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Forecasting Model Based on Neutrosophic Logical Relationship and Jaccard Similarity

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Abstract: The daily fluctuation trends of a stock market are illustrated by three statuses: up, equal, and down. These can be represented by a neutrosophic set which consists of three functions—truth-membership, indeterminacy-membership, and falsity-membership. In this paper, we propose a novel forecasting model based on neutrosophic set theory and the fuzzy logical relationships between the status of historical and current values. Firstly, the original time series of the stock market is converted to a fluctuation time series by comparing each piece of data with that of the previous day. The fluctuation time series is then fuzzified into a fuzzy-fluctuation time series in terms of the pre-defined up, equal, and down intervals. Next, the fuzzy logical relationships can be expressed by two neutrosophic sets according to the probabilities of different statuses for each current value and a certain range of corresponding histories. Finally, based on the neutrosophic logical relationships and the status of history, a Jaccard similarity measure is employed to find the most proper logical rule to forecast its future. The authentic Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) time series datasets are used as an example to illustrate the forecasting procedure and performance comparisons. The experimental results show that the proposed method can successfully forecast the stock market and other similar kinds of time series. We also apply the proposed method to forecast the Shanghai Stock Exchange Composite Index (SHSECI) to verify its effectiveness and universality.

Keywords: fuzzy time series; forecasting; fuzzy logical relationship; neutrosophic set; Jaccard similarity

1. Introduction

It is well known that there is a statistical long-range dependency between current values and historical values at different times in certain time series [1]. Therefore, many researchers have developed various models to predict the future of such time series based on historical data sets, for example the regression analysis model [2], the autoregressive moving average (ARIMA) model [3], the autoregressive conditional heteroscedasticity (ARCH) model [4], the generalized ARCH (GARCH) model [5], and so on. However, crisp data used in those models are sometimes unavailable as such time series contain many uncertainties. In fact, models that satisfy the constraints precisely can miss the true optimal design within the confines of practical and realistic approximations. Therefore, Song and Chissom proposed the fuzzy time series (FTS) forecasting model [6–8] to predict the future of such nonlinear and complicated problems. In a financial context, FTS approaches have been widely applied to stock index forecasting [9–13]. In order to improve the accuracy of forecasts for stock market indices, some researchers combine fuzzy and non-fuzzy time series with heuristic optimization methods in their forecasting strategies [14]. Other approaches even introduce neural networks and machine learning procedures in order to find forecasting rules from historical time series [15–17].

The major points in FTS models are related to the fuzzifying of original time series, the establishment of fuzzy logical relationships from historical training datasets, and the forecasting and defuzzification of the outputs. Various proposals have been considered to determine the basic steps of the fuzzifying method, such as the effective length of intervals—e.g., determining the optimal interval length based on averages and distribution methods [18], using statistical theory [18–23], the unequal interval length method based on ratios of data [24], or the length determination method based on particle swarm optimization (PSO) techniques [10], etc. To state appropriate fuzzy logical relationships, Yu [25] proposed a weight assignation model, based on the recurrent fuzzy relationships, for each individual relationship. Aladag et al. [26] considered artificial neural networks to be a basic high-order method for the establishment of logical relationships. Fuzzy auto regressive (AR) models and fuzzy auto regressive and moving average (ARMA) models are also widely used to reflect the recurrence and weights of different fuzzy logical relationships [9,10,27–35]. These obtained logical relationships will be used as rules during the forecasting process. However, the proportions of the lagged variables in AR or ARMA models only represent the general best fitness for certain training datasets, without taking into account the differences between individual relationships. Although the weight assignation model considers the differences between individual relationships, it has to deal with special relationships that appear in the testing dataset but never happen in the training dataset. These FTS methods look for point forecasts without taking into account the implicit uncertainty in the ex post forecasts.

For a financial system, if anything, future fluctuation is more important than the indicated number itself. Therefore, the crucial ingredients for financial forecasting are the fluctuation orientations (including up, equal, and down) and to what extent the trends would be realized. Inspired by this, we first changed the original time series into a fluctuation time series for further rule generation. Meanwhile, comparing the three statuses with the concept of the neutrosophic set, the trends and weights of the relationships between historical and current statuses can be represented by the different dimensions of the neutrosophic sets, respectively. The concept of the neutrosophic set was originally proposed from a philosophical point of view by Smarandache [36]. A neutrosophic set is characterized independently by a truth-membership function, an indeterminacy-membership function and a falsity-membership function. Its similarity measure plays a key role in decision-making in uncertain environments. Researchers have proposed various similarity measures and mainly applied them to decision-making—e.g., Jaccard, Dice and Cosine similarity measures [37], distance-based similarity measures [38], entropy measures [39], etc. Although neutrosophic sets have been successfully applied to decision-making [37–42], they have rarely been applied to forecasting problems.

In this paper, we introduce neutrosophic sets to stock market forecasting. We propose a novel forecasting model based on neutrosophic set theory and the fuzzy logical relationships between current and historical statuses. Firstly, the original time series of the stock market is converted to a fluctuation time series by comparing each piece of data with that of the previous day. The fluctuation time series is then fuzzified into a fuzzy-fluctuation time series in terms of the pre-defined up, equal, and down intervals. Next, the fuzzy logical relationships can be expressed by two neutrosophic sets according to the probabilities for different statuses of each current value and a certain range of corresponding histories. Finally, based on the neutrosophic logical relationships and statuses of recent history, the Jaccard similarity measure is employed to find the most proper logical rule with which to forecast its future.

The remaining content of this paper is organized as follows: Section 2 introduces some preliminaries of fuzzy-fluctuation time series and concepts, and the similarity measures of neutrosophic sets. Section 3 describes a novel approach for forecasting based on fuzzy-fluctuation trends and logical relationships. In Section 4, the proposed model is used to forecast the stock market using Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) datasets from 1997 to 2005 and Shanghai Stock Exchange Composite Index (SHSECI) from 2007 to 2015. Conclusions and potential issues for future research are summarized in Section 5.

2. Preliminaries

2.1. Definition of Fuzzy-Fluctuation Time Series (FFTS)

Song and Chissom [6–8] combined fuzzy set theory with time series and defined fuzzy time series. In this section, we extend fuzzy time series to fuzzy-fluctuation time series (FFTS) and propose the related concepts.

Definition 1. *Let* $L = \{l_1, l_2, \ldots, l_g\}$ *be a fuzzy set in the universe of discourse U; it can be defined by its membership function,* $\mu_L : U \to [0,1]$, *where* $\mu_L(u_i)$ *denotes the grade of membership of* u_i, $U = \{u_1, u_2, \ldots u_i, \ldots, u_l\}$.

The fluctuation trends of a stock market can be expressed by a linguistic set $L = \{l_1, l_2, l_3\}$ = {*down, equal, up*}. The element l_i and its subscript i are strictly monotonically increasing [43], so the function can be defined as follows: $f : l_i = f(i)$.

Definition 2. *Let* $F(t)(t = 1, 2, \ldots, T)$ *be a time series of real numbers, where T is the number of the time series.* $G(t)$ *is defined as a fluctuation time series, where* $G(t) = F(t) - F(t-1), (t = 2, 3, \ldots, T)$. *Each element of* $G(t)$ *can be represented by a fuzzy set* $S(t)(t = 2, 3, \ldots, T)$ *as defined in Definition 1. Then we call the time series* $G(t)$, *which is to be fuzzified into a fuzzy-fluctuation time series* (FFTS), $S(t)$.

Definition 3. *Let* $S(t)$ $(t = n+1, n+2, \ldots, T, n \geq 1)$ *be a FFTS. If* $S(t)$ *is determined by* $S(t-1), S(t-2), \ldots, S(t-n)$, *then the fuzzy-fluctuation logical relationship is represented by*:

$$S(t-1), S(t-2), \ldots, S(t-n) \to S(t) \tag{1}$$

and it is called the nth-order fuzzy-fluctuation logical relationship (FFLR) of the fuzzy-fluctuation time series, where $S(t-n), \ldots, S(t-2)S(t-1)$ *is called the left-hand side (LHS) and* $S(t)$ *is called the right-hand side (RHS) of the FFLR, and* $S(k)(k = t, t-1, t-2, \ldots, t-n) \in L$.

2.2. Basic Concept of Neutrosophic Logical Relationship (NLR)

Smarandache [36] originally presented the neutrosophic set theory. Based on neutrosophic set theory, we propose the concept of the fuzzy-neutrosophic logical relationship, which employs the three terms of a neutrosophic set to reflect the fuzzy-fluctuation trends and weights of an *n*th-order FFLR.

