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Entropy and Similarity Measure for T2SVNSs and Its Application

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Abstract: The objective of this paper is to present a new approach for solving the multi-criteria group decision-making (MCGDM) problems in type-2 single valued neutrosophic set (T2SVNS) environment. Firstly, we give the concepts SVNS, T2SVNS and tangent similarity measure with T2SVN information. Secondly, we define a new entropy function for determining unknown attribute weights. In addition, a MCGDM method is developed based on entropy and tangent similarity measure of T2SVNSs. Finally, an illustrative example and comparative analysis are given to confirm the rationality and feasibility of the proposed method.

Keywords: Type-2 single valued neutrosophic set; Entropy; Tangent function; Similarity measure; MCGDM

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§1. Introduction

Neutrosophic set [32] is an important tool for dealing with problems involving uncertainty, indeterminacy and inconsistency. Wang et al. [33] developed the concept of SVNSs, which is a subclass of the neutrosophic sets (NSs) for solving scientific and engineering problems. SVNSs have been widely used in different fields, like engineering problems, [12, 34] medical problems, [1, 2, 11] image processing problems, [10, 17, 18] decision-making problems, [25, 29, 37]

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social problems [22, 26], conflict problems [27]. Many scholars have also studied type-2 fuzzy sets, such as Yang et al. [35] introduced the similarity of type-2 fuzzy sets, they also investigated its properties, Hung et al. [13] proposed similarity methods between two type-2 fuzzy sets, at this moment, the properties of these methods were obtained. Sing [31] introduced two type-2 fuzzy sets based on the distances between Euclidean and Hamming. Zhao et al. [39] have studied type-2 intuitionistic fuzzy set (T2IFS), they gave the concept of T2IFS, and discussed the relation of T2IFS. Cuong et al. [8] introduced some operations between two T2IFSs.

Similarity measure is becoming important in decision making problems. Some strategies [7, 14] are proposed to measure the similarity between fuzzy sets, whereas these strategies can not deal with the similarity measures involving uncertainty and inconsistency. In the references, some scholars have discussed the similarity measures of NSs. [3, 4] Mondal et al. [23, 36] proposed sine hyperbolic similarity measure and tangent similarity measure methods to deal with MADM problems. Lu et al. [24] proposed logarithmic similarity measure and applied it in fault diagnosis strategy under interval valued fuzzy set environment [19]. In addition to similarity measurement, there are other aspects of research, such as: correlation coefficient [30], TOPSIS method [5], aggregating operators [9, 15, 21]. Based on the above analysis, few scholars have studied the MCGDM method using tangent similarity measure, so the main contents of this paper are:

(1) To define a new similarity measure under T2SVNS environment and prove its basic properties.

(2) To define a new entropy function of T2SVNSs to determine the weight of unknown attributes.

(3) To develop a MCGDM model based on proposed entropy and similarity measures.

(4) To present an illustrative example and comparative analysis to illustrate effectiveness and feasibility of the proposed method.

The rest of this paper is structured as follows. In section 2, the concepts of SVNSs and T2SVNSs are given. In section 3, we define tangent similarity measure between two T2SVNSs and prove its properties. In section 4, a new entropy funtion to compute unknown attribute weights for T2SVNSs is proposed. In section 5, we propose a MCGDM method based on entropy and tangent similarity measures of T2SVNSs. In section 6, an example and comparative analysis are given to illustrate effectiveness and feasibility of the proposed method. In section 7, we come to the conclusion.

§2. Preliminaries

2.1. Single valued neutrosophic sets (SVNS)

Definition 2.1. *[38] Let X be a universal space of points (objects), with a generic element in X denoted by x, single valued neutrosophic set (SVNS) $Q \subset X$ is characterized by truth-membership function $t_q(x)$, indeterminacy-membership function $i_q(x)$ and falsity-membership function $f_q(x)$.*

A SVNS can be expressed as

$$Q=\{[\langle x,t_q(x),i_q(x),f_q(x)\rangle]|x\in X\}. \tag{2.1}$$

where $t_q(x)$, $i_q(x)$, $f_q(x)$ *are real standard or nonstandard subsets of* $[0,1]$, *so that it means* $t_q(x)$: $X\to[0,1]$, $i_q(x)$: $X\to[0,1]$, $f_q(x)$: $X\to[0,1]$, *with the condition of* $0\le\sup t_q(x)+\sup i_q(x)+\sup f_q(x)\le 3$, *for all* $x\in X$.

When X *is continuous, a SVNS* Q *can be written as*

$$Q=\int_X \langle t_q(x),i_q(x),f_q(x)\rangle/x,\quad x\in X. \tag{2.2}$$

When X *is discrete, a SVNS* Q *can be written as*

$$Q=\sum_{i=1}^{n}\langle t_q(x_i),i_q(x_i),f_q(x_i)\rangle/x_i,\quad x_i\in X. \tag{2.3}$$

Definition 2.2. *[33] Let* P *and* Q *be two SVNSs,*

$$P=\langle t_p(x),i_p(x),f_p(x)\rangle,\quad Q=\langle t_q(x),i_q(x),f_q(x)\rangle,$$

then, for all $x\in X$, *operations can be defined as follows:*

(1) $P\subseteq Q$, *iff,* $t_q(x)\ge t_p(x), i_q(x)\le i_p(x), f_q(x)\le f_p(x)$.

(2) $P=Q$, *iff,* $P\subseteq Q$ *and* $Q\subseteq P$.

(3) The complement of a SVNS P *is denoted as* P^c, *which is defined as*$t_{p^c}(x)=f_p(x)$, $i_{p^c}(x)=1-i_p(x)$, $f_{p^c}(x)=t_p(x)$.

(4) $Q\bigcup P=\langle max(t_p(x),t_q(x)),min(i_p(x),i_q(x)),min(f_p(x),f_q(x))\rangle$.

(5) $Q\bigcap P=\langle min(t_p(x),t_q(x)),max(i_p(x),i_q(x)),max(f_p(x),f_q(x))\rangle$.

