Applied Soft Computing 60 (2017) 737–749

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Entropy-based weights on decision makers in group decision-making setting with hybrid preference representations

Chuan Yue

College of Mathematics and Computer Science, Guangdong Ocean University, Zhanjiang 524088, China

ARTICLE INFO

Article history:
Received 7 January 2017
Received in revised form 25 May 2017
Accepted 16 July 2017
Available online 21 July 2017

Keywords:
Group decision-making
Weights of decision makers
Entropy measure
Normalized projection measure
Interval data
Hybrid information

ABSTRACT

The weights of decision makers play an important role in group decision-making problems. Entropy is a very important measure in information science. This work models an approach to determine the weights of decision makers by using an entropy measure. A new normalized projection as a separation measure, along with TOPSIS (technique for order preference by similarity to ideal solution) technique, is used for current decision model. The attribute values in current model are characterized by exact values and intervals. A comparison and experimental analysis show the applicability, feasibility, effectiveness and advantages of the proposed method.

1. Introduction

Entropy is originally developed by Shannon [26], which is used for measuring the degree of disorder in a system. So it is also called Shannon entropy. This is a concept from thermodynamics. According to this theory, the natural trend, without exertion of work, is toward disorder and increase in entropy. The relationship between increase in disorder and increase in entropy has been noted by specialists in information science and applied in their field [25]. Specially, entropy has become a very important measure which is used for measuring the uncertainty in a system. So it is also called information entropy. Subsequent research on Shannon entropy has contributed to the resolution of a range of problems in decision science [8,27]. For example, the entropy was often used to determine the weight of attribute [15,65]. This paper provides two methodologies to group decision-making (GDM) problems, they are that (1) employ Shannon entropy to determine the weight of decision maker (DM) or expert under a GDM [18,19,21,22,24] environment; and (2) develop a new normalized projection as a separation measure in TOPSIS (technique for order preference by similarity to ideal solution) technique. The work (1) is motivated by the following example.

Example 1. Suppose that

$$X_1 = \begin{matrix} & u_1 & u_2 \\ A_1 & 36 & 58 \\ A_2 & 78 & 59 \\ A_3 & 66 & 98 \end{matrix}, \quad X_2 = \begin{matrix} & u_1 & u_2 \\ A_1 & 80 & 80 \\ A_2 & 80 & 80 \\ A_3 & 80 & 80 \end{matrix}, \quad X_3 = \begin{matrix} & u_1 & u_2 \\ A_1 & 63 & 88 \\ A_2 & 82 & 69 \\ A_3 & 96 & 58 \end{matrix}$$

are the individual decisions of DMs d_1, d_2 and d_3 respectively, where the A_1, A_2 and A_3 are alternatives, the u_1 and u_2 are benefit attributes (the larger the better), which are characterized by the hundred-mark system. It is obvious that the decision matrix X_2, provided by DM d_2, is insignificant because it is only to add a constant 80 to this decision system over A_1, A_2 and A_3. Therefore it does not affect the order of A_1, A_2 and A_3. The decision matrix X_2 can be observed from

E-mail address: yuechuan-1988@163.com

http://dx.doi.org/10.1016/j.asoc.2017.07.033
1568-4946/

information entropy. Entropy is a measure of disorder. Usually the greater the entropy is, the lower the weight is [15]. X_2 is a decision matrix (here a small decision system) with high entropy (see the computational result in Example 7 below), which implies that the weight of DM d_2 is very low according to his/her decision X_2.

All systems are subject to the laws of thermodynamics [25]. The weights of DMs in a decision system should be subject to this law. As aforementioned, the interest of this paper is to use the information entropy to determine the importance of DM based on his/her own decision.

Another interest is the separation measures in TOPSIS technique. In general, the Euclidean distance and the Hamming distance are the best known and most widely used separation measures. However, both of them consider only the distance between two evaluation objectives, the angle between them are ignored. Consider that the projections [28,48,56,59,58,53] are the separation measures with the angle between two decision objectives, which are more comprehensive considerations for measuring some separations. This review finds that the current projection measure is not always reasonable in real and interval vector settings (see Examples 8–10 below). To solve these problems, this paper interests in consideration that the separation is replaced by a new normalized projection measure in a GDM environment.

In addition, considering the increasing complexity in many real decision situations, only one of information representations is sometimes difficult to characterize all attributes. Many approaches have been proposed to solve GDM problems with different preference representations. Chen et al. [6] provided a constructive survey for GDM, in which three types of fusion approaches are reviewed. Moreover, with respect to insights gained from prior researches, several open problems are proposed for the future research. Dong et al. [10] developed a consensus reaching model to solve GDM problems, in which the attribute values, provided by DM, are hybrid, and the individual sets of alternatives, provided by DM, can change dynamically in the decision process. Chiclana et al. [7] presented a GDM method with hybrid preference relations, where the self-confidence levels of DMs are considered in decision process. Dong and Zhang [9] proposed a direct consensus framework for GDM with different preference representations (preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations). Using the transformation functions among different preference representation structures, Dong and Zhang's method avoids internal inconsistency issue.

These works have made great contributions to GDM with different preference representations. However, as far as we know, there is a gap in GDM with hybrid preferences. This gap is that the weight of DM is based on the entropy of his/her own decision. To fill it, this work considers that: in some cases, the crisp values may be suitable for characterizing some attributes; in others, the interval data may be more suitable for characterizing other attributes [59]. Because of this kind of background, this paper intends to establish a complex GDM model with crisp values and interval data, in which the weight of DM is based on the entropy.

In a word, the motivations in this work are intended to achieve a threefold contribution to existing literature, which is listed as follows:

(1) This work intends to develop an entropy-based approach to determine the weight of DM in a GDM setting.
(2) This article intends to explore a new normalized projection to measure the separation between individual decision and ideal decision(s).
(3) This paper intends to propose a new GDM method with hybrid decision information.

The rest of the paper is structured as follows. Section 2 introduces the related work. Section 3 briefly reviews some relevant concepts and problems, and presents an approach to determine the weights of DMs for GDM [37,38] with hybrid information. Section 4 introduces an approach to GDM based on entropy-weight on DMs, a new projection measurement and TOPSIS technique for selecting alternative. Section 5 establishes a GDM algorithm introduced model in this paper. Section 6 introduces a comparison of weight methods of DM. To illustrate the practicability, feasibility, effectiveness and advantages of introduced method, Section 7 gives an illustrative example and Section 8 gives an experimental analysis. And Section 9 draws our conclusions and future research.

2. Related work

This section introduces the related work, which includes the approaches to determine the weights of DMs and projection measures.

2.1. Approaches to determine the weights of decision makers

The weights of DMs play a crucial role in the GDM process. How to determine the weight of DM is one of key techniques. There are many methods to obtain the weights of DMs. For example, Dong et al. [11] proposed a dynamic approach to generate the DMs' weights based on multi-attribute mutual evaluation matrices. This consensus framework can effectively manage the non-cooperative behaviors of some DMs. A good idea for allocating the weight of DM can been found in [10], where DMs have different individual attributes and different individual alternatives. This work has solved a real-world GDM problem. Herrera-Viedma et al. [13] suggested a method to allocate the weights of DMs with incomplete fuzzy preference relations based on additive consistency. Urena et al. [29] described a method to model the weight of DM with incomplete fuzzy preference relations based on DM's confidence from which a GDM procedure. Meng et al. [17] utilized a distance measure to derive the weights of DMs. Yue [50,51,57] proposed a series of methods to derive the DMs' weights for the GDM problems under real number environments [50,53], interval fuzzy environments [51,55,48,54] and intuitionistic fuzzy environments [52]. Wan and Dong [32] and Wan et al. [33] introduced two methods for determining the DMs' weights based on similarity degree. Zhang and Xu [63] constructed a goal-programming model to derive the weights of DMs. Pérez et al. [23] proposed a consensus model to determine the DMs' weights. How to assign objectively the DMs' weights becomes a critical step in the GDM process [63]. Bodily [1] derived the DMs' weights as a result of designation of voting weights from an expert to a delegation subcommittee made up of other DMs of the group. Brock [2] used a Nash bargaining-based method to estimate the DMs' weights intrinsically. Utilized some deviation measures, Xu [41] gave some straightforward formulas for determining the DMs' weights. Xu and Cai [42] presented a novel method based on minimizing group discordance to determine the DMs' weights.

The aforementioned methods have made great contributions to measure the weights of DMs. However, the information in a decision system may be very complex. The importance of a DM in group of experts is associated with various factors, in which the most taboo thing is that the decision of a DM has the same information, as shown the decision X_2 in Example 1, provided by 2th DM. If so, the amount of information from 2th DM is very small. In face, the weights of DMs can be determined by information entropy, this is a knowledge gap. To fill it, this paper intends to determine the DMs' weights based on an entropy measure.

2.2. Approaches to projection measures

Projection can consider not only the distance but also the angle between two decision objectives [45]. Projection methods have attracted great attention from researchers [60,62] and successfully applied to many multi-attribute decision-making (MADM) [4] and GDM problems [39,64,16]. For example, Xu and Da [44] and Xu [40] established two projection models in uncertain MADM context. Xu and Hu [45], Wang et al. [34] and Xu and Cai [43] explored projection model-based approaches to MADM with intuitionistic fuzzy information. Zheng et al. [64] and Fu et al. [12] proposed grey relational projection methods and applied them to MADM. Wei [35] proposed a projection method in intuitionistic fuzzy setting. Tsao and Chen [28] proposed a projection-based compromising method for MADM with interval-valued intuitionistic fuzzy information. Ji et al. [14] introduced a projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection.

Ju and Wang [16] proposed a projection method for GDM with incomplete weight information in linguistic setting. Wei [36] proposed a projection method for GDM in two-tuple linguistic setting. Zeng et al. [60], Yue and Jia [58] and Yue [56,53] developed GDM methods based on projection measurement with intuitionistic fuzzy information. Xu and Liu [39] and Yue [48] established projection methods in uncertain GDM context.

These methods are useful to deal with decision problems. However, this research finds that the existing projection formulae are unreasonable in real and interval vector settings. Some problems of existing projection formulae are shown in Examples 8–10 in Section 4 below. To solve these problems, this paper intends to propose new projection formulae. It is expected that (1) new projection formulae can resolve the existing projection questions eluded researchers; (2) new projection formulae satisfy normalization conditions; and (3) this work shows a new projection-based GDM methodology with crisp values and interval data.

3. Determination of the weights of decision makers

The following sets are used for GDM problem in this paper:

- Alternative, or evaluation objective: A set of m feasible alternatives is written as $A=\{A_i|i\in M\}$, where the $M=\{1,2,\ldots,m\}$;
- Attribute, or criterion: A set of attributes is written as $U=\{u_j|j\in N\}$, where the $N=\{1,2,\ldots,n\}$;
- Weight of attribute, or importance of attribute: A weight vector of attributes is written as $w=(w_1,\ldots,w_j,\ldots,w_n)$, with $0\le w_j\le 1$ and $\sum_{j=1}^{n} w_j=1$;
- Decision maker (DM), or expert: A set of DMs is written as $D=\{d_k|k\in T\}$, where the $T=\{1,2,\ldots,t\}$;
- Weight of DM, or importance of DM: A weight vector over DMs is written as $\lambda=(\lambda_1,\ldots,\lambda_k,\ldots,\lambda_t)$, with $0\le\lambda_k\le 1$ and $\sum_{k=1}^{t}\lambda_k=1$.

