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Evidential cognitive maps

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ABSTRACT

In order to handle uncertain information, this paper proposes evidential cognitive maps (ECMs), similar to the fuzzy cognitive maps (FCMs). ECMs are uncertain-graph structures for representing causal reasoning through the combination of cognitive maps and Dempster Shafer evidence theory. The framework of ECMs is developed in detail and an application to socio-economic model is used to illustrate the application of the proposed methodology.

1. Introduction

The concept of fuzzy cognitive map has received special attention in recent years as a powerful tool to manipulate knowledge by imitating human reasoning and thinking. Many complex problems like fuzzy control [1–3], approximate reasoning [4–7], strategic planning [8–11], data mining [12], virtual worlds and network models [13] have been dealt with using FCMs. Especially, in the field of medical decision making [14,15], Kannappan [16] models and predicts autistic spectrum disorder using FCM, and an unsupervised non-linear Hebbian learning algorithm is applied to improve it's efficiency. Papageorgiou [17] presents a novel framework for the construction of augmented FCMs based on fuzzy rule-extraction methods for decisions in medical informatics. The study extracted the available knowledge from data in the form of fuzzy rules and inserted them into the FCM, contributing to the development of a dynamic decision support system. FCM has also been investigated for risk analysis of pulmonary infections during patient admission into the hospital [18–23].

Although FCM has achieved success in many fields, there are some limitations inherent in FCM, such as lack of adequate capability to handle uncertain information and lack of enough ability to aggregate the information from different sources. Some attention has been paid to the first issue by some researchers. For example, Salmeron [24] proposes an innovative and flexible model based on Grey Systems Theory, called fuzzy grey cognitive maps (FGCM), which can be adapted to a wide range of problems, especially in multiple meaning-based environments. Iakovidis and Papageorgiou [25] propose an approach based on cognitive maps and intuitionistic fuzzy logic, which is called intuitionistic fuzzy cognitive map (IFCM) to extend the existing FCM by considering the expert's hesitancy in the determination of the causal relations between the concepts of a domain. Similarly, after the introduction of neutrosophic logic (similar to intuitionistic fuzzy sets) by Samarandache [26], indeterminacy has been introduced into causal relationships between some of concepts of FCMs. This is a generalization of FCMs and the structure is called neutrosophic cognitive maps (NCMs) [27]. However, how to extend the ability of FCM to aggregate the information from different sources under uncertain environment is a significant question in the application of FCM and is still an open issue.

Uncertain information fusion has been studied for many years [28–41], indicating that Dempster Shafer theory (DS theory or evidence theory) is an effective framework to represent and fuse uncertain information. Therefore this paper combines FCM and evidential theory to the concept development of evidential cognitive maps that not only remains the ability to represent uncertainty but also contributes to aggregating knowledge from different sources (experts/commanders). The combination of evidence theory and FCM is shown to be a valuable approach through illustrations.

This paper is organized as follows: Section 2 briefly presents FCM and basic evidence theory and some operations of interval numbers. Section 3 develops the mathematical model of the proposed ECM concept. Section 4 describes the implementation

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of ECM. Section 5 briefly presents qualitative comparison of ECM with FCM and NCM. An application of ECM to socio-economic model is presented in Section 6.

2. Preliminaries

In this section, we briefly introduce FCM and evidence theory.

2.1. FCM

Political scientist Robert Axelrod [42] introduced cognitive maps in the 1970's for representing social scientific knowledge. Fuzzy cognitive map (FCM), an extension of the cognitive map, is a causal description in order to model the behavior of the system [43]. FCM is an interactive structure of concepts, each of which interacts with the rest showing the dynamics and different aspects of the behavior of the system. Human experience and knowledge of the operation of complex systems are embedded in FCM, i.e., knowledge gained about the operation of the system and its behavior under different circumstances by human experts.

FCM consists of nodes (concepts, agents) and weighted arcs (connection, edge), which are graphically illustrated as signed weighted graph with optional feedback loops. Nodes on the graph represent concepts describing behavioral characteristics of the system. Concepts can be inputs, outputs, variables, states, events, actions, goals, and trends of the system. Signed weighted arcs represent causal relationships (cause and effect) that exist among concepts.

Fig. 1 illustrates a simple FCM consisting of six concepts C_i ($i = 1, \ldots, 6$). The value of C_i is denoted by A_i ($i = 1, \ldots, 6$), where A_i is mapped in the interval [0, 1]. Weight $\omega_{ij} \in [-1, 1]$ represents the causal relationship between concept i and concept j, where a negative sign represents inverse causation. This scheme may give rise to the following three types of interactions:

(1) $\omega_{ij} > 0$ indicates a positive causality, where an increase in the value of the ith concept causes an increase in the value of the jth concept;
(2) $\omega_{ij} < 0$ indicates a negative causality, where an increase in the value of the ith concept causes an decrease in the value of the jth concept;
(3) $\omega_{ij} = 0$ indicates that there is no causal relationship between the ith concept and the jth concept.

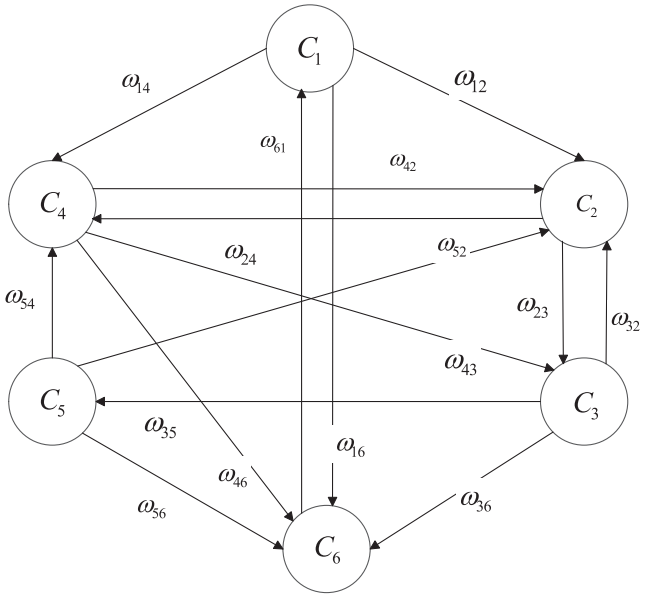

Fig. 1. The structural diagram of fuzzy cognitive map.

The edge matrix of six concept fuzzy cognitive map is denoted as in Eq. (1).

