



Research article

Distance measures between interval complex neutrosophic sets and their applications in multi-criteria group decision making

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Abstract: As an extension of neutrosophic set, interval complex neutrosophic set is a new research topic in the field of neutrosophic set theory, which can handle the uncertain, inconsistent and incomplete information in periodic data. Distance measure is an important tool to solve some problems in engineering and science. Hence, this paper presents some interval complex neutrosophic distance measures to deal with multi-criteria group decision-making problems. Firstly, this paper shows the definitions of interval complex neutrosophic set, and especially some novel set theoretic properties. Then, some new distance measures based on Hamming, Euclidean and Hausdorff metrics are proposed. Next, an approach is developed to rank the alternatives in multi-criteria group decision-making problems. Finally, a numerical example is given to demonstrate the practicality and effectiveness of these distance measures.

Keywords: interval complex neutrosophic set; distance measures; multi-criteria group decision making

Mathematics Subject Classification: 28E10, 90B50

1. Introduction

Multi-criteria group decision-making (MCGDM) is the process of ranking a series of alternatives and find the optimal one from them. During the last decades, most decision makers tend to evaluate the rating values of each criterion with crisp number. However, due to the fuzziness of human thinking and the complexity and uncertainty of objective things in real life, the information in MCGDM problem is either vague, imprecise or uncertain [1]. To deal with it, the theory of fuzzy set (FS) [2], intuitionistic fuzzy set (IFS) [3] and interval-valued IFS (IVIFS) [4], which can express the evaluation values more reasonably, were proposed. Based on these theories, some decision making (DM) methodologies have been presented and applied in various disciplines. For instance, Xu [5, 6] presented some aggregation operators to aggregation information such as geometric aggregation operator and weighted averaging

operator. However, FS, IFS and IVIFS can only deal with incomplete information, but not uncertain and inconsistent information.

Therefore, Smarandache [7] firstly introduced the neutrosophic set (NS) as a generalization of classical set, FS, IFS and IVIFS. However, NS was mainly put forward from a philosophical viewpoint which is difficult to be applied in real life. So single-valued neutrosophic set (SVNS) [8] and interval neutrosophic set (INS) [9] were proposed. Based on SVNS, Ye [10] proposed the correlation coefficient and weighted correlation coefficient for SVNS and used them to handle multi-criteria decision-making (MCDM) problems. And a single-valued neutrosophic cross-entropy for MCDM problem was proposed in [11]. Besides, some researchers studied the neutrosophic algebraic structures or its extensions. Singh [12] proposed a method to generate three-way fuzzy concept lattice using neutrosophic set. Cetkin and Aygun [13] proposed neutrosophic subgroup and neutrosophic normal subgroup with their basic properties and characterizations.

In recent years, "big data" which is frequently characterized by uncertainty and periodicity has become a new research trend. However, FS, IFS, IVIFS, SVNS, INS can only deal with the data with uncertainty but not periodicity. In order to handle this situation, Ramot et al. [14] introduced a seminal concept and called it complex fuzzy set (CFS) which is a combination of fuzzy set and complex number. In CFS, the degree of membership is represented in the form of complex value, denoted as $\mu = r_s(x) \cdot e^{j\omega_s(x)}$ where $j = \sqrt{-1}$ and $r_s(x) \in [0, 1]$, $\omega_s(x)$ is a real value. Later on, Alkouri and Salleh [15] introduced the concept of complex intuitionistic fuzzy set (CIFS) by adding the degree of non-membership with complex-valued form and defined the basic operations of CIFS. To provide the more freedom to the decision-makers, it is advisable to ask the experts to describe their preferences by means of intervals. So Garg and Rani [16] introduced the complex interval-valued intuitionistic fuzzy set (CIVIFS) with their algebraic operators and corresponding aggregation operators, then established a model to address MCGDM problems. Soon after, complex neutrosophic set (CNS) as an extension of CFS, CIFS, CIVIFS was proposed by Ali and Smarandache [17], and applied it in the signal processing. Later on, Ali et al. [18] firstly attempted to defined the interval complex neutrosophic set (ICNS). In ICNS, its set theoretic properties, operations and operational rules were introduced, then a MCGDM model was established under interval complex neutrosophic environment.

The selection of distance formula is a crucial step in MCGDM problems and several classical distance formulas are usually used, such as Hamming distance, Euclidean distance and Hausdorff distance. Rani and Garg [19] proposed some distance measures between the CIFSs based on Hamming, Euclidean and Hausdorff metrics and discussed various desirable relations in detail. Ali and Smarandache [17] proposed distance measure and δ -equalities of CNS, and then discussed their properties. Kumar and Bajaj [20] proposed some distance and entropy measures for complex intuitionistic fuzzy soft sets (CIFSSs) and applies them in MCDM problems. In view of the Cartesian representation of pure complex fuzzy grade of membership, Ali [21] proposed some new operations of complex fuzzy classes, and introduced the δ -equalities of complex fuzzy classes with corresponding implication operators. Based on Hamming and Euclidean metrics, Dai et al. [22] proposed some interval-valued complex fuzzy distance measures and discussed their geometric properties. Based on traditional distance metrics, Hong et al. [23] presented a weighted parameter interval neutrosophic distance measure, and applied it into TODIM method to deal with multi-attribute decision making problems.

The purpose of this paper is to construct some interval complex neutrosophic distance measures and apply them into MCGDM problem. The specific arrangements of this article are structured as follows. In section 2, we introduce the concept of ICNS. Section 3 proposes some set theoretic properties of ICNS, such as operational rules, aggregated operator and comparison method. In section 4, we present some distance measures which are satisfied with the axiomatic conditions. A MCGDM approach based on the operator and distance measure is proposed in section 5. In section 6, this paper illustrates the practicality and effectiveness of the proposed approach via a numerical example. In section 7, a conclusion of this paper is given.

2. Preliminaries

2.1. Neutrosophic set

Definition 2.1. [7] Let X be a space of points(objects), with a generic element in X denoted by x . A NS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is $T_{A(x)}: X \rightarrow]0^-, 1^+[$, $I_{A(x)}: X \rightarrow]0^-, 1^+[$, $F_{A(x)}: X \rightarrow]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [7] The complement of A is denoted by A^C and is defined as

$$T_{A^C}(x) = \{1^+\} \ominus T_A(x), I_{A^C}(x) = \{1^+\} \ominus I_A(x), F_{A^C}(x) = \{1^+\} \ominus F_A(x)$$

for every x in X .

Definition 2.3. [7] A NS A is contained in the other NS B , $A \subseteq B$ if and only if

$$\inf T_A(x) \leq \inf T_B(x), \sup T_A(x) \leq \sup T_B(x),$$

$$\inf I_A(x) \geq \inf I_B(x), \sup I_A(x) \geq \sup I_B(x),$$

$$\inf F_A(x) \geq \inf F_B(x), \sup F_A(x) \geq \sup F_B(x)$$

for every x in X .