Definition 2.3. *[20] Let* Q *and* P *be two SVNSs,*

$$Q=\langle t_q(x),i_q(x),f_q(x)\rangle,\quad P=\langle t_p(x),i_p(x),f_p(x)\rangle,$$

then, $\forall\ k\in R$, *there is*

(1) $Q\oplus P=\langle t_q(x)+t_p(x)-t_q(x)\cdot t_p(x), i_q(x)\cdot i_p(x), f_q(x)\cdot f_p(x)\rangle$.

(2) $Q\otimes P=\langle t_q(x)\cdot t_p(x), i_q(x)+i_p(x)-i_q(x)\cdot i_p(x), f_q(x)+f_p(x)-f_q(x)\cdot f_p(x)\rangle$.

(3) $\lambda Q=(1-(1-t_q(x))^k, i_q(x)^k, f_q(x)^k)$.

(4) $Q^k=(t_q(x)^k, 1-(1-i_q(x))^k, 1-(1-f_q(x))^k)$.

2.2. Type-2 single valued neutrosophic set (T2SVNS)

Definition 2.4. *[16] A T2SVNS* $\tilde{N}$ *is a set of pairs* $\{\mu_N(a),\eta_N(a),\nu_N(a)\}$, $a\in A$, $\mu_N(a)$, $\eta_N(a)$ *and* $\nu_N(a)$ *are respectively called true membership, uncertain membership and false membership, which are defined as follows:*

$$\begin{aligned}\mu_N(a)&=\int_{u_N\in j_a^T} t_a(u_N)/u_N;\\ \eta_N(a)&=\int_{n_N\in j_a^I} i_a(n_N)/n_N;\\ \nu_N(a)&=\int_{v_N\in j_a^F} f_a(v_N)/v_N\end{aligned} \tag{2.4}$$

where u_N, n_N and v_N are named Primary truth-membership function(Ptmf), Primary indeterminacy membership function (Pimf) and Primary falsity-membership function (Pfmf). $t_a(u_N)$, $i_a(n_N)$ and $f_a(v_N)$ are called Secondary truth membership function (Stmf), Secondary indeterminacy membership function (Simf) and Secondary falsity-membership function (Sfmf). j_a^T, j_a^I and j_a^F are called as primary truth membership, primary indeterminant membership and primary falsity membership, respectively.

T2SVNS $\tilde{N}$ can be shown as:

$$\tilde{N}=\{\langle(a,u_N,n_N,v_N),(t_a(u_N),i_a(n_N),f_a(v_N))\rangle|a\in A,\ u_N\in j_a^T,\ n_N\in j_a^I,\ v_N\in j_a^F\}. \tag{2.5}$$

For convenience, $\tilde{N}$ can be abbreviated as $\tilde{N}=\langle(u_N,t_a(u_N),n_N,i_a(n_N),v_N,f_a(v_N))\rangle$, which is called type-2 single valued neutrosophic number(T2SVNN). From now on, the set of all T2SVNS over the universe A will be denoted by $SV_2(A)$.

Definition 2.5. *[16] Let $\tilde{N}_1=\langle(u_{N_1},t_a(u_{N_1}),n_{N_1},i_a(n_{N_1}),v_{N_1},f_a(v_{N_1}))\rangle$ and $\tilde{N}_2=\langle(u_{N_2},t_a(u_{N_2}),n_{N_2},i_a(n_{N_2}),v_{N_2},f_a(v_{N_2}))\rangle$ be two T2SVNSs, $\forall a\in A$. Then,*

(1) $\tilde{N}_1\subseteq\tilde{N}_2$ if and only if $u_{N_1}\le u_{N_2}$, $t_a(u_{N_1})\le t_a(u_{N_2})$, $n_{N_1}\ge n_{N_2}$, $i_a(n_{N_1})\ge i_a(n_{N_2})$, $v_{N_1}\ge v_{N_2}$, $f_a(v_{N_1})\ge f_a(v_{N_2})$.

(2) $\tilde{N}_1=\tilde{N}_2$ if and only if $\tilde{N}_1\supseteq\tilde{N}_2$ and $\tilde{N}_1\subseteq\tilde{N}_2$.

(3) $\tilde{N}_1^c=\langle v_{N_1},f_a(v_{N_1}),1-n_{N_1},1-i_a(n_{N_1}),u_{N_1},t_a(u_{N_1})\rangle$.

(4) $\tilde{N}_1\bigcup\tilde{N}_2=\langle max(u_{N_1},u_{N_2}),max(t_a(u_{N_1}),t_a(u_{N_2})),min(n_{N_1},n_{N_2}),min(i_a(n_{N_1}),i_a(n_{N_2})),min(v_{N_1},v_{N_2}),min(f_a(v_{N_1}),f_a(v_{N_2}))\rangle$.

(5) $\tilde{N}_1\bigcap\tilde{N}_2=\langle min(u_{N_1},u_{N_2}),min(t_a(u_{N_1}),t_a(u_{N_2})),max(n_{N_1},n_{N_2}),max(i_a(n_{N_1}),i_a(n_{N_2})),max(v_{N_1},v_{N_2}),max(f_a(v_{N_1}),f_a(v_{N_2}))\rangle$.

§3. Tangent similarity measures for T2SVNSs

Definition 3.1. *Assume that $\tilde{N}_1,\tilde{N}_2\in SV_2(A)$, similarity measure based on tangent function between two T2SVNSs is defined as follows:*

$$\begin{aligned}T(\tilde{N}_1,\tilde{N}_2)=1-\frac{1}{m}\sum_{i=1}^{m}tan[\frac{\pi}{12}(|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})\\ -\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|+|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})\\ -\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|+|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})\\ -\lambda v_{N_2}(a_i)-(1-\lambda)f_{a_i}(v_{N_2})|)],\quad 0\le\lambda\le 1.\end{aligned} \tag{3.1}$$

Theorem 3.1. *The defined tangent similarity measure $T(\tilde{N}_1,\tilde{N}_2)$ of two T2SVNSs, the basic operations are satisfied as follows:*

(1) $1\ge T(\tilde{N}_1,\tilde{N}_2)\ge 0$.