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{pmatrix} 0 & \omega_{12} & 0 & \omega_{14} & 0 & \omega_{16} \\ 0 & 0 & \omega_{23} & \omega_{24} & 0 & 0 \\ 0 & \omega_{32} & 0 & 0 & \omega_{35} & \omega_{36} \\ 0 & \omega_{42} & \omega_{43} & 0 & 0 & \omega_{46} \\ 0 & \omega_{52} & 0 & \omega_{54} & 0 & \omega_{56} \\ \omega_{61} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{1}$$

Kosko [43] proposed a rule to calculate the value of each concept based on the influence of the interconnected concepts, where the content of the following function is normalized in the interval $[-1, 1]$:

$$A_j^t = f\left(k_1 \sum_{\substack{i=1 \\ i \neq j}}^{n} A_i^{t-1}\omega_{ij} + k_2 A_j^{t-1}\right) \quad 0 \leqslant k_1 \leqslant 1 \quad 0 \leqslant k_2 \leqslant 1 \tag{2}$$

where A_j^t is the normalized ($A_j^t \in [0, 1]$) value (a.k.a activation level) of concept C_j at time step t, and $f(x)$ is a threshold function. Generally, a sigmoidal function $f(x) = \frac{1}{1+e^{-\lambda x}}$ is used to constrain the value of $f(x)$ in the interval [0,1], where $\lambda > 0$ determines the steepness of $f(x)$. The coefficient k_1 expresses the influence of interconnected concepts in the configuration of the new value A_i of concept C_i. For example, in Fig. 1, the concept C_6 receives inputs from concepts C_1, C_3, C_4, and C_5. If experts perceive that C_4 and C_5 interact in such a way that both are fully participating in impacting C_6 then the k_1 associated with them will be closer to 1. Similarly, k_2 accounts for the importance of C_6 being at its activation level in the previous time step. The selection of coefficients k_1 and k_2 depends on the nature and type of each concept, and may differ from concept to concept.

2.2. Dempster–Shafer (DS) theory of evidence

The DS theory of evidence, which was first proposed by Dempster [28] and then developed by Shafer [31], is regarded as a generalization of the Bayesian theory of probability. Due to its ability to handle uncertainty or imprecision embedded in the evidence, the DS theory has been increasingly applied in recent years [44–47,41,48–52], and applied to multiple attribute decision analysis problems [53–57].

The introduction of DS theory are briefly summarized as following:

(1) "Frame of discernment" [31]:
Let $\Theta = \{H_1, H_2, \ldots, H_N\}$ be a finite set of n elements, and $P(\Theta)$ denote the power set composed of 2^N elements of Θ.

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \ldots, \Theta\} \tag{3}$$

(2) "Basic probability assignment (BPA)" [31]:
The BPA function is defined as a mapping of the power set $P(\Theta)$ to a number between 0 and 1.

$$m : P(\Theta) \to [0, 1] \tag{4}$$

and which satisfies the following conditions:

$$m(\emptyset) = 0, \quad \sum_{A \subseteq P(\Theta)} m(A) = 1 \tag{5}$$

The mass $m(A)$ represents how strongly the evidence supports A.

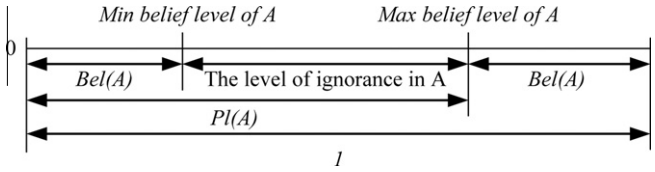

Fig. 2. The relation between Bel and Pl.

(3) "Belief and plausibility functions" [31]:
The belief function *Bel* is defined as

$$Bel : P(\Theta) \to [0,1] \text{ and } Bel(A) = \sum_{B \subseteq A} m(B) \tag{6}$$

and the plausibility function *Pl* is defined as

$$Pl : P(\Theta) \to [0,1] \text{ and } Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \tag{7}$$

Bel (A) and *Pl (A)* are the lower limit and the upper limit, respectively, of the belief level of hypothesis *A* which is illustrated in Fig. 2. Both imprecision and uncertainty can be represented by them.

(4) "Dempster's combination rule":
Two bodies of evidence *X* and *Y* regarding Θ can be used to calculate the belief level for some new hypothesis *C* as follows:
The measure of conflict *K* is given as

$$K = \sum_{X \cap Y = \emptyset, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y) \tag{8}$$

and the mass function after combination is

$$m(C) = m_i(X) \oplus m_{i'}(Y) = \begin{cases} 0, & \text{if } X \cap Y = \emptyset, \\ \frac{\sum_{X \cap Y = C, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y)}{1-K}, & \text{if } X \cap Y \neq \emptyset. \end{cases} \tag{9}$$

2.3. Basic operations of interval numbers

Let $\widetilde{A}, \widetilde{B}$ be two interval numbers, $\widetilde{A} = [A_l, A_u], \widetilde{B} = [B_l, B_u]$.

(1) The representation of the addition operation $\oplus$ on interval numbers $\widetilde{A}$ and $\widetilde{B}$ can be defined as

$$\widetilde{A} \oplus \widetilde{B} = [A_l + B_l, A_u + B_u] \tag{10}$$

Note that $\oplus_1^n(\widetilde{A_i}) = \widetilde{A_1} \oplus \widetilde{A_2} \oplus \cdots \oplus \widetilde{A_n}$.

(2) The representation of the multiplication operation $\otimes$ on interval numbers $\widetilde{A}$ and $\widetilde{B}$ can be defined as

$$\widetilde{A} \otimes \widetilde{B} = [Z_l, Z_u] \tag{11}$$

such that

$$Z_l = \min\{A_l B_l, A_l B_u, A_u B_l, A_u B_u\}$$
$$Z_u = \max\{A_l B_l, A_l B_u, A_u B_l, A_u B_u\}$$

where *min* and *max* are the minimum and maximum of the denoted values.

3. Evidential cognitive map (ECM)

ECM is also a directed graph with feedback, consisting of nodes and weighted arcs. Nodes of the graph stand for the concepts that are used to describe the behavior of the system and they are connected by weighted arcs representing the causal relationships that exist between the concepts. Each concept $\widetilde{C_i}$ is characterized by an interval $\widetilde{A_i}$ that represents its value, and it results from the transformation of the fuzzy value of the system variable. In this way, the representation of the concept is more flexible than the representation of the concept in FCM that uses crisp numbers.