Suppose that kth ($k\in T$) DM provides his/her decision information, which includes the following individual decision matrix

$$X_k=(x_{ij}^k)_{m\times n}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}u_1 & u_2 & \ldots & u_n\end{array}\\ \begin{pmatrix} x_{11}^k & x_{12}^k & \ldots & x_{1n}^k\\ x_{21}^k & x_{22}^k & \ldots & x_{2n}^k\\ \vdots & \vdots & \vdots & \vdots\\ x_{m1}^k & x_{m2}^k & \ldots & x_{mn}^k\end{pmatrix}\end{array}, k\in T, \quad (1)$$

and the weight vector of attributes

$$w_k=(w_1^k,w_2^k,\ldots,w_n^k),\ k\in T, \quad (2)$$

where the $x_{ij}^k\ge 0$, $0\le w_j^k\le 1$ and $\sum_{j=1}^{n} w_j^k=1$.

For convenience, the attribute values x_{ij}^k in this model are allowed to be characterized by crisp values and/or intervals. A decision matrix with hybrid information is given in the following definition.

Definition 1 *([59]).* Let $X=(x_{kj})_{t\times n}$ be a decision matrix and let $U=\{u_1,u_2,\ldots,u_n\}$ be the set of attributes. If C and I are two subsets of U such that $C\cup I=U$ and $C\cap I=\emptyset$, then X is called a hybrid decision, where if $u_j\in C$, then the attribute values x_{kj} are characterized by crisp values; if $u_j\in I$, then the attribute values x_{kj} are characterized by interval data.

The interval mentioned here is in ordinary meaning, as can be stated in the following definition.

Definition 2 *([61]).* Let $a=[a^l,a^u]=\{x|a^l\le x\le a^u\}$, then a is called an interval. Especially, if $a^l=a^u$, then a is reduced to a real number.

In general, the normalization of attribute values in Eq. (1) is necessary. Without losing generality, suppose that there are only benefit attributes (the larger the better) and/or cost attributes (the smaller the better) in our model. The normalization will be conducted according to the following formulas.

If the x_{kj}^i in Eq. (1) is a real number, and u_j is a benefit attribute, then the x_{ij}^k is normalized by:

$$y_{ij}^k=\frac{x_{ij}^k-\min_{S_j}}{\max_{S_j}-\min_{S_j}},\ i\in M,\quad j\in N,\quad k\in T, \quad (3)$$

where $\max_{S_j}$ and $\min_{S_j}$, respectively, are the maximum grade and minimum grade of the attribute(s) u_j, in which the attribute values are characterized by the same measure system.

If the x_{ij}^k in Eq. (1) is a real number, and u_j is a cost attribute, then the x_{ij}^k is normalized by:

$$y_{ij}^k=\frac{\max_{S_j}-x_{ij}^k}{\max_{S_j}-\min_{S_j}},\ i\in M,\quad j\in N,\quad k\in T, \quad (4)$$

where $\max_{S_j}$ and $\min_{S_j}$ are the same as in Eq. (3).

If the x_{ij}^k in Eq. (1) is an interval, and u_j is a benefit attribute, then the x_{ij}^k is normalized by:

$$\begin{cases} y_{ij}^{kl}=\dfrac{x_{ij}^{kl}}{\max_{S_j}}\\ y_{ij}^{ku}=\dfrac{x_{ij}^{ku}}{\max_{S_j}},\ i\in M,\quad j\in N,\quad k\in T,\end{cases} \quad (5)$$

where $\max_{S_j}$ is the same as in Eq. (3).

If the x_{ij}^k in Eq. (1) is an interval, and u_j is a cost attribute, then the x_{ij}^k is normalized by:

$$\begin{cases} y_{ij}^{kl}=\dfrac{\min_{S_j}}{x_{ij}^{kl}}\\ y_{ij}^{ku}=\dfrac{\min_{S_j}}{x_{ij}^{ku}},\ i\in M, j\in N, k\in T,\end{cases} \quad (6)$$

where the $\min_{S_j}$ is the same as in Eq. (3).

Example 2. Suppose that

$$X=(x_{ij})_{3\times3}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}u_1 & u_2 & u_3\end{array}\\ \begin{pmatrix}7 & [79,85] & 80\\ 8 & [59,80] & 76\\ 6 & [82,98] & 92\end{pmatrix}\end{array},$$

is an individual decision, where the A_1, A_2 and A_3 are alternatives, and the u_1, u_2 and u_3 are benefit attributes. The evaluation scores in u_1 are characterized by real numbers in the ten-point scale (0–10), the evaluation scores in u_2 are characterized by intervals in the hundred-mark system (0–100) and the evaluation scores in u_3 are characterized by real numbers in the hundred-mark system. The $\max_{S_1}=10$, $\max_{S_2}=\max_{S_3}=100$ and the $\min_{S_1}=\min_{S_2}=\min_{S_3}=0$.

The normalized decision matrices are written as:

$$Y_k=(y_{ij}^k)_{m\times n},\ k\in T. \tag{7}$$

Example 3. For the X given in Example 2, the normalized decision matrix is as follows:

$$Y=(y_{ij})_{3\times3}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}u_1 & u_2 & u_3\end{array}\\ \begin{pmatrix}0.7 & [0.79,0.85] & 0.80\\ 0.8 & [0.59,0.80] & 0.76\\ 0.6 & [0.82,0.98] & 0.92\end{pmatrix}\end{array},$$

For the weight vector of attributes in Eq. (2), the weighted decision matrices are obtained as:

$$R_k=(r_{ij}^k)_{m\times n},\ k\in T, \tag{8}$$

where, if the y_{ij}^k in Eq. (7) is a real number, then $r_{ij}^k=w_j^k y_{ij}^k$; if the y_{ij}^k in Eq. (7) is an interval $\left[y_{ij}^{kl},y_{ij}^{ku}\right]$, then $r_{ij}^k=\left[w_j^k y_{ij}^{kl},w_j^k y_{ij}^{ku}\right]$.

Example 4. For the Y given in Example 3, if the the weight vector of attributes is $w=(0.3,0.4,0.3)$, then the weighted decision matrix is as follows:

$$R=(r_{ij})_{3\times3}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}u_1 & u_2 & u_3\end{array}\\ \begin{pmatrix}0.21 & [0.316,0.340] & 0.240\\ 0.24 & [0.236,0.320] & 0.228\\ 0.18 & [0.328,0.392] & 0.276\end{pmatrix}\end{array}.$$

As aforementioned, the weights of DMs is crucial in the GDM problems. The following research focuses on rating DMs based on Shannon/information entropy.

To establish the entropy of R_k in Eq. (8), we firstly transform the interval $\left[r_{ij}^{kl},r_{ij}^{ku}\right]$ in R_k into a real value u_{ij}^k by using the following formula [31]:

$$u_{ij}^k=\frac{\min\{r_{ij}^{kl},1-r_{ij}^{ku}\}+\min\{1-r_{ij}^{kl},r_{ij}^{ku}\}}{\max\{r_{ij}^{kl},1-r_{ij}^{ku}\}+\max\{1-r_{ij}^{kl},r_{ij}^{ku}\}},\ i\in M,\quad j\in N,\quad k\in T. \tag{9}$$

After this transformation, the hybrid decisions R_k are transformed into the crisp decisions as follows:

$$U_k=(u_{ij}^k)_{m\times n},\ k\in T, \tag{10}$$

where, if the r_{ij}^k in Eq. (8) is a real number, then $u_{ij}^k=r_{ij}^k$.

Example 5. For the R given in Example 4, the entropy matrix is as follows:

$$U=(u_{ij})_{3\times3}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}u_1 & u_2 & u_3\end{array}\\ \begin{pmatrix}0.210 & 0.488 & 0.240\\ 0.240 & 0.385 & 0.228\\ 0.180 & 0.563 & 0.276\end{pmatrix}\end{array},$$

where, for example, $u_{12}=(min(0.316,\ 1-0.340)+min(1-0.316,\ 0.340))/(max(0.316,\ 1-0.340)+max(1-0.316,\ 0.340))=0.488$.

Then the the entropy of U_k is defined as:

$$E_k=-\frac{1}{\ln(mn)}\sum_{i=1}^{m}\sum_{j=1}^{n}p_{ij}^k\ln p_{ij}^k,\ k\in T, \tag{11}$$

where the $p_{ij}^k=u_{ij}^k/\sum_{i=1}^m\sum_{j=1}^n u_{ij}^k$ and $0\le E_k\le 1$.

Example 6. For the U given in Example 5, the $\sum_{i=1}^3\sum_{j=1}^3 u_{ij}=2.810$ and $p_{ij}=u_{ij}/2.180$ for all i, $j=1$, 2, 3. For example, $p_{11}=0.21/2.81=0.0747$. If all p_{ij} are written as the elements of P as follows:

$$P=(p_{ij})_{3\times3}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}u_1 & u_2 & u_3\end{array}\\ \begin{pmatrix}0.0747 & 0.1737 & 0.0854\\ 0.0854 & 0.1370 & 0.0811\\ 0.0641 & 0.2004 & 0.0982\end{pmatrix}\end{array},$$

then the entropy of U in Example 5 can be calculated as $E=-(\sum_{i=1}^m\sum_{j=1}^n p_{ij}\ln p_{ij})/\ln(mn)=-(0.0747\times\ln 0.0747+0.1737\times\ln 0.1737+0.0854\times\ln 0.0854+0.0854\times\ln 0.0854+0.1370\times\ln 0.1370+0.0811\times\ln 0.0811+0.0641\times\ln 0.0641+0.2004\times\ln 0.2004+0.0982\times\ln 0.0982)/\ln(3\times3)=2.1203/2.1972=0.9650$.

Remark 1. If all attribute values r_{ij}^k in Eq. (8) are characterized by real numbers, then the entropy of R_k is calculated by Eq. (11), where the $p_{ij}^k=r_{ij}^k/\sum_{i=1}^m\sum_{j=1}^n r_{ij}^k$.

The entropy-based weights should be subject to the law that the lower the information entropy E_k, the higher the weight λ_k is [15]. That is, the larger the value $1-E_k$, the higher the weight λ_k is. Therefore the entropy-based weights of DMs can be defined as $\lambda_k=(1-E_k)/\sum_{k=1}^t(1-E_k)$. So the weights of DMs are expressed by the following equation:

$$\lambda_k=\frac{1-E_k}{t-\sum_{k=1}^t E_k}, \tag{12}$$

where it is obvious that the $0\le\lambda_k\le1$ and $\sum_{k=1}^t\lambda_k=1$.