2.2. Interval complex neutrosophic set

Definition 2.4. [18] Let X be a space of points(objects) with generic elements in X denoted by x . An ICNS S in X is characterized by a truth-membership function $T_S(x)$, an indeterminacy-membership function $I_S(x)$, and a falsity-membership function $F_S(x)$, which are satisfied the following conditions:

$$T_S(x) : X \rightarrow \Gamma^{[0,1]} \times R, T_S(x) = t_S(x) \cdot e^{j\omega_S(x)}$$

$$I_S(x) : X \rightarrow \Gamma^{[0,1]} \times R, I_S(x) = i_S(x) \cdot e^{j\psi_S(x)}$$

$$F_S(x) : X \rightarrow \Gamma^{[0,1]} \times R, F_S(x) = f_S(x) \cdot e^{j\phi_S(x)}$$

where $\Gamma^{[0,1]}$ is the collection of interval neutrosophic sets and R is the set of real numbers, $t_S(x)$ is the interval truth membership function, $i_S(x)$ is the interval indeterminate membership function and $f_S(x)$

is the interval falsehood membership function, while $e^{j\omega_S(x)}$, $e^{j\psi_S(x)}$ and $e^{j\phi_S(x)}$ are the corresponding interval-valued phase terms, respectively, with $j = \sqrt{-1}$. In set theoretic form, an interval complex neutrosophic set can be written as:

$$S = \left\{ \left\langle \frac{t_S(x) \cdot e^{j\omega_S(x)}, i_S(x) \cdot e^{j\psi_S(x)}, f_S(x) \cdot e^{j\phi_S(x)}}{x} \right\rangle : x \in X \right\} \quad (2.1)$$

In (2.1), the amplitude interval-valued terms $T_S(x)$, $I_S(x)$, $F_S(x)$ can be further split as $t_S(x) = [t_S^L(x), t_S^U(x)]$, $i_S(x) = [i_S^L(x), i_S^U(x)]$ and $f_S(x) = [f_S^L(x), f_S^U(x)]$. Similarly, for the phase terms: $\omega_S(x) = [\omega_S^L(x), \omega_S^U(x)]$, $\psi_S(x) = [\psi_S^L(x), \psi_S^U(x)]$ and $\phi_S(x) = [\phi_S^L(x), \phi_S^U(x)]$.

Definition 2.5. [18] The complement of an ICNS S is denoted by S^c and is defined as

$$S^c = \left\{ \left\langle \frac{t_{S^c}(x) \cdot e^{j\omega_{S^c}(x)}, i_{S^c}(x) \cdot e^{j\psi_{S^c}(x)}, f_{S^c}(x) \cdot e^{j\phi_{S^c}(x)}}{x} \right\rangle : x \in X \right\} \quad (2.2)$$

where $t_{S^c}(x) = f_S(x)$, $\omega_{S^c}(x) = 2\pi - \omega_S(x)$, $i_{S^c}(x) = [1 - i_S^U(x), 1 - i_S^L(x)]$, $\psi_{S^c}(x) = 2\pi - \psi_S(x)$, $f_{S^c}(x) = t_S(x)$, $\phi_{S^c}(x) = 2\pi - \phi_S(x)$.

3. Set theoretic properties of interval complex neutrosophic set

Definition 3.1. Let A and B be two ICNSs, then the operational rules of ICNS are defined as follows:

(1) The addition of A and B , denoted as $A + B$, and is defined as:

$$\begin{aligned} T_{A+B}(x) &= [t_A^L(x) + t_B^L(x) - t_A^L(x)t_B^L(x), t_A^U(x) + t_B^U(x) - t_A^U(x)t_B^U(x)] \cdot e^{j[\omega_{A+B}^L(x), \omega_{A+B}^U(x)]} \\ I_{A+B}(x) &= [i_A^L(x)i_B^L(x), i_A^U(x)i_B^U(x)] \cdot e^{j[\psi_{A+B}^L(x), \psi_{A+B}^U(x)]} \\ F_{A+B}(x) &= [f_A^L(x)f_B^L(x), f_A^U(x)f_B^U(x)] \cdot e^{j[\phi_{A+B}^L(x), \phi_{A+B}^U(x)]} \end{aligned}$$

the addition of phase terms is defined below:

$$\begin{aligned} \omega_{A+B}^L(x) &= 2\pi \left(\frac{\omega_A^L(x)}{2\pi} + \frac{\omega_B^L(x)}{2\pi} - \frac{\omega_A^L(x)\omega_B^L(x)}{(2\pi)^2} \right) \\ \omega_{A+B}^U(x) &= 2\pi \left(\frac{\omega_A^U(x)}{2\pi} + \frac{\omega_B^U(x)}{2\pi} - \frac{\omega_A^U(x)\omega_B^U(x)}{(2\pi)^2} \right) \\ \psi_{A+B}^L(x) &= 2\pi \left(\frac{\psi_A^L(x)}{2\pi} \cdot \frac{\psi_B^L(x)}{2\pi} \right), \psi_{A \times B}^U(x) = 2\pi \left(\frac{\psi_A^U(x)}{2\pi} \cdot \frac{\psi_B^U(x)}{2\pi} \right) \\ \phi_{A+B}^L(x) &= 2\pi \left(\frac{\phi_A^L(x)}{2\pi} \cdot \frac{\phi_B^L(x)}{2\pi} \right), \phi_{A \times B}^U(x) = 2\pi \left(\frac{\phi_A^U(x)}{2\pi} \cdot \frac{\phi_B^U(x)}{2\pi} \right) \end{aligned}$$

(2) The product of A and B , denoted as $A \times B$, and is defined as:

$$\begin{aligned}
T_{A \times B}(x) &= [t_A^L(x)t_B^L(x), t_A^U(x)t_B^U(x)] \cdot e^{j\pi[\omega_{A \times B}^L(x), \omega_{A \times B}^U(x)]} \\
I_{A \times B}(x) &= [i_A^L(x) + i_B^L(x) - i_A^L(x)i_B^L(x), i_A^U(x) + i_B^U(x) - i_A^U(x)i_B^U(x)] \cdot e^{j\pi[\psi_{A \times B}^L(x), \psi_{A \times B}^U(x)]} \\
F_{A \times B}(x) &= [f_A^L(x) + f_B^L(x) - f_A^L(x)f_B^L(x), f_A^U(x) + f_B^U(x) - f_A^U(x)f_B^U(x)] \cdot e^{j\pi[\phi_{A \times B}^L(x), \phi_{A \times B}^U(x)]}
\end{aligned}$$

the product of phase terms is defined below:

$$\begin{aligned}
\omega_{A \times B}^L(x) &= 2\pi \left(\frac{\omega_A^L(x)}{2\pi} \cdot \frac{\omega_B^L(x)}{2\pi} \right), \omega_{A \times B}^U(x) = 2\pi \left(\frac{\omega_A^U(x)}{2\pi} \cdot \frac{\omega_B^U(x)}{2\pi} \right) \\
\psi_{A \times B}^L(x) &= 2\pi \left(\frac{\psi_A^L(x)}{2\pi} + \frac{\psi_B^L(x)}{2\pi} - \frac{\psi_A^L(x)\psi_B^L(x)}{(2\pi)^2} \right) \\
\psi_{A \times B}^U(x) &= 2\pi \left(\frac{\psi_A^U(x)}{2\pi} + \frac{\psi_B^U(x)}{2\pi} - \frac{\psi_A^U(x)\psi_B^U(x)}{(2\pi)^2} \right) \\
\phi_{A \times B}^L(x) &= 2\pi \left(\frac{\phi_A^L(x)}{2\pi} + \frac{\phi_B^L(x)}{2\pi} - \frac{\phi_A^L(x)\phi_B^L(x)}{(2\pi)^2} \right) \\
\phi_{A \times B}^U(x) &= 2\pi \left(\frac{\phi_A^U(x)}{2\pi} + \frac{\phi_B^U(x)}{2\pi} - \frac{\phi_A^U(x)\phi_B^U(x)}{(2\pi)^2} \right)
\end{aligned}$$

