Energy and Spectrum Analysis of Interval Valued Neutrosophic Graph using MATLAB

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Abstract. In recent time graphical analytics of uncertainty and indeterminacy has become major concern for data analytics researchers. In this direction, the mathematical algebra of neutrosophic graph is extended to interval-valued neutrosophic graph. However, building the interval-valued neutrosophic graphs, its spectrum and energy computation is addressed as another issues by research community of neutrosophic environment. To resolve this issue the current paper proposed some related mathematical notations to compute the spectrum and energy of interval-valued neutrosophic graph using the MATAB.

Keywords: Interval valued neutrosophic graphs. Adjacency matrix. Spectrum of IVNG. Energy of IVNG. Complete-IVNG.

1 Introduction

The handling uncertainty in the given data set is considered as one of the major issues for the research communities. To deal with this issue the mathematical algebra of neutrosophic set is introduced [1]. The calculus of neutrosophic sets (NSs)[1, 2] given a way to represent the uncertainty based on acceptation, rejection and uncertain part, independently. It is nothing but just an extension of fuzzy set [3], intuitionistic fuzzy set [4-6], and interval valued fuzzy sets [7] beyond the unipolar fuzzy space. It characterizes the uncertainty based on a truth-membership function (T), an indeterminate-membership function (I) and a falsity-membership function(F) independently of a defined neutrosophic set via real a standard or non-standard unit interval$]^{-}0, 1^{+}[$. One of the best suitable example is for the neutrosophic logic is win/loss and draw of a match, opinion of people towards an event is based on its acceptance, rejection and uncertain values. These properties of neutrosophic set differentiate it from any of the available approaches in fuzzy set theory while measuring the indeterminacy. Due to which mathematics of single valued neutrosophic sets (abbr. SVNS) [8] as well as interval valued neutrosophic sets (abbr.IVNS) [9-10] is introduced for precise analysis of indeterminacy in the given interval. The IVNS represents the acceptance, rejection and uncertain membership functions in the unit interval [0, 1] which helped a lot for knowledge processing tasks using different classifier [11], similarity method [12-14] as well as multi-decision making process [15-17] at user defined weighted method [18-24]. In this process a problem is addressed while drawing the interval-valued neutrosophic graph, its spectrum and energy analysis. To achieve this goal, the current paper tried to focus on introducing these related properties and its analysis using MATLAB.

2 Literature Review

There are several applications of graph theory which is a mathematical tool provides a way to visualize the given data sets for its precise analysis. It is utilized for solving several mathematical problems. In this process, a problem is addressed while representing the uncertainty and vagueness exists in any given attributes (i.e. vertices) and their corresponding relationship i.e edges. To deal with this problem, the properties of fuzzy graph [25-26] theory is extended to intuitionistic fuzzy graph [28-30], interval valued fuzzy graphs [31] is studied with applications [32—33]. In this case a problem is addressed while measuring with indeterminacy and its situation. Hence, the neutrosophic graphs and its properties is introduced by Smaranadache [34-37] to characterizes them using their truth, falsity, and indeterminacy membership-values (T, I, F) with its applications [38-40]. Broumi et al. [41] introduced neutrosophic graph theory considering (T, I, F) for vertices and edges in the graph specially termed as "Single valued neutrosophic graph theory (abbr. SVNG)" with its other properties [42-44]. Afterwards several researchers studied the neutrosophic graphs and its applications [65, 68]. Broumi et al. [50] utilized the

SVNGs to find the shortest path in the given network subsequently other researchers used it in different fields [51-53, 59-60, 65]. To measure the partial ignorance, Broumi et al. [45] introduced interval valued-neutrosophic graphs and its related operations [46-48] with its application in decision making process in various extensions[49, 54, 57 61, 62, 64,73-84].

Some other researchers introduced antipodal single valued neutrosophic graphs [63, 65], single valued neutrosophic digraph [68] for solving multi-criteria decision making. Naz et al.[69] discussed the concept of energy and laplacian energy of SVNGs. This given a major thrust to introduce it into interval-valued neutrosophic graph and its matrix. The matrix is a very useful tool in representing the graphs to computers, matrix representation of SVNG, some researchers study adjacency matrix and incident matrix of SVNG. Varol et al. [70] introduced single valued neutrosophic matrix as a generalization of fuzzy matrix, intuitionistic fuzzy matrix and investigated some of its algebraic operations including subtraction, addition, product, transposition. Uma et al. [66] proposed a determinant theory for fuzzy neutrosophic soft matrices. Hamidiand Saeid [72] proposed the concept of accessible single-valued neutrosophic graphs.

It is observed that, few literature have shown the study on energy of IVNG. Hence this paper, introduces some basic concept related to the interval valued neutrosophic graphs are developed with an interesting properties and its illustration for its various applications in several research field.

3 Preliminaries

This section consists some of the elementary concepts related to the neutrosophic sets, single valued neutrosophic sets, interval-valued neutrosophic sets, single valued neutrosophic graphs and adjacency matrix for establishing the new mathematical properties of interval-valued neutrosophic graphs. Readers can refer to following references for more detail about basics of these sets and their mathematical representations [1, 8, 41].

Definition 3.1:[1] Suppose ξ be a nonempty set. A neutrosophic set (abbr.NS) N in$\xi$is an object taking the form N_{NS}= {<x: $T_N(k)$, $I_N(k)$, $F_N(k)$>, k∈ ξ} (1)

Where $T_N(k)$:ξ →]$^-$0,1$^+$[, $I_N(k)$:ξ →]$^-$0,1$^+$[,$F_N(k)$:ξ →]$^-$0,1$^+$[are known as truth-membership function, indeterminate –membership function and false-membership unction, respectively. The neutrosophic sets is subject to the following condition:

$$^-0 \leq T_N(k)+I_N(k)+F_N(k) \leq 3^+ \qquad (2)$$

Definition 3.2:[8]Suppose ξ be a nonempty set. A single valued neutrosophic sets N (abbr. SVNs) inξ is an object taking the form:

$$N_{SVNs}=\{<k:T_N(k), I_N(k), F_N(k)>, k\in \xi\} \quad (3)$$

where $T_N(k)$, $I_N(k)$, $F_N(k)$ ∈ [0, 1] are mappings. $T_N(k)$denote the truth-membership function of an element x ∈ ξ, $I_N(k)$denote the indeterminate –membership function of an element k ∈ ξ.$F_N(k)$denote the false–membership function of an element k ∈ ξ. The SVNs subject to condition

$$0 \leq T_N(k)+I_N(k)+F_N(k) \leq 3 \qquad (4)$$

Example 3.3: Let us consider following example to understand the indeterminacy and neutrosophic logic:

In a given mobile phone suppose 100 calls came at end of the day.

