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Decision process in MCDM with large number of criteria and heterogeneous risk preferences

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ABSTRACT

A new decision process is proposed to address the challenge that a large number criteria in the multi-criteria decision making (MCDM) problem and the decision makers with heterogeneous risk preferences. First, from the perspective of objective data, the effective criteria are extracted based on the similarity relations between criterion values and the criteria are weighted, respectively. Second, the corresponding types of theoretic model of risk preferences expectations will be built, based on the possibility and similarity between criterion values to solve the problem for different interval numbers with the same expectation. Then, the risk preferences (Risk-seeking, risk-neutral and risk-aversion) will be embedded in the decision process. Later, the optimal decision object is selected according to the risk preferences of decision makers based on the corresponding theoretic model. Finally, a new algorithm of information aggregation model is proposed based on fairness maximization of decision results for the group decision, considering the coexistence of decision makers with heterogeneous risk preferences. The scientific rationality verification of this new method is given through the analysis of real case.

1. Introduction

This paper studies the decision problems that consider decision makers (DMs) with heterogeneous risk preferences (RPs) (there are two or more than two different RPs types of DMs in decision making) facing large amounts of data. The heterogeneous of RPs leads to an impact on the decision process (such as: extract effective criteria, obtain criteria weight, information aggregation, ranking and prioritizing the alternatives) and generating some new characteristics, which needs to be studied in detail. The large amounts of data in the decision tables mainly implies three kinds of situations [13], (a) a large number of criteria; (b) a large number of decision objects; (c) a large number of decision objects and criteria. This paper studies the dynamic decision process of the first situation and considers the heterogeneous assumption of RPs, then proposes a dynamic decision strategy. The proposed method can address the problems that consider the coexistence of DMs with heterogeneous RPs.

In this paper, first, effective criteria are extracted based on the criterion reduction algorithm via the similarity relation between criterion values. Similarity analysis matrix is used to weight the effective criteria. The RPs (risk-aversion, risk-neutral and risk-seeking) of DMs are incorporated afterwards. In the process of classification decisions, the RPs expectations theoretical model, based on the measure of possibility and similarity, is proposed to solve the decision problems, in which criterion values are interval numbers. The corresponding risk expectations models is selected in the following process of information aggregation and alternatives ranking. Finally, a new algorithm of information aggregation model is proposed based on maximization of decision results fairness. The proposed decision strategy can address the decompose decision tasks while making classification decisions based on the RPs of DMs.

With the development of information technology, how to extract useful information from large amounts of data to support decision making, becomes very important research content and also an important challenge in decision science [11,23]. Criterion reduction [10,28,29] is a useful method to extract useful knowledge from large amounts of information. Researches show that in the process of extracting effective criteria from data tables containing numbers of criteria, the attitudes of the DMs can be divided into two cate-

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gories: (a) the RPs of DMs play a very important role in extracting the effective criteria (i.e. decision-makers with different RPs extract different effective criteria from the same decision table); (b) the RPs of DMs is associated with the ranking and selecting process of decision objects, but has nothing to do with the selection and weighting of effective criteria, which based on the objective data [28]. The literatures [1,14,20] have studied the impact of DMs' RPs on decision results. Liu et al. [14] have proposed a strategy corresponding to the first situation that classifies DMs before making decisions, then extract the effective criteria.

In this paper, our decision process corresponding to the second situation. This paper introduces the criterion reduction technology to decision-making problems that contains a lot of criteria. Under this scenario, a proper reduction algorithm will be selected based on the RPs of DMs and the characteristics of decision data itself. The criterion reduction algorithm is proposed based on similarity relations between criterion values. For decision tables whose criterion values are all real numbers, if the criterion values of two decision objects on the same criterion are equal, then this criterion is not effective in comparing these two decision objects. For decision tables whose criterion values are all interval number, if the criterion similarity (information coincidence degree) between two decision objects' criterion values (as interval numbers) of the same criterion reaches a threshold (such as: 85% or 95%), then these two criterion values provide the same decision information for DMs. That also means that the criterion is not effective (redundant criteria) in comparing these two decision objects. Due to the similarity between two interval numbers doesn't change with the RPs of DMs, there is no need to classify the DMs according to their RPs, so we use the similarity relations between criterion values to extract effective criteria. Overall, the criterion reduction threshold is firstly determined. Then, the effective criteria are weighted. Finally, the decision results are ranked and prioritized. Due to the particularity of interval numbers, a new definition of interval number similarity is given in this paper.

Obtaining the criteria weight and information aggregation are two important steps in multi-criterion decision making (MCDM). General speaking, the proposed methods of obtaining criterion weights to solve the MCDM problems fall into three categories: subjective weighting method [7,21,30], objective weighting method [5,26,27], subjective and objective weighting method [12,25]. For the information aggregation, Liu and Jin [16] propose some new operators and relative group decision making methods for the multi-attribute decision-making (MADM) problems in which attribute values are generalized interval-valued trapezoidal fuzzy numbers. Liu and Yu [18] and Liu et al. [15] proposed some aggregation operators, including 2-dimension uncertain linguistic power generalized aggregation operator and 2-dimension uncertain linguistic power generalized weighted aggregation operator to solve the multiple attribute group decision making problems with 2-dimension uncertain linguistic information. Liu and Liu [17] and Liu and Shi [19] also proposed generalized aggregation operators to solve the MCDM problems with intuitionistic trapezoidal fuzzy information and interval neutrosophic hesitant fuzzy information.