Definition 4. *Let* $P^i_{A(t)}$ *be the probabilities of each element* $l_i (l_i \in L)$ *in the LHS of an nth-order FFLR* $S(t-1), S(t-2), \ldots, S(t-n) \to S(t)$, *and it can be generated by:*

$$P^i_{A(t)} = \frac{\sum_{j=1}^{n} w_{i,j}}{n} \qquad i = 1, 2, 3 \tag{2}$$

where $w_{i,j} = 1$ *if* $S(t-j) = i$ *and 0 otherwise. Let X be a universal set, and the left-hand side of a neutrosophic logical relationship is defined by:*

$$A(t) = \left\{ \left\langle x, P^1_{A(t)}, P^2_{A(t)}, P^3_{A(t)} \right\rangle | x \in X \right\} \tag{3}$$

Definition 5. *For S(t)(t = n + 1, n + 2, ... , T) is a FFTS and A(t) is the LHS of a neutrosophic logical relationship. The FFLRs with the same A(t) can be grouped into a FFLRG by putting all their RHSs together as on the RHS of the FFLRG. The RHSs of the FFRLG for A(t) can be represented by a neutrosophic set as described by Definition 4:*

$$B_{A(t)} = \left\{ \left\langle x, P^1_{B_{A(t)}}, P^2_{B_{A(t)}}, P^3_{B_{A(t)}} \right\rangle | x \in X \right\} \tag{4}$$

where $P^i_{B_{A(t)}}$ $(i = 1,2,3)$ *represent the down, equal or up probability of the RHSs of the FFRLG for* $A(t)$*.* $P^i_{B_{A(t)}}$ $(i = 1,2,3)$ *is called the righ-hand side of a neutrosophic logical relationship.*

In this way, the FFLR $S(t-1), S(t-2), \ldots, S(t-n) \rightarrow S(t)$ can be represented by a neutrosophic logical relationship (NLR) $A(t) \rightarrow B_{A(t)}$.

Definition 6 [37]. *Let* $A(t_1)$ *and* $A(t_2)$ *be two neutrosophic sets. The Jaccard similarity measure between* $A(t_1)$ *and* $A(t_2)$ *in vector space is defined as follows:*

$$J(A(t_1), A(t_2)) = \frac{\sum_{i=1}^{3} P^i_{A(t_1)} P^i_{A(t_2)}}{\sum_{i=1}^{3} (P^i_{A(t_1)})^2 + \sum_{i=1}^{3} (P^i_{A(t_2)})^2 - \sum_{i=1}^{3} P^i_{A(t_1)} P^i_{A(t_2)}} \tag{5}$$

3. A Novel Forecasting Model Based on Neutrosophic Logical Relationships

In this paper, we propose a novel forecasting model based on high-order neutrosophic logical relationships and Jaccard similarity measures. In order to compare the forecasting results with other researchers' work [9,17,23,25,44–48], the authentic TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) is employed to illustrate the forecasting process. The data from January 1999 to October 1999 are used as the training time series and the data from November 1999 to December 1999 are used as the testing dataset. The basic steps of the proposed model are shown in Figure 1.

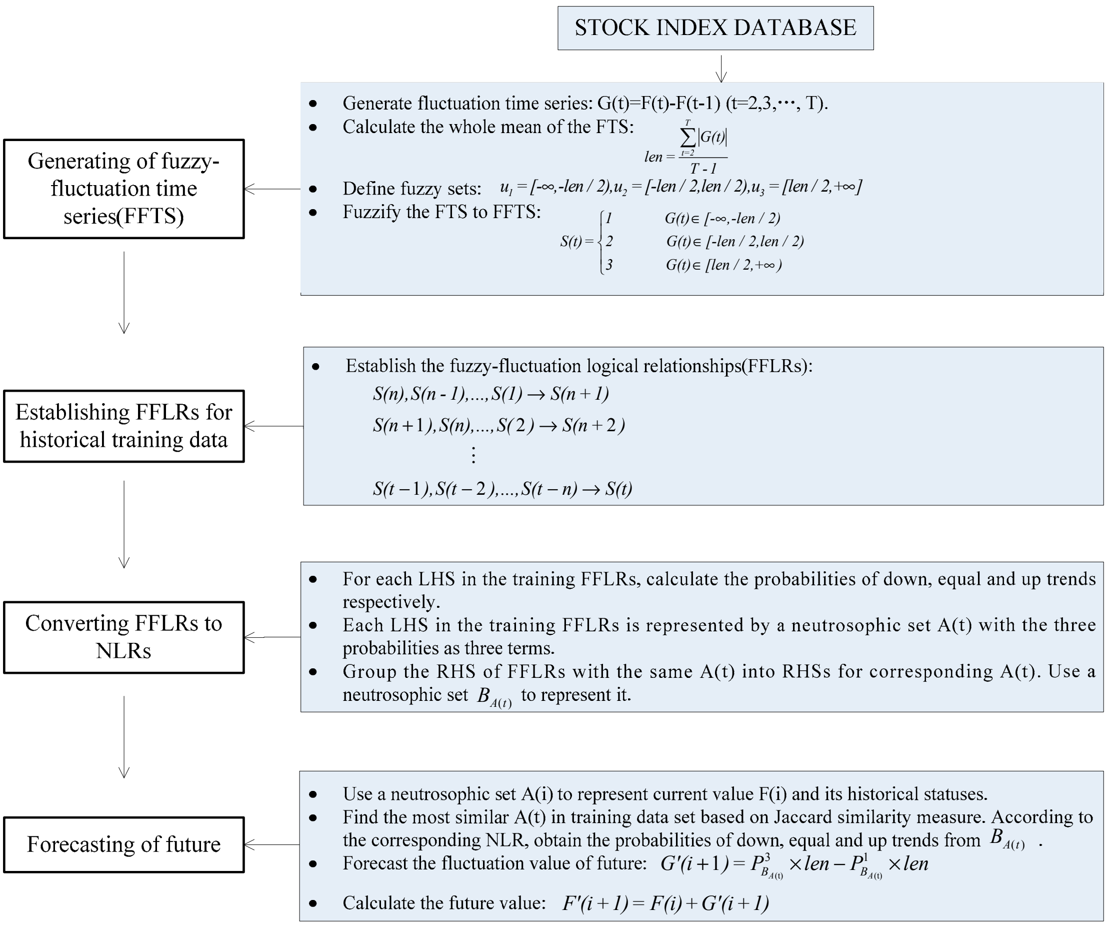

Figure 1. Flowchart of our proposed forecasting model.

Step 1. Construct FFTS for the historical training data

For each element $F(t)(t = 1, 2, \ldots, T)$ in the historical training time series, its fluctuation trend is determined by $G(t) = F(t) - F(t-1), (t = 2, 3, \ldots, T)$. According to the range and orientation of the fluctuations, $G(t)(t = 2, 3, \ldots, T)$ can be fuzzified into a linguistic set {down, equal, up}. Let *len* be the whole mean of all elements in the fluctuation time series $G(t)(t = 2, 3, \ldots, T)$, define $u_1 = [-\infty, -len/2]$, $u_2 = [-len/2, len/2]$, $u_3 = [len/2, +\infty]$, and then $G(t)(t = 2, 3, \ldots, T)$ can be fuzzified into a fuzzy-fluctuation time series $S(t)(t = 2, 3, \ldots, T)$.

Step 2. Establish *n*th-order FFLRs for the training data set

According to Definition 3, each $S(t)(t > n)$ in the historical training data set can be represented by its previous *n* days' fuzzy-fluctuation numbers to establish the training FFLRs.

Step 3. Convert the FFLRs to NLRs

According to Definition 4, the LHS of each FFLR can be expressed by a neutrosophic set $A(t)$. Then, we can generate the RHSs $B_{A(t)}$ for different LHSs respectively, as described in Definition 5. Thus, the FFLRs for the historical training dataset are converted into NLRs.

Step 4. Forecast test time series

For each observed point $F(i)$ in the test time series, we can use a neutrosophic set $A(i)$ to represent its *n*th-order fuzzy-fluctuation trends. Then, for each $A(t)$ obtained in step 3, compare $A(i)$ with $A(t)$ respectively, and find the most similar one based on the Jaccard similarity measure method described in Definition 6. Next, use the corresponding $B_{A(t)}$ as the forecasting rule to predict the fluctuation value $G'(i+1)$ of the next point. Finally, obtain the forecasting value by $F'(i+1) = F(i) + G'(i+1)$.

4. Empirical Analysis

4.1. Forecasting Taiwan Stock Exchange Capitalization Weighted Stock Index

Many studies use TAIEX1999 as an example to illustrate their proposed forecasting methods [9,17,25,34,44–48]. In order to compare the accuracy with their models, we also use TAIEX1999 to illustrate the proposed method.