3.1. Edge weight of the cognitive map

In a cognitive map, experts' opinions are reflected by the estimate of the degree of causation between nodes in the referred concept set, namely weight estimate. Each expert's estimate of some causal relation can be regarded as evidence. Generally, due to the complexity of the relations between concepts and limitation of knowledge and experience of experts, the causal relation of two concepts could be described by the following four cases in the evidence theory:

(1) Negative causal relation, which can be described as m{−1}, where an increase in the value of the *i*th concept causes a decrease in the value of the *j*th concept.
(2) Positive causal relation, which can be described as m{+1}, where an increase in the value of the *i*th concept causes a increase in the value of the *j*th concept.
(3) No causal relation between the *i*th concept and the *j*th concept, which can be described as *m*{0}.
(4) No idea or abstaining from voting, which can be described as *m*{−1, 0, 1}.

Hence, the combined influence from concept *i*th to concept *j*th can be described as

$$BPA_{ij} = \begin{pmatrix} m\{-1\} = a \\ m\{1\} = b \\ m\{0\} = c \\ m\{-1,0,1\} = 1 - a - b - c \end{pmatrix} \tag{12}$$

such that $a \geqslant 0$, $b \geqslant 0$, $c \geqslant 0$, $1 - a - b - c \geqslant 0$.

With this representation, the uncertainty of the causal relation between two concepts is more clearly described and handled compared to the frame of FCM. The idea of evidential cognitive map is illustrated by Fig. 3, and the weights are shown as in Eq. (13).

$$W = \begin{array}{c} \\ \widetilde{C_1} \\ \widetilde{C_2} \\ \widetilde{C_3} \\ \widetilde{C_4} \\ \widetilde{C_5} \\ \widetilde{C_6} \end{array} \begin{array}{c} \begin{array}{cccccc} \widetilde{C_1} & \widetilde{C_2} & \widetilde{C_3} & \widetilde{C_4} & \widetilde{C_5} & \widetilde{C_6} \end{array} \\ \begin{pmatrix} 0 & BPA_{12} & 0 & BPA_{14} & 0 & BPA_{16} \\ 0 & 0 & BPA_{23} & BPA_{24} & 0 & 0 \\ 0 & BPA_{32} & 0 & 0 & BPA_{35} & BPA_{36} \\ 0 & BPA_{42} & BPA_{43} & 0 & 0 & BPA_{46} \\ 0 & BPA_{52} & 0 & BPA_{54} & 0 & BPA_{56} \\ BPA_{61} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{13}$$

3.2. Aggregate knowledge under conflict environment

In the process of collection of knowledge, the experts' opinions may be inconsistent with each other. How to combine the knowledge to reach consensus is a critical problem. In this paper, the method of combining belief functions based on distance of evidence proposed by Deng et al. [58] is applied. The main process is as follows:

Suppose the distance between two bodies of evidence (R_i, m_i) and (R_j, m_j) can be calculated by the algorithm in Ref. [59] and is denoted as $d(m_i, m_j)$.

$$d(m_i, m_j) = \sqrt{\frac{1}{2}(\vec{m}_i - \vec{m}_j)^T \overline{D}(\vec{m}_i - \vec{m}_j)} \tag{14}$$

such that $\overline{D}$ is a matrix $(2^N \times 2^N)$, and $D(A,B) = \frac{|A \cap B|}{|A \cup B|}$.

The similarity measure Sim_{ij} between the two bodies of evidence (R_i, m_i) and (R_j, m_j) is defined as:

$$Sim(m_i, m_j) = 1 - d(m_i, m_j) \tag{15}$$

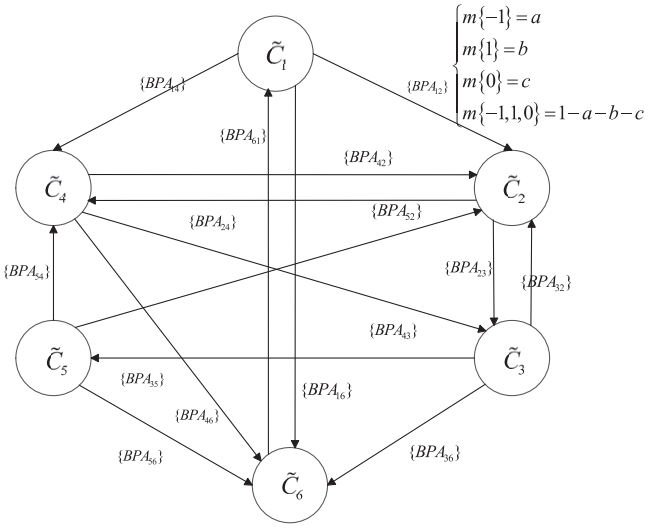

Fig. 3. The structural diagram of evidential cognitive map.

Suppose the number of bodies of evidence is k. After all the degrees of similarity between the bodies of evidence are obtained, we can construct a *similarity measure matrix* (SMM), which gives us insight into the agreement between the bodies of evidence.

$$SMM = \begin{pmatrix} 1 & S_{12} & \cdots & S_{1j} & \cdots & S_{1k} \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ S_{i1} & S_{i2} & & S_{ij} & & S_{ik} \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ S_{k1} & S_{k2} & & S_{kj} & & 1 \end{pmatrix} \tag{16}$$

The support degree of the body of evidence $(R_i, m_i)(i = 1, 2 \ldots, k)$ is defined as:

$$Sup(m_i) = \sum_{\substack{j=1 \\ j \neq i}}^{k} Sim(m_i, m_j) \tag{17}$$

The credibility degree Crd_i of the body of evidence (R_i, m_i) $(i = 1, 2 \ldots, k)$ is defined as:

$$Crd_i = \frac{Sup(m_i)}{\sum_{i=1}^{k} Sup(m_i)} \tag{18}$$

It can be easily seen that $\sum_{i=1}^{n} Crd_i = 1$, thus, the credibility degree is actually a weight, which shows the relative importance of the collected evidence.

If the maximum of the credibility degree ($MaxCrd_i$) of the body of evidence is 1, which can be the discounting coefficient for the ith evidence, and the discounting coefficient of the body of evidence can be defined as:

$$\alpha_i = \frac{Crd_i}{MaxCrd_i}, (i = 1, \ldots, k) \tag{19}$$

If a source of evidence provides a mass function m which has probability α of reliability, then the discounted belief m' on Θ is defined as [28]:

$$m'(A) = \alpha m(A), \quad \forall A \subset \Theta, A \neq \Theta \tag{20}$$

$$m'(\Theta) = 1 - \alpha + \alpha m(\Theta) \tag{21}$$

All mass functions are discounted by α, the discounting coefficient. Then the modified (discounted) evidence can be combined with the classical combination rule Eq. (9) to get the aggregated result.

