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FUZZY MAGIC LABELLING OF NEUTROSOPHIC PATH AND STAR GRAPH

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ABSTRACT. Graph Labelling is the assignment of labels (integers) to the edges or vertices or to both. Fuzzy labelling of graphs is more detailed and compatible when compared with the classical models. In this paper, fuzzy magic and bi-magic labelling of Neutrosophic path graph is examined. In addition to that the existence of magic value of Intuitionistic and Neutrosophic star graph is also investigated.

1. INTRODUCTION

The phenomena of uncertainty and vagueness are known as fuzzy and the logical approach to this great idea was introduced by Lotfi.A Zadeh, in 1965 [1]. This fuzzy theory has more functional representation of vague concepts in natural languages. The concept of fuzzy has an extensive application in mathematical fields. The concept of graph was presented by Euler in 1936. Graph theory is more useful in modelling the features of system with finite components. A way of representing information involving relationship between objects is called graph, where the vertices and edges of a graph represents the objects and their relation respectively. The graphical models are used to represent telephone network, railway network, communication problems, traffic network, etc.

If there is vagueness in the matter of objects or its relations or in both, we are in need of designing a fuzzy graph model. Fuzzy graph was first defined by Kauffman in 1973, and Rosenfeld developed the theory of fuzzy graph theory

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in 1975 based on the relations [2]. Crisp and fuzzy graphs are similar in structure. Fuzzy graph has more applications in data mining, image segmentation, clustering, image capturing, networking, planning and scheduling.

Graph labelling was introduced by Rosa [3]. A graph labelling is the mapping that carries a set of graph elements onto a set of numbers called labels. There are numerous labelling in graphs and a few of them are graceful, cordial and mean labelling. The concept of fuzzy labelling was introduced by A NagoorGani et al [4]. The notion of bi-magic labelling was introduced by Basker Babujee, in which there are two magic values.

In this paper, taking the lead of magic labelling, we have investigated fuzzy magic, bi-magic labelling of Neutrosophic Path graph and also fuzzy magic labelling of Intuitionistic and Neutrosophic Star graphs.

2. Preliminaries

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \to [0,1]$, where $\forall \quad u, v \in V, \quad \mu(u,v) \leq \sigma(u) \wedge \sigma(v)$. A labelling of a graph is an assignment of values to the vertices and edges of a graph. The graph G is said to be a fuzzy labelling graph if it is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

A fuzzy labelling graph $G = (\sigma, \mu)$ is said to be a fuzzy magic labelling graph, if there exist an m such that $\sigma(x) + \sigma(y) + \mu(xy) = m$ for all $xy \in E$ and $x, y \in V$. A fuzzy labelling graph G is said to be a fuzzy bi-magic labelling graph if there exist m_1 and m_2 such that $\sigma(x) + \sigma(y) + \mu(xy) = m_1$ or m_2 for all $xy \in E \quad and \quad x, y \in V$.

An Intuitionistic Fuzzy Graph is of the form $G = (V, E)$ where $V = \{v_1, v_2, v_3, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively, and

$$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \tag{1}$$

for every $v_i \in V \, (i = 1, 2, 3, \ldots .n)$, $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)],$$

$$\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$$

$0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E \; (i, j = 1, 2, 3, \ldots\ldots, n)$.

An Intuitionistic fuzzy labelling graph is an Intuitionistic fuzzy magic labelling graph if there exists an M such that

$$M = \{\mu_1(v_i) + \mu_1(v_j) + \mu_2(v_i, v_j), \gamma_1(v_i) + \gamma_1(v_j) + \gamma_2(v_i, v_j)\}.$$

An Intuitionistic fuzzy labelling graph is a Intuitionistic fuzzy magic labelling graph $\exists$ M_1, M_2 such that M_1 or $M_2 = \{\mu_1(v_i) + \mu_1(v_j) + \mu_2(v_i, v_j), \gamma_1(v_i) + \gamma_1(v_j) + \gamma_2(v_i, v_j)\}$.

