Received: 7 September 2019
DOI: 10.1002/mma.6189

SPECIAL ISSUE PAPER

WILEY

Fixed point results for a new metric space

Murat Kirişci[1] | **Necip Şimşek**[2] | **Mahmut Akyiğit**[3]

[1]Department of Mathematical Education, Istanbul University-Cerrahpaşa, Istanbul, Turkey
[2]Department of Mathematics, Istanbul Commerce University, Istanbul, Turkey
[3]Department of Mathematics, Sakarya University, Sakarya, Turkey

Correspondence
Murat Kirişci, Department of Mathematical Education, Istanbul University-Cerrahpaşa, Istanbul, Turkey.
Email: mkirisci@hotmail.com

Communicated by: M. Tosun

In this paper, we introduce the neutrosophic contractive and neutrosophic mapping. We establish some results on fixed points of a neutrosophic mapping.

KEYWORDS
complete neutrosophic metric space, fixed point, neutrosophic contraction

MSC CLASSIFICATION
54H25; 03E72; 47H10

1 | INTRODUCTION

Fuzzy sets (FSs) put forward by Zadeh[1] has influenced deeply all the scientific fields since the publication of the paper. It is seen that this concept, which is very important for real-life situations, had not enough solution to some problems in time. New quests for such problems have been coming up. Atanassov[2] initiated intuitionistic fuzzy sets (IFSs) for such cases. Neutrosophic set (NS) is a new version of the idea of the classical set that is defined by Smarandache.[3] Examples of other generalizations are FS[1] interval-valued FS,[4] IFS,[2] interval-valued IFS,[5] the sets paraconsistent, dialetheist, paradoxist, and tautological,[6] Pythagorean fuzzy sets.[7]

Using the concepts probabilistic metric space and fuzzy, fuzzy metric space (FMS) is introduced in Kramosil and Michalek.[8] Kaleva and Seikkala[9] have defined the FMS as a distance between two points to be a non-negative fuzzy number. In George and Veeramani,[10] some basic properties of FMS were studied, and the Baire category theorem for FMS were proved. Further, some properties such as separability and countability are given, and uniform limit theorem is proved in George and Veramani. [11] Afterward, FMS has used in the applied sciences such as fixed point theory, image and signal processing, medical imaging, and decision-making. After defined of the intuitionistic fuzzy set (IFS), it was used in all areas where FS theory was studied. Park[12] defined IF metric space (IFMS), which is a generalization of FMSs. Park used George and Veeramani's[10] idea of applying t-norm and t-conorm to the FMS meanwhile defining IFMS and studying its basic features.

Fixed point theorem for fuzzy contraction mappings is initiated by Heilpern.[13] Bose and Sahani[14] extended the Heilperns' study. Alaca et al[15] gave fixed point theorems related to IFMSs. Fixed point results for fuzzy metric spaces and IFMSs are studied by many researchers.[16-20] For more detail on fixed point theory and related concepts, see previous studies[17-22] as examples.

In this paper, fixed point results for neutrosophic metric spaces (NMSs) are given.

2 | PRELIMINARIES

Some definitions related to the fuzziness, intuitionistic fuzziness, and neutrosophy are given as follows:

The fuzzy subset F of $\mathbb{R}$ is said to be a fuzzy number (FN). The FN is a mapping $F : \mathbb{R} \to [0, 1]$ that corresponds to each real number a to the degree of membership $F(a)$.

Let F is a FN. Then it is known that[23]

- If $F(a_0) = 1$, for $a_0 \in \mathbb{R}$, F is said to be normal,
- If for each $\mu > 0$, $F^{-1}\{[0, \tau + \mu)\}$ is open in the usual topology $\forall \tau \in [0, 1)$, F is said to be upper semi continuous,
- The set $[F]^\tau = \{a \in \mathbb{R} : F(a) \geq \tau\}$, $\tau \in [0, 1]$ is called τ-cuts of F.