Example 7. For the individual decisions X_1, X_2, X_3 in Example 1, the attribute values are normalized by Eq. (3), and the normalized decisions Y_1, Y_2, Y_3 are shown as:

$$Y_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.36 & 0.58 \\ 0.78 & 0.59 \\ 0.66 & 0.98 \end{pmatrix}\end{array}, Y_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.80 & 0.80 \\ 0.80 & 0.80 \\ 0.80 & 0.80 \end{pmatrix}\end{array}, Y_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.63 & 0.88 \\ 0.82 & 0.69 \\ 0.96 & 0.58 \end{pmatrix}\end{array}.$$

$$R_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.180 & 0.290 \\ 0.390 & 0.295 \\ 0.330 & 0.490 \end{pmatrix}\end{array}, R_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.400 & 0.400 \\ 0.400 & 0.400 \\ 0.400 & 0.400 \end{pmatrix}\end{array}, R_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.315 & 0.440 \\ 0.410 & 0.345 \\ 0.480 & 0.290 \end{pmatrix}\end{array},$$

where $R_k = (r^k_{ij})_{3\times2} (k = 1, 2, 3)$, and $s_1 = \sum_{i=1}^{3}\sum_{j=1}^{2} r^1_{ij} = 1.975$, $s_2 = \sum_{i=1}^{3}\sum_{j=1}^{2} r^2_{ij} = 2.400$, $s_3 = \sum_{i=1}^{3}\sum_{j=1}^{2} r^3_{ij} = 2.280$.

$$P_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.0911 & 0.1468 \\ 0.1975 & 0.1494 \\ 0.1671 & 0.2481 \end{pmatrix}\end{array}, P_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.1667 & 0.1667 \\ 0.1667 & 0.1667 \\ 0.1667 & 0.1667 \end{pmatrix}\end{array}, P_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} u_1 \quad u_2 \\ \begin{pmatrix} 0.1382 & 0.1930 \\ 0.1798 & 0.1513 \\ 0.2105 & 0.1272 \end{pmatrix}\end{array}.$$

4. Normalized projection approach to group decision-making

For the weight λ_k of DMs, we assign the λ_k to R_k in Eq. (8), then the individual weighted decision $H_k (k \in T)$ are calculated by:

$$H_k = (h^k_{ij})_{m\times n}, \ k \in T, \tag{13}$$

where if the r^k_{ij} in Eq. (8) is a real number, then $h^k_{ij} = \lambda_k r^k_{ij}$; if the x^k_{ij} in Eq. (8) is an interval $[r^{kl}_{ij}, x^{ku}_{ij}]$, then $h^k_{ij} = [\lambda_k r^{kl}_{ij}, \lambda_k r^{ku}_{ij}]$.

For convenience, we convert the H_k in Eq. (13) into a group decision as the following expression:

$$H_i = (h^i_{kj})_{t\times n} = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_t \end{array}\begin{array}{c} u_1 \quad u_2 \quad \cdots \quad u_n \\ \begin{pmatrix} h^i_{11} & h^i_{12} & \cdots & h^i_{1n} \\ h^i_{21} & h^i_{22} & \cdots & h^i_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ h^i_{t1} & h^i_{t2} & \cdots & h^i_{tn} \end{pmatrix}\end{array}, i \in M, \tag{14}$$

where the elements h^i_{kj} are the same as h^k_{ij} in H_k of Eq. (13).

According to the idea of the TOPSIS technique [30], the positive and negative ideal decisions for all $H_i (i \in M)$ are obvious, as the following matrices show.

$$H_+ = (h^+_{kj})_{t\times n}, \ H_- = (h^-_{kj})_{t\times n}, \tag{15}$$

where if the h^i_{kj} in Eq. (14) is a real number, then the $h^+_{kj} = \max_{i\in M}\{h^i_{kj}\}$ and $h^-_{kj} = \min_{i\in M}\{h^i_{kj}\}$; if y^i_{kj} in Eq. (14) is an interval $[h^{il}_{kj}, h^{iu}_{kj}]$, then $h^+_{kj} = [h^{+l}_{kj}, h^{+u}_{kj}] = [\max_{i\in M}\{h^{il}_{kj}\}, \max_{i\in M}\{h^{iu}_{kj}\}]$ and $h^-_{kj} = [h^{-l}_{kj}, h^{-u}_{kj}] = [\min_{i\in M}\{h^{il}_{kj}\}, \min_{i\in M}\{h^{iu}_{kj}\}]$.

As mentioned in the Introduction, the separation measures are main tools in TOPSIS technique. To improve and optimize the separation measures in real and interval vector settings, we first review the classical projection between two real vectors.

Definition 3. ([59]) Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_t)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_t)$ be two real vectors, then

$$Proj_\beta(\alpha) = \frac{\alpha\beta}{|\beta|} \tag{16}$$

is called a projection of vector α onto vector β, where the $|\beta|$ is the module of vector β, and the $\alpha\beta$ is the inner product between α and β.

Generally, the larger the value $Proj_\beta(\alpha)$ is, the closer the α is to the β [64,12]. However, this research finds the following counterexample.

Example 8. If we take $\alpha = (2, 0)$ and $\beta = (1, 0)$, then the $\alpha\beta = 2 + 0 = 2$, $|\beta| = 1$ and $\beta\beta = |\beta|^2 = 1$. According to Eq. (16), we have the $Proj_\beta(\alpha) = 2$ and $Proj_\beta(\beta) = 1$. In this case, we have $Proj_\beta(\beta) < Proj_\beta(\alpha)$. However, it is obvious that the β is much closer to itself than to the α since $\alpha \neq \beta$.

To solve this problem, we give the following normalized projection measure.

Definition 4. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_t)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_t)$ be two real vectors, then the normalized projection of vector α onto vector β is defined as:

$$NProj_\beta(\alpha) = \frac{\alpha\beta}{\alpha\beta + |\alpha\beta - |\beta|^2|}. \tag{17}$$

It is obvious that if α and β are non-negative vectors, then the Eq. (17) satisfies $0 \le NProj_\beta(\alpha) \le 1$. The closer the $NProj_\beta(\alpha)$ is to 1, the closer the vector α is to the β.

For the $\alpha = (2, 0)$ and $\beta = (1, 0)$ in Example 3, according to Eq. (17), we have $NProj_\beta(\alpha) = 2/(2 + |2 - 1|) = 2/3$ and $NProj_\beta(\beta) = 1/(1 + |1 - 1|) = 1$. Hereby, we can see that the $NProj_\beta(\beta)$ is closer to (here, equal to) 1 than the $NProj_\beta(\alpha)$. Therefore, the β here is much closer to itself than to the α.

However, we are puzzled by the following example.

Example 9. If we take $\alpha = (0, 1, 2)$ and $\beta = (1, 0, 1)$, then the $\alpha\beta = 2$ and $\beta\beta = |\beta|^2 = 2$. According to Eq. (17), we have the $NProj_\beta(\alpha) = 2/(2 + |2 - 2|) = 1$ and $NProj_\beta(\beta) = 2/(2 + |2 - 2|) = 1$. In this case, we have $NProj_{\beta}(\beta) = NProj_{\beta}(\alpha)$. However, the instinct tells us that the β should be much closer to itself than to the α since $\alpha \neq \beta$.

And for this, we give the following normalized projection.

Definition 5. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_t)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_t)$ be two real vectors, then the normalized projection of vector α onto vector β is defined as:

$$NProj_{\beta}(\alpha) = \frac{\min\{|\alpha\beta|, |\alpha|^2, |\beta|^2\}}{\max\{|\alpha\beta|, |\alpha|^2, |\beta|^2\} + |\alpha\beta - |\beta|^2|}. \quad (18)$$

For the $\alpha = (0, 1, 2)$ and $\beta = (1, 0, 1)$ in Example 4, we have the $\alpha\beta = 2$, $|\alpha|^2 = 1 + 4 = 5$ and $\beta\beta = |\beta|^2 = 2$. It follows that the $\min\{|\alpha\beta|, |\alpha|^2, |\beta|^2\} = \min\{2, 5, 2\} = 2$, $\max\{|\alpha\beta|, |\alpha|^2, |\beta|^2\} = 5$. According to Eq. (18), we have the $Proj_{\beta}(\alpha) = 2/(5+0) = 2/5$, $Proj_{\beta}(\beta) = 2/(2+0) = 1$. This is what we would expect.

Similar to Eq. (16), a previous research [59] introduced the classical projection between two interval vectors.

Definition 6. Let $\tilde{\alpha} = ([\alpha_1^l, \alpha_1^u], [\alpha_2^l, \alpha_2^u], \ldots, [\alpha_t^l, \alpha_t^u])$ and $\tilde{\beta} = ([\beta_1^l, \beta_1^u], [\beta_2^l, \beta_2^u], \ldots, [\beta_t^l, \beta_t^u])$ be two interval vectors, then

$$Proj_{\tilde{\beta}}(\tilde{\alpha}) = \frac{\tilde{\alpha}\tilde{\beta}}{|\tilde{\beta}|} \quad (19)$$

is called a projection of vector $\tilde{\alpha}$ onto vector $\tilde{\beta}$, where $|\tilde{\beta}| = \sqrt{\sum_{k=1}^{t}((\beta_k^l)^2 + (\beta_k^u)^2)}$ and $\tilde{\alpha}\tilde{\beta} = \sum_{k=1}^{t}(\alpha_k^l\beta_k^l + \alpha_k^u\beta_k^u)$.

In general, the larger the value $Proj_{\tilde{\beta}}(\tilde{\alpha})$ is, the closer the $\tilde{\alpha}$ is to the $\tilde{\beta}$. However, the following example shows a counterexample.

Example 10. Suppose that $\tilde{\alpha} = ([0, 1], [0, 2])$ and $\tilde{\beta} = ([0, 1], [0, 1])$. Since the $\tilde{\alpha}\tilde{\beta} = 3$, $\tilde{\beta}\tilde{\beta} = |\tilde{\beta}|^2 = 2$ and $|\tilde{\beta}| = \sqrt{2}$, according to Eq. (19), we have that $Proj_{\tilde{\beta}}(\tilde{\alpha}) = 3/\sqrt{2}$ and $Proj_{\tilde{\beta}}(\tilde{\beta}) = 2/\sqrt{2}$. In this case, we have $Proj_{\tilde{\beta}}(\tilde{\beta}) < Proj_{\tilde{\beta}}(\tilde{\alpha})$. As mentioned in Example 1, since $\tilde{\alpha} \neq \tilde{\beta}$, the $\tilde{\beta}$ should be much closer to itself than to the $\tilde{\alpha}$.

Similar to Eq. (18), we give the following normalized projection measure between two interval vectors.

Definition 7. Let $\tilde{\alpha} = ([\alpha_1^l, \alpha_1^u], [\alpha_2^l, \alpha_2^u], \ldots, [\alpha_t^l, \alpha_t^u])$ and $\tilde{\beta} = ([\beta_1^l, \beta_1^u], [\beta_2^l, \beta_2^u], \ldots, [\beta_t^l, \beta_t^u])$ be two interval vectors, then the normalized projection of vector $\tilde{\alpha}$ onto vector $\tilde{\beta}$ is defined as follows:

$$NProj_{\tilde{\beta}}(\tilde{\alpha}) = \frac{\min\{|\tilde{\alpha}\tilde{\beta}|, |\tilde{\alpha}|^2, |\tilde{\beta}|^2\}}{\max\{|\tilde{\alpha}\tilde{\beta}|, |\tilde{\alpha}|^2, |\tilde{\beta}|^2\} + |\tilde{\alpha}\tilde{\beta} - |\tilde{\beta}|^2|}. \quad (20)$$

It is obvious that if $\tilde{\alpha}$ and $\tilde{\beta}$ are non-negative vectors, then the Eq. (20) satisfies $0 \leq NProj_{\tilde{\beta}}(\tilde{\alpha}) \leq 1$. The closer the $NProj_{\tilde{\beta}}(\tilde{\alpha})$ is to 1, the closer the vector $\tilde{\alpha}$ is to the $\tilde{\beta}$.

For the $\tilde{\alpha} = ([0, 1], [0, 2])$ and $\tilde{\beta} = ([0, 1], [0, 1])$ in Example 5, we next use the Eq. (20) to calculate the normalized projection of $\tilde{\alpha}$ on $\tilde{\beta}$. Since the $\tilde{\alpha}\tilde{\beta} = 3$, $|\tilde{\alpha}|^2 = 5$ and $\tilde{\beta}\tilde{\beta} = |\tilde{\beta}|^2 = 2$, it follows that $\min\{|\tilde{\alpha}\tilde{\beta}|, |\tilde{\alpha}|^2, |\tilde{\beta}|^2\} = \min\{3, 5, 2\} = 2$, $\max\{|\tilde{\alpha}\tilde{\beta}|, |\tilde{\alpha}|^2, |\tilde{\beta}|^2\} = \max\{3, 5, 2\} = 5$. We have the $NProj_{\tilde{\beta}}(\tilde{\alpha}) = 2/(5 + |3 - 2|) = 1/3$, $NProj_{\tilde{\beta}}(\tilde{\beta}) = 2/(2 + |2 - 2|) = 1$. In this case, we have $Proj_{\tilde{\beta}}(\tilde{\alpha}) < Proj_{\tilde{\beta}}(\tilde{\beta})$. This is also what we would expect.