A neutrosophic Fuzzy Graph is $G = (V, E)$ where $V = \{v_1, v_2, v_3, ..., v_n\}$ such that $T_1 : V \to [0,1]$, $I_1 : V \to [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the element $v_i \in V$ respectively, and $0 \leq T_1(v_i) + I_1(v_i) + F_1(v_i) \leq 3$ for every $v_i \in V\,(i = 1, ...n)$, $E \subseteq V \times V$ where $T_2 : V \times V \to [0,1]$, $I_2 : V \times V \to [0,1]$ and $F_2 : V \times V \to [0,1]$ are such that

$$T_2(v_i, v_j) \leq \min[T_1(v_i), T_1(v_j)],$$
$$I_2(v_i, v_j) \leq \min[I_1(v_i), I_1(v_j)],$$
$$F_2(v_i, v_j) \leq \min[F_1(v_i), F_1(v_j)]$$

$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$ for every $(v_i, v_j) \in E\,(i, j = 1, ..., n)$.
A Neutrosophic fuzzy labelling graph is a Neutrosophic fuzzy magic labelling graph if there exists an M such that M equals

$$\begin{aligned}&\{T_1(v_i) + T_1(v_j) + T_2(v_i, v_j),\\ &I_1(v_i) + I_1(v_j) + I_2(v_i, v_j),\\ &F_1(v_i) + F_1(v_j) + F_2(v_i, v_j)\}.\end{aligned}$$

3. Fuzzy Magic and Bi-magic Labelling of Neutrosophic Path Graph

A Neutrosophic path graph satisfies the conditions defined for a neutrosophic graph. And in this section we are going to investigate the magic and bi-magic value of the neutrosophic path graph.

Theorem 3.1. *Any Neutrosophic Path graph $P_n, n \geq 2 \quad (n \in N)$, admits fuzzy magic labelling.*

Proof. Let $n \in N, n \geq 2$, $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{e_j = v_j v_{(j+1)} : 1 \leq j \leq n-1\}$ and Path graph be $P_n = (V, E)$.

For all j, $(1 \leq j \leq n)$, the truth, indeterminacy and false membership functions $\sigma : V \to [0,1]$, $\rho : V \to [0,1]$ $\mu : V \to [0,1]$ respectively for the vertices are defined as follows: $\sigma(v_j) = \begin{cases} \frac{4n-(j+1)}{2(2n-1)} & if \quad j \quad is \quad odd \\ \frac{3n-(j+1)}{2(2n-1)} & if \quad j \quad is \quad even \end{cases}$

$$\rho(v_j) = \begin{cases} \frac{4n-(j+(n-1))}{2(2n-1)} & if\ j\ is\ odd \\ \frac{3n-(j+(n-1))}{2(2n-1)} & if\ j\ is\ even \end{cases}$$

$$\mu(v_j) = \begin{cases} \frac{4n-(j+(n-1))}{10(n-1)} & if \quad j \quad is \quad odd \\ \frac{3n-(j+(n-1))}{10(n-1)} & if \quad j \quad is \quad even \end{cases}$$

The membership of edges $\forall j, \quad (1 \leq j \leq n-1)$, are defined by the functions $\alpha : E \to [0,1]$ by $\alpha(e_j) = \left\{\frac{j}{2n-1}\right\}$, $\beta : E \to [0,1]$ by $\beta(e_j) = \left\{\frac{j}{2n-1}\right\}$, $\gamma : E \to [0,1]$ by $\gamma(e_j) = \left\{\frac{j}{5(n-1)}\right\}$ where $e_j = v_j v_{j+1} \in E$. A fuzzy labelling graph, satisfies the condition $\mu(e_j) < \sigma(v_j) \wedge \sigma(v_{j+1})$.

Truth Value: If j is odd then $\sigma(v_j) = \frac{4n-(j+1)}{2(2n-1)}$ and $j = j+1$, becomes even, then $\sigma(v_{j+1}) = \frac{3n-(j+1)}{2(2n-1)}$.