3.3. Transformation using the belief function and plausibility function

The next step is to estimate each connection weight of the evidential cognitive map in Fig. 3. Take any BPA_{ij} for example. It can be denoted explicitly by

$$\begin{aligned} &m\{-1\} = a \\ &m\{1\} = b \\ &m\{0\} = c \\ &m\{-1, 0, 1\} = 1 - a - b - c \end{aligned} \tag{22}$$

such that $a \geqslant 0, b \geqslant 0, c \geqslant 0, 1 - a - b - c \geqslant 0$. According to Eq. (6),

$$Bel\{-1\} = m\{-1\} = a \tag{23}$$

$$Pl\{-1\} = m\{-1\} + m\{-1, 0, 1\} = 1 - b - c \tag{24}$$

Hence, the possibility of the positive casual relation $P\{-1\}$ is denoted as

$$P\{-1\} = [Bel\{-1\}, Pl\{-1\}] = [a, 1 - b - c] \tag{25}$$

Similarly, $P\{1\} = [b, 1 - a - c]$, $P\{0\} = [c, 1 - a - b]$. And the connection weight from $\widetilde{C_1}$ and $\widetilde{C_4}$ can be calculated as follows (Here the 4th influence is ignored since it is abstention from voting):

$$\begin{aligned} \widetilde{\omega_{ij}} &= P\{1\} \times 1 \oplus P\{0\} \times 0 \oplus P\{-1\} \times (-1) \\ &= 1 * [b, 1 - a - c] \oplus 0 * [a, 1 - b - c] \oplus (-1) * [a, 1 - b - c] \\ &= [b, 1 - a - c] \oplus [-1 + b + c, -a] = [2b + c - 1, 1 - 2a - c] \end{aligned} \tag{26}$$

$$W = \begin{array}{c} \\ \widetilde{C_1} \\ \widetilde{C_2} \\ \widetilde{C_3} \\ \widetilde{C_4} \\ \widetilde{C_5} \\ \widetilde{C_6} \end{array} \begin{array}{c} \begin{array}{cccccc} \widetilde{C_1} & \widetilde{C_2} & \widetilde{C_3} & \widetilde{C_4} & \widetilde{C_5} & \widetilde{C_6} \end{array} \\ \begin{pmatrix} 0 & \widetilde{\omega_{12}} & 0 & \widetilde{\omega_{14}} & 0 & \widetilde{\omega_{16}} \\ 0 & 0 & \widetilde{\omega_{23}} & \widetilde{\omega_{24}} & 0 & 0 \\ 0 & \widetilde{\omega_{32}} & 0 & 0 & \widetilde{\omega_{35}} & \widetilde{\omega_{36}} \\ 0 & \widetilde{\omega_{42}} & \widetilde{\omega_{43}} & 0 & 0 & \widetilde{\omega_{46}} \\ 0 & \widetilde{\omega_{52}} & 0 & \widetilde{\omega_{54}} & 0 & \widetilde{\omega_{56}} \\ \widetilde{\omega_{61}} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{27}$$

3.4. ECM dynamics

At each simulation step, the value $\widetilde{A_i}$ of a concept $\widetilde{C_i}$ is calculated by computing the influence of the interconnected concepts $\widetilde{C_j}$'s on the specific concept $\widetilde{C_i}$ following the calculation rule:

$$\widetilde{A_j^t} = f\left(k_1 \overset{\substack{n \\ i=1 \\ i \neq j}}{\oplus} (\widetilde{A_i^{t-1}} \otimes \widetilde{\omega_{ij}}) \oplus k_2 \widetilde{A_j^{t-1}} \right) \quad 0 \leqslant k_1 \leqslant 1 \quad 0 \leqslant k_2 \leqslant 1 \tag{28}$$

where $\widetilde{A_j^t}$ is the value of concept $\widetilde{C_j}$ at simulation step t, $\widetilde{A_i^{t-1}}$ is the value of concept $\widetilde{C_i}$ at simulation step $t - 1$. The meaning of k_1 and k_2 here is the same as meaning of k_1 and k_2 in FCM. $\widetilde{\omega_{ij}}$ is the weight of the interconnection from concept $\widetilde{C_i}$ to concept $\widetilde{C_j}$ and f is nonlinear mapping function as

$$f(\widetilde{X}) = \frac{1}{1 + e^{-\lambda \widetilde{X}}} \tag{29}$$

where $\widetilde{X}$ is an interval number and λ is a parameter determining its steepness. The output of f is also an interval number. It can approximatively handle the uncertain information from concepts and connection weights. The meaning of f is illustrated in Fig. 4 ($x > 0$).

4. Application framework of ECMs

The Framework of ECM is shown as Fig. 5, and its application is detailed as follows:

4.1. Knowledge acquisition

Suppose we have m experts, and they are divided into n groups according to their knowledge and background. For each group, the relationship (or the edge weight) between the concepts (or the nodes) is decided by voting. For example, may there are ten experts to evaluate the relation from the ith concept to jth concept, and six of them think that it is positive, one of them thinks that it is negative, one of them has no idea, and others are abstain from voting. Hence, using evidence theory, the relation between concept $\widetilde{C}_i$ and concept $\widetilde{C}_j$ is described as: $m_1\{-1\} = 0.1$; $m_1\{0\} = 0.1$; $m_1\{+1\} = 0.6$; $m_1\{-1, 0, +1\} = 0.2$. This evidence can also be given by a single expert/commander directly.

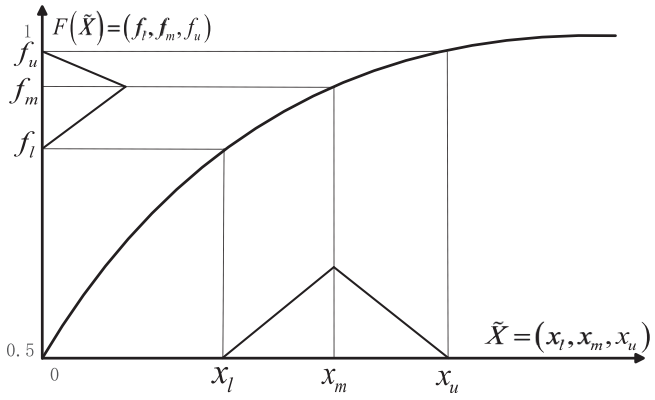

Fig. 4. The nonlinear mapping function.

