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Fuzzy neutrosophic Soft Ideal Theory Fuzzy neutrosophic Soft Local Function and Generated Fuzzy neutrosophic Soft Topological Spaces

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Abstract*:* The purpose of this paper is to introduce the notion of fuzzy neutrosophic soft ideal in fuzzy neutrosophic soft set theory. The concept of fuzzy neutrosophic soft local function is also introduced. These concepts are discussed with a view to find new fuzzy neutrosophic soft topologies from the original one. The basic structure, especially a basis for such generated fuzzy neutrosophic soft topologies also studied here. Finally, the notion of compatibility of fuzzy neutrosophic soft ideals with fuzzy neutrosophic soft topologies is introduced and some equivalent conditions concerning this topic are established here.

Keywords: Soft set, Neutrosophic Soft, fuzzy neutrosophic Soft topological space , fuzzy neutrosophic Soft ideal, fuzzy neutrosophic Soft local function, $*$- fuzzy neutrosophic soft topology, Compatible fuzzy neutrosophic soft ideal.

1. Introduction:

The fuzzy set was presented by Zadeh [16] in 1965 where every element had a degree of membership. The intuitionistic fuzzy set on a universe X was presented by K. Atanaasov [3] in 1983 as a generalization of fuzzy set. The idea of Neutrosophic set was presented by F. Smarandache [14] which is a mathematical instrument for dealing with issues including uncertain, indeterminacy and conflicting information. In 1999, Molodtsov [10], initiated the novel concept of soft set theory, which was a totally new methodology for demonstrating vulnerability and had a rich potential for application in a few ways is free from the troubles influencing existing techniques. P. K. Maji [9] had joined the Neutrosophic set with delicate sets and presented another scientific model 'Neutrosophic soft set'. A new notion of Fuzzy Neutrosophic soft set and its basic operations and results in Fuzzy Neutrosophic soft spaces are obtained.[1]. In 1966 Kuratowski [8] and Vaidyanathaswamy [15], introduced the concept of ideal in topological space and defined local function in ideal topological space. Further Hamlett and Jankovic in [6] studied the properties of ideal topological spaces.

Salama at el [11] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [13]. Also, in 2014 A.A. Salama et al.[12] introduced and studied the concept of neutrosophic crisp local function, and construct a new type of neutrosophic crisp topological space via neutrosophic crisp ideals.

The purpose behind this paper is to present the idea of fuzzy neutrosophic soft ideal in fuzzy neutrosophic soft set theory which is the generalization of the concept of fuzzy intuitionistic ideal topological concepts and soft ideal topological concepts, first initiated by Salama [11], Kandil et al.[7] sequentially. The concept of fuzzy neutrosophic soft local function is also introduced. These ideas are talked about so as to discover new fuzzy neutrosophic soft topologies from the original one. The basic structure, especially a basis for such generated fuzzy neutrosophic soft topologies also studied here. At long last, the thought of compatibility of fuzzy neutrosophic soft ideals with fuzzy neutrosophic soft topologies is presented and some equivalent conditions concerning this theme are built up here.

2. Preliminaries

For the definitions and results on fuzzy neutrosophic soft set theory, we refer to [1,4,5]. However, we recall some definitions and results on fuzzy neutrosophic soft set theory and fuzzy neutrosophic soft topology.

Definition 2.1 [2]: A fuzzy neutrosophic set A on the universe X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >; x \in X\}$ where the function $T, I, F: X \to [0,1]$. denote the degree of membership function (namely $T_A(x)$), the degree of indeterminacy function (namely $I_A(x)$) and the degree of non-membership (namely $F_A(x)$) respectively of each element $x \in X$ to the set A and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

Definition 2.2 [10]: Let X be a universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a function from E into the set of all subsets of X , i.e. $: E \to P(X)$, where $P(X)$ is the power set of X. The set of all soft sets over X is denoted by $SS(X, E)$.

Definition 2.3 [1]: Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, $A \subseteq E$. Let $FNS(U)$ denote the set of all fuzzy neutrosophic sets of U. The collection $(N\tilde{F}, A)$ is termed to be the fuzzy neutrosophic soft set (FNS set or FNSS for short) over U , where $N\tilde{F}$ is a mapping given by $N\tilde{F}: A \to FNS(U)$. The family of all these fuzzy neutrosophic soft set over U with parameters from E denoted by $FNSS(U)_E$.

Definition 2.4 [1]: The complement of a fuzzy neutrosophic soft set $(N\tilde{F}, E)$ denoted by $(N\tilde{F}, A)^c$and is defined as$(N\tilde{F}, A)^c = (N\tilde{F}^c, A)$ where $N\tilde{F}^c: A \to FNS(U)$ is a mapping given by $N\tilde{F}^c(e) = \{< x, T_{N\tilde{F}^c(e)}(x) = F_{N\tilde{F}(e)}(x), I_{N\tilde{F}^c(e)}(x) = 1 - I_{N\tilde{F}(e)}(x), F_{N\tilde{F}^c(e)}(x) = T_{\tilde{F}(e)}(x) >\}$.

Definition 2.5 [1]: A fuzzy neutrosophic soft set $(N\tilde{F}, E)$ over the universe U is said to be

(1) empty fuzzy neutrosophic soft set with respect to the parameter E if $T_{N\tilde{F}(e)} = 0, I_{N\tilde{F}(e)} = 0, F_{N\tilde{F}(e)} = 1, \forall x \in U, \forall e \in E \cdot it\ is\ denoted\ by\ \tilde{0}_E^U$.

(2) universe fuzzy neutrosophic soft set with respect to the parameter E if $T_{N\tilde{F}(e)} = 1\,, I_{N\tilde{F}(e)} = 1\,, F_{N\tilde{F}(e)} = 0\,, \forall x \in U, \forall e \in E \cdot$ *it is denoted by* $\tilde{1}_E^U$.

Note: $(\tilde{0}_E^U)^c = \tilde{1}_E^U$ *and* $(\tilde{1}_E^U)^c = \tilde{0}_E^U$

Definition 2.6 [4]:

1. A fuzzy neutrosophic soft set $(N\tilde{F}, E)$ is said to be a fuzzy neutrosophic soft point, denoted by $e_{N\tilde{F}}$, if for the element e ∈ E , $N\tilde{F}(e) \neq \tilde{0}_N$ and $N\tilde{F}(e') = \tilde{0}_N$, ∀e' ∈ E − {e}. A class of all fuzzy neutrosophic soft points in U is denoted as $FNSP(U)_E$

2. The complement of a fuzzy neutrosophic soft point $e_{N\tilde{F}}$ is a fuzzy neutrosophic soft point $e^c_{N\tilde{F}}$ such that $N\tilde{F}^c$(e) = $\left(N\tilde{F}(e)\right)^c$

3. A fuzzy neutrosophic soft point $e_{N\tilde{F}}$ is said to be in a fuzzy neutrosophic soft set $(N\tilde{G}, E)$ (or $e_{N\tilde{F}}$ is a fuzzy neutrosophic soft point of $(N\tilde{G}, E)$), denoted by $e_{N\tilde{F}} \in (N\tilde{G}, E)$ if for the element e ∈ E, $N\tilde{F}(e) \subseteq N\tilde{G}(e)$.