1. 60 calls were received truly among them 50 numbers are saved and 10 were unsaved in mobile. In this case these 60 calls will be considered as truth membership i.e. 0.6.

2. 30 calls were not-received by mobile holder. Among them 20 calls which are saved in mobile contacts were not received due to driving, meeting, or phone left in home, car or bag and 10 were not received due to uncertain numbers. In this case all 30 not received numbers by any cause (i.e. driving, meeting or phone left at home) will be considered as Indeterminacy membership i.e. 0.3.

3. 10 calls were those number which was rejected calls intentionally by mobile holder due to behavior of those saved numbers, not useful calls, marketing numbers or other cases for that he/she do not want to pick or may be blocked numbers. In all cases these calls can be considered as false i.e. 0.1 membership value.

The above situation can be represented as (0.6, 0.3, 0.1) as neutrosophic set.

Definition 3.4: [10] Suppose ξ be a nonempty set. An interval valued neutrosophic sets N (abbr.IVNs) in $\xi$is an object taking the form:

$$N_{IVNs}=\{<k:\tilde{T}_N(k), \tilde{I}_N(k),\tilde{F}_N(k)>,k\in \xi>\} \quad (5)$$

Where $\tilde{T}_N(k)$, $\tilde{I}_N(k)$, $\tilde{F}_N(k) \subseteq int[0,1]$ are mappings. $\tilde{T}_N(k)=[T_N^L(k), T_N^U(k)]$ denote the interval truth-membership function of an element k$\in \xi$.$\tilde{I}_N(k)=[I_N^L(x), I_N^U(k)]$ denote the interval indeterminate-membership function of an element k$\in \xi$.$\tilde{F}_N(k)=[F_N^L(k), F_N^U(k)]$ denote the false-membership function of an element k$\in \xi$.

Definition 3.4: [10]For every two interval valued-neutrosophic sets A and B in ξ, we define

$$(N \cup M)(k)= ([T_C^L(k), T_C^U(k)], [I_C^L(k), I_C^U(k)], [F_C^L(k), F_C^U(k)]) \text{ for all } k \in \xi \quad (6)$$

Where

$T_C^L(k)= T_N^L(k) \vee T_M^L(k)$, $T_C^U(k)= T_N^U(k) \vee T_M^U(k)$

$I_C^L(k)= I_N^L(k) \wedge I_M^L(k)$, $I_C^U(k)= I_N^U(k) \wedge I_M^U(k)$

$F_C^L(k)= F_N^L(k) \wedge F_M^L(k)$, $F_C^U(k)= F_N^U(k) \wedge F_M^U(k)$

Definition 3.5: [41]A pair G=(V,E) is known as single valued neutrosophic graph (abbr.SVNG) if the following holds:

1. V= $\{k_i$:i=1,..,n$\}$ such as T_1:V→ [0,1] is the truth-membership degree, I_1:V→[0,1] is the indeterminate –membership degree and F_1:V→[0,1]is the false membership degree of $k_i \in$ V subject to condition

$$0 \leq T_1(k_i)+I_1(k_i)+F_1(k_i) \leq 3 \quad (7)$$

2. E=$\{(k_i, k_j)$: $(k_i, k_j) \in V \times V\}$ such as $T_2:V \times V \to [0,1]$ is the truth-memebership degree, $I_2:V \times V \to [0,1]$ is the indeterminate –membership degree and $F_2:V \times V \to [0,1]$ is the false-memebership degree of $(k_i,k_j) \in$ E defined as

$$T_2(k_i,k_j) \leq T_1(k_i) \wedge T_1(k_j) \quad (8)$$

$$I_2(k_i, k_j) \geq I_1(k_i) \vee I_1(k_j) \quad (9)$$

$$F_2(k_i, k_j) \geq F_1(k_i) \vee F_1(k_j) \quad (10)$$

Subject to condition $0 \leq T_2(k_ik_2)+I_2(k_1k_2)+F_2(k_ik_j) \leq 3 \;\forall\, (k_i, k_j) \in$ E. (11)

The Fig. 1 shows an illustration of SVNG.

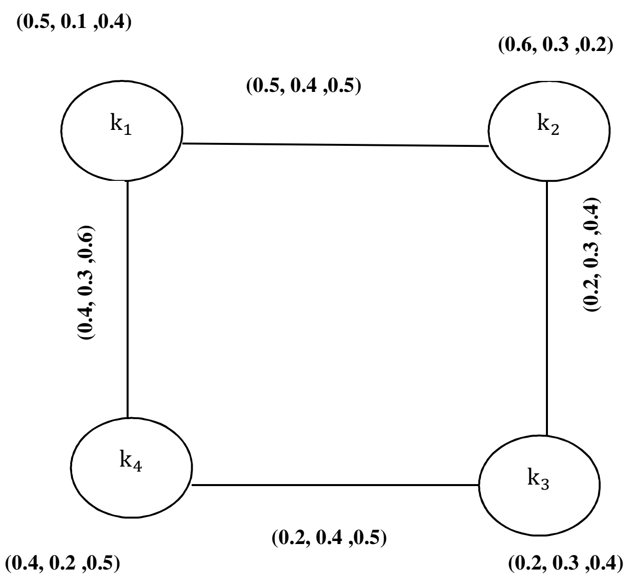

Fig. 1. An illustration of single valued neutrosophic graph

Definition 3.6[41]. A single valued neutrosophic graph G=(N, M) of G^*= (V, E) is termed strong single valued neutrosophic graph if the following holds:

$T_M(k_ik_j)$= $T_N(k_i) \wedge T_N(k_j)$ (12)

$I_M(k_ik_j)$ = $I_N(k_i) \vee I_N(k_i)$ (13)

$F_M(k_ik_j)$= $F_N(k_i) \vee F_N(k_j)$ (14)

$\forall$ (k_i , k_j) $\in$ E.