Next we extend the projection of one vector onto another to the projection of one matrix onto another.

Definition 8. Suppose that $X = (x_{kj})_{t\times n}$ and $Y = (y_{kj})_{t\times n}$ be two decision matrices, then

$$NProj_{Y}(X) = \frac{\min\{|XY|, |X|^2, |Y|^2\}}{\min\{|XY|, |X|^2, |Y|^2\} + |XY - |X|^2|} \quad (21)$$

is called the projection of matrix X onto matrix Y, where the $XY = \sum_{k=1}^{t}\sum_{j=1}^{n} x_{kj}y_{kj}$ is the inner product between X and Y, the $|X|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}(x_{kj})^2$ and $|Y|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}(y_{kj})^2$, respectively, are the squared norm of matrix X and Y.

According to Definition 8, the normalized projections of H_i onto H_+ and H_-, respectively, are calculated by:

$$NProj_{H_+}(H_i) = \frac{\min\{|H_iH_+|, |H_i|^2, |H_+|^2\}}{\max\{|H_iH_+|, |H_i|^2, |H_+|^2\} + |H_iH_+ - |H_+|^2|},$$

$$NProj_{H_-}(H_i) = \frac{\min\{|H_iH_-|, |H_i|^2, |H_-|^2\}}{\max\{|H_iH_-|, |H_i|^2, |H_-|^2\} + |H_iH_- - |H_-|^2|}, \quad i \in M, \quad (22)$$

where the $H_iH_+ = \sum_{k=1}^{t}(\sum_{u_j\in C, j=1}^{n} h_{kj}^i h_{kj}^+ + \sum_{u_j\in I, j=1}^{n}(h_{kj}^{il}h_{kj}^{+l} + h_{kj}^{iu}h_{kj}^{+u}))$, $H_iH_- = \sum_{k=1}^{t}(\sum_{u_j\in C, j=1}^{n} h_{kj}^i h_{kj}^- + \sum_{u_j\in I, j=1}^{n}(h_{kj}^{il}h_{kj}^{-l} + h_{kj}^{iu}h_{kj}^{-u}))$, $|H_i|^2 = \sum_{k=1}^{t}(\sum_{u_j\in C, j=1}^{n}(h_{kj}^i)^2 + \sum_{u_j\in I, j=1}^{n}((h_{kj}^{il})^2 + (h_{kj}^{iu})^2))$, $|H_+|^2 = \sum_{k=1}^{t}(\sum_{u_j\in C, j=1}^{n}(h_{kj}^+)^2 + \sum_{u_j\in I, j=1}^{n}((h_{kj}^{+l})^2 + (h_{kj}^{+u})^2))$ and $|H_-|^2 = \sum_{k=1}^{t}(\sum_{u_j\in C, j=1}^{n}(h_{kj}^-)^2 + \sum_{u_j\in I, j=1}^{n}((h_{kj}^{-l})^2 + (h_{kj}^{-u})^2))$. The C and I are the same as in Definition 1.

The relative closeness of each group decision H_i with respect to ideal decisions H_+ and H_- is based upon their normalized projection measurements as follows:

$$NPRC_i = \frac{NProj_{H_+}(H_i)}{NProj_{H_+}(H_i) + NProj_{H_-}(H_i)}, \quad i \in M. \quad (23)$$

All alternatives are ranked in accordance with the order of their relative closeness. The larger $NPRC_i$ means the better alternative A_i.

5. Presented algorithm

This section intends to establish a GDM algorithm with hybrid information representations. The following steps are involved in the decision procedure.

Step 1. Establish the individual decisions and the weights of attributes.

The individual decisions $X_k = (x_{ij}^k)_{t\times n}$ are established by Eq. (1), where the $X_1, X_2, \ldots, X_m$ are provided by DMs, respectively, in which the attribute values are allowed to have different preference representations. Specifically, the attribute values x_{ij}^k in this model are allowed to be characterized by crisp values and/or intervals.

Step 2. Normalize the individual decisions.

The individual decisions $X_k (k \in T)$ are normalized by using Eqs. (3)–(6), and the normalized decisions $Y_k (k \in T)$ are shown in Eq. (7).

Eqs. (3)–(6) provide the normalization methods of attribute values in GDM setting.

Step 3. Construct the weighted decisions.

For the weight vector $w_k = (w_1^k, w_2^k, \ldots, w_n^k)$ of attributes, given by kth DM and shown in Eq. (2), the weighted decisions $R_k (k \in T)$ are constructed by Eq. (8).

There are many methods to determine the weights of attributes. Interested reader can refer to the literature [10].

Step 4. Transform the decisions with hybrid information into the crisp decisions.

For the convenience of calculation of entropy, the intervals in $R_k (k \in T)$ are transformed into real values by Eq. (9). Then the crisp decisions $U_k (k \in T)$ are calculated by Eq. (10).

Step 5. Calculate the entropies of the crisp decisions.

The entropies of crisp decisions $U_k (k \in T)$ are calculated by Eq. (11).

The entropy is a measure in a system. In this context, a individual decision can be thought of as an evaluation system.

Step 6. Determine the weights of DMs.

The weights of DMs are determined by Eq. (12).

The DM's weight λ_k in Eq. (12) is based on the entropy of individual decision $R_k(k \in T)$ in Eq. (8), i.e., the R_k in Eq. (8) is thought of as a quantitative decision system of kth DM.

Step 7. Calculate the individual weighted decisions.

The individual weighted decisions are calculated by Eq. (13). In this way, the individual importance of DM has been considered in decision process.

Step 8. Convert the individual weighted decisions into the group decisions.

The individual weighted decision matrices $H_k(k \in T)$ in Eq. (13) can be converted into the group decision matrices $H_i(i \in M)$ by Eq. (14), in which each alternative A_i is evaluated by a group of DMs. The concerns of individual decisions are the decisions of DMs, while the concerns of group decisions are the alternatives.

Step 9. Determine the ideal decisions.

The ideal decisions H_+ and H_- of the group decisions $H_i(i \in M)$ are determined by Eq. (15).

The ideal decision is a relative concept. There are many ideal decisions in different decision settings. Interested reader can refer to the literature [49].

Step 10. Calculate the separation measures.

Based on the normalized projection measurement, the separations between each group decision H_i and their ideal decisions H_+ and H_-, $NProj_{H_+}(H_i)$ and $NProj_{H_-}(H_i)$, are calculated by Eq. (22).

The separation measures here are more comprehensive considerations. Specifically, both distance and angle factors are considered by a normalized projection measure.

Step 11. Calculate the relative closeness.

For each group decision, the relative closeness is calculated by Eq. (23).

In face, the $NProj_{H_+}(H_i)$ or $NProj_{H_-}(H_i)$ is also able to provide a separation measure. The larger the value $NProj_{H_+}(H_i)$, or smaller the value $NProj_{H_-}(H_i)$ is, the better the alternative A_i is. The TOPSIS technique is a compromise method [3,46]. The compromise between $NProj_{H_+}(H_i)$ and $NProj_{H_-}(H_i)$ is achieved by a relative closeness $NPRC_i$ in Eq. (23).

Step 12. Rank the preference order of alternatives.

The alternatives are ranked in descending order in accordance with the order of their relative closeness.

6. Advantages comparison of weight methods of decision maker

This section makes a comparison of weight methods of DM. We compare the differences and advantages between this proposal and other two models.

Herrera-Viedma et al. [13] introduced an approach that use DMs' consistency to allocate the weights of the DMs. In this model, the weight of the DM is based on the consistency of decision information of DM, and in which the incomplete fuzzy preference relations are provided by DMs. The consistency level of fuzzy preference relations are estimated by the additive consistency and completeness of preference values. The higher the consistency level of preference values provided by a DM is, the larger the weight of DM is.

Urena et al. [29] also introduced an approach to GDM, in which the confidence level associated to a reciprocal intuitionistic fuzzy preference relation is used for different importance degrees to DMs. In Urena's model, the hesitancy degree of the reciprocal intuitionistic fuzzy preference relation is used to determine the DM's confidence from which a GDM procedure. The importance of DM is estimated by the consistency and confidence degrees in decision process. The confidence of DMs can increase the consistency. The higher the consistency and confidence levels is, the larger the weight of DM is. This is a good idea for determining the weight of DM, although it is not obviously focused in the weight of expert.

To illustrate and compare the entropy-based weight presented in this work, firstly, the differences between this proposal and above-mentioned methods [13,29] are reported as follows:

(1) There is no the missing information in the decision matrices of DMs in this work; while there is the missing information in the decision matrices of DMs in above-mentioned methods. The incomplete fuzzy preference relation is beneficial to the human being decisions. However, it affects the calculation of weight of DM.
(2) The weight of DM is based on the disorder of his/her decision matrix in this work. The greater the disorder a decision is, the higher the entropy a decision is, the lower the weight this DM is; while the weight of DM is based on the consistency in decision matrix and the confidence in group of DMs in above-mentioned methods. The higher the consistency and confidence levels is, the larger the weight of DM is.
(3) The weight of DM is allocated by using entropy measure in this work. The higher the entropy a decision is, the lower the weight this DM is; while the weight of DM is allocated by using the additive consistency and completeness of preference values in [13]. And the weight of DM is allocated by using the hesitancy degree [29].
(4) The preference values are provided by DM in real number or interval in this work; while the preference values are provided by DMs in incomplete reciprocal intuitionistic preference relations in [13]. And the preference values are provided by DMs in reciprocal intuitionistic fuzzy preference relation in [29].

Each approach has its advantages and disadvantages. The advantages in this work and above-mentioned methods [13,29] are reported as follows:

(1) In some cases, where the decision information provided by DM is complete, the entropy-based weight in this work can reflect the overall quality of his/her decision matrix. The weight is comprehensive since the decision information is complete. That is one of its main advantages. However, it is difficult that the determination of decision information is characterized by exact values. That is one of its main disadvantages.
(2) In some cases, where the decision information provided by DM is incomplete, the missing information is estimated from the existing information. The incomplete fuzzy preference relation is more suitable for all the human being decisions. That is one of its main advantages. However, it is difficult that the weights of DMs is determined in incomplete decision information setting. That is one of its main disadvantages.

7. Illustrative example

An evaluation of trust between members in a virtual enterprise is given in this section. Virtual enterprises represent a dynamic networking alliance that can react sensitively to changing market opportunities and gather knowledge resources from a wide range of enterprises using Internet technology to develop, design, manufacture, and market goods and services [66]. The success of a virtual enterprise depends on the effective integration and sharing of knowledge and information among alliance enterprises. The partner selection and the related trust issues such as trust evaluation, mutual trust, and trust building have largely been affecting the success of a virtual enterprise [20]. Trust is an important facilitator of knowledge sharing [5]. Now the trust models have focused on qualitative measurement due to its complexity.

The current trusts of four members in a virtual enterprise are evaluated in this example. The four members evaluated here com-

Table 1
Evaluation matrices and weights of attributes.