(3) The scalar multiplication of A is an interval complex neutrosophic set denoted as $C = \lambda A$, $\lambda > 0$, and defined as:

$$\begin{aligned}
T_C(x) &= [1 - (1 - t_A^L(x))^\lambda, 1 - (1 - t_A^U(x))^\lambda] \cdot e^{j[\omega_C^L(x), \omega_C^U(x)]} \\
I_C(x) &= [(i_A^L(x))^\lambda, (i_A^U(x))^\lambda] \cdot e^{j[\psi_C^L(x), \psi_C^U(x)]} \\
F_C(x) &= [(f_A^L(x))^\lambda, (f_A^U(x))^\lambda] \cdot e^{j[\phi_C^L(x), \phi_C^U(x)]}
\end{aligned}$$

The scalar multiplication of phase terms is defined below:

$$\begin{aligned}
\omega_C^L(x) &= 2\pi \left(1 - \left(1 - \frac{\omega_A^L(x)}{2\pi} \right)^\lambda \right), \omega_C^U(x) = 2\pi \left(1 - \left(1 - \frac{\omega_A^U(x)}{2\pi} \right)^\lambda \right) \\
\psi_C^L(x) &= 2\pi \left(\frac{\psi_A^L(x)}{2\pi} \right)^\lambda, \psi_C^U(x) = 2\pi \left(\frac{\psi_A^U(x)}{2\pi} \right)^\lambda \\
\phi_C^L(x) &= 2\pi \left(\frac{\phi_A^L(x)}{2\pi} \right)^\lambda, \phi_C^U(x) = 2\pi \left(\frac{\phi_A^U(x)}{2\pi} \right)^\lambda
\end{aligned}$$

(4) The power of A is denoted as $D = A^\lambda$, $\lambda > 0$ and defined as:

$$\begin{aligned}
T_D(x) &= [(t_A^L(x))^\lambda, (t_A^U(x))^\lambda] \cdot e^{j[\omega_D^L(x), \omega_D^U(x)]} \\
I_D(x) &= [1 - (1 - i_A^L(x))^\lambda, 1 - (1 - i_A^U(x))^\lambda] \cdot e^{j[\psi_D^L(x), \psi_D^U(x)]} \\
F_D(x) &= [1 - (1 - f_A^L(x))^\lambda, 1 - (1 - f_A^U(x))^\lambda] \cdot e^{j[\phi_D^L(x), \phi_D^U(x)]}
\end{aligned}$$

The power of phase terms is defined below:

$$\begin{aligned}\omega_D^L(x) &= 2\pi \left(\frac{\omega_A^L(x)}{2\pi} \right)^\lambda, \omega_D^U(x) = 2\pi \left(\frac{\omega_A^U(x)}{2\pi} \right)^\lambda \\ \psi_D^L(x) &= 2\pi \left(1 - \left(1 - \frac{\psi_A^L(x)}{2\pi} \right)^\lambda \right), \psi_D^U(x) = 2\pi \left(1 - \left(1 - \frac{\psi_A^U(x)}{2\pi} \right)^\lambda \right) \\ \phi_D^L(x) &= 2\pi \left(1 - \left(1 - \frac{\phi_A^L(x)}{2\pi} \right)^\lambda \right), \phi_D^U(x) = 2\pi \left(1 - \left(1 - \frac{\phi_A^U(x)}{2\pi} \right)^\lambda \right)\end{aligned}$$

Definition 3.2. Let $a_k = \left([t_k^L, t_k^U] \cdot e^{j[\omega_k^L, \omega_k^U]}, [i_k^L, i_k^U] \cdot e^{j[\psi_k^L, \psi_k^U]}, [f_k^L, f_k^U] \cdot e^{j[\phi_k^L, \phi_k^U]} \right)$ ($k = 1, 2, \dots, n$) be a collection of interval complex neutrosophic numbers (ICNNs), the interval complex neutrosophic weighted averaging (ICNWA) operator can be defined as follows:

$$ICNWA_w(a_1, a_2, \dots, a_n) = w_1 a_1 + w_2 a_2 + \dots + w_n a_n$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of a_k , such that $0 < w_k < 1$, $\sum_{k=1}^n w_k = 1$. Then the ICNWA operator is denoted as following:

$$\begin{aligned}ICNWA_w(a_1, a_2, \dots, a_n) &= \sum_{k=1}^n w_k a_k \\ &= \left[\left[1 - \prod_{k=1}^n (1 - t_k^L)^{w_k}, 1 - \prod_{k=1}^n (1 - t_k^U)^{w_k} \right] \cdot e^{j \left[2\pi \left(1 - \prod_{k=1}^n \left(1 - \frac{\omega_k^L(x)}{2\pi} \right)^{w_k} \right), 2\pi \left(1 - \prod_{k=1}^n \left(1 - \frac{\omega_k^U(x)}{2\pi} \right)^{w_k} \right) \right]} \right], \\ &\left[\prod_{k=1}^n (i_k^L)^{w_k}, \prod_{k=1}^n (i_k^U)^{w_k} \right] \cdot e^{j \left[2\pi \left(\prod_{k=1}^n \left(\frac{\psi_k^L}{2\pi} \right)^{w_k} \right), 2\pi \left(\prod_{k=1}^n \left(\frac{\psi_k^U}{2\pi} \right)^{w_k} \right) \right]}, \\ &\left[\prod_{k=1}^n (f_k^L)^{w_k}, \prod_{k=1}^n (f_k^U)^{w_k} \right] \cdot e^{j \left[2\pi \left(\prod_{k=1}^n \left(\frac{\phi_k^L}{2\pi} \right)^{w_k} \right), 2\pi \left(\prod_{k=1}^n \left(\frac{\phi_k^U}{2\pi} \right)^{w_k} \right) \right]}\end{aligned} \quad (3.1)$$

Especially when the weight vector is $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n} \right)$, the ICNWA operator will reduce to interval complex neutrosophic average (ICNA) operator.

Definition 3.3. Let A be an ICNN, then the score function $S(A)$ of A is defined as:

$$S(A) = \frac{1}{12} \left((2 + t_A^L - i_A^L - f_A^L) + (2 + t_A^U - i_A^U - f_A^U) + \frac{1}{2\pi} \left((4\pi + \omega_A^L - \psi_A^L - \phi_A^L) + (4\pi + \omega_A^U - \psi_A^U - \phi_A^U) \right) \right) \quad (3.2)$$

Definition 3.4. Let A be an ICNN, then the accuracy function $H(A)$ of A is defined as:

$$H(A) = \frac{1}{3} \left(\frac{t_A^L - f_A^L + t_A^U - f_A^U}{2} + \frac{\omega_A^L - \phi_A^L + \omega_A^U - \phi_A^U}{2\pi} \right) \quad (3.3)$$

Definition 3.5. Let A_1 and A_2 be two ICNNs, and S be the score functions, H be the accuracy functions. If $S(A_1) < S(A_2)$ then $A_1 < A_2$, if $S(A_1) = S(A_2)$ then

- (1) If $H(A_1) < H(A_2)$, then $A_1 < A_2$;
- (2) If $H(A_1) = H(A_2)$, then $A_1 = A_2$.