Where the operator $\wedge$denote minimum and the operator $\vee$denote the maximum

Definition 3.8[41]. A single valued neutrosophic graph G=(N, M) of G^*= (V, E) is termed complete single valued neutrosophic graph if the following holds:

$T_M(k_ik_j)$= $T_N(k_i) \wedge T_N(k_j)$ (15)

$I_M(k_ik_j)$ = $I_N(k_i) \vee I_N(k_i)$ (16)

$F_M(k_ik_j)$= $F_N(k_i) \vee F_N(k_j)$ (17)

$\forall k_i$, k_j $\in$V.

Definition 3.9:[70] The Eigen value of a graph G are the Eigen values of its adjacency matrix.

Definition 3.10:[70]The spectrum of a graph is the set of all Eigen values of its adjacency matrix

$$\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n \qquad (18)$$

Definition 3.11:[70]The energy of the graph G is defined as the sum of the absolute values of its eigenvalues and denoted it by E(G):

$$\mathrm{E(G)}=\sum_{i=1}^{n}|\lambda_i| \qquad (19)$$

4.Some Basic Concepts of Interval Valued Neutrosophic Graphs

Throughout this paper, we abbreviate G^*=(V, E) as a crisp graph, and G=(N, M) an interval valued neutrosophic graph.In this section we have defined some basic concepts of interval valued neutrosophic graphs and discuses some of their properties.

Definition 4.1:[45] A pair G=(V,E) is called an interval valued neutrosophic graph (abbr.IVNG) if the following holds:

1. V= {k_i:i=1,..,n} such as T_1^L:V→ [0,1] is the lower truth-membership degree,T_1^U:V→ [0,1] is the upper truth-membership degree,I_1^L:V→ [0,1] is the lower indeterminate-membership degree,I_1^U:V→ [0,1] is the upper indterminate-membership degree, and F_1^L:V→ [0,1] is the lower false-membership degree,F_1^U:V→ [0,1] is the upper false-membership degree,of v_i $\in$ V subject to condition

$$0 \leq T_1^U(k_i)+I_1^U(k_i)+F_1^U(k_i) \leq 3 \qquad (20)$$

2. E={(k_i, k_j): $(k_i, k_j) \in V \times V$} such as T_2^L:$V \times V \to$ [0,1] is the lower truth-memebership degree, as T_2^U:$V \times V \to$ [0,1] is the upper truth-memebership degree, I_2^L:$V \times V \to$ [0,1] is the lower indeterminate-memebership degree, I_2^U:$V \times V \to$ [0,1] is the upper indeterminate-memebership degree and F_2^L:$V \times V \to$ [0,1] is the lower false-memebership degree, F_2^U:$V \times V \to$ [0,1] is the upper false-memebership degree of (k_i,k_j) $\in$ E defined as

$$T_2^L(k_i,k_j) \leq T_1^L(k_i) \wedge T_1^L(k_j) \text{ ,} T_2^U(k_i,k_j) \leq T_1^U(k_i) \wedge T_1^U(k_j) \qquad (21)$$

$$I_2^L(k_i,k_j) \geq I_1^L(k_i) \vee I_1^L(k_j) \text{ ,} I_2^U(k_i,k_j) \geq I_1^U(k_i) \vee I_1^U(k_j) \qquad (22)$$

$$F_2^L(k_i,k_j) \geq F_1^L(k_i) \vee F_1^L(k_j) \text{ ,} F_2^U(k_i,k_j) \geq F_1^U(k_i) \vee F_1^U(k_j) \qquad (23)$$

Subject to condition $0 \leq T_2^U(k_ik_2)+I_2^U(k_1k_2)+F_2^U(k_ik_j) \leq 3$ $\forall$ (k_i, k_j) $\in$ E. (24)

Example 4.2. Consider a crisp graph G^* such that V= $\{k_1, k_2, k_3\}$, E=$\{k_1k_2, k_2k_3, k_3k_4\}$. Suppose N be an interval valued neutrosophic subset of V and suppose M an interval valued neutrosophic subset of E denoted by:

	k_1	k_2	k_3
T_N^L	0.3	0.2	0.1
T_N^U	0.5	0.3	0.3
I_N^L	0.2	0.2	0.2
I_N^U	0.3	0.3	0.4
F_N^L	0.3	0.1	0.3
F_N^U	0.4	0.4	0.5

	k_1k_2	k_2k_3	k_3k_1
T_M^L	0.1	0.1	0.1
T_M^U	0.2	0.3	0.2
I_M^L	0.3	0.4	0.3
I_M^U	0.4	0.5	0.5
F_M^L	0.4	0.4	0.4
F_M^U	0.5	0.5	0.6

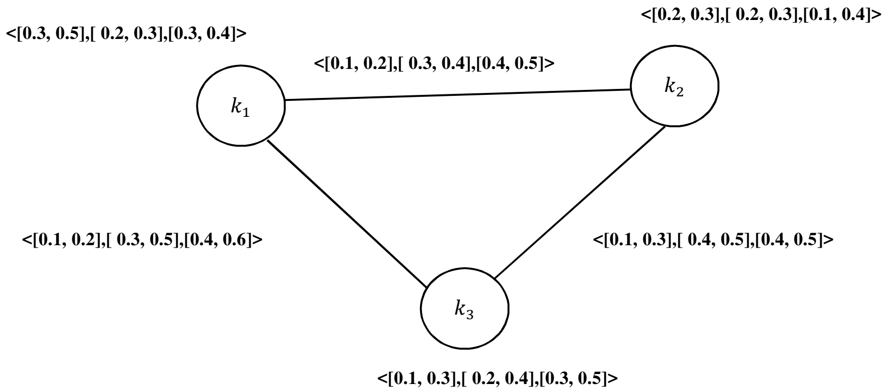

Fig. 2.Example of an interval valued neutrosophic graph

Definition 4.3A graph G=(N , M) is termed simple interval valued neutrosophic graph if it has neither self lops nor parallel edges in an interval valued neutrosophic graph.

