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Finding a Basic Feasible Solution for Neutrosophic Linear Programming Models: Case Studies, Analysis, and Improvements

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Abstract: Since the inception of operations research, linear programming has received the attention of researchers in this field due to the many areas of its use. The focus was on the methods used to find the optimal solution for linear models. The direct simplex method, with its three basic stages, begins by writing the linear model in standard form and then finding a basic solution that is improved according to the simplex steps until We get the optimal solution, but we encounter many linear models that do not give us a basic solution after we put it in a standard form, and here we need to solve a rule through which we reach the optimal solution. For these models, researchers and scholars in the field of operations research introduced the simplex method with an artificial basis, which helped to Find the optimal solution for linear models, given the importance of this method and as a complement to the previous research we presented using the concepts of neutrosophic science. In this research, we will reformulate the simplex algorithm with an artificial basis using concepts of neutrosophic science.

Keywords: Linear Programming; Simplex Method; Neutrosophic Science; Simplex Neutrosophic Method; Artificial Variable.

1. Introduction

The great scientific development that our contemporary world has witnessed has led to the emergence of what is called operations research. This name refers to the group of scientific methods used in analyzing problems and searching for optimal solutions. Operations research is considered one of the modern sciences whose applications have achieved wide success in various fields of life. One of the methods of operations research is the linear programming method that allows us to model, analyze, and solve a wide range of issues that have resulted from the great scientific development that our contemporary world is witnessing [1]. In all previous studies, we have reached the optimal solution for solvable models, and this solution has a specific value resulting from specific data provided by the study. The field studies that were conducted are linked to the conditions that existed, but the reality of the situation indicates that the conditions surrounding the work environment are not fixed and the future cannot be predicted. These specific values for profits and available resources are subject to instantaneous change. Out of interest in keeping pace with scientific development, we have in this research reformulated one of the most important methods used to find the optimal solution for linear models: the simplex method with an artificial basis using the concepts of neutrosophic science, the science that has proven its ability to provide the best solution in many fields of science. Therefore, researchers' interest has focused on providing studies and research in various fields according to the concepts of this science [2-6].

The purpose of solving linear models is to choose the optimal solution from the set of acceptable solutions. This is done based on a base solution that is improved using the direct simplex algorithm that was presented according to the concepts of neutrosophic science in the research [7]. It consists of three basic stages:

(i). The stage of converting the imposed model into an equivalent systematic form [8].
(ii). The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.
(iii). The stage of searching for the ideal solution requires from among the non-negative basic solutions [7].

Therefore, the process of searching for the optimal solution does not begin until after obtaining a base solution, but in many linear models we face great difficulty in obtaining the base solution, so the simplex method with an artificial base was proposed, where a base is formed consisting of a set of artificial variables that are not negativity is added to constraints that do not contain a basic variable, thus obtaining the basic solution. Then we improve it using the direct simplex algorithm until we obtain the optimal solution. In this research, we will reformulate the simplex algorithm with an artificial basis to find the optimal solution for linear models for which it is difficult to obtain a basic solution, using the concepts of neutrosophic.

2. Problem statement

Find the optimal solution for the following neutrosophic linear model:

$$Max\ Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n + NC_0$$

Constraints:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m \\ x_1, x_2, \dots, x_n \geq 0 \end{gathered}$$

where $NC_j = C_j \pm \varepsilon_j$, $Nb_i = b_i \pm \delta_i$, a_{ij}, $j = 1,2,\dots,n$, $i = 1,2,\dots,m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables. It is worth mentioning that the coefficients subscribed by the index N are of neutrosophic values. The objective function coefficients $NC_1, NC_2, \dots, NC_n$ have neutrosophic meaning are intervals of possible values: That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2,\dots,n$. Also, we have the values of the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$ are regarded as neutrosophic interval values:
$Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1,2,\dots,m$.

In the previous model, we note that the number of variables is n and the number of constraints is m, and this model is in the standard form.
We move to the second stage, which is to find a basic solution. Here we use the simplex algorithm with an artificial base, which is represented by the following:

(i). From the standard form, we form an artificial base form by adding to the left side of each of the constraint equations a non-negative artificial variable ε_i. Thus, we form a base consisting of the non-negative variables $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$.
(ii). Since the artificial variables are introduced into constraints that were originally linear equations, these variables must take the value of zero so that the linear constraints are not affected.

(iii). Therefore, we must move all of them from the base until they become non-base variables, and to be able to make this transition, we use the direct simplex algorithm.