Matrix	Member	u_1	u_2	u_3	u_4
X_1	A_1	7.0	[79.0, 85.0]	8.0	[72.0, 82.0]
	A_2	8.0	[80.0, 86.0]	6.0	[65.0, 67.0]
	A_3	6.0	[61.0, 67.0]	7.0	[80.0, 83.0]
	A_4	5.0	[82.0, 90.0]	9.0	[84.0, 92.0]
	w_1	0.2	0.3	0.2	0.3
X_2	A_1	7.0	[80.0, 86.0]	9.0	[59.0, 63.0]
	A_2	7.0	[55.0, 62.0]	9.0	[70.0, 98.0]
	A_3	6.0	[68.0, 85.0]	10.0	[82.0, 85.0]
	A_4	8.0	[73.0, 85.0]	5.0	[58.0, 95.0]
	w_2	0.3	0.2	0.3	0.2
X_3	A_1	8.0	[70.0, 72.0]	7.0	[84.0, 89.0]
	A_2	9.0	[80.0, 87.0]	8.0	[61.0, 83.0]
	A_3	7.0	[68.0, 82.0]	9.0	[58.0, 80.0]
	A_4	6.0	[58.0, 61.0]	8.0	[73.0, 82.0]
	w_3	0.2	0.3	0.3	0.2

Table 2
Normalized evaluation matrices of members.

Matrix	Member	u_1	u_2	u_3	u_4
Y_1	A_1	0.70	[0.79, 0.85]	0.80	[0.72, 0.82]
	A_2	0.80	[0.80, 0.86]	0.60	[0.65, 0.67]
	A_3	0.60	[0.61, 0.67]	0.70	[0.80, 0.83]
	A_4	0.50	[0.82, 0.90]	0.90	[0.84, 0.92]
	w_1	0.2	0.3	0.2	0.3
Y_2	A_1	0.70	[0.80, 0.86]	0.90	[0.59, 0.63]
	A_2	0.70	[0.55, 0.62]	0.90	[0.70, 0.98]
	A_3	0.60	[0.68, 0.85]	1.00	[0.82, 0.85]
	A_4	0.80	[0.73, 0.85]	0.50	[0.58, 0.95]
	w_2	0.3	0.2	0.3	0.2
Y_3	A_1	0.80	[0.70, 0.72]	0.70	[0.84, 0.89]
	A_2	0.90	[0.80, 0.87]	0.80	[0.61, 0.83]
	A_3	0.70	[0.68, 0.82]	0.90	[0.58, 0.80]
	A_4	0.60	[0.58, 0.61]	0.80	[0.73, 0.82]
	w_3	0.2	0.3	0.3	0.2

prise a set denoted by $A = \{A_1, A_2, A_3, A_4\}$. The set of DMs is written as $D = \{d_1, d_2, d_3\}$, where the $d_k (k = 1, 2, 3)$ is a group of experts from this alliance. The attributes of evaluation are based on the concerns of DMs, which are $U = \{u_1, u_2, u_3, u_4\}$ = {collaborative relation, activity correlation, collaborative time, past trust value}. The collaborative relation includes outsourcing, collaboration, and corporate relations. The activity correlation is an interactive relationship between members, participating in activities are associated with trust between members. A long period of collaboration can accumulate great trust between members, therefore, collaborative time is an important attribute. Trust between members is affected by previous experiences of collaboration. Therefore, past trust value is adopted to evaluate current trust [5].

The individual decision matrices $X_k = (x_{ij}^k)_{4\times4} (k = 1, 2, 3)$ and the weights of attributes are given by Step 1, in which the x_{ij}^k in u_1 and u_3 are characterized by real numbers in ten-point scale; and the x_{ij}^k in u_2 and u_4 are characterized by interval data $[x_{ij}^{kl}, x_{ij}^{ku}]$ in hundred-mark system. The evaluation matrices and the weights of attributes are shown in Table 1.

According to Step 2, the normalized matrices of trust evaluations are calculated, which are shown in Table 2.

For the attributes' weight vector given by each DM, the weighted (on attributes) evaluation matrices of the members can be obtained by Step 3, which are shown in Table 3.

By Step 4, the intervals in R_k shown in Table 3 can be converted into real values, which are shown in Table 4.

The entropies $E_k (k \in T)$ of the weighted evaluation matrices are calculated by Step 5, which are shown in Table 5. The DMs' weights and their ranking are calculated by Step 6, which are also shown in Table 5.

For the weights of DMs, assigning λ_k to the weighted evaluation matrix R_k, we have individual weighted decisions H_k by Step 7, which are shown in Table 6.

For convenience to rank alternatives, we convert the individual weighted decisions $H_k (k = 1, 2, 3)$ into the group decisions $H_i (i = 1, 2, 3, 4)$ by Step 8, which are shown in Table 7.

The ideal decisions of $H_i (i = 1, 2, 3, 4)$ are determined by Step 9, which are shown in Table 8.

Table 3
Weighted evaluation matrices of members.

Matrix	Member	u_1	u_2	u_3	u_4
R_1	A_1	0.140	[0.237, 0.255]	0.160	[0.216, 0.246]
	A_2	0.160	[0.240, 0.258]	0.120	[0.195, 0.201]
	A_3	0.120	[0.183, 0.201]	0.140	[0.240, 0.249]
	A_4	0.100	[0.246, 0.270]	0.180	[0.252, 0.276]
R_2	A_1	0.210	[0.160, 0.172]	0.270	[0.118, 0.126]
	A_2	0.210	[0.110, 0.124]	0.270	[0.140, 0.196]
	A_3	0.180	[0.136, 0.170]	0.300	[0.164, 0.170]
	A_4	0.240	[0.146, 0.170]	0.150	[0.116, 0.190]
R_3	A_1	0.160	[0.210, 0.216]	0.210	[0.168, 0.178]
	A_2	0.180	[0.240, 0.261]	0.240	[0.122, 0.166]
	A_3	0.140	[0.204, 0.246]	0.270	[0.116, 0.160]
	A_4	0.120	[0.174, 0.183]	0.240	[0.146, 0.164]

Table 4
Crisp evaluation matrices of members.

Matrix	Member	u_1	u_2	u_3	u_4
U_1	A_1	0.1400	0.3263	0.1600	0.3004
	A_2	0.1600	0.3316	0.1200	0.2469
	A_3	0.1200	0.2376	0.1400	0.3236
	A_4	0.1000	0.3477	0.1800	0.3587
U_2	A_1	0.2100	0.1990	0.2700	0.1390
	A_2	0.2100	0.1325	0.2700	0.2019
	A_3	0.1800	0.1806	0.3000	0.2005
	A_4	0.2400	0.1876	0.1500	0.1806
U_3	A_1	0.1600	0.2706	0.2100	0.2092
	A_2	0.1800	0.3342	0.2400	0.1682
	A_3	0.1400	0.2903	0.2700	0.1601
	A_4	0.1200	0.2173	0.2400	0.1834

Table 5
Entropies, DMs' weights and their ranking.

DM	E_k	λ_k	Ranking
d_1	0.9697	0.5795	1
d_2	0.9910	0.1721	3
d_3	0.9870	0.2484	2

Table 6
Individual weighted evaluation matrices $H_k (k = 1, 2, 3)$ of members.

Matrix	Member	u_1	u_2	u_3	u_4
H_1	A_1	0.0811	[0.1373, 0.1478]	0.0927	[0.1252, 0.1426]
	A_2	0.0927	[0.1391, 0.1495]	0.0695	[0.1130, 0.1165]
	A_3	0.0695	[0.1061, 0.1165]	0.0811	[0.1391, 0.1443]
	A_4	0.0580	[0.1426, 0.1565]	0.1043	[0.1460, 0.1599]
H_2	A_1	0.0361	[0.0275, 0.0296]	0.0465	[0.0203, 0.0217]
	A_2	0.0361	[0.0189, 0.0213]	0.0465	[0.0241, 0.0337]
	A_3	0.0310	[0.0234, 0.0293]	0.0516	[0.0282, 0.0293]
	A_4	0.0413	[0.0251, 0.0293]	0.0258	[0.0200, 0.0327]
H_3	A_1	0.0397	[0.0522, 0.0537]	0.0522	[0.0417, 0.0442]
	A_2	0.0447	[0.0596, 0.0648]	0.0596	[0.0303, 0.0412]
	A_3	0.0348	[0.0507, 0.0611]	0.0671	[0.0288, 0.0397]
	A_4	0.0298	[0.0432, 0.0455]	0.0596	[0.0363, 0.0407]

Table 7
Group evaluation matrices $H_i (i = 1, 2, 3, 4)$ of members.

Matrix	DM	u_1	u_2	u_3	u_4
H_1	d_1	0.0811	[0.1373, 0.1478]	0.0927	[0.1252, 0.1426]
	d_2	0.0361	[0.0275, 0.0296]	0.0465	[0.0203, 0.0217]
	d_3	0.0397	[0.0522, 0.0537]	0.0522	[0.0417, 0.0442]
H_2	d_1	0.0927	[0.1391, 0.1495]	0.0695	[0.1130, 0.1165]
	d_2	0.0361	[0.0189, 0.0213]	0.0465	[0.0241, 0.0337]
	d_3	0.0447	[0.0596, 0.0648]	0.0596	[0.0303, 0.0412]
H_3	d_1	0.0695	[0.1061, 0.1165]	0.0811	[0.1391, 0.1443]
	d_2	0.0310	[0.0234, 0.0293]	0.0516	[0.0282, 0.0293]
	d_3	0.0348	[0.0507, 0.0611]	0.0671	[0.0288, 0.0397]
H_4	d_1	0.0580	[0.1426, 0.1565]	0.1043	[0.1460, 0.1599]
	d_2	0.0413	[0.0251, 0.0293]	0.0258	[0.0200, 0.0327]
	d_3	0.0298	[0.0432, 0.0455]	0.0596	[0.0363, 0.0407]

Table 8
Ideal decisions of members.

Decision	DMs	u_1	u_2	u_3	u_4
H_+	d_1	0.0927	[0.1426, 0.1565]	0.1043	[0.1460, 0.1599]
	d_2	0.0413	[0.0275, 0.0296]	0.0516	[0.0282, 0.0337]
	d_3	0.0447	[0.0596, 0.0648]	0.0671	[0.0417, 0.0442]
H_-	d_1	0.0580	[0.1061, 0.1165]	0.0695	[0.1130, 0.1165]
	d_2	0.0310	[0.0189, 0.0213]	0.0258	[0.0200, 0.0217]
	d_3	0.0298	[0.0432, 0.0455]	0.0522	[0.0288, 0.0397]

Table 9
Normalized projections, relative closeness and rankings of members.

Member	$NProj_{H_+}(H_i)$	Ranking	$NProj_{H_-}(H_i)$	Ranking	$NPRC_i$	Ranking
A_1	0.7391	2	0.5654	2	0.5666	2
A_2	0.6617	3	0.6242	3	0.5146	3
A_3	0.6121	4	0.6700	4	0.4774	4
A_4	0.8413	1	0.5047	1	0.6250	1

The normalized projections of group decision matrices $H_i (i = 1, 2, 3, 4)$ on the ideal decisions H_+ and H_- are calculated by Step 10; the relative closeness of each $H_i (i = 1, 2, 3, 4)$ to ideal decisions H_+ and H_- are calculated by Step 11, and the trusts of four members are ranked by Step 12, which are summarized in Table 9.

Table 9 shows that the preference order of trusts of four members as follows:

$A_4 \succ A_1 \succ A_2 \succ A_3,$

that is to say, the best trust is A_4, followed by A_1, A_2 and A_3.

8. Experimental analysis

In this section, we show an experimental comparison with other measures to illustrate the effectiveness and advantages of introduced method in this paper.