Definition 4.4 The degree d(k) of any vertex k of an interval valued neutrosophic graph G=(N, M) is defined as follow:

$$d(v)= [d_T^L(k),d_T^U(k)],[d_I^L(k),d_I^U(k)],[d_F^L(k),d_F^U(k)] \quad (25)$$

Where
$d_T^L(k)=\sum_{k_i\neq k_j} T_M^L(k_ik_j)$ known as the degree of lower truth-membership vertex
$d_T^U(k)=\sum_{k_i\neq k_j} T_M^U(k_ik_j)$ known as the degree of upper truth-membership vertex
$d_I^L(k)=\sum_{k_i\neq k_j} I_M^L(k_ik_j)$ known as the degree of lower indterminate-membership vertex
$d_I^U(k)=\sum_{k_i\neq k_j} I_M^U(k_ik_j)$ known as the degree of upperindeterminate-membership vertex
$d_F^L(k)=\sum_{k_i\neq k_j} F_M^L(k_ik_j)$ known as the degree of lower false-membership vertex
$d_F^U(k)=\sum_{k_i\neq k_j} F_M^U(k_ik_j)$ known as the degree of upperfalse-membership vertex

Example 4.5 Consider an IVNG G=(N, M) presented in Fig. 4 with vertices set V=$\{k_i: i = 1,..,4\}$ and edges set E =$\{k_1k_4 ,k_4k_3, k_3k_2,k_2k_1\}$.

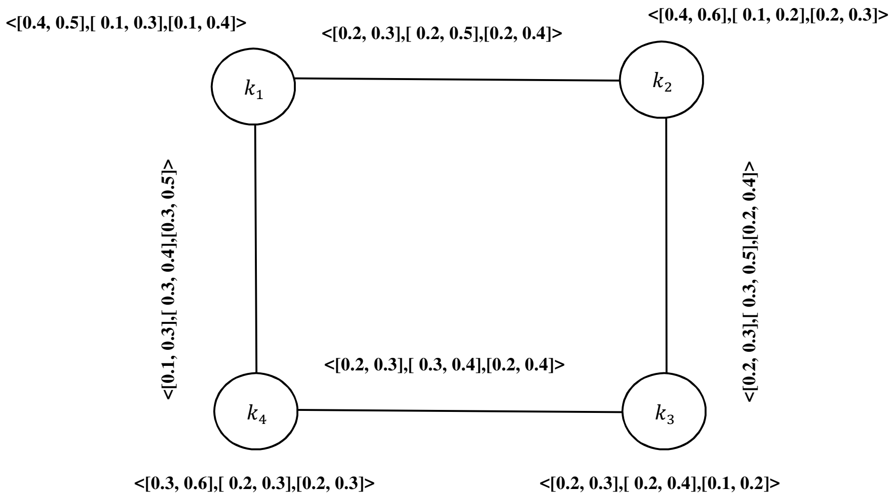

Fig. 4.Illutstrationof an interval valued neutrosophic graph

The degree of each vertex $k_i$is given as follows:

$d(\mathrm{k}_1)$= ([0.3, 0.6], [0.5, 0.9], [0.5, 0.9]),

$d(\mathrm{k}_2)$= ([0.4, 0.6], [0.5, 1.0], [0.4, 0.8]),

$d(\mathrm{k}_3)$= ([0.4, 0.6], [0.6, 0.9], [0.4, 0.8]),

$d(\mathrm{k}_4)$= ([0.3, 0.6], [0.6, 0.8], [0.5, 0.9]).

Definition 4.6. A graph G=(N, M) is termed regular interval valued neutrosophic graph if d(k)=r=([r_{1L}, r_{1U}], [r_{2L}, r_{2U}], [r_{3L}, r_{3U}]), ∀ k ∈ V.
(i.e.) if each vertex has same degree r, then G is said to be a regular interval valued neutrosophic graph of degree r.

Definition 4.7. A graph G=(N,M) is termed irregular interval valued neutrosophic graph if the degree of some vertices are different than other.

Example 4.8 Let us Suppose, G is a regular interval-valued neutrosophic graph as portrayed in Fig. 5 having vertex set V={k_1, k_2, k_3, k_4} and edge sets E={k_1k_2,k_2k_3, k_3k_4 ,k_4k_1} as follows.

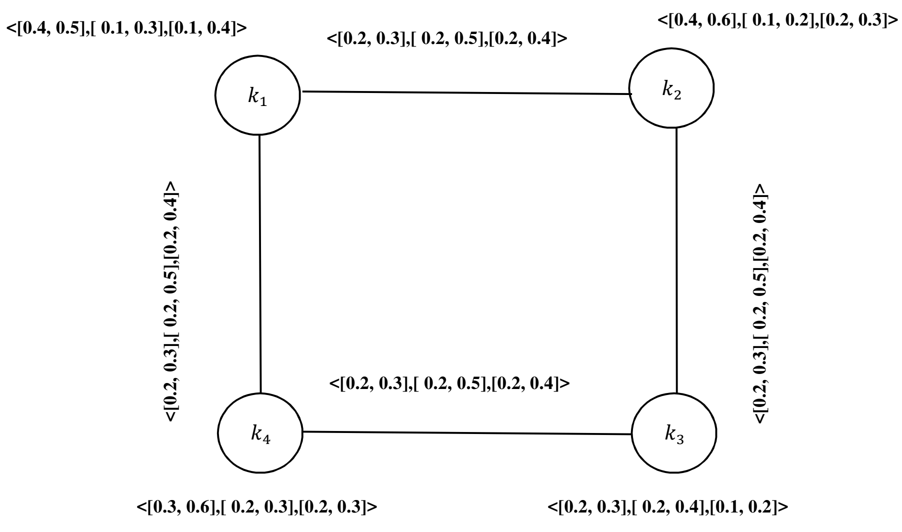

Fig.5 .Regular IVN-graph.

In the Fig. 5. All adjacent vertices k_1k_4 , k_4k_3 , k_3k_2 , k_2k_1 have the same degree equal <[0.4,0.6],[0.4,1],[0.4,0.8]>. Hence, the graph G is a regular interval valued neutrosophic graph.