The criterion reduction based on the similarity relation is introduced as the first step in decision process. Therefore, it's necessary to adopt an algorithm consistent with the criterion reduction based on similarity relation, to ensure the consistency of algorithm thinking in the decision sub-processes. On this basis, an algorithm of criterion weighting based on the discrimination matrix of similarity relations is proposed. The RPs of DMs are incorporated gradually in the decision process after the criterion reductions and obtaining the criteria weight for effective criteria. The expectation theoretical model based on the measure of possibility and similarity is built corresponding to the RPs to address the problems for different interval numbers with the same expectation. In this paper, we propose an information aggregation model based on weighted combinatorial advantage value (WCAV) considering the DMs with heterogeneous RPs and fairness maximization of decision results.

Table 1
The result of "yes" answers from 100 experts.

U	C_1	C_2	......	C_m	......
A_1	[75, 85]	[55, 75]	......	[18, 18]	......
A_2	[76, 95]	[60, 85]	......	[12, 85]	......
A_3	[88, 100]	[65, 85]	......	[25, 70]	......
A_4	[75, 85]	[65, 80]	......	[22, 22]	......
A_5	[89, 100]	[86, 90]	......	[25, 25]	......

The innovations of this paper as follows: Considering the coexistence of DMs with heterogeneous RPs facing large number of criteria, this paper proposes a dynamic decision strategy and the corresponding model, which classifies DMs while approaching the solution according to the study of decision process, following by information aggregation [3,8]. First, the effective criteria are extracted through discrimination matrix, which is built based on the similarity relations between criterion values. Second, the effective criteria are weighted based on the discrimination matrix of similarity relations. Then, decision makers are classified based on the RPs of DMs, which are embedded into the decision process afterwards. The proper algorithm corresponding with the specific RPs are selected to conduct the information aggregation. In this process, the expectation theoretical model corresponding with the specific risk preference is built to determine the dominance relations between criteria values. Finally, based on fairness maximization of decision results, a new information aggregation model and the ranking algorithm are proposed, considering the coexistence of DMs with heterogeneous RPs.

2. The decision problem of big data table

In this paper, we focus on the decision problems considering decision table with numbers of criteria and the criterion values can be expressed as interval numbers.

2.1. Decision problems of massive criteria and criterion values as interval numbers

Supposing that in the process of EU's seventh framework QB50 project double-cell standard atmosphere to explore a cubic star, five control solutions are selected for the next round of competition after a preliminary screening, denoting as $\boldsymbol{U} = \{A_1, A_2, A_3, A_4, A_5\}$. 100 experts are invited to vote on all entries' status of each criterion. Then votes on the corresponding criteria can be divided into three categories: affirmative, negative and abstention. Obviously, some experts has made abstention votes when the sum of affirmative votes and negative votes is less than 100. The possible maximum of affirmative votes can be obtained by the number of experts minus the number of negative votes. The criterion values can be expressed as interval numbers [26]. The optimal solution is selected based on the data in Table 1.

Obviously, Table 1 contains numbers of decision criteria and the criterion values are all denoted as interval numbers. How to choose the appropriate criterion reduction algorithm to extract effective criteria from Table 1 becomes the emphasis of this study.

2.2. Basic knowledge of interval numbers

Definition 1. If $\tilde{a} = [a^L, a^U] = \{x | a^L \le x \le a^U; a^L, a^U \in \mathbf{R}\}$, then $\tilde{a}$ is an interval number [9,14]. Here, $l_{\tilde{a}} = a^U - a^L$ denotes the length of interval number $\tilde{a}$. If $l_{\tilde{a}} = 0$, then $\tilde{a}$ is a real number.

Set $\tilde{a} = [a^L, a^U]$, $\tilde{b} = [b^L, b^U]$, then

Rule 1: If and only if: $a^U \ge b^L$ or $b^U \ge a^L$, $\tilde{a} \cap \tilde{b} = [\max\{a^L, b^L\}, \min\{a^U, b^U\}]$

Rule 2: If and only if: $a^U \ge b^L$ or $b^U \ge a^L$, $\tilde{a} \cup \tilde{b} = [\min\{a^L, b^L\}, \max\{a^U, b^U\}]$

Rule 3: If and only if: $a^L = b^L$ and $a^U = b^U$, $\tilde{a} = \tilde{b}$

Definition 2. Interval number $\tilde{a} = [a^L, a^U]$, $\tilde{b} = [b^L, b^U]$, if $a^U \ge b^L$ or $b^U \ge a^L$, $\tilde{a} \cap \tilde{b} = [\max\{a^L, b^L\}, \min\{a^U, b^U\}]$, $\tilde{a} \cup \tilde{b} = [\min\{a^L, b^L\}, \max\{a^U, b^U\}]$, then

$$S(\tilde{a}, \tilde{b}) = \begin{cases} 0 & b^U < a^L \quad \text{or} \quad a^U < b^L \\ \dfrac{l_{\tilde{a}\cap\tilde{b}}}{l_{\tilde{a}\cup\tilde{b}}} & b^U \ge a^L \quad \text{or} \quad a^U \ge b^L \\ 1 & \tilde{a} = \tilde{b} \end{cases} \tag{1}$$

Denote $S(\tilde{a}, \tilde{b})$ as the similarity degree between $\tilde{a}$ and $\tilde{b}$.