Choose non-empty set F. An IFS in F is an object U defined by

$$U = \{< a, G_U(a), Y_U(a) >: a \in F\},$$

where $G_U(a) : F \to [0, 1]$ and $Y_U(a) : F \to [0, 1]$ are functions for all $a \in F$ such that $0 \leq G_U(a) + Y_U(a) \leq 1$.[2] Let U be an IFN. Then

- an IF subset of the $\mathbb{R}$,
- If $G_U(a_0) = 1$ and $Y_U(a_0) = 0$, for $a_0 \in \mathbb{R}$, normal,
- If $G_U(\lambda a_1 + (1 - \lambda)a_2) \geq \min(G_U(a_1), G_U(a_2))$, $\forall a_1, a_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$, then the membership function(MF) $G_U(a)$ is called convex,
- If $Y_U(\lambda a_1 + (1 - \lambda)a_2) \geq \min(Y_U(a_1), Y_U(a_2))$, $\forall a_1, a_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$, then the nonmembership function(NMF)$Y_U(a)$ is concave,
- G_U is upper semicontinuous and Y_U is lower semicontinuous,
- $suppU = cl(\{a \in F : Y_U(a) < 1\})$ is bounded.

An IFS $U = \{< a, G_U(a), Y_U(a) >: a \in F\}$ such that $G_U(a)$ and $1 - Y_U(a)$ are FNs, where $(1 - Y_U)(a) = 1 - Y_U(a)$, and $G_U(a) + Y_U(a) \leq 1$ is called an IFN.

Let us consider that F is a space of points (objects). Denote the $G_U(a)$ is a truth-MF, $B_U(a)$ is an indeterminacy-MF and $Y_U(a)$ is a falsity-MF, where U is a set in F with $a \in F$. Then, if we take $I =]0^-, 1^+[$

$$G_U(a) : F \to I,$$
$$B_U(a) : F \to I,$$
$$Y_U(a) : F \to I.$$

There is no restriction on the sum of $G_U(a)$, $B_U(a)$, and $Y_U(a)$. Therefore,

$$0^- \leq \sup \quad G_U(a) + \sup \quad B_U(a) + \sup Y_U(a) \leq 3^+.$$

The set U that consist of with $G_U(a)$, $B_U(a)$ and $Y_U(a)$ in F is called a neutrosophic sets (NS) and can be denoted by

$$U = \{< a, (G_U(a), B_U(a), Y_U(a)) >: a \in F, G_U(a), B_U(a), Y_U(a) \in I\}. \tag{1}$$

Clearly, NS is an enhancement of $[0, 1]$ of IFSs.

An NS U is included in another NS V, $(U \subseteq V)$, if and only if,

$$\inf G_U(a) \leq \inf G_V(a), \quad \sup G_U(a) \leq \sup G_V(a),$$
$$\inf B_U(a) \geq \inf B_V(a), \quad \sup B_U(a) \geq \sup B_V(a),$$
$$\inf Y_U(a) \geq \inf Y_V(a), \quad \sup Y_U(a) \geq \sup Y_V(a).$$

for any $a \in F$. However, NSs are inconvenient to practice in real problems. To cope with this inconvenient situation, Wang et al[24] customized NS's definition and single-valued NSs (SVNSs) suggested.

To cope with this inconvenient situation, Wang et al[24] customized NS's definition and single-valued NSs suggested. Ye[25] described the notion of simplified NSs (SNSs), which may be characterized by three real numbers in the $[0, 1]$. At the same time, the simplified NSs' operations may be impractical, in some cases.[25] Hence, the operations and comparison way between SNSs and the aggregation operators for SNSs are redefined in Peng et al.[26]

According to the Ye,[25] a simplification of an NS U, in (1), is

$$U = \{< a, (G_U(a), B_U(a), Y_U(a)) >: a \in F\},$$

which is called a simplified NS. Especially, if F has only one element $< G_U(a), B_U(a), Y_U(a) >$ is said to be an simplified neutrosophic number (SNN). Expressly, we may see simplified NSs as a subclass of NSs.

An simplified NS U is comprised in another simplified NS V ($U \subseteq V$), iff $G_U(a) \leq G_V(a)$, $B_U(a) \geq B_V(a)$ and $Y_U(a) \geq Y_V(a)$ for any $a \in F$. Then the following operations are given by Ye[25]:

$$\begin{aligned}
U + V &= \langle G_U(a) + G_V(a) - G_U(a).G_V(a), B_U(a) + B_V(a) - B_U(a).B_V(a), Y_U(a) + Y_V(a) - Y_U(a).Y_V(a)\rangle,\\
U.V &= \langle G_U(a).G_V(a), B_U(a).B_V(a), Y_U(a).Y_V(a)\rangle,\\
\alpha.U &= \langle 1-(1-G_U(a))^\alpha, 1-(1-B_U(a))^\alpha, 1-(1-Y_U(a))^\alpha\rangle, \quad for \quad \alpha > 0,\\
U^\alpha &= \langle G_U^\alpha(a), B_U^\alpha(a), Y_U^\alpha(a)\rangle, \quad for \quad \alpha > 0.
\end{aligned}$$

Triangular norms (t-norms) (TN) were initiated by Menger.[27] In the problem of computing the distance between two elements in space, Menger offered using probability distributions instead of using numbers for distance. TNs are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conorms (t-conorms) (TC) know as dual operations of TNs. TNs and TCs are very significant for fuzzy operations (intersections and unions).