Definition 4.9 A graph G= (N, M) on $G^*$is termed strong interval valued neutrosophic graph if the following holds:

$$T_M^L(k_i,k_j) = T_N^L(k_i)\wedge T_N^L(k_j)$$
$$T_M^U(k_i,k_j) = T_N^U(k_i)\wedge T_N^U(k_j)$$
$$I_M^L(k_i,k_j) = I_N^L(k_i)\vee I_N^L(k_j)$$
$$I_M^U(k_i,k_j) = I_N^U(k_i)\vee I_N^U(k_j)$$
$$F_M^L(k_i,k_j) = F_N^L(k_i)\vee F_N^L(k)$$
$$F_M^U(k_i,k_j) = F_N^U(k_i)\vee F_N^U(k_j)\ \forall\ (k_i ,k_j)\in E \qquad (26)$$

Example 4.10. Consider the strong interval valued neutrosophic graph G=(N, M) in Fig. 6 with vertex set N ={k_1, k_2, k_3, k_4}and edge set M={k_1k_2, k_2k_3, k_3k_4, k_4k_1} as follows:

	k_1	k_2	k_3
T_N^L	0.3	0.2	0.1
T_N^U	0.5	0.3	0.3
I_N^L	0.2	0.2	0.2
I_N^U	0.3	0.3	0.4
F_N^L	0.3	0.1	0.3
F_N^U	0.4	0.4	0.5

	k_1k_2	k_2k_3	k_3k_1
T_M^L	0.2	0.1	0.1
T_M^U	0.3	0.3	0.3
I_M^L	0.2	0.2	0.2
I_M^U	0.3	0.4	0.4
F_M^L	0.3	0.3	0.3
F_M^U	0.4	0.4	0.5

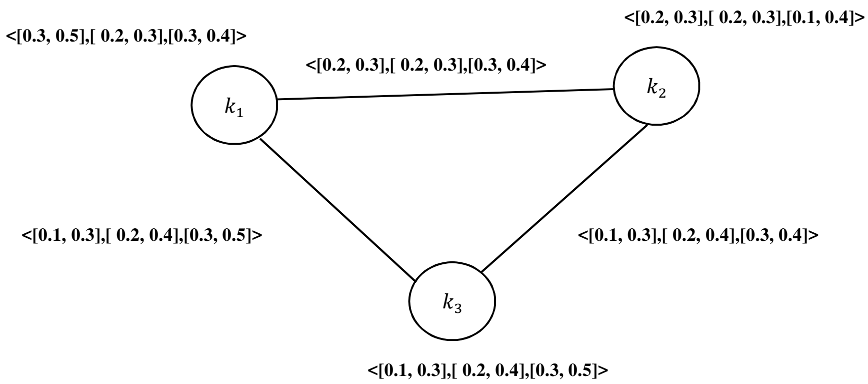

Fig.6.Illustration of strong IVNG

Proposition 4.11 For everyk_i,k_j ∈ *V*, we have

$$T_M^L(k_i,k_j) = T_M^L(k_j,k_i) \text{ and } \quad T_M^U(k_i,k_j) = T_M^U(k_j,k_i)$$
$$I_M^L(k_i,k_j) = I_M^L(k_j,k_i) \textit{ and } \quad I_M^U(k_i,k_j) = I_M^U(k_j,k_i)$$
$$F_M^L(k_i,k_j) = F_M^L(k_j,k_i) \text{ and } \quad F_M^U(k_i,k_j) = F_M^U(k_j,k_i) \qquad (27)$$

Proof. Suppose G =(N, M) be an interval valued neutrosophic graph, suppose $\boldsymbol{k_i}$ is a neigbourhood of $\boldsymbol{k_j}$ in G.Then , we have

$\boldsymbol{T_M^L(k_i, k_j)}$ =min [$\boldsymbol{T_N^L(k_i)}$, $\boldsymbol{T_N^L(k_j)}$] and $\boldsymbol{T_M^U(k_i, k_j)}$ =min [$\boldsymbol{T_N^U(k_i)}$, $\boldsymbol{T_N^U(k_j)}$]

$\boldsymbol{I_M^L(k_i, k_j)}$ =max [$\boldsymbol{I_N^L(k_i)}$, $\boldsymbol{I_N^L(k_j)}$] and $\boldsymbol{I_M^U(k_i, k_j)}$ =max[$\boldsymbol{I_N^U(k_i)}$, $\boldsymbol{I_N^U(k_j)}$]

$\boldsymbol{F_M^L(k_i, k_j)}$ =max [$\boldsymbol{F_N^L(k_i)}$, $\boldsymbol{F_N^L(k_j)}$] and $\boldsymbol{F_M^U(k_i, k_j)}$ =max [$\boldsymbol{F_N^U(k_i)}$, $\boldsymbol{F_N^U(k_j)}$]

Similarly we have also for

$\boldsymbol{T_M^L(k_j, k_i)}$ =min [$\boldsymbol{T_N^L(k_j)}$, $\boldsymbol{T_N^L(k_i)}$] and $\boldsymbol{T_M^U(k_j, k_i)}$ =min [$\boldsymbol{T_N^U(k_j)}$, $\boldsymbol{T_N^U(k_i)}$]

$\boldsymbol{I_M^L(k_j, k_i)}$ =max [$\boldsymbol{I_N^L(k_j)}$, $\boldsymbol{I_N^L(k_i)}$] and $\boldsymbol{I_M^U(k_j, k_i)}$ =max[$\boldsymbol{I_N^U(k_j)}$, $\boldsymbol{I_N^U(k_i)}$]

$\boldsymbol{F_M^L(k_j, k_i)}$ =max [$\boldsymbol{F_N^L(k_j)}$, $\boldsymbol{F_N^L(k_i)}$] and $\boldsymbol{F_M^U(k_j, k_i)}$ =max [$\boldsymbol{F_N^U(k_j)}$, $\boldsymbol{F_N^U(k_i)}$]

Thus

$\boldsymbol{T_M^L(k_i, k_j)}$ =$\boldsymbol{T_M^L(k_j, k_i)}and\boldsymbol{T_M^U(k_i, k_j)}$ =$\boldsymbol{T_M^U(k_j, k_i)}$
$\boldsymbol{I_M^L(k_i, k_j)}$ =$\boldsymbol{I_M^L(k_j, k_i)}$***and***$\boldsymbol{I_M^U(k_i, k_j)}$ =$\boldsymbol{I_M^U(k_j, k_i)}$
$\boldsymbol{F_M^L(k_i, k_j)}$ =$\boldsymbol{F_M^L(k_j, k_i)}and\boldsymbol{F_M^U}(\boldsymbol{k_i}, k)$ =$\boldsymbol{F_M^U(k_j, k_i)}$