We know $3n < 4n$, and also $3n-(j+1) < 4n-(j+1)$,

$$\Rightarrow \frac{3n-(j+1)}{2(2n-1)} < \frac{4n-(j+1)}{2(2n-1)},$$

$$\Rightarrow \sigma(v_j) \wedge \sigma(v_{j+1}) = \frac{3n-(j+1)}{2(2n-1)}.$$

The membership of edge is

$$\alpha(e_j) = \left\{\frac{j}{2n-1}\right\} \quad and \quad \frac{j}{2n-1} < \frac{3n-(j+1)}{2(2n-1)}$$

$$\Rightarrow \alpha(e_j) < \sigma(v_j) \wedge \sigma(v_{j+1}).$$

The membership value of neutrosophic graph satisfies fuzzy labelling condition. We check the existence of magic value of neutrosophic path graph.

Existence of Magic value: We have $\sigma(v_j) = \frac{4n-(j+1)}{2(2n-1)}$ and $\sigma(v_{j+1}) = \frac{3n-(j+1)}{2(2n-1)}$.

Thus, $M = \frac{4n-(j+1)}{2(2n-1)} + \frac{3n-(j+1)}{2(2n-1)} + \frac{j}{2n-1} = \frac{7n-2}{2(2n-1)}$.

Indeterminacy Value: If j is odd, we have $\rho(v_j) = \frac{4n-(j+(n-1))}{2(2n-1)}$ and $\rho(v_{j+1}) = \frac{3n-(j+(n-1))}{2(2n-1)}$.

We know $3n < 4n$, and also $3n-(j+(n-1)) < 4n-(j+(n-1))$

$$\Rightarrow \frac{3n-(j+(n-1))}{2\,(2n-1)} < \frac{4n-(j+(n-1))}{2\,(2n-1)}$$

$$\Rightarrow \rho\,(v_j) \wedge \rho\,(v_{j+1}) = \frac{3n-(j+(n-1))}{2\,(2n-1)}.$$

The membership of edge is

$$\beta\,(e_j) = \left\{\frac{j}{2n-1}\right\} \; and \; \frac{j}{2n-1} < \frac{3n-(j+(n-1))}{2\,(2n-1)}$$

$$\Rightarrow \beta\,(e_j) < \rho\,(v_j) \wedge \rho\,(v_{j+1})\,.$$

The membership value is checked and it is verified that neutrosophic graph satisfies fuzzy labelling condition. Now we check the existence of magic value of neutrosophic path graph.

Existence of Magic value: We get $\rho\,(v_j) = \frac{4n-(j+(n-1))}{2(2n-1)}$ and $\rho\,(v_{j+1}) = \frac{3n-(j+(n-1))}{2(2n-1)}$.

Thus, $M = \frac{4n-(j+(n-1))}{2(2n-1)} + \frac{3n-(j+(n-1))}{2(2n-1)} + \frac{j}{2n-1} = \frac{5n+2}{2(2n-1)}$.

False Value: If j is odd, $\mu\,(v_j) = \frac{4n-(j+(n-1))}{10(n-1)}$ and $\mu\,(v_{j+1}) = \frac{3n-(j+(n-1))}{10(n-1)}$.

We know $3n < 4n$, and also $3n-(j+(n-1)) < 4n-(j+(n-1))$

$$\Rightarrow \frac{3n-(j+(n-1))}{10\,(n-1)} < \frac{4n-(j+(n-1))}{10\,(n-1)}$$

$$\Rightarrow \mu\,(v_j) \wedge \mu\,(v_{j+1}) = \frac{3n-(j+(n-1))}{10\,(n-1)}.$$

The membership of edge is

$$\gamma\,(e_j) = \frac{j}{5\,(n-1)} \;\text{ and }\; \frac{j}{5\,(n-1)} < \frac{3n-(j+(n-1))}{10\,(n-1)}$$

$$\Rightarrow \gamma\,(e_j) < \mu\,(v_j) \wedge \mu\,(v_{j+1})\,.$$

Thus the neutrosophic graph satisfies fuzzy labelling condition. Next we check the existence of magic value of neutrosophic path graph.