Step 1: Calculate the fluctuation trend for each element in the historical training dataset of TAIEX1999. Then, we use the whole mean of the fluctuation numbers of the training dataset to fuzzify the fluctuation trends into FFTS. For example, the whole mean of the historical dataset of TAIEX1999 from January to October is 85. That is to say, *len* = 85. For $F(1) = 6152.43$ and $F(2) = 6199.91$, $G(2) = 47.48$, $S(2) = 3$. In this way, the historical training dataset can be represented by a fuzzified fluctuation dataset as shown in Table A1.

Step 2: Based on the FFTS from 5 January to 30 October 1999—shown in Table A1—the *n*th-order FFLRs for the forecasting model are established as shown in Table A2. The subscript *i* is used to represent element l_i in the FFLRs for convenience.

Step 3: In order to convert the FFLRs to NLRs, first of all the LHSs of the FFLRs in Table A2 are represented by a neutrosophic set, respectively (shown in Table A2). Then, the RHSs of the FFLRs are grouped with the same LHS neutrosophic set value into the RHSs group. A neutrosophic set is used to represent the RHSs group. For example, the LHS of FFLR 2,3,1,1,1,2,2,3,3→1 can be represented by the neutrosophic set (0.33,0.33,0.33). The detailed grouping and converting processes are shown in Figure 2.

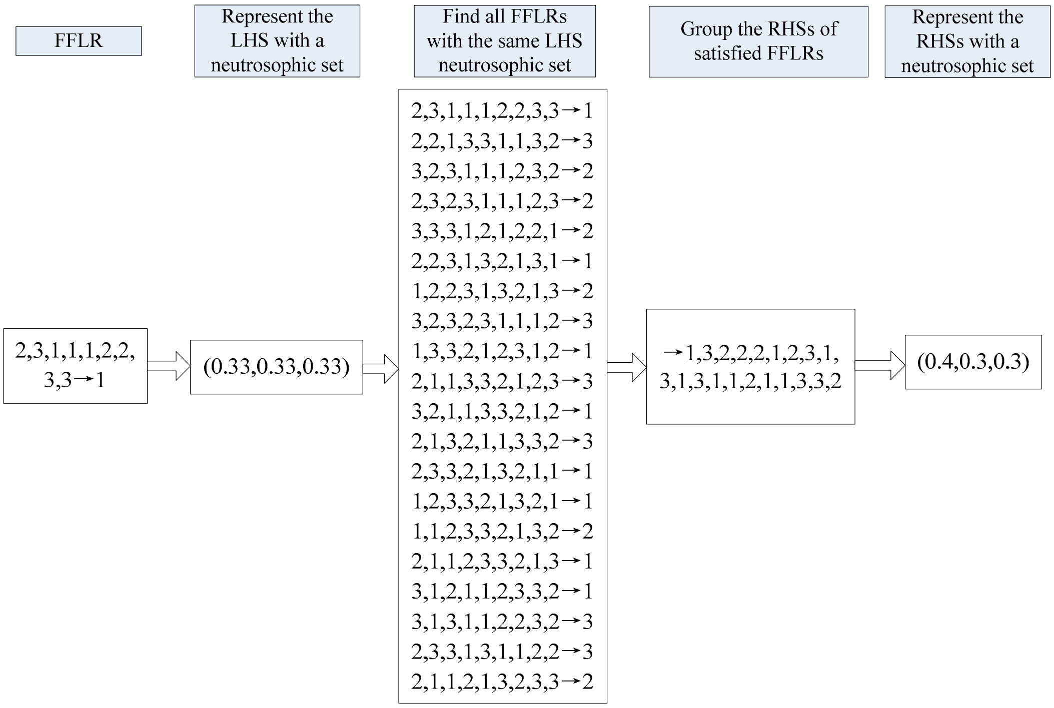

Figure 2. Group and converting processes for FFLR 2,3,1,1,1,2,2,3,3→2.

In this way, the FFLR 2,3,1,1,1,2,2,3,3→1 and other members of the same group are converted into an NLR (0.33,0.33,0.33)→(0.4,0.3,0.3). Therefore, the FFLRs in Table A2 can be converted into NLRs as shown in Table 1.

Table 1. Neutrosophic logical relationships (NLRs) for the historical training data of TAIEX1999.

NLRs	NLRs	NLRs
(0.33,0.33,0.33)→(0.4,0.3,0.3)	(0.22,0.33,0.44)→(0,0.6,0.4)	(0.22,0.78,0)→(0.5,0.5,0)
(0.44,0.33,0.22)→(0.23,0.46,0.31)	(0.22,0.44,0.33)→(0.33,0.33,0.33)	(0.33,0.67,0)→(0,0,1)
(0.44,0.44,0.11)→(0.4,0.33,0.27)	(0.11,0.56,0.33)→(0.17,0.5,0.33)	(0.11,0.11,0.78)→(0,1,0)
(0.33,0.44,0.22)→(0.54,0.23,0.23)	(0.11,0.67,0.22)→(0.17,0.33,0.5)	(0,0.22,0.78)→(0,1,0)
(0.33,0.56,0.11)→(0.25,0.5,0.25)	(0.22,0.56,0.22)→(0.25,0.5,0.25)	(0,0.33,0.67)→(0,1,0)
(0.56,0.33,0.11)→(0.36,0.27,0.36)	(0.11,0.44,0.44)→(0,0.38,0.63)	(0.56,0.22,0.22)→(0.25,0.25,0.5)
(0.67,0.22,0.11)→(0,1,0)	(0.11,0.33,0.56)→(0.33,0.17,0.5)	(0.44,0.11,0.44)→(0.5,0.5,0)
(0.56,0.44,0)→(0,0,1)	(0.11,0.22,0.67)→(0.43,0.43,0.14)	(0.56,0.11,0.33)→(1,0,0)
(0.44,0.22,0.33)→(0.29,0.43,0.29)	(0,0.56,0.44)→(0.33,0,0.67)	(0.67,0,0.33)→(0,1,0)
(0.33,0.22,0.44)→(0.31,0.38,0.31)	(0,0.44,0.56)→(0.14,0.43,0.43)	(0.67,0.11,0.22)→(0.5,0.25,0.25)
(0.22,0.22,0.56)→(0.25,0.25,0.5)	(0.11,0.78,0.11)→(0,0.8,0.2)	(0.22,0.67,0.11)→(0,0,1)
(0.33,0.11,0.56)→(0,0.5,0.5)	(0,0.89,0.11)→(0.25,0.75,0)	
(0.22,0.11,0.67)→(0.29,0.29,0.43)	(0.11,0.89,0)→(0.5,0.5,0)	

Step 4: Use the NLRs obtained from historical training data to forecast the test dataset from 1 November to 30 December 1999. For example, the forecasting value of the TAIEX on 1 November 1999 is calculated as follows:

First, the ninth-order historical fuzzy-fluctuation trends 3,2,2,2,2,3,1,2,2 on 1 November 1999 can be represented by a neutrosophic set (0.11,0.67,0.22). Then, we use the Jaccard similarity measure method as described by Definition 6 to choose the most optimal NLR from the NLRs listed in Table 1. The NLR (0.11,0.67,0.22)→(0.17,0.33,0.5) is evidently the best rule for further forecasting. Therefore, the forecasted fuzzy-fluctuation number is:

$$S'(i+1) = (-0.17) + 0.5 = 0.33$$

The forecasted fluctuation from the current value to the next value can be obtained by defuzzifying the fluctuation fuzzy number:

$$G'(i+1) = S'(i+1) \times len = 0.33 \times 85 = 28.05$$

Finally, the forecasted value can be obtained by the current value and the fluctuation value:

$$F'(i+1) = F(i) + G'(i+1) = 7854.85 + 28.05 = 7882.9$$

The other forecasting results are shown in Table 2 and Figure 3.

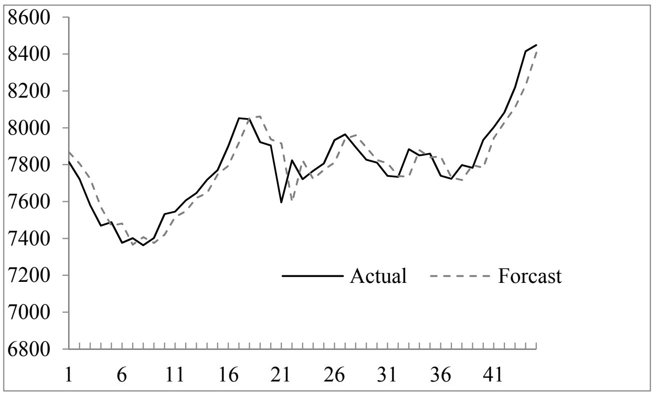

Figure 3. Forecasting results from 1 November 1999 to 30 December 1999.

Table 2. Forecasting results from 1 November 1999 to 30 December 1999.