Definition 1. Give an operation $\circ : [0,1] \times [0,1] \to [0,1]$. If the operation $\circ$ is satisfying the following conditions, then it is called that the operation $\circ$ is continuous TN(CTN): For $s, t, u, v \in [0,1]$,

i. $s \circ 1 = s$,
ii. If $s \leq u$ and $t \leq v$, then $s \circ t \leq u \circ v$,
iii. $\circ$ is continuous,
iv. $\circ$ is commutative and associative.

Definition 2. Give an operation $\bullet : [0,1] \times [0,1] \to [0,1]$. If the operation $\bullet$ is satisfying the following conditions, then it is called that the operation $\bullet$ is continuous TC(CTC):

i. $s \bullet 0 = s$,
ii. If $s \leq u$ and $t \leq v$, then $s \bullet t \leq u \bullet v$,
iii. $\bullet$ is continuous,
iv. $\bullet$ is commutative and associative.

From above definitions, we note that if we choose $0 < \varepsilon_1, \varepsilon_2 < 1$ for $\varepsilon_1 > \varepsilon_2$, then there exist $0 < \varepsilon_3, \varepsilon_4 < 0, 1$ such that $\varepsilon_1 \circ \varepsilon_3 \geq \varepsilon_2$, $\varepsilon_1 \geq \varepsilon_4 \bullet \varepsilon_2$. Further, if we choose $\varepsilon_5 \in (0,1)$, then there exist $\varepsilon_6, \varepsilon_7 \in (0,1)$ such that $\varepsilon_6 \circ \varepsilon_6 \geq \varepsilon_5$ and $\varepsilon_7 \bullet \varepsilon_7 \leq \varepsilon_5$.

Remark 1. [12] Take $\circ$ and $\bullet$ are a CTN and CTC, respectively. For $p, s, t, u, v \in [0,1]$,

a. If $s > t$, then there are u, v such that $s \circ u \geq t$ and $s \geq t \bullet v$.
b. There are p, t such that $t \circ t \geq s$ and $s \geq p \bullet p$.

Definition 3. [28] Take F be an arbitrary set, $V = \mathcal{N} = \{< a, G(a), B(a), Y(a) >: a \in F\}$ be a NS such that $\mathcal{N} : F \times F \times \mathbb{R}^+ \to [0,1]$. Let $\circ$ and $\bullet$ show the continuous TN and continuous TC, respectively. The four-tuple $(F, \mathcal{N}, \circ, \bullet)$ is called neutrosophic metric space (NMS) when the following conditions are satisfied. $\forall a, b, c \in F$,

i. $0 \leq G(a,b,\lambda) \leq 1, 0 \leq B(a,b,\lambda) \leq 1, 0 \leq Y(a,b,\lambda) \leq 1, \forall \lambda \in \mathbb{R}^+$,
ii. $G(a,b,\lambda) + B(a,b,\lambda) + Y(a,b,\lambda) \leq 3$, (for $\lambda \in \mathbb{R}^+$),
iii. $G(a,b,\lambda) = 1$, (for $\lambda > 0$) if and only if $a = b$,
iv. $G(a,b,\lambda) = G(b,a,\lambda)$, (for $\lambda > 0$),
v. $G(a,b,\lambda) \circ G(b,c,\mu) \leq G(a,c,\lambda+\mu)$, ($\forall \lambda, \mu > 0$),
vi. $G(a,b,.) : [0,\infty) \to [0,1]$ is continuous,