4. Distance measures between ICNSs

Definition 4.1. [24] Let $X = \{x_1, x_2, \dots, x_n\}$ be the universe of discourse, $\Phi(X)$ be the family of ICNSs and \mathbb{R}^+ be the set of non-negative real numbers. A distance measure of interval complex neutrosophic set is a function $d : \Phi(X) \times \Phi(X) \rightarrow \mathbb{R}^+$, which satisfies the following three conditions: for any $A, B, C \in \Phi(X)$,

- (1) $d(A, B) \geq 0$ and $d(A, B) = 0$ if and only if $A = B$;
- (2) $d(A, B) = d(B, A)$;
- (3) $d(A, C) \leq d(A, B) + d(B, C)$.

Now, we define the some distance measures with interval complex neutrosophic sets as follows: for any two ICNSs A and B ,

Definition 4.2. The normalized Hamming distance:

$$d_{Hm}(A, B) = \frac{1}{12n} \sum_{j=1}^n \left\{ \left| t_A^L(x_j) - t_B^L(x_j) \right| + \left| t_A^U(x_j) - t_B^U(x_j) \right| + \left| i_A^L(x_j) - i_B^L(x_j) \right| + \left| i_A^U(x_j) - i_B^U(x_j) \right| + \left| f_A^L(x_j) - f_B^L(x_j) \right| + \left| f_A^U(x_j) - f_B^U(x_j) \right| + \frac{1}{2\pi} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right| + \left| \omega_A^U(x_j) - \omega_B^U(x_j) \right| + \left| \psi_A^L(x_j) - \psi_B^L(x_j) \right| + \left| \psi_A^U(x_j) - \psi_B^U(x_j) \right| + \left| \phi_A^L(x_j) - \phi_B^L(x_j) \right| + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right| \right) \right\} \quad (4.1)$$

Definition 4.3. The normalized Euclidean distance:

$$d_E(A, B) = \left\{ \frac{1}{12n} \sum_{j=1}^n \left[\left| t_A^L(x_j) - t_B^L(x_j) \right|^2 + \left| t_A^U(x_j) - t_B^U(x_j) \right|^2 + \left| i_A^L(x_j) - i_B^L(x_j) \right|^2 + \left| i_A^U(x_j) - i_B^U(x_j) \right|^2 + \left| f_A^L(x_j) - f_B^L(x_j) \right|^2 + \left| f_A^U(x_j) - f_B^U(x_j) \right|^2 + \frac{1}{(2\pi)^2} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right|^2 + \left| \omega_A^U(x_j) - \omega_B^U(x_j) \right|^2 + \left| \psi_A^L(x_j) - \psi_B^L(x_j) \right|^2 + \left| \psi_A^U(x_j) - \psi_B^U(x_j) \right|^2 + \left| \phi_A^L(x_j) - \phi_B^L(x_j) \right|^2 + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right|^2 \right) \right] \right\}^{\frac{1}{2}} \quad (4.2)$$

Definition 4.4. The normalized Hausdorff distance:

$$d_{Hd}(A, B) = \frac{1}{n} \sum_{j=1}^n \max \left\{ \max \left(\frac{1}{2} \left(\left| t_A^L(x_j) - t_B^L(x_j) \right| + \left| t_A^U(x_j) - t_B^U(x_j) \right| \right), \frac{1}{2} \left(\left| i_A^L(x_j) - i_B^L(x_j) \right| + \left| i_A^U(x_j) - i_B^U(x_j) \right| \right) \right), \frac{1}{2} \left(\left| f_A^L(x_j) - f_B^L(x_j) \right| + \left| f_A^U(x_j) - f_B^U(x_j) \right| \right), \frac{1}{2\pi} \max \left(\frac{1}{2} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right| + \left| \omega_A^U(x_j) - \omega_B^U(x_j) \right| \right), \frac{1}{2} \left(\left| \psi_A^L(x_j) - \psi_B^L(x_j) \right| + \left| \psi_A^U(x_j) - \psi_B^U(x_j) \right| \right), \frac{1}{2} \left(\left| \phi_A^L(x_j) - \phi_B^L(x_j) \right| + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right| \right) \right) \right\} \quad (4.3)$$

Definition 4.5. *The normalized generalized distance:*

$$\begin{aligned}
 d_E(A, B) = & \left\{ \frac{1}{12n} \sum_{j=1}^n \left[\left| t_A^L(x_j) - t_B^L(x_j) \right|^\lambda + \left| t_A^U(x_j) - t_B^U(x_j) \right|^\lambda + \left| i_A^L(x_j) - i_B^L(x_j) \right|^\lambda + \left| i_A^U(x_j) - i_B^U(x_j) \right|^\lambda + \right. \right. \\
 & \left. \left| f_A^L(x_j) - f_B^L(x_j) \right|^\lambda + \left| f_A^U(x_j) - f_B^U(x_j) \right|^\lambda + \frac{1}{(2\pi)^\lambda} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right|^\lambda + \left| \omega_A^U(x_j) - \omega_B^U(x_j) \right|^\lambda + \right. \right. \\
 & \left. \left. \left| \psi_A^L(x_j) - \psi_B^L(x_j) \right|^\lambda + \left| \psi_A^U(x_j) - \psi_B^U(x_j) \right|^\lambda + \left| \phi_A^L(x_j) - \phi_B^L(x_j) \right|^\lambda + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right|^\lambda \right) \right] \right\}^{\frac{1}{\lambda}} \quad (4.4)
 \end{aligned}$$

with $\lambda > 0$.

Theorem 4.1. *All functions defined in Eqs (4.1)–(4.4) are distance measures of ICNSs.*

Proof. Take Hamming distance $d_{Hm}(A, B)$ as an example, it is easy to see that $d_{Hm}(A, B)$ satisfies the conditions (1)–(2). Thus, we just go to prove the condition (3), i.e., the triangular inequality.