(2) $T(\tilde{N}_1,\tilde{N}_2)$=1 if and only if $\tilde{N}_1=\tilde{N}_2$.

(3) $T(\tilde{N}_1,\tilde{N}_2)$=$T(\tilde{N}_2,\tilde{N}_1)$.

(4) if $\tilde{N}_3\in SV_2(A)$, $\tilde{N}_1\subseteq\tilde{N}_2\subseteq\tilde{N}_3$, then $T(\tilde{N}_1,\tilde{N}_3)\le T(\tilde{N}_1,\tilde{N}_2)$ and $T(\tilde{N}_1,\tilde{N}_3)\le T(\tilde{N}_2,\tilde{N}_3)$.

Proof. (1) Tangent function increases monotonically on the interval $[0, \frac{\pi}{4}]$. It also depends on the interval [0,1]. Therefore, $0 \leq T(\tilde{N}_1, \tilde{N}_2) \leq 1$.

(2) Assume that two T2SVNS, $0 \leq \lambda \leq 1$,

$$\begin{aligned}\tilde{N}_1 = \tilde{N}_2 \Rightarrow & \lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) = \lambda u_{N_2}(a_i) + (1-\lambda)t_{a_i}(u_{N_2}),\\ & \lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) = \lambda n_{N_2}(a_i) + (1-\lambda)i_{a_i}(n_{N_2}),\\ & \lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) = \lambda v_{N_2}(a_i) + (1-\lambda)f_{a_i}(v_{N_2})\\ & \Rightarrow |\lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) - \lambda u_{N_2}(a_i) - (1-\lambda)t_{a_i}(u_{N_2})| = 0,\\ & |\lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) - \lambda n_{N_2}(a_i) - (1-\lambda)i_{a_i}(n_{N_2})| = 0,\\ & |\lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) - \lambda v_{N_2}(a_i) - (1-\lambda)f_{a_i}(v_{N_2})| = 0.\\ & \Rightarrow T(\tilde{N}_1, \tilde{N}_2) = 1.\end{aligned}$$

$$\begin{aligned}T(\tilde{N}_1, \tilde{N}_2) = 1 \Rightarrow & |\lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) - \lambda u_{N_2}(a_i) - (1-\lambda)t_{a_i}(u_{N_2})| = 0,\\ & |\lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) - \lambda n_{N_2}(a_i) - (1-\lambda)i_{a_i}(n_{N_2})| = 0,\\ & |\lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) - \lambda v_{N_2}(a_i) - (1-\lambda)f_{a_i}(v_{N_2})| = 0\\ & \Rightarrow \lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) = \lambda u_{N_2}(a_i) + (1-\lambda)t_{a_i}(u_{N_2}),\\ & \lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) = \lambda n_{N_2}(a_i) + (1-\lambda)i_{a_i}(n_{N_2}),\\ & \lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) = \lambda v_{N_2}(a_i) + (1-\lambda)f_{a_i}(v_{N_2}),\\ & \Rightarrow \tilde{N}_1 = \tilde{N}_2.\end{aligned}$$

$$\begin{aligned}(3)\ T(\tilde{N}_1, \tilde{N}_2) = & 1 - \frac{1}{m}\sum_{i=1}^{m} tan[\frac{\pi}{12}(|\lambda_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) - \lambda u_{N_2}(a_i) - (1-\lambda)t_{a_i}(u_{N_2})|\\ & + |\lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) - \lambda n_{N_2}(a_i) - (1-\lambda)i_{a_i}(n_{N_2})|\\ & + |\lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) - \lambda v_{N_2}(a_i) - (1-\lambda)f_{a_i}(v_{N_2})|)]\\ = & 1 - \frac{1}{m}\sum_{i=1}^{m} tan[\frac{\pi}{12}(|\lambda u_{N_2}(a_i) + (1-\lambda)t_{a_i}(u_{N_2}) - \lambda u_{N_1}(a_i) - (1-\lambda)t_{a_i}(u_{N_1})|\\ & + |\lambda n_{N_2}(a_i) + (1-\lambda)i_{a_i}(n_{N_2}) - \lambda n_{N_1}(a_i) - (1-\lambda)i_{a_i}(n_{N_1})|\\ & + |\lambda v_{N_2}(a_i) + (1-\lambda)f_{a_i}(v_{N_2}) - \lambda v_{N_1}(a_i) - (1-\lambda)f_{a_i}(v_{N_1})|)] = T(\tilde{N}_2, \tilde{N}_1).\end{aligned}$$

(4) If $\tilde{N}_1 \subseteq \tilde{N}_2 \subseteq \tilde{N}_3$, then

$$\begin{aligned}& |\lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) - \lambda u_{N_2}(a_i) - (1-\lambda)t_{a_i}(u_{N_2})|\\ \leq & |\lambda u_{N_1}(a_i) + (1-\lambda)t_{a_i}(u_{N_1}) - \lambda u_{N_3}(a_i) - (1-\lambda)t_{a_i}(u_{N_3})|,\\ & |\lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) - \lambda n_{N_2}(a_i) - (1-\lambda)i_{a_i}(n_{N_2})|\\ \leq & |\lambda n_{N_1}(a_i) + (1-\lambda)i_{a_i}(n_{N_1}) - \lambda n_{N_3}(a_i) - (1-\lambda)i_{a_i}(n_{N_3})|,\\ & |\lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) - \lambda v_{N_2}(a_i) + (1-\lambda)f_{a_i}(v_{N_2})|\\ \leq & |\lambda v_{N_1}(a_i) + (1-\lambda)f_{a_i}(v_{N_1}) - \lambda v_{N_3}(a_i) + (1-\lambda)f_{a_i}(v_{N_3})|\end{aligned}$$