(iv). We introduce these variables into the objective function with the likes of M (where M is a sufficiently large positive number that is at least greater than any $|Nc_j|$) and preceded by a minus sign (because the objective function is a maximization function) so that we do not transfer them back to the base variables again.

(v). We obtain the following basic form of the neutrosophic linear model:

$$Max\, Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n - M\varepsilon_1 - M\,\varepsilon_2 - \cdots - M\,\varepsilon_m + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$

$$x_j \geq 0\ , \varepsilon_i > 0\ , Nb_i > 0\ ; j = 1,2, \dots, n \text{ and } i = 1,2, \dots, m$$

(vi). After obtaining the basic solution, we use the direct simplex algorithm to improve this solution to reach the optimal solution. Therefore, we arrange the previous information in Table 1.

Table 1. General data of the model.

Basic \ Variables	x_1	x_2	….	x_n	ε_1	ε_2	….	ε_m	b_i
ε_1	a_{11}	a_{12}	….	a_{1n}	1	0	….	0	b_1
ε_2	a_{21}	a_{22}	…	a_{2n}	0	1	0	0	b_2
….	…	…	…	…	…	…	…	…	…
ε_m	a_{m1}	a_{m2}	…	a_{mn}	0	0	…	1	b_m
objective function	NC_1	NC_2	….	NC_n	$-M$	$-M$	…	$-M$	$Z - NC_0$

We get rid of the artificial variables. Here we study the constants b_i corresponding to the artificial variables and choose the largest of them, let it be b_t corresponding to the variable ε_t and we consider its row to be the pivot row. Then we determine the pivot element in it by dividing the elements of the objective function row (elements NC_j) by the elements of the ε_t row and then we take the smallest positive ratio θ where:

$$\theta = \underset{j}{\text{Min}} \left[\frac{NC_j}{a_{tj}} > 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, then the pivot element is a_{ts}, and we exchange the variables x_s and ε_t, According to the direct neutrosophic Simplex algorithm instructions, see [7]. We repeat step (vi) until we get rid of all the artificial variables and obtain a normal base consisting of the basic variables.

After getting rid of the artificial variables, we return to working according to the direct neutrosophic simplex algorithm.

3. Examples

3.1 Example: All constraints are of type equals

Find the ideal solution for the following linear model:

$$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$6x_1 - 3x_2 + 3x_3 = [-12, -9]$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

1. We convert the model to the standard form, multiply the second equation by (-1) and we obtain the following model:
   Find a rule solution for the following neutrosophic linear model:
   $$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$
   Constraints:
   $$8x_1 - x_2 + 4x_3 = [4,6]$$
   $$-6x_1 + 3x_2 - 3x_3 = [9,12]$$
   $$x_1, x_2, x_3 \geq 0$$

2. We add the artificial variables and enter them into the objective function with a capital letter M preceded by a minus sign. Here we take $M = 15$.
   Find a rule solution for the following neutrosophic linear model:
   $$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3 - 15\varepsilon_1 - 15\varepsilon_2$$
   Constraints:
   $$8x_1 - x_2 + 4x_3 = [4,6]$$
   $$-6x_1 + 3x_2 - 3x_3 = [9,12]$$
   $$x_1, x_2, x_3, \varepsilon_1, \varepsilon_2 \geq 0$$

   We arrange the previous information in Table 2.

Table 2. Artificial base.

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	8	-1	4	1	0	[4,6]
ε_2	-6	3	-3	0	1	[9,12]
objective function	-12	[6,9]	3	-15	-15	$Z - 0$

Since the rule is artificial, we study the constants $b_i$and find that the largest of them belong to the group [9,12] corresponding to the variable ε_2. Therefore, we divide the objective function row by the positive elements in the ε_2 row and calculate the index θ, and we find that:

$$\theta = \operatorname*{Min}_{j}\left[\frac{[6,9]}{3}\right] = \frac{[6,9]}{3}$$

Thus, the pivot element is (3) corresponding to x_2. Therefore, we replace x_2 with ε_2, then the variable x_2 becomes a base variable and ε_2 comes out of the base. We perform the necessary calculations and obtain Table 3.

Table 3. The first change table in the base.