Apparently, the similarity degree between two interval numbers satisfies: $0 \le S(\tilde{a}, \tilde{b}) \le 1$.

2.3. The principles and methods of criterion reduction

In Table 1, the criterion values of decision objects A_3, A_5 on criterion C_1 are [88, 100] and [89, 100], respectively. Obviously, these two interval numbers are not equal. While the similarity degree between these two interval numbers reaches to $11/12 = 0.9167$, which means these two criterion values of decision objects A_3, A_5 on criterion C_1 show a contact ratio of their mutual information and possible information as 91.67%. When the similarity degree between this two criterion values reaches a certain threshold (standard) (such as: 85% or 95%), these two decision objects may provide the same information on this criterion. That means this criterion doesn't work in changing the result of decisions, then this criterion is marked as unnecessary and should be removed from decision table. At the same time, two criteria are equivalently if the similarity degree between two interval numbers reaches to a threshold in decision. According to the mentioned hypothesis analysis, criterion C_1 is not effective in distinguishing decision objects A_3, A_5. However, the similarity degree between decision objects A_1, A_3 on criterion C_1 is 0, thus C_1 is effective in distinguishing A_1, A_3. Therefore, the analysis of all the decision objects in the decision table is necessary in the construction of the corresponding discrimination matrix. After that, the effective criteria can be extracted from the decision table with the corresponding algorithm.

Definition 3. Assuming $\{A_1, A_2, \cdots, A_n\}$ as a set of decision objects in the decision table, $\{C_1, C_2, \cdots, C_m\}$ as a set of the criteria, and the criterion values of decision objects A_i, $A_k\{i, k \in 1, 2, \cdots, n\}$ on criteria $C_j\{j = 1, 2, \cdots, m\}$ are $\tilde{f}(A_i, C_j)$ and $\tilde{f}(A_k, C_j)$, respectively. Where, $\tilde{f}(A_i, C_j)$ and $\tilde{f}(A_k, C_j)$ are interval numbers, M^S is the discrimination matrix based on the similarity relationship between criterion values; $\alpha_j (j \in 1, 2, \cdots, m)$ is the reduction standard (threshold) of DMs on criteria $C_j\{j = 1, 2, \cdots, m\}$. Then, we will get

$$M^S = (m_{ik})_{n\times n} = \begin{cases} \{C_j \in \mathbf{C} : S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)) < \alpha_j \\ \phi \qquad\qquad\qquad else \end{cases} \tag{2}$$

In Eq. (2), m_{ik} is the criteria set whose similarities degree between criterion values of decision objects A_i and $A_k\{i, k \in 1, 2, \cdots, n\}$ on the corresponding criteria are smaller than the reduction standard $\alpha_j (j \in 1, 2, \cdots, m)$. (The reduction standard is determined with practical problems, psychological thresholds and RPs of DMs.)

Extract the effective criteria: The effective criteria are extracted with the discrimination function from the discrimination matrix [9,22].

Definition 4. Assuming f_{M^S} as a discrimination function of the discrimination matrix M^S based on the similarity relations between criterion values, $\{C_1^*, C_2^*, \cdots, C_m^*\}$ as a group of effective criteria of criteria set $\{C_1, C_2, \cdots, C_m\}$. Then, we will get

$$f_{M^S}(C_1^*, C_2^*, \cdots, C_m^*) = \bigwedge_{1\le j<i\le m,\, C\in C_{ij}^*, m_{ij}\ne\phi} \bigvee C \tag{3}$$

In Eq. (3), $C_{ij}^* = \{C^* : C \in m_{ij}\}$, M^S is denoted in Definition 3.

2.4. Obtaining weights for criteria

In Definition 2, the similarity degree is the contact ratio of the common information between two interval numbers. According to the above principles and methods of criterion reduction, the greater the similarity degree is, the weaker the influence of this criterion on distinguishing the decision objects from DMs is. Then, the DMs show less concern for this decision criterion accordingly. That is, the corresponding criterion weights in decision making process become smaller. While, the criterion will have a stronger impact for the DMs to distinguish the decision objects if the decision objects have a smaller similarity degree on a certain criterion. Thus, the criterion weights will be larger. Therefore, the size of the criterion weights is inversely related to the similarities between criterion values.

