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*Research Article*

# Design of a Control Chart Based on COM-Poisson Distribution for the Uncertainty Environment

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This paper will introduce the neutrosophic COM-Poisson (NCOM-Poisson) distribution. Then, the design of the attribute control chart using the NCOM-Poisson distribution is given. The structure of the control chart under the neutrosophic statistical interval method will be given. The algorithm to determine the average run length under neutrosophic statistical interval system will be given. The performance of the proposed control chart is compared with the chart based on classical statistics in terms of neutrosophic average run length (NARL). A simulation study and a real example are also added. From the comparison of the proposed control chart with the existing chart, it is concluded that the proposed control chart is more efficient in detecting a shift in the process. Therefore, the proposed control chart will be helpful in minimizing the defective product. In addition, the proposed control chart is more adequate and effective to apply in uncertainty environment.

## 1. Introduction

Control chart is an important tool of the statistical process control (SPC) that has been widely used in the industry and service company for the monitoring of the manufacturing process. In the industry, specifications are set to manufacture the product. The manufacturing process away from the target causes the production of the nonconforming items. Therefore, an increase in the nonconforming items causes the minimizing of the profit of the company. The control chats provide the signal when the process shifted from the target parameters. A timely signal about the shift in the process helps the industrial engineers to sort out the problem and bring back the process to the in-control state. The operational procedure of the control chart is decided based on data obtained from the production process. The production data is either discrete data or continuous data. The discrete data is obtained from the counting process while the continuous data is obtained from the measurement process. The control charts using both data have been widely used in the industry for the monitoring of the process. Although the control charts based on the variable data are more informative than the control charts based on the attribute data, the variable control chart cannot be applied when the purpose is to monitor the number of nonconforming items. According to [1], for modelling the count data, the Poisson distribution that has only one parameter has been widely used and is the best-fitted model when the mean and the variance are identical. The Poisson distribution cannot apply when mean is smaller than variance (overdispersed data) or variance is smaller than mean (underdispersed data). Therefore, this distribution has limitation and may mislead the experimenters when the assumption of identical mean and variance is violated. To overcome this issue, [2] proposed the COM-Poisson distribution which can be applied for underdispersed data, overdispersed data, and equal dispersed data. Reference [3] studied the properties of the COM-Poisson distribution. The COM-Poisson distribution is more efficient than the generalized Poisson distribution proposed by [4] and the weighted Poisson distribution proposed by [5]. Due to the comparative edge and flexibility over the Poisson distribution, generalized Poisson distribution, and the weighted Poisson distribution, the COM-Poisson distribution has been widely used in the industry for the monitoring of the process. Reference [6] designed control chart using the COM-Poisson distribution. Reference [7] discussed the application of this distribution.

Reference [8] designed attribute control chart using the multivariate COM-Poisson distribution. References [9] and [10] proposed chart for the count data. Reference [11] proposed the control chart based on COM-Poisson distribution using the resampling approach. Reference [12] designed modified EWMA chart based on the COM-Poisson distribution. More details on such control charts can be seen in [13, 14].

Usually, the attribute control charts are designed when the proportion defective parameter is determined or crisp value. In practice, it is not always possible that the industrial engineers know about the proportion defective parameter. In this situation, the attribute control charts based on fuzzy approach are applied for the monitoring of nonconformities. Reference [15] studied fuzzy variable and attribute control charts. Reference [16] proposed the fuzzy chart for multistage processes. Reference [17] designed various fuzzy attribute control charts. Reference [18] worked on X-bar and range chart using the fuzzy approach. Reference [19] studied the fuzzy np control chart. Reference [20] presented the control chart for the fuzzy score number. Reference [21] presented the algorithm for the control chart under the fuzzy logic. Reference [22] studied the fuzzy attribute chart using Monte Carlo simulation. More information on the application of fuzzy logic can be seen in [23–25].

The existing control chart based on the COM-Poisson distribution is designed under the classical statistics. The classical statistics assumed no indeterminacy in the proportion defective parameters or the observations. Fuzzy logic is based on the degree of truth/false, rather than "false or true." Reference [26] argued that the neutrosophic logic is the extension of the fuzzy logic. According to [26], "neutrosophic set and neutrosophic logic are generalizations of the fuzzy set and respectively fuzzy logic. In neutrosophic logic, a proposition has a degree of truth, a degree of indeterminacy, and a degree of falsity." The neutrosophic statistics (NS) which is the generalization of the classical statistics is proposed by [27]. The NS deals with the observations or the parameters are in the indeterminacy interval rather than the determined value. The NS means that the statistical analysis of population or the sample has imprecise, incomplete, and unknown values in the data. Reference [28, 29] used the NS in the engineering rock mass. Recently, [30–32] introduced the NS in the area of acceptance sampling plans. Reference [33] introduced the NS in the area of control chart. Reference [34] proposed the variance control chart under the NS. Reference [35] proposed the control for monitoring the reliability under the NS. Reference [36] worked on the gamma chart under the uncertainty environment. More analysis to deal with the uncertainty can be seen [37, 38].