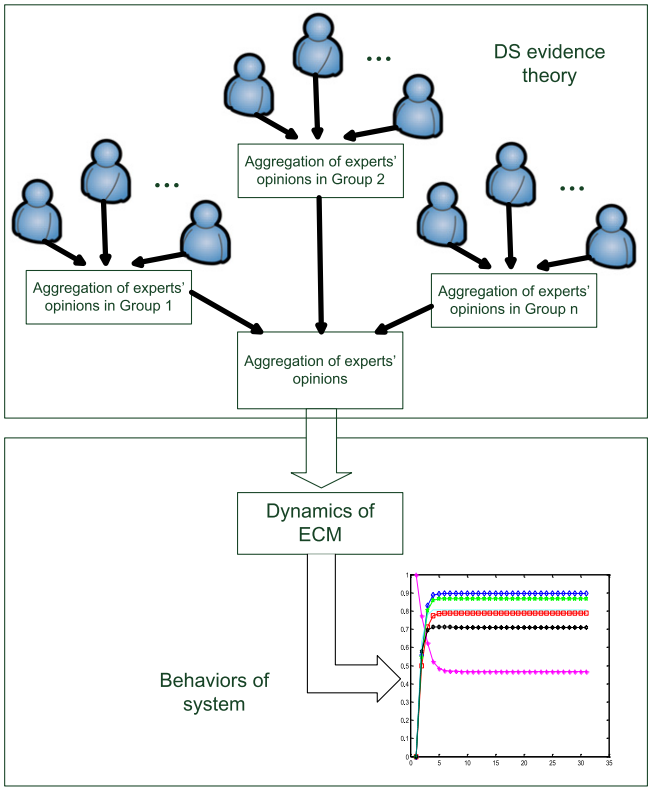

Fig. 5. Framework of ECM.

4.2. Knowledge aggregation

This step allows the aggregation of knowledge acquired from various sources to develop a comprehensive ECM, which will represent the understanding of the experts about the special issue. The comprehensive ECM combines partial ECMs from inner groups and outer groups. The aggregation of knowledge from inner groups is for the opinions of the experts of each group and the aggregation of knowledge from outer groups is for the edges of the partial ECMs.

In this part, a critical problem is how to deal with the conflicting evidences since the opinions from different experts are not always consistent. The method dealing with conflicting evidence in evidence theory is mature, here the method of combing conflicting evidence proposed by Deng [58] is applied here.

4.3. Training ECM

ECM is a dynamic system, and the procedure is described in Algorithm 1 in detail. After converting the evidence into interval numbers with the belief and plausibility functions. The state vector can be handled easily, which has more flexibility to deal with uncertain and fuzzy information when compared to the classical fuzzy cognitive maps. Whether the dynamic is reaches equilibrium, the paper [60] has provided some inspiration for the interpretation of the condition.

Algorithm 1. The convergent procedure of state vector of ECMs

Input: the initial state vector of nodes, $X_i = [A_1, A_2, \ldots, A_n]$;
the matrix of edge weight $W_{n\times n}$;
Output: the equilibrium state vector of nodes,
$X_e = [A_1, A_2, \ldots, A_n]$;

```
Temp = [] = Xi;
While not satisfy the condition do
  Xi = Temp;
  for j = 1; j ⩽ n; do
    sum_l = 0;
    sum_u = 0;
    for j = 1; j ⩽ n; do
      if i ≠ j then
        a = Xi(i, 1) * W(i, j, 1);
        b = Xi(i, 1) * W(i, j, 2);
        c = Xi(i, 2) * W(i, j, 1);
        d = Xi(i, 2) * W(i, j, 2);
        sum_l = sum_l + min{a, b, c, d};
        sum_u = sum_u + max{a, b, c, c};
      end
    end
    sum_l = sum_l + Xi(j, 1);
    sum_u = sum_u + Xi(j, 2);
    Temp (j,1) = function (sum_l);
    Temp (j,2) = function (sum_u);
    if Temp(j, 1) ⩽ 0.5 then
      Temp (j, 1) = 0;
      Temp (j, 2) = 0;
    end
  end
  Xe = Temp;
end
return Xe;
```

4.4. Interpreting ECM

The outcome of an ECM is in the form of concepts being "activated" at different levels after reaching equilibrium. The interpretation of these concepts will determine the judgement for a given scenario.

5. Qualitative comparison with FCM and NCM

Here (in ECM) we use the fact that between any two concepts/nodes the existing relation may be an indeterminate (as) in reality, FCM do not reflect the notion of indeterminacy. Some differences between ECM and FCM are listed as follows:

Table 1
Experts' knowledge in group 1 (values of aggregated knowledge are shown in bold).

	$\widetilde{C_1}$	$\widetilde{C_3}$	$\widetilde{C_5}$
$\widetilde{C_1}$	–	E1: (0.7, 0.1, 0.0, 0.2) E2: (0.8, 0.0, 0.0, 0.2) E3: (0.2, 0.5, 0.1, 0.2) **E123: (0.8782, 0.0759, 0.0076, 0.0383)**	E1: (0.1, 0.6, 0.1, 0.3) E2: (0.3, 0.3, 0.0, 0.4) E3: (0.1, 0.5, 0.2, 0.2) **E123: (0.0980,0.7978, 0.0699, 0.0342)**
$\widetilde{C_3}$	–	–	E1: (0.6, 0.1, 0.1, 0.2) E2: (0.7, 0.1, 0.0, 0.2) E3: (0.3, 0.5, 0.0, 0.2) **E123: (0.8369, 0.1225, 0.0135, 0.0271)**
$\widetilde{C_5}$	–	–	–

Table 2
Experts' knowledge in group 2 (values of aggregated knowledge are shown in bold).