Definition 5. Supposing $\{A_1, A_2, \cdots, A_n\}$ as a set of decision objects in the decision tables, $\{C_1, C_2, \cdots, C_m\}$ as a set of all the criteria, the criterion value of decision criteria $C_j\{j = 1, 2, \cdots, m\}$ on decision objects A_i, $A_k\{i, k \in 1, 2, \cdots, n\}$ are $\tilde{f}(A_i, C_j)$ and $\tilde{f}(A_k, C_j)$ respectively. $S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j))$ is the similarity degree between the criterion values of decision objects A_i, $A_k\{i, k \in 1, 2, \cdots, n\}$ on criteria $C_j\{j = 1, 2, \cdots, m\}$. If $S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)) \ge \alpha_j$, then these two decision objects provide the same information for DMs, so this criterion doesn't work in changing the decision results. Thus, the criterion weighting considers the similarity degree $S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)) < \alpha_j$ only.

According to the above descriptions, if $S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)) < \alpha_j$, the criterion weighting algorithm based on similarities can be expressed as follows.

$$\omega_j = \frac{\sum_{k=1}^{n}\sum_{i=1}^{n}(\alpha_j - S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)))}{\sum_{j=1}^{m}\sum_{k=1}^{n}\sum_{i=1}^{n}(\alpha_j - S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j)))} \tag{4}$$

If $\sum_{k=1}^{n}\sum_{i=1}^{n}(\alpha_j - S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j))) \le 0$, then the criterion values of all the decision objects in the decision table on criterion $C_j\{j = 1, 2, \cdots, m\}$ are same. That is, when DMs comparing the decision objects in the decision table, this criterion doesn't work, so the weight of this criterion is 0.

If $\sum_{j=1}^{m}\sum_{k=1}^{n}\sum_{i=1}^{n}(\alpha_j - S(\tilde{f}(A_i, C_j), \tilde{f}(A_k, C_j))) \le 0$, then the criterion values of all the decision objects from the decision table on the same criterion are the same. That is, all the decision objects in the decision table are equivalent.

2.5. Information aggregation and the expectation theoretical model

Based on the weighted advantage degree matrix (WADM) [13], the corresponding risk type (RT) of DMs, and the ranking algorithm are expressed in Eqs. (5)–(8).

$$WADM = \begin{pmatrix} d_{A_1 \succ A_1} & d_{A_1 \succ A_2} & \cdots & d_{A_1 \succ A_n} \\ d_{A_2 \succ A_1} & d_{A_2 \succ A_2} & \cdots & d_{A_2 \succ A_n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{A_n \succ A_1} & d_{A_n \succ A_2} & \cdots & d_{A_n \succ A_n} \end{pmatrix} \quad (5)$$

Where, $d_{A_1 \succ A_2}$ is the weighted comprehensive dominance value of decision object A_1 and A_2 in comparisons. Then, Eq. (6) can be obtained.

$$d_{A_1 \succ A_2} = d_{A_1 \succ A_2/C_1} \cdot \omega_1 + \cdots + d_{A_1 \succ A_2/C_{m'}} \cdot \omega_{m'} \quad (6)$$

If $d_{A_1 \succ A_2} > 0$, then it shows that decision object $A_1 \succ A_2$.

Where, $d_{A_1 \succ A_2/C_1}$ is the dominance value of decision objects A_1 and A_2 comparatively on criterion C_1, the dominance relation between criteria are expressed as dominance value in Eq. (7).

$$d_{A_1 \succ A_2/C_1} = \begin{cases} 1 & A_1 \succ A_2/C_1 \\ 0 & A_1 \cong A_2/C_1 \\ -1 & A_2 \prec A_1/C_1 \end{cases} \quad (7)$$

The decision objects are ranked based on $RT_{WACV_{A_k}^{\succ}}$, which indicates the weighted combinatorial advantage values (WCAV) of the decision objects A_k corresponding to the risk types (RT) of the DMs, then the following equation can be obtained.

$$RT_{WCAV_{A_k}^{\succ}} = \frac{1}{n-1} \sum_{i \neq k} d_{A_k \succ A_i} \quad (8)$$

Definition 6. Set $\tilde{a}$ and $\tilde{b}$ are two interval numbers, $\tilde{a} = [a^L, a^U]$, $\tilde{b} = [b^L, b^U]$, and $l_{\tilde{a}} = a^U - a^L$, $l_{\tilde{b}} = b^U - b^L$, then we will get the follows [26]

$$P(\tilde{a} \geq \tilde{b}) = \begin{cases} 1 & a^L \geq b^U \\ \frac{a^U - b^L}{l_{\tilde{a}} + l_{\tilde{b}}} & a^U > b^L \text{ or } a^L < b^U \\ 0 & a^U \leq b^L \end{cases} \quad (9)$$

Eq. (9) represents the possibility of $\tilde{a} \geq \tilde{b}$.

Based on prospect theory [2,4], DMs with different RPs have different attitude between different criteria with the same expectation values.

In Eq. (2), $\alpha_j (j \in 1, 2, \cdots, m)$ indicates the standard of criterion reduction. If the similarity degree between two criteria values is higher than or equal to $\alpha_j (j \in 1, 2, \cdots, m)$, these two criteria provide the same information for DMs.

If the similarity degree between the decision object and the positive ideal object is larger, then, this decision object is superior, and if the similarity between the decision object and the negative ideal object is smaller, then, this decision object is superior.