vii. $lim_{\lambda \to \infty} G(a, b, \lambda) = 1, (\forall \lambda > 0)$,
viii. $B(a, b, \lambda) = 0$, (for $\lambda > 0$) if and only if $a = b$,
ix. $B(a, b, \lambda) = B(b, a, \lambda)$, (for $\lambda > 0$),
x. $B(a, b, \lambda) \bullet B(b, c, \mu) \geq B(a, c, \lambda + \mu), (\forall \lambda, \mu > 0)$,
xi. $B(a, b, .) : [0, \infty) \to [0, 1]$ is continuous,
xii. $lim_{\lambda \to \infty} B(a, b, \lambda) = 0, (\forall \lambda > 0)$,
xiii. $Y(a, b, \lambda) = 0$, (for $\lambda > 0$) if and only if $a = b$,
xiv. $Y(a, b, \lambda) = Y(b, a, \lambda), (\forall \lambda > 0)$,
xv. $Y(a, b, \lambda) \bullet Y(b, c, \mu) \geq Y(a, c, \lambda + \mu), (\forall \lambda, \mu > 0)$,
xvi. $Y(a, b, .) : [0, \infty) \to [0, 1]$ is continuous,
xvii. $lim_{\lambda \to \infty} Y(a, b, \lambda) = 0$, (for $\lambda > 0$),
xviii. If $\lambda \leq 0$, then $G(a, b, \lambda) = 0$, $B(a, b, \lambda) = 1$ and $Y(a, b, \lambda) = 1$.

Then, $\mathcal{N} = (G, B, Y)$ is called neutrosophic metric (NM) on F.

The functions $G(a, b, \lambda)$, $B(a, b, \lambda)$, and $Y(a, b, \lambda)$ denote the degree of nearness, the degree of neutralness, and the degree of non-nearness between a and b with respect to λ, respectively.

Definition 4. [28] Give V be a NMS, $0 < \varepsilon < 1$, $\lambda > 0$ and $a \in F$. The set $O(a, \varepsilon, \lambda) = \{b \in F : G(a, b, \lambda) > 1 - \varepsilon, \quad B(a, b, \lambda) < \varepsilon, \quad Y(a, b, \lambda) < \varepsilon\}$ is said to be the open ball (OB) (center a and radius ε with respect to λ).

Lemma 1. [28] *Every OB $O(a, \varepsilon, \lambda)$ is an open set (OS).*

3 | FIXED POINT RESULTS

Definition 5. Let F be a set. A non-negative real-valued function h on $F \times F$ is called as a quasi-metric on F if it satisfies the following axioms:

i. $h(a, b) = h(b, a) = 0$ if and only if $a = b$,
ii. $h(a, b) \leq h(a, c) + h(c, b)$,

for all $a, b, c \in F$.

From this definition, we can understand: It is possible $h(a, b) \neq h(b, a)$ for some $a, b \in F$.

A quasi-metric is a distance function that satisfies the triangle inequality but is not symmetric in general. Quasi-metrics are a subject of comprehensive investigation in both pure and applied mathematics in areas such as in functional analysis, topology, and computer science.

Proposition 1. *Let V be the NMS. For any $\varepsilon \in (0, 1]$, define $h : F \times F \to R^+$ as follows:*

$$h_\varepsilon(a, b) = \inf\{\lambda > 0 : G(a, b, \lambda) > 1 - \varepsilon, \quad B(a, b, \lambda) < \varepsilon, \quad Y(a, b, \lambda) < \varepsilon\} \tag{2}$$

Then,

i. $(F, h_\varepsilon : \varepsilon \in (0, 1])$ is a generating space of quasi-metric family.
ii. The topology $\tau_{\mathcal{N}}$ on $(F, h_\varepsilon : \varepsilon \in (0, 1])$ coincides with the $\mathcal{N}-$topology on V, that is, h_ε is a compatible symmetric for $\tau_{\mathcal{N}}$.

Proof. Firstly, we prove that (i.). It can be easily seen that h_ε suffices the conditions of the definition of quasi-metric. Let us show the condition (ii.) of quasi-metric. We know that the operations $\circ, \bullet$ are continuous. If we consider Remark 1, for any given $\varepsilon \in (0, 1)$, we can take $\varepsilon^* \in (0, \varepsilon)$ such that $(1 - \varepsilon^*) \circ (1 - \varepsilon^*) > 1 - \varepsilon$ and $\varepsilon^* \bullet \varepsilon^* < \varepsilon$. Given $h_\varepsilon(a, b) = x$ and $h_\varepsilon(b, c) = y$. From (2),

$$G(a, b, x + \lambda) > 1 - \varepsilon^*, B(a, b, x + \lambda) < \varepsilon^*, \quad Y(a, b, x + \lambda) < \varepsilon^*$$
$$G(a, c, y + \lambda) > 1 - \varepsilon^*, B(a, c, y + \lambda) < \varepsilon^*, \quad Y(a, c, y + \lambda) < \varepsilon^*.$$

