledge, concepts, and arithmetic operations on TNNs are introduced in Section 2. In Section 3, we review a basic DEA model and propose some ranking functions. Section 4 introduces a new DEA model with TNNs data and proposed a new strategy to solve it. In Sections 5 and 6, numerical tests are presented to show the reliability and efficiency of the method. Finally, some conclusions are stated in Section 7.

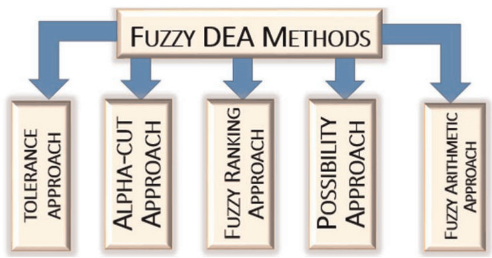

Fig. 1 *Classification of fuzzy DEA methods*

2 Prerequisite

Here, we addressed some basic definitions regarding NSs and the related concepts.

Definition 1: Neutrosophic set [46, 47]. A NS in universal X is defined by three membership functions for the truth, indeterminacy, and falsity of x in the real non-standard $]^{-}0, 1^{+}[$, where their summation belong to [0, 3].

Definition 2: Single-valued Neutrosophic set [51]. If the three membership functions of NS are singleton in the real standard [0, 1], then, a single-valued NS ψ is denoted by

$$\psi = \{(x, \ \tau_{\psi}(x), \ \iota_{\psi}(x), \ \nu_{\psi}(x)) | x \in X\}$$

which satisfies the following condition:

$$0 \le \tau_{\psi}(x) + \iota_{\psi}(x) + \ \nu_{\psi}(x) \le 3. \tag{1}$$

Definition 3: TNN [51]. A triangular neutrosophic number (TNN) is denoted by $A^{\aleph} = \langle (a^l, a^m, a^u), (\mu, i, \omega) \rangle$, where the three membership functions for the truth, indeterminacy, and falsity of x can be defined as follows:

$$\tau_{A^{\aleph}}(x) = \begin{cases} \dfrac{(x - a^l)}{(a^m - a^l)}\mu & a^l \le x < a^m, \\ \mu & x = a^m, \\ \dfrac{(a^u - x)}{(a^u - a^m)}\mu & a^m \le x < a^u, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

$$\iota_{A^{\aleph}}(x) = \begin{cases} \dfrac{(a^m - x)}{(a^m - a^l)}i, & a^l \le x < a^m, \\ i, & x = a^m, \\ \dfrac{(x - a^u)}{(a^u - a^m)}i, & a^m \le x < a^u, \\ 1, & \text{otherwise.} \end{cases} \tag{3}$$

$$\nu_{A^{\aleph}}(x) = \begin{cases} \dfrac{(a^m - x)}{(a^m - a^l)}\omega, & a^l \le x < a^m, \\ \omega, & x = a^m, \\ \dfrac{(x - a^u)}{(a^u - a^m)}\omega, & a^m \le x < a^u, \\ 1, & \text{otherwise.} \end{cases} \tag{4}$$

where $0 \le \tau_{A^{\aleph}}(x) + \iota_{A^{\aleph}}(x) + \nu_{A^{\aleph}}(x) \le 3, x \in A^{\aleph}$.

Definition 4: Arithmetic operations [51]. Suppose $A_1^{\aleph} = \langle (a_1, b_1, c_1), (\mu_1, \nu_1, \omega_1) \rangle$ and $A_2^{\aleph} = \langle (a_2, b_2, c_2), (\mu_2, \nu_2, \omega_2) \rangle$ be two TNNs. Then the arithmetic relations are defined as

(*i*) $A_1^{\aleph} \oplus A_2^{\aleph} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2), (\mu_1 \wedge \mu_2, \nu_1 \vee \nu_2, \omega_1 \vee \omega_2) \rangle$

