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FUZZY-ROUGH MULTI-OBJECTIVE PRODUCT BLENDING FIXED-CHARGE TRANSPORTATION PROBLEM WITH TRUCK LOAD CONSTRAINTS THROUGH TRANSFER STATION

Shyamali Ghosh and Sankar Kumar Roy*

Abstract. In this contribution, for the first time, an efficient model of multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station is formulated. Transfer station inserts transfer cost and type-I fixed-charge. Our aim is to analyze an extra cost that treats as type-II fixed-charge and truck load constraints in the designed model that required when the amount of items exceeds the capacity of vehicle for fulfilling the shipment by more than one trip. Type-II fixed-charge is added with transportation cost and other cost from transfer station. We consider here an important issue of the multi-objective transportation problem as product blending constraints for transporting raw materials with different purity levels for customers' satisfaction. In realistic point of view, the parameters of the model are imprecise in nature due to existing several unpredictable factors. These factors are apprehended by incorporating the fuzzy-rough environment on the parameters. Expected-value operator is utilized to derive the deterministic form of fuzzy-rough data, and the model is experienced with help of fuzzy programming, neutrosophic linear programming and global criteria method. Two numerical examples are illustrated to determine the applicability of the proposed model.

Mathematics Subject Classification. 90B06, 90C08, 90C29, 90C70.

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1. Introduction

Transportation problem (TP) is a particular type of linear programming problem (LPP) initiated by Hitchcock [13] and also known as Hitchcock-Koopmans TP. This transportation system becomes more eco-friendly by considering the transfer station (TS). It optimizes the productivity of vehicles by giving the report of passing average number of vehicles per day, per month, per year. It reduces maintenance cost of collected vehicles which stay on station with minimum transfer cost and encourage for fuel saving. Also air pollution is reduced by using the less number of vehicles being on the road and reduced the traffic. Therefore transfer cost exists for existing TS that defined an optimal strategy. Transfer cost is the total opportunity cost for transporting the products from various sources to different destinations through some TSs. TS considers fixed-charge by adding

Keywords. Transportation problem, fixed-charge and transfer cost, truck load constraints and product blending, multi-objective optimization, fuzzy-rough uncertainty, fuzzy programming and neutrosophic linear programming, global criteria method.

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, West Bengal, India.
*Corresponding author: sankroy2006@gmail.com

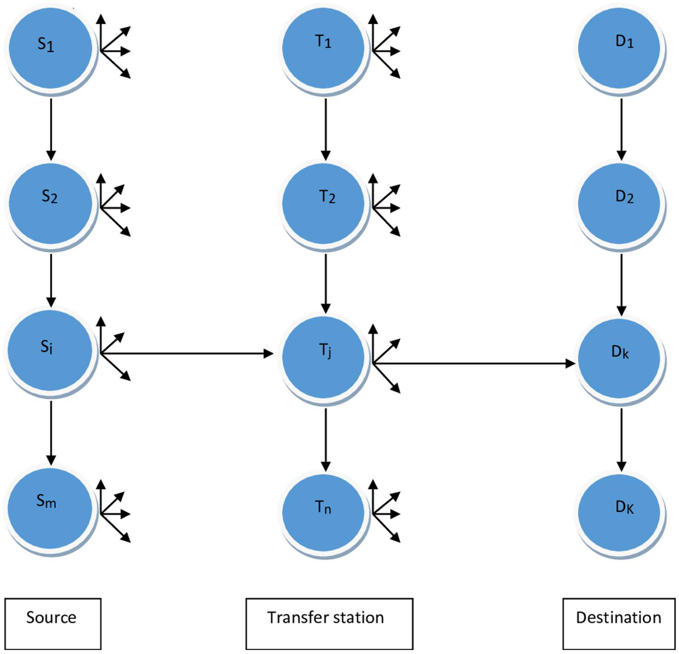

FIGURE 1. Graphical presentation of source, TS and destination for TP.

with transfer cost, and this fixed-charge is assumed as type-I fixed-charge which is independent of transporting items. Therefore the items are transferred with transportation cost, transfer cost and fixed-charge (type-I and type-II). Zhao and Pan [39] introduced transfer cost for a TP with uncertain situation. Hashmi *et al.* [11] analyzed two stage TP with fixed-charge under fuzzy nature with multi-objective ground. Kaur *et al.* [14] investigated a capacitated TP with two-stage and restricted flow for the aim of time minimization. A graphical presentation of transportation from sources to destinations through TS is depicted in Figure 1.

To carry the items, full vehicles such as heavy duty trucks, light duty vehicles or medium duty vehicles are used. Whenever lot size may exceed the capacity of vehicles then the shipment is to be completed by performing more than one trip. Due to increasing number of trips for transporting the product, there exists fixed-charge which is proportional to the number of trips performed. In this case, truck load constraints are considered into the TP. Hence this charge depends on transported amount of items and vehicle capacity. Based on the real-life scenario, we choose this charge as type-II fixed-charge for the first time, and for another realization we assume it as a general charge in the case of truck load constraints. Balaji *et al.* [3] analyzed truck load constraints on a TP with fixed-charge. Generally fixed-charge transportation problem (FCTP) is an extension of general TP that consists of binary variables which specify whether the product is transported from the supply points to the demand centres, and this charge is independent on the transported amount of items. However, while including the truck load constraints in TP, the binary variables reduced into the integer variables which are the number of trips and it depends on the transported amount of items. Hence the problem is more critical and important than the general FCTP. Fixed-charge appears as renting cost of vehicle, landing fees at an airport, toll charges on a high way, establishment cost for machines, permit fees, set up cost of the process, etc. FCTP has a wide variety of applications, such as facility location, manufacturing and transportation. At first, Hirsch and Dantzig [12] defined the concept of FCTP. Also there exist some methods [23, 24, 31] on FCTP which are provided in the literature review. This type of FCTPs was solved by interval programming or rough programming or other type of programming. Midya and Roy [23] also introduced a stochastic FCTP with multi-objective, and with single-sink. Midya and Roy [24] discussed FCTP in different situations and analyzed by interval programming.