Let $A, B, C \in \Phi(X)$, for the Hamming distance, we have

$$\begin{aligned}
 d_{Hm}(A, B) + d_{Hm}(B, C) = & \frac{1}{12n} \sum_{j=1}^n \left\{ \left| t_A^L(x_j) - t_B^L(x_j) \right| + \left| t_A^U(x_j) - t_B^U(x_j) \right| + \left| i_A^L(x_j) - i_B^L(x_j) \right| + \left| i_A^U(x_j) - i_B^U(x_j) \right| + \left| f_A^L(x_j) - f_B^L(x_j) \right| \right. \\
 & + \left| f_A^U(x_j) - f_B^U(x_j) \right| + \frac{1}{2\pi} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right| + \left| \omega_A^U(x_j) - \omega_B^U(x_j) \right| + \left| \psi_A^L(x_j) - \psi_B^L(x_j) \right| \right. \\
 & \left. \left. + \left| \psi_A^U(x_j) - \psi_B^U(x_j) \right| + \left| \phi_A^L(x_j) - \phi_B^L(x_j) \right| + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right| \right) \right\} + \\
 & \frac{1}{12n} \sum_{j=1}^n \left\{ \left| t_B^L(x_j) - t_C^L(x_j) \right| + \left| t_B^U(x_j) - t_C^U(x_j) \right| + \left| i_B^L(x_j) - i_C^L(x_j) \right| + \left| i_B^U(x_j) - i_C^U(x_j) \right| + \left| f_B^L(x_j) - f_C^L(x_j) \right| \right. \\
 & + \left| f_B^U(x_j) - f_C^U(x_j) \right| + \frac{1}{2\pi} \left(\left| \omega_B^L(x_j) - \omega_C^L(x_j) \right| + \left| \omega_B^U(x_j) - \omega_C^U(x_j) \right| + \left| \psi_B^L(x_j) - \psi_C^L(x_j) \right| \right. \\
 & \left. \left. + \left| \psi_B^U(x_j) - \psi_C^U(x_j) \right| + \left| \phi_B^L(x_j) - \phi_C^L(x_j) \right| + \left| \phi_B^U(x_j) - \phi_C^U(x_j) \right| \right) \right\} \\
 = & \frac{1}{12n} \sum_{j=1}^n \left\{ \left| t_A^L(x_j) - t_B^L(x_j) \right| + \left| t_B^L(x_j) - t_C^L(x_j) \right| + \cdots + \left| f_A^U(x_j) - f_B^U(x_j) \right| + \left| f_B^U(x_j) - f_C^U(x_j) \right| \right. \\
 & \left. + \frac{1}{2\pi} \left(\left| \omega_A^L(x_j) - \omega_B^L(x_j) \right| + \left| \omega_B^L(x_j) - \omega_C^L(x_j) \right| + \cdots + \left| \phi_A^U(x_j) - \phi_B^U(x_j) \right| + \left| \phi_B^U(x_j) - \phi_C^U(x_j) \right| \right) \right\} \\
 \geq & \frac{1}{12n} \sum_{j=1}^n \left\{ \left| t_A^L(x_j) - t_C^L(x_j) \right| + \left| t_A^U(x_j) - t_C^U(x_j) \right| + \left| i_A^L(x_j) - i_C^L(x_j) \right| + \left| i_A^U(x_j) - i_C^U(x_j) \right| + \left| f_A^L(x_j) - f_C^L(x_j) \right| \right. \\
 & + \left| f_A^U(x_j) - f_C^U(x_j) \right| + \frac{1}{2\pi} \left(\left| \omega_A^L(x_j) - \omega_C^L(x_j) \right| + \left| \omega_A^U(x_j) - \omega_C^U(x_j) \right| + \left| \psi_A^L(x_j) - \psi_C^L(x_j) \right| \right. \\
 & \left. \left. + \left| \psi_A^U(x_j) - \psi_C^U(x_j) \right| + \left| \phi_A^L(x_j) - \phi_C^L(x_j) \right| + \left| \phi_A^U(x_j) - \phi_C^U(x_j) \right| \right) \right\} \\
 = & d_{Hm}(A, C)
 \end{aligned}$$

So we can consider $d_{Hm}(A, B)$ as a distance measure. Analogously, the normalized Euclidean distance $d_E(A, B)$, normalized Hausdorff distance $d_{Hd}(A, B)$ and normalized generalized distance $d_G(A, B)$ be proved as valid distance measures. \square

5. An approach for MCGDM

In this section, a MCGDM approach is presented by using the operational rules and above-defined distance measures for ICNSs.

Assume that a committee of l decision makers ($D_h, h = 1, 2, \dots, l$) is responsible for evaluating m alternatives ($A_p, p = 1, 2, \dots, m$) under n selection criteria ($C_q, q = 1, 2, \dots, n$), where the performance ratings are ICNSs. The weight vector of the criteria is $w_q (q = 1, 2, \dots, n)$ which satisfies $0 < w_q < 1$ and $\sum_{q=1}^n w_q = 1$. Then the steps of the proposed MAGDM method as follows:

Step 1: Aggregate the ratings of alternatives versus criteria.

Let $x_{hpq} = \left([T_{hpq}^L, T_{hpq}^U], [I_{hpq}^L, I_{hpq}^U], [F_{hpq}^L, F_{hpq}^U] \right)$, $h = 1, 2, \dots, l$; $p = 1, 2, \dots, m$; $q = 1, 2, \dots, n$ be the performance ratings evaluated by decision maker D_h for alternative A_p versus criterion C_q , then its decision-making matrix can be denoted as $M^{(h)} = (x_{hpq})_{m \times n}$. By using the ICNA operator, we can get the aggregating decision making matrix $M = (x_{pq})_{m \times n} = (T_{pq}, I_{pq}, F_{pq})_{m \times n}$ with aggregation ratings, where x_{pq} is defined as follows:

$$x_{pq} = \sum_{h=1}^l \frac{1}{l} x_{hpq} = \frac{1}{l} x_{1pq} + \frac{1}{l} x_{2pq} + \dots + \frac{1}{l} x_{lpq} = \frac{1}{l} (x_{1pq} + x_{2pq} + \dots + x_{lpq})$$

where

$$\begin{aligned} T_{pq} &= \left[1 - \prod_{h=1}^l (1 - T_{hpq}^L)^{\frac{1}{l}}, 1 - \prod_{h=1}^l (1 - T_{hpq}^U)^{\frac{1}{l}} \right] \cdot e \left[2\pi \left(1 - \prod_{h=1}^l \left(1 - \frac{\omega_{hpq}^L}{2\pi} \right)^{\frac{1}{l}} \right), 2\pi \left(1 - \prod_{h=1}^l \left(1 - \frac{\omega_{hpq}^U}{2\pi} \right)^{\frac{1}{l}} \right) \right] \\ I_{pq} &= \left[\prod_{h=1}^l (I_{hpq}^L)^{\frac{1}{l}}, \prod_{h=1}^l (I_{hpq}^U)^{\frac{1}{l}} \right] \cdot e \left[2\pi \left(\prod_{h=1}^l \left(\frac{\psi_{hpq}^L}{2\pi} \right)^{\frac{1}{l}} \right), 2\pi \left(\prod_{h=1}^l \left(\frac{\psi_{hpq}^U}{2\pi} \right)^{\frac{1}{l}} \right) \right] \\ F_{pq} &= \left[\prod_{h=1}^l (F_{hpq}^L)^{\frac{1}{l}}, \prod_{h=1}^l (F_{hpq}^U)^{\frac{1}{l}} \right] \cdot e \left[2\pi \left(\prod_{h=1}^l \left(\frac{\phi_{hpq}^L}{2\pi} \right)^{\frac{1}{l}} \right), 2\pi \left(\prod_{h=1}^l \left(\frac{\phi_{hpq}^U}{2\pi} \right)^{\frac{1}{l}} \right) \right] \end{aligned} \quad (5.1)$$

Step 2: Calculating the weighted decision-making matrix R according to the criteria weights.