Existence of Magic value: We have

$$\mu\,(v_j) = \frac{4n-(j+(n-1))}{10\,(n-1)}$$

and

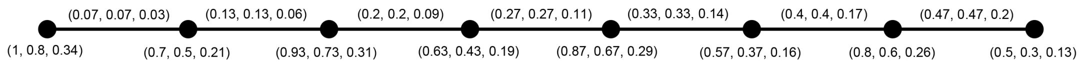
$$\mu\,(v_{j+1}) = \frac{3n-(j+(n-1))}{10\,(n-1)}.$$

Thus, $M = \frac{4n-(j+(n-1))}{10(n-1)} + \frac{3n-(j+(n-1))}{10(n-1)} + \frac{j}{5(n-1)} = \frac{5n+2}{10(n-1)}$.

Therefore the Magic value of Neutrosophic path graph is

$$M = \left(\frac{7n-2}{2(2n-1)}, \frac{5n+2}{2(2n-1)}, \frac{5n+2}{10(n-1)}\right). \qquad \square$$

Example 3.2.



The magic value of the above Neutrosophic Path graph P_8 is $(1.8, 1.4, 0.6)$.

Theorem 3.3. *Any Neutrosophic Path graph $P_n, n \geq 3 \quad (n \in N)$, admits fuzzy bi - magic labelling.*

Proof. Let $n \in N, n \geq 3, V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{e_j = v_j v_{j+1} : 1 \leq j \leq n-1\}$ and path graph be $P_n = (V, E)$.

For all j $(1 \leq j \leq n)$, the truth, indeterminacy and false membership for the vertices $\sigma : V \to [0,1]$, $\rho : V \to [0,1]$, $\mu : V \to [0,1]$ are defined respectively as follows:

$$\sigma(v_j) = \begin{cases} \frac{4n-(j+1)}{2(2n-1)} & if \quad j \quad is \quad odd \\ \frac{3n-(j+1)}{2(2n-1)} & if \quad j \quad is \quad even \end{cases}$$

$$\rho(v_j) = \begin{cases} \frac{4n-(j+(n-1))}{2(2n-1)} & if \quad j \quad is \quad odd \\ \frac{3n-(j+(n-1))}{2(2n-1)} & if \quad j \quad is \quad even \end{cases}$$

$$\mu(v_j) = \begin{cases} \frac{4n-(j+(n-1))}{10(n-1)} & if \quad j \quad is \quad odd \\ \frac{3n-(j+(n-1))}{10(n-1)} & if \quad j \quad is \quad even \end{cases}$$

The truth, indeterminacy and false membership of edges $\forall j, \quad (1 \leq j \leq n-1)$, are defined by the function

$$\alpha : E \to [0,1] \;\; by \; \alpha(e_j) = \begin{cases} \frac{j}{2n-4} & j - odd \\ \frac{j}{2n+5} & j - even \end{cases}$$

$$\beta : E \to [0,1] \;\; by \; \beta(e_j) = \begin{cases} \frac{j}{2n-4} & j - odd \\ \frac{j}{2n+3} & j - even \end{cases}$$

$$\gamma : E \to [0,1] \;\; by \; \gamma(e_j) = \begin{cases} \frac{j}{5(n-4)} & j - odd \\ \frac{j}{5(n+5)} & j - even \end{cases}$$

where $e_j = v_j v_{j+1} \in E$.

A fuzzy labelling graph, satisfies the condition $\alpha\left(e_j\right) < \sigma\left(v_j\right) \wedge \sigma\left(v_{j+1}\right)$.

Truth Value: If j is odd then $\sigma\left(v_j\right) = \frac{4n-(j+1)}{2(2n-1)}$ and $\sigma\left(v_{j+1}\right) = \frac{3n-(j+1)}{2(2n-1)}$.