Roy *et al.* [31] presented a FCTP with random rough variables. They analyzed the problem in multi-objective ground under rough programming.

In TP product blending is an important concept and common issue. In many planning industries such as chemical, petroleum, gasoline, etc., blending of raw materials with different attributes and purity levels which meet with minimum quality of final products is called product blending. Whenever the blending of raw materials has been declined then the profit becomes high. An important part of product blending for raw materials is to minimize transportation shipping cost by producing a variety of products that satisfy the market demand with keeping minimum quality of items. An organization finds an opportunity to realize considerable cost saving by blending raw materials. The intrinsic flexibility of such blending process can be utilized to optimize the allocation and transportation of raw materials to production centres through TS. Some important studies related on product blending are given as: Papageorgiou *et al.* [27] who first analyzed TP in the presence of product blending including fixed-charge. Gao and Kar [9] introduced product blending on a solid transportation problem (STP). Roy and Midya [29] represented product blending on a FCTP in intuitionistic fuzzy situations with solid nature under multi-objective environment.

For transporting items, different types of conveyances are used such as bus, train, truck, car, ship, flight, etc., and engines of such conveyances cause the emission of CO_2 gas and other green house gasses. The most parts of green house gasses emit by light-duty vehicles, passenger car, minibus, etc. and the remaining part are from heavy-duty vehicles such as truck, ships, freight transport, etc. Nowadays the greenhouse gas emission is high risk for the environmental and air pollution. The carbon emission depends on its fuel type, engine type, traffic rule, road condition, driving rules, etc. Here we include carbon emission as an important fact of TP to minimize the rate of air pollution/CO_2 emission during transportation. Many researchers have studied on carbon emission in TP, some of them [4, 5, 20, 34] are delineated here. Das and Roy [4] studied the effect of carbon emission, treat as variable on a multi-objective problem with neutrosophic field and p-facility location TP. Das *et al.* [5] applied carbon tax, cap and offset policy for carbon emission about a green STP with multiple objectives. Also they discussed dwell time for the problem of transportation-location in the presence of type-2 fuzzy uncertainty. Maity *et al.* [20] represented time variant for sustainable development by minimizing pollution factor of TP with interval valued in multi-objective scenario. Sengupta *et al.* [34] considered carbon emission on a STP and solved by Gamma type-2 defuzzification approach.

For some economic systems on TP, there exist transportation cost, transfer cost, fixed-charge, carbon emission, average delivery time of product, product blending constraints, etc. at the time of passing a homogeneous product from a source to different destinations through TS, and so multi-objective functions are invited in the formulated model. Hence multi-objective optimization problem is included by extending the traditional single objective TP to tackle the conflicting system. There exist some researches studying on multi-objective transportation problem (MOTP) in some realistic environments. A few of them [1, 2, 15, 19, 25, 32] are presented including their works. Most of them were analyzed by fuzzy programming (FP), rough programming, neutrosophic linear programming (NLP), goal programming (GP), etc. Allah *et al.* [1] defined a multi-objective transportation model in the presence of neutrosophic environment. Anukokila and Radhakrishan [2] formulated multi-objective fractional TP in fuzzy environment and then solved it by GP approach. Li and Lai [15] analyzed multi-objective problem on transportation system using FP. Maity *et al.* [19] briefly defined a TP in uncertain system related with multi-objective ground. Midya and Roy [25] discussed a MOTP with fixed-charge and solved with the help of rough programming. A STP with fixed-charge was provided by Roy *et al.* [32] in the presence of two-fold uncertainty.

In general multi-objective fixed-charge transportation problem (MOFCTP), where the decision maker (DM) always confirms the nature of parameters. In industrial problems/realistic applications, where all the parameters may not be clearly (*i.e.*, not precisely) defined but they may be imprecise nature due to existence of insufficient information, lack of evidence, competitive economic condition, fluctuations of financial market, etc. There exist various uncertainties provided by literature review such as fuzzy, interval, rough, stochastic, randomness, intuitionistic fuzzy, neutrosophic, etc. Between randomness and fuzziness, roughness is also another type of uncertainty. The roughness introduces in a problem whenever the DM has not precise information about data and coefficients of problem. Then the feasible region of the problem is more flexible due to this roughness. Thus rough MOFCTP has a realistic background. Many investigations have been done on MOFCTP under various uncertainties, but there are some critical realistic situations/industrial problems where single uncertainty is not enough to tackle the situation. Due to this fact, we introduce fuzzy-rough variable in our proposed MOFCTP. Also to transform the fuzzy-rough MOFCTP into crisp MOFCTP, fuzzy-rough expected-value operator is used. Zadeh [38] first defined fuzzy set theory, and Zimmermann [41] introduced the FP for different objective functions. Also Menni and Chaabane [22] investigated possibilistic optimization on fuzzy environment with integer efficient set. There exist several methodologies in literature for solving crisp or fuzzy problems with single objective or multi-objectives. Ebrahimnejad [8] addressed a problem including interval-valued trapezoidal fuzzy variables and provided a method for linear programming. Newton method for multi-objective optimization problems was developed by Ghaznavi *et al.* [10] and obtained Pareto-optimal solutions in fuzzy environment. Roy *et al.* [30] defined a new approach for solving a TP in intuitionistic fuzzy nature with multi-objective ground. Ebrahimnejad [7] represented LR flat fuzzy numbers on TP and defined a new approach for solving such problem. For solving linear programming problem, a stepwise solution procedure of FP described by Marbini *et al.* [21]. They used possibility and necessity relations for fuzzy constraints and then extended the method for objective function with fuzzy parameter. Pawlak [28] was the first mathematician who introduced rough set theory, and he provided an important mathematical tool for the analyzing such vagueness. Xu and Yo [36] represented random rough coefficients for presenting a multi-objective programming problem. Zhimiao and Xu [40] applied rough programming for solving STP. Ebrahimnejad [6] defined a new approach to solve TP in fuzzy environment.

A list of some recent remarkable articles on TP in different environments is summarised in Table 1 for clear comparison of current study with previous ones.