Definition 4.12 The graph G= (N, M) is termed an interval valued neutrosophic graph if the following holds

$\boldsymbol{T_M^L(k_i, k_j)}$ =min [$\boldsymbol{T_N^L(k_i)}$, $\boldsymbol{T_N^U(k_j)}$] and $\boldsymbol{T_M^U(k_i, k_j)}$ =min [$\boldsymbol{T_N^U(k_i)}$, $\boldsymbol{T_N^U(k_j)}$]

$\boldsymbol{I_M^L(k_i, k_j)}$ =max [$\boldsymbol{I_N^L(k_i)}$, $\boldsymbol{I_N^L(k_j)}$] and $\boldsymbol{I_M^U(k_i, k_j)}$ =max[$\boldsymbol{I_N^U(k_i)}$, $\boldsymbol{I_N^U(k_j)}$]

$\boldsymbol{F_M^L(k_i, k_j)}$ =max [$\boldsymbol{F_N^L(k_i)}$, $\boldsymbol{F_N^L(k_j)}$] and $\boldsymbol{F_M^U(k_i, k_j)}$ =max [$\boldsymbol{F_N^U(k_i)}$, $\boldsymbol{F_N^U(k_j)}$] $\forall \boldsymbol{k_i},\boldsymbol{k_j} \in$ V (28)

Example 4.13. Consider the complete interval valued neutrosophic graph G=(N, M) portrayed in Fig. 7 with vertex set A ={k_1, k_2, k_3, k_4}and edge set E={k_1k_2,k_1k_3 ,k_2k_3, k_1k_4, k_3k_4 ,k_2k_4}as follows

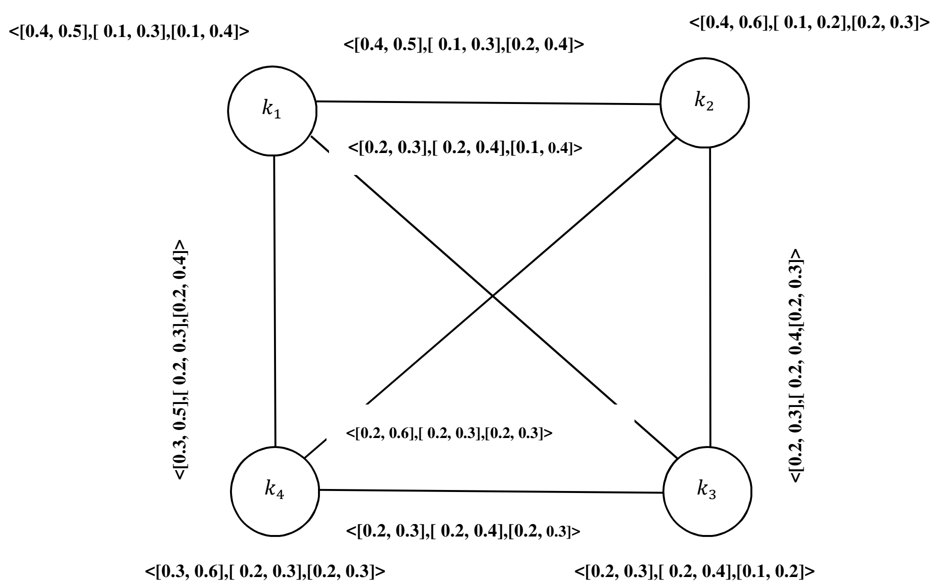

Fig.7 .Illustration of complete IVN-graph

In the following based on the extension of the adjacency matrix of SVNG [69], we defined the concept of adjacency matrix of IVNG as follow:

Definition 4.14: The adjacency matrix M(G) of IVNG G= (N, M) is defined as a square matrix M(G)=$[a_{ij}]$, with a_{ij}=<$\tilde{T}_M(k_i,k_j)$,$\tilde{I}^L_M(k_i,k_j)$,$\tilde{F}^L_M(k_i,k_j)$>, where

$\tilde{T}_M(k_i,k_j)$= [$T^L_M(k_i,k_j)$,$T^U(k_i,k_j)$] denote the strength of relationship

$\tilde{I}_M(k_i,k_j)$= [$I^L_M(k_i,k_j)$,$I^U_M(k_i,k_j)$] denote the strength of undecided relationship

$\tilde{F}_M(k_i,k_j)$=[$F^L_M(k_i,k_j)$,$F^U_M(k_i,k_j)$] denote the strength of non-relationship between k_i and k_j (29)

The adjacency matrix of an IVNG can be expressed as sixth matrices, first matrix contain the entries as lower truth-membership values, second contain upper truth-membership values, third contain lower indeterminacy-membership values, forth contain upper indeterminacy-membership, fifth contains lower non-membership values and the sixth contain the upper non-membership values, i.e.,

$$M(G)=<[T^L_M(k_i,k_j),T^U_M(k_i,k_j)]\,,\,[I^L_M(k_i,k_j),I^U_M(k_i,k_j)]\,,\,[F^L_M(k_i,k_j),F^U_M(k_i,k_j)]>, \qquad (30)$$

From the Fig. 1, the adjacency matrix of IVNG is defined as:

$$\boldsymbol{M_G}=\begin{bmatrix} \mathbf{0} & <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}]> & <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.6}]> \\ <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}]> & \mathbf{0} & <[\mathbf{0.1},\mathbf{0.3}],[\mathbf{0.4},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.5}]> \\ <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.6}]> & <[\mathbf{0.1},\mathbf{0.3}],[\mathbf{0.4},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.5}]> & \mathbf{0} \end{bmatrix}$$

In the literature, there is no Matlab toolbox deals with neutrosophic matrix such as adjacency matrix and so on. Recently Broumi et al [58] developed a Matlab toolbox for computing operations on interval valued neutrosophicmatrices.So, we can inputted the adjacency matrix of IVNG in the workspace Matlab as portrayed in Fig. 8.