	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	$\widetilde{C_4}$
$\widetilde{C_1}$	–	–	E1: (0.5, 0.1, 0.1, 0.2) E2: (0.7, 0.1, 0.1, 0.1) E3: (0.3, 0.6, 0.0, 0.1) **E123: (0.7851, 0.1575, 0.0313, 0.0260)**	–
$\widetilde{C_2}$	–	–		E1: (0.5, 0.1, 0.1, 0.2) E2: (0.4, 0.1, 0.1, 0.1) E3: (0.3, 0.0, 0.2, 0.5) **E123: (0.7088, 0.0649, 0.1263, 0.1001)**
$\widetilde{C_3}$	–	E1: (0.5, 0.1, 0.1, 0.2) E2: (0.5, 0.2, 0.1, 0.2) E3: (0.7, 0.0, 0.0, 0.3) **E123: (0.8848, 0.0520, 0.0313, 0.0319)**	–	–
$\widetilde{C_4}$	E1: (0.7, 0.1, 0.0, 0.2) E2: (0.2, 0.5, 0.1, 0.2) E3: (0.6, 0.1, 0.0, 0.3) **E123: (0.8055, 0.1478, 0.0106, 0.0362)**	E1: (0.1, 0.5, 0.2, 0.2) E2: (0.1, 0.5, 0.2, 0.2) E3: (0.0, 0.7, 0.0, 0.3) **E123: (0.0295, 0.8763, 0.0707, 0.0236)**	–	–

Table 3
Experts' knowledge in group 3 (values of aggregated knowledge are shown in bold).

	$\widetilde{C_1}$	$\widetilde{C_4}$	$\widetilde{C_5}$
$\widetilde{C_1}$	–	–	E1: (0.1, 0.7, 0.1, 0.1) E2: (0.0, 0.6, 0.0, 0.4) E3: (0.5, 0.2, 0.2, 0.1) **E123: (0.1090, 0.8078, 0.0585, 0.0248)**
$\widetilde{C_4}$	E1: (0.6, 0.1, 0.1, 0.2) E2: (0.4, 0.2, 0.2, 0.2) E3: (0.5, 0.1, 0.2, 0.2) **E123: (0.8146, 0.0681, 0.0984, 0.0189)**	–	–
$\widetilde{C_5}$	–	E1: (0.5, 0.4, 0.0, 0.1) E2: (0.2, 0.6, 0.0, 0.2) E3: (0.1, 0.7, 0.0, 0.2) **E123: (0.1346, 0.8600, 0.0000, 0.0054)**	–

(1) FCM measures the existence of causal relation between two concepts using crisp number between −1 and 1 and if no relation exists it is denoted by 0. ECM measures not only the existence or absence of causal relations between two concepts but also give representation to the indeterminacy of the relation.
(2) Because ECM measures the indeterminacy, the expert of the model can give due and careful representation while implementing the results of the model.
(3) Being able to aggregate multiple sources and conflicting information is an important advantage in ECM.

The advantage of ECM compared with NCM can be listed as follows:

(1) More accurate representation of knowledge and dynamics of system. In NCM, every edge is weighted with a number in the set {−1, 0, 1}, and if there is a hesitating decision, **I**, which is a symbo, is directly used to represent the indeterminacy, but in ECM, every edge is weighted with belief and plausibility dimension using BPA {$m\{-1\}$, $m\{0\}$, $m\{1\}$, $m\{-1, 0, 1\}$}, where $m\{-1\}$, $m\{0\}$, $m\{1\}$, m{−1, 0, 1} all belong to the interval [0, 1] and they are all crisp number, and there is no symbolic computation. Therefore, it is a more accurate representation and can describe the behavior of the system more accurately.
(2) Being able to aggregate multiple, conflicting information using evidence theory is an important advantage in ECM.

6. An application of ECM to a socio-economic model

This section illustrates the application of the proposed method to a socio-economic model. It is constructed with Population, Crime, Economic condition, Poverty, and Unemployment as nodes or concepts. Our purpose is to evaluate the trend of factors changing with any one factor using ECM.

First, the structure of ECM should be established using several sources of partial knowledge. All the available experts are divided into three groups (group1, group2, and grou3). and the opinions are provided in Tables 1–3. They can be described as Figs. 6–8 accordingly.

Next, the opinions from different experts and the partial ECMs are combined together considering the conflict in the evidence. (See Table 4 and Fig. 9).

Take group1 for example; The relationships between $\widetilde{C_1}$ and $\widetilde{C_3}$ is provided by three evidences (*Evidence1* (E1), *Evidence2* (E2) and *Evidence3* (E3)) as follows:

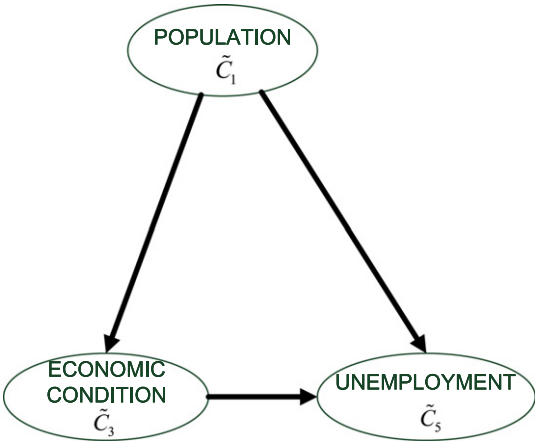

Fig. 6. Partial ECM from group 1.

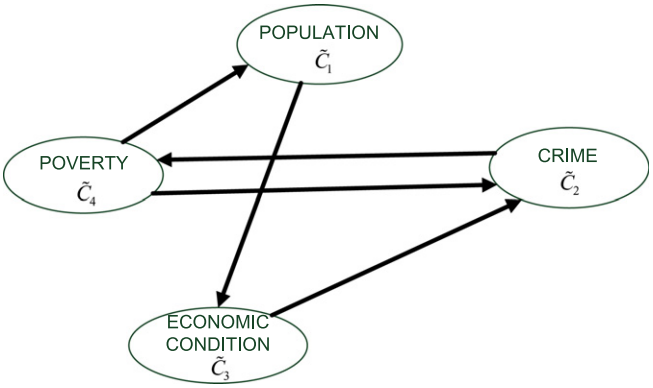

Fig. 7. Partial ECM from group 2.

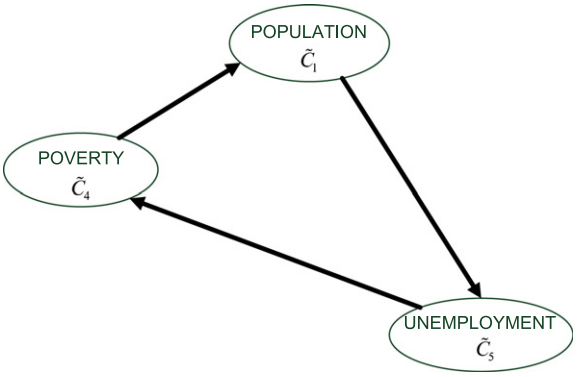

Fig. 8. Partial ECM from group 3.