$R = (r_{pq})_{m \times n} = \left([\bar{T}_{pq}^L, \bar{T}_{pq}^U], [\bar{I}_{pq}^L, \bar{I}_{pq}^U], [\bar{F}_{pq}^L, \bar{F}_{pq}^U] \right)_{m \times n}$ where $r_{pq} = w_q * x_{pq}$ and

$$\begin{aligned} [\bar{T}_{pq}^L, \bar{T}_{pq}^U] &= \left[1 - (1 - T_{pq}^L)^{w_q}, 1 - (1 - T_{pq}^U)^{w_q} \right] \cdot e \left[2\pi \left(1 - \left(1 - \frac{\omega_{pq}^L}{2\pi} \right)^{w_q} \right), 2\pi \left(1 - \left(1 - \frac{\omega_{pq}^U}{2\pi} \right)^{w_q} \right) \right] \\ [\bar{I}_{pq}^L, \bar{I}_{pq}^U] &= \left[(I_{pq}^L)^{w_q}, (I_{pq}^U)^{w_q} \right] \cdot e \left[2\pi \left(\left(\frac{\psi_{pq}^L}{2\pi} \right)^{w_q} \right), 2\pi \left(\left(\frac{\psi_{pq}^U}{2\pi} \right)^{w_q} \right) \right] \\ [\bar{F}_{pq}^L, \bar{F}_{pq}^U] &= \left[(F_{pq}^L)^{w_q}, (F_{pq}^U)^{w_q} \right] \cdot e \left[2\pi \left(\left(\frac{\phi_{pq}^L}{2\pi} \right)^{w_q} \right), 2\pi \left(\left(\frac{\phi_{pq}^U}{2\pi} \right)^{w_q} \right) \right] \end{aligned} \quad (5.2)$$

Step 3: Determination of the positive ideal solution (PIS) and negative ideal solution (NIS).

$PIS = R^+ = (R_1^+, R_2^+, \dots, R_n^+)$ and $NIS = R^- = (R_1^-, R_2^-, \dots, R_n^-)$ where

$$R_q^+ = \left(\left[\max_p (\bar{T}_{pq}^L), \max_p (\bar{T}_{pq}^U) \right] \cdot e^{j \left[\max_p (\bar{\omega}_{pq}^L), \max_p (\bar{\omega}_{pq}^U) \right]}, \right. \\ \left[\min_p (\bar{I}_{pq}^L), \min_p (\bar{I}_{pq}^U) \right] \cdot e^{j \left[\min_p (\bar{\psi}_{pq}^L), \min_p (\bar{\psi}_{pq}^U) \right]} \\ \left. \left[\min_p (\bar{F}_{pq}^L), \min_p (\bar{F}_{pq}^U) \right] \cdot e^{j \left[\min_p (\bar{\phi}_{pq}^L), \min_p (\bar{\phi}_{pq}^U) \right]} \right) \quad (5.3)$$

$$R_q^- = \left(\left[\min_p (\bar{T}_{pq}^L), \min_p (\bar{T}_{pq}^U) \right] \cdot e^{j \left[\min_p (\bar{\omega}_{pq}^L), \min_p (\bar{\omega}_{pq}^U) \right]}, \right. \\ \left[\max_p (\bar{I}_{pq}^L), \max_p (\bar{I}_{pq}^U) \right] \cdot e^{j \left[\max_p (\bar{\psi}_{pq}^L), \max_p (\bar{\psi}_{pq}^U) \right]}, \\ \left. \left[\max_p (\bar{F}_{pq}^L), \max_p (\bar{F}_{pq}^U) \right] \cdot e^{j \left[\max_p (\bar{\phi}_{pq}^L), \max_p (\bar{\phi}_{pq}^U) \right]} \right) \quad (5.4)$$

Step 4: Calculating the distance between each alternative and PIS, NIS.

$$d_p^+ = \sum_{q=1}^n d(r_{pq}, R_q^+) \quad (5.5)$$

$$d_p^- = \sum_{q=1}^n d(r_{pq}, R_q^-) \quad (5.6)$$

where $d(r_{pq}, R_q^+)$ and $d(r_{pq}, R_q^-)$ are defined in section 3.

Step 5: Calculating the closeness coefficients of alternatives.

$$CC_p = \frac{d_p^-}{d_p^+ + d_p^-} \quad (5.7)$$

Step 6: Ranking the alternatives. The larger value of closeness coefficients CC_p , the better alternative A_p is.

6. Numerical example

In this section, we apply the proposed MCGDM method for green supplier selection.

The managers would like to manage the suppliers effectively, due to an increasing number of them [18]. Data were collected by conducting semi-structured interviews with managers and department heads. Three managers (decision-makers), i.e., $D_1 - D_3$, were requested to separately proceed to their own evaluation. Five criteria, namely Price/cost (C_1), Quality (C_2), Delivery (C_3), Relationship Closeness (C_4) and Environmental Management Systems (C_5), were selected to evaluate the green suppliers. The weight of criteria is $w = \{0.2, 0.3, 0.25, 0.15, 0.1\}$. Three managers

determined the performance ratings of three suppliers A_1 , A_2 and A_3 under the criteria by using the linguistic set $S = \{VL, L, F, G, VG\}$, where

$$VL = \text{VeryLow} = ([0.1, 0.2] e^{j\pi[0.7,0.8]}, [0.7, 0.8] e^{j\pi[0.9,1.0]}, [0.6, 0.7] e^{j\pi[1.0,1.1]})$$

$$L = \text{Low} = ([0.3, 0.4] e^{j\pi[0.8,0.9]}, [0.6, 0.7] e^{j\pi[1.0,1.1]}, [0.5, 0.6] e^{j\pi[0.9,1.0]})$$

$$F = \text{Fair} = ([0.4, 0.5] e^{j\pi[0.8,0.9]}, [0.5, 0.6] e^{j\pi[0.9,1.0]}, [0.4, 0.5] e^{j\pi[0.8,0.9]})$$

$$G = \text{Good} = ([0.6, 0.7] e^{j\pi[0.9,1.0]}, [0.4, 0.5] e^{j\pi[0.9,1.0]}, [0.3, 0.4] e^{j\pi[0.7,0.8]})$$

$$VG = \text{VeryGood} = ([0.7, 0.8] e^{j\pi[1.1,1.2]}, [0.2, 0.3] e^{j\pi[0.8,0.9]}, [0.1, 0.2] e^{j\pi[0.6,0.7]})$$

to evaluate the suitability of the suppliers under each criteria. Tables 1–3 show the three managers' performance ratings.

Table 1. The Performance Rating From Decision maker D_1 .

	C_1	C_2	C_3	C_4	C_5
A_1	G	F	L	G	L
A_2	F	VG	G	F	G
A_3	VG	F	F	G	G

Table 2. The Performance Rating From Decision maker D_2 .

	C_1	C_2	C_3	C_4	C_5
A_1	G	F	L	G	L
A_2	F	VG	G	F	G
A_3	VG	F	F	G	G

Table 3. The Performance Rating From Decision maker D_3 .

	C_1	C_2	C_3	C_4	C_5
A_1	G	F	L	G	L
A_2	F	VG	G	F	G
A_3	VG	F	F	G	G

Then the complete MCGDM procedure is characterized by the following steps:

Step 1: Aggregated performance ratings.