8.1. Experimental comparison with other entropy models

In this subsection, we analyze the impacts of different entropy models of U_k in Eq. (10) on the results in illustrative example.

If the entropy value of U_k is based on the attribute (u_j) vector, or column vector, $U_j^k = (u_{1j}^k, u_{2j}^k, \ldots, u_{mj}^k) (j \in N, k \in T)$, then the entropy of U_j^k can be calculated by

$$E_j^k = -\frac{1}{\ln m}\sum_{i=1}^{m} p_{ij}^k \ln p_{ij}^k, \quad k \in T, \quad j \in N, \tag{24}$$

where the $p_{ij}^k = u_{ij}^k / \sum_{i=1}^{m} u_{ij}^k$.

Table 10
Entropies based on attributes.

Entropy	u_1	u_2	u_3	u_4
E_j^1	0.9892	0.9927	0.9919	0.9936
E_j^2	0.9963	0.9920	0.9784	0.9925
E_j^3	0.9919	0.9917	0.9972	0.9962

Table 11
Overall entropy, weights of DMs and their ranking based on attributes.

DM	E_k	λ_k	Ranking
d_1	0.9919	0.3378	2
d_2	0.9898	0.4228	1
d_3	0.9942	0.2394	3

Table 12
Entropies based on categories of attributes.

Attribute	E_j^1	E_j^2	E_j^3
u_1 and u_3	0.9926	0.9895	0.9839
u_2 and u_4	0.9954	0.9948	0.9846

Table 13
Overall entropy, weights of DMs and their ranking based on categories of attributes.

DM	E_k	λ_k	Ranking
d_1	0.9940	0.2025	3
d_2	0.9921	0.2652	2
d_3	0.9842	0.5323	1

The entropies based on attribute $u_j (j = 1, 2, 3, 4)$ are shown in Table 10.

Therefore, the overall entropy of U_k is defined as follows:

$$E_k = \frac{1}{n}\sum_{j=1}^{n} E_j^k, \quad k \in T, \tag{25}$$

where the E_j^k is shown in Eq. (24).

The weights of DMs are also calculated by Eq. (12). The overall entropy E_k of U_k, the weights of DMs and their ranking are shown in Table 11.

Table 11 shows that the order of DMs is different than the order in Table 5.

In our model, the hybrid information in R_k shown in Eq. (8) includes two categories: real numbers and interval data. If the entropy value of U_k is based on different categories, then we investigate the changes of the entropies.

For the category characterized by real numbers in R_k, the entropy of category can be calculated by

$$E_{jC}^k = -\frac{1}{\ln(mn_1)}\sum_{i=1}^{m} p_{ij}^k \ln p_{ij}^k, \quad k \in T, \quad j \in N, \tag{26}$$

where the $p_{ij}^k = r_{ij}^k / \sum_{i=1}^{m}\sum_{u_j \in C, j=1}^{n} r_{ij}^k$, the r_{ij}^k in R_k are real numbers and the $n_1 = |C|$.

For the category characterized by intervals in R_k, the entropy of category can be calculated by

$$E_{jI}^k = -\frac{1}{\ln(mn_2)}\sum_{i=1}^{m} p_{ij}^k \ln p_{ij}^k, \quad k \in T, \quad j \in N, \tag{27}$$

where the $p_{ij}^k = u_{ij}^k / \sum_{i=1}^{m}\sum_{u_j \in I, j=1}^{n} u_{ij}^k$, the u_{ij}^k are shown in U_k, and the $n_2 = |I|$.

The entropies based on categories of attributes are shown in Table 12, where the $n_1 = |C| = |\{u_1, u_3\}| = 2$ and $n_2 = |I| = |\{u_2, u_4\}| = 2$.

Table 14
Normalized projections, relative closeness and rankings of members based on different entropy models.

Member	NP_{A+}	NP_{A-}	$NPRC_A$	Ranking	NP_{C+}	NP_{C-}	$NPRC_C$	Ranking	$NPRC_i$	Ranking
A_1	0.8790	0.8308	0.5141	1	0.8706	0.8620	0.5025	3	0.5170	2
A_2	0.8700	0.8385	0.5092	3	0.9153	0.8334	0.5234	1	0.4975	3
A_3	0.8760	0.8368	0.5115	2	0.8865	0.8511	0.5102	2	0.4864	4
A_4	0.8561	0.8328	0.5069	4	0.8107	0.9024	0.4732	4	0.5346	1

Notes: The NP_{A+} and NP_{A-} indicate $NProj_{H_+}(H_i)$ and $NProj_{H_-}(H_i)$ respectively based on attributes. The $NPRC_A$ indicates the relative closeness based on attributes. The NP_{C+} and NP_{C-} indicate $NProj_{H_+}(H_i)$ and $NProj_{H_-}(H_i)$ respectively based on categories. The $NPRC_C$ indicates the relative closeness based on categories. The $NPRC_i$ is shown in Eq. (23) and the ranking is shown in Table 9.

The overall entropy of U_k is defined as follows:

$$E_k = \frac{1}{2}(E_{jC}^k + E_{jI}^k), \ \ k \in T, \tag{28}$$

where the E_{jC}^k is shown in Eq. (26) and E_{jI}^k is shown in Eq. (27).

The weights of DMs are also calculated by Eq. (12). The overall entropy E_k of U_k, the weights of DMs and their ranking are shown in Table 13.

Table 13 shows that the order of DMs is also different than the order in Table 5. So it must be that the scores of alternatives are different than the scores in Table 9. The normalized projections, relative closeness and their rankings based on attributes, categories and overall entropy in Eq. (11) are summarized in Table 14.

As mentioned in Introduction section, the information entropy is a measure of uncertainty. The Eq. (24) measures the uncertainty of attribute values in an attribute u_j vector, which is partial. The entropy in Eq. (25) is an average of partial entropies, it exists a large deviation with overall entropy in Eq. (11). Eqs. (26) and (27) have partial completeness for determining the entropy. However, they are only an entropy measure in a category. And the Eq. (27) is also an average of entropies of two categories, it is not as accurate as the entropy in Eq. (11). Table 14 shows the difference of rankings based on three entropy measures.

8.2. Experimental comparison with other projection measures

In this subsection, we show the experimental comparisons of normalized projection with other measures: the classical projection measure and a new projection measure [59] proposed by Yue and Jia.

Firstly, we make an experimental comparison with the classical projection measure, and verify the rankings' changes of four members' trusts in the above-mentioned example. In order to do so, we review the classical projection of a decision matrix on another. Let $X = (x_{kj})_{t\times n}$ and $Y = (y_{kj})_{t\times n}$ be two decision matrices, then

$$Proj_Y(X) = \frac{XY}{|Y|}, \tag{29}$$

is called the classical projection [58] of X on Y.

Then we employ the classical projection to replace the Eq. (22), which are shown as follows.

$$Proj_{H_+}(H_i) = \frac{H_iH_+}{|H_+|}, \ \ Proj_{H_-}(H_i) = \frac{H_iH_-}{|H_-|}, \ \ i \in M. \tag{30}$$

The relative closeness in Step 5 is expressed by the following equation:

$$PRC_i = \frac{Proj_{H_+}(H_i)}{Proj_{H_+}(H_i) + Proj_{H_-}(H_i)}, \ \ i \in M. \tag{31}$$

The classical projections, relative closeness and trusts' ranking of four members in the above-mentioned example are shown in Table 15.

Table 15 shows that the trusts' ranking of four members differs from the ranking based on the normalized projection in Table 9.

Table 15
Projections, relative closeness and trusts' ranking of members based on the classical projection.

Member	$Proj_{H_+}(H_i)$	Ranking	$Proj_{H_-}(H_i)$	Ranking	PRC_i	Ranking
A_1	0.3332	2	0.3325	3	0.5006	2
A_2	0.3185	3	0.3174	2	0.5008	1
A_3	0.3103	4	0.3098	1	0.5004	3
A_4	0.3468	1	0.3483	4	0.4990	4

Table 16
Separations, relative closeness and trusts' ranking of members based on the projection proposed by Yue and Jia [59].

Member	$NP_{H_+}(H_i)$	Ranking	$NP_{H_-}(H_i)$	Ranking	NRC_i	Ranking
A_1	0.9000	2	0.8407	2	0.5170	2
A_2	0.8601	3	0.8687	3	0.4975	3
A_3	0.8380	4	0.8848	4	0.4864	4
A_4	0.9367	1	0.8155	1	0.5346	1

We have know from Examples 3 and 5 that the classical projection is unreliable. Our main work is to present an empirical evidence to show that the proposed approach is better than existing approaches. In order to do so, we further compare this work with other measures.

Let X and Y be two matrices, then a new projection measure of vector X onto vector Y is proposed by Yue and Jia [59] as follows:

$$NP_Y(X) = \frac{RP_Y(X)}{RP_Y(X) + |1 - RP_Y(X)|}, \tag{32}$$

where the $RP_Y(X) = XY/|Y|^2$, the XY and $|Y|$ are the same as in Eq. (21). It is similar to the normalized projection proposed in Eq. (21). Next we compare it with the normalized projection proposed in Eq. (21).

If the Eq. (22) is replace by the projection measure in Eq. (32), then the projection of H_i onto H_+ and H_- are shown respectively as follows:

$$NP_{H_+}(H_i) = \frac{RP_{H_+}(H_i)}{RP_{H_+}(H_i) + |1 - RP_{H_+}(H_i)|}, \ \ NP_{H_-}(H_i) = \frac{RP_{H_-}(H_i)}{RP_{H_-}(H_i) + |1 - RP_{H_-}(H_i)|}, \ \ i \in M, \tag{33}$$

where the $RP_{H_+}(H_i) = H_iH_+/|H_+|^2$, $RP_{H_-}(H_i) = H_iH_-/|H_-|^2$; the H_iH_+, $|H_+|^2$ and $|H_-|^2$ are the same as in Eq. (22).

The relative closeness in Eq. (23) is modified accordingly as follows:

$$NRC_i = \frac{NP_{H_+}(H_i)}{NP_{H_+}(H_i) + NP_{H_-}(H_i)}, \ \ i \in M. \tag{34}$$

The separations, relative closeness and trusts' ranking of four members in above-mentioned example, based on the projection measure proposed in [59], are shown in Table 16.

Table 16 shows that the ranking of trusts of four members is consistent with the ranking based on the normalized projection in Table 9.

Three different projection measures lead to two different rankings. Although we never really know the actual ranking of trusts

Table 17
Statistics of rankings based on nine measures.

Measure	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_3 \succ A_2 \succ A_1 \succ A_4$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$NProj_{Y_+}(\cdot)$	√		
$NProj_{Y_-}(\cdot)$	√		
$NPRC_{(\cdot)}$	√		
$Proj_{Y_+}(\cdot)$	√		
$Proj_{Y_-}(\cdot)$		√	
$PRC_{(\cdot)}$			√
$NP_{H_+}(\cdot)$	√		
$NP_{H_-}(\cdot)$	√		
$NRC_{(\cdot)}$	√		

of four members in above-mentioned example, an ideal ranking is based on the following idea. If a ranking is selected by multiply measures, then the most widely preferred choice a ranking is, the highest reliability the ranking is. The most widely preferred choice should be an ideal ranking.

Based on this idea, we further review above-mentioned measures. Firstly, we review the normalized projection measure. It is noted that the $NProj_{H_+}(\cdot)$ in Eq. (22) is also a measure for selecting alternatives. By Eqs. (18) and (20), the larger the value $NProj_{H_+}(H_i)$, the better the alternative A_i is. Similarly, the $NProj_{H_-}(\cdot)$ in Eq. (22) is also a measure to rank the alternatives. The smaller the value $NProj_{H_-}(H_i)$, the better the alternative A_i is. The rankings based on $NProj_{H_+}(\cdot)$ and $NProj_{H_-}(\cdot)$ are also shown in Table 9.