(*ii*) $A_1^{\aleph} - A_2^{\aleph} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2), (\mu_1 \wedge \mu_2, \nu_1 \vee \nu_2, \omega_1 \vee \omega_2) \rangle$

(*iii*) $A_1^{\aleph} \otimes A_2^{\aleph} = \langle (a_1 a_2, b_1 b_2, c_1 c_2), (\mu_1 \wedge \mu_2, \nu_1 \vee \nu_2, \omega_1 \vee \omega_2) \rangle$, if $a_1 > 0, b_1 > 0$,

(*iv*) $\lambda A_1^{\aleph} = \begin{cases} \langle (\lambda a_1, \lambda b_1, \lambda c_1), (\mu_1, \nu_1, \omega_1) \rangle, \text{ if } \lambda > 0 \\ \langle (\lambda c_1, \lambda b_1, \lambda a_1), (\mu_1, \nu_1, \omega_1) \rangle, \text{ if } \lambda < 0 \end{cases}$

Definition 5: Ranking function [22]. Suppose $A^{\aleph}$ and $B^{\aleph}$ be two TNNs, then:

(i) $A^{\aleph} \le B^{\aleph}$ if and only if $R(\tilde{A}) \le R(B^{\aleph})$,
(ii) $A^{\aleph} < B^{\aleph}$ if and only if $R(\tilde{A}) < R(B^{\aleph})$,

where $R(.)$ is a ranking function.
Now, we present a new ranking function

Definition 6: One can compare any two TNNs based on the ranking functions. Let $A^{\aleph} = \langle (a, b, c), (\mu, \nu, \omega) \rangle$ be a TNN; then $R(A^{\aleph}) = \dfrac{a + b + c}{9}(\mu + (1 - \nu) + (1 - \omega))$.

Example 1: Let $A^{\aleph} = ((4, 8, 10); 0.5, 0.3, 0.6)$ then $R(A^{\aleph}) = 3.91$. Also, for $1^{\aleph} = ((1, 1, 1); 1, 0, 0)$, $0^{\aleph} = ((0, 0, 0); 1, 1, 1)$ we have $R(\tilde{1}) = 1$ and $R(\tilde{0}) = 0$.
Also, since $R(\tilde{a} + \tilde{b}) \neq R(\tilde{a}) + R(\tilde{b})$, we define an aggregation ranking function as follows:

Definition 7: Let $A_i^{\aleph} = ((a_i, b_i, c_i); \mu_i, \nu_i, \omega_i)$ be n TNNs. Then the aggregation ranking function is as follows:

$$\bar{R}\left(\sum_{i=1}^{n} A_i^{\aleph}\right) = \frac{(2 + \min \mu_i - \max \nu_i - \max \omega_i)}{9} \sum_{i=1}^{n} (a_i + b_i + c_i). \tag{5}$$

Example 2: Let $A^{\aleph} = ((4, 8, 10); 0.5, 0.3, 0.6)$ and $B^{\aleph} = ((3, 7, 11); 0.4, 0.5, 0.6)$ then based on Definition 5:

$$R(A^{\aleph}) = 3.91,$$
$$R(B^{\aleph}) = 3.03.$$

However, by Definition 3

$$A^{\aleph} + B^{\aleph} = ((7, 15, 21); 0.4, 0.5, 0.6),$$
$$R(A^{\aleph} + B^{\aleph}) = 6.21 \neq R(A^{\aleph}) + R(B^{\aleph}) = 6.94.$$

However

$$\bar{R}(A^{\aleph} + B^{\aleph}) = \frac{(2 + 0.4 - 0.5 - 0.6)}{9} \times (22 + 21) = 6.94$$
$$= R(A^{\aleph} + B^{\aleph}).$$

3 Dual CCR model

Suppose that we have n DMUs (DMU_j: $j = 1, 2, \ldots, n$), with m inputs x_{ij} $(i = 1, 2, \ldots, m)$ and s outputs $y_{rj}(r = 1, 2, \ldots, s)$.