The comparisons from Table 1 and literature survey are focused on research gap which are traced out as:

- Literature survey reveals that most of the researchers analyzed MOTP with various conditions, but they did not think about the case of low vehicle capacity for transporting the items as truck load constraints and type-II fixed-charge that depend on transported amount of items.
- Also for transporting the items from sources to destinations, researchers (*cf.* [25,31,32]) did not consider a stage that separates the whole system into two different steps, which may help to obtain some facilities for TP.
- For environment improvement and for customers satisfaction, minimum CO_2 emission from transportation with minimum time of transportation are required as these are essential objectives as of minimum transportation cost. But most of researchers did not include all these objectives together. Also product blending constraints did not incorporate together, but this is also an important fact of customer satisfaction.
- Most of the research studies on TP are considered on single type of uncertainty but some critical realistic situations occur which cannot tackle by such single type of uncertainty, so it is essential to incorporate a couple of uncertainty to tackle critical realistic situations.
- Balaji *et al.* [3] introduced truck load constraints with FCTP. They analyzed that the case of truck load constraints with fixed-charge which provide less transportation cost then the case of general fixed-charge (*i.e.*, type-I fixed-charge). They did not separately consider two cases as the truck load constraints with general charge, and fixed-charge (*i.e.*, type-II) depending on transported amount and vehicle capacity. Also they analyzed only single objective TP as of authors (*cf.* [9,27,34,39]), did not include other objectives (such as transportation time, carbon emission, deterioration, etc. together) which are conflicting nature, and may be effected on the cost objective function. In fact they (except [39]) did not consider any station in between source and demand centre that may be essential for some situations.
- Zhao and Pan [39] considered TP in single uncertainty with transfer cost. The authors of [11,39] represented the problem in two stages but they did not think a couple of uncertainty that may be required in some situations. Again the authors of [39] considered only single objective as cost function, but they did not include other objective or constraints related with TP such as product blending constraints, truck load constraints, safety factor/budget constraints, etc. Literature survey revealed that many authors (*cf.* [1,4,11,19,20,30,34,40]) did not consider extra constraints relating with TP.

TABLE 1. Survey of research works of TP under various environments.

References	Nature of problem	Environment	No. of objective function	Additional functions
Allah *et al.* [1]	TP	Neutrosophic	Multi	No
Balaji *et al.* [3]	TP	Crisp	Single	Fixed-charge, truck load constraints
Das *et al.* [4]	TP	Crisp	Multi	No
Gao and Kar [9]	STP	Uncertain	Single	Product blending constraints
Hashmi *et al.* [11]	TP	Fuzzy linguistic	Multi	Fixed-charge
Maity *et al.* [19]	TP	Uncertain	Multi	No
Maity *et al.* [20]	TP	Interval	Multi	No
Midya and Roy [25]	TP	Rough	Multi	Fixed-charge
Papageorgiou *et al.* [27]	TP	Crisp	Single	Fixed-charge, product blending constraints
Roy *et al.* [30]	TP	Intuitionistic fuzzy	Multi	No
Roy *et al.* [31]	TP	Random rough	Multi	Fixed-charge
Roy and Midya [29]	STP	Intuitionistic fuzzy	Multi	Fixed-charge, product blending constraints
Roy *et al.* [32]	STP	Two-fold uncertainty	Multi	Fixed-charge
Sengupta *et al.* [34]	STP	Gamma type-2	Single	No
Zhao and Pan [39]	TP	Uncertain	Single	Transfer cost
Zhimiao and Xu [40]	STP	Rough	Multi	No
Proposed model	TP	Fuzzy-rough uncertainty	Multi	Product blending constraints, truck load constraints, transfer cost, fixed-charge

– Roy and Midya [29] proposed MOFCTP with conveyance constraints, product blending constraints in intuitionistic fuzzy uncertainty, but did not analyze the case of low vehicle capacity as truck load constraints and the type-II fixed-charge which depend on transported items and vehicle capacity. Also they included only single uncertainty, not type-2 uncertainty which may be required on some critical situations. They analyzed the problem in single stage, not in two/multi stage situation which are significant in some positions. Again they did not consider carbon emission case which is a global problem arisen from transportation sector.

Now studying the research gaps in mentioned above, as well as the work of [39] on transfer cost, the work of [3] on truck load constraints and the work of [29] about product blending constraints, we are motivated to formulate a new mathematical model on MOFCTP. The main focuses of the proposed problem are described as follows:

– To add transfer cost and type-I fixed-charge from the TS for transferring the items of transportation system in the proposed model on MOFCTP.

- To impact truck load constraints and type-II fixed-charge in the suggested model on MOFCTP for transportation policies and for analyzing the situation to select a suitable choice among them by providing the advantages or disadvantages.
- To incorporate product blending constraints in the designed model of MOFCTP for purity levels of the customers' satisfaction and for some economical facility of related demand company.
- To introduce minimum transportation cost with other extra cost in the considered model on MOFCTP for the beneficial effect by optimum delivery time to improve customers' demand and minimum carbon emission that helpful for the government policy to reduce air pollution.
- To apprehend the realistic situations of the expressed model, fuzzy-rough variable is chosen as two-fold uncertainty that overcomes the gap of single type uncertainty.
- To extract the deterministic form of the exposed model, expected value operator is used.
- To verify the presented model, three advanced and updated techniques as FP, NLP and global criteria method (GCM) are utilized in presence of fuzzy-rough environment.

The remaining part of the paper is structured as follows: Section 2 defines the motivation of our study. Some basic definitions on fuzzy set, rough set, neutrosophic set, fuzzy-rough set are presented in Section 3. In Section 4, notations, assumptions and the mathematical model are described. Section 5 introduces the solution methodology and the main contributions with limitations of our proposed model. Case study with two numerical examples are illustrated in Section 6. Section 7 depicts the results and discussion. Managerial insights are covered in Section 8. Conclusions and future research scopes are provided in Section 9.