The existing control charts to monitor the nonconforming items can be applied only when all observations in the data are precise, exact, and determined. Therefore, the existing control charts using COM-Poisson distribution under classical statistics cannot be applied for the monitoring of the process when uncertain observations are in the data. By exploring the literature, and according to the best of our knowledge, there is no work on the design of attribute control charts based on COM-Poisson distribution under the neutrosophic statistical interval method. In this paper, we will first introduce the neutrosophic COM-Poisson (NCOM-Poisson) distribution. Then, the design of the attribute control chart using the NCOM-Poisson distribution will be given. We will present the structure of the control chart under the neutrosophic statistical interval method. We expect that the proposed control chart will be more effective, informative, flexible, and adequate in uncertainty environment. The algorithm to determine the average run length under neutrosophic statistical interval system will be given. The performance of the proposed control chart is compared with the chart based on classical statistics in terms of neutrosophic average run length (NARL). A simulation study and a real example are also added. The findings of this current study will redound to the benefit of industry where the statistical quality control plays an important role. Thus, the industries that apply the proposed control chart will be able to produce a high-quality product. For the researcher, the current study will help them uncover areas in the neutrosophic statistics that many researchers are not able to explore. Thus, a new methodology and its application on control chart using neutrosophic COM-Poisson in uncertainty may be arrived at. It is hoped that the proposed chart using neutrosophic COM-Poisson will be more efficient in detecting a shift in the process. It is expected that the proposed chart will be more flexible and informative under uncertainty than the existing competitor's chart. The rest of the paper is set as follows: a brief introduction about NCOM-Poisson distribution is given in Section 2. The design of the proposed chart is given in Section 3. The advantages and simulation are presented in Sections 4 and 5, respectively. An example is given in Section 6 and some concluding remarks are given in the last section.

## 2. The NCOM-Poisson Distribution

In this section, the introduction of the NCOM-Poisson Distribution is given, which is the generalization of the classical COM-Poisson distribution proposed by [1]. Suppose that a neutrosophic random variable $X_N = S_N + u_N I$, $I\epsilon\{infI, supI\}$, which consists of determinate part $S_N$ and indeterminate part $u_N I$ follows the NCOM-Poisson Distribution. Note here that $I$ presents the indeterminacy. Therefore, the neutrosophic random $X_N \epsilon \{S_N, u_N I\}$ consists of $S_N$ which is the lower value, say $X_L$, and $u_N I$ which is the upper value, say $X_U$. Based on $X_N \epsilon \{X_L, X_U\}$, let $\mu_N \epsilon \{u_N, u_N\} (> 0)$ be a neutrosophic scale parameter and $\nu_N \epsilon \{\nu_N, \nu_N\} (\geq 0)$ denotes the neutrosophic dispersion parameter. The neutrosophic probability mass function (npmf) of the neutrosophic random variable $X_N \epsilon \{S_N, u_N I\}$ is defined by

$$P\left(X_N = \frac{x_N}{\mu_N}, \nu_N\right) = \frac{u_N^{x_N}}{(X_N!)^{\mathrm{v}}\, z_N(\mu_N, \nu_N)}; \qquad X_N \epsilon \{S_N, u_N I\} \text{ for } x_N = [0,0], [1,1], [2,2] \ldots \quad (1)$$

Here $z_N(\mu_N, \nu_N) = \sum_{s_N=0}^{\infty} (u_N^{s_N}/(s_N!)^{\mathrm{v}})$ where $s_N = [0,0], [1,1], [2,2] \ldots$, shows the neutrosophic normalizing constant. The NCOM-Poisson Distribution reduces to the classical COM-Poisson distribution with no uncertainty in the population or in the sample. The NCOM-Poisson distribution becomes the neutrosophic Poisson distribution when $\nu_N \epsilon \{1, 1\}$, neutrosophic geometric distribution when $\nu_N \epsilon \{0, 0\}$, and Bernoulli distribution when $\nu_N \epsilon \{\infty, \infty\}$. Some more details about neutrosophic attribute distributions can be seen in [27]. The mean and variance of NCOM-Poisson distribution are given by

$$\mu_{X_N} = \mu_{\mathrm{N}}^{1/\mathrm{v}_N} - \frac{\mathrm{v}_N - 1}{2\mathrm{v}_N}; \quad \mu_N \epsilon \{u_N, u_N\}, \ \nu_N \epsilon \{\nu_N, \nu_N\} \tag{2}$$

$$\sigma^2{}_{X_N} = \frac{\mu_{\mathrm{N}}^{1/\mathrm{v}_N}}{\mathrm{v}_N}; \quad \mu_N \epsilon \{u_N, u_N\}, \ \nu_N \epsilon \{\nu_N, \nu_N\} \tag{3}$$

## 3. Design of Chart for NCOM-Poisson Distribution

This section presents the design of the proposed control chart for the NCOM-Poisson distribution. The proposed control chart to monitor the number of nonconformities under the neutrosophic statistics is stated as follows.

*Step#1.* From the production process, select a random sample $X_N \epsilon \{X_L, X_U\}$ of size 1 and record the number of nonconformities, say $X_{tN} \epsilon \{X_{tL}, X_{tU}\}$.

*Step#2.* The process is said to be in an in-control state if $LCL_N < X_{tN} < UCL_N$; $X_{tN} \epsilon \{X_{tL}, X_{tU}\}$, where $LCL_N$ and $UCL_N$ are lower control limit and upper control limit under the neutrosophic statistical interval method, respectively.