If DMU_0 is under consideration, then the dual CCR model is defined as [1]

$$\theta_o^* = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$
$$\text{s.t.}$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \forall j \tag{6}$$
$$u_r,\ v_i \ge 0 \quad \forall r,\ i$$

where u_r and v_i are the related weights.

The above non-linear programming may be converted as follows to simplify the computation:

$$\theta_o^* = \max \sum_{r=1}^{s} u_r y_{ro}$$
$$\text{s.t.}$$
$$\sum_{i=1}^{m} v_i x_{io} = 1 \tag{7}$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,$$
$$u_r,\ v_i \ge 0 \quad \forall r,\ i$$

The DMU_o is efficient if $\theta_o^* = 1$, otherwise, it is inefficient.

4 Neutrosophic DEA based on TNNs

Consider the input and output for the jth DMU as the following TNNs:

$$x_{ij}^{\aleph} = \left(\left(x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u}\right);\ \mu_{x_{ij}},\ \nu_{x_{ij}},\ \omega_{x_{ij}}\right),$$
$$y_{rj}^{\aleph} = \left(\left(y_{rj}^{l}, y_{rj}^{m}, y_{rj}^{u}\right);\ \mu_{y_{rj}},\ \nu_{y_{rj}},\ \omega_{y_{rj}}\right).$$

Then the triangular neutrosophic CCR model called TNN–CCR is defined as follows:

$$\theta_o^* = \max \sum_{r=1}^{s} u_r y_{ro}^{\aleph}$$
$$\text{s.t.}$$
$$\sum_{i=1}^{m} v_i x_{io}^{\aleph} = 1^{\aleph} \tag{8}$$
$$\sum_{r=1}^{s} u_r y_{rj}^{\aleph} - \sum_{i=1}^{m} v_i x_{ij}^{\aleph} \le 0^{\aleph},$$
$$u_r,\ v_i \ge 0 \quad \forall r,\ i.$$

Here, to solve model (8), we propose the following algorithm (Fig. 2):

Theorem 1: Models (7) and (8) are equivalent.

Proof: By considering the aggregation ranking function and Algorithm 1, it is easy to see that each optimal feasible solution of model (8) is an optimal feasible solution of model (7) and vice versa. □

Step 1. Construct the problem as the Model (8).

Step 2. By Definition 3, concert the Model (8) into Model (9):

$$\theta_o^* = \max \sum_{r=1}^{s} u_r \left(\left(y_{ro}^{l}, y_{ro}^{m}, y_{ro}^{u}\right); \mu_{y_{ro}}, \nu_{y_{ro}}, \omega_{y_{ro}}\right) \tag{9}$$
$$s.t.$$
$$\sum_{i=1}^{m} v_i \left(\left(x_{io}^{l}, x_{io}^{m}, x_{io}^{u}\right); \mu_{x_{io}}, \nu_{x_{io}}, \omega_{x_{io}}\right) = 1^{\aleph}$$
$$\sum_{r=1}^{s} u_r \left(\left(y_{rj}^{l}, y_{rj}^{m}, y_{rj}^{u}\right); \mu_{y_{rj}}, \nu_{y_{rj}}, \omega_{y_{rj}}\right) \le \sum_{i=1}^{m} v_i \left(\left(x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u}\right); \mu_{x_{ij}}, \nu_{x_{ij}}, \omega_{x_{ij}}\right),$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