We know $3n < 4n$, and also $3n - (j+1) < 4n - (j+1)$

$$\Rightarrow \frac{3n-(j+1)}{2\left(2n-1\right)} < \frac{4n-(j+1)}{2\left(2n-1\right)}$$

$$\Rightarrow \sigma\left(v_j\right) \wedge \sigma\left(v_{j+1}\right) = \frac{3n-(j+1)}{2\left(2n-1\right)}.$$

The membership of edge is $\alpha\left(e_j\right) = \begin{cases} \frac{j}{2n-4} \\ \frac{j}{2n+5} \end{cases}$, $\frac{j}{2n-4} < \frac{3n-(j+1)}{2(2n-1)} \Rightarrow \frac{j}{2n+5} < \frac{3n-(j+1)}{2(2n-1)}$

$\Rightarrow \alpha\left(e_j\right) < \sigma\left(v_j\right) \wedge \sigma\left(v_{j+1}\right)$.

Thus the truth value of neutrosophic path graph satisfies fuzzy labelling condition. Now we check the existence of magic value of truth membership value.

Existence of Bi-Magic value: We have $\sigma\left(v_j\right) = \frac{4n-(j+1)}{2(2n-1)}$ and $\sigma\left(v_{j+1}\right) = \frac{3n-(j+1)}{2(2n-1)}$

Subcase a: when j is odd, $\alpha\left(e_j\right) = \frac{j}{2n-4}$. Thus,

$$M = \frac{4n-(j+1)}{2(2n-1)} + \frac{3n-(j+1)}{2(2n-1)} + \frac{j}{2n-4}$$

$$M = \frac{7n-4}{2(2n-1)} + \frac{1}{2n-4}$$

Subcase b: when j is even, $\alpha\left(e_j\right) = \frac{j}{2n+5}$. Thus,

$$M = \frac{4n-(j+1)}{2(2n-1)} + \frac{3n-(j+1)}{2(2n-1)} + \frac{j}{2n+5}$$

$$M = \frac{7n-6}{2(2n-1)} + \frac{1}{2n+5}.$$

Indeterminacy Value: If j is odd, we have $\rho\left(v_j\right) = \frac{4n-(j+(n-1))}{2(2n-1)}$ and $\rho\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{2(2n-1)}$. We know $3n < 4n$, and also

$$3n - (j + (n-1)) < 4n - (j + (n-1)),$$

$$\Rightarrow \frac{3n-(j+(n-1))}{2(2n-1)} < \frac{4n-(j+(n-1))}{2(2n-1)},$$

$$\Rightarrow \rho\left(v_j\right) \wedge \rho\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{2(2n-1)}.$$

The membership of edge is $\beta\left(e_j\right) = \begin{cases} \frac{j}{2n-4} \\ \frac{j}{2n+3} \end{cases}$, $\frac{j}{2n-4} < \frac{3n-(j+(n-1))}{2(2n-1)} \Rightarrow \frac{j}{2n+3} < \frac{3n-(j+(n-1))}{2(2n-1)} \Rightarrow \beta\left(e_j\right) < \rho\left(v_j\right) \wedge \rho\left(v_{j+1}\right)$.

It is verified that neutrosophic graph satisfies fuzzy labelling condition. Now we check the existence of magic value of neutrosophic path graph.

Existence of Bi-Magic value:

We have $\rho\left(v_j\right) = \frac{4n-(j+(n-1))}{2(2n-1)}$ and $\rho\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{2(2n-1)}$

Subcase a: when j is odd, $\beta\left(e_j\right) = \frac{j}{2n-4}$. Thus, $M = \frac{4n-(j+(n-1))}{2(2n-1)} + \frac{3n-(j+(n-1))}{2(2n-1)} + \frac{j}{2n-4}$. $M = \frac{5n}{2(2n-1)} + \frac{1}{2n-4}$.

Subcase b: when j is even, $\beta\left(e_j\right) = \frac{j}{2n+3}$. Thus, $M = \frac{4n-(j+(n-1))}{2(2n-1)} + \frac{3n-(j+(n-1))}{2(2n-1)} + \frac{j}{2n+3}$. $M = \frac{5n-2}{2(2n-1)} + \frac{1}{2n+3}$.

False Value: If j is odd, $\mu\left(v_j\right) = \frac{4n-(j+(n-1))}{10(n-1)}$ and $\mu\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{10(n-1)}$.