The proposed control chart for the NCOM-Poisson distribution under the neutrosophic statistics is the extension of the control chart for the COM-Poisson distribution under the classical statistics proposed by [6]. The proposed chart reduces to [6] chart when there is no indeterminacy in the observations; that is, $X_L = X_U$. Two neutrosophic control limits are given by

$$\mathrm{LCL}_N = \left(\mu_{\mathrm{N}}^{1/\mathrm{v}_N} - \frac{\mathrm{v}_N - 1}{2\mathrm{v}_N}\right) - \mathrm{k}_N \sqrt{\left[\frac{\mu_{\mathrm{N}}^{1/\mathrm{v}_N}}{\mathrm{v}_N}\right]}; \quad k_N \epsilon \{k_N, k_N\} \tag{4}$$

$$\mathrm{LCL}_N = \left(\mu_{\mathrm{N}}^{1/\mathrm{v}_N} - \frac{\mathrm{v}_N - 1}{2\mathrm{v}_N}\right) - \mathrm{k}_N \sqrt{\left[\frac{\mu_{\mathrm{N}}^{1/\mathrm{v}_N}}{\mathrm{v}_N}\right]}; \quad k_N \epsilon \{k_N, k_N\} \tag{5}$$

where $k_N \epsilon \{k_N, k_N\}$ is the neutrosophic control chart coefficient and will be determined through the neutrosophic algorithm later. Suppose that $\mu_{0N} \epsilon \{\mu_{0L}, \mu_{0U}\}$ is the target neutrosophic mean. The probability that the process is at $\mu_{0N} \epsilon \{\mu_{0L}, \mu_{0U}\}$ under the neutrosophic statistical interval method is derived as

$$P_N^{in} = \mathrm{P}\left(\mathrm{LCL}_N \leq X_{tN} \leq \mathrm{UCL}_N\right) \tag{6}$$

or

$$P_N^{in} = \mathrm{P}\left(\frac{\left(\mathrm{LCL}_N - \mu_{X_N}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}} \leq \frac{\left(X_{tN} - \mu_{X_N}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}} \leq \frac{\left(\mathrm{UCL}_N - \mu_{X_N}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}}\right) \tag{7}$$

where $(X_{tN} - \mu_{X_N})/\sqrt{[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N]} = Z_N$ is the neutrosophic standard normal variable; see [27, 30]. After some simplification, (7) can be written as

$$\begin{aligned} P_N^{in} &= \mathrm{P}\left(\mathrm{LCL}_N \leq \mathrm{Z_t} \leq \mathrm{UCL}_N\right) = \Phi\left(\mathrm{k}_N\right) - \Phi\left(-\mathrm{k}_N\right) \\ &= 2\Phi\left(\mathrm{k}_N\right) - 1; \quad k_N \epsilon \{k_N, k_N\} \end{aligned} \tag{8}$$

where $\Phi(\mathrm{r})$ shows the neutrosophic cumulative standard normal distribution.

The performance of any control chart is measured with the average run length (ARL) which is the indication when on the average the process will be out of control. The smaller the values of ARL, the better the performance of the control chart. The ARL under the neutrosophic statistics is termed as neutrosophic average run length (NARL) which is defined by

$$\mathrm{ARL_{0N}} = \frac{1}{1 - P_N^{in}}; \quad \mathrm{ARL_{0N}} \epsilon \{ARL_{0L}, ARL_{0U}\} \tag{9}$$

Now, suppose that due to some uncontrollable factors such as the temperature, machines, and workers the process has shifted to a new target value $\mu_{1N} = \mathrm{c}\mu_{0\mathrm{N}}$, where c is a shift and is constant. The probability that the process is at $\mu_{1N} \epsilon \{\mu_{1L}, \mu_{1U}\}$ under the neutrosophic statistical interval method is derived as

$$P_N^{out} \mid \mu_{1N} = \mathrm{P}\left(\mathrm{LCL}_N \leq X_{tN} \leq \mathrm{UCL}_N \mid \mu_{1N}\right) \tag{10}$$

or

$$P_N^{out} \mid \mu_{1N} = \mathrm{P}\left(\frac{\left(\mathrm{LCL}_N - \mathrm{c}\mu_{0\mathrm{N}}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}} \leq \frac{\left(X_{tN} - \mathrm{c}\mu_{0\mathrm{N}}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}} \leq \frac{\left(\mathrm{UCL}_N - \mathrm{c}\mu_{0\mathrm{N}}\right)}{\sqrt{\left[\mu_{\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}}\right) \tag{11}$$

After some simplification, the $P_N^{out} \mid \mu_{1N}$ is given by

$$\begin{aligned} P_N^{out} \mid \mu_{1N} &= \Phi\left(\frac{\left(1 - c^{1/\mathrm{v}_N}\right)\mu_{0\mathrm{N}}^{1/\mathrm{v}_N} + \mathrm{k}_N\sqrt{\left[\mu_{0\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}}{\sqrt{\mu_{0\mathrm{N}}^{1/\mathrm{v}_N} c^{1/\mathrm{v}_N}/\mathrm{v}_N}}\right) \\ &\quad - \Phi\left(\frac{\left(1 - c^{1/\mathrm{v}_N}\right)\mu_{0\mathrm{N}}^{1/\mathrm{v}_N} - \mathrm{k}_N\sqrt{\left[\mu_{0\mathrm{N}}^{1/\mathrm{v}_N}/\mathrm{v}_N\right]}}{\sqrt{\mu_{0\mathrm{N}}^{1/\mathrm{v}_N} c^{1/\mathrm{v}_N}/\mathrm{v}_N}}\right) \end{aligned} \tag{12}$$