2. Motivation for this study

Most of the commonly used vehicles are low duty, medium duty and heavy duty vehicles. All times the capacity of vehicle may not be equal to the amount of items that to be transported. Generally a TP has been analyzed for two cases of vehicle capacities. At a time when vehicle capacity is greater than the amount of items that to be carried out from sources to destinations and this case includes as STP. Hence the transported cost is defined as c_{ijk} for transporting x_{ijk} unit of items from ith source to jth destination using kth type of conveyance. But in other case, when amount of items is greater than the vehicle capacity then the amount cannot be transported with single slot and slot will be repeated times. For example, if a vehicle capacity is 5 ton and amount that to be transported is 8.5 ton then item will be transported in 2 slots. As $8.5/5 = 1.7 \equiv 2$ (integer variable) and for this slot the cost is added with transportation cost. Again when vehicle capacity is 4 ton then $8.5/4 = 2.125 \equiv 3$ (integer variable) and hence 3 slots arise. Hence the slots are represented as integer variables and for such existing integer variables, the types of problems are referred to as truck load constraints. Whenever vehicle capacity decreases then number of slot increases and total transportation cost increases, *i.e.*, cost varies as inversely proportional to the capacity of vehicle. Again whenever the vehicle capacity gradually increases in compare to the amount of items that to be transported and the problem is treated as STP. Now we analyze this vehicle cost as in the following. From above example let transported amount (x) be 8.5 ton, vehicle capacity $(W) = 5\,\text{ton}$, carrying cost (F) per slot = 75\$. Hence $8.5/5 = 1.7 \equiv 2$ (integer variable). Therefore two types of cost exist which are $1.7 \times 75\$ = 127.5\$$ and $2 \times 75\$ = 150\$$. It is obvious that the cost for second case is greater than first case. If the second case includes as truck load constraints and then the total cost depends on the number of slots. Other case is the general problem where cost considers as fixed-charge (*i.e.*, type-II fixed-charge) that related per unit of weight. Therefore by our investigation, we analyze truck load constraints for MOTP in the presence of TS. We always prefer vehicle capacity depending on the weight of items, and as a result we choose the cost as the type-II fixed-charge not on the number of slots for truck load constraints.

3. Basic fundamental definitions

In the purpose of model formulation, we describe some useful definitions and theorems which are related with fuzzy set, rough set, neutrosophic set and fuzzy-rough set.

Definition 3.1 ([16])**.** Considering a non-empty set Λ and Δ be an element of $\mathbb{A}$, where $\mathbb{A}$ be a σ-algebra of subsets of Λ. Considering π as a non-negative, real-valued and additive set function. Thereafter $(\Lambda,\ \Delta,\ \mathbb{A}, \pi)$ is defined as *rough space*.

Trust theory is initiated from rough programming and the possibility theory for FP. Liu [16] generated the trust measure with both the probability measure and the possibility measure to describe two-fold uncertainty, such as random-rough uncertainty and fuzzy-rough uncertainty.

Definition 3.2 ([28])**.** Let U is the set of objects defined as the universe. Again let $R \subseteq U \times U$ be an indiscernibility relation with R is an equivalence relation that induced by any element x, denoted as $R(x)$. Now let X be a subset of U and the elementary notions of rough set theory are defined as follows:

- Xu and Tao [35]. The *lower approximation* of a set X with respect to R is the set of all objects, which are certainly as X with respect to R. This approximation is denoted by $\underline{R}(X)$ and is defined as follows: $\underline{R}(X) = \cup\{R(x) : R(x) \subseteq X, X \in U\}$
- Xu and Tao [35]. The *upper approximation* of a set X with respect to R is the set of all objects which are possibly classified as X with respect to R. It is denoted by $\overline{R}(X)$ and is described as : $\overline{R}(X) = \cup\{R(x) : R(x) \cap X \neq \phi, X \in U\}$.
- Xu and Tao [35]. The *boundary region* of a set X with respect to R is the set of all objects, which can be classified neither as X nor as $\overline{X}$ (*i.e.*, not-X) with respect to R. The boundary region is denoted by $BN_R(X)$ and is described as: $BN_R(X) = \underline{R}(X) - \overline{R}(X)$.
- Pawlak [28]. Now the set X is defined *rough* (*i.e.*, *imprecise* with respect to R), if the boundary region of X is non-empty, otherwise the set X is defined *crisp* (*i.e.*, exact with respect to R), if the boundary region of X is empty.

3.1. Arithmetics operations on rough intervals

Let there be two rough intervals in the form: $r^* = ([\underline{r_1^*}, \overline{r_1^*}], [\underline{r_2^*}, \overline{r_2^*}])$; $\underline{r_2^*} \leq \underline{r_1^*} < \overline{r_1^*} \leq \overline{r_2^*}$ and $r' = ([\underline{r_1'}, \overline{r_1'}], [\underline{r_2'}, \overline{r_2'}])$; $\underline{r_2'} \leq \underline{r_1'} < \overline{r_1'} \leq \overline{r_2'}$. Let $\circ \in \{+, -, \times, /\}$ be a binary operation on the set of crisp intervals. Then the rough interval arithmetic operations are defined by $r^* \circ r' = ([\underline{r^*} \circ \underline{r'}], [\overline{r^*} \circ \overline{r'}])$, where $r^* \circ r'$ becomes a rough interval.

- **Addition**: $r^* + r' = ([\underline{r^*} + \underline{r'}], [\overline{r^*} + \overline{r'}])$.
- **Subtraction**: $r^* - r' = ([\underline{r^*} - \overline{r'}], [\overline{r^*} - \underline{r'}])$.
- **Multiplication**: $r^* \times r' = ([\underline{r^*} \times \underline{r'}], [\overline{r^*} \times \overline{r'}])$.
- **Division**: $r^*/r' = ([\underline{r_1^*}/\overline{r_1'}, \overline{r_1^*}/\underline{r_1'}], [\underline{r_2^*}/\overline{r_2'}, \overline{r_2^*}/\underline{r_2'}])$ if $0 \notin [\underline{r_2'}, \overline{r_2'}]$.

Definition 3.3 ([38])**.** A fuzzy set $\tilde{A}$ in a universal set X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X, a real number in the interval [0, 1]. A fuzzy set $\tilde{A}$ is normal iff sup $\mu_{\tilde{A}}(x)$=1. A fuzzy set $\tilde{A}$ is convex iff for every pair of points x_1, x_2 in X, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\delta \in [0, 1]$. A fuzzy number (FN) is a convex normalized fuzzy set of the real line $\mathbb{R}$ with continuous membership function.