Example 2.7:

Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then,

$$e_{1N\tilde{F}} = \{< x_1, (0.6, 0.4, 0.8) >, < x_2, (0.8, 0.3, 0.5) >, < x_3, (0.3, 0.7, 0.6) >\}$$

is a fuzzy neutrosophic soft point whose complement is

$$e^c_{1N\tilde{F}} = \{< x_1, (0.8, 0.6, 0.6) >, < x_2, (0.5, 0.7, 0.8) >, < x_3, (0.6, 0.3, 0.3) >\}.$$

For another FNSS $(N\tilde{G}, E)$ defined on same (U, E),

let, $N\tilde{G}\,(e_1) = \{< x_1, (0.7, 0.4, 0.7) >, < x_2\,, (0.8, 0.5, 0.4) >, < x_3, (0.5, 0.8, 0.5) >\}$.
Then, $N\tilde{F}(e_1) \leq N\tilde{G}(e_1)$ i.e., $e_{1N\tilde{F}} \in (N\tilde{G}, E)$.

Theorem 2.8: if $(N\tilde{F}, E)$ be a fuzzy neutrosophic soft set of U. implies $\left(N\tilde{F}, E\right) = \tilde{\cup}\{e_{N\tilde{F}} : e_{N\tilde{F}} \in (N\tilde{F}, E)\}$

Proof. Let $E = \{e_i : i \in I\}$, $(N\tilde{F}, E)$ be a fuzzy neutrosophic soft set and $\{e_{N\tilde{G}_k}\}_{k\in\Omega}$ be family of all fuzzy neutrosophic soft points of $(N\tilde{F}, E)$. For all $e_i \in E$, because of

$$N\tilde{F}(e_i) = \cup_{k\in\Omega} \tilde{G}_k(e_i)$$

we have $\left(N\tilde{F}, E\right) = \{N\tilde{F}(e_i) : e_i \in E\} = \tilde{\cup}_{k\in\Omega}\tilde{G}_k$.

Theorem 2.9: Let $(N\tilde{F}, E)$ and $(N\tilde{G}, E)$ be two fuzzy neutrosophic soft sets in $FNSS(U)_E$. Then $(N\tilde{F}, E) \tilde{\subseteq} (N\tilde{G}, E)$ if and only if $e_{N\tilde{H}} \in (N\tilde{F}, E)$ implies $e_{N\tilde{H}} \in (N\tilde{G}, E)$.

Proof. (⇒) : Let $(N\tilde{F}, E) \tilde{\subseteq} (N\tilde{G}, E)$. Therefore, for all e ∈ E, $N\tilde{F}(e) \tilde{\subseteq} N\tilde{G}(e)$. If $e_{N\tilde{H}} \in (N\tilde{F}, E)$, then $N\tilde{H}(e) \tilde{\subseteq} N\tilde{F}(e)$. Because of $N\tilde{H}(e) \tilde{\subseteq} N\tilde{F}(e) \tilde{\subseteq} N\tilde{G}(e)$, we have $N\tilde{H}(e) \tilde{\subseteq} N\tilde{G}(e)$ and so, $e_{N\tilde{H}} \in (N\tilde{G}, E)$.

(⇐) : If, for every $e_{N\tilde{H}} \in (N\tilde{F}, E)$ implies $e_{N\tilde{H}} \in (N\tilde{G}, E)$ implies, then in accordance with Theorem 2.10,

$\tilde{\cup}_{e_{N\tilde{H}} \in (\tilde{F}.E)} e_{N\tilde{H}} = (N\tilde{F}, E)$ and this implies $\tilde{\cup}_{e_{N\tilde{H}} \in (N\tilde{G},E)} e_{N\tilde{H}} = (N\tilde{G}, E)$. So, $(N\tilde{F}, E) \tilde{\subseteq} (N\tilde{G}, E)$.

Definition 2.10 [5]: Let $e_{N\tilde{F}}$ and $e'_{N\tilde{G}}$ be two fuzzy neutrosophic soft points. For the fuzzy neutrosophic soft points $e_{N\tilde{F}}$ and $e'_{N\tilde{G}}$ over a common universe U , we say that the fuzzy neutrosophic soft points are distinct points if $e_{N\tilde{F}} \tilde{\cap} e'_{N\tilde{G}} = \tilde{0}_E^U$. It is clear that $e_{N\tilde{F}}$ and $e'_{N\tilde{G}}$ are distinct fuzzy neutrosophic soft points if and only if $e \neq e'$ or $\neq y$.

Definition 2.11 [1]: Let $(N\tilde{F}, E)$be FNS set on (U, E) and τ be a collection of fuzzy neutrosophic soft subsets of $(N\tilde{F}, E)$. $(N\tilde{F}, E)$ is called fuzzy neutrosophic soft topology (FNST) if the following conditions hold.

(i) $\tilde{0}_E^U$, $\tilde{1}_E^U \in \tau$

(ii) $(N\tilde{F},E), (N\tilde{G},E) \in \tau$ implies $(N\tilde{F},E) \,\tilde{\cap}\, (N\tilde{G},E) \in \tau$

(iii) $\{ (N\tilde{F}_\alpha, E) : \alpha \in \Gamma \} \in \tau$ implies $\tilde{\bigcup}\{(N\tilde{F}_\alpha, E) : \alpha \in \Gamma\} \in \tau$

The triplet (U, τ, E) is called an Fuzzy neutrosophic soft topological space (FNSTS) over U. Every member of τ is called an fuzzy neutrosophic soft open set in U. $(N\tilde{F},E)$ is called an fuzzy neutrosophic soft closed set in U if $(N\tilde{F},E) \in \tau^c$, where $\tau^c = \{(N\tilde{F},E)^c : (N\tilde{F},E) \in \tau\}$.

Definition 2.12: Let $\hat{\mathcal{J}}$ be a non-null collection of fuzzy neutrosophic soft sets over a universe U with a fixed set of parameters E is called a fuzzy neutrosophic soft ideal (FNSL for short) on U if

(1) $(N\tilde{F},E) \in \hat{\mathcal{J}}$ and $(N\tilde{G},E) \in \hat{\mathcal{J}} \Rightarrow (N\tilde{F},E) \,\tilde{\cup}\, (N\tilde{G},E) \in \hat{\mathcal{J}}$ [Finite additivity],

(2) $(N\tilde{F},E) \in \hat{\mathcal{J}}$ and $(N\tilde{G},E) \,\tilde{\subseteq}\, (N\tilde{F},E) \Rightarrow (N\tilde{G},E) \in \hat{\mathcal{J}}$ [heredity].

If (U,τ,E) be a Fuzzy neutrosophic soft topological space , then $(U,\tau,E,\hat{\mathcal{J}})$ is called a Fuzzy neutrosophic soft ideal topological space (FNSITS, for short).

3. Fuzzy Neutrosophic Soft Local Function and Generated Fuzzy Neutrosophic Soft Topology

Definition 3.1: Let (U,τ,E) be a fuzzy neutrosophic soft topological space and $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. $(N\tilde{F},E)^*(\hat{\hat{\mathcal{J}}},\tau)$ *or* $(N\tilde{F},E)^* = \tilde{\bigcup}\{ e_{N\tilde{F}} \in FNSP(U)_E : (N\tilde{O},E)_{e_{N\tilde{F}}} \,\tilde{\cap}\, (N\tilde{F},E) \notin \hat{\mathcal{J}},\ \forall\, (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau\}$

is called the fuzzy neutrosophic soft local function of $(N\tilde{F},E)$ with respect to $\hat{\mathcal{J}}$ and τ, where $(N\tilde{O},E)_{e_{N\tilde{F}}}$ is a τ-open fuzzy neutrosophic soft set containing $e_{N\tilde{F}}$.