According to Eq (5.1), the aggregated ratings of three suppliers versus five criteria from three decision makers are given in Table 4.

Table 4. The Aggregated Ratings of Suppliers versus The Criteria.

C_q	A_p	Aggregation ratings
C_1	A_1	$([0.5421, 0.6443] e^{j2\pi[0.4338, 0.4839]}, [0.4309, 0.5313] e^{j2\pi[0.45, 0.50]}, [0.3302, 0.4309] e^{j2\pi[0.3659, 0.4160]})$
	A_2	$([0.4759, 0.5783] e^{j2\pi[0.4172, 0.4672]}, [0.4642, 0.5646] e^{j2\pi[0.45, 0.50]}, [0.3646, 0.4642] e^{j2\pi[0.3826, 0.4327]})$
	A_3	$([0.6698, 0.7711] e^{j2\pi[0.5189, 0.5691]}, [0.2520, 0.3557] e^{j2\pi[0.4160, 0.4661]}, [0.1442, 0.2520] e^{j2\pi[0.3158, 0.3659]})$
C_2	A_1	$([0.4, 0.5] e^{j2\pi[0.40, 0.45]}, [0.5, 0.6] e^{j2\pi[0.45, 0.50]}, [0.4, 0.5] e^{j2\pi[0.40, 0.45]})$
	A_2	$([0.6366, 0.7379] e^{j2\pi[0.4856, 0.5358]}, [0.3175, 0.4217] e^{j2\pi[0.4327, 0.4827]}, [0.2080, 0.3175] e^{j2\pi[0.3325, 0.3826]})$
	A_3	$([0.5421, 0.6443] e^{j2\pi[0.4338, 0.4839]}, [0.4309, 0.5313] e^{j2\pi[0.45, 0.50]}, [0.3302, 0.4309] e^{j2\pi[0.3659, 0.4160]})$
C_3	A_1	$([0.3351, 0.4354] e^{j2\pi[0.40, 0.45]}, [0.5646, 0.6649] e^{j2\pi[0.4827, 0.5328]}, [0.4642, 0.5646] e^{j2\pi[0.4327, 0.4827]})$
	A_2	$([0.6, 0.7] e^{j2\pi[0.45, 0.50]}, [0.4, 0.5] e^{j2\pi[0.45, 0.50]}, [0.3, 0.4] e^{j2\pi[0.35, 0.4]})$
	A_3	$([0.4759, 0.5783] e^{j2\pi[0.4172, 0.4672]}, [0.4642, 0.5646] e^{j2\pi[0.45, 0.50]}, [0.3646, 0.4642] e^{j2\pi[0.3826, 0.4327]})$
C_4	A_1	$([0.5421, 0.6443] e^{j2\pi[0.4338, 0.4839]}, [0.4309, 0.5313] e^{j2\pi[0.45, 0.50]}, [0.3302, 0.4309] e^{j2\pi[0.3659, 0.4160]})$
	A_2	$([0.3684, 0.4687] e^{j2\pi[0.40, 0.45]}, [0.5313, 0.6316] e^{j2\pi[0.4661, 0.5161]}, [0.4309, 0.5313] e^{j2\pi[0.4160, 0.4661]})$
	A_3	$([0.6366, 0.7379] e^{j2\pi[0.4672, 0.5173]}, [0.3175, 0.4217] e^{j2\pi[0.4327, 0.4827]}, [0.2080, 0.3175] e^{j2\pi[0.3325, 0.3826]})$
C_5	A_1	$([0.3351, 0.4354] e^{j2\pi[0.40, 0.45]}, [0.5646, 0.6649] e^{j2\pi[0.4827, 0.5328]}, [0.4642, 0.5646] e^{j2\pi[0.4327, 0.4827]})$
	A_2	$([0.6366, 0.7379] e^{j2\pi[0.4856, 0.5358]}, [0.3175, 0.4217] e^{j2\pi[0.4327, 0.4827]}, [0.2080, 0.3175] e^{j2\pi[0.3325, 0.3826]})$
	A_3	$([0.4759, 0.5783] e^{j2\pi[0.4172, 0.4672]}, [0.4642, 0.5646] e^{j2\pi[0.45, 0.50]}, [0.3646, 0.4642] e^{j2\pi[0.3826, 0.4327]})$

Step 2: Calculating the weighted decision making matrix $R = (r_{pq})_{m \times n}$.

According to the attribute weight $w = \{0.2, 0.3, 0.25, 0.15, 0.1\}$ and Eq (5.2), the weighted decision making matrix is obtained and shown in Table 5.

Table 5. The Weighted Decision Making Matrix R .

C_q	A_p	Aggregation ratings
C_1	A_1	$([0.1446, 0.1868] e^{j2\pi[0.1075, 0.1239]}, [0.8450, 0.8812] e^{j2\pi[0.8524, 0.8706]}, [0.8012, 0.8450] e^{j2\pi[0.8178, 0.8391]})$
	A_2	$([0.1212, 0.1586] e^{j2\pi[0.1024, 0.1183]}, [0.8577, 0.8920] e^{j2\pi[0.8524, 0.8706]}, [0.8173, 0.8577] e^{j2\pi[0.8252, 0.8457]})$
	A_3	$([0.1988, 0.2554] e^{j2\pi[0.1361, 0.1550]}, [0.7591, 0.8132] e^{j2\pi[0.8391, 0.8584]}, [0.6789, 0.7591] e^{j2\pi[0.7941, 0.8178]})$
C_2	A_1	$([0.1421, 0.1877] e^{j2\pi[0.1421, 0.1642]}, [0.8123, 0.8579] e^{j2\pi[0.7870, 0.8123]}, [0.7597, 0.8123] e^{j2\pi[0.7597, 0.7870]})$
	A_2	$([0.2619, 0.3308] e^{j2\pi[0.1808, 0.2057]}, [0.7088, 0.7718] e^{j2\pi[0.7778, 0.8037]}, [0.6243, 0.7088] e^{j2\pi[0.7157, 0.7496]})$
	A_3	$([0.2089, 0.2666] e^{j2\pi[0.1569, 0.1800]}, [0.7768, 0.8272] e^{j2\pi[0.7870, 0.8123]}, [0.7172, 0.7768] e^{j2\pi[0.7396, 0.7686]})$
C_3	A_1	$([0.0970, 0.1332] e^{j2\pi[0.1199, 0.1388]}, [0.8668, 0.9030] e^{j2\pi[0.8335, 0.8544]}, [0.8254, 0.8668] e^{j2\pi[0.8110, 0.8335]})$
	A_2	$([0.2407, 0.2599] e^{j2\pi[0.1388, 0.1591]}, [0.7953, 0.8409] e^{j2\pi[0.8190, 0.8409]}, [0.7401, 0.7953] e^{j2\pi[0.7692, 0.7953]})$
	A_3	$([0.1491, 0.1942] e^{j2\pi[0.1263, 0.1456]}, [0.8254, 0.8668] e^{j2\pi[0.8190, 0.8409]}, [0.7764, 0.8254] e^{j2\pi[0.7883, 0.8110]})$
C_4	A_1	$([0.1106, 0.1436] e^{j2\pi[0.0818, 0.0945]}, [0.8814, 0.9095] e^{j2\pi[0.8871, 0.9013]}, [0.8469, 0.8814] e^{j2\pi[0.8600, 0.8767]})$
	A_2	$([0.0666, 0.0905] e^{j2\pi[0.0738, 0.0858]}, [0.9095, 0.9334] e^{j2\pi[0.8918, 0.9055]}, [0.8814, 0.9095] e^{j2\pi[0.8767, 0.8918]})$
	A_3	$([0.1409, 0.1820] e^{j2\pi[0.0901, 0.1035]}, [0.8419, 0.8785] e^{j2\pi[0.8819, 0.8965]}, [0.7901, 0.8419] e^{j2\pi[0.8478, 0.8658]})$
C_5	A_1	$([0.0400, 0.0556] e^{j2\pi[0.0498, 0.0580]}, [0.9444, 0.9600] e^{j2\pi[0.9298, 0.9390]}, [0.9261, 0.9444] e^{j2\pi[0.9196, 0.9298]})$
	A_2	$([0.0963, 0.1253] e^{j2\pi[0.0643, 0.0739]}, [0.8916, 0.9173] e^{j2\pi[0.9196, 0.9298]}, [0.8547, 0.8916] e^{j2\pi[0.8957, 0.9084]})$
	A_3	$([0.0626, 0.0827] e^{j2\pi[0.0526, 0.0610]}, [0.9261, 0.9444] e^{j2\pi[0.9233, 0.9390]}, [0.9037, 0.9261] e^{j2\pi[0.9092, 0.9196]})$