In fuzzy set, trapezoidal fuzzy number (TzFN) is a quadruplet defined as $\tilde{a} = (a_1, a_2, a_3, a_4)$ where $a_1 \leq a_2 \leq a_3 \leq a_4$. Therefore for a TzFN $\tilde{A}$, the membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\ 1, & \text{if } a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{if } a_3 \leq x \leq a_4 \\ 0, & \text{if } x > a_4. \end{cases}$$

Definition 3.4 ([33])**.** The concept of *neutrosophic set* is an analytical sight to represent the indeterminate and inconsistent information and applied in scientific and engineering applications. Let X be the universal set. A single valued neutrosophic set $\tilde{A}^n$ over X is of the form $\tilde{A}^n = \{\langle x, \mu_{\tilde{A}^n}(x), \sigma_{\tilde{A}^n}(x), \gamma_{\tilde{A}^n}(x)\rangle : x \in X\}$, where $\mu_{\tilde{A}^n}(x) : X \to [0,1]$, $\sigma_{\tilde{A}^n}(x) : X \to [0,1]$, $\gamma_{\tilde{A}^n}(x) : X \to [0,1]$ with $0 \le \sup\{\mu_{\tilde{A}^n}(x)\} + \sup\{\sigma_{\tilde{A}^n}(x)\} + \sup\{\gamma_{\tilde{A}^n}(x)\} \le 3, \forall\, x \in X$. Here $\mu_{\tilde{A}^n}(x)$, $\sigma_{\tilde{A}^n}(x)$ and $\gamma_{\tilde{A}^n}(x)$ are the degrees of truth membership, indeterminacy membership and falsity membership of x in $\tilde{A}^n$ respectively.

Definition 3.5 ([16])**.** A *fuzzy-rough variable* is a function ζ from a rough space $(\Lambda,\ \Delta,\ \mathbb{A}, \pi)$ to a collection of fuzzy variables such that for any Borel set B of $\mathbb{R}$, the function $\zeta(B)(\lambda)$=Pos $\{\zeta(\lambda) \in B\}$, where the abbreviation Pos is a mapping that represents possibility measure of λ.

For example let $\zeta = (x-3, x-2, x+2, x+3)$ be a trapezoidal fuzzy-rough number with $x = ([p,q],[r,s])$, $0 \le r \le p < q \le s$, where x is a rough variable then, ζ is a fuzzy-rough variable, and symbolically it is denoted by $x \models ([p,q],[r,s])$.

Definition 3.6 ([16])**.** Let ζ be a fuzzy-rough variable, on the rough space $(\Lambda,\ \Delta,\ \mathbb{A}, \pi)$. Then, the *expected value* of ζ is described as follows:

$$E[\zeta] = \int_0^{\infty} Tr\{\lambda \in \Lambda : E[\zeta(\lambda)] \ge r\}\mathrm{d}r - \int_{-\infty}^{0} Tr\{\lambda \in \Lambda : E[\zeta(\lambda)] \le r\}\mathrm{d}r,$$

provided that at least one of the integrals exist, where E is the expected-value operator and the abbreviation Tr represents the trust measure of ζ [16].

Definition 3.7 ([16])**.** If $\zeta = ([p,q][r,s])$ is a rough variable, then trust measure of ζ can be defined as:

$$Tr\{\zeta \ge x\} = \begin{cases} 0, & \text{for} \quad x \ge s \\ \frac{(s-x)}{2(s-r)}, & \text{for} \quad q \le x \le s \\ \frac{1}{2}\big(\frac{(s-x)}{(s-r)} + \frac{(q-x)}{(q-p)}\big), & \text{for} \quad p \le x \le q \\ \frac{1}{2}\big(\frac{(s-x)}{(s-r)} + 1\big), & \text{if} \quad r \le x \le p \\ 1, & \text{if} \quad x \le r. \end{cases}$$

$$Tr\{\zeta \le x\} = \begin{cases} 0, & \text{for} \quad x \le r \\ \frac{(x-r)}{2(s-r)}, & \text{for} \quad r \le x \le p \\ \frac{1}{2}\big(\frac{(x-r)}{(s-r)} + \frac{(x-p)}{(q-p)}\big), & \text{for} \quad p \le x \le q \\ \frac{1}{2}\big(\frac{(x-r)}{(s-r)} + 1\big), & \text{if} \quad q \le x \le s \\ 1, & \text{if} \quad x \ge s. \end{cases}$$

So the expected value of ζ calculated from Definition 3.6 is $\frac{1}{4}(p+q+r+s)$.

Definition 3.8 ([16])**.** Let $\tilde{X}$ is a normalized fuzzy variable. The expected value of this fuzzy variable is presented by the help of Credibility measure (presented by Liu and Liu [18]) which is defined as: $E(\tilde{X}) = \int_0^{\infty} Cr\{\tilde{X} \ge r\}\mathrm{d}r - \int_{-\infty}^{0} Cr\{\tilde{X} \le r\}\mathrm{d}r$, provided that at least one of two integral is finite. If $\tilde{A} = (a,b,c,d)$ be a TzFN, then by the above definition the expected value of $\tilde{A}$ is $\frac{1}{4}(a+b+c+d)$.

Here E is the expected-value operator and the abbreviation Cr represents the Credibility measure.

Theorem 3.9 ([17])**.** *Let ζ be the fuzzy-rough variable with finite expected value. As a useful expression of expected value, we have $E[\zeta] = \int_0^1 \phi^{-1}(\beta)\mathrm{d}\beta$, where ϕ^{-1} is the inverse of uncertainty distribution of the fuzzy-rough variable ζ.*

Proposition 3.10 ([37]). *Let ζ be a trapezoidal fuzzy-rough variable with $\zeta = (a_1, a_2, a_3, a_4)$, where a_1, a_2, a_3 and a_4 are rough variables (i.e., rough interval) defined on a rough space $(\Lambda,\ \Delta,\ \mathbb{A}, \pi)$, and we have*

$$\begin{aligned}
a_1 &= ([p_2, p_3], [p_1, p_4]), \quad p_1 \le p_2 < p_3 \le p_4,\\
a_2 &= ([q_2, q_3], [q_1, q_4]), \quad q_1 \le q_2 < q_3 \le q_4,\\
a_3 &= ([r_2, r_3], [r_1, r_4]), \quad r_1 \le r_2 < r_3 \le r_4,\\
a_4 &= ([s_2, s_3], [s_1, s_4]), \quad s_1 \le s_2 < s_3 \le s_4.
\end{aligned}$$

Then, the expected value of ζ is given by: $E[\zeta] = \frac{1}{16}\sum_{i=1}^{4}(p_i + q_i + r_i + s_i)$.