so $T(\tilde{N}_1,\tilde{N}_3)\leq T(\tilde{N}_1,\tilde{N}_2)$. In the same way,

$$\begin{aligned}
&|\lambda u_{N_2}(a_i)+(1-\lambda)t_{a_i}(u_{N_2})-\lambda u_{N_3}(a_i)-(1-\lambda)t_{a_i}(u_{N_3})|\\
\leq&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_3}(a_i)-(1-\lambda)t_{a_i}(u_{N_3})|,\\
&|\lambda n_{N_2}(a_i)+(1-\lambda)i_{a_i}(n_{N_2})-\lambda n_{N_3}(a_i)-(1-\lambda)i_{a_i}(n_{N_3})|\\
\leq&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_3}(a_i)-(1-\lambda)i_{a_i}(n_{N_3})|,\\
&|\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2})-\lambda v_{N_3}(a_i)+(1-\lambda)f_{a_i}(v_{N_3})|\\
\leq&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_3}(a_i)+(1-\lambda)f_{a_i}(v_{N_3})|
\end{aligned}$$

so $T(\tilde{N}_1,\tilde{N}_3)\leq T(\tilde{N}_2,\tilde{N}_3)$. □

Definition 3.2. *Assume that $\tilde{N}_1,\tilde{N}_2\in SV_2(A)$, weighted similarity measure based on tangent function between two T2SVNSs is defined as follows:*

$$\begin{aligned}
T^W(\tilde{N}_1,\tilde{N}_2)=1-\sum_{i=1}^{m}\omega_i tan[\frac{\pi}{12}(&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|\\
&+|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|\\
&+|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_2}(a_i)-(1-\lambda)f_{a_i}(v_{N_2})|)]
\end{aligned}\tag{3.2}$$

Here, $0\leq\lambda\leq1$, $\sum\limits_{i=1}^{m}\omega_i=1$.

Theorem 3.2. *The defined tangent similarity measure $T(\tilde{N}_1,\tilde{N}_2)$ of two T2SVNS, the basic operations are satisfied as follows::*

(1) $1\geq T^W(\tilde{N}_1,\tilde{N}_2)\geq0$,

(2) $T^W(\tilde{N}_1,\tilde{N}_2)$=1 if and only if $\tilde{N}_1=\tilde{N}_2$,

(3) $T^W(\tilde{N}_1,\tilde{N}_2)$=$T^W(\tilde{N}_2,\tilde{N}_1)$,

(4) if $\tilde{N}_3\in SV_2(A)$ and $\tilde{N}_1\subseteq\tilde{N}_2\subseteq\tilde{N}_3$, then $T^W(\tilde{N}_1,\tilde{N}_3)\leq T^W(\tilde{N}_1,\tilde{N}_2)$ and $T^W(\tilde{N}_1,\tilde{N}_3)\leq T^W(\tilde{N}_2,\tilde{N}_3)$.

Proof. (1) Tangent function increases monotonically on the interval $[0,\frac{\pi}{4}]$. It also depends on the interval [0,1] and $\sum\limits_{i=1}^{m}\omega_i=1$. So, $0\leq T^W(\tilde{N}_1,\tilde{N}_2)\leq1$.

(2) For any two T2SVNS $\tilde{N}_1$ and $\tilde{N}_2$, $0\leq\lambda\leq1$,

$$\begin{aligned}
&\tilde{N}_1=\tilde{N}_2\\
\Rightarrow&\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})=\lambda u_{N_2}(a_i)+(1-\lambda)t_{a_i}(u_{N_2}),\\
&\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})=\lambda n_{N_2}(a_i)+(1-\lambda)i_{a_i}(n_{N_2}),\\
&\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})=\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2})\\
\Rightarrow&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|=0,\\
&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|=0,\\
&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_2}(a_i)-(1-\lambda)f_{a_i}(v_{N_2})|=0.
\end{aligned}$$

Therefore, $T^W(\tilde{N}_1,\tilde{N}_2)=1$ for $0\leq\lambda\leq 1$ and $\sum\limits_{i=1}^{m}\omega_i=1$.

Conversely,

$$\begin{aligned}T(\tilde{N}_1,\tilde{N}_2)=1\Rightarrow&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|=0,\\&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|=0,\\&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_2}(a_i)-(1-\lambda)f_{a_i}(v_{N_2})|=0\\\Rightarrow&\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})=\lambda u_{N_2}(a_i)+(1-\lambda)t_{a_i}(u_{N_2}),\\&\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})=\lambda n_{N_2}(a_i)+(1-\lambda)i_{a_i}(n_{N_2}),\\&\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})=\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2}),\\\Rightarrow&\tilde{N}_1=\tilde{N}_2\end{aligned}$$

(3) Lets prove the third question

$$\begin{aligned}T(\tilde{N}_1,\tilde{N}_2)=&1-\sum_{i=1}^{m}\omega_i tan[\frac{\pi}{12}(|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|\\&+|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|\\&+|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_2}(a_i)-(1-\lambda)f_{a_i}(v_{N_2})|)]\\=&1-\sum_{i=1}^{m}\omega_i tan[\frac{\pi}{12}(|\lambda u_{N_2}(a_i)+(1-\lambda)t_{a_i}(u_{N_2})-\lambda u_{N_1}(a_i)-(1-\lambda)t_{a_i}(u_{N_1})|\\&+|\lambda n_{N_2}(a_i)+(1-\lambda)i_{a_i}(n_{N_2})-\lambda n_{N_1}(a_i)-(1-\lambda)i_{a_i}(n_{N_1})|\\&+|\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2})-\lambda v_{N_1}(a_i)-(1-\lambda)f_{a_i}(v_{N_1})|)]=T(\tilde{N}_2,\tilde{N}_1).\end{aligned}$$