Date (MM/DD/YYYY)	Actual	Forecast	(Forecast − Actual)^2	Date (MM/DD/YYYY)	Actual	Forecast	(Forecast − Actual)^2
11/1/1999	7814.89	7882.90	4625.36	12/1/1999	7766.20	7720.87	2054.81
11/2/1999	7721.59	7842.94	14,725.82	12/2/1999	7806.26	7766.20	1604.80
11/3/1999	7580.09	7721.59	20,022.25	12/3/1999	7933.17	7797.76	18,335.87
11/4/1999	7469.23	7580.09	12,289.94	12/4/1999	7964.49	7924.67	1585.63
11/5/1999	7488.26	7469.23	362.14	12/6/1999	7894.46	7955.99	3785.94
11/6/1999	7376.56	7488.26	12,476.89	12/7/1999	7827.05	7885.96	3470.39
11/8/1999	7401.49	7365.51	1294.56	12/8/1999	7811.02	7827.05	256.96
11/9/1999	7362.69	7390.44	770.06	12/9/1999	7738.84	7802.52	4055.14
11/10/1999	7401.81	7351.64	2517.03	12/10/1999	7733.77	7745.64	140.90
11/11/1999	7532.22	7486.82	2061.16	12/13/1999	7883.61	7707.42	31,042.92
11/15/1999	7545.03	7521.17	569.30	12/14/1999	7850.14	7857.26	50.69
11/16/1999	7606.20	7545.03	3741.77	12/15/1999	7859.89	7823.79	1303.21
11/17/1999	7645.78	7606.20	1566.58	12/16/1999	7739.76	7859.89	14,431.22
11/18/1999	7718.06	7673.83	1956.29	12/17/1999	7723.22	7728.71	30.14
11/19/1999	7770.81	7731.66	1532.72	12/18/1999	7797.87	7723.22	5572.62
11/20/1999	7900.34	7799.71	10,126.40	12/20/1999	7782.94	7797.87	222.90
11/22/1999	8052.31	7924.99	16,210.38	12/21/1999	7934.26	7782.94	22,897.74
11/23/1999	8046.19	8052.31	37.45	12/22/1999	8002.76	7947.86	3014.01
11/24/1999	7921.85	8046.19	15,460.44	12/23/1999	8083.49	8056.32	738.21
11/25/1999	7904.53	7936.30	1009.33	12/24/1999	8219.45	8137.05	6789.76
11/26/1999	7595.44	7918.98	104,678.13	12/27/1999	8415.07	8233.90	32,822.57
11/29/1999	7823.90	7629.44	37,814.69	12/28/1999	8448.84	8390.42	3412.90
11/30/1999	7720.87	7845.15	15,445.52	Root Mean Square Error(RMSE)			98.76

The forecasting performance can be assessed by comparing the difference between the forecasted values and the actual values. The widely used indicators in time series model comparisons are the mean squared error (MSE), the root of the mean squared error (RMSE), the mean absolute error (MAE), and the mean percentage error (MPE), etc. To compare the performance of different forecasting methods, the Diebold-Mariano test statistic (S) is also widely used [49]. These indicators are defined by Equations (6)–(10):

$$MSE = \frac{\sum_{t=1}^{n} (forecast(t) - actual(t))^2}{n} \quad (6)$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (forecast(t) - actual(t))^2}{n}} \quad (7)$$

$$MAE = \frac{\sum_{t=1}^{n} |(forecast(t) - actual(t))|}{n} \quad (8)$$

$$MPE = \frac{\sum_{t=1}^{n} |(forecast(t) - actual(t))|/actual(t)}{n} \quad (9)$$

$$S = \frac{\bar{d}}{(Variance(\bar{d}))^{1/2}}, \ \bar{d} = \frac{\sum_{t=1}^{n} (error \quad of \quad forecast1)_t^2 - \sum_{t=1}^{n} (error \quad of \quad forecast2)_t^2}{n} \quad (10)$$

where n denotes the number of values forecasted, *forecast*(t) and *actual*(t) denote the predicted value and actual value at time t, respectively. S is a test statistic of the Diebold method that is used to compare the predictive accuracy of two forecasts obtained by different methods. *Forecast*1 represents the dataset obtained by method 1, and *Forecast*2 represents another dataset from method 2. If $S > 0$ and $|S| > Z = 1.64$ at the 0.05 significance level, then *Forecast*2 has better predictive accuracy than *Forecast*1. With respect to the proposed method for the ninth order, the MSE, RMSE, MAE, and MPE are 9753.63, 98.76, 76.32, and 0.01, respectively.

Let the order number n vary from two to 10; the RMSEs for different nth-order forecasting models are listed in Table 3. The item "Average" refers to the RMSE for the average forecasting results of these different nth-order (n = 2, 3, ..., 10) models.

Table 3. Comparison of forecasting errors for different nth orders.

	n									**Average**
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
RMSE	100.22	100.9	100.66	99.81	102.83	103.48	100.36	98.76	108.99	99.03

In practical forecasting, the average of the results of different nth-order (n = 2, 3, ..., 9) forecasting models is adopted to avoid the uncertainty. The proposed method is employed to forecast the TAIEX from 1997 to 2005. The forecasting results and errors are shown in Figure 4 and Table 4.

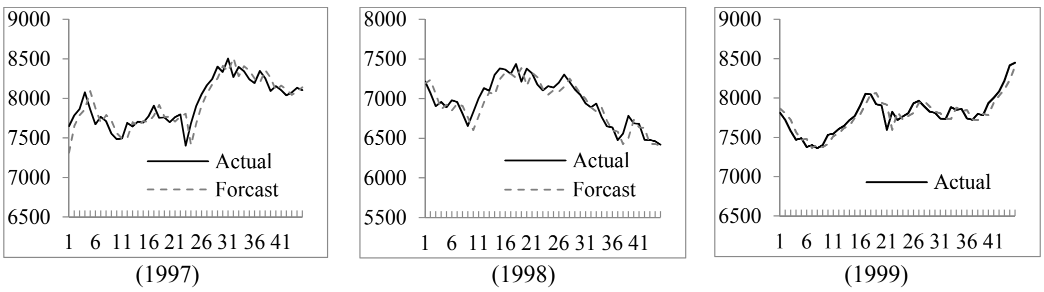

Figure 4. *Cont.*

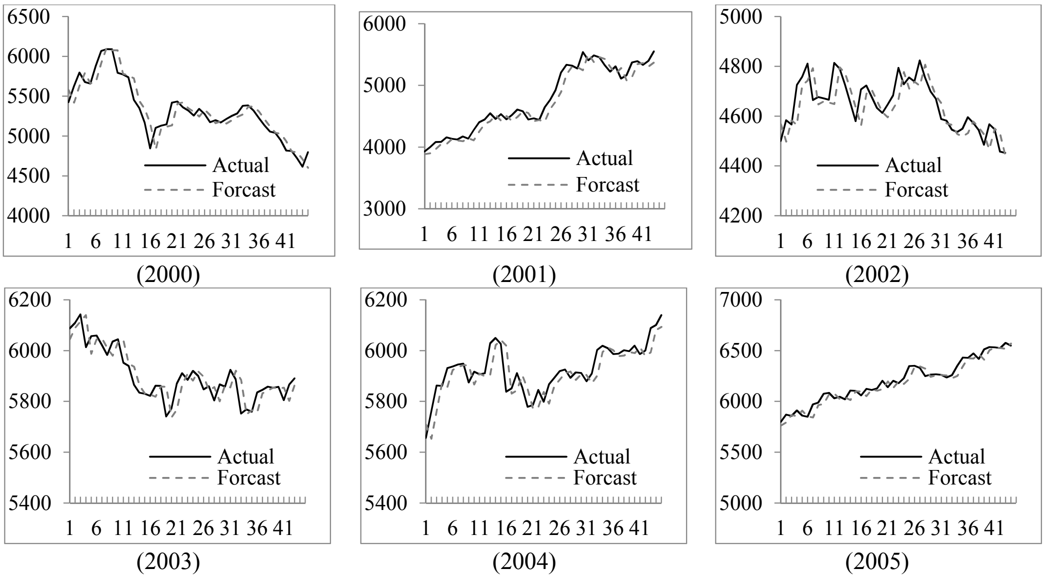

Figure 4. The stock market fluctuation for the TAIEX test dataset (1997–2005).

Table 4. RMSEs of forecast errors for TAIEX 1997 to 2005.

	Year								
	1997	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
RMSE	141.89	119.85	99.03	128.62	125.64	66.29	53.2	56.11	55.83

Table 5 shows a comparison between the RMSEs of different methods for forecasting the TAIEX1999. From this table, we can see that the performance of the proposed method is acceptable. The greatest advantage of the proposed method is that it does not need to determine the boundary of discourse or the intervals for number fuzzifying. Meanwhile, the introduction of neutrosophic sets into the expression of logical relationships makes it possible to employ a similar comparison method to locate the most appropriate rules for further forecasting. Therefore, the proposed method, to some extent, is more rigorous than other methods that just use meaningless values in the case of missing rules in the training data. Though the RMSEs of some of the other methods outperform the proposed method, they often need to determine complex discretization partitioning rules or use adaptive expectation models to justify the final forecasting results. The method proposed in this paper is simpler and more easily realized by a computer program.