We know $3n < 4n$, and also $3n - (j + (n-1)) < 4n - (j + (n-1))$

$\Rightarrow \frac{3n-(j+(n-1))}{10(n-1)} < \frac{4n-(j+(n-1))}{10(n-1)} \Rightarrow \mu\left(v_j\right) \wedge \mu\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{10(n-1)}$.

The membership of edge is $\gamma\left(e_j\right) = \begin{cases} \frac{j}{5(n-4)} \\ \frac{j}{5(n+5)} \end{cases}$

$$\frac{j}{5\left(n-4\right)} < \frac{3n-\left(j+\left(n-1\right)\right)}{10\left(n-1\right)} \Rightarrow \frac{j}{5\left(n+5\right)} < \frac{3n-\left(j+\left(n-1\right)\right)}{10\left(n-1\right)}$$

$\Rightarrow \gamma\left(e_j\right) < \mu\left(v_j\right) \wedge \mu\left(v_{j+1}\right)$. It is verified that neutrosophic graph satisfies fuzzy labelling condition. Now we check the existence of magic value of neutrosophic path graph.

Existence of Bi-Magic value:

We have $\mu\left(v_j\right) = \frac{4n-(j+(n-1))}{10(n-1)}$ and $\mu\left(v_{j+1}\right) = \frac{3n-(j+(n-1))}{10(n-1)}$

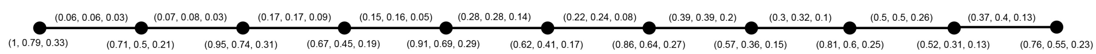
Subcase a: when j is odd, $\gamma\left(e_j\right) = \frac{j}{5(n-4)}$. Thus, $M = \frac{4n-(j+(n-1))}{10(n-1)} + \frac{3n-(j+(n-1))}{10(n-1)} + \frac{j}{5(n-4)}$. $M = \frac{5n}{10(n-1)} + \frac{1}{5(n-4)}$.

Subcase b: when j is even, $\gamma\left(e_j\right) = \frac{j}{5(n+5)}$. Thus, $M = \frac{4n-(j+(n-1))}{10(n-1)} + \frac{3n-(j+(n-1))}{10(n-1)} + \frac{j}{5(n+5)}$. $M = \frac{5n-2}{10(n-1)} + \frac{1}{5(n+5)}$.

Thus the bi-magic value of Neutrosophic path graph is

$$M_1, M_2 = \begin{cases} \left(\frac{7n-4}{2(2n-1)} + \frac{1}{2n-4}, \frac{5n}{2(2n-1)} + \frac{1}{2n-4}, \frac{5n}{10(n-1)} + \frac{1}{5(n-4)}\right) & if \quad j \quad is \quad odd \\ \left(\frac{7n-6}{2(2n-1)} + \frac{1}{2n+5}, \frac{5n-2}{2(2n-1)} + \frac{1}{2n+3}, \frac{5n-2}{10(n-1)} + \frac{1}{5(n+5)}\right) & if \quad j \quad is \quad even \end{cases} \quad \square$$

Example 3.4.



The bi-magic value of the above Neutrosophic Path graph P_{11} *is* $(1.8, 1.4, 0.6)$ *and* $(1.7, 1.3, 0.5)$.

4. Fuzzy Magic Labelling of Intuitionistic & Neutrosophic star Graph

Now we are going to find the existence of magic value for intuitionistic and neutrosophic star graph.

Theorem 4.1. *Any Intuitionistic Star graph* S_n, $n \geq 2$ *admits fuzzy magic labelling.*

Proof. Let $n \in N, n \geq 2$, $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{e_j = v_j v_{j+1} : 1 \leq j \leq n-1\}$ and Path graph be $S_n = (V, E)$.

For all j, $(1 \leq j \leq n)$, the membership value and the non-membership value for the vertices $\sigma : V \to [0,1]$, $\rho : V \to [0,1]$ are defined respectively as follows: $\sigma(v_j) = \frac{2n-(j-1)}{2(2n-1)}$ and $\rho(v_j) = \frac{2n-(j-1)}{3(2n-1)}$.