Proof. The proof of the proposition is apparent by taking the expected values of fuzzy-rough variable from Definitions 3.7 and 3.8. □

Theorem 3.11 ([16]). *Let ζ and η be the fuzzy-rough variables with finite expected values. Then, for any real numbers p and q, we have $E[p\zeta + q\eta] = pE[\zeta] + qE[\eta]$.*

Proof. Let $\zeta = (a_1, a_2, a_3, a_4)$ and $\eta = (b_1, b_2, b_3, b_4)$ be two fuzzy-rough variables with a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are rough variables defined on a rough space $(\Lambda,\ \Delta,\ \mathbb{A}, \pi)$, and we have

$$\begin{aligned}
a_1 &= ([p_2, p_3], [p_1, p_4]), \quad p_1 \le p_2 < p_3 \le p_4, &\quad a_2 &= ([q_2, q_3], [q_1, q_4]), \quad q_1 \le q_2 < q_3 \le q_4,\\
a_3 &= ([r_2, r_3], [r_1, r_4]), \quad r_1 \le r_2 < r_3 \le r_4, &\quad a_4 &= ([s_2, s_3], [s_1, s_4]), \quad s_1 \le s_2 < s_3 \le s_4;\\
b_1 &= ([x_2, x_3], [x_1, x_4]), \quad x_1 \le x_2 < x_3 \le x_4, &\quad b_2 &= ([y_2, y_3], [y_1, y_4]), \quad y_1 \le y_2 < y_3 \le y_4,\\
b_3 &= ([u_2, u_3], [u_1, u_4]), \quad u_1 \le u_2 < u_3 \le u_4, &\quad b_4 &= ([v_2, v_3], [v_1, v_4]), \quad v_1 \le v_2 < v_3 \le v_4.
\end{aligned}$$

Therefore $[p\zeta + q\eta] = (p \cdot a_1 + q \cdot b_1, p \cdot a_2 + q \cdot b_2, p \cdot a_3 + q \cdot b_3, p \cdot a_4 + q \cdot b_4)$, where

$$\begin{aligned}
(p \cdot a_1 + q \cdot b_1) &= ([p \cdot p_2 + q \cdot x_2, p \cdot p_3 + q \cdot x_3], [p \cdot p_1 + q \cdot x_1, p \cdot p_4 + q \cdot x_4]),\\
&\quad p \cdot p_1 + q \cdot x_1 \le p \cdot p_2 + q \cdot x_2 < p \cdot p_3 + q \cdot x_3 \le p \cdot p_4 + q \cdot x_4,\\
(p \cdot a_2 + q \cdot b_2) &= ([p \cdot q_2 + q \cdot y_2, p \cdot q_3 + q \cdot y_3], [p \cdot q_1 + q \cdot y_1, p \cdot q_4 + q \cdot y_4]),\\
&\quad p \cdot q_1 + q \cdot y_1 \le p \cdot q_2 + q \cdot y_2 < p \cdot q_3 + q \cdot y_3 \le p \cdot q_4 + q \cdot y_4,\\
(p \cdot a_3 + q \cdot b_3) &= ([p \cdot r_2 + q \cdot u_2, p \cdot r_3 + q \cdot u_3], [p \cdot r_1 + q \cdot u_1, p \cdot r_4 + q \cdot u_4]),\\
&\quad p \cdot r_1 + q \cdot u_1 \le p \cdot r_2 + q \cdot u_2 < p \cdot r_3 + q \cdot u_3 \le p \cdot r_4 + q \cdot u_4,\\
(p \cdot a_4 + q \cdot b_4) &= ([p \cdot s_2 + q \cdot v_2, p \cdot s_3 + q \cdot v_3], [p \cdot s_1 + q \cdot v_1, p \cdot s_4 + q \cdot v_4]),\\
&\quad p \cdot s_1 + q \cdot v_1 \le p \cdot s_2 + q \cdot v_2 < p \cdot s_3 + q \cdot v_3 \le p \cdot s_4 + q \cdot v_4.
\end{aligned}$$

From the Proposition 3.10 we have that expected value

$$\begin{aligned}
E[p\zeta + q\eta] &= \frac{1}{16}\sum_{i=1}^{4}((p \cdot p_i + q \cdot x_i) + (p \cdot q_i + q \cdot y_i) + (p \cdot r_i + q \cdot u_i) + (p \cdot s_i + q \cdot v_i))\\
&= \frac{1}{16}\sum_{i=1}^{4}(p \cdot (p_i + q_i + r_i + s_i) + q \cdot (x_i + y_i + u_i + v_i))\\
&= p\frac{1}{16}\sum_{i=1}^{4}(p_i + q_i + r_i + s_i) + q\frac{1}{16}\sum_{i=1}^{4}(x_i + y_i + u_i + v_i)\\
&= pE[\zeta] + qE[\eta].
\end{aligned}$$

□

Theorem 3.12 ([37]). *Let* $\tilde{\overline{x}}_{ijk}$ *are a trapezoidal fuzzy-rough variable, defined as:* $\tilde{\overline{x}}_{ijk} = (\overline{x}_{ijk1}, \overline{x}_{ijk2}, \overline{x}_{ijk3}, \overline{x}_{ijk4})$ *with* $\overline{x}_{ijks} \models ([x_{ijks2}, x_{ijks3}], [x_{ijks1}, x_{ijks4}])$*, for* $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, K; s = 1, 2, 3, 4$*, where* $x_{ijks1} \leq x_{ijks2} < x_{ijks3} \leq x_{ijks4}$*, then* $E[\tilde{\overline{x}}_{ijk}] = \frac{1}{16} \sum_{s=1}^{4} \sum_{u=1}^{4} x_{ijksu} \; \forall \; i, j, k$.