Table 5. A comparison of RMSEs for different methods for forecasting the TAIEX1999.

Methods	**RMSE**	**S**
Yu's Method (2005) [25]	145	1.82 **
Hsieh et al.'s Method (2011) [48]	94	−0.42
Chang et al.'s Method (2011) [45]	100	0.21
Cheng et al.'s Method (2013) [47]	103	0.42
Chen et al.'s Method (2013) [46]	102.11	0.39
Chen and Chen's Method (2015) [9]	103.9	0.29
Chen and Chen's Method (2015) [44]	92	−0.51
Zhao et al.'s Method (2016) [23]	110.85	1.16
Jia et al.'s Method (2017) [17]	99.31	0.11
The Proposed Method	99.03	-

** The proposed method has better predictive accuracy than the method at the 5% significance level.

4.2. Forecasting Shanghai Stock Exchange Composite Index

The SHSECI is the most famous stock market index in China. In the following, we apply the proposed method to forecast the SHSECI from 2007 to 2015. For each year, the authentic datasets of the historical daily SHSECI closing prices between January and October are used as the training data, and the datasets from November to December are used as the testing data. The RMSEs of forecast errors are shown in Table 6.

From Table 6, we can see that the proposed method can successfully predict the SHSECI stock market.

Table 6. RMSEs of forecast errors for SHSECI from 2007 to 2015.

	Year								
	2007	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
RMSE	113.47	71.6	49.14	45.35	27.74	25.83	19.95	41.42	64.6

5. Conclusions

In this paper, a novel forecasting model is proposed based on neutrosophic logical relationships, the Jaccard similarity measure, and on fluctuations of the time series. The high-order fuzzy-fluctuation logical relationships are represented by neutrosophic logical relationships. Therefore, we can use the Jaccard similarity measure method to find the optimal forecasting rules. The biggest advantage of this method is that it can deal with the problem of lack of rules. Considering the fact that future fluctuation is more important than the indicated number itself, this method focuses on the forecasting of fluctuation orientations in terms of the extent of the fluctuation rather than on the real numbers. Meanwhile, utilizing NLRs instead of FLRs makes it possible to select the most appropriate rules for further forecasting. Therefore, the proposed method is more rigorous and interpretable. Experiments show that the parameters generated by the training dataset can be successfully used for future datasets as well. In order to compare the performance with that of other methods, we took the TAIEX 1999 as an example. We also forecasted TAIEX 1997–2005 and SHSECI 2007–2015 to verify its effectiveness and universality. In the future, we will consider other factors that might affect the fluctuation of the stock market, such as the trade volume, the beginning value, the end value, etc. We will also consider the influence of other stock markets, such as the Dow Jones, the National Association of Securities Dealers Automated Quotations (NASDAQ), the M1b, and so on.

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Author Contributions: Hongjun Guan designed the experiments and wrote the paper; Shuang Guan performed the experiments and analyzed the data; and Aiwu Zhao conceived the main idea of the method.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Historical training data and fuzzified fluctuation data of TAIEX 1999.

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
1/5/1999	6152.43	-	-	4/17/1999	7581.5	114.68	3	7/26/1999	7595.71	−128.81	1
1/6/1999	6199.91	47.48	3	4/19/1999	7623.18	41.68	2	7/27/1999	7367.97	−227.74	1
1/7/1999	6404.31	204.4	3	4/20/1999	7627.74	4.56	2	7/28/1999	7484.5	116.53	3
1/8/1999	6421.75	17.44	2	4/21/1999	7474.16	−153.58	1	7/29/1999	7359.37	−125.13	1
1/11/1999	6406.99	−14.76	2	4/22/1999	7494.6	20.44	2	7/30/1999	7413.11	53.74	3
1/12/1999	6363.89	−43.1	1	4/23/1999	7612.8	118.2	3	7/31/1999	7326.75	−86.36	1
1/13/1999	6319.34	−44.55	1	4/26/1999	7629.09	16.29	2	8/2/1999	7195.94	−130.81	1
1/14/1999	6241.32	−78.02	1	4/27/1999	7550.13	−78.96	1	8/3/1999	7175.19	−20.75	2
1/15/1999	6454.6	213.28	3	4/28/1999	7496.61	−53.52	1	8/4/1999	7110.8	−64.39	1
1/16/1999	6483.3	28.7	2	4/29/1999	7289.62	−206.99	1	8/5/1999	6959.73	−151.07	1
1/18/1999	6377.25	−106.05	1	4/30/1999	7371.17	81.55	3	8/6/1999	6823.52	−136.21	1
1/19/1999	6343.36	−33.89	2	5/3/1999	7383.26	12.09	2	8/7/1999	7049.74	226.22	3
1/20/1999	6310.71	−32.65	2	5/4/1999	7588.04	204.78	3	8/9/1999	7028.01	−21.73	2
1/21/1999	6332.2	21.49	2	5/5/1999	7572.16	−15.88	2	8/10/1999	7269.6	241.59	3
1/22/1999	6228.95	−103.25	1	5/6/1999	7560.05	−12.11	2	8/11/1999	7228.68	−40.92	2
1/25/1999	6033.21	−195.74	1	5/7/1999	7469.33	−90.72	1	8/12/1999	7330.24	101.56	3
1/26/1999	6115.64	82.43	3	5/10/1999	7484.37	15.04	2	8/13/1999	7626.05	295.81	3
1/27/1999	6138.87	23.23	2	5/11/1999	7474.45	−9.92	2	8/16/1999	8018.47	392.42	3
1/28/1999	6063.41	−75.46	1	5/12/1999	7448.41	−26.04	2	8/17/1999	8083.43	64.96	3
1/29/1999	5984	−79.41	1	5/13/1999	7416.2	−32.21	2	8/18/1999	7993.71	−89.72	1
1/30/1999	5998.32	14.32	2	5/14/1999	7592.53	176.33	3	8/19/1999	7964.67	−29.04	2
2/1/1999	5862.79	−135.53	1	5/15/1999	7576.64	−15.89	2	8/20/1999	8117.42	152.75	3
2/2/1999	5749.64	−113.15	1	5/17/1999	7599.76	23.12	2	8/21/1999	8153.57	36.15	2
2/3/1999	5743.86	−5.78	2	5/18/1999	7585.51	−14.25	2	8/23/1999	8119.98	−33.59	2
2/4/1999	5514.89	−228.97	1	5/19/1999	7614.6	29.09	2	8/24/1999	7984.39	−135.59	1
2/5/1999	5474.79	−40.1	2	5/20/1999	7608.88	−5.72	2	8/25/1999	8127.09	142.7	3
2/6/1999	5710.18	235.39	3	5/21/1999	7606.69	−2.19	2	8/26/1999	8097.57	−29.52	2
2/8/1999	5822.98	112.8	3	5/24/1999	7588.23	−18.46	2	8/27/1999	8053.97	−43.6	1
2/9/1999	5723.73	−99.25	1	5/25/1999	7417.03	−171.2	1	8/30/1999	8071.36	17.39	2
2/10/1999	5798	74.27	3	5/26/1999	7426.63	9.6	2	8/31/1999	8157.73	86.37	3
2/20/1999	6072.33	274.33	3	5/27/1999	7469.01	42.38	2	9/1/1999	8273.33	115.6	3
2/22/1999	6313.63	241.3	3	5/28/1999	7387.37	−81.64	1	9/2/1999	8226.15	−47.18	1
2/23/1999	6180.94	−132.69	1	5/29/1999	7419.7	32.33	2	9/3/1999	8073.97	−152.18	1
2/24/1999	6238.87	57.93	3	5/31/1999	7316.57	−103.13	1	9/4/1999	8065.11	−8.86	2
2/25/1999	6275.53	36.66	2	6/1/1999	7397.62	81.05	3	9/6/1999	8130.28	65.17	3
2/26/1999	6318.52	42.99	3	6/2/1999	7488.03	90.41	3	9/7/1999	7945.76	−184.52	1
3/1/1999	6312.25	−6.27	2	6/3/1999	7572.91	84.88	3	9/8/1999	7973.3	27.54	2