Step 3. Using the aggregation ranking function, convert Model (9) into Model (10):

$$\bar{R}(\theta_o^*) = \max \bar{R}\left(\sum_{r=1}^{s} u_r \left(\left(y_{ro}^{l}, y_{ro}^{m}, y_{ro}^{u}\right); \mu_{y_{ro}}, \nu_{y_{ro}}, \omega_{y_{ro}}\right)\right) \tag{10}$$
$$s.t.$$
$$\bar{R}\left(\sum_{i=1}^{m} v_i \left(\left(x_{io}^{l}, x_{io}^{m}, x_{io}^{u}\right); \mu_{x_{io}}, \nu_{x_{io}}, \omega_{x_{io}}\right)\right) = \bar{R}(1^{\aleph})$$
$$\bar{R}\left(\sum_{r=1}^{s} u_r \left(\left(y_{rj}^{l}, y_{rj}^{m}, y_{rj}^{u}\right); \mu_{y_{rj}}, \nu_{y_{rj}}, \omega_{y_{rj}}\right)\right) \le$$
$$\bar{R}\left(\sum_{i=1}^{m} v_i \left(\left(x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u}\right); \mu_{x_{ij}}, \nu_{x_{ij}}, \omega_{x_{ij}}\right)\right),$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

Step 4. Using Definition 6, convert the Model (10) into the next crisp model:

$$\bar{R}(\theta_o^*) = \max \left[\frac{(2 + \min_{1\le r\le s} \mu_{y_{ro}} - \max_{1\le r\le s} \nu_{y_{ro}} - \max_{1\le r\le s} \omega_{y_{ro}})}{9} \sum_{r=1}^{s} u_r (y_{ro}^{l} + y_{ro}^{m} + y_{ro}^{u})\right]$$
$$s.t.$$
$$\frac{(2 + \min_{1\le i\le m} \mu_{x_{io}} - \max_{1\le i\le m} \nu_{x_{io}} - \max_{1\le i\le m} \omega_{x_{io}})}{9} \sum_{i=1}^{m} v_i (x_{io}^{l} + x_{io}^{m} + x_{io}^{u}) = 1,$$
$$\frac{(2 + \min_{1\le r\le s} \mu_{y_{rj}} - \max_{1\le r\le s} \nu_{y_{rj}} - \max_{1\le r\le s} \omega_{y_{rj}})}{9} \sum_{r=1}^{s} u_r (y_{rj}^{l} + y_{rj}^{m} + y_{rj}^{u}) \le$$
$$\frac{(2 + \min_{1\le i\le m} \mu_{x_{ij}} - \max_{1\le i\le m} \nu_{x_{ij}} - \max_{1\le i\le m} \omega_{x_{ij}})}{9} \sum_{i=1}^{m} v_i (x_{ij}^{l} + x_{ij}^{m} + x_{ij}^{u}),$$
$$u_r, v_i \ge 0 \qquad \forall r, i.$$

Step 5. Run the crisp model of Step 4 and get the optimal efficiency of each DMUs.

Fig. 2 *Algorithm*

Table 1 Inputs and outputs of five DMUs

DMU	DMU 1	DMU 2	DMU 2	DMU 4	DMU 5
input 1	⟨(3.5, 4, 4.5); 0.7, 0.4, 0.3⟩	⟨(2.9, 2.9, 2.9); 0.6, 0.5, 0.2⟩	⟨(4.4, 4.9, 5.4); 0.6, 0.4, 0.1⟩	⟨(3.4, 4.1, 4.8); 0.4, 0.3, 0.2⟩	⟨(5.9, 6.5, 7.1); 0.7, 0.4, 0.3⟩
input 2	⟨(1.9, 2.1, 2.3); 0.4, 0.3, 0.5⟩	⟨(1.4, 1.5, 1.6); 0.8, 0.2, 0.1⟩	⟨(2.2, 2.6, 3.0); 0.7, 0.4, 0.2⟩	⟨(2.2, 2.3, 2.4); 1.0, 0.0, 0.0⟩	⟨(3.6, 4.1, 4.6); 0.9, 0.1, 0.1⟩
output 1	⟨(2.4, 2.6, 2.8); 0.9, 0.2, 0.1⟩	⟨(2.2, 2.2, 2.2); 0.9, 0.3, 0.0⟩	⟨(2.7, 3.2, 3.7); 0.7, 0.5, 0.2⟩	⟨(2.5, 2.9, 3.3); 0.7, 0.5, 0.1⟩	⟨(4.4, 5.1, 5.8); 0.8, 0.4, 0.2⟩
output 2	⟨(3.8, 4.1, 4.4); 0.8, 0.5, 0.1⟩	⟨(3.3, 3.5, 3.7); 1.0, 0.0, 0.0⟩	⟨(4.3, 5.1, 5.9); 0.7, 0.5, 0.1⟩	⟨(5.5, 5.7, 5.9); 0.4, 0.2, 0.1⟩	⟨(6.5, 7.4, 8.3); 0.5, 0.0, 0.2⟩