From here,

$$G(a,c,x+y+2\lambda) \geq G(a,b,x+\lambda) \circ G(b,c,y+\lambda) > (1-\varepsilon^*) \circ (1-\varepsilon^*) > 1-\varepsilon,$$
$$B(a,c,x+y+2\lambda) \leq B(a,b,x+\lambda) \bullet B(b,c,y+\lambda) < \varepsilon^* \bullet \varepsilon^* < \varepsilon,$$
$$Y(a,c,x+y+2\lambda) \leq Y(a,b,x+\lambda) \bullet Y(b,c,y+\lambda) < \varepsilon^* \bullet \varepsilon^* < \varepsilon.$$

Therefore, we have $h_\varepsilon(a,c) \leq x+y+2\lambda = h_\varepsilon(a,b)+h_\varepsilon(b,c)+2\lambda$. Since $\lambda > 0$ is arbitrary, $h_\varepsilon(a,c) \leq h_\varepsilon(a,b)+h_\varepsilon(b,c)$.

Now, we prove that (ii.). We must show that

$$h_\varepsilon(a,c) < \lambda \quad \Leftrightarrow \quad G(a,b,\lambda) > 1-\varepsilon, \quad B(a,b,\lambda) < \varepsilon, \quad Y(a,b,\lambda) < \varepsilon,$$

for any $\lambda > 0$ and $\varepsilon \in (0,1)$. If $h_\varepsilon(a,b) < \lambda$, then $G(a,b,\lambda) > 1-\varepsilon, \quad B(a,b,\lambda) < \varepsilon, \quad Y(a,b,\lambda) < \varepsilon$ from (2).

Conversely, consider $G(a,b,\lambda) > 1-\varepsilon, \quad B(a,b,\lambda) < \varepsilon, \quad Y(a,b,\lambda) < \varepsilon$. Since the functions G, B, Y are continuous from the definition, then there exists an $\eta > 0$ such that $G(a,b,\lambda-\eta) > 1-\varepsilon, \quad B(a,b,\lambda-\eta) < \varepsilon, \quad Y(a,b,\lambda-\eta) < \varepsilon$. From here, we have $h_\varepsilon(a,b) \leq \lambda-\eta < \lambda$. □

Definition 6. Let V be an NMS. The mapping $f : F \to F$ is called neutrosophic contraction (NC) if there exists $k \in (0,1)$ such that

$$\frac{1}{G(f(a),f(b),\lambda)} - 1 \leq k(\frac{1}{G(a,b,\lambda)-1}), \quad B(f(a),f(b),\lambda) \leq kB(a,b,\lambda), \quad Y(f(a),f(b),\lambda) \leq kY(a,b,\lambda),$$

for each $a, b \in F$ and $\lambda > 0$.

Definition 7. Let V be a NMS and let $f : F \to F$ be a NC mapping. Then there exists $c \in F$ such that $c = f(c)$. That is, c is called neutrosophic fixed point (NFP) of f.

Generally, we claim that the contractions have fixed point. If all contractions (including NC) have fixed points, then we can easily say that f^2 should have a fixed point. In below proposition, we will show that if f^n is a NC then, f^n has fixed point.

Proposition 2. *Suppose that f is a NC. Then f^n is also a NC. Furthermore, if k is the constant for f, then k^n is the constant for f^n.*

Proof. We will use the induction for proof. We take $n = 2$. If f is a NC, then, it is clear that

$$h(f(x), f(y)) \leq k \times h(x,y) \tag{3}$$

for $k \in (0,1)$. If we apply f to both of sides of inequality (3), we have

$$h(f^2(a), f^2(b)) \leq k \times h(f(a), f(b)). \tag{4}$$

From (3), we can write $k \times h(f(a), f(b)) \leq k^2 \times h(a,b)$. Thus, if we combine the (3) and (4) and last inequality, we have

$$h(f^2(a), f^2(b)) \leq k \times h(f(a), f(b)) \leq k^2 \times h(a,b)$$

which leads us to the fact that f^2 is a NC. If we consider that f^n is a NC, then we can say that f^{n+1} is a NC with above processes.