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	6	0	3	1	$1/3$	$[\mathbf{7}, \mathbf{10}]$
x_2	-2	1	-1	0	$1/3$	$[\mathbf{3}, \mathbf{4}]$
objective function	$[0,6]$	0	$[9,12]$	-15	$[-18,-17]$	$\boldsymbol{Z} - [\mathbf{18}, \mathbf{36}]$

The artificial variable ε_1 is still present in the base, so we perform another substitution, adopting the pivot line as the line opposite it. To determine the pivot column, we calculate the index θ, we find:

$$\theta = \underset{j}{\mathrm{Min}} \left[\frac{[0,6]}{6}, \frac{[9,12]}{3} \right] \in \frac{[0,6]}{6}$$

Thus, the pivot element is (6) corresponding to x_1, so we move x_1 to the base instead of ε_1, so we get the following Table 4.

Table 4. The second change in the base.

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_1	1	0	$\frac{1}{2}$	$\frac{3}{6}$	$\frac{1}{18}$	$\left[\frac{7}{6}, \frac{10}{6}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
objective function	0	0	9	$[-18,-15]$	$\left[-18, \frac{-50}{3}\right]$	$Z - [18,46]$

From the previous table, we note that the base variables x_1, x_2, and thus we have obtained an initial solution for the linear model, which gives us the following rule solution:

$$\left(x_1 \in \left[\frac{7}{6}, \frac{10}{6}\right], x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 = 0, \varepsilon_1 = 0, \varepsilon_2 = 0 \right)$$

But it is clear from the table that this solution is not the ideal solution because, in the target function line, there is a positive value corresponding to the variable x_3. Therefore, we apply the direct simplex algorithm to improve the basic solution. We obtain the ideal solution from Table 5.

Table 5. The optimal solution for the model.

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_3	2	0	1	1	$\frac{1}{9}$	$\left[\frac{7}{3},\frac{10}{3}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3},\frac{22}{3}\right]$
objective function	-18	0	0	$[-27,-24]$	$\left[-19,\frac{-53}{3}\right]$	$Z-[39,76]$

The optimal solution for the linear model:

$$x_1 = 0, x_2 \in \left[\frac{16}{3},\frac{22}{3}\right], x_3 \in \left[\frac{7}{3},\frac{10}{3}\right], \varepsilon_1 = 0, \varepsilon_2 = 0$$

In this solution, the goal function takes its greatest value, which is:

$$Z \in [39,76]$$

The solution can be verified by substituting the constraints and the objective function statement, we note that the values in the ideal solution of the previous linear model are neutrosophic values.

3.2 Example: Constraints are mixed

Find the ideal solution for the following linear model:

$$Min\,Z = -3x_1 + [8,10]x_2 + [0,6]x_3$$

Constraints:

$$x_1 - 2x_2 + x_3 \le [3,7]$$
$$-4x_1 + x_2 + 2x_3 \ge [9,6]$$
$$2x_1 - x_3 = 1$$
$$x_1, x_2, x_3 \ge 0$$

Converting this model to standard form the problem becomes:

Find the ideal solution for the following linear model:

$$Min\,Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$
$$-4x_1 + x_2 + 2x_3 - y_2 = [9,6]$$
$$2x_1 - x_3 = 1$$
$$x_1, x_2, x_3, y_1, y_2 \ge 0$$

The variable y_1 in the first constraint is a basic variable, and since there are no other basic variables, we add artificial variables to the second and third restrictions and enter them into the objective function in sufficiently positive times because the model is a minimization model, and thus we obtain the following basic form:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2 + 12\varepsilon_1 + 12\varepsilon_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 + \varepsilon_1 = [9,6]$$

$$2x_1 - x_3 + \varepsilon_2 = 1$$

$$x_1, x_2, x_3, y_1, y_2, \varepsilon_1, \varepsilon_2 \geq 0$$

We follow the same steps mentioned in Example 1 to remove the artificial variables from the base and insert the basic variables. After obtaining the base solution, we use the direct simplex method to find the optimal solution.

Important Notes:

- If the row ε_i does not include a positive element and $b_t > 0$, this indicates a conflict of constraints and the problem is unsolvable.
- If we cannot find a positive ratio $\frac{NC_j}{a_{tj}}$, we calculate the largest negative ratio θ' where:

$$\theta' = Max\left[\frac{NC_j}{a_{tj}} < 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, so a_{ts} is the pivot element and it is a positive element.

4. Conclusions

In this study, we presented one of the important methods for finding the optimal solution for neutrosophic linear models, which is the synthetic simplex method that we resort to when we are unable to find a rule solution. We found that the optimal solution that we obtained is neutrosophic values, indeterminate values, perfectly defined, belonging to a field that represents its minimum. The smallest value that the objective function can take and the highest alone represent the highest value of the objective function, which is proportional to the conditions surrounding the system's operating environment, which can be represented by the linear model.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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