The RPs expectation theoretical models based on the possibilities of interval numbers and the measurement of similarities is shown as followings.

Risk aversion expectation theoretical model 1:

$\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ are the criterion values of two interval numbers. So, the risk aversion type of DMs will choose the negative ideal interval number as the reference point. Set the negative ideal number as $\tilde{c}^{-*}$, the expectation theoretical model is expressed in Eq. (10).

$$\begin{cases} \tilde{a} \succ \tilde{b} \Leftrightarrow \begin{cases} P(\tilde{a} \geq \tilde{b}) > 1/2 \\ P(\tilde{a} \geq \tilde{b}) = 1/2 \text{ and } S(\tilde{a}, \tilde{x}^{-*}) < S(\tilde{b}, \tilde{x}^{-*}) \end{cases}, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \\ \tilde{a} \cong \tilde{b} \Leftrightarrow S(\tilde{a}, \tilde{b}) \geq \alpha_j \\ \tilde{a} \prec \tilde{b} \Leftrightarrow \begin{cases} P(\tilde{a} \geq \tilde{b}) < 1/2 \\ P(\tilde{a} \geq \tilde{b}) = 1/2 \text{ and } S(\tilde{a}, \tilde{x}^{-*}) > S(\tilde{b}, \tilde{x}^{-*}) \end{cases}, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \end{cases} \quad (10)$$

Risk neutral expectation theoretical model 2:

$\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ are the criterion values of two interval numbers. Due to the DMs of risk neutral type don't care the risk, so the expectation theoretical model is shown in Eq. (11).

$$\begin{cases} \tilde{a} \succ \tilde{b} \Leftrightarrow & P(\tilde{a} \geq \tilde{b}) > 1/2, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \\ \tilde{a} \cong \tilde{b} \Leftrightarrow & P(\tilde{a} \geq \tilde{b}) = P(\tilde{b} \geq \tilde{a}) = 1/2 \text{ or } S(\tilde{a}, \tilde{b}) \geq \alpha_j \\ \tilde{a} \prec \tilde{b} \Leftrightarrow & P(\tilde{a} \geq \tilde{b}) < 1/2, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \end{cases} \quad (11)$$

Risk seeking expectation theoretical model 3:

$\tilde{a} = [a^L, a^U]$, $\tilde{b} = [b^L, b^U]$are the criterion values of two interval numbers. For the DMs of risk seeking type will choose the positive ideal interval number as the reference point. Set the positive ideal number as $\tilde{c}^{+*}$, the expectation theoretical model is expressed in Eq. (12).

$$\begin{cases} \tilde{a} \succ \tilde{b} \Leftrightarrow \begin{cases} P(\tilde{a} \geq \tilde{b}) > 1/2 \\ P(\tilde{a} \geq \tilde{b}) = 1/2 \text{ and } S(\tilde{a}, \tilde{x}^{+*}) > S(\tilde{b}, \tilde{x}^{+*}) \end{cases}, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \\ \tilde{a} \cong \tilde{b} \Leftrightarrow S(\tilde{a}, \tilde{b}) \geq \alpha_j \\ \tilde{a} \prec \tilde{b} \Leftrightarrow \begin{cases} P(\tilde{a} \geq \tilde{b}) < 1/2 \\ P(\tilde{a} \geq \tilde{b}) = 1/2 \text{ and } S(\tilde{a}, \tilde{x}^{+*}) < S(\tilde{b}, \tilde{x}^{+*}) \end{cases}, \text{if } S(\tilde{a}, \tilde{b}) < \alpha_j \end{cases} \quad (12)$$

Where, $P(\tilde{a} \geq \tilde{b})$ represents the possibility between interval numbers $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$. $\alpha_j (j \in 1, 2, \cdots, m)$ is the reduction standard on the corresponding criteria.

Obviously, if DMs consider that non-voters will vote for "yes" in the future, the best way is the least negative votes by now, while if DMs considering the non-voters will vote for "no" in the future, then the most affirmative votes will be the best. It is determined by the attitude of DMs for DMs to predict that whether non-voters vote for "yes" or "no" . For the situation that different values hold the same possibilities (expectation values), if DMs tend to regard as everything at worst, namely, having pessimistic attitudes toward the future, who clearly belong to the type of risk aversion. Then, if DMs always think of good development directions, namely, having positive attitudes toward the future, who clearly belong to risk seeking, then the best in the future is the optimal object.

2.6. The model of information aggregation based on preference difference

In the actual decisions, when two or more than two RPs types exist in the decision problems, how to make decisions or information aggregation, which will be introduced in this section. This paper will propose the information aggregation algorithm based on fairness maximization of decision results [6,24], considering the coexistence of DMs with heterogeneous RPs.

Definition 7. Set $\{DM_1, DM_2, \cdots, DM_n\}$ represents the DMs in the decision process, n_{RA}, n_{RN}, n_{RS} as the number of DMs of risk-aversion (RA), risk-neutral (RN) and risk-seeking (RS), respectively.