Example 3.2:

(1) If $\hat{\mathcal{J}} = \{ \tilde{0}_E^U \}$, then $(N\tilde{F},E)^*(\hat{\mathcal{J}},\tau) = FNScl(N\tilde{F},E)$ for all $(N\tilde{F},E) \in FNSS(U)_E$.

(2) If $\hat{\mathcal{J}} = FNSS(U)_E$, then $(N\tilde{F},E)^*(\hat{\mathcal{J}},\tau) = \tilde{0}_E^U$ for all $(N\tilde{F},E) \in FNSS(U)_E$.

Theorem 3.3: Let $\hat{\mathcal{J}}$ and $\hat{\mathcal{J}}_1$ be any two fuzzy neutrosophic soft ideals with the same set of parameters E on a fuzzy neutrosophic soft topological space (U,τ,E). Let $(N\tilde{F},E),(N\tilde{G},E) \in FNSS(U)_E$. Then

(1) $(\tilde{0}_E^U)^* = \tilde{0}_E^U$,

(2) $(N\tilde{F},E) \tilde{\subseteq} (N\tilde{G},E) \Longrightarrow (N\tilde{F},E)^* \tilde{\subseteq} (N\tilde{G},E)^*$,

(3) $\hat{\mathcal{J}} \tilde{\subseteq} \hat{\mathcal{J}}_1 \Longrightarrow (N\tilde{F},E)^*(\hat{\mathcal{J}}_1,\tau) \tilde{\subseteq} (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau)$,

(4) $(N\tilde{F},E)^* = FNScl(N\tilde{F},E)^* \tilde{\subseteq} FNScl(N\tilde{F},E)$, where $FNScl$ is the fuzzy neutrosophic soft closure w.r.t. τ,

(5) $(N\tilde{F},E)^*$ is τ-closed fuzzy neutrosophic soft set.

(6) $((N\tilde{F},E)^*)^* \tilde{\subseteq} (N\tilde{F},E)^*$,

(7) $((N\tilde{F},E) \tilde{\cup} (N\tilde{G},E))^* = (N\tilde{F},E)^* \tilde{\cup} (N\tilde{G},E)^*$,

(8) $((N\tilde{F},E) \tilde{\cap} (N\tilde{G},E))^* \tilde{\subseteq} (N\tilde{F},E)^* \tilde{\cap} (N\tilde{G},E)^*$,

(9) $(N\tilde{H},E) \in \hat{\mathcal{J}} \Longrightarrow ((N\tilde{F},E) \tilde{\cup} (N\tilde{H},E))^* = (N\tilde{F},E)^*$

Proof.

(1) $(\tilde{0}_E^U)^* = \tilde{\cup}\{ e_{N\tilde{F}} \in \tilde{1}_E^U : (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} \tilde{0}_E^U = \tilde{0}_E^U \in \hat{\mathcal{J}} \ \forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau\} = \tilde{0}_E^U$.

(2) Let $e_{N\tilde{F}} \in (N\tilde{F},E)^*$. Then
$(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}} \ \forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. Since

$(N\tilde{F},E) \tilde{\subseteq} (N\tilde{G},E) \Longrightarrow$
$(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \tilde{\subseteq} (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E)$ and
$(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$. Then $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E) \notin \hat{\mathcal{J}}$
$\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$ from Definition 3.1. Hence $e_{N\tilde{F}} \in (N\tilde{G},E)^*$. Thus
$(N\tilde{F},E)^* \tilde{\subseteq} (N\tilde{G},E)^*$.

(3) Let $e_{N\tilde{F}} \in (N\tilde{F},E)^*(\hat{\mathcal{J}}_1,\tau)$. Then $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}_1$
$\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. Since $\hat{\mathcal{J}} \tilde{\subseteq} \hat{\mathcal{J}}_1$. Then $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$
$\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. Hence $e_{N\tilde{F}} \in (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau)$. Thus
$(N\tilde{F},E)^*(\hat{\mathcal{J}}_1,\tau) \tilde{\subseteq} (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau)$.

(4) since $\{\tilde{0}_E^U\} \tilde{\subseteq} \hat{\mathcal{J}}$ for any FNS ideal on U, therefore by (3) and Example 3.2, $(N\tilde{F},E)^*(\hat{\mathcal{J}}) \tilde{\subseteq} (N\tilde{F},E)^*(\tilde{0}_E^U) = FNScl(N\tilde{F},E)$ for any FNSS $(N\tilde{F},E)$ on U.

Suppose , $e_{N\tilde{F}} \in FNScl(N\tilde{F},E)^*$. So for every $(N\tilde{O},E)_{e_{N\tilde{F}}}$ open FNS set contains $e_{N\tilde{F}}$, $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E)^* \neq \tilde{0}_E^U$, there exists $e_{1N\tilde{G}} \in (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E)^*$ such that for every $(N\tilde{V},E)_{e_{1N\tilde{G}}}$ open FNS set contains $e_{1N\tilde{G}}$, $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$. Since $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{V},E)_{e_{1N\tilde{G}}}$ open FNS set contains $e_{1N\tilde{G}}$ then $\left((N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{V},E)_{e_{1N\tilde{G}}}\right) \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$ which leads to $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$ $\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$, therefore $e_{N\tilde{F}} \in (N\tilde{F},E)^*$, and so, $FNScl(N\tilde{F},E)^* \tilde{\subseteq} (N\tilde{F},E)^*$ and since $(N\tilde{F},E)^* \tilde{\subseteq} FNScl(N\tilde{F},E)^*$. Hence $(N\tilde{F},E)^* = FNScl(N\tilde{F},E)^*$.

(5) from (4) $(N\tilde{F},E)^* = FNScl(N\tilde{F},E)^*$ then $(N\tilde{F},E)^*$ is τ-closed fuzzy neutrosophic soft set.

(6) By (4), we have
$((N\tilde{F},E)^*)^* = FNScl((N\tilde{F},E)^*)^* \tilde{\subseteq} FNScl(N\tilde{F},E)^* = (N\tilde{F},E)^*$.

(7) Let $e_{N\tilde{F}} \in \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^*$. Then $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right) \notin \hat{\mathcal{J}}\ \forall\ (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$, i.e $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \tilde{\cup} (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E) \notin \hat{\mathcal{J}}\ \ \forall\ (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. then, we have two cases $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$ and $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E) \in \hat{\mathcal{J}}$ or the converse, if $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \in \hat{\mathcal{J}}$ and $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E) \in \hat{\mathcal{J}}$,this means that $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \tilde{\cup} (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{G},E) \in \hat{\mathcal{J}}$, which contradicts the hypothesis. Hence
$\left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^* \tilde{\subseteq} (N\tilde{F},E)^* \tilde{\cup} (N\tilde{G},E)^*$.

For the reverse inclusion, since $\left(N\tilde{F},E\right), (N\tilde{G},E) \tilde{\subseteq} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)$. Then $(N\tilde{F},E)^* \tilde{\subseteq} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^*$ and $(N\tilde{G},E)^* \tilde{\subseteq} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^*$ from (2). Hence $(N\tilde{F},E)^* \tilde{\cup} (N\tilde{G},E)^* \tilde{\subseteq} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^*$ and it is implies that
$\left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{G},E\right)\right)^* = (N\tilde{F},E)^* \tilde{\cup} (N\tilde{G},E)^*$.