Table A1. *Cont.*

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
3/2/1999	6263.54	−48.71	1	6/4/1999	7590.44	17.53	2	9/9/1999	8025.02	51.72	3
3/3/1999	6403.14	139.6	3	6/5/1999	7639.3	48.86	3	9/10/1999	8161.46	136.44	3
3/4/1999	6393.74	−9.4	2	6/7/1999	7802.69	163.39	3	9/13/1999	8178.69	17.23	2
3/5/1999	6383.09	−10.65	2	6/8/1999	7892.13	89.44	3	9/14/1999	8092.02	−86.67	1
3/6/1999	6421.73	38.64	2	6/9/1999	7957.71	65.58	3	9/15/1999	7971.04	−120.98	1
3/8/1999	6431.96	10.23	2	6/10/1999	7996.76	39.05	2	9/16/1999	7968.9	−2.14	2
3/9/1999	6493.43	61.47	3	6/11/1999	7979.4	−17.36	2	9/17/1999	7916.92	−51.98	1
3/10/1999	6486.61	−6.82	2	6/14/1999	7973.58	−5.82	2	9/18/1999	8016.93	100.01	3
3/11/1999	6436.8	−49.81	1	6/15/1999	7960	−13.58	2	9/20/1999	7972.14	−44.79	1
3/12/1999	6462.73	25.93	2	6/16/1999	8059.02	99.02	3	9/27/1999	7759.93	−212.21	1
3/15/1999	6598.32	135.59	3	6/17/1999	8274.36	215.34	3	9/28/1999	7577.85	−182.08	1
3/16/1999	6672.23	73.91	3	6/21/1999	8413.48	139.12	3	9/29/1999	7615.45	37.6	2
3/17/1999	6757.07	84.84	3	6/22/1999	8608.91	195.43	3	9/30/1999	7598.79	−16.66	2
3/18/1999	6895.01	137.94	3	6/23/1999	8492.32	−116.59	1	10/1/1999	7694.99	96.2	3
3/19/1999	6997.29	102.28	3	6/24/1999	8589.31	96.99	3	10/2/1999	7659.55	−35.44	2
3/20/1999	6993.38	−3.91	2	6/25/1999	8265.96	−323.35	1	10/4/1999	7685.48	25.93	2
3/22/1999	7043.23	49.85	3	6/28/1999	8281.45	15.49	2	10/5/1999	7557.01	−128.47	1
3/23/1999	6945.48	−97.75	1	6/29/1999	8514.27	232.82	3	10/6/1999	7501.63	−55.38	1
3/24/1999	6889.42	−56.06	1	6/30/1999	8467.37	−46.9	1	10/7/1999	7612	110.37	3
3/25/1999	6941.38	51.96	3	7/2/1999	8572.09	104.72	3	10/8/1999	7552.98	−59.02	1
3/26/1999	7033.25	91.87	3	7/3/1999	8563.55	−8.54	2	10/11/1999	7607.11	54.13	3
3/29/1999	6901.68	−131.57	1	7/5/1999	8593.35	29.8	2	10/12/1999	7835.37	228.26	3
3/30/1999	6898.66	−3.02	2	7/6/1999	8454.49	−138.86	1	10/13/1999	7836.94	1.57	2
3/31/1999	6881.72	−16.94	2	7/7/1999	8470.07	15.58	2	10/14/1999	7879.91	42.97	3
4/1/1999	7018.68	136.96	3	7/8/1999	8592.43	122.36	3	10/15/1999	7819.09	−60.82	1
4/2/1999	7232.51	213.83	3	7/9/1999	8550.27	−42.16	2	10/16/1999	7829.39	10.3	2
4/3/1999	7182.2	−50.31	1	7/12/1999	8463.9	−86.37	1	10/18/1999	7745.26	−84.13	1
4/6/1999	7163.99	−18.21	2	7/13/1999	8204.5	−259.4	1	10/19/1999	7692.96	−52.3	1
4/7/1999	7135.89	−28.1	2	7/14/1999	7888.66	−315.84	1	10/20/1999	7666.64	−26.32	2
4/8/1999	7273.41	137.52	3	7/15/1999	7918.04	29.38	2	10/21/1999	7654.9	−11.74	2
4/9/1999	7265.7	−7.71	2	7/16/1999	7411.58	−506.46	1	10/22/1999	7559.63	−95.27	1
4/12/1999	7242.4	−23.3	2	7/17/1999	7366.23	−45.35	1	10/25/1999	7680.87	121.24	3
4/13/1999	7337.85	95.45	3	7/19/1999	7386.89	20.66	2	10/26/1999	7700.29	19.42	2
4/14/1999	7398.65	60.8	3	7/20/1999	7806.85	419.96	3	10/27/1999	7701.22	0.93	2
4/15/1999	7498.17	99.52	3	7/21/1999	7786.65	−20.2	2	10/28/1999	7681.85	−19.37	2
4/16/1999	7466.82	−31.35	2	7/22/1999	7678.67	−107.98	1	10/29/1999	7706.67	24.82	2
4/17/1999	7581.5	114.68	3	7/23/1999	7724.52	45.85	3	10/30/1999	7854.85	148.18	3

Table A2. The FFLRs and the converted left hand side of NLRs for historical training data of TAIEX1999.

Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR
1/18/1999	2,3,1,1,1,2,2,3,3→2	(0.33,0.33,0.33)	4/3/1999	3,3,2,2,1,3,3,1,1→3	(0.33,0.22,0.44)	6/11/1999	2,3,3,3,3,2,3,3,3→2	(0,0.22,0.78)	8/21/1999	3,2,1,3,3,3,3,2,3→3	(0.11,0.22,0.67)
1/19/1999	1,2,3,1,1,1,2,2,3→1	(0.44,0.33,0.22)	4/6/1999	1,3,3,2,2,1,3,3,1→1	(0.33,0.22,0.44)	6/14/1999	2,2,3,3,3,3,2,3,3→2	(0,0.33,0.67)	8/23/1999	2,3,2,1,3,3,3,3,2→2	(0.11,0.33,0.56)
1/20/1999	2,1,2,3,1,1,1,2,2→2	(0.44,0.44,0.11)	4/7/1999	2,1,3,3,2,2,1,3,3→2	(0.22,0.33,0.44)	6/15/1999	2,2,2,3,3,3,3,2,3→2	(0,0.44,0.56)	8/24/1999	2,2,3,2,1,3,3,3,3→2	(0.11,0.33,0.56)
1/21/1999	2,2,1,2,3,1,1,1,2→2	(0.44,0.44,0.11)	4/8/1999	2,2,1,3,3,2,2,1,3→2	(0.22,0.44,0.33)	6/16/1999	2,2,2,2,3,3,3,3,2→2	(0,0.56,0.44)	8/25/1999	1,2,2,3,2,1,3,3,3→1	(0.22,0.33,0.44)
1/22/1999	2,2,2,1,2,3,1,1,1→2	(0.44,0.44,0.11)	4/9/1999	3,2,2,1,3,3,2,2,1→3	(0.22,0.44,0.33)	6/17/1999	3,2,2,2,2,3,3,3,3→3	(0,0.44,0.56)	8/26/1999	3,1,2,2,3,2,1,3,3→3	(0.22,0.33,0.44)
1/25/1999	1,2,2,2,1,2,3,1,1→1	(0.44,0.44,0.11)	4/12/1999	2,3,2,2,1,3,3,2,2→2	(0.11,0.56,0.33)	6/21/1999	3,3,2,2,2,2,3,3,3→3	(0,0.44,0.56)	8/27/1999	2,3,1,2,2,3,2,1,3→2	(0.22,0.44,0.33)
1/26/1999	1,1,2,2,2,1,2,3,1→1	(0.44,0.44,0.11)	4/13/1999	2,2,3,2,2,1,3,3,2→2	(0.11,0.56,0.33)	6/22/1999	3,3,3,2,2,2,2,3,3→3	(0,0.44,0.56)	8/30/1999	1,2,3,1,2,2,3,2,1→1	(0.33,0.44,0.22)
1/27/1999	3,1,1,2,2,2,1,2,3→3	(0.33,0.44,0.22)	4/14/1999	3,2,2,3,2,2,1,3,3→3	(0.11,0.44,0.44)	6/23/1999	3,3,3,3,2,2,2,2,3→3	(0,0.44,0.56)	8/31/1999	2,1,2,3,1,2,2,3,2→2	(0.22,0.56,0.22)
1/28/1999	2,3,1,1,2,2,2,1,2→2	(0.33,0.56,0.11)	4/15/1999	3,3,2,2,3,2,2,1,3→3	(0.11,0.44,0.44)	6/24/1999	1,3,3,3,3,2,2,2,2→1	(0.11,0.44,0.44)	9/1/1999	3,2,1,2,3,1,2,2,3→3	(0.22,0.44,0.33)
1/29/1999	1,2,3,1,1,2,2,2,1→1	(0.44,0.44,0.11)	4/16/1999	3,3,3,2,2,3,2,2,1→3	(0.11,0.44,0.44)	6/25/1999	3,1,3,3,3,3,2,2,2→3	(0.11,0.33,0.56)	9/2/1999	3,3,2,1,2,3,1,2,2→3	(0.22,0.44,0.33)
1/30/1999	1,1,2,3,1,1,2,2,2→1	(0.44,0.44,0.11)	4/17/1999	2,3,3,3,2,2,3,2,2→2	(0,0.56,0.44)	6/28/1999	1,3,1,3,3,3,3,2,2→1	(0.22,0.22,0.56)	9/3/1999	1,3,3,2,1,2,3,1,2→1	(0.33,0.33,0.33)
2/1/1999	2,1,1,2,3,1,1,2,2→2	(0.44,0.44,0.11)	4/19/1999	3,2,3,3,3,2,2,3,2→3	(0,0.44,0.56)	6/29/1999	2,1,3,1,3,3,3,3,2→2	(0.22,0.22,0.56)	9/4/1999	1,1,3,3,2,1,2,3,1→1	(0.44,0.22,0.33)
2/2/1999	1,2,1,1,2,3,1,1,2→1	(0.56,0.33,0.11)	4/20/1999	2,3,2,3,3,3,2,2,3→2	(0,0.44,0.56)	6/30/1999	3,2,1,3,1,3,3,3,3→3	(0.22,0.11,0.67)	9/6/1999	2,1,1,3,3,2,1,2,3→2	(0.33,0.33,0.33)
2/3/1999	1,1,2,1,1,2,3,1,1→1	(0.67,0.22,0.11)	4/21/1999	2,2,3,2,3,3,3,2,2→2	(0,0.56,0.44)	7/2/1999	1,3,2,1,3,1,3,3,3→1	(0.33,0.11,0.56)	9/7/1999	3,2,1,1,3,3,2,1,2→3	(0.33,0.33,0.33)
2/4/1999	2,1,1,2,1,1,2,3,1→2	(0.56,0.33,0.11)	4/22/1999	1,2,2,3,2,3,3,3,2→1	(0.11,0.44,0.44)	7/3/1999	3,1,3,2,1,3,1,3,3→3	(0.33,0.11,0.56)	9/8/1999	1,3,2,1,1,3,3,2,1→1	(0.44,0.22,0.33)
2/5/1999	1,2,1,1,2,1,1,2,3→1	(0.56,0.33,0.11)	4/23/1999	2,1,2,2,3,2,3,3,3→2	(0.11,0.44,0.44)	7/5/1999	2,3,1,3,2,1,3,1,3→2	(0.33,0.22,0.44)	9/9/1999	2,1,3,2,1,1,3,3,2→2	(0.33,0.33,0.33)
2/6/1999	2,1,2,1,1,2,1,1,2→2	(0.56,0.44,0)	4/26/1999	3,2,1,2,2,3,2,3,3→3	(0.11,0.44,0.44)	7/6/1999	2,2,3,1,3,2,1,3,1→2	(0.33,0.33,0.33)	9/10/1999	3,2,1,3,2,1,1,3,3→3	(0.33,0.22,0.44)
2/8/1999	3,2,1,2,1,1,2,1,1→3	(0.56,0.33,0.11)	4/27/1999	2,3,2,1,2,2,3,2,3→2	(0.11,0.56,0.33)	7/7/1999	1,2,2,3,1,3,2,1,3→1	(0.33,0.33,0.33)	9/13/1999	3,3,2,1,3,2,1,1,3→3	(0.33,0.22,0.44)
2/9/1999	3,3,2,1,2,1,1,2,1→3	(0.44,0.33,0.22)	4/28/1999	1,2,3,2,1,2,2,3,2→1	(0.22,0.56,0.22)	7/8/1999	2,1,2,2,3,1,3,2,1→2	(0.33,0.44,0.22)	9/14/1999	2,3,3,2,1,3,2,1,1→2	(0.33,0.33,0.33)
2/10/1999	1,3,3,2,1,2,1,1,2→1	(0.44,0.33,0.22)	4/29/1999	1,1,2,3,2,1,2,2,3→1	(0.33,0.44,0.22)	7/9/1999	3,2,1,2,2,3,1,3,2→3	(0.22,0.44,0.33)	9/15/1999	1,2,3,3,2,1,3,2,1→1	(0.33,0.33,0.33)
2/20/1999	3,1,3,3,2,1,2,1,1→3	(0.44,0.22,0.33)	4/30/1999	1,1,1,2,3,2,1,2,2→1	(0.44,0.44,0.11)	7/12/1999	2,3,2,1,2,2,3,1,3→2	(0.22,0.44,0.33)	9/16/1999	1,1,2,3,3,2,1,3,2→1	(0.33,0.33,0.33)
2/22/1999	3,3,1,3,3,2,1,2,1→3	(0.33,0.22,0.44)	5/3/1999	3,1,1,1,2,3,2,1,2→3	(0.44,0.33,0.22)	7/13/1999	1,2,3,2,1,2,2,3,1→1	(0.33,0.44,0.22)	9/17/1999	2,1,1,2,3,3,2,1,3→2	(0.33,0.33,0.33)
2/23/1999	3,3,3,1,3,3,2,1,2→3	(0.22,0.22,0.56)	5/4/1999	2,3,1,1,1,2,3,2,1→2	(0.44,0.33,0.22)	7/14/1999	1,1,2,3,2,1,2,2,3→1	(0.33,0.44,0.22)	9/18/1999	1,2,1,1,2,3,3,2,1→1	(0.44,0.33,0.22)
2/24/1999	1,3,3,3,1,3,3,2,1→1	(0.33,0.11,0.56)	5/5/1999	3,2,3,1,1,1,2,3,2→3	(0.33,0.33,0.33)	7/15/1999	1,1,1,2,3,2,1,2,2→1	(0.44,0.44,0.11)	9/20/1999	3,1,2,1,1,2,3,3,2→3	(0.33,0.33,0.33)
2/25/1999	3,1,3,3,3,1,3,3,2→3	(0.22,0.11,0.67)	5/6/1999	2,3,2,3,1,1,1,2,3→2	(0.33,0.33,0.33)	7/16/1999	2,1,1,1,2,3,2,1,2→2	(0.44,0.44,0.11)	9/27/1999	1,3,1,2,1,1,2,3,3→1	(0.44,0.22,0.33)
2/26/1999	2,3,1,3,3,3,1,3,3→2	(0.22,0.11,0.67)	5/7/1999	2,2,3,2,3,1,1,1,2→2	(0.33,0.44,0.22)	7/17/1999	1,2,1,1,1,2,3,2,1→1	(0.56,0.33,0.11)	9/28/1999	1,1,3,1,2,1,1,2,3→1	(0.56,0.22,0.22)
3/1/1999	3,2,3,1,3,3,3,1,3→3	(0.22,0.11,0.67)	5/10/1999	1,2,2,3,2,3,1,1,1→1	(0.44,0.33,0.22)	7/19/1999	1,1,2,1,1,1,2,3,2→1	(0.56,0.33,0.11)	9/29/1999	1,1,1,3,1,2,1,1,2→1	(0.67,0.22,0.11)
3/2/1999	2,3,2,3,1,3,3,3,1→2	(0.22,0.22,0.56)	5/11/1999	2,1,2,2,3,2,3,1,1→2	(0.33,0.44,0.22)	7/20/1999	2,1,1,2,1,1,1,2,3→2	(0.56,0.33,0.11)	9/30/1999	2,1,1,1,3,1,2,1,1→2	(0.67,0.22,0.11)
3/3/1999	1,2,3,2,3,1,3,3,3→1	(0.22,0.22,0.56)	5/12/1999	2,2,1,2,2,3,2,3,1→2	(0.22,0.56,0.22)	7/21/1999	3,2,1,1,2,1,1,1,2→3	(0.56,0.33,0.11)	10/1/1999	2,2,1,1,1,3,1,2,1→2	(0.56,0.33,0.11)
3/4/1999	3,1,2,3,2,3,1,3,3→3	(0.22,0.22,0.56)	5/13/1999	2,2,2,1,2,2,3,2,3→2	(0.11,0.67,0.22)	7/22/1999	2,3,2,1,1,2,1,1,1→2	(0.56,0.33,0.11)	10/2/1999	3,2,2,1,1,1,3,1,2→3	(0.44,0.33,0.22)
3/5/1999	2,3,1,2,3,2,3,1,3→2	(0.22,0.33,0.44)	5/14/1999	2,2,2,2,1,2,2,3,2→2	(0.11,0.78,0.11)	7/23/1999	1,2,3,2,1,1,2,1,1→1	(0.56,0.33,0.11)	10/4/1999	2,3,2,2,1,1,1,3,1→2	(0.44,0.33,0.22)
3/6/1999	2,2,3,1,2,3,2,3,1→2	(0.22,0.44,0.33)	5/15/1999	3,2,2,2,2,1,2,2,3→3	(0.11,0.67,0.22)	7/26/1999	3,1,2,3,2,1,1,2,1→3	(0.44,0.33,0.22)	10/5/1999	2,2,3,2,2,1,1,1,3→2	(0.33,0.44,0.22)
3/8/1999	2,2,2,3,1,2,3,2,3→2	(0.11,0.56,0.33)	5/17/1999	2,3,2,2,2,2,1,2,2→2	(0.11,0.78,0.11)	7/27/1999	1,3,1,2,3,2,1,1,2→1	(0.44,0.33,0.22)	10/6/1999	1,2,2,3,2,2,1,1,1→1	(0.44,0.44,0.11)
3/9/1999	2,2,2,2,3,1,2,3,2→2	(0.11,0.67,0.22)	5/18/1999	2,2,3,2,2,2,2,1,2→2	(0.11,0.78,0.11)	7/28/1999	1,1,3,1,2,3,2,1,1→1	(0.56,0.22,0.22)	10/7/1999	1,1,2,2,3,2,2,1,1→1	(0.44,0.44,0.11)
3/10/1999	3,2,2,2,2,3,1,2,3→3	(0.11,0.56,0.33)	5/19/1999	2,2,2,3,2,2,2,2,1→2	(0.11,0.78,0.11)	7/29/1999	3,1,1,3,1,2,3,2,1→3	(0.44,0.22,0.33)	10/8/1999	3,1,1,2,2,3,2,2,1→3	(0.33,0.44,0.22)
3/11/1999	2,3,2,2,2,2,3,1,2→2	(0.11,0.67,0.22)	5/20/1999	2,2,2,2,3,2,2,2,2→2	(0,0.89,0.11)	7/30/1999	1,3,1,1,3,1,2,3,2→1	(0.44,0.22,0.33)	10/11/1999	1,3,1,1,2,2,3,2,2→1	(0.33,0.44,0.22)
3/12/1999	1,2,3,2,2,2,2,3,1→1	(0.22,0.56,0.22)	5/21/1999	2,2,2,2,2,3,2,2,2→2	(0,0.89,0.11)	7/31/1999	3,1,3,1,1,3,1,2,3→3	(0.44,0.11,0.44)	10/12/1999	3,1,3,1,1,2,2,3,2→3	(0.33,0.33,0.33)
3/15/1999	2,1,2,3,2,2,2,2,3→2	(0.11,0.67,0.22)	5/24/1999	2,2,2,2,2,2,3,2,2→2	(0,0.89,0.11)	8/2/1999	1,3,1,3,1,1,3,1,2→1	(0.56,0.11,0.33)	10/13/1999	3,3,1,3,1,1,2,2,3→3	(0.33,0.22,0.44)
3/16/1999	3,2,1,2,3,2,2,2,2→3	(0.11,0.67,0.22)	5/25/1999	2,2,2,2,2,2,2,3,2→2	(0,0.89,0.11)	8/3/1999	1,1,3,1,3,1,1,3,1→1	(0.67,0,0.33)	10/14/1999	2,3,3,1,3,1,1,2,2→2	(0.33,0.33,0.33)
3/17/1999	3,3,2,1,2,3,2,2,2→3	(0.11,0.56,0.33)	5/26/1999	1,2,2,2,2,2,2,2,3→1	(0.11,0.78,0.11)	8/4/1999	2,1,1,3,1,3,1,1,3→2	(0.56,0.11,0.33)	10/15/1999	3,2,3,3,1,3,1,1,2→3	(0.33,0.22,0.44)
3/18/1999	3,3,3,2,1,2,3,2,2→3	(0.11,0.44,0.44)	5/27/1999	2,1,2,2,2,2,2,2,2→2	(0.11,0.89,0)	8/5/1999	1,2,1,1,3,1,3,1,1→1	(0.67,0.11,0.22)	10/16/1999	1,3,2,3,3,1,3,1,1→1	(0.44,0.11,0.44)
3/19/1999	3,3,3,3,2,1,2,3,2→3	(0.11,0.33,0.56)	5/28/1999	2,2,1,2,2,2,2,2,2→2	(0.11,0.89,0)	8/6/1999	1,1,2,1,1,3,1,3,1→1	(0.67,0.11,0.22)	10/18/1999	2,1,3,2,3,3,1,3,1→2	(0.33,0.22,0.44)
3/20/1999	3,3,3,3,3,2,1,2,3→3	(0.11,0.22,0.67)	5/29/1999	1,2,2,1,2,2,2,2,2→1	(0.22,0.78,0)	8/7/1999	1,1,1,2,1,1,3,1,3→1	(0.67,0.11,0.22)	10/19/1999	1,2,1,3,2,3,3,1,3→1	(0.33,0.22,0.44)