Step 4: Determination of the PIS and NIS.

According to Eqs (5.3) and (5.4), we can get the PIS and NIS as following:

$$R_1^+ = \left([0.1988, 0.2554] e^{j2\pi[0.1361, 0.1550]}, [0.7591, 0.8132] e^{j2\pi[0.8391, 0.8584]}, \right. \\ \left. [0.6789, 0.7591] e^{j2\pi[0.7941, 0.8178]} \right)$$

$$R_2^+ = \left([0.2619, 0.3308] e^{j2\pi[0.1808, 0.2057]}, [0.7088, 0.7718] e^{j2\pi[0.7778, 0.8037]}, \right. \\ \left. [0.6243, 0.7088] e^{j2\pi[0.7187, 0.7496]} \right)$$

$$R_3^+ = \left([0.2407, 0.2599] e^{j2\pi[0.1388, 0.1591]}, [0.7593, 0.8409] e^{j2\pi[0.8190, 0.8409]}, \right. \\ \left. [0.7401, 0.7953] e^{j2\pi[0.7692, 0.7953]} \right)$$

$$R_4^+ = \left([0.1409, 0.1820] e^{j2\pi[0.0901, 0.1035]}, [0.8419, 0.8785] e^{j2\pi[0.8819, 0.8965]}, \right. \\ \left. [0.7901, 0.8419] e^{j2\pi[0.8478, 0.8658]} \right)$$

$$R_5^+ = \left([0.0963, 0.1253] e^{j2\pi[0.0643, 0.0739]}, [0.8916, 0.9173] e^{j2\pi[0.9196, 0.9298]}, \right. \\ \left. [0.8547, 0.8916] e^{j2\pi[0.8597, 0.9084]} \right)$$

$$R_1^- = \left([0.1212, 0.1586] e^{j2\pi[0.1024, 0.1183]}, [0.8577, 0.8920] e^{j2\pi[0.8524, 0.8706]}, \right. \\ \left. [0.8173, 0.8577] e^{j2\pi[0.8252, 0.8457]} \right)$$

$$R_2^- = \left([0.1421, 0.1877] e^{j2\pi[0.1421, 0.1642]}, [0.8123, 0.8579] e^{j2\pi[0.7870, 0.8123]}, \right. \\ \left. [0.7597, 0.8123] e^{j2\pi[0.7597, 0.7870]} \right)$$

$$R_3^- = \left([0.0970, 0.1332] e^{j2\pi[0.1199, 0.1388]}, [0.8668, 0.9030] e^{j2\pi[0.8335, 0.8544]}, \right. \\ \left. [0.8254, 0.8668] e^{j2\pi[0.8110, 0.8335]} \right)$$

$$R_4^- = \left([0.0666, 0.0905] e^{j2\pi[0.0738, 0.0858]}, [0.9095, 0.9334] e^{j2\pi[0.8918, 0.9055]}, \right. \\ \left. [0.8814, 0.9095] e^{j2\pi[0.8767, 0.8918]} \right)$$

$$R_5^- = \left([0.0400, 0.0556] e^{j2\pi[0.0498, 0.0580]}, [0.9444, 0.9600] e^{j2\pi[0.9298, 0.9390]}, \right. \\ \left. [0.9261, 0.9444] e^{j2\pi[0.9196, 0.9298]} \right)$$

Step 5: Calculating the distance measures between alternatives and PIS, NIS.

We compute the distance measures by Eqs (5.5) and (5.6) which are shown in Table 6.

Table 6. The Distance Measures by Different Methods.

	d_1^+	d_2^+	d_3^+	d_1^-	d_2^-	d_3^-
d_{Hm}	0.2484	0.1082	0.0974	0.0331	0.1684	0.1819
d_E	0.2972	0.1288	0.1191	0.0406	0.2012	0.2176
d_{Hd}	0.4819	0.2014	0.2009	0.0744	0.3297	0.3552

Step 6: Computing the closeness coefficients.

According to Eq (5.7), the closeness coefficients are obtained and shown in Table 7.

Table 7. The Closeness Coefficient.

	CC_1	CC_2	CC_3	Final Ranking
d_{Hm}	0.1176	0.6088	0.6513	$A_3 > A_2 > A_1$
d_E	0.1202	0.6097	0.6463	$A_3 > A_2 > A_1$
d_{Hd}	0.1337	0.6208	0.6387	$A_3 > A_2 > A_1$

From these results, it is obvious that the ranking of three suppliers is $A_3 > A_2 > A_1$, and the optimal supplier is A_3 .

By comparing with the method proposed in [18], the biggest difference is the criteria weights which are the interval complex neutrosophic numbers in [18] whereas real numbers in our paper, but one similarity is that the weights are satisfied that $w_2 > w_3 > w_1 > w_4 > w_5$, so the ranking result and optimal supplier are in the same way which can show our approach is practical and effective.

7. Conclusions

It is obvious that interval complex neutrosophic set is a useful tool for dealing with the uncertain, inconsistent and incomplete information in periodic data. The aim of this paper is to introduce some interval complex neutrosophic distance measures and apply them into MCGDM problems. Hence, based on the Hamming, Euclidean, Hausdorff metrics, we present some distance measures for ICNSs, and an approach is developed to handle the MCGDM problems. At the beginning of this article, we briefly introduce some definitions and set theoretic properties of ICNS. Next, in order to obtain the best alternative(s), we propose an approach based on some distance measures for MCGDM problems. Finally, we illustrate the application of the proposed method thorough a numerical example. From the result we can see the practicality and effectiveness of this method.

As further work, we may develop more information measures and techniques for decision-making problems under interval complex neutrosophic environment and apply them into different fields, such as venture capital, pattern recognition and comprehensive evaluation.

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Conflict of interest

The authors declare that there is no conflict of interest.

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