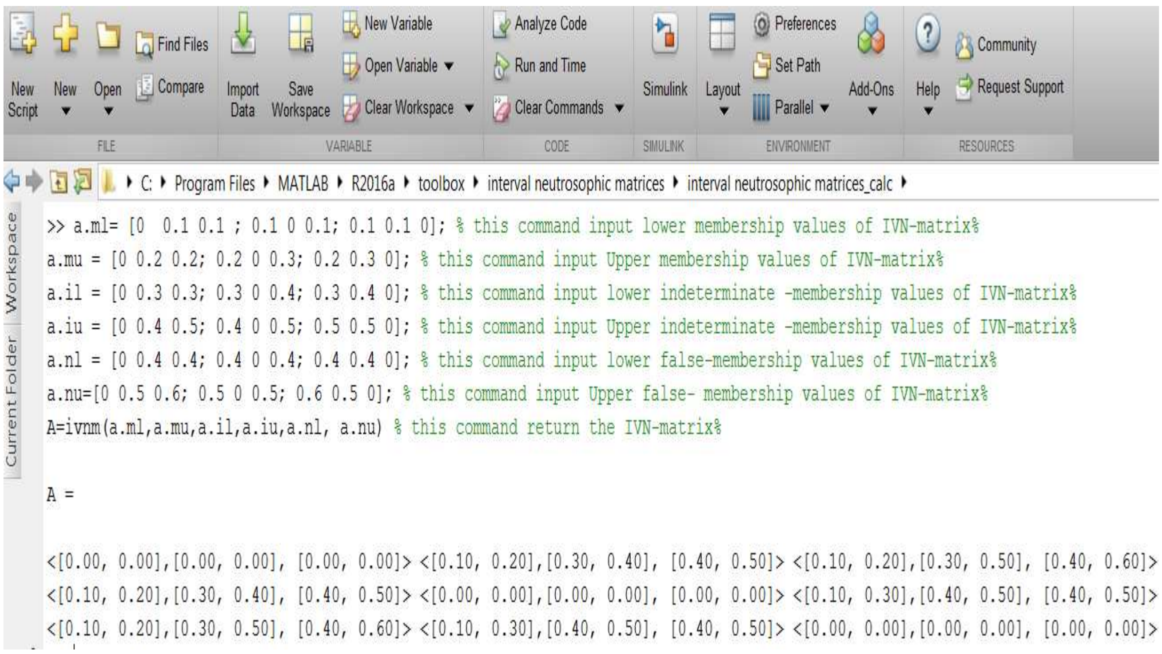

Fig. .8 Screen shot of Workspace MATLAB

Definition 4.15: The spectrum of adjacency matrix of an IVNG M(G) is defined as

$$<\tilde{R}, \tilde{S},\tilde{Q}>=<[\tilde{R}^L,\tilde{R}^U],[\tilde{S}^L,\tilde{S}^U],<[\tilde{Q}^L,\tilde{Q}^U]> \quad (31)$$

Where $\tilde{R}^L$is the set of eigenvalues of $M(T_M^L(k_i,k_j))$,$\tilde{R}^U$ is the set of eigenvalues of $M(T_B^U(k_i,k_j))$,$\tilde{S}^L$ is the set of eigenvalues of $M(I_M^L(k_i,k_j))$,$\tilde{S}^U$ is the set of eigenvalues of $M(I_M^U(k_i,k_j))$, $\tilde{Q}^L$ is the set of eigenvalues of $M(F_M^L(k_i,k_j))$ and $\tilde{Q}^U$ is the set of eigenvalue of $M(F_B^U(k_i,k_j))$respectively.

Definition 4.16: The energy of an IVNG G= (N,M) is defined as

$$\mathbf{E(G)=<E(\tilde{T}_M(k_i,k_j)),E(\tilde{I}_M^L(k_i,k_j)),E(\tilde{F}_M^L(k_i,k_j))>} \quad (32)$$

Where

$$\mathrm{E}(\tilde{T}_M(k_i,k_j)) = [\mathrm{E}(T_M^L(k_ik_j)),\mathrm{E}(T_M^U(k_ik_j))]=[\sum_{\substack{i=1\\ \lambda_i^L\in\tilde{R}^L}}^{n} |\lambda_i^L|, \sum_{\substack{i=1\\ \lambda_i^U\in\tilde{R}^U}}^{n} |\lambda_i^U|]$$

$$\mathrm{E}(\tilde{I}_M(k_i,k_j)) = [\mathrm{E}(I_M^L(k_ik_j)),\mathrm{E}(I_M^U(k_ik_j))]=[\sum_{\substack{i=1\\ \delta_i^L\in\tilde{S}^L}}^{n} |\delta_i^L|, \sum_{\substack{i=1\\ \delta_i^U\in\tilde{S}^U}}^{n} |\delta_i^U|]$$

$$\mathrm{E}(\tilde{F}_M(k_i,k_j))= [\mathrm{E}(F_M^L(k_ik_j)),\mathrm{E}(F_M^U(k_ik_j))] =[\sum_{\substack{i=1\\ \zeta_i^L\in\tilde{Q}^L}}^{n} |\zeta_i^L|, \sum_{\substack{i=1\\ \zeta_i^U\in\tilde{Q}^U}}^{n} |\zeta_i^U|]$$

Definition 4.17: Two interval valued neutrosophic graphsG_1 and G_2 are termed equienergetic, if they have the same number of vertices and the same energy.

Proposition4.18:If an interval valued neutrosophic G is both regular and totally regular, then the eigen values are balanced on the energy.

$$\sum_{i=1}^{n} \pm\lambda_i^L= 0, \sum_{i=1}^{n} \pm\lambda_i^U= 0, \sum_{i=1}^{n} \pm\delta_i^L= 0, \sum_{i=1}^{n} \pm\delta_i^U= 0, \sum_{i=1}^{n} \pm\zeta_i^L= 0 \text{ and} \sum_{i=1}^{n} \pm\zeta_i^U= 0. \quad (33)$$

4.19. MATLAB program for findingspectrum of an interval valued neutrosophic graph

To generate the MATLAB program for finding the spectrum of interval valued neutrosophic graph. The program termed "Spec.m" is written as follow:

```
Function SG=Spec(A);
% Spectrum of an interval valued neutrosophic matrix A
% "A" have to be an interval valued neutrosophic  matrix - "ivnm" object:
a.ml=eig(A.ml);            % eigenvalues of lower membership of ivnm%
a.mu=eig(A.mu);            % eigenvalues of upper membership of ivnm%
a.il=eig(A.il);            % eigenvalues of lower rindeterminate-membership of ivnm%
a.iu=eig(A.iu);            % eigenvalues of upper indterminate- membership of ivnm%
a.nl=eig(A.nl);            % eigenvalues of lower false-membership of ivnm%
a.nu=eig(A.nu);            % eigenvalues of upper false-membership of ivnm%
SG=ivnm(a.ml,a.mu,a.il,a.iu,a.nl,a.nu);
```