(4) If $\tilde{N}_1\subseteq\tilde{N}_2\subseteq\tilde{N}_3$, then

$$\begin{aligned}&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_2}(a_i)-(1-\lambda)t_{a_i}(u_{N_2})|\\\leq&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_3}(a_i)-(1-\lambda)t_{a_i}(u_{N_3})|,\\&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_2}(a_i)-(1-\lambda)i_{a_i}(n_{N_2})|\\\leq&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_3}(a_i)-(1-\lambda)i_{a_i}(n_{N_3})|,\\&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2})|\\\leq&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_3}(a_i)+(1-\lambda)f_{a_i}(v_{N_3})|\end{aligned}$$

for $\sum\limits_{i=1}^{m}\omega_i=1$. From Equation (3.2), $T^W(\tilde{N}_1,\tilde{N}_3)\leq T^W(\tilde{N}_1,\tilde{N}_2)$ is getted. In the same way,

$$\begin{aligned}&|\lambda u_{N_2}(a_i)+(1-\lambda)t_{a_i}(u_{N_2})-\lambda u_{N_3}(a_i)-(1-\lambda)t_{a_i}(u_{N_3})|\\\leq&|\lambda u_{N_1}(a_i)+(1-\lambda)t_{a_i}(u_{N_1})-\lambda u_{N_3}(a_i)-(1-\lambda)t_{a_i}(u_{N_3})|,\\&|\lambda n_{N_2}(a_i)+(1-\lambda)i_{a_i}(n_{N_2})-\lambda n_{N_3}(a_i)-(1-\lambda)i_{a_i}(n_{N_3})|\\\leq&|\lambda n_{N_1}(a_i)+(1-\lambda)i_{a_i}(n_{N_1})-\lambda n_{N_3}(a_i)-(1-\lambda)i_{a_i}(n_{N_3})|,\\&|\lambda v_{N_2}(a_i)+(1-\lambda)f_{a_i}(v_{N_2})-\lambda v_{N_3}(a_i)+(1-\lambda)f_{a_i}(v_{N_3})|\\\leq&|\lambda v_{N_1}(a_i)+(1-\lambda)f_{a_i}(v_{N_1})-\lambda v_{N_3}(a_i)+(1-\lambda)f_{a_i}(v_{N_3})|\end{aligned}$$

for $\sum_{i=1}^{m}\omega_i=1$. From Equation (3.2), $T^W(\tilde{N}_1,\tilde{N}_3)\le T^W(\tilde{N}_2,\tilde{N}_3)$ is also getted. □

§4. A new entropy measure for T2SVNSs

Definition 4.1. *The entropy function of a T2SVNS*

$$\tilde{N}=\langle u_{N_j}(a_i),t_{a_i}(u_{N_j}),n_{N_j}(a_i),i_{a_i}(n_{N_j}),v_{N_j}(a_i),f_{a_i}(v_{N_j})\rangle$$

is defined as follows:

$$E_j(N)=1-\frac{1}{n}\sum_{i=1}^{n}[\frac{u_{N_j}(a_i)+t_{a_i}(u_{N_j})}{2}+\frac{v_{N_j}(a_i)+f_{a_i}(v_{N_j})}{2}]\cdot[1-(n_{N_j}(a_i)+i_{a_i}(n_{N_j}))]^2 \quad (4.1)$$

$$\omega_j=\frac{1-E_j(N)}{n-\sum_{j=1}^{n}E_j(N)},(j=1,2,\ldots,n) \quad (4.2)$$

Here, $\sum_{j=1}^{n}\omega_j=1$.

Theorem 4.1. *The entropy function $E_j(N)$ satisfies the following properties:*

(1) $E_j(N)=0$. if $u_{N_j}(a_i)+t_{a_i}(u_{N_j})=1$, $v_{N_j}(a_i)+f_{a_i}(v_{N_j})=0$,

(2) $E_j(N)=1$. if $N=(0.5,0.5,0.5,0.5,0.5,0.5)$,

(3) $E_j(N)\ge E_j(N^{'})$. if

$$u_{N_j}(a_i)+t_{a_i}(u_{N_j})+v_{N_j}(a_i)+f_{a_i}(v_{N_j})\le u_{N'_j}(a_i)+t_{a_i}(u_{N'_j})+v_{N'_j}(a_i)+f_{a_i}(v_{N'_j}),$$

$$n_{N_j}(a_i)+i_{a_i}(n_{N_j})\ge n_{N'_j}(a_i)+i_{a_i}(n_{N'_j}).$$

(4) $E_j(N)=E_j(N^c)$.

Proof. (1) $u_{N_j}(a_i)+t_{a_i}(u_{N_j})=1,v_{N_j}(a_i)+f_{a_i}(v_{N_j})=0\Rightarrow E_j(N)=1-\frac{1}{n}\sum_{i=1}^{n}[(\frac{1}{2}+\frac{1}{2})]=0$.

(2) $N=(0.5,0.5,0.5,0.5,0.5,0.5)\Rightarrow E_j(N)=1-\frac{1}{n}\sum_{i=1}^{n}[(\frac{1}{2}+\frac{1}{2})\times 0]=1-0=1$.

(3) Lets prove the third question

$$u_{N_j}(a_i)+t_{a_i}(u_{N_j})+v_{N_j}(a_i)+f_{a_i}(v_{N_j})\le u_{N'_j}(a_i)+t_{a_i}(u_{N'_j})+v_{N'_j}(a_i)+f_{a_i}(v_{N'_j}),$$

$$n_{N_j}(a_i)+i_{a_i}(n_{N_j})\ge n_{N'_j}(a_i)+i_{a_i}(n_{N'_j})$$

$$\Rightarrow\sum_{i=1}^{n}[\frac{u_{N_j}(a_i)+t_{a_i}(u_{N_j})}{2}+\frac{v_{N_j}(a_i)+f_{a_i}(v_{N_j})}{2}]\cdot[1-(n_{N_j}(a_i)+i_{a_i}(n_{N_j}))]^2$$

$$\le\sum_{i=1}^{n}[\frac{u_{N'_j}(a_i)+t_{a_i}(u_{N'_j})}{2}+\frac{v_{N'_j}(a_i)+f_{a_i}(v_{N'_j})}{2}]\cdot[1-(n_{N'_j}(a_i)+i_{a_i}(n_{N'_j}))]^2$$

$$\Rightarrow 1-\frac{1}{n}\sum_{i=1}^{n}[\frac{u_{N_j}(a_i)+t_{a_i}(u_{N_j})}{2}+\frac{v_{N_j}(a_i)+f_{a_i}(v_{N_j})}{2}]\cdot[1-(n_{N_j}(a_i)+i_{a_i}(n_{N_j}))]^2$$

$$\ge 1-\frac{1}{n}\sum_{i=1}^{n}[\frac{u_{N'_j}(a_i)+t_{a_i}(u_{N'_j})}{2}+\frac{v_{N'_j}(a_i)+f_{a_i}(v_{N'_j})}{2}]\cdot[1-(n_{N'_j}(a_i)+i_{a_i}(n_{N'_j}))]^2$$

$$\Rightarrow E_j(N)\ge E_j(N^{'})$$

(4) Since $\langle u_N,t_a(u_N),n_N,i_a(n_N),v_N,f_a(v_N)\rangle^c=\langle v_N,f_a(v_N),1-n_N,1-i_a(n_N),u_N,t_a(u_N)\rangle$, we have $E_j(N)=E_j(N^c)$. □