Table 1: The NARLs of proposed chart when $v_N \epsilon [0.4, 0.6]$ and $u_{0N} \epsilon [2.5, 3.5]$.

| $k_N$ | [2.8071,2.8079] | [2.9355,2.9373] | [2.9997,3.0019] |
|---|---|---|---|
| $c$ | | NARL | |
| 1 | [200.02,200.55] | [300.24,302.03] | [370.03,372.73] |
| 1.0125 | [172.25,181.78] | [255.28,271.45] | [312.54,333.53] |
| 1.025 | [144.95,162.56] | [211.69,240.42] | [257.22,293.95] |
| 1.0375 | [119.88,143.74] | [172.33,210.39] | [207.70,255.84] |
| 1.05 | [98.03,125.99] | [138.64,182.38] | [165.69,220.54] |
| 1.0625 | [79.67,109.73] | [110.85,157.05] | [131.38,188.80] |
| 1.075 | [64.63,95.16] | [88.50,134.65] | [104.03,160.92] |
| 1.125 | [28.95,53.30] | [37.41,72.18] | [42.70,84.35] |
| 1.15 | [20.12,40.26] | [25.36,53.43] | [28.58,61.81] |
| 1.2 | [10.62,23.88] | [12.83,30.57] | [14.14,34.72] |
| 1.25 | [6.28,15.03] | [7.31,18.65] | [7.92,20.84] |
| 1.3 | [4.09,10.02] | [4.63,12.10] | [4.93,13.34] |
| 1.35 | [2.90,7.05] | [3.20,8.31] | [3.37,9.05] |
| 1.4 | [2.21,5.20] | [2.38,6.00] | [2.48,6.46] |
| 1.45 | [1.78,4.00] | [1.89,4.53] | [1.95,4.83] |
| 1.5 | [1.51,3.19] | [1.58,3.55] | [1.62,3.76] |
| 1.6 | [1.22,2.23] | [1.25,2.41] | [1.26,2.51] |
| 1.8 | [1.03,1.43] | [1.03,1.49] | [1.04,1.52] |
| 1.9 | [1.01,1.25] | [1.01,1.29] | [1.01,1.31] |
| 2 | [1.00,1.15] | [1.00,1.17] | [1.00,1.18] |

The NARL for the shifted process is given by

$$\mathrm{ARL}_{1\mathrm{N}} = \frac{1}{1 - P_N^{out}}; \quad \mathrm{ARL}_{1\mathrm{N}} \epsilon \{ARL_{1L}, ARL_{1U}\} \tag{13}$$

Suppose that $r_{0N}$ denotes the specified value of $\mathrm{ARL}_{0\mathrm{N}}$. The values of $\mathrm{ARL}_{1\mathrm{N}}$ for the various shift constants $c$, the neutrosophic scale parameter, and dispersion parameter are reported in Tables 1–3. Table 1 shows $\mathrm{ARL}_{1\mathrm{N}}$ when $r_{0N}$=200,300,370, $v_N \epsilon [0.4, 0.6]$ and $u_N \in [2.5, 3.5]$. Table 2 shows $\mathrm{ARL}_{1\mathrm{N}}$ when $r_{0N}$=200,300,370, $v_N \epsilon [0.9, 1.1]$, and $u_{0N} \in [3.5, 4.5]$. Table 3 shows $\mathrm{ARL}_{1\mathrm{N}}$ when $r_{0N}$= 200,300,370, $v_N \epsilon [0.36, 0.37]$, and $u_{0N} \in [2.85, 2.90]$. From Tables 1–3, it is noted that the indeterminacy interval increases when $r_{0N}$ shifted from 200 to 300. The indeterminacy also increases when both the scale and the dispersion parameters increase.

To determine the neutrosophic control chart coefficients $k_N \epsilon \{k_N, k_N\}$ and $\mathrm{ARL}_{1\mathrm{N}} \epsilon \{ARL_{1L}, ARL_{1U}\}$, the following algorithm has been applied:

(1) Predefine the values of $c$, $r_{0N}$, $v_N$, and $u_N$.

(2) Determine the suitable indeterminacy interval of $k_N \epsilon \{k_N, k_N\}$ where $\mathrm{ARL}_{0\mathrm{N}}$ is close to $r_{0N}$.

(3) Repeat the process 10,000 times and select values of $k_N \epsilon \{k_N, k_N\}$ where $\mathrm{ARL}_{0\mathrm{N}} \geq r_{0N}$.

(4) Find indeterminacy interval of $\mathrm{ARL}_{1\mathrm{N}} \epsilon \{ARL_{1L}, ARL_{1U}\}$ using $k_N \epsilon \{k_N, k_N\}$.