Proof. This theorem is the generalized form of Proposition 3.10. The proof of this theorem is also apparent by taking the expected values of fuzzy-rough variable from Definitions 3.7 and 3.8. □

Authors [32] used this defuzzification technique in their proposed study to transform two-fold uncertainty into deterministic form and applied in a MOFCTP with multi-item and conveyance constraints.

Whenever $\tilde{\overline{x}}_{ijk}$ are triangular fuzzy-rough variable, defined as: $\tilde{\overline{x}}_{ijk} = (\overline{x}_{ijk1}, \overline{x}_{ijk2}, \overline{x}_{ijk3})$ with $\overline{x}_{ijks} \models ([x_{ijks2}, x_{ijks3}], [x_{ijks1}, x_{ijks4}])$, for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, K$; $s = 1, 2, 3$, where $x_{ijks1} \leq x_{ijks2} < x_{ijks3} \leq x_{ijks4}$, then $E[\tilde{\overline{x}}_{ijk}] = \frac{1}{12} \sum_{s=1}^{3} \sum_{u=1}^{4} x_{ijksu} \; \forall \; i, j, k$.

4. Problem background

In this section, the first subsection contains a list of notations with their usual meanings, and assumptions are considered in next subsection to formulate a model of MOFCTP with truck load constraints and product blending constraints in fuzzy-rough environment.

4.1. Notations

The following notations are used to describe our proposed mathematical model as:

Parameters

i: number of sources,
j: number of TSs,
k: number of destinations,
$\tilde{\overline{c}}^1_{ij}$: transportation cost for unit quantity of the products from ith source to jth TS,
$\tilde{\overline{c}}^2_{jk}$: transportation cost for unit quantity of the products from jth TS to kth destination,
D^1_{ij}: distance from ith source to jth TS,
D^2_{jk}: distance from jth TS to kth destination,
$\tilde{\overline{F}}^1_{ij}$: type-II fixed-charge or general charge for truck load constraints of the product transported from ith source to jth TS,
$\tilde{\overline{F}}^2_{jk}$: type-II fixed-charge or general charge for truck load constraints of the product transported from jth TS to kth destination,
W: weight capacity of the vehicle,
$\tilde{\overline{\alpha}}_j$: transfer cost for unit product at jth TS,
$\tilde{\overline{f}}_j$: type-I fixed-charge for unit product at jth TS,
$\tilde{\overline{t}}^1_{ij}$: time of transportation of the products from ith source to jth TS,
$\tilde{\overline{t}}^2_{jk}$: time of transportation of the products from jth TS to kth destination,
$\tilde{\overline{a}}_i$: amount of product available at ith source,
$\tilde{\overline{b}}_k$: demand of product at kth destination,
e_{CO_2}: rate of carbon emission of the vehicle per unit item and per unit distance,
p_i: normal purity of product available at ith source,
$p^{\min}_j$: minimum purity of product required at jth TS,
$p^{\min}_k$: minimum purity of product required at kth destination,
$\tilde{\overline{Z}}_l$: the objective function in fuzzy-rough (*i.e.*, two-fold uncertainty) nature ($l = 1, 2, 3$),
Z_l: the expected value of objective function $\tilde{\overline{Z}}_l, (l = 1, 2, 3)$.

Decision variables

x_{ij}: amount of products that to be transported from ith source to jth TS,
y_{jk}: amount of products that to be transported from jth TS to kth destination,
δ^1_{ij}: integer variable occurs from ith source to jth TS for truck load constraints which is defined as:

$$\delta^1_{ij} = \begin{cases} 0, & \text{if } \ x_{ij} = 0, \\ \text{integral part of } \ \left(\frac{x_{ij}}{W}\right) + 1, & \text{if } \ x_{ij} > 0 \ \text{ and remainder of } \ \left(\frac{x_{ij}}{W}\right) > 0, \\ \text{integral part of } \ \left(\frac{x_{ij}}{W}\right), & \text{if } \ x_{ij} > 0 \ \text{ and remainder of } \ \left(\frac{x_{ij}}{W}\right) = 0, \end{cases}$$

δ^2_{jk}: integer variable uses from jth TS to kth destination for truck load constraints which is stated as:

$$\delta^2_{jk} = \begin{cases} 0, & \text{if } \ y_{jk} = 0, \\ \text{integral part of } \ \left(\frac{y_{jk}}{W}\right) + 1, & \text{if } \ y_{jk} > 0 \ \text{ and remainder of } \ \left(\frac{y_{jk}}{W}\right) > 0, \\ \text{integral part of } \ \left(\frac{y_{jk}}{W}\right), & \text{if } \ y_{jk} > 0 \ \text{ and remainder of } \ \left(\frac{y_{jk}}{W}\right) = 0. \end{cases}$$

Binary variables

$\phi\left(\sum_i x_{ij}\right)$: binary variable takes the value "1" if $\sum_i x_{ij} > 0$ used and "0" otherwise, *i.e.*,

$$\phi\left(\sum\nolimits_i x_{ij}\right) = \begin{cases} 1, & \text{if } \ \sum_i x_{ij} > 0, \\ 0, & \text{otherwise}, \end{cases}$$

η^1_{ij}: binary variable selects the value "1" if the $x_{ij} > 0$ and "0" otherwise, *i.e.*,

$$\eta^1_{ij} = \begin{cases} 1, & \text{if } \ x_{ij} > 0, \\ 0, & \text{otherwise}, \end{cases}$$

η^2_{jk}: binary variable chooses the value "1" if the $y_{jk} > 0$ and "0" otherwise, *i.e.*,

$$\eta^2_{jk} = \begin{cases} 1, & \text{if } \ y_{jk} > 0, \\ 0, & \text{otherwise}. \end{cases}$$

4.2. Assumptions

The following assumptions are taken into consideration to describe our proposed mathematical model as:

- $\tilde{\tilde{a}}_i > 0, \tilde{\tilde{b}}_k > 0 \ \forall \ i, k$.
- $x, \ y$ are the vectors consisting of $x_{ij}, \ y_{jk} \ \forall \ i, j, k$ respectively.
- No items deteriorate during transportation.
- Elapsed time of TS is not considered.