§5. MCGDM method based on the entropy and tangent similarity measures of T2SVNSs

In this section, a MCGDM approach is presented by tangent similarity measures for T2SVNSs. Assume that $P=\{p_1,p_2,\ldots,p_d\}$ be a committee of decision makers, $A=\{A_1,A_2,\ldots,A_k\}$ be the alternatives, $C=\{C_1,C_2,\ldots,C_s\}$ be the attributes of each alternative. Then, the following steps are described for finding the best alternative(s).

Step 1: Determination of the T2SVN decision matrix of the decision makers (DMs).

When an expert evaluate the given alternatives A_i under different attributes C_j made by decision makers $P_m(m=1,2,\ldots,d)$ and represent their values in terms of T2SVNNs p_{ij}^m. Hence, decision matrix $P_m=(p_{ij}^m)_{k\times s}$ can be written as follows:

$$P_m=(p_{ij}^m)_{k\times s}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_k\end{array}\begin{array}{c}\begin{array}{cccc}C_1 & C_2 & \cdots & C_s\end{array}\\ \begin{pmatrix} p_{11}^m & p_{12}^m & \cdots & p_{1s}^m\\ p_{21}^m & p_{22}^m & \cdots & p_{2s}^m\\ \vdots & \vdots & \cdots & \vdots\\ p_{k1}^m & p_{k2}^m & \cdots & p_{ks}^m\end{pmatrix}\end{array} \tag{5.1}$$

where $p_{ij}^m=\langle u_{ij}^m,t_{A_i}^m(u_{ij}^m),n_{ij}^m,i_{A_i}^m(n_{ij}^m),v_{ij}^m,f_{A_i}^m(v_{ij}^m)\rangle$.

Step 2: Determination of the aggregating decision matrix.

The aggregating matrix $B=(b_{ij})_{k\times s}$ is expressed as follows:

$$B=(b_{ij})_{k\times s}=\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1s}\\ b_{21} & b_{22} & \cdots & b_{2s}\\ \vdots & \vdots & \cdots & \vdots\\ b_{k1} & b_{k2} & \cdots & b_{ks}\end{pmatrix} \tag{5.2}$$

where $b_{ij}=\bigoplus_{m=1}^{d}(\delta_m d_{ij}^m)=(1-\prod_{m=1}^{d}(1-u_{ij}^m)^{\delta_m},1-\prod_{m=1}^{d}(1-t_{A_i}^m(u_{ij}))^{\delta_m},\prod_{m=1}^{d}(n_{ij}^m)^{\delta_m},\prod_{m=1}^{d}(i_{A_i}^m(n_N))^{\delta_m},$ $\prod_{m=1}^{d}(v_{ij}^m)^{\delta_m},\prod_{m=1}^{d}(f_{A_i}^m(v_{ij}))^{\delta_m})$, $\delta_m(m=1,2,\cdots,d)$ is the weight of p_m.

Step 3: Determination of the ideal solution.

T2SVN local positive ideal solution (T2SVNPIS) and the T2SVN negative ideal solution

(T2SVNNIS) are defined as follows:

$$\begin{aligned} b_j^* = \langle & \max_i(1-\prod_{m=1}^{d}(1-u_{ij}^m)^{\delta_m}, \max_i(1-\prod_{m=1}^{d}(1-t_{A_i}^m(u_{ij}))^{\delta_m}), \\ & \min_i(\prod_{m=1}^{d}(n_{ij}^m)^{\delta_m}), \min_i(\prod_{m=1}^{d}(i_{A_i}^m(n_N))^{\delta_m}), \\ & \min_i(\prod_{m=1}^{d}(v_{ij}^m)^{\delta_m}), \min_i(\prod_{m=1}^{d}(f_{A_i}^m(v_{ij}))^{\delta_m})\rangle \end{aligned} \tag{5.3}$$

$$\begin{aligned} b_j^- = \langle & \min_i(1-\prod_{m=1}^{d}(1-u_{ij}^m)^{\delta_m}, \min_i(1-\prod_{m=1}^{d}(1-t_{A_i}^m(u_{ij}))^{\delta_m}), \\ & \max_i(\prod_{m=1}^{d}(n_{ij}^m)^{\delta_m}), \max_i(\prod_{m=1}^{d}(i_{A_i}^m(n_N))^{\delta_m}), \\ & \max_i(\prod_{m=1}^{d}(v_{ij}^m)^{\delta_m}), \max_i(\prod_{m=1}^{d}(f_{A_i}^m(v_{ij}))^{\delta_m})\rangle \end{aligned} \tag{5.4}$$

Step 4: Determination the weights of attribute.

By Equation (4.2), we can calculate the attribute weights.

Step 5: Determination of separation measures from ideal solutions to each alternatives.

Separation measures d_i^* and d_i^- of each alternative from ideal solutions can be found by using weighted similarity distance measure formula given in Section 3. Then,

$$\begin{cases} d_i^* = \sum_{j=1}^{s} \omega_j T(b_{ij}, b_j^*) \\ d_i^- = \sum_{j=1}^{s} \omega_j T(b_{ij}, b_j^-) \end{cases} \quad for \quad i=1,2,\ldots,k. \tag{5.5}$$

Step 6: Calculating the closeness coefficients of alternatives.