Table 9 shows that three rankings are consistent. So we may say that the normalized projection is a robust measure. Further, it is an ideal measure.

Similar to the normalized projection, we further review the classical projection. It is noted that the $Proj_{H_+}(\cdot)$ in Eq. (30) is also a measure. The larger the value $Proj_{H_+}(H_i)$, the better the alternative A_i is. Also the $Proj_{H_-}(\cdot)$ in Eq. (30) is also a measure. The smaller the value $Proj_{H_-}(H_i)$, the better the alternative A_i is. The trusts' ranking of four members based on the Eq. (30) are also shown in Table 15.

Table 15 shows that three rankings are not consistent. In this sense, we say that the normalized projection in Eq. (22) is superior to the classical projection measure in Eq. (30).

Similar to above discussion, next discuss the new projection measure [59] proposed by Yue and Jia. The $NP_{H_+}(\cdot)$ and $NP_{H_-}(\cdot)$ in Eq. (33) are also measures. Specifically, the larger the value $NP_{H_+}(\cdot)$, the better the alternative A_i is; and the smaller the value $NP_{H_-}(\cdot)$, the better the alternative A_i is. The trusts' rankings of four members in the above-mentioned example, based on $NP_{H_+}(\cdot)$ and $NP_{H_-}(\cdot)$, are also shown in Table 16.

Table 16 shows that the rankings based on $NP_{H_+}(\cdot)$ and $NP_{H_-}(\cdot)$ are also consistent with the ranking based on $NPRC_i$ in Eq. (23).

To find an ideal ranking of alternatives, we list these rankings based on nine measures in Table 17.

Table 17 shows that the ranking $A_4 \succ A_1 \succ A_2 \succ A_3$ is the most widely preferred ranking from nine measures. In this sense, it is an ideal ranking. And it is consistent with the normalized projection measurement shown in Eq. (23). This result confirms the superiority of normalized projection measure provided in this paper. However, we do not known which ranking between the projection in Eq. (21) and the projection in Eq. (32) is the proper one. For this, we further compare them in the following section.

8.3. A dynamic comparison with other projection measures

In this subsection, we show a dynamic comparison with the classical projection measure in Eq. (29) and the new projection measure in Eq. (32), and further validate the results based on normalized projection measure.

We now review the $x_{12}^1 = [x_{12}^{1l}, x_{12}^{1u}] = [79, 85]$ in X_1 in Table 1. To show a dynamic comparison, let $x_{12}^1 = [\alpha, 6+\alpha]$, where the $\alpha \in [0,$ 94] is a parameter. Other values are the same as in Table 1. Then let α increase from 0 to 94, and the normalized projection in Eq. (23) is employed to rank the alternatives A_1, A_2, A_3, A_4, we observe the changes of their rankings. The curves of rankings of A_1, A_2, A_3 and A_4 are shown in Fig. 1.

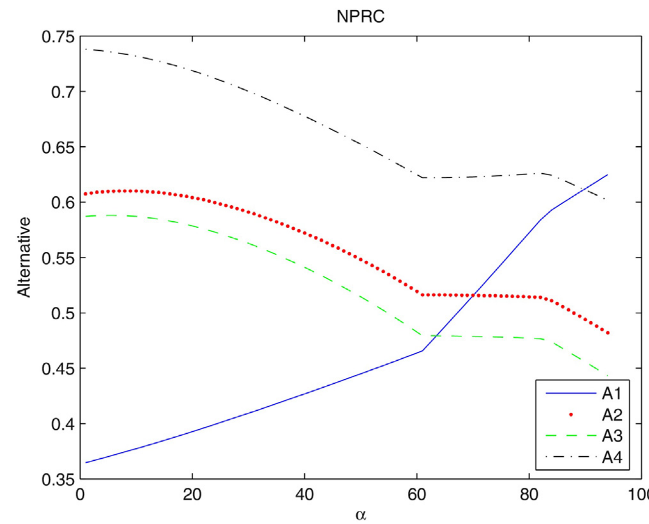

Fig. 1. Rankings of the trusts A_1, A_2, A_3, A_4 based on the normalized projection in Eq. (23).

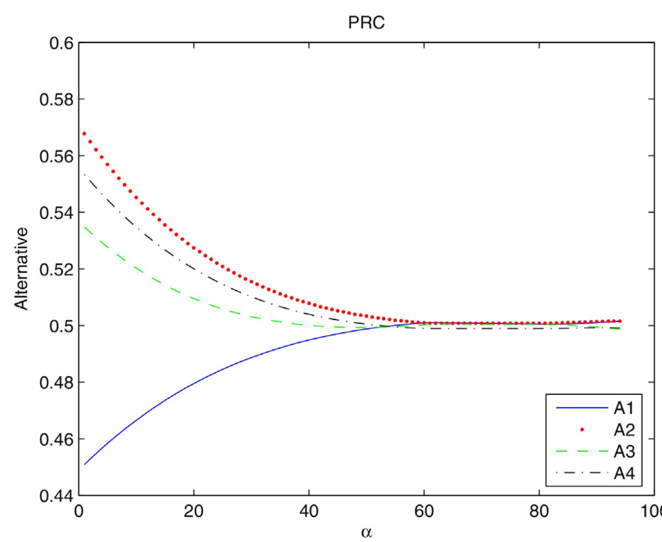

Fig. 2. Rankings of trusts A_1, A_2, A_3, A_4 based on the classical projection in Eq. (31).

We make a dynamic comparison based on this figure with other two projection measures.

Similar to the normalized projection measure, next we employ the classical projection measure to rank the trusts of members A_1, A_2, A_3, A_4, then we observe the changes of their rankings. Their rankings are shown in Fig. 2.

We can see that the rankings based on the classical projection are very different from the changes based on the normalized projection, which further shows the necessity to improve the classical projection.

Next we make a comparison with the projection measure in Eq. (34). If we employ the Eq. (34) to rank the alternatives A_1, A_2, A_3, A_4, we obtain rankings shown in Fig. 3.

Fig. 3 is very similar to the changes based on the normalized projection. That is to say, the normalized projection in Eq. (21) is supported by the projection measure in Eq. (32) in this dynamic

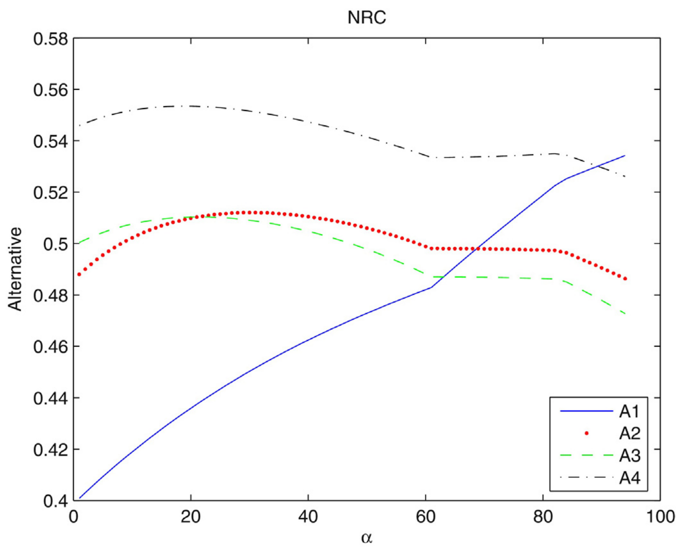

Fig. 3. Rankings of trusts A_1, A_2, A_3, A_4 based on the projection measure in Eq. (34).

comparison. However, if Fig. 1 is compared carefully with Fig. 3, it can be found that (1) the ranking based on normalized projection in Eq. (23) is stable. The ranking $A_4 \succ A_2 \succ A_3$ is stable from beginning to end as α approaches 94 from the left; the ranking based on another projection measure in Eq. (34) is unstable. The order of A_3 is alternated with A_2 in this dynamic process. Therefore, comparing with the projection measure in Eq. (32), the normalized projection in Eq. (21) is a robust measure. And (2) the distinction of rankings of 4 alternatives in Fig. 1 is obvious, its values is from 0.35 to 0.75; the distinction of rankings of 4 alternatives in Fig. 3 is not enough, its values is from 0.4 to 0.56. Based on the above, we can see that the normalized projection measure in Eq. (21) is superior to the projection measure in Eq. (32).

9. Conclusion

This paper has successfully developed an entropy-weight-based approach to determine the weights of DMs. This method has promoted the quantity of information from a single attribute vector to whole decision matrix. The former, focused on an attribute vector, is used for determining the weight of attribute; the latter, focused on a decision matrix, is used for determining the weight of DM. This paper has successfully developed three normalized projections of one real vector on another, one interval vector on another, and one decision matrix on another, respectively, and applied them to GDM with hybrid information. There are four main highlights in our method:

(1) Novelty. The weights of DMs are determined by a novel entropy measure; the separation measures in TOPSIS technique are replaced by a new normalized projection measure. These are innovations.
(2) Comprehensive performance. The performance of our model has dealt with a complex GDM problem with hybrid information. The hybrid information has promoted the wide applications of developed model, although it is more difficult to establish the entropy-based model and normalized projection measures than previous work.
(3) Simplicity. The normalized projection is a simple measure because its values are bound on [0, 1] and it is monotonous.
(4) Comprehensiveness. The model proposed in this paper has summarized the entropy measure to determine the weights of DMs and the normalized projective measure to rank alternatives in a GDM environment with hybrid information.

However, hybrid information deals with real numbers and intervals only. That should be a severe limitation. In the future, we will continue working on other information representations, such as the ordinary fuzzy numbers, the intuitionistic fuzzy numbers and interval-valued intuitionistic fuzzy numbers [47], and so on.

Acknowledgments

The author would like to thank the editors and the anonymous reviewers for their insightful and constructive comments and suggestions that have led to this improved version of the paper. This work was supported partially by the Young Creative Talents Project from Department of Education of Guangdong Province (No. 2016KQNCX064), the Education and Teaching Reform Program of Guangdong Ocean University (No. XJG201644) and the Project of Enhancing School with Innovation of Guangdong Ocean University (No. GDOU2017052802).