## 4. Advantages of the Proposed Chart

A control chart having the smaller values of NARL is said to be the more efficient control chart. This section presents the comparison of the proposed control chart with the control chart under the classical statistics in terms of NARL. The values of NARL for the proposed control chart under the neutrosophic statistics and the existing control chart proposed by [6] under the classical statistics are presented in Table 4. From Table 4, it can be noted that the proposed control chart has smaller values of NARL than the existing control chart proposed by [6] for all values of $c$ and $\mathrm{ARL}_{0\mathrm{N}} \epsilon \{ARL_{0L}, ARL_{0U}\}$. For example, when $c$=1.0125, the proposed control chart has indeterminacy interval $\mathrm{ARL}_{1\mathrm{N}} \epsilon \{312.5, 333.5\}$ while the existing chart has a determined value which is 323.5. From this comparison, we note that the proposed control chart has the advantage to detect shift earlier than the existing control chart under the classical statistics. According to [29], a method which provides parameters in an interval under the uncertainty environment is called the most effective and adequate method compared to the method which provides a determinate value. The proposed control chart provides NARL in the indeterminacy interval while the existing control chart provides the determinate values of ARL. Therefore, the proposed control chart under the neutrosophic statistical interval method is more effective and adequate to monitor the process having uncertainty, unclear, and imprecise observations.

Table 2: The NARLs of proposed chart when $v_N \epsilon[0.9, 1.1]$ and $u_{0N} \epsilon[3.5, 4.5]$.

| $k_N$ | [2.807,2.814] | [2.935,2.938] | [3,3.004] |
|---|---|---|---|
| $c$ | | NARL | |
| 0 | [200.14,204.36] | [300.03,303.05] | [370.59,374.97] |
| 1.0125 | [187.97,194.09] | [280.28,286.60] | [345.22,353.8] |
| 1.025 | [175.81,183.70] | [260.66,270.04] | [320.10,332.53] |
| 1.0375 | [163.84,173.34] | [241.46,253.58] | [295.59,311.46] |
| 1.05 | [152.20,163.11] | [222.91,237.43] | [271.99,290.83] |
| 1.0625 | [141.01,153.12] | [205.19,221.74] | [249.53,270.85] |
| 1.075 | [130.35,143.45] | [188.44,206.64] | [228.38,251.68] |
| 1.125 | [93.92,108.98] | [132.20,153.59] | [158.01,184.87] |
| 1.15 | [79.49,94.57] | [110.43,131.83] | [131.08,157.75] |
| 1.2 | [57.2,71.22] | [77.50,97.20] | [90.81,115.00] |
| 1.25 | [41.75,54.04] | [55.29,72.31] | [64.02,84.64] |
| 1.3 | [31.07,41.53] | [40.3,54.56] | [46.16,63.24] |
| 1.35 | [23.62,32.40] | [30.05,41.86] | [34.09,48.09] |
| 1.4 | [18.34,25.68] | [22.93,32.67] | [25.78,37.23] |
| 1.45 | [14.52,20.68] | [17.88,25.94] | [19.94,29.33] |
| 1.5 | [11.72,16.91] | [14.23,20.93] | [15.74,23.50] |
| 1.6 | [8.04,11.78] | [9.50,14.25] | [10.38,15.80] |
| 1.8 | [4.46,6.58] | [5.06,7.66] | [5.42,8.32] |
| 1.9 | [3.55,5.20] | [3.96,5.96] | [4.20,6.42] |
| 2 | [2.92,4.24] | [3.22,4.80] | [3.39,5.13] |

Table 3: The NARLs of proposed chart when $v_N \epsilon[0.36, 0.37]$ and $u_{0N} \epsilon[2.85, 2.90]$.

| $k_N$ | [2.807,2.811] | [2.935,2.936] | [3,3.003] |
|---|---|---|---|
| $c$ | | NARL | |
| 0 | [200.10,202.48] | [300.21,301.13] | [370.11,374.02] |
| 1.0125 | [167.01,169.90] | [246.73,249.01] | [301.76,306.82] |
| 1.025 | [132.70,136.05] | [192.32,195.81] | [232.89,238.88] |
| 1.0375 | [102.10,105.59] | [144.89,148.98] | [173.56,179.79] |
| 1.05 | [77.27,80.61] | [107.35,111.45] | [127.19,133.03] |
| 1.0625 | [58.21,61.21] | [79.22,82.99] | [92.89,98.01] |
| 1.075 | [44.01,46.61] | [58.74,62.01] | [68.19,72.50] |
| 1.125 | [15.99,17.25] | [19.95,21.51] | [22.38,24.28] |
| 1.15 | [10.42,11.29] | [12.65,13.71] | [13.98,15.25] |
| 1.2 | [5.15,5.59] | [5.95,6.47] | [6.42,7.02] |
| 1.25 | [3.03,3.274] | [3.37,3.65] | [3.56,3.88] |
| 1.3 | [2.05,2.20] | [2.22,2.38] | [2.31,2.49] |
| 1.35 | [1.56,1.65] | [1.64,1.75] | [1.69,1.80] |
| 1.4 | [1.30,1.36] | [1.34,1.41] | [1.37,1.44] |
| 1.45 | [1.15,1.19] | [1.18,1.22] | [1.19,1.23] |
| 1.5 | [1.07,1.10] | [1.09,1.11] | [1.09,1.12] |
| 1.6 | [1.01,1.02] | [1.02,1.02] | [1.02,1.03] |
| 1.8 | [1.00,1.00] | [1.00,1.00] | [1.00,1.00] |
| 1.9 | [1.00,1.00] | [1.00,1.00] | [1.00,1.00] |
| 2 | [1.00,1.00] | [1.00,1.00] | [1.00,1.00] |

Table 4: Comparison of NARLs when $v_N \epsilon [0.4, 0.6]$ and $u_{0N} \epsilon [2.5, 3.5]$.