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \quad for \quad i=1,2,\ldots,k. \tag{5.6}$$

Step 7: Ranking the alternatives.

The highest value of closeness coefficients CC_i, the best alternative A_i is.

§6. Illustrative of the proposed method and comparative analysis

In this part, we first give a numerical example of the low carbon logistics service provider selection problem provided by Chen et al. [6]. There are three DMs (D_1, D_2, D_3) to evaluate with four alternatives $A_i (i=1,2,3)$ and three attributes: C_1: low-carbon technology, C_2: risk factor, C_3: capacity.

6.1. Illustration of the proposed approach

The complete MCGDM model tangent similarity measure is summarized by the following steps:

Step 1: Evaluation of alternatives for each criteria by the linguistic terms shown in Table 1. Tables 2-4 show their evaluations matrix.

Table 1 Evaluations of the alternatives by the linguistic variables.

Grades	Ptmf	Stmf	Pimf	Simf	Pfmf	Sfmf
Very good (Vg)	1.000	1.000	0.000	0.000	0.000	0.000
Good (G)	0.858	0.858	0.238	0.238	0.132	0.132
Medium good (Mg)	0.762	0.762	0.400	0.400	0.238	0.238
Fairly (F)	0.500	0.500	0.500	0.500	0.500	0.500
Medium poor (Mp)	0.248	0.248	0.600	0.600	0.762	0.762
Poor (P)	0.142	0.142	0.762	0.762	0.868	0.868
Very poor (Vp)	0.000	0.000	1.000	1.000	1.000	1.000

Table 2 Linguistic decision matrix by Decision maker D_1.

Alternatives	C_1	C_2	C_3
A_1	(Vg, g, P, Vp, Vp, P)	(Vg, Mg, G, Mg, Vp, Mp)	(Vg, G, G, Mp, Vp, P)
A_2	(G, Mg, F, Vp, P, Mp)	(Vg, G, Mp, F, Vp, p)	(Mg, F, Mg, G, Mp, F)
A_3	(Mg, Mg, Vp, P, F, F)	(Vg, Mg, Mg, G, Vp, Mp)	(Vg, G, F, F, Vp, P)

Table 3 Linguistic decision matrix by Decision maker D_2.

Alternatives	C_1	C_2	C_3
A_1	(G, Mp, P, Mg, P, Mg)	(G, Vg, Vg, G, P, Vp)	(G, Vg, Vp, Vg, P, Vp)
A_2	(Mg, Mp, Mg, Mp, Mp, Mg)	(G, F, P, Mp, P, F)	(G, F, G, G, P, F)
A_3	(G, Mg, Mg, F, P, Mp)	(G, Mg, Mg, P, P, Vp)	(G, Vg, Mg, Vp, P, Vp)

$$D_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{21} \\ d_{22} \\ d_{23} \\ d_{31} \\ d_{32} \\ d_{33} \end{bmatrix} = \begin{bmatrix} \langle 1.000, 0.858, 0.762, 1.000, 1.000, 0.868 \rangle \\ \langle 1.000, 0.762, 0.238, 0.400, 1.000, 0.762 \rangle \\ \langle 1.000, 0.858, 0.238, 0.600, 1.000, 0.868 \rangle \\ \langle 0.858, 0.762, 0.500, 1.000, 0.868, 0.762 \rangle \\ \langle 1.000, 0.858, 0.600, 0.500, 1.000, 0.868 \rangle \\ \langle 0.762, 0.500, 0.400, 0.238, 0.762, 0.500 \rangle \\ \langle 0.762, 0.762, 1.000, 0.762, 0.500, 0.500 \rangle \\ \langle 0.762, 0.400, 0.238, 1.000, 0.762, 1.000 \rangle \\ \langle 1.000, 0.858, 0.500, 0.500, 1.000, 0.868 \rangle \end{bmatrix}$$

Table 4 Linguistic decision matrix by Decision maker D_3.

Alternatives	C_1	C_2	C_3
A_1	(G, Mg, Mp, Mp, P, Mp)	(G, G, Mg, MP, P, P)	(G, F, Vg, Vg, P, F)
A_2	(Mp, F, F, F, Mg, F)	(Mp, Mg, G, G, Mg, Mp)	(F, Mg, Mg, Mg, F, Mp)
A_3	(P, F, Mp, Mg, G, F)	(Vg, G, Vp, Vp, Vp, P)	(Vg, Mg, G, F, Vp, Mp)

$$D_2=\begin{bmatrix}d_{11}\\d_{12}\\d_{13}\\d_{21}\\d_{22}\\d_{23}\\d_{31}\\d_{32}\\d_{33}\end{bmatrix}=\begin{bmatrix}\langle 0.858,0.248,0.762,0.400,0.868,0.238\rangle\\\langle 0.858,1.000,0.000,0.238,0.868,1.000\rangle\\\langle 0.858,1.000,1.000,0.000,0.868,1.000\rangle\\\langle 0.762,0.248,0.400,0.600,0.762,0.238\rangle\\\langle 0.858,0.500,0.762,0.600,0.868,0.500\rangle\\\langle 0.858,0.500,0.238,0.238,0.868,0.500\rangle\\\langle 0.858,0.762,0.400,0.500,0.868,0.762\rangle\\\langle 0.858,0.762,0.400,0.762,0.868,1.000\rangle\\\langle 0.858,1.000,0.400,1.000,0.868,1.000\rangle\end{bmatrix}$$

$$D_3=\begin{bmatrix}d_{11}\\d_{12}\\d_{13}\\d_{21}\\d_{22}\\d_{23}\\d_{31}\\d_{32}\\d_{33}\end{bmatrix}=\begin{bmatrix}\langle 0.858,0.762,0.600,0.600,0.868,0.762\rangle\\\langle 0.858,0.858,0.400,0.600,0.868,0.868\rangle\\\langle 0.858,0.500,0.000,0.000,0.868,0.500\rangle\\\langle 0.248,0.500,0.500,0.500,0.238,0.500\rangle\\\langle 0.248,0.762,0.238,0.238,0.238,0.762\rangle\\\langle 0.500,0.762,0.400,0.400,0.500,0.762\rangle\\\langle 0.142,0.500,0.600,0.400,0.132,0.500\rangle\\\langle 1.000,0.858,1.000,1.000,1.000,0.868\rangle\\\langle 1.000,0.762,0.238,0.500,1.000,0.762\rangle\end{bmatrix}$$

Step 2: Determination of the aggregating decision matrix $B=(b_{ij})_{k\times s}$.