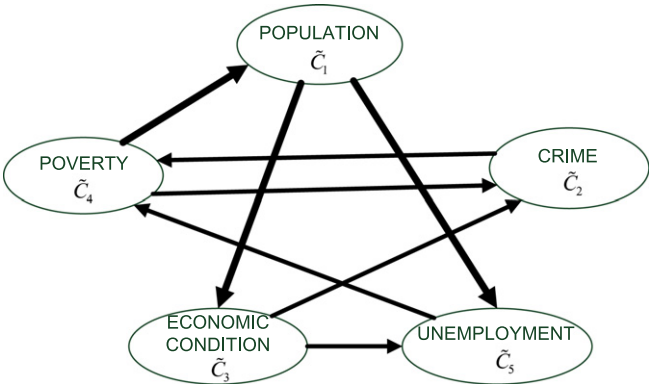

Fig. 9. The aggregation of all three partial ECMs from 3 groups.

$$E1: m_1\{-1\} = 0.7; m_1\{1\} = 0.1; m_1\{0\} = 0.0; m_1\{-1,1,0\} = 0.2;$$
$$E2: m_2\{-1\} = 0.8; m_2\{1\} = 0.0; m_2\{0\} = 0.0; m_2\{-1,1,0\} = 0.2;$$
$$E3: m_3\{-1\} = 0.2; m_3\{1\} = 0.5; m_3\{0\} = 0.1; m_3\{-1,1,0\} = 0.2;$$

It is easy to conclude that *Evidence3* is not consistent with *Evidence1* and *Evidence2*. The combination with evidential distance is applied here, and the process is as follows:

(1) The distance matrix of the evidences is

$$d = \begin{bmatrix} 0.00 & 0.02 & 0.42 \\ 0.02 & 0.00 & 0.62 \\ 0.42 & 0.62 & 0.00 \end{bmatrix}$$

Table 4
Aggregation of experts' knowledge from 3 groups (values of aggregated knowledge are shown in bold).

	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	$\widetilde{C_4}$	$\widetilde{C_5}$
$\widetilde{C_1}$	–	–	E1: (0.8782, 0.0759, 0.0076, 0.0383) E2: (0.7851, 0.1575, 0.0313, 0.0260) **E12: (0.9703, 0.0261, 0.0021, 0.0013)**	–	E1: (0.0980, 0.7978, 0.0699, 0.0342) E2: (0.1090, 0.8078, 0.0585, 0.0248) **E12: (0.0235, 0.9644, 0.0109, 0.0012)**
$\widetilde{C_2}$	–	–	–	E1: (0.7088, 0.0649, 0.1263, 0.1001)	–
$\widetilde{C_3}$	–	E1: (0.8848, 0.0520, 0.0313, 0.0319)	–	–	E1: (0.8369, 0.1225, 0.0135, 0.0271)
$\widetilde{C_4}$	E1: (0.8055, 0.1478, 0.0106, 0.0362) E2: (0.8146, 0.0681, 0.0984, 0.0189) **E12: (0.9713, 0.0212, 0.0067, 0.0009)**	E1: (0.0295, 0.8763, 0.0707, 0.0236)	–	–	–
$\widetilde{C_5}$	–	–	–	E1: (0.1346, 0.8600, 0.0000, 0.0054)	–

(2) The similarity matrix of the evidences is

$$SMM = \begin{bmatrix} 1.00 & 0.98 & 0.58 \\ 0.98 & 1.00 & 0.38 \\ 0.58 & 0.38 & 1.00 \end{bmatrix}$$

(3) The credibility degree of the evidences is

$$Cred = [0.3721,\ 0.3430,\ 0.2849]$$

(4) The discounting coefficient is

$$\alpha = [1.0000,\ 0.9219,\ 0.7656]$$

(5) Now we combine the three evidences using DS theory.

$$E12 : m_1\{-1\} = 0.9150; m_1\{1\} = 0.0283;$$
$$m_1\{0\} = 0.0; m_1\{-1,1,0\} = 0.0567;$$
$$E123 : m_1\{-1\} = 0.8782; m_1\{1\} = 0.0759;$$
$$m_1\{0\} = 0.0076; m_1\{-1,1,0\} = 0.0383;$$

Then, the partial ECMs can be combined with each other in a similar way. Taking Figs. 6 and 7 for example, the shared edge is from concept $\widetilde{C_1}$ to $\widetilde{C_3}$, and the result of combination is shown in Tables 4 and 5.

Let $A^0 = [\widetilde{A}_1^0, \widetilde{A}_2^0, \widetilde{A}_3^0, \widetilde{A}_4^0, \widetilde{A}_5^0] (A_1^0 = [0.9, 1], \widetilde{A}_i^0 = [0, 0], i = 2, \ldots, 5)$ be an initial vector state, and let $k_1 = 1$ and $k_2 = 1$. Let the sigmoidal function with $\lambda = 1$ be used as a threshold function. Figs. 10–12 represent the results of Eq. (28) simulated iteratively thirty times.

Table 5
Aggregation of experts' knowledge in 3 groups (values shown in bold are transformed using the belief function and plausibility function).

	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	$\widetilde{C_4}$	$\widetilde{C_5}$
$\widetilde{C_1}$	[0.0000, 0.0000]	[0.0000, 0.0000]	**[−0.9457, −0.9427]**	[0.0000, 0.0000]	**[0.9397, 0.9421]**
$\widetilde{C_2}$	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]	**[−0.7439, −0.5439]**	[0.0000, 0.0000]
$\widetilde{C_3}$	[0.0000, 0.0000]	**[−0.8647, −0.8009]**	[0.0000, 0.0000]	[0.0000, 0.0000]	**[−0.7415, −0.6873]**
$\widetilde{C_4}$	**[−0.9509, −0.9493]**	**[0.8233, 0.8703]**	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
$\widetilde{C_5}$	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]	**[0.7200, 0.7308]**	[0.0000, 0.0000]