| | Proposed Chart | | | Existing Chart | | |
|---|---|---|---|---|---|---|
| $c$ | NARL | | | ARL | | |
| 1 | [200.0,200.5] | [300.2,302.0] | [370.0,372.7 | 200.0 | 300.0 | 370.0 |
| 1.0125 | [172.2,181.7] | [255.2,271.4] | [312.5,333.5] | 177.5 | 263.7 | 323.5 |
| 1.025 | [144.9,162.5] | [211.6,240.4] | [257.2,293.9] | 154.9 | 227.4 | 277.3 |
| 1.0375 | [119.8,143.7] | [172.3,210.3] | [207.7,255.8] | 133.2 | 193.1 | 234.0 |
| 1.05 | [98.0,125.9] | [138.6,182.3] | [165.6,220.5] | 113.4 | 162.3 | 195.3 |
| 1.0625 | [79.6,109.7] | [110.8,157.0] | [131.3,188.8] | 95.9 | 135.4 | 161.8 |
| 1.075 | [64.6,95.1] | [88.5,134.6] | [104.0,160.9] | 80.8 | 112.5 | 133.6 |
| 1.125 | [28.9,53.3] | [37.4,72.1] | [42.7,84.3] | 40.9 | 54.2 | 62.7 |
| 1.15 | [20.1,40.2] | [25.3,53.4] | [28.5,61.8] | 29.7 | 38.5 | 44.0 |
| 1.2 | [10.6,23.8] | [12.8,30.5] | [14.1,34.7] | 16.6 | 20.7 | 23.2 |
| 1.25 | [6.2,15.0] | [7.3,18.6] | [7.9,20.8] | 10.0 | 12.1 | 13.4 |
| 1.3 | [4.0,10.0] | [4.6,12.1] | [4.9,13.3] | 6.6 | 7.7 | 8.4 |
| 1.35 | [2.9,7.0] | [3.2,8.3] | [3.3,9.0] | 4.6 | 5.2 | 5.6 |
| 1.4 | [2.2,5.2] | [2.3,6.0] | [2.4,6.4] | 3.4 | 3.8 | 4.0 |
| 1.45 | [1.7,4.0] | [1.8,4.5] | [1.9,4.8] | 2.6 | 2.9 | 3.0 |
| 1.5 | [1.5,3.1] | [1.5,3.5] | [1.6,3.7] | 2.1 | 2.3 | 2.4 |
| 1.6 | [1.2,2.2] | [1.2,2.4] | [1.2,2.5] | 1.6 | 1.6 | 1.7 |
| 1.8 | [1.0,1.4] | [1.0,1.4] | [1.0,1.5] | 1.1 | 1.1 | 1.1 |
| 1.9 | [1.0,1.2] | [1.0,1.2] | [1.0,1.3] | 1.0 | 1.0 | 1.0 |
| 2 | [1.0,1.1] | [1.0,1.1] | [1.0,1.18] | 1.0 | 1.0 | 1.0 |

## 5. Simulation Study

The efficiency of the proposed control chart over the chart proposed by [6] will be discussed using the data generated from the NCOM-Poisson distribution. For this study, let $v_N \epsilon [0.4, 0.6]; u_{0N} \in [2.5, 3.5]$ and $\text{ARL}_{0N} \epsilon \{370, 370\}$. The first twenty values are generated by assuming that the process is at the in-control state and next thirty observations are generated when the process has shifted with $c$=1.2. For these specified parameters, the tabulated NARL is $\text{ARL}_{1N} \epsilon \{14.14, 34.72\}$. It means that it is expected that the first out-of-control sample will be between 14.14 and 34.72 samples. The neutrosophic statistics $X_{tN}$ is computed and plotted in Figure 1. From Figure 1, it can be seen that the 34th sample is out of the control limit. It is noted that six points in Figure 1 are in indeterminacy interval. Figure 2 shows the plotting of the existing control chart for the same level of all specified parameters. From 2, it can be noted that all values of the plotting statistic are between the control limits. It is noted from Figure 2 that no point is near the control limits. By comparing both figures, it is concluded that the proposed control chart detects a shift in the process under the uncertainty environment while the existing control chart does not provide any signal about the shift in the process. Therefore, the proposed control chart under the NS is more efficient, effective, informative, and adequate to be used in uncertainty than [6] chart.

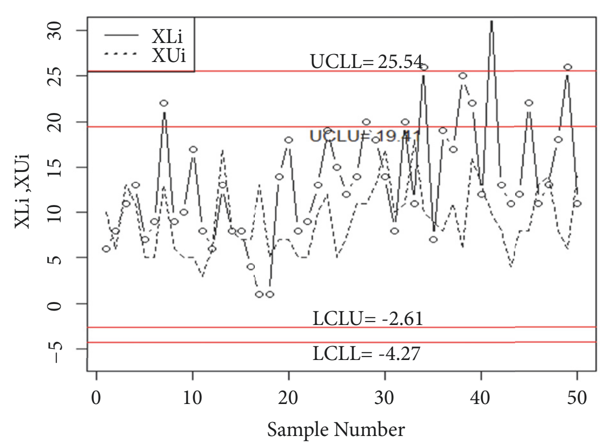


Figure 1: The proposed control chart for simulated data.