4.20. MATLAB program for finding energy of an interval valued neutrosophic graph

To generate the MATLAB program for finding the energy of interval valued neutrosophic graph. The program termed "ENG.m"iswritten as follow:

```
function EG=ENG(A);
% energy of an interval valued neutrosophic matrix A
% "A" have to be an interval valued neutrosophic  matrix - "ivnm" object:
a.ml=sum(abs(eig(A.ml)));
a.mu=sum(abs(eig(A.mu)));
a.il=sum(abs(eig(A.il)));
a.iu=sum(abs(eig(A.iu)));
a.nl=sum(abs(eig(A.nl)));
a.nu=sum(abs(eig(A.nu)));
EG=ivnm(a.ml,a.mu,a.il,a.iu,a.nl,a.nu);
```

Example4.21: The spectrum and the energy of an IVNG, illustrated in Fig. 6, are given below:

Spec($T_M^L(k_ik_j)$)={ -0.10, -0.10,0.20}, Spec($T_M^U(k_ik_j)$)={-0.30,-0.17,0.47}
Spec($I_M^L(k_ik_j)$)={-0.40,-0.27,0.67}, Spec($I_M^U(k_ik_j)$)={-0.53,-0.40,0.93]}
Spec($F_M^L(k_ik_j)$)={-0.40,-0.40,0.80}, Spec($F_M^U(k_ik_j)$)={ -0.60,-0.47,1.07}

Hence,
Spec(G)={<[-0.10, -0.30], [-0.40, -0.53],[-0.40, -0.60]>, <[-0.10, -0.17], [-0.27, -0.40],[-0.40, -0.47]>, <[0.20, 0.47], [0.67, 0.93],[0.80, 1.07]>}
Now ,
E($T_M^L(k_ik_j)$)=0.40, E($T_M^U(k_ik_j)$)=0.94

E($I_M^L(k_ik_j)$)=1.34,E($I_M^U(k_ik_j)$)=1.87

E($F_M^L(k_ik_j)$)=1.60,E($F_M^U(k_ik_j)$)=2.14

Therefore
E(G)= <[0.40, 0.94],[1.34, 1.87], [1.60, 2.14]>
Based on toolbox MATLAB developed in [58], the readers can run the program termed “Spec.m”, for computing the spectrum of graph, by writing in command window “Spec (A)” as described below:

```
>> Spec(A) % this command return the spectrum of IVN-matrix%
Warning! The created new object is NOT an interval valued neutrosophic matrix

ans =

<[-0.10, -0.30],[-0.40, -0.53], [-0.40, -0.60]>
<[-0.10, -0.17],[-0.27, -0.40], [-0.40, -0.47]>
<[0.20, 0.47],[0.67, 0.93], [0.80, 1.07]>
```

Similarly, the readers can also run the program termed “ENG.m”, for computing the energy of graph, by writing in command window “ENG (A) as described below:

```
>> ENG(A) % this command return the Energy of IVN-matrix%
Warning! The created new object is NOT an interval valued neutrosophic matrix

ans =

<[0.40, 0.94],[1.34, 1.87], [1.60, 2.14]>
```

In term of the number of vertices and the sum of interval truth-membership, interval indeterminate-membership and interval false-membership, we define the upper and lower bounds on energy of an IVNG.

Proposition 4.22. Suppose G= (N, M) be an IVNG on n vertices and the adjacency matrix of G.then

$$\sqrt{2\sum_{1\le ij\le n}\left(T_M^L(k_ik_j)\right)^2+n(n-1)|T^L|^{2/N}}\le E\left(T_M^L\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(T_M^L(k_ik_j)\right)^2} \quad (34)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(T_M^U(k_ik_j)\right)^2+n(n-1)|T^U|^{2/N}}\le E\left(T_M^U\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(T_M^U(k_ik_j)\right)^2} \quad (35)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(I_M^L(k_ik_j)\right)^2+n(n-1)|I^L|^{2/N}}\le E\left(I_M^L\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(I_M^L(k_ik_j)\right)^2} \quad (36)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(I_M^U(k_ik_j)\right)^2+n(n-1)|I^U|^{2/N}}\le E\left(I_M^U\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(I_M^U(k_ik_j)\right)^2} \quad (37)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(F_M^L(k_ik_j)\right)^2+n(n-1)|F^L|^{2/N}}\le E\left(F_M^L\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(F_M^L(k_ik_j)\right)^2} \quad (38)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(F_M^U(k_ik_j)\right)^2+n(n-1)|F^U|^{2/N}}\le E\left(F_M^U\left(k_ik_j\right)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(F_M^U(k_ik_j)\right)^2} \quad (39)$$

Where $|\boldsymbol{T^L}|$,$|\boldsymbol{T^U}|$,$|\boldsymbol{I^L}|$,$|\boldsymbol{I^U}|$,$|\boldsymbol{F^L}|$and $|\boldsymbol{F^U}|$ are the determinant of $\boldsymbol{M(T_M^L(k_i,k_j))}$, $\boldsymbol{M(T_M^U(k_i,k_j))}$, $\boldsymbol{M(I_M^L(k_i,k_j))}$, $\boldsymbol{M(I_M^U(k_i,k_j))}$, $\boldsymbol{M(F_M^L(k_i,k_j))}$ and$\boldsymbol{M(F_M^U(k_i,k_j))}$,respectively.

Proof: proof is similar as in Theorem 3.2 [69]

Conclusion

This paper introduces some basic operations on interval-valued neutrosophic set to increase its utility in various fields for multi-decision process. To achieve this goal, a new mathematical algebra of interval-valued neutrosophic graphs, its energy as well as spectral computation is discussed with mathematical proof using MATLAB. In the near future, we plan to extend our research to interval valued neutrosophic digraphs and developed the concept of domination in interval valued-neutrosophic graphs. Same time the author will focus on handling its necessity for knowledge representation and processing tasks [85-87].

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