References

[1] S.E. Bodily, Note – a delegation process for combining individual utility functions, Manag. Sci. 25 (10) (1979) 1035–1041.
[2] H.W. Brock, The problem of "utility weights" in group preference aggregation, Oper. Res. 28 (1) (1980) 176–187.
[3] J. Chai, J.N.K. Liu, E.W.T. Ngai, Application of decision-making techniques in supplier selection: a systematic review of literature, Expert Syst. Appl. 40 (10) (2013) 3872–3885.
[4] T.-Y. Chen, An interval-valued intuitionistic fuzzy permutation method with likelihood-based preference functions and its application to multiple criteria decision analysis, Appl. Soft Comput. 42 (2016) 390–409.
[5] T.-Y. Chen, Y.-M. Chen, C.-J. Lin, P.-Y. Chen, A fuzzy trust evaluation method for knowledge sharing in virtual enterprises, Comput. Ind. Eng. 59 (4) (2010) 853–864.
[6] X. Chen, H. Zhang, Y. Dong, The fusion process with heterogeneous preference structures in group decision making: a survey, Inf. Fusion 24 (2015) 72–83.
[7] F. Chiclana, E. Herrera-Viedma, F.J. Cabrerizo, Y. Dong, W. Liu, Group decision-making based on heterogeneous preference relations with self-confidence, Fuzzy Optim. Decis. Mak. (2016), http://dx.doi.org/10.1007/s10700-016-9254-8.
[8] A. Delgado, I. Romero, Environmental conflict analysis using an integrated grey clustering and entropy-weight method: a case study of a mining project in Peru, Environ. Model. Softw. 77 (2016) 108–121.
[9] Y. Dong, H. Zhang, Multiperson decision making with different preference representation structures: a direct consensus framework and its properties, Knowl.-Based Syst. 58 (2014) 45–57.
[10] Y. Dong, H. Zhang, E. Herrera-Viedma, Consensus reaching model in the complex and dynamic MAGDM problem, Knowl.-Based Syst. 106 (2016) 206–219.
[11] Y. Dong, H. Zhang, E. Herrera-Viedma, Integrating experts' weights generated dynamically into the consensus reaching process and its applications in managing non-cooperative behaviors, Decis. Support Syst. 84 (2016) 1–15.
[12] C. Fu, X. Gao, M. Liu, X. Liu, L. Han, J. Chen, GRAP: grey risk assessment based on projection in ad hoc networks, J. Parallel Distrib. Comput. 71 (9) (2011) 1249–1260.
[13] E. Herrera-Viedma, F. Chiclana, F. Herrera, S. Alonso, Group decision-making model with incomplete fuzzy preference relations based on additive consistency, IEEE Trans. Syst., Man, Cybern., Part B (Cybern.) 37 (1) (2007) 176–189.
[14] P. Ji, H.Y. Zhang, J.Q. Wang, A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection, Neural Comput. Appl. (2016) 1–14.
[15] Y. Ji, G.H. Huang, W. Sun, Risk assessment of hydropower stations through an integrated fuzzy entropy-weight multiple criteria decision making method: a case study of the Xiangxi River, Expert Syst. Appl. 42 (12) (2015) 5380–5389.
[16] Y. Ju, A. Wang, Projection method for multiple criteria group decision making with incomplete weight information in linguistic setting, Appl. Math. Model. 37 (20) (2013) 9031–9040.
[17] F.Y. Meng, Q. Zhang, H. Cheng, Approaches to multiple-criteria group decision making based on interval-valued intuitionistic fuzzy choquet integral with respect to the generalized λ-shapley index, Knowl.-Based Syst. 37 (2013) 237–249.
[18] S. Meysam Mousavi, S. Ali Torabi, R. Tavakkoli-Moghaddam, A hierarchical group decision-making approach for new product selection in a fuzzy environment, Arab. J. Sci. Eng. 38 (11) (2013) 3233–3248.
[19] S. Meysam Mousavi, B. Vahdani, R. Tavakkoli-Moghaddam, N. Tajik, Soft computing based on a fuzzy grey group compromise solution approach with

an application to the selection problem of material handling equipment, Int. J. Comput. Integr. Manuf. 27 (6) (2014) 547–569.
[20] J. Mun, M. Shin, K. Lee, M. Jung, Manufacturing enterprise collaboration based on a goal-oriented fuzzy trust evaluation model in a virtual enterprise, Comput. Ind. Eng. 56 (3) (2009) 888–901.
[21] I. Palomares, L. Martínez, Low-dimensional visualization of experts' preferences in urgent group decision making under uncertainty, Proc. Comput. Sci. 29 (2014) 2090–2101.
[22] I. Palomares, L. Martínez, F. Herrera, MENTOR: a graphical monitoring tool of preferences evolution in large-scale group decision making, Knowl.-Based Syst. 58 (2014) 66–74.
[23] I. Javier Pérez, F.J. Cabrerizo, S. Alonso, E. Herrera-Viedma, A new consensus model for group decision making problems with non-homogeneous experts, IEEE Trans. Syst., Man, Cybern.: Syst. 44 (4) (2014) 494–498.
[24] F.J. Quesada, I. Palomares, L. Martínez, Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators, Appl. Soft Comput. 35 (2015) 873–887.
[25] S.D. Rasberry, Entropy in quality systems, Accredit. Qual. Assur. 14 (2) (2009) 65–66.
[26] C.E. Shannon, W. Weaver, The Mathematical Theory of Communication, University of Illinois Press, Urbana, 1947.
[27] A. Shemshadi, H. Shirazi, M. Toreihi, M.J. Tarokh, A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting, Expert Syst. Appl. 38 (10) (2011) 12160–12167.
[28] C.-Y. Tsao, T.-Y. Chen, A projection-based compromising method for multiple criteria decision analysis with interval-valued intuitionistic fuzzy information, Appl. Soft Comput. 45 (2016) 207–223.
[29] R. Ureña, F. Chiclana, H. Fujita, E. Herrera-Viedma, Confidence-consistency driven group decision making approach with incomplete reciprocal intuitionistic preference relations, Knowl.-Based Syst. 89 (2015) 86–96.
[30] B. Vahdani, S. Meysam Mousavi, R. Tavakkoli-Moghaddam, Group decision making based on novel fuzzy modified TOPSIS method, Appl. Math. Model. 35 (9) (2011) 4257–4269.
[31] I.K. Vlachos, G.D. Sergiadis, Subsethood, entropy, and cardinality for interval-valued fuzzy sets – an algebraic derivation, Fuzzy Sets Syst. 158 (12) (2007) 1384–1396.
[32] S.P. Wan, J.Y. Dong, A possibility degree method for interval-valued intuitionistic fuzzy multi-attribute group decision making, J. Comput. Syst. Sci. 80 (1) (2014) 237–256.
[33] S.P. Wan, G.L. Xu, F. Wang, J.Y. Dong, A new method for Atanassov's interval-valued intuitionistic fuzzy MAGDM with incomplete attribute weight information, Inf. Sci. 316 (2015) 329–347.
[34] Z.J. Wang, K.W. Li, An interval-valued intuitionistic fuzzy multiattribute group decision making framework with incomplete preference over alternatives, Expert Syst. Appl.: Int. J. 39 (18) (2012) 13509–13516.
[35] G.W. Wei, Decision-making based on projection for intuitionistic fuzzy multiple attributes, Chin. J. Manag. 6 (9) (2009) 1154–1156.
[36] G.W. Wei, Project method for multiple attribute group decision making in two-tuple linguistic setting, Oper. Res. Manag. Sci. 18 (5) (2009) 59–63.
[37] S. Wibowo, Interval-valued intuitionistic fuzzy multicriteria group decision making approach for hotel selection, Int. J. Mach. Learn. Comput. 3 (2013) 65–69.
[38] S. Wibowo, H. Deng, Multi-criteria group decision making for evaluating the performance of e-waste recycling programs under uncertainty, Waste Manag. 40 (2015) 127–135.
[39] G.L. Xu, F. Liu, An approach to group decision making based on interval multiplicative and fuzzy preference relations by using projection, Appl. Math. Model. 37 (6) (2013) 3929–3943.
[40] Z.S. Xu, On method for uncertain multiple attribute decision making problems with uncertain multiplicative preference information on alternatives, Fuzzy Optim. Decis. Mak. 4 (2) (2005) 131–139.
[41] Z.S. Xu, Group decision making based on multiple types of linguistic preference relations, Inf. Sci. 178 (2) (2008) 452–467.
[42] Z.S. Xu, X.Q. Cai, Minimizing group discordance optimization model for deriving expert weights, Group Decis. Negot. 21 (6) (2012) 863–875.
[43] Z.S. Xu, X.Q. Cai, Projection model-based approaches to intuitionistic fuzzy multi-attribute decision making, in: Intuitionistic Fuzzy Information Aggregation, Springer, 2012, pp. 249–258.
[44] Z.S. Xu, Q.L. Da, Projection method for uncertain multi-attribute decision making with preference information on alternatives, Int. J. Inf. Technol. Decis. Mak. 3 (03) (2004) 429–434.
[45] Z.S. Xu, H. Hu, Projection models for intuitionistic fuzzy multiple attribute decision making, Int. J. Inf. Technol. Decis. Mak. 9 (2) (2010) 267–280.
[46] P.-L. Yu, A class of solutions for group decision problems, Manag. Sci. 19 (8) (1973) 936–946.
[47] C. Yue, A geometric approach for ranking interval-valued intuitionistic fuzzy numbers with an application to group decision-making, Comput. Ind. Eng. 102 (2016) 233–245.
[48] Z. Yue, Application of the projection method to determine weights of decision makers for group decision making, Sci. Iran. 19 (3) (2012) 872–878.
[49] Z.L. Yue, TOPSIS-based group decision-making methodology in intuitionistic fuzzy setting, Inf. Sci. 277 (2014) 141–153.
[50] Z.L. Yue, A method for group decision-making based on determining weights of decision makers using TOPSIS, Appl. Math. Model. 35 (4) (2011) 1926–1936.
[51] Z.L. Yue, An extended TOPSIS for determining weights of decision makers with interval numbers, Knowl.-Based Syst. 24 (1) (2011) 146–153.
[52] Z.L. Yue, Deriving decision maker's weights based on distance measure for interval-valued intuitionistic fuzzy group decision making, Expert Syst. Appl. 38 (9) (2011) 11665–11670.
[53] Z.L. Yue, Approach to group decision making based on determining the weights of experts by using projection method, Appl. Math. Model. 36 (7) (2012) 2900–2910.
[54] Z.L. Yue, Developing a straightforward approach for group decision making based on determining weights of decision makers, Appl. Math. Model. 36 (9) (2012) 4106–4117.
[55] Z.L. Yue, Extension of TOPSIS to determine weight of decision maker for group decision making problems with uncertain information, Expert Syst. Appl. 39 (7) (2012) 6343–6350.
[56] Z.L. Yue, An intuitionistic fuzzy projection-based approach for partner selection, Appl. Math. Model. 37 (23) (2013) 9538–9551.
[57] Z.L. Yue, Y.Y. Jia, An application of soft computing technique in group decision making under interval-valued intuitionistic fuzzy environment, Appl. Soft Comput. 13 (5) (2013) 2490–2503.
[58] Z.L. Yue, Y.Y. Jia, A group decision making model with hybrid intuitionistic fuzzy information, Comput. Ind. Eng. 87 (2015) 202–212.
[59] Z.L. Yue, Y.Y. Jia, A direct projection-based group decision-making methodology with crisp values and interval data, Soft Comput. 21 (9) (2017) 2395–2405.
[60] S. Zeng, T. Baležentis, J. Chen, G. Luo, A projection method for multiple attribute group decision making with intuitionistic fuzzy information, Informatica 24 (3) (2013) 485–503.
[61] J. Zhang, D. Wu, D.L. Olson, The method of grey related analysis to multiple attribute decision making problems with interval numbers, Math. Comput. Model. 42 (9–10) (2005) 991–998.
[62] X. Zhang, F. Jin, P. Liu, A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number, Appl. Math. Model. 37 (5) (2013) 3467–3477.
[63] X.L. Zhang, Z.S. Xu, Soft computing based on maximizing consensus and fuzzy TOPSIS approach to interval-valued intuitionistic fuzzy group decision making, Appl. Soft Comput. 26 (2015) 42–56.
[64] G. Zheng, Y. Jing, H. Huang, Y. Gao, Application of improved grey relational projection method to evaluate sustainable building envelope performance, Appl. Energy 87 (2) (2010) 710–720.
[65] J. Zhou, L. Chen, C.L. Philip Chen, Y. Zhang, H.-X. Li, Fuzzy clustering with the entropy of attribute weights, Neurocomputing 198 (2016) 125–134.
[66] Y. Zunqi, L. Hai, Assessment of knowledge shared risk in virtual enterprise based on modified analytic hierarchy process, in: International Conference on Information and Automation, IEEE, Zhangjiajie, China, 2008, pp. 1056–1060.