## 6. Case Study

This section presents the application of the proposed control chart in a well-known electrical company in Saudi Arabia. This company manufactured the printed circuits boards (PCB) which have been used in several electronic goods, electrical items, and computers. The main function of PCB is to connect the features with each other and to provide the mechanical support to the electrical product. The company is not sure about the proportion defective parameter for the monitoring of PCB product. In addition, due to the complex system of PCB, there is uncertainty in a number of nonconformities in a sample. Due to uncertainty in proportion defective parameter, it is not possible to apply the control chart designed under the classical statistics. The company is interested to apply the proposed control chart

Table 5: Number of nonconformities in 100 samples.

| Sample# | nonconformities | Sample# | nonconformities |
|---|---|---|---|
| 1 | [1,1] | 16 | [3,3] |
| 2 | [2,2] | 17 | [5,5] |
| 3 | [3,3] | 18 | [5,5] |
| 4 | [3,3] | 19 | [4,4] |
| 5 | [1,1] | 20 | [6,6] |
| 6 | [1,1] | 21 | [5,5] |
| 7 | [8,9] | 22 | [7,7] |
| 8 | [2,2] | 23 | [5,5] |
| 9 | [5,5] | 24 | [8,8] |
| 10 | [11,11] | 25 | [2,2] |
| 11 | [2,3] | 26 | [5,6] |
| 12 | [1,1] | 27 | [6,6] |
| 13 | [0,0] | 28 | [8,9] |
| 14 | [2,2] | 29 | [3,3] |
| 15 | [5,5] | 30 | [7,7] |

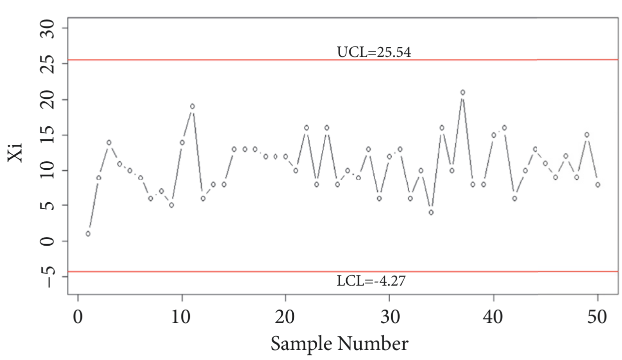


Figure 2: The exiting control chart for simulated data.

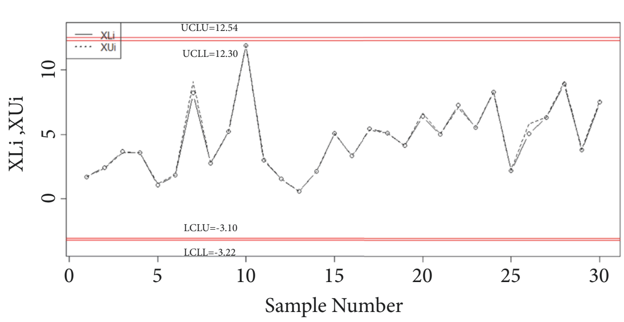


Figure 3: The proposed control chart for the real data.

for the monitoring of a number of nonconformities. Let the company decide $ARL_{0N}\epsilon\{370, 370\}$ and sample size is 100. The data on PCB is given in Table 5.

The data presented in Table 5 follows the NCOM-Poisson distribution with parameters $\mu_N\epsilon\{2.5485, 2.5485\}$ and $\nu_N\epsilon\{0.634222, 0.657101\}$. The number of nonconformities is plotted in Figure 3. From Figure 3, it can be seen that, although the PCB manufacturing process is in control,

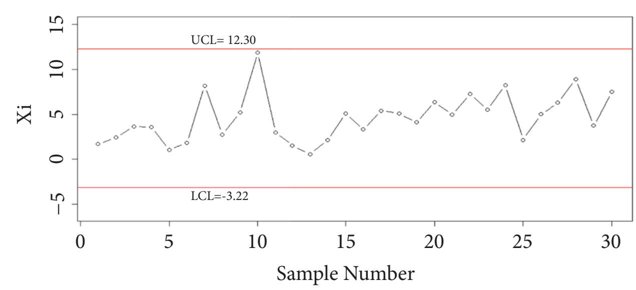


Figure 4: The existing control chart for the real data.

points 7 and 9 are near the control limits. These points near the control limit need the engineer's attention. The chart under the classical statistics proposed by [6] is also shown in Figure 4. By comparing Figure 3 with Figure 4, the existing control chart does not reflect the indeterminacy interval in the proportion parameter and control limits. The existing chart only provides the determined values of all parameters, which are not reasonable in uncertainty. Therefore, the proposed control chart is more effective and flexible to be used under uncertainty environment.

## 7. Concluding Remarks

This paper introduced the NCOM-Poisson distribution first. Then, we proposed the control chart using this distribution under the neutrosophic statistics. The proposed chart is the extension of the control chart using the COM-Poisson under the classical statistics. The NARLs are derived under the neutrosophic statistical method. From the comparison, it is concluded that the proposed control chart performs better than the existing control chart in detecting the shift in the process. The proposed control chart can be applied when observations are unclear, fuzzy, and imprecise. The proposed control chart is more adequate and is an effective method under the uncertainty environment. The proposed control chart can be only applied when the data follows the NCOM-Poisson distribution. The proposed control chart can be applied in the electronics industry, the food industry, and automobile industry. The results of the proposed control chart can be improved using the repetitive sampling and the multiple dependent state sampling as future research.