(8) Since $\left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{G},E\right)\right) \tilde{\subseteq} (N\tilde{F},E), (N\tilde{G},E)$. Then
$\left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{G},E\right)\right)^* \tilde{\subseteq} (N\tilde{F},E)^*$ and
$\left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{G},E\right)\right)^* \tilde{\subseteq} (N\tilde{G},E)^*$ from (2). Hence
$\left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{G},E\right)\right)^* \tilde{\subseteq} (N\tilde{F},E)^* \tilde{\cap} (N\tilde{G},E)^*$.

(9) Let $e_{N\tilde{F}} \in \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{H},E\right)\right)^*$. Then $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} \left(\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{H},E\right)\right) \notin \hat{\mathcal{J}}, \forall\ (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$, i.e $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \tilde{\cup} (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{H},E) \notin \hat{\mathcal{J}}\ ,\ \forall\ (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. then, we have two cases $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \in \hat{\mathcal{J}}$ and $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{H},E) \notin \hat{\mathcal{J}}$ or the converse, if $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{H},E) \notin \hat{\mathcal{J}}$

gives $(N\tilde{H},E) \notin \hat{\mathcal{J}}$ which is a contradiction. Thus $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$ $\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau$. So $e_{N\tilde{F}} \in (N\tilde{F},E)^*$ hence $\left((N\tilde{F},E) \tilde{\cup} (N\tilde{H},E)\right) \tilde{\subseteq} (N\tilde{F},E)^*$.

For the reverse inclusion, since $(N\tilde{F},E) \tilde{\subseteq} \left((N\tilde{F},E) \tilde{\cup} (N\tilde{G},E)\right)$. Then $(N\tilde{F},E)^* \tilde{\subseteq} \left((N\tilde{F},E) \tilde{\cup} (N\tilde{G},E)\right)^*$ from (2). Hence $\left((N\tilde{F},E) \tilde{\cup} (N\tilde{H},E)\right) = (N\tilde{F},E)^*$.

Definition 3.4: Let (U,τ,E) be a fuzzy neutrosophic soft topological space and $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. Then the operator $FNScl^*: FNSS(U)_E \to FNSS(U)_E$ defined by: $FNScl^*(N\tilde{F},E) = (N\tilde{F},E) \tilde{\cup} (N\tilde{F},E)^*$ is a fuzzy neutrosophic soft closure operator.

Theorem 3.5:

(1) $FNScl^*(\tilde{0}_E^U) = (\tilde{0}_E^U)$.

(2) $(N\tilde{F},E) \tilde{\subseteq} FNScl^*(N\tilde{F},E), \forall (N\tilde{F},E) \in FNSS(U)_E$.

(3) $FNScl^*\left((N\tilde{F},E) \tilde{\cup} (N\tilde{G},E)\right) = FNScl^*(N\tilde{F},E) \tilde{\cup} FNScl^*(N\tilde{G},E)$, $\forall (N\tilde{F},E), (N\tilde{G},E) \in FNSS(U)_E$.

(4) $FNScl^*\left(FNScl^*(N\tilde{F},E)\right) \tilde{\subseteq} FNScl^*(N\tilde{F},E), \forall (N\tilde{F},E) \in FNSS(U)_E$.

Proof.

(1) since $FNScl^*(\tilde{0}_E^U) = (\tilde{0}_E^U) \tilde{\cup} (\tilde{0}_E^U)^* = \tilde{0}_E^U \tilde{\cup} \tilde{0}_E^U = \tilde{0}_E^U$ from Theorem 3.3.(1).

(2) since $(N\tilde{F},E) \tilde{\subseteq} (N\tilde{F},E) \tilde{\cup} (N\tilde{F},E)^* = FNScl^*(N\tilde{F},E)$

Thus $(N\tilde{F},E) \tilde{\subseteq} FNScl^*(N\tilde{F},E)$.

(3)
$FNScl^*\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{G},E)\right) =$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{G},E)\right)\tilde{\cup}\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{G},E)\right)^* =$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{G},E)\right)\tilde{\cup}\left((N\tilde{F},E)^*\,\tilde{\cup}\,(N\tilde{G},E)^*\right) =$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)\tilde{\cup}\left((N\tilde{G},E)\,\tilde{\cup}\,(N\tilde{G},E)^*\right) =$
$FNScl^*(N\tilde{F},E)\,\tilde{\cup}\,FNScl^*(N\tilde{G},E)$ from Theorem 3.3. (7).

(4) $FNScl^*\left(FNScl^*(N\tilde{F},E)\right) = FNScl^*\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)$

$=\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)\tilde{\cup}\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)^* =$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)\tilde{\cup}\left((N\tilde{F},E)^*\,\tilde{\cup}\,((N\tilde{F},E)^*)^*\right) \tilde{\subseteq}$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)\tilde{\cup}\left((N\tilde{F},E)^*\,\tilde{\cup}\,(N\tilde{F},E)^*\right) =$
$\left((N\tilde{F},E)\,\tilde{\cup}\,(N\tilde{F},E)^*\right)$

$= FNScl^*(N\tilde{F},E)$ from Theorem 3.3. (6),(7) ,thus
$FNScl^*\left(FNScl^*(N\tilde{F},E)\right) \tilde{\subseteq} FNScl^*(N\tilde{F},E)$.

Definition 3.6 **:** Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E and $FNScl^*: FNSS(U)_E \to FNSS(U)_E$ be the fuzzy neutrosophic soft closure operator. Then there exists a unique fuzzy neutrosophic soft topology over U with the same set of parameters E, finer than τ, called the ∗-fuzzy neutrosophic soft topology, denoted by $\tau^*(\hat{\mathcal{J}})$ or τ^* , given by

$$\tau^*(\hat{\mathcal{J}}) = \{\,(N\tilde{F},E) \in FNSS(U)_E : FNScl^*(N\tilde{F},E)^c = (N\tilde{F},E)^c\}.$$

Example 3.7: Let U = { r_1, r_2 } , E = { e } and $\tau = \{\tilde{1}_E^U, \tilde{0}_E^U, (N\tilde{F}, E)\}$ where $(N\tilde{F}, E)$ is a fuzzy neutrosophic soft set over U defined by

$N\tilde{F}(e) = \{< r_1, (0.8,0.7,0.5) >, < r_2, (0.9,0.6,0.5) >\}$. Then τ defines a Fuzzy neutrosophic soft topology on U.

Let $\hat{\mathcal{J}} = \{\tilde{0}_E^U, (N\tilde{G}, E)\}$ be a fuzzy neutrosophic soft ideal over U where $(N\tilde{G}, E)$ is a fuzzy neutrosophic soft set over U defined by

$N\tilde{G}(e) = \{< r_1, (0.8,0.7,0.5) >, < r_2, (0.9,0.6,0.5) >\}$. Then,
$\tau^* = \{\tilde{1}_E^U, \tilde{0}_E^U, (N\tilde{F}_1, E), (N\tilde{F}_2, E)\}$, where $(N\tilde{F}_1, E), (N\tilde{F}_2, E)$ are fuzzy neutrosophic soft sets over U where

$N\tilde{F}_1(e) = \{< r_1, (0.8,0.7,0.5) >, < r_2, (0.9,0.6,0.5) >\}$

$N\tilde{F}_2(e) = \{< r_1, (0.5,0.3,0.8) >, < r_2, (0.5,0.4,0.9) >\}$.

Example 3.8:

(1) If $\hat{\mathcal{J}} = \{\tilde{0}_E^U\}$, then $FNScl^*(N\tilde{F}, E) = FNScl(N\tilde{F}, E)$ for all $(N\tilde{F}, E) \in FNSS(U)_E$ and $\tau^* = \tau$.