5 Numerical test

Here, we select an example of [63] to illustrate the results obtained in previous sections.

Consider five DMUs as Table 1 that consume two inputs to produce two outputs.

Next, we apply Algorithm 1 for the mentioned problem. For example, Algorithm 1 for DMU_1 can be applied as follows:

First, By Table 1, we have the following TNN-DEA model:

$$\begin{aligned}
\theta_1^* = \max\,(&< 2.4, 2.6, 2.8; 0.9, 0.2, 0.1 > u_1 \\
&+ < 3.8, 4.1, 4.4; 0.8, 0.5, 0.1 > u_2) \\
&\text{s.t.} \\
&< 3.5, 4.0, 4.5; 0.7, 0.4, 0.3 > v_1 \\
&+ < 1.9, 2.1, 2.3; 0.4, 0.3, 0.5 > v_2 = 1^{\aleph}, \\
&< 2.4, 2.6, 2.8; 0.9, 0.2, 0.1 > u_1 \\
&+ < 3.8, 4.1, 4.4; 0.8, 0.5, 0.1 > u_2 \leq \\
&< 3.5, 4.0, 4.5; 0.7, 0.4, 0.3 > v_1 \\
&+ < 1.9, 2.1, 2.3; 0.4, 0.3, 0.5 > v_2, \\
&< 2.2, 2.2, 2.2; 0.9, 0.3, 0.0 > u_1 \\
&+ < 3.3, 3.5, 3.7; 1.0, 0.0, 0.0 > u_2 \leq \\
&< 2.9, 2.9, 2.9; 0.6, 0.5, 0.2 > v_1 \\
&+ < 1.4, 1.5, 1.6; 0.8, 0.2, 0.1 > v_2, \\
&< 2.7, 3.2, 3.7; 0.7, 0.5, 0.2 > u_1 \\
&+ < 4.3, 5.1, 5.9; 0.7, 0.5, 0.1 > u_2 \leq \\
&< 4.4, 4.9, 5.4; 0.6, 0.4, 0.1 > v_1 \\
&+ < 2.2, 2.6, 3.0; 0.7, 0.4, 0.2 > v_2, \\
&< 2.5, 2.9, 3.3; 0.7, 0.5, 0.1 > u_1 \\
&+ < 5.5, 5.7, 5.9; 0.4, 0.2, 0.1 > u_2 \leq \\
&< 3.4, 4.1, 4.8; 0.4, 0.3, 0.2 > v_1 \\
&+ < 2.2, 2.3, 2.4; 1.0, 0.0, 0.0 > v_2, \\
&< 4.4, 5.1, 5.8; 0.8, 0.4, 0.2 > u_1 \\
&+ < 6.5, 7.4, 8.3; 0.5, 0.0, 0.2 > u_2 \leq \\
&< 5.9, 6.5, 7.1; 0.7, 0.4, 0.3 > v_1 \\
&+ < 3.6, 4.1, 4.6; 0.9, 0.1, 0.1 > v_2, \\
&u_r, v_i \geq 0 \quad r, i = 1, 2.
\end{aligned}$$