Table A2. *Cont.*

Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR	Date (MM/DD/YYYY)	FFLR	LHS of NLR
3/22/1999	2,3,3,3,3,3,2,1,2→2	(0.11,0.33,0.56)	5/31/1999	2,1,2,2,1,2,2,2,2→2	(0.22,0.78,0)	8/9/1999	3,1,1,1,2,1,1,3,1→3	(0.67,0.11,0.22)	10/20/1999	1,1,2,1,3,2,3,3,1→1	(0.44,0.22,0.33)
3/23/1999	3,2,3,3,3,3,3,2,1→3	(0.11,0.22,0.67)	6/1/1999	1,2,1,2,2,1,2,2,2→1	(0.33,0.67,0)	8/10/1999	2,3,1,1,1,2,1,1,3→2	(0.56,0.22,0.22)	10/21/1999	2,1,1,2,1,3,2,3,3→2	(0.33,0.33,0.33)
3/24/1999	1,3,2,3,3,3,3,3,2→1	(0.11,0.22,0.67)	6/2/1999	3,1,2,1,2,2,1,2,2→3	(0.33,0.56,0.11)	8/11/1999	3,2,3,1,1,1,2,1,1→3	(0.56,0.22,0.22)	10/22/1999	2,2,1,1,2,1,3,2,3→2	(0.33,0.44,0.22)
3/25/1999	1,1,3,2,3,3,3,3,3→1	(0.22,0.11,0.67)	6/3/1999	3,3,1,2,1,2,2,1,2→3	(0.33,0.44,0.22)	8/12/1999	2,3,2,3,1,1,1,2,1→2	(0.44,0.33,0.22)	10/25/1999	1,2,2,1,1,2,1,3,2→1	(0.44,0.44,0.11)
3/26/1999	3,1,1,3,2,3,3,3,3→3	(0.22,0.11,0.67)	6/4/1999	3,3,3,1,2,1,2,2,1→3	(0.33,0.33,0.33)	8/13/1999	3,2,3,2,3,1,1,1,2→3	(0.33,0.33,0.33)	10/26/1999	3,1,2,2,1,1,2,1,3→3	(0.44,0.33,0.22)
3/29/1999	3,3,1,1,3,2,3,3,3→3	(0.22,0.11,0.67)	6/5/1999	2,3,3,3,1,2,1,2,2→2	(0.22,0.44,0.33)	8/16/1999	3,3,2,3,2,3,1,1,1→3	(0.33,0.22,0.44)	10/27/1999	2,3,1,2,2,1,1,2,1→2	(0.44,0.44,0.11)
3/30/1999	1,3,3,1,1,3,2,3,3→1	(0.33,0.11,0.56)	6/7/1999	3,2,3,3,3,1,2,1,2→3	(0.22,0.33,0.44)	8/17/1999	3,3,3,2,3,2,3,1,1→3	(0.22,0.22,0.56)	10/28/1999	2,2,3,1,2,2,1,1,2→2	(0.33,0.56,0.11)
3/31/1999	2,1,3,3,1,1,3,2,3→2	(0.33,0.22,0.44)	6/8/1999	3,3,2,3,3,3,1,2,1→3	(0.22,0.22,0.56)	8/18/1999	3,3,3,3,2,3,2,3,1→3	(0.11,0.22,0.67)	10/29/1999	2,2,2,3,1,2,2,1,1→2	(0.33,0.56,0.11)
4/1/1999	2,2,1,3,3,1,1,3,2→2	(0.33,0.33,0.33)	6/9/1999	3,3,3,2,3,3,3,1,2→3	(0.11,0.22,0.67)	8/19/1999	1,3,3,3,3,2,3,2,3→1	(0.11,0.22,0.67)	10/30/1999	2,2,2,2,3,1,2,2,1→2	(0.22,0.67,0.11)
4/2/1999	3,2,2,1,3,3,1,1,3→3	(0.33,0.22,0.44)	6/10/1999	3,3,3,3,2,3,3,3,1→3	(0.11,0.11,0.78)	8/20/1999	2,1,3,3,3,3,2,3,2→2	(0.11,0.33,0.56)			

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