(2) If $\hat{\mathcal{J}} = FNSS(U)_E$, then $FNScl^*(N\tilde{F}, E) = (N\tilde{F}, E)$ for all $(N\tilde{F}, E) \in FNSS(U)_E$ and $\tau^* = FNSS(U)_E$ (the fuzzy neutrosophic soft discrete topology).

Proof.

(1) Let $\hat{\mathcal{J}} = \{\tilde{0}_E^U\} \Longrightarrow (N\tilde{F}, E)^*(\hat{\mathcal{J}}, \tau) = FNScl(N\tilde{F}, E)$ from Example 3.2

And since
$FNScl^*(N\tilde{F}, E) = (N\tilde{F}, E) \tilde{\cup} (N\tilde{F}, E)^* = (N\tilde{F}, E) \tilde{\cup} FNScl(N\tilde{F}, E)$

Since $(N\tilde{F}, E) \tilde{\subseteq} FNScl(N\tilde{F}, E)$ then $(N\tilde{F}, E) \tilde{\cup} FNScl(N\tilde{F}, E) = FNScl(N\tilde{F}, E)$, Thus $FNScl^*(N\tilde{F}, E) = FNScl(N\tilde{F}, E)$.

Now, let $\left(N\tilde{F},E\right) \in \tau^*$, $FNScl^*\left(N\tilde{F},E\right) = FNScl\left(N\tilde{F},E\right)$

Since $\left(N\tilde{F},E\right) \in \tau^* \Longrightarrow FNScl^*\left(N\tilde{F},E\right)^c = \left(N\tilde{F},E\right)^c$

So, $FNScl^*\left(N\tilde{F},E\right)^c = FNScl\left(N\tilde{F},E\right)^c = \left(N\tilde{F},E\right)^c$

Thus $\left(N\tilde{F},E\right)^c$ is closed FNS set $\Longrightarrow \left(N\tilde{F},E\right)$ is open FNS set i.e $\left(N\tilde{F},E\right) \in \tau$

Hence $\tau^* \tilde{\subseteq} \tau$. But from Definition 3.6. $\tau \tilde{\subseteq} \tau^*$, hence $\tau^* = \tau$.

(2) Let $\hat{\mathcal{J}} = FNSS(U)_E \Longrightarrow (N\tilde{F},E)^*\left(\hat{\mathcal{J}},\tau\right) = \tilde{0}_E^U$ from Example 3.3.
since $FNScl^*\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right) \tilde{\cup} (N\tilde{F},E)^* = \left(N\tilde{F},E\right) \tilde{\cup} \tilde{0}_E^U = \left(N\tilde{F},E\right)$

So, $FNScl^*\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right)$.

Now, let $\left(N\tilde{F},E\right) \in FNSS(U)_E$, $FNScl^*\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right)$ So, $FNScl^*\left(N\tilde{F},E\right)^c = \left(N\tilde{F},E\right)^c$,thus $\left(N\tilde{F},E\right) \in \tau^*$

Hence $FNSS(U)_E \tilde{\subseteq} \tau^*$, and from Definition 3.6. $\tau^* \tilde{\subseteq} FNSS(U)_E$

Thus $\tau^* = FNSS(U)_E$.

Theorem 3.9: Let (U,τ,E) be a fuzzy neutrosophic soft topological space and $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. Then

$\sigma\left(\hat{\mathcal{J}},\tau\right) = \left\{\left(N\tilde{F},E\right) - \left(N\tilde{G},E\right) : \left(N\tilde{F},E\right) \in \tau, \left(N\tilde{G},E\right) \in \hat{\mathcal{J}}\right\}$ forms a basis for the generated fuzzy neutrosophic soft topology $\tau^*(\hat{\mathcal{J}})$.

Proof. Since $\tilde{1}_E^U \in \tau$, $\tilde{0}_E^U \in \hat{\mathcal{J}}$. Then $\tilde{1}_E^U - \tilde{0}_E^U \in \sigma$. Hence $\tilde{1}_E^U \in \sigma$ and $\tilde{\cup}_{j\in J}\left(\left(N\tilde{F}_j,E\right) - \left(N\tilde{G}_j,E\right)\right) = \tilde{1}_E^U$.

Also, let $\left(\left(N\tilde{F}_1,E\right) - \left(N\tilde{G}_1,E\right)\right)$, $\left(\left(N\tilde{F}_2,E\right) - \left(N\tilde{G}_2,E\right)\right) \in \sigma$ such that $e_{N\tilde{F}} \in \left(\left(N\tilde{F}_1,E\right) - \left(N\tilde{G}_1,E\right)\right) \tilde{\cap} \left(\left(N\tilde{F}_2,E\right) - \left(N\tilde{G}_2,E\right)\right)$. Then

$$e_{N\tilde{F}} \in \left(\left(N\tilde{F}_1, E\right) - \left(N\tilde{G}_1, E\right)\right) \tilde{\cap} \left(\left(N\tilde{F}_2, E\right) - \left(N\tilde{G}_2, E\right)\right)$$

$$= \left(\left(N\tilde{F}_1, E\right) \tilde{\cap} \left(N\tilde{G}_1, E\right)^c\right) \tilde{\cap} \left(\left(N\tilde{F}_2, E\right) \tilde{\cap} \left(N\tilde{G}_2, E\right)^c\right)$$

$$= \left(\left(N\tilde{F}_1, E\right) \tilde{\cap} \left(N\tilde{F}_2, E\right)\right) \tilde{\cap} \left(\left(N\tilde{G}_1, E\right)^c \tilde{\cap} \left(N\tilde{G}_2, E\right)^c\right)$$

$$= \left(\left(N\tilde{F}_1, E\right) \tilde{\cap} \left(N\tilde{F}_2, E\right)\right) \tilde{\cap} \left(\left(N\tilde{G}_1, E\right) \tilde{\cup} \left(N\tilde{G}_2, E\right)\right)^c$$

$$= \left(\left(N\tilde{F}_1, E\right) \tilde{\cap} \left(N\tilde{F}_2, E\right)\right) - \left(\left(N\tilde{G}_1, E\right) \tilde{\cap} \left(N\tilde{G}_2, E\right)\right) \in \sigma\left(\hat{\mathcal{J}}, \tau\right)$$

Thus σ is a fuzzy neutrosophic soft basis of τ^*.

Corollary 3.10: Let (U, τ, E) be a fuzzy neutrosophic soft topological space and $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. Then $\tilde{\subseteq} \sigma\left(\hat{\mathcal{J}}, \tau\right) \tilde{\subseteq} \tau^*(\hat{\mathcal{J}})$.

Proof. Let $\left(N\tilde{F}, E\right) \in \tau$, since $\tilde{0}_E^U \in \hat{\mathcal{J}}$

Thus $\left(N\tilde{F}, E\right) = \left(N\tilde{F}, E\right) - \tilde{0}_E^U \in \sigma\left(\hat{\mathcal{J}}, \tau\right)$ $i.e$ $\left(N\tilde{F}, E\right) \in \sigma\left(\hat{\mathcal{J}}, \tau\right)$

So, $\tau \tilde{\subseteq} \sigma\left(\hat{\mathcal{J}}, \tau\right)$, and since $\sigma\left(\hat{\mathcal{J}}, \tau\right)$ is a fuzzy neutrosophic soft basis of $\tau^*(\hat{\mathcal{J}})$, Then $\sigma\left(\hat{\mathcal{J}}, \tau\right) \tilde{\subseteq} \tau^*(\hat{\mathcal{J}})$, hence $\tau \tilde{\subseteq} \sigma\left(\hat{\mathcal{J}}, \tau\right) \tilde{\subseteq} \tau^*(\hat{\mathcal{J}})$.