Finally based on Step 4 of Algorithm 1, we convert the above model to the following model:

$$\begin{aligned}
&\theta_1^* = \max 1.9063u_1 + 3.0061u_2 \\
&\text{s.t.} \\
&2.0004v_1 + 1.0502v_2 = 1, \\
&(1.9063u_1 + 3.0061u_2) - (2.0004v_1 + 1.0502v_2) \leq 0, \\
&(1.9067u_1 + 3.0334u_2) - (1.8366v_1 + 0.95v_2) \leq 0, \\
&(2.1331u_1 + 3.3997u_2) - (3.2663v_1 + 1.7332v_2) \leq 0, \\
&(1.74u_1 + 3.42u_2) - (2.5965v_1 + 1.4566v_2) \leq 0, \\
&(3.2298u_1 + 4.6864u_2) - (4.3329v_1 + 2.7331v_2) \leq 0, \\
&u_r, v_i \geq 0 \quad r, i = 1, 2.
\end{aligned}$$

After computations with Matlab, we obtain $\theta_1^* = 0.9183$ for DMU_1.

6 Results analysis

Here, we illustrate the results. If we solve the mentioned example for all DMUs, we obtain the results of Table 2.

So, by our model, we have

$$DMU_2 > DMU_1 > DMU_4 > DMU_5 > DMU_3$$

However, by the existing model of [63] we have

$$DMU_4 > DMU_2 > DMU_5 > DMU_1 > DMU_3.$$

Owing to the existence of several production frontiers that adopted in the process of efficiency measure of model [63], the mentioned results are incompatible with our results. Moreover, both results show that DMU_3 is the worst DMU.

Table 2 Efficiencies of the DMUs based on TNN-DEA

DMUs	1	2	3	4	5
efficiency	0.9183	1.0000	0.6302	0.7975	0.7180
ranking	2	1	5	3	4

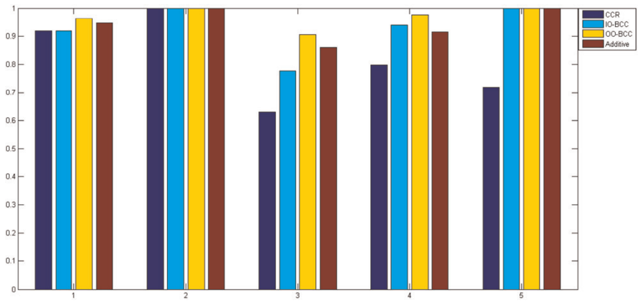

Fig. 3 *Comparison of various models of DEA under neutrosophic environment*

Table 3 Results of the DMUs based on intuitionistic fuzzy data

DMUs	1	2	3	4	5
efficiency	1.0000	0.8587	0.5760	0.7779	0.5934
ranking	1	2	5	3	4

Next, we test input-oriented BCC [3], output-oriented BCC [3], and SBM [7] models of DEA by our algorithm. Fig. 3 shows these results.

From Fig. 3, we can see that DMU_2 is the best.

Next, we investigate the impact of the neutrosophic parameter in the ranking of DMUs. To this end, we obtain the result based on the triangular intuitionistic fuzzy information. Table 3 shows the efficiencies of DMUs according to the triangular intuitionistic fuzzy information.

It is interesting to see that without considering the indeterminacy information $DMU_2 < DMU_1$. So, it can conclude that with neutrosophic information, we can obtain realistic results.

7 Conclusion

In this study, we introduce the neutrosophic DEA and propose a novel model to solve it. Since the existing arithmetic operations of TNNs are complicated and could not consider the relationship between three membership functions for the truth, indeterminacy, and falsity of different TNNs easily, a new ranking function is introduced to overcome the existing problem. Besides, an aggregation ranking function is presented to obtain the summation of TNNs reliably. A new algorithm using these ranking functions is also presented to obtain the efficiency of each DMU without any unreasonable evaluation values. Finally, we use an example to illustrate the practicality and validity of the proposed method. We also implement some other models of DEA such as input-oriented BCC, output-oriented BCC, and SBM models by our algorithm, and obtain promising results. Furthermore, we investigate the impact of the indeterminacy parameter in the ranking of DMUs. Finally, from the obtained results, it can be concluded that the model is efficient and convenient.

8 References

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