The membership and non-membership values of edges $\forall j, (1 \leq j \leq n-1)$, are defined by the functions $\alpha : E \to [0,1]$ by $\alpha(e_j) = \left\{\frac{j}{2(2n-1)}\right\}$ and $\beta : E \to [0,1]$ by $\beta(e_j) = \left\{\frac{j}{3(2n-1)}\right\}$ where $e_j = v_j v_{j+1} \in E$.

Clearly, the fuzzy labelling condition is satisfied for the defined membership and non-membership values. Thus now the crux of the theorem is the existence of Magic value of Intuitionistic star graph S_n.

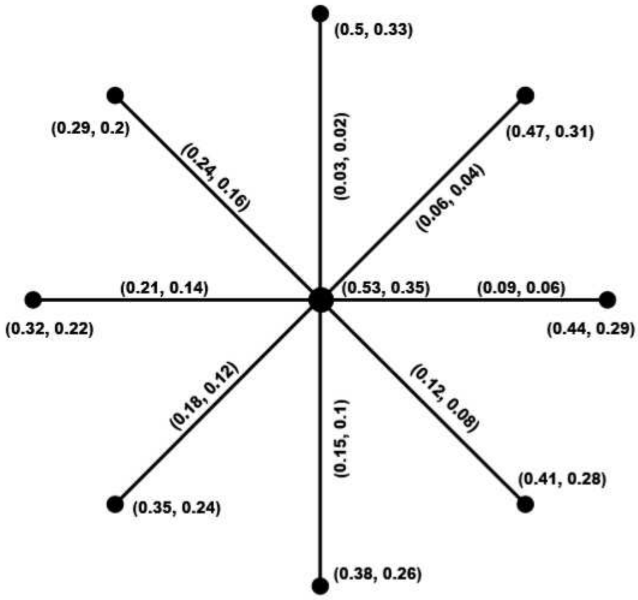
Existence of Magic value for membership value:

In star graph, all the vertices are incidented with the vertex v_1 and thus we have $\sigma(v_1) = \frac{2n}{2(2n-1)}$ and $\sigma(v_j) = \frac{2n-(j-1)}{2(2n-1)}$. Thus, $M = \frac{2n}{2(2n-1)} + \frac{2n-(j-1)}{2(2n-1)} + \frac{j}{2(2n-1)} = \frac{4n+1}{2(2n-1)}$.

Existence of Magic value for non-membership value:

In star graph all the vertices are incidented with the vertex v_1 and thus we have $\sigma(v_1) = \frac{2n}{3(2n-1)}$ and $\sigma(v_j) = \frac{2n-(j-1)}{3(2n-1)}$. Thus, $M = \frac{2n}{3(2n-1)} + \frac{2n-(j-1)}{3(2n-1)} + \frac{j}{3(2n-1)} = \frac{4n+1}{3(2n-1)}$. Therefore the magic value of the Intuitionistic star graph is $M = \left(\frac{4n+1}{2(2n-1)}, \frac{4n+1}{3(2n-1)}\right)$. □

Example 4.2.



The magic value of the above Intuitionistic Star graph S_9 *is* $(1.1, 0.7)$.

Theorem 4.3. *Any Neutrosophic star graph* S_n*,* $n \geq 2$ *has fuzzy magic labelling.*

Proof. Let $n \in N, n \geq 2$, $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{e_j = v_j v_{j+1} : 1 \leq j \leq n-1\}$ and Path graph be $S_n = (V, E)$.

For all $j \quad (1 \leq j \leq n)$, the truth, indeterminacy and false membership for the vertices $\sigma : V \to [0,1], \quad \rho : V \to [0,1], \quad \mu : V \to [0,1]$ are defined as follows: $\sigma(v_j) = \frac{4n-(j+(n-1))}{2(2n+1)}, \quad \rho(v_j) = \frac{2n-(j-1)}{2(2n-1)}, \quad \mu(v_j) = \frac{2n-(j-1)}{3(2n-1)}$.