Theorem 3.11: Let $\hat{\mathcal{J}}_1, \hat{\mathcal{J}}_2$ are a fuzzy neutrosophic soft ideals over U, $\hat{\mathcal{J}}_1 \tilde{\subseteq} \hat{\mathcal{J}}_2$ then $\tau^*(\hat{\mathcal{J}}_1) \tilde{\subseteq} \tau^*(\hat{\mathcal{J}}_2)$.

Proof. Let $\left(N\tilde{F}, E\right) \in \tau^*(\hat{\mathcal{J}}_1) \Longrightarrow FNScl^*\left(N\tilde{F}, E\right)^c = \left(N\tilde{F}, E\right)^c$

$\Longrightarrow FNScl^*\left(N\tilde{F}, E\right)^c = \left(N\tilde{F}, E\right)^c \tilde{\cup} \left(\left(N\tilde{F}, E\right)^c\right)^*(\hat{\mathcal{J}}_1, \tau) = \left(N\tilde{F}, E\right)^c$ from definition 3.6.

$$\Longrightarrow [(N\tilde{F}, E)^c]^c = \left[\left(N\tilde{F}, E\right)^c \tilde{\cup} \left(\left(N\tilde{F}, E\right)^c\right)^*(\hat{\mathcal{J}}_1, \tau)\right]^c .$$

$\Longrightarrow (N\tilde{F},E) = (N\tilde{F},E) \tilde{\cap} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_1,\tau)$

Since $\hat{\mathcal{J}}_1 \tilde{\subseteq} \hat{\mathcal{J}}_2$ then $\left(\left(N\tilde{F},E\right)^c\right)^* (\hat{\mathcal{J}}_2,\tau) \tilde{\subseteq} \left(\left(N\tilde{F},E\right)^c\right)^* (\hat{\mathcal{J}}_1,\tau)$ from Theorem 3.3. (3)

$\Longrightarrow \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_1,\tau) \tilde{\subseteq} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_2,\tau)$

$\Longrightarrow (N\tilde{F},E) =$
$(N\tilde{F},E) \tilde{\cap} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_1,\tau) \tilde{\subseteq} (N\tilde{F},E) \tilde{\cap} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_2,\tau)$

But $(N\tilde{F},E) \tilde{\cap} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_2,\tau) \tilde{\subseteq} (N\tilde{F},E)$,
thus$(N\tilde{F},E) = (N\tilde{F},E) \tilde{\cap} \left[\left(\left(N\tilde{F},E\right)^c\right)^*\right]^c (\hat{\mathcal{J}}_2,\tau) \Longrightarrow$
$(N\tilde{F},E)^c \tilde{\cup} \left(\left(N\tilde{F},E\right)^c\right)^* (\hat{\mathcal{J}}_2,\tau) = (N\tilde{F},E)^c$

Thus, $FNScl^*(N\tilde{F},E)^c = (N\tilde{F},E)^c$ $i.e$ $(N\tilde{F},E) \in \tau^*(\hat{\mathcal{J}}_2)$, hence $\tau^*(\hat{\mathcal{J}}_1) \tilde{\subseteq} \tau^*(\hat{\mathcal{J}}_2)$.

Theorem 3.12: Let τ_1, τ_2 be two fuzzy neutrosophic soft topologies on U. Then for any fuzzy neutrosophic soft ideal $\hat{\mathcal{J}}$ on U, $\tau_1 \tilde{\subseteq} \tau_2$ implies $(N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_2) \tilde{\subseteq} (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_1)$, $\forall (N\tilde{F},E) \in \hat{\mathcal{J}}$ and $\tau_1^* \tilde{\subseteq} \tau_2^*$

Proof. Let $e_{N\tilde{F}} \in (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_2) \Longrightarrow (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$
$\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau_2$ Since $\tau_1 \tilde{\subseteq} \tau_2 \Longrightarrow (N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \notin \hat{\mathcal{J}}$
$\forall (N\tilde{O},E)_{e_{N\tilde{F}}} \in \tau_2$, Thus $e_{N\tilde{F}} \in (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_1)$, hence
$(N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_2) \tilde{\subseteq} (N\tilde{F},E)^*(\hat{\mathcal{J}},\tau_1)$.

Next, we proved $\tau_1^* \tilde{\subseteq} \tau_2^*$, Let $(N\tilde{F},E) \in \tau_1^* \Longrightarrow FNScl^*_{\tau_1}(N\tilde{F},E)^c =$
$(N\tilde{F},E)^c$

$\Rightarrow FNScl^*_{\tau_1}(N\tilde{F},E)^c = (N\tilde{F},E)^c \tilde{\cup} \left(\left((N\tilde{F},E)^c\right)^*(\hat{\mathcal{J}},\tau_1)\right) = (N\tilde{F},E)^c$ from definition 3.6.

$$\Rightarrow \left((N\tilde{F},E)^c\right)^c = \left((N\tilde{F},E)^c \tilde{\cup} \left((N\tilde{F},E)^c\right)^*(\hat{\mathcal{J}},\tau_1)\right)^c$$

$$\Rightarrow (N\tilde{F},E) = (N\tilde{F},E) \tilde{\cap} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_1)$$

Since $\tau_1 \tilde{\subseteq} \tau_2$ then $\left((N\tilde{F},E)^c\right)^*(\hat{\mathcal{J}},\tau_2) \tilde{\subseteq} \left((N\tilde{F},E)^c\right)^*(\hat{\mathcal{J}},\tau_1)$ from above,

$$\Rightarrow \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_1) \tilde{\subseteq} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_2),$$

$\Rightarrow (N\tilde{F},E) = (N\tilde{F},E) \tilde{\cap} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_1) \tilde{\subseteq} (N\tilde{F},E) \tilde{\cap} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_2)$, But $(N\tilde{F},E) \tilde{\cap} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_2) \tilde{\subseteq} (N\tilde{F},E)$, so, $(N\tilde{F},E) = (N\tilde{F},E) \tilde{\cap} \left[\left((N\tilde{F},E)^c\right)^*\right]^c(\hat{\mathcal{J}},\tau_2)$,

$$\Rightarrow (N\tilde{F},E)^c \tilde{\cup} \left((N\tilde{F},E)^c\right)^*(\hat{\mathcal{J}},\tau_2) = (N\tilde{F},E)^c$$

Thus, $FNScl^*_{\tau_2}(N\tilde{F},E)^c = (N\tilde{F},E)^c$ $i.e$ $(N\tilde{F},E) \in \tau_2^*$.

4. *Compatibility of Fuzzy Neutrosophic Soft Ideals with Fuzzy Neutrosophic Soft Topology*

Definition 4.1: Let (U,τ,E) be a fuzzy neutrosophic soft topological space and $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal on U. τ is said to be compatible with $\hat{\mathcal{J}}$, denoted by $\tau \approx \hat{\mathcal{J}}$, if for every fuzzy neutrosophic soft set $(N\tilde{F},E)$ of U and for all fuzzy neutrosophic soft point $e_{N\tilde{F}} \in (N\tilde{F},E)$, there exists $(N\tilde{O},E)_{e_{N\tilde{F}}}$ open fuzzy neutrosophic soft contains $e_{N\tilde{F}}$ such that $(N\tilde{O},E)_{e_{N\tilde{F}}} \tilde{\cap} (N\tilde{F},E) \in \hat{\mathcal{J}}$, then $(N\tilde{F},E) \in \hat{\mathcal{J}}$.