## Abbreviations

| | |
|---|---|
| SPC: | Statistical process control |
| COM-Poisson: | Conway and Maxwell distribution |
| NCOM-Poisson: | Neutrosophic Conway and Maxwell distribution |
| NS: | Neutrosophic statistics |
| NARL: | Neutrosophic average run length |
| $X_N$: | Neutrosophic random variable |
| $S_N$: | Determinate part |
| $u_N I$: | Indeterminate part |
| $X_L$: | Lower value |
| $X_U$: | Upper value |
| $\mu_N \epsilon \{u_N, u_N\}$: | Neutrosophic scale parameter |
| $\nu_N \epsilon \{\nu_N, \nu_N\}$: | Neutrosophic dispersion parameter |
| npmf: | Neutrosophic probability mass function |
| $z_N(\mu_N, \nu_N)$: | Neutrosophic normalizing constant |
| $\mu_{X_N}$: | Neutrosophic mean |
| $\sigma^2{}_{X_N}$: | Neutrosophic variance |
| $LCL_N$: | Neutrosophic lower control limit |
| $UCL_N$: | Neutrosophic upper control limit |
| $k_N \epsilon \{k_N, k_N\}$: | Neutrosophic control chart coefficient |
| $P_N^{in}$: | The probability of in-control process |
| $P_N^{out} \mid \mu_{1N}$: | The probability of out-of-control process. |

## Data Availability

The data is given in the paper.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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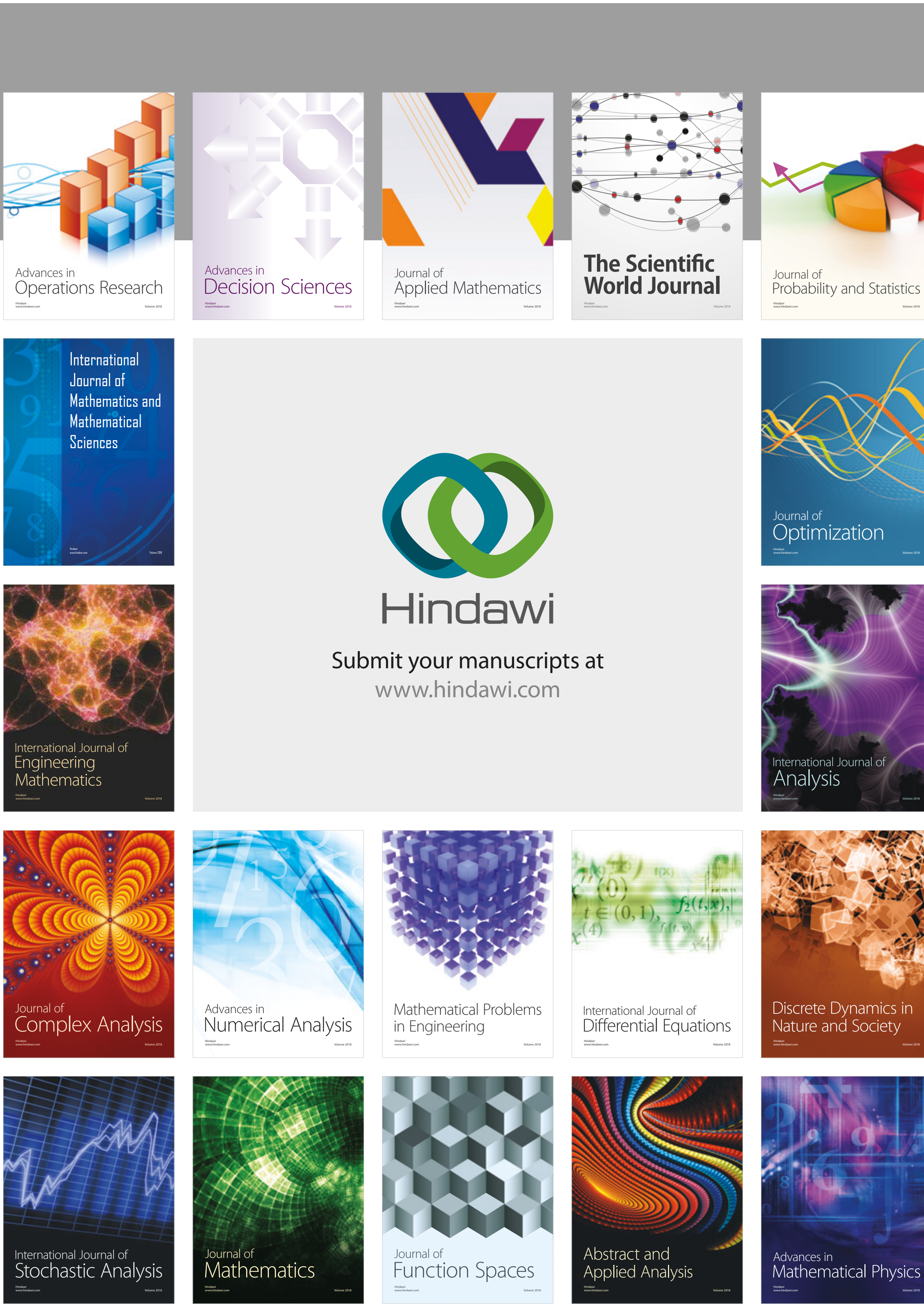
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