For simplify, we assume the weight of DMs are equal in this paper. Considering the coexistence of DMs with heterogeneous RPs, the weighted combinatorial advantage value (WCAV) based on fairness maximization of decision results is expressed in Eq. (13).

$$WCAV_{A_k}^{\succ} = \frac{n_{RA} \cdot RA_{WCAV_{A_k}^{\succ}} + n_{RN} \cdot RN_{WCAV_{A_k}^{\succ}} + n_{RS} \cdot RS_{WCAV_{A_k}^{\succ}}}{n} \quad (13)$$

Where, $n_{RA} + n_{RN} + n_{RS} = n (n_{RA}, n_{RN}, n_{RS} \geq 0)$.

3. Dynamic decision process based on the heterogeneous assumption of RPs

In this paper, the criteria reduction is embedded into the decision process also as the first step. Thus, some adjusts are made in this basis existing traditional decision problem solving steps.

Table 2
The final result of "yes" answers from 100 experts (nine criteria).

U	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	[50, 80]	[55, 75]	[65, 95]	[20, 70]	[50, 90]	[40, 80]	[50, 70]	[75, 85]	[61, 70]
A_2	[65, 75]	[60, 85]	[60, 90]	[55, 75]	[50, 87]	[45, 70]	[50, 75]	[76, 95]	[72, 85]
A_3	[70, 90]	[65, 85]	[35, 55]	[65, 80]	[45, 90]	[40, 80]	[60, 90]	[88, 100]	[60, 70]
A_4	[65, 86]	[65, 80]	[35, 60]	[65, 65]	[70, 90]	[65, 90]	[65, 85]	[75, 85]	[80, 90]
A_5	[71, 92]	[86, 90]	[70, 91]	[65, 65]	[65, 80]	[50, 70]	[50, 72]	[89, 100]	[60, 65]

Step1 criterion reduction

The effective criteria are extracted via discrimination matrix is built based on the similarity relations of criteria.

Step2 criterion weighting

The effective criteria are weighted based on the discrimination matrix.

Step3 preference incorporate the decision process

The DMs are classified corresponding to RPs types. Then the information aggregation, and ranking can be conducted after choosing the corresponding algorithm.

Step4 information aggregation based on the risk heterogeneous assumption of DMs

The information aggregation model is built according to the fairness maximization of decision results, and the decision objects ranking can be obtained accordingly.

Step5 confirmation and analysis

Confirm and analyze decision results.

4. The decision case based on the heterogeneous assumption of RPs

We use the case that mentioned in Section 2. Supposing the DMs will consider nine criteria $\boldsymbol{C} = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9\}$, where C_1 stands for process capability criteria (quality of components and parts), C_2 stands for risk, C_3 stands for reliability, C_4 stands for cost (repair, processing and development), C_5 stands for feasibility, C_6 stands for time, C_7 stands for the return rate of investments, C_8 stands for the assembly ability of parts, C_9 stands for customer satisfaction. 100 experts are invited to vote on all entries' status of each criterion. The results can be divided into three categories: affirmative vote, negative vote and abstention vote.

In order to simplify, we assume that the reduction standards of DMs on all criteria in the decision table are the same based on the data in Table 2. And the reduction standard is 85%.

Step1 criterion reduction

Using Eqs. (1) and (2), the discrimination matrix based on the similarity relation is symmetric. The upper triangular part is shown as followings. When the reduction standards of all the criteria are 85%, the discrimination matrix is shown.

$$M_{0.85}^{S} = \begin{pmatrix} \phi & C_1C_2C_3C_4C_6C_7C_8C_9 & C_1C_2C_3C_4C_7C_8 & C_1C_2C_3C_4C_5C_6C_7C_9 & C_1C_2C_3C_4C_5C_6C_8C_9 \\ & \phi & \boldsymbol{C} & \boldsymbol{C} & C_1C_2C_3C_4C_5C_6C_8C_9 \\ & & \phi & \boldsymbol{C} & C_2C_3C_4C_5C_6C_7C_9 \\ & & & \phi & C_1C_2C_3C_5C_6C_7C_8C_9 \\ & & & & \phi \end{pmatrix}$$

The effective criteria $\{C_1, C_2, C_3, C_4, C_7, C_8\}$ can be extracted from the discrimination matrix via the discrimination function [9,22] from Table 2 (*Effective criteria contains 6 criteria, while the origin data contains 9 criteria*).

When the decision objects are prioritized, these six criteria are necessary as the effective criteria $\{C_1, C_2, C_3, C_4, C_7, C_8\}$. Three criteria $\{C_5, C_6, C_9\}$ can be neglected.

Step2 criteria weighting

The weights of each effective criterion are expressed as followings,

$$\omega_1 = 0.1349,\ \omega_2 = 0.1537,\ \omega_3 = 0.1809,\ \omega_4 = 0.2103,$$
$$\omega_7 = 0.1484, \omega_8 = 0.1718$$

Step3 Incorporate RPs and making classification decisions

(1) DMs of risk-aversion type

The decision objects WADM of effective criteria $\{C_1, C_2, C_3, C_4, C_7, C_8\}$ is built based on Eqs. (5)–(8), and (10), the decision objects ranking result is shown.