Theorem 4.2: Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E and $\tau \approx \hat{\mathcal{J}}$. Then For any fuzzy neutrosophic soft set

$(N\tilde{F},E)$ of U, $(N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*} = \tilde{0}_E^U$ implies $(N\tilde{F},E)^{*} = \tilde{0}_E^U$ iff $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*}$.

Proof. ($\Longrightarrow$) Suppose that $(N\tilde{F},E)$ any fuzzy neutrosophic soft set of U,

$(N\tilde{F},E)^{*} = \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*}$ and $(N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*} = \tilde{0}_E^U$.

Then $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*} = (\tilde{0}_E^U)^{*} = \tilde{0}_E^U$. Hence $(N\tilde{F},E)^{*} = \tilde{0}_E^U$.

($\Longleftarrow$) Suppose that $(N\tilde{F},E)$ any fuzzy neutrosophic soft set of U, $(N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*} = \tilde{0}_E^U$ and $(N\tilde{F},E)^{*} = \tilde{0}_E^U$. Since $(N\tilde{F},E)^{*} = \tilde{0}_E^U$ and $(N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*} = \tilde{0}_E^U \Longrightarrow \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*} = (\tilde{0}_E^U)^{*} = \tilde{0}_E^U$, Hence $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*}$.

Corollary 4.3: Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E, For any fuzzy neutrosophic soft set $(N\tilde{F},E)$ of U and $\tau \approx \hat{\mathcal{J}}$. Then $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)^{*}\right)^{*}$.

Proof. Suppose that $(N\tilde{F},E)$ any fuzzy neutrosophic soft set of U. Since $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*}\right)^{*} \,\tilde{\subseteq}\, (N\tilde{F},E)^{*}\,\tilde{\cap}\,\left((N\tilde{F},E)^{*}\right)^{*} = \left((N\tilde{F},E)^{*}\right)^{*}$ from Theorem 3.3. (8), (6). thus $(N\tilde{F},E)^{*} = \left((N\tilde{F},E)^{*}\right)^{*}$.

Theorem 4.4: Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. Then the following are equivalent:

(a) $\tau \approx \hat{\mathcal{J}}$.

(b) For every $(N\tilde{F},E) \in FNSS(U)_E$ such that $(N\tilde{F},E)\,\tilde{\cap}\,(N\tilde{F},E)^{*} = \tilde{0}_E^U$. then $(N\tilde{F},E) \in \hat{\mathcal{J}}$.

(c) For every $(N\tilde{F},E) \in FNSS(U)_E$, $(N\tilde{F},E) - (N\tilde{F},E)^{*} \in \hat{\mathcal{J}}$.

Proof. (a) $\Longrightarrow$ (b) Let $(N\tilde{F},E) \in FNSS(U)_E$ such that $(N\tilde{F},E) \,\tilde{\cap}\, (N\tilde{F},E)^* = \tilde{0}_E^U$, so, $(N\tilde{F},E) \,\tilde{\subseteq}\, ((N\tilde{F},E)^*)^c$

Since $(N\tilde{F},E)^*$ is τ-closed fuzzy neutrosophic soft set , Then for all $e_{N\tilde{F}} \in (N\tilde{F},E)$ and $e_{N\tilde{F}} \notin (N\tilde{F},E)^*$ such that $e_{N\tilde{F}} \in ((N\tilde{F},E)^*)^c$, we have $((N\tilde{F},E)^*)^c \,\tilde{\cap}\, (N\tilde{F},E) \in \hat{\mathcal{J}}$ for some $((N\tilde{F},E)^*)^c \in \tau$. Thus $(N\tilde{F},E) \in \hat{\mathcal{J}}$ by (a).

(b) $\Longrightarrow$ (c) Let $(N\tilde{F},E) \in FNSS(U)_E$. Since

$$(N\tilde{F},E) - (N\tilde{F},E)^* \,\tilde{\cap}\, \left((N\tilde{F},E) \,\tilde{\cap}\, (N\tilde{F},E)^*\right)^* =$$

$$\left((N\tilde{F},E) \,\tilde{\cap}\, ((N\tilde{F},E)^*)^c\right) \tilde{\cap} \left((N\tilde{F},E) \,\tilde{\cap}\, (N\tilde{F},E)^*\right)^* \tilde{\subseteq}$$

$$\left((N\tilde{F},E) \,\tilde{\cap}\, ((N\tilde{F},E)^*)^c\right) \tilde{\cap}\, (N\tilde{F},E)^* \,\tilde{\cap}\, ((N\tilde{F},E)^*)^*$$

$$\tilde{\subseteq} \left((N\tilde{F},E) \,\tilde{\cap}\, ((N\tilde{F},E)^*)^c\right) \tilde{\cap}\, (N\tilde{F},E)^* = \tilde{0}_E^U$$

Then $(N\tilde{F},E) - (N\tilde{F},E)^* \in \hat{\mathcal{J}}$ by (b).

$e_{N\tilde{F}} \in (N\tilde{F},E)$, $((N\tilde{F},E)^*)^c \,\tilde{\cap}\, (N\tilde{F},E) \in \hat{\mathcal{J}}$ such that $e_{N\tilde{F}} \in ((N\tilde{F},E)^*)^c$

$\Longrightarrow (N\tilde{F},E) \,\tilde{\subseteq}\, ((N\tilde{F},E)^*)^c$. Since $(N\tilde{F},E)^*$ is τ-closed fuzzy neutrosophic soft set $\Longrightarrow$ $((N\tilde{F},E)^*)^c \in \tau$. Thus $(N\tilde{F},E) \in \hat{\mathcal{J}}$ by (b).

$(c) \Longrightarrow (a)$ suppose that $(N\tilde{F},E) \in FNSS(U)_E$ and assume that for every $e_{N\tilde{F}} \in (N\tilde{F},E)$ there exists $(N\tilde{O},E)_{e_{N\tilde{F}}}$ open fuzzy neutrosophic soft contains $e_{N\tilde{F}}$ such that $(N\tilde{O},E)_{e_{N\tilde{F}}} \,\tilde{\cap}\, (N\tilde{F},E) \in \hat{\mathcal{J}}$ and $(N\tilde{F},E) - (N\tilde{F},E)^* \in \hat{\mathcal{J}}$. Then $e_{N\tilde{F}} \notin (N\tilde{F},E)^*$ such that $(N\tilde{F},E) \,\tilde{\cap}\, (N\tilde{F},E)^* = \tilde{0}_{\mathrm{E}}^{\mathrm{U}}$. Hence $(N\tilde{F},E) \,\tilde{\subseteq}\, ((N\tilde{F},E)^*)^c$. Since $(N\tilde{F},E)^*$ is τ-closed fuzzy neutrosophic soft set $((N\tilde{F},E)^*)^c \in \tau$ such that $e_{N\tilde{F}} \in \left(\left(N\tilde{F},E\right)^{*}\right)^{c}$. Let $\left(\left(N\tilde{F},E\right)^{*}\right)^{c} = (N\tilde{O},E)_{e_{N\tilde{F}}}$ such that $e_{N\tilde{F}} \in \left(\left(N\tilde{F},E\right)^{*}\right)^{c}$. thus $\left(\left(N\tilde{F},E\right)^{*}\right)^{c} \tilde{\cap} (N\tilde{F},E) \in \hat{\mathcal{J}}$. Since $\left(N\tilde{F},E\right) \tilde{\subseteq} \left(\left(N\tilde{F},E\right)^{*}\right)^{c} \Rightarrow \left(N\tilde{F},E\right) = \left(N\tilde{F},E\right) \tilde{\cap} \left(\left(N\tilde{F},E\right)^{*}\right)^{c} = \left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*}$. by hypothesis, $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*} \in \hat{\mathcal{J}}$. Thus $\left(N\tilde{F},E\right) \in \hat{\mathcal{J}}$. hence $\tau \approx \hat{\mathcal{J}}$.