We must prove that the constant for f^n is k^n. Consider $h(f^n(a), f^n(b)) \leq k^n \times h(a,b)$. As similar to the above process, we can apply f to both of sides of this inequality, we get

$$h(f^{n+1}(a), f^{n+1}(b)) \leq k^{n+1} \times h(f(a), f(b)) \leq k^{n+1} \times h(a,b).$$

From this inequality, $h(f^{n+1}(a), f^{n+1}(b)) \leq k^{n+1} \times h(a,b)$. Therefore, we understand that the theorem is true for all n. □

Remark 2. From Proposition 2, we can say that each f^n has the same fixed point. Because, if we take $f(a) = a$, then $f^2 = f(f(a)) = f(a) = a$ and by induction, $f^n(a) = a$.

Proposition 3. *Let f be a NC and $a \in F$. $f[O(a,\varepsilon,\lambda)] \subset O(a,\varepsilon,\lambda)$, for large enough values of ε.*

Proof. Let $b \in O(a,\varepsilon,\lambda)$. We must find ε such that $f(b) \in O(a,\varepsilon,\lambda)$ and so $h(a, f(b)) < \varepsilon$. We can write

$$h(a, f(b)) \leq h(a, f(a)) + h(f(a), f(b)).$$

Further, $h(f(a), f(b)) \leq k \times h(a, b)$ and $h(a, b) \leq \varepsilon$. Therefore,

$$h(f(a), f(b)) \leq k \times \varepsilon. \tag{5}$$

From this inequalities, we have $h(a, f(b)) \leq h(a, f(a)) + k \times \varepsilon$. We can choose ε so that $h(a, f(a)) + k \times \varepsilon$. Then, for any $b \in O(a,\varepsilon,\lambda)$, $h(x, f(b)) < \varepsilon$ and $f(b) \in O(a,\varepsilon,\lambda)$. □

Remark 3. From Proposition 3 and the definition neutrosophic OB, if the inclusion $f[O(a,\varepsilon,\lambda)] \subset O(a,\varepsilon,\lambda)$ is hold, then the inclusion also $\overline{f[O(a,\varepsilon,\lambda)]} \subset \overline{O(a,\varepsilon,\lambda)}$ is hold.

Proposition 4. *The inclusion $f^n[O(a,\varepsilon,\lambda)] \subset O(f^n(a),\varepsilon^*,\lambda)$ is hold for all n, where $\varepsilon^* = k^n \times \varepsilon$.*

The proof of this proposition is similar to Proposition 2.

Remark 4. It is fact that if the inclusion $f^n[O(a,\varepsilon,\lambda)] \subset O(f^n(a),\varepsilon^*,\lambda)$ is hold, then the inclusion also $\overline{f^n[O(a,\varepsilon,\lambda)]} \subset \overline{O(f^n(a),\varepsilon^*,\lambda)}$ is hold.

Theorem 1. *Let V be a complete NMS. Let $f : F \to F$ be a NC mapping. Then, f has a unique NFP.*

Theorem 1 is a consequence of Theorem 3.6 in Rafi and Noorani.[29] Hence, using the concept of neutrosophy, Theorem 1 is proved as similar Theorem 3.6 in Rafi and Noorani.[29]

For the alternative proof, we can use $\overline{f^n(O(a,\varepsilon,\lambda))}$. That is, if we choose $b \in \overline{f^n(O(a,\varepsilon,\lambda))}$, we can see that $f(b) \in \overline{f^n(O(a,\varepsilon,\lambda))}$. Therefore, the distance between b and $f(b)$ is ϵ and so $f(b) = b$. Thus b is a fixed point.

4 | CONCLUSION

The purpose of this paper is to apply the NMS that is defined by Kirisci and Simsek.[28] NC mapping is defined. After the properties related to NC are proved, fixed point theorem is given.

ACKNOWLEDGEMENTS

We have benefited much from the constructive reports of the anonymous reviewers, and we are grateful for their valuable comments on the first draft of this paper, which improved the presentation and readability.

CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

AUTHOR CONTRIBUTIONS

All authors contributed equally to the manuscript. All authors approved the final version of this manuscript.

ORCID

Murat Kirişci https://orcid.org/0000-0003-4938-5207
Necip Şimşek https://orcid.org/0000-0003-3061-5770
Mahmut Akyiğit https://orcid.org/0000-0002-8398-365X

REFERENCES

1. Zadeh LA. Fuzzy sets. *Inf Comp*. 1965;8:338-353.
2. Atanassov K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst*. 1986;20:87-96.