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

(2) DMs of risk-neutral type

The decision objects WADM of effective criteria $\{C_1, C_2, C_3, C_4, C_7, C_8\}$ is built based on Eqs. (5)–(8), and (11), the decision objects ranking result for risk-neutral types of DMs is as the following.

$$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$$

Thus, the optimal decision object is A_5.

(3) DMs of risk-seeking type

The decision objects WADM of effective criteria $\{C_1, C_2, C_3, C_4, C_7, C_8\}$ is built based on Eqs. (5)–(8), and (12), the decision objects ranking result for risk-seeking types of DMs is as the following.

$$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$$

Thus, the optimal decision object is A_5.

(4) DMs of heterogeneous RPs

Assuming there is n_{RA}=n_{RN}=n_{RS} in this paper, by using the information aggregation algorithm via Eq. (13), the WCAV of decision objects can be obtained.

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

5. Confirmation and analysis of decision results

5.1. Compare confirmations of decision results from effective criteria and all criteria

By using Eq. (4), all the criteria are assigned and the corresponding weights as follows.

$\omega_{C_1} = 0.0948, \omega_{C_2} = 0.1081, \omega_{C_3} = 0.1272, \omega_{C_4} = 0.1479,$
$\omega_{C_5} = 0.0698, \omega_{C_6} = 0.0849, \omega_{C_7} = 0.1044, \omega_{C_8} = 0.1208,$
$\omega_{C_9} = 0.1421.$

(1) DMs of risk-aversion type

By using Eqs. (5)–(8), and (10), the ranking results of decision objects are expressed as follows:

$$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We get the same optimal result based on the effective criteria or all the criteria are the same, while the ranking results are different.

(2) DMs of risk-neutral type

By using Eqs. (5)–(8) and (11), the ranking results of decision objects are expressed as follows,

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We will get the same optimal result based on the effective criteria or all the criteria are the same, while the ranking results are different.

(3) DMs of risk-seeking type

By using Eqs. (5)–(8) and (12), the ranking results of decision objects are expressed as follows,

$$A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_3.

The ranking results and the optimal decision objects based on the data in Table 2, which makes decision based on the effective criteria or all the criteria are totally different.

(4) DMs of heterogeneous RPs

In order to simplify, we assume that each group of RPs has the same number of DMs, namely, n_{RA}=n_{RN}=n_{RS}, the ranking result of the decision objects can be expressed as follows,

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

When DMs of heterogeneous RPs all participant in the decision, the optimal decision objects after the extraction of effective criteria based on the data in Table 2, which makes decision based on the effective criteria or all the criteria are the same, while the ranking results are different.

5.2. Compare confirmations based on dominance relations

The proposed criterion reduction algorithm based on the similarity relations between criterion values is different from the principles and methods of the criterion reduction algorithm based on the dominance relations [14], when a decision object has relative evident advantages on a certain criterion over other decision objects, this criterion is an element in the corresponding position of discrimination matrix. (i.e. if two criterion values on a certain criterion are differently, this criterion may become a corresponding element in the discrimination matrix.) Overall, the criterion reduction standard is lower than the proposed criterion reduction algorithm in this paper based on the similarity relations between criterion values, which lose less information in effective information extractions. To verify the above arguments, the dominance relations between criterion values will be introduced in detail in the following chapters.

(1) DMs of risk-aversion type

Using the proposed method [14], the corresponding discrimination matrix based on the dominance relations between criteria values can be obtained from the data in Table 2.

$$M_{RA}^{\succ} = \begin{pmatrix} \phi & C_3C_5C_6 & C_3C_5C_9 & C_3 & C_9 \\ C_1C_2C_4C_7C_8C_9 & \phi & C_3C_5C_9 & C_3C_8 & C_7C_9 \\ C_1C_2C_4C_7C_8 & C_1C_2C_4C_6C_7C_8 & \phi & C_1C_2C_4C_8 & C_4C_7C_9 \\ C_1C_2C_4C_5C_6C_7C_9 & C_1C_2C_4C_5C_6C_7C_9 & C_3C_5C_6C_7C_9 & \phi & C_5C_6C_7C_9 \\ C_1C_2C_3C_4C_5C_6C_7C_8 & C_1C_2C_3C_4C_5C_6C_8 & C_1C_2C_3C_5C_6C_8 & C_1C_2C_3C_8 & \phi \end{pmatrix}$$

The effective criteria $\{C_1, C_2, C_3, C_4, C_8, C_9\}$ can be obtained. Decision is made based on the effective criteria, the following result can be obtained as follows.

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We get the same optimal decision objects and ranking results.

(2) DMs of risk-neutral type

The discrimination matrix based on the dominance relations can be obtained as follows:

$$M_{RN}^{\succ} = \begin{pmatrix} \phi & C_3C_5C_6 & C_3C_5C_9 & C_3 & C_9 \\ C_1C_2C_4C_7C_8C_9 & \phi & C_3C_5C_9 & C_3C_8 & C_7C_9 \\ C_1C_2C_4C_7C_8 & C_1C_2C_4C_6C_7C_8 & \phi & C_1C_2C_4C_8 & C_4C_7C_9 \\ C_1C_2C_4C_5C_6C_7C_9 & C_1C_2C_5C_6C_7C_9 & C_3C_5C_6C_9 & \phi & C_5C_6C_7C_9 \\ C_1C_2C_3C_4C_5C_7C_8 & C_1C_2C_3C_5C_6C_8 & C_1C_2C_3C_5C_8 & C_1C_2C_3C_8 & \phi \end{pmatrix}$$

The effective criteria $\{C_1, C_2, C_3, C_4, C_8, C_9\}$ can be obtained. Decision is made based on the effective criteria, and the following result can be obtained.