***Theorem 4.5*:** Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. then For every $\left(N\tilde{F},E\right) \in FNSS(U)_{E}$, $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*} \in \hat{\mathcal{J}}$ if and only if $\left(N\tilde{F},E\right)$ contains no non-null soft set $\left(N\tilde{G},E\right)$ with $\left(N\tilde{G},E\right) \tilde{\subseteq} \left(N\tilde{G},E\right)^{*}$, then $\left(N\tilde{F},E\right) \in \hat{\mathcal{J}}$.

Proof. $(\Longrightarrow)$ Suppose that $\left(N\tilde{F},E\right) \in FNSS(U)_{E}$ such that $\left(N\tilde{F},E\right)$ contains no non-null fuzzy neutrosophic soft set $\left(\tilde{G},E\right)$ with $\left(N\tilde{G},E\right) \tilde{\subseteq} \left(N\tilde{G},E\right)^{*}$ and $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*} \in \hat{\mathcal{J}}$. Since $\left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{F},E\right)^{*}\right) \tilde{\subseteq} \left(N\tilde{F},E\right)^{*} = \left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{F},E\right)^{*}\right)^{*}$ from Theorem(2.3.2). It follows that $\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{F},E\right)^{*} \tilde{\subseteq} \left(\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{F},E\right)^{*}\right)^{*}$. By assumption, $\left(N\tilde{F},E\right) \tilde{\cap} \left(N\tilde{F},E\right)^{*} = \tilde{0}_{E}^{U}$, Thus $\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*} \in \hat{\mathcal{J}}$.

$(\Longleftarrow)$ Suppose that $\left(N\tilde{F},E\right) \in FNSS(U)_{E}$ such that $\left(N\tilde{F},E\right)$ contains no non-null fuzzy neutrosophic soft set $\left(N\tilde{G},E\right)$ with $\left(N\tilde{G},E\right) \tilde{\subseteq} \left(N\tilde{G},E\right)^{*}$ and $\left(N\tilde{F},E\right) \in \hat{\mathcal{J}}$. Since $\left(\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*}\right) \tilde{\cap} \left(\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*}\right)^{*} = \tilde{0}_{E}^{U}$ and $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*}$ contains no non-null fuzzy neutrosophic soft set $\left(N\tilde{G},E\right)$ with $\left(N\tilde{G},E\right) \tilde{\subseteq} \left(N\tilde{G},E\right)^{*}$. Hence $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^{*} \in \hat{\mathcal{J}}$.

Theorem 4.6: If (U,τ,E) is a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E and compatible with τ. Then a fuzzy neutrosophic soft set is

τ^*-closed if and only if it is the union of a τ-closed fuzzy neutrosophic soft set and a fuzzy neutrosophic soft set in $\hat{\mathcal{J}}$.

Proof. Let $\left(N\tilde{F},E\right)$ be a τ^*-closed fuzzy neutrosophic soft set. Then $FNScl^*\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right)$ and $\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{F},E\right)^* = \left(N\tilde{F},E\right)$.Hence $\left(N\tilde{F},E\right)^* \tilde{\subseteq} \left(N\tilde{F},E\right)$. Thus $\left(N\tilde{F},E\right) = \left(\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^*\right) \tilde{\cup} \left(N\tilde{F},E\right)^*$, $\left(N\tilde{F},E\right) - \left(N\tilde{F},E\right)^* \in \hat{\mathcal{J}}$ from Theorem 4.4. and $\left(N\tilde{F},E\right)^*$ is τ-closed fuzzy neutrosophic soft set from Theorem 3.3. (5).

Conversely, let $\left(N\tilde{F},E\right) = \left(N\tilde{G},E\right) \tilde{\cup} (NI,E)$, where $\left(N\tilde{G},E\right)$ is τ-closed fuzzy neutrosophic soft set and $(NI,E) \in \hat{\mathcal{J}}$. Then $\left(N\tilde{F},E\right)^* = \left(\left(N\tilde{G},E\right) \tilde{\cup} (NI,E)\right)^* = \left(N\tilde{G},E\right)^* \tilde{\subseteq} FNScl\left(N\tilde{G},E\right) = \left(N\tilde{G},E\right) \tilde{\subseteq} \left(N\tilde{F},E\right)$ from Theorem 3.3. (2), (4). Hence $\left(N\tilde{F},E\right) \tilde{\cup} \left(N\tilde{F},E\right)^* = \left(N\tilde{F},E\right)$. Thus $FNScl^*\left(N\tilde{F},E\right) = \left(N\tilde{F},E\right)$. It follows that $\left(N\tilde{F},E\right)$ is a τ^*-closed fuzzy neutrosophic soft set.

***Theorem 4.7*:** Let (U,τ,E) be a fuzzy neutrosophic soft topological space, $\hat{\mathcal{J}}$ be a fuzzy neutrosophic soft ideal over U with the same set of parameters E. If $\tau \approx \hat{\mathcal{J}}$, then $\sigma(\hat{\mathcal{J}},\tau)$ is a fuzzy neutrosophic soft topology and hence $\sigma = \tau^*$.

Proof. Let $\left(N\tilde{G},E\right) \in \tau^*$. Then $\left(N\tilde{G},E\right)^c = \left(N\tilde{F},E\right) \tilde{\cup} (NI,E)$, where $\left(N\tilde{F},E\right)$ is τ-closed fuzzy neutrosophic soft set and $(NI,E) \in \hat{\mathcal{J}}$. Hence

$$\left(N\tilde{G},E\right) = \tilde{1}_{\mathrm{E}}^{\mathrm{U}} - \left(\left(N\tilde{F},E\right) \tilde{\cup} (NI,E)\right) = \left(\tilde{1}_{\mathrm{E}}^{\mathrm{U}} - \left(N\tilde{F},E\right)\right) \tilde{\cap} \left(\tilde{1}_{\mathrm{E}}^{\mathrm{U}} - (NI,E)\right) = \left(\tilde{1}_{\mathrm{E}}^{\mathrm{U}} - \left(N\tilde{F},E\right)\right) - (NI,E)$$

, where $\left(N\tilde{F},E\right) \in \tau$ and $(NI,E) \in \hat{\mathcal{J}}$. Thus

$$\left(N\tilde{G},E\right) = \left(\tilde{1}_{\mathrm{E}}^{\mathrm{U}} - \left(N\tilde{F},E\right)\right) - (NI,E) \in \sigma(\hat{\mathcal{J}},\tau).$$ So $\tau^* \tilde{\subseteq} \sigma(\hat{\mathcal{J}},\tau)$.

And since $\sigma(\hat{\mathcal{J}},\tau) \tilde{\subseteq} \tau^*$ from Corollary (2.2.11) Hence $\sigma = \tau^*$.

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