The truth, indeterminacy and false membership of edges $\forall j, \quad (1 \leq j \leq n-1)$, are defined by the functions $\alpha : E \to [0,1]$ by $\alpha(e_j) = \left\{\frac{j}{2(2n+1)}\right\}, \quad \beta : E \to [0,1]$ by $\beta(e_j) = \left\{\frac{j}{2(2n-1)}\right\}, \quad \gamma : E \to [0,1]$ by $\gamma(e_j) = \left\{\frac{j}{3(2n-1)}\right\}$ respectively, where $e_j = v_j v_{j+1} \in E$.

Clearly, the fuzzy labelling condition is satisfied for the truth, indeterminacy and false membership defined. Thus now the crux of the theorem is the existence of Magic value of Neutrosophic star graph S_n.

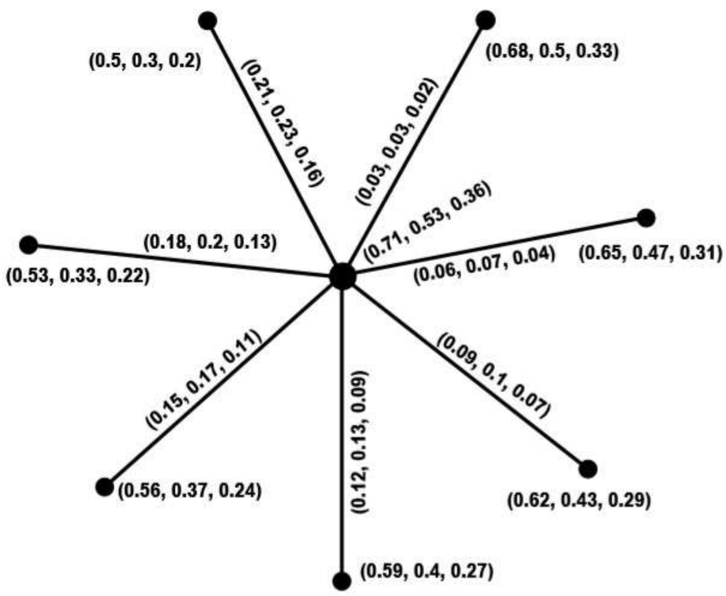
Existence of Magic value for truth value: In star graph, all vertices are incidented with the vertex v_1 and thus we have $\sigma(v_1) = \frac{3n}{2(2n+1)}$ and $\sigma(v_j) =$

$\frac{4n-(j+(n-1))}{2(2n+1)}$. Thus, $M = \frac{3n}{2(2n+1)} + \frac{4n-(j+(n-1))}{2(2n+1)} + \frac{j}{2(2n+1)} = \frac{6n+1}{2(2n+1)}$.

Existence of Magic value for indeterminacy value: All the vertices are incidented with the vertex v_1 and thus we have $\sigma(v_1) = \frac{2n}{2(2n-1)}$ and $\sigma(v_j) = \frac{2n-(j-1)}{2(2n-1)}$. Thus, $M = \frac{2n}{2(2n-1)} + \frac{2n-(j-1)}{2(2n-1)} + \frac{j}{2(2n-1)} = \frac{4n+1}{2(2n-1)}$.

Existence of Magic value for false value: All the vertices are incidented with the vertex v_1, thus we have $\sigma(v_1) = \frac{2n}{3(2n-1)}$ and $\sigma(v_j) = \frac{2n-(j-1)}{3(2n-1)}$. Thus, $M = \frac{2n}{3(2n-1)} + \frac{2n-(j-1)}{3(2n-1)} + \frac{j}{3(2n-1)} = \frac{4n+1}{3(2n-1)}$. Therefore the magic value of Neutrosophic star graph is $M = \left(\frac{6n+1}{2(2n+1)}, \frac{4n+1}{2(2n-1)}, \frac{4n+1}{3(2n-1)}\right)$. □

Example 4.4.



The magic value of the above Neutrosophic Star graph S_8 *is* $(1.4, 1.1, 0.7)$.

5. Conclusion

In this paper, the existence of magic value for Neutrosophic path graph and the bi-magic value of Neutrosophic path graph are investigated. In addition to that magic values of Intuitionistic and Neutrosophic star graphs have been found.

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