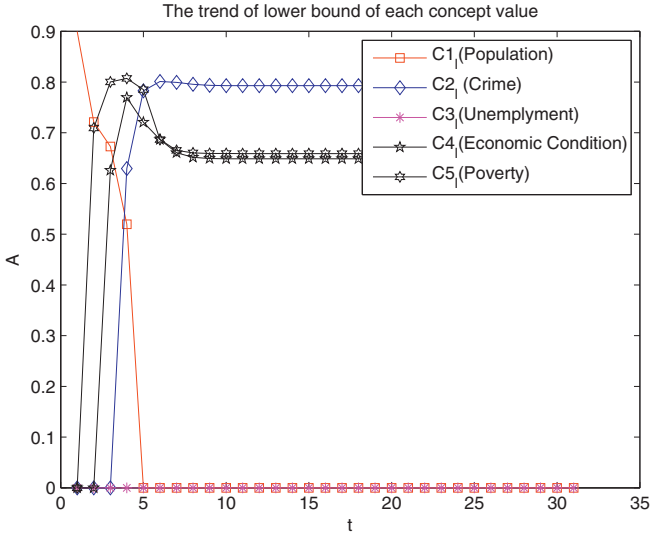

Fig. 10. Results of ECM simulations (Trend of lower bound of each concept value).

It can be seen that the ECM reaches an equilibrium state approximately after 11 iterations. The values of the concepts reach an equilibrium state vector $\widetilde{A}^{11} = \left[\widetilde{A}_1^{11}, \widetilde{A}_2^{11}, \widetilde{A}_3^{11}, \widetilde{A}_4^{11}, \widetilde{A}_5^{11}\right]$. (See Table 6).

Once the ECM reaches equilibrium, the activation values provide the "triggering" or "firing" strength of those concepts for a given scenario. Generally, when the FCM reaches equilibrium, the activation levels are transformed back to the corresponding values. These activation levels may be interpreted quantitatively or qualitatively. For example, the ECM shown in Fig. 11 reaches an equibrium state vector $A_{mean}^{11} = [0.0000, 0.7929, 0.0000, 0.6488, 0.6591]$, which implies that, concept $\widetilde{C_2}$, for example, is 79.29% (fired) of its maximum normalized value. And the whole procedure can be interpreted as a process of inference. The result can be "When the population is initially triggering, the rate of crime is increasing, the poverty is more serious, and the economic condition (volume of economic) may be improved; the population is decreasing gradually at the same time".

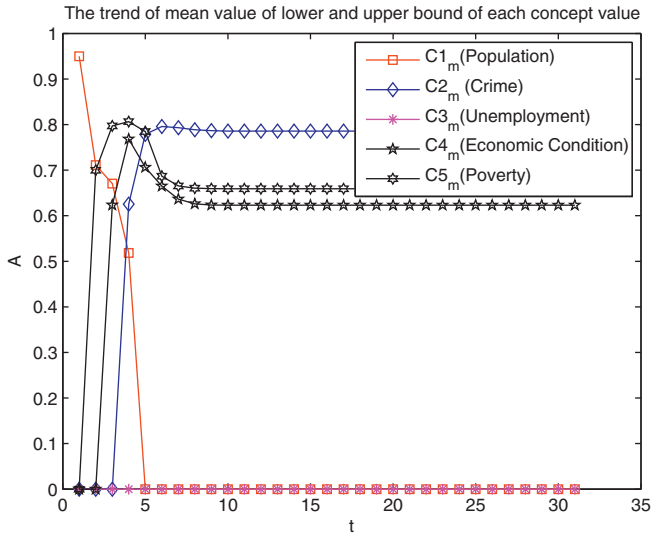

Fig. 11. Results of ECM simulations (Trend of mean value of each concept value).

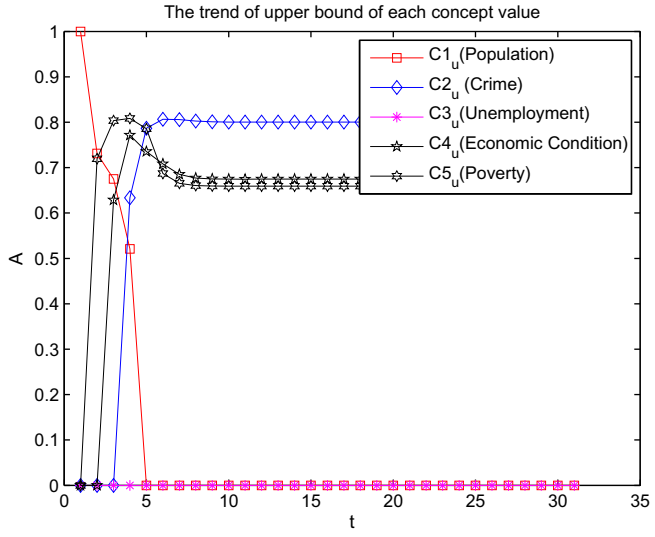

Fig. 12. Results of ECM simulations (Trend of upper bound of each concept value).

Table 6
Equilibrium values of the concepts.

Item	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	$\widetilde{C_4}$	$\widetilde{C_5}$
A^0	[0.9000, 0.1000] **0.9500**	[0.0000, 0.0000] **0.0000**	[0.0000, 0.0000] **0.0000**	[0.0000, 0.0000] **0.0000**	[0.0000, 0.0000] **0.0000**
A^1	[0.7109, 0.7311] **0.9500**	[0.0000, 0.0000] **0.0000**	[0.0000, 0.0000] **0.0000**	[0.0000, 0.0000] **0.0000**	[0.6997, 0.7195] **0.0000**
	... ...	... ...	... ...	... ...	... ...
A^{11}	[0.0000, 0.0000] **0.0000**	[0.7856, 0.8002] **0.7929**	[0.0000, 0.0000] **0.0000**	[0.6230, 0.6746] **0.6488**	[0.6591, 0.6591] **0.6591**
A^{12}	[0.0000, 0.0000] **0.0000**	[0.7856, 0.8002] **0.7929**	[0.0000, 0.0000] **0.0000**	[0.6230, 0.6746] **0.6488**	[0.6591, 0.6591] **0.6591**

7. Conclusions

Evidential cognitive maps (ECMs) are uncertain-graph structures for representing causal reasoning. They can be considered as the exploration of cognitive maps (CMs) and fuzzy cognitive maps (FCMs). ECMs can not only deal with the uncertain information but can also handle the fuzzy information with the advantage of evidence theory, and can be used in many applications involving decision making and uncertain reasoning. The framework of ECMs is developed in this paper and a simple application is shown to illustrate the implementation. Future work needs to enhance the learning ability of ECMs to handle problems in recognition and classification.

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