Assume that the weights of the experts are $\delta_1=0.36,\delta_2=0.29$ and $\delta_3=0.35$, respectively. B matrix can be constructed. For example

$$\begin{aligned}b_{11}=&\langle 1-(1-1.000)^{0.36}\cdot(1-0.858)^{0.29}\cdot(1-0.858)^{0.35},\\&1-(1-0.858)^{0.36}\cdot(1-0.248)^{0.29}\cdot(1-0.762)^{0.35},\\&0.762^{0.36}\cdot 0.762^{0.29}\cdot 0.600^{0.35},\\&1.000^{0.36}\cdot 0.400^{0.29}\cdot 0.600^{0.35},\\&1.000^{0.36}\cdot 0.868^{0.29}\cdot 0.868^{0.35},\\&0.868^{0.36}\cdot 0.238^{0.29}\cdot 0.762^{0.35}\rangle\\=&\langle 1.000,0.724,0.701,0.641,0.913,0.570\rangle\end{aligned}$$

Using the same way, we can calculate other values in B matrix as follows:

$$B=\begin{bmatrix}\langle 1.000,0.724,0.701,0.641,0.913,0.570\rangle\\ \langle 1.000,1.000,0.000,0.340,0.913,0.863\rangle\\ \langle 1.000,1.000,0.000,0.000,0.913,0.746\rangle\\ \langle 0.704,0.569,0.469,0.677,0.531,0.469\rangle\\ \langle 1.000,0.755,0.465,0.407,0.581,0.707\rangle\\ \langle 0.734,0.614,0.344,0.285,0.683,0.579\rangle\\ \langle 0.679,0.691,0.641,0.538,0.368,0.564\rangle\\ \langle 1.000,0.723,0.457,0.924,0.870,0.952\rangle\\ \langle 1.000,1.000,0.361,0.611,0.960,0.864\rangle\end{bmatrix}$$

Step 3: Determination of the ideal solution. By using the aggregating matrix, the T2SVN local positive ideal solution (T2SVNPIS) and the T2SVN negative ideal solution (T2SVNNIS) are obtained as follows:

$$B^*=\begin{bmatrix}\langle 1.000,0.724,0.469,0.538,0.368,0.469\rangle\\ \langle 1.000,1.000,0.000,0.340,0.581,0.707\rangle\\ \langle 1.000,1.000,0.000,0.000,0.683,0.579\rangle\end{bmatrix}$$

$$B^-=\begin{bmatrix}\langle 0.679,0.569,0.701,0.677,0.913,0.570\rangle\\ \langle 1.000,0.723,0.465,0.924,0.913,0.952\rangle\\ \langle 0.734,0.614,0.361,0.611,0.960,0.864\rangle\end{bmatrix}.$$

Step 4: Determination of the attribute weights ω by entropy.

By using Equation (4.2), the weights of the attribute can be calculated: $\omega=[\omega_1,\omega_2,\omega_3]=[0.7909,0.1213,0.0878]$

Step 5: Determination of separation measures from ideal solutions to each alternatives and relative closeness coefficient.

By Equation (5.5), the separation measures d_i^* and d_i^- are indicated. Relative closeness coefficient CC_i is calculated by using Equation (5.6). lets say that $\lambda=0.55$. These results are listed in Table 5.

Table 5 Distance measure and relative closeness coefficient of each alternative.

Proposed approach	d_i^*	d_i^-	CC_i
A_1	0.7842	0.8290	0.5138
A_2	0.7168	0.4241	0.3717
A_3	0.7014	0.2763	0.2826

Step 6: Ranking of the three alternatives.

According to Table 5, we can get the final ranking of three alternatives, which is $A_1 \succ A_2 \succ A_3$. Thus, A_1 is the best alternative.

6.2. Comparative analysis and discussion

From Table 5, we know that A_1 is the best alternative for different values of λ. However, the ranking results are different. For confirming the reasonableness and feasibility of the proposed method, we will compare with current methods to solve the same decision-making problem. The ranking results from other methods are shown in Table 6. Ranking results from proposed method with λ=0.10, 0.25, 0.40 are the same as the ranking result of Mondal's method [23]. Ranking results from proposed method with λ=0.55, 0.70, 0.90 are the same as the ranking result of Karaaslan's method [16] and Sahin's method [28], which are able to show that the proposed approach is practical and effective.

Table 6 Comparison of other methods.

Methods	Final Ranking	The Chosen Alternative
Proposed method(λ=0.10, 0.25, 0.40)	$A_2 \prec A_3 \prec A_1$	A_1
Proposed method(λ=0.55, 0.70, 0.90)	$A_3 \prec A_2 \prec A_1$	A_1
Mondal et al. [23]	$A_2 \prec A_3 \prec A_1$	A_1
Karaaslan [16]	$A_3 \prec A_2 \prec A_1$	A_1
Sahin [28]	$A_3 \prec A_2 \prec A_1$	A_1

§7. Conclusion

In this paper, we proposed the concepts of SVNS and T2SVNS. Then, we defined tangent similarity measure, which are also proved in T2SVN environment. We also defined a new entropy function for determining unknown attribute weights. A new approach for solving the (MCGDM) problems under T2SVNSenvironment was developed. Finally, we provided an illustrative example to illustrate the application of the proposed method. The comparative analysis with the current methods were given to confirm the rationality and feasibility of the proposed method. It enriches and develops the theory and method of MCGDM, and provides a new way to solve MCGDM problem. In future research, we will further develop the proposed similarity measures of the T2SVNS and their application.

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