3. Smarandache F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. *Inter J Pure Appl Math*. 2005;24:287-297.
4. Turksen I. Interval valued fuzzy sets based on normal forms. *Fuzzy Sets Syst*. 1996;20:191-210.
5. Atanassov K, Gargov G. Interval valued intuitionistic fuzzy sets. *Inf Comp*. 1989;31:343-349.
6. Smarandache F. *A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability and statistics*. Xiquan: Phoenix; 2003.
7. Yager RR. Pythagorean fuzzy subsets. In: 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS) IEEE; 2013; Edmonton, Canada.
8. Kramosil I, Michalek J. Fuzzy metric and statistical metric spaces. *Kybernetika*. 1975;11:336-344.
9. Kaleva O, Seikkala S. On fuzzy metric spaces. *Fuzzy Sets Syst*. 1984;12:215-229.
10. George A, Veeramani P. On some results in fuzzy metric spaces. *Fuzzy Sets Syst*. 1994;64:395-399.
11. George A, Veeramani P. On some results of analysis for fuzzy metric spaces. *Fuzzy Sets Syst*. 1997;90:365-368.
12. Park JH. Intuitionistic fuzzy metric spaces. *Chaos Solitons Fractals*. 2004;22:1039-1046.
13. Heilpern S. Fuzzy mappings and fixed point theorems. *J Math Anal Appl*. 1981;83:566-569.
14. Bose RK, Sahani D. Fuzzy mappings and fixed point theorems. *Fuzzy Sets Syst*. 1987;231:53-58.
15. Alaca C, Turkoglu D, Yildiz C. Fixed points in intuitionistic fuzzy metric spaces. *Chaos Solitons Fractals*. 2006;29:1073-1078.
16. Gregori V, Sapane A. On fixed-point theorems in fuzzy metric spaces. *Fuzzy Sets Syst*. 2002;125:245-252.
17. Imdad M, Ali J. Some common fixed point theorems in fuzzy metric spaces. *Math Commun*. 2006;11:153-163.
18. Mihet D. A banach contraction theorem in fuzzy metric spaces. *Fuzzy Sets Syst*. 2004;44:431-439.
19. Turkoglu D, Alaca C, Cho YJ, Yildiz C. Common fixed point theorems in intuitionistic fuzzy metric spaces. *J Appl Math Comput*. 2006;22:411-424.
20. Hussain N, Khaleghizadeh S, Salimi P, Abdou AAN. A new approach to fixed point results in triangular intuitionistic fuzzy metric spaces. *Abstr Appl Anal*. 2014;2014:1-16.
21. Kirişci M. Multiplicative generalized metric spaces and fixed point theorems. *J Math Anal*. 2017b;8:212-224.
22. Mursaleen M, Mouhiddine SA, Agarwal RP. Coupled fixed point theorems for alpha-psi contractive type mappings in partially ordered metric spaces. *Fixed Point Theory Appl*. 2012;2012:228.
23. Kirişci M. Integrated and differentiated spaces of triangular fuzzy numbers. *Fas Math*. 2017a;59:75-89.
24. Wang H, Smarandache F, Zhang YQ, Sunderraman R. Single valued neutrosophic sets. *Fuzzy Sets Syst*. 1996;4:410-413.
25. Ye J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J Intell Fuzzy Syst*. 2014;26:2459-2466.
26. Peng JJ, Wang JQ, Wang J, Zhang HY, Chen XH. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *Int J Syst Sci*. 2016;47:2342-2358.
27. Menger KM. Statistical metrics. *Proc Nat Acad Sci*. 1942;28:535-537.
28. Kirişci M, Simsek N. Neutrosophic metric spaces. arXiv:1907.00798; 2019.
29. Rafi M, Noorani SM. Fixed point theorem on intuitionistic fuzzy metric spaces. *Iranian J Fuzzy Syst*. 2006;3:23-29.
30. Ilkhan M, Kara EE. On statistical convergence in quasi-metric spaces. *Demonstr Math*. 2019;52.

How to cite this article: Kirişci M, Şimşek N, Akyiğit M. Fixed point results for a new metric space. *Math Meth Appl Sci*. 2020;1-7. https://doi.org/10.1002/mma.6189