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We get the same optimal decision objects based on the dominance relations proposed, while the ranking results are different.

(3) DMs of risk-seeking type

The discrimination matrix based on the dominance relations can be obtained as follows:

$$M_{RS}^{\succ} = \begin{pmatrix} \phi & C_3C_5C_6 & C_3C_5C_9 & C_3 & C_6C_9 \\ C_1C_2C_4C_7C_8C_9 & \phi & C_3C_5C_9 & C_3C_4C_8 & C_7C_4C_9 \\ C_1C_2C_4C_7C_8 & C_1C_2C_4C_6C_7C_8 & \phi & C_1C_2C_4C_7C_8 & C_4C_6C_7C_9 \\ C_1C_2C_4C_5C_6C_7C_9 & C_1C_2C_5C_6C_7C_9 & C_3C_5C_6C_9 & \phi & C_5C_6C_7C_9 \\ C_1C_2C_3C_4C_5C_7C_8 & C_1C_2C_3C_5C_6C_8 & C_1C_2C_3C_5C_8 & C_1C_2C_3C_8 & \phi \end{pmatrix}$$

The effective criteria $\{C_1, C_2, C_3, C_4, C_8, C_9\}$ can be obtained. Decision is made based on the effective criteria, and the following result can be obtained.

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We get the same optimal decision objects, while the ranking results are different.

(4) DMs of heterogeneous RPs

If n_{RA}=n_{RN}=n_{RS}, the ranking result of the decision objects can be expressed as follows,

$$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$$

Thus, the optimal decision object is A_5.

We get the same optimal decision objects and ranking results.

5.3. Decision results comparisons and discussions

In Section 5.1: Under one condition (DMs of RA only) shows the same ranking results and optimal decision objects, and two conditions (DMs of RN and DMs of heterogeneous RPs) show the same optimal decision objects, while the ranking results are different. One condition (DMs of RS only) shows totally different ranking results and optimal decision objects.

In Section 5.2: Two conditions (DMs of RA type and DMs of heterogeneous RPs) show the same ranking results and optimal decision objects, and two conditions (DMs of RN type and DMs of RS type) show the same optimal decision objects, while the ranking results are different.

It is reasonable for different RPs types of DMs to obtain different ranking results of decision objects from the same data. In Sections 5.1 and 5.2, there are the same optimal decision objects and different ranking results for the same RPs of DMs. Four possible reasons are shown.

(1) The decision objects are not comparable. DMs cannot find a decision objects which has dominance over the other decision objects on all the criteria.
(2) The problem of setting the criterion reduction standard. The DMs should set an effective threshold value. On this basis, it useful to enhance the consistency of the decision objects ranking results, and furthermore improve the quality of decision.
(3) Setting the ratios of DMs of heterogeneous RPs. This paper assumes the numbers of three different types of DMs are same. Because, most of DMs belong to the RA, and only a few DMs belongs to the RS in the decision problems. Therefore, it will influence the ranking results of DMs to determine the proportions of different RPs types of DMs.
(4) Due to the attitude of RS DMs, this type of DMs may accept different decision results.

Furthermore, the attention DMs paid to the optimal decision objects is more important than that to the ranking results of decision objects in the real decision-making processes, which illustrates the algorithm proposed in this paper, is of scientific rationality.

6. Conclusion and future work

This paper proposed a decision process that considers the scenario of DMs with heterogeneous RPs, large number of criteria in decision table and the criteria values presented as interval numbers, this method involved the criteria reduction technology and the prospect theory.

The contributions of this paper can be mainly summed up as five aspects: (1) Built the discrimination matrix based on the criteria similarity relations to extract the effective criteria, and assign weights for the effective criteria through the discrimination matrix based on the similarity relations; (2) Incorporated the RPs of DMs corresponding algorithm models, which classifies DMs and make decisions while decomposing the decision tasks, following by information aggregation; (3) Built the RPs expectation theoretical models based on the similarity relation and possibility between interval numbers to address the challenge of different interval numbers hold the same expectation, then ranked and prioritized the decision objects via WCAV; (4) Proposed the information aggregation algorithm based on the maximization of fairness and utilities for decision situations that contains two or more than two different RPs types of DMs; (5) Verified the effectiveness and rationality of the proposed method through comparisons and analysis between decision results based on the effective criteria and all the criteria, and comparisons and analysis between decision results through the proposed method in this paper and the existed method, respectively.

The future research will be based on the new preference models of DMs' heterogeneous assumption of RPs, to build the corresponding algorithm and solve the balance point of fairness and utility balance in group decision-making process.

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