4.3. Model formulation

Transportation cost has a major role on the total network charge and therefore we concentrate our focus on transportation cost and also on transportation time, carbon emission. These objects are optimized through proper planning and management. The best optimal allocation for transportation with low vehicle capacity based on source, demand, product blending constraints and truck load constraints are used in this model. To formulate the mathematical model, we consider x_{ij} and y_{jk} are two continuous decision variables which denote the amount of product transported from ith source to jth TS and jth TS to kth destination respectively. Also p_i be the normal quality of product such that the average value quality of product meets at jth TS and kth destination as $\frac{\sum_i p_i x_{ij}}{\sum_i x_{ij}}$ and $\frac{\sum_j p_j^{\min} y_{jk}}{\sum_j y_{jk}}, j = 1, 2, \ldots, n, \ k = 1, 2, \ldots, K$ respectively.

Since, $p_j^{\min}$ and $p_k^{\min}$ are the minimum quality of the product which are required at jth TS and kth destination, therefore we introduce the following constraints as: $\frac{\sum_i p_i x_{ij}}{\sum_i x_{ij}} \geq p_j^{\min}$; $\frac{\sum_j p_j^{\min} y_{jk}}{\sum_j y_{jk}} \geq p_k^{\min}$, *i.e.*, the linear forms are $\sum_i (p_i - p_j^{\min}) x_{ij} \geq 0$ and $\sum_j (p_j^{\min} - p_k^{\min}) y_{jk} \geq 0,\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, K$. In our formulated model, we add these constraints as product blending constraints which are defined the average quality of all products received at TSs and destinations. The aim of our formulated model is to minimize total transportation cost including type-I fixed-charge and transfer cost. Also to minimize carbon emission and time which are required for transporting products from supplier to consumer through TS. Hence three objective functions have different units. The respective units are as cost in \$, total carbon emission in ton and time in hour. Therefore three objective functions cannot follow the arithmetic addition and we are not able to minimize them by summing them adjacently. Again we consider the parameter related to carbon emission in crisp nature, whereas cost parameter and time parameter are considered in fuzzy-rough nature depending on several unpredictable data. Also transportation time and carbon emission during transportation are contradict to each other. To reduce transportation time, the vehicle speed is to be increased and then total carbon emission with total cost of transportation are increased. Therefore transportation time is inversely proportional to carbon emission and cost. With such circumstances, the problem becomes multi-objective with each objective function contradicts to each other. Henceforth the formulated model of MOFCTP with product blending linear constraints, TS and truck load constraints is defined as:

Model 1

$$
\begin{aligned}
\text{minimize}\ \ \tilde{\tilde{Z}}_1(x, y) &= \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\tilde{\tilde{c}}_{ij}^{1} x_{ij} + \tilde{\tilde{F}}_{ij}^{1} \delta_{ij}^{1} \right) + \sum_{j=1}^{n} \sum_{k=1}^{K} \left(\tilde{\tilde{c}}_{jk}^{2} y_{jk} + \tilde{\tilde{F}}_{jk}^{2} \delta_{jk}^{2} \right) \\
&\quad + \sum_{j=1}^{n} \tilde{\tilde{f}}_j \cdot \phi \left(\sum_{i=1}^{m} x_{ij} \right) + \sum_{j=1}^{n} \tilde{\tilde{\alpha}}_j \cdot \left(\sum_{i=1}^{m} x_{ij} \right) \\
\text{minimize}\ \ Z_2(x, y) &= \left(\sum_{i=1}^{m} \sum_{j=1}^{n} D_{ij}^{1} x_{ij} + \sum_{j=1}^{n} \sum_{k=1}^{K} D_{jk}^{2} y_{jk} \right) e_{\mathrm{CO_2}} \\
\text{minimize}\ \ \tilde{\tilde{Z}}_3(x, y) &= \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\tilde{t}}_{ij}^{1} \eta_{ij}^{1} + \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{\tilde{t}}_{jk}^{2} \eta_{jk}^{2} \\
\text{subject to}\ \ & \sum_{j=1}^{n} x_{ij} \leq \tilde{\tilde{a}}_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{j=1}^{n} y_{jk} \geq \tilde{\tilde{b}}_k \quad (k = 1, 2, \ldots, K), \\
& \sum_{i=1}^{m} \left(p_i - p_j^{\min} \right) x_{ij} \geq 0 \quad (j = 1, 2, \ldots, n), \\
& \sum_{j=1}^{n} \left(p_j^{\min} - p_k^{\min} \right) y_{jk} \geq 0 \quad (k = 1, 2, \ldots, K), \\
& \sum_{i=1}^{m} x_{ij} = \sum_{k=1}^{K} y_{jk} \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0, y_{jk} \geq 0, \forall\ i,\ j,\ k.
\end{aligned}
$$

The feasibility condition of this TP is chosen as follows:

$$\sum_{i=1}^{m} \tilde{\bar{a}}_i \geq \sum_{k=1}^{K} \tilde{\bar{b}}_k.$$

In Model 1, we consider most of parameters are in fuzzy-rough nature. In this model, the first and second part of objective function of $\tilde{\bar{Z}}_1(x,y)$ represent the transportation cost and type-II fixed-charge or general charge for truck load constraints from ith source to jth TS and from jth TS to kth destination respectively. Hence fixed-charge is considered for vehicle capacity that reduces into truck load constraints, and this fixed-charge depends on the transported amount of items. The third part is the general fixed-charge (*i.e.*, type-I fixed-charge) at jth TS which is independent on transported amount of items, and fourth part exists for transfer cost at jth TS. Second objective function $Z_2(x,y)$ considers the carbon emission from ith source to kth destination through jth TS. Also third objective function $\tilde{\bar{Z}}_3(x,y)$ presents the total transportation time from ith source to jth TS and then jth TS to kth destination. In this model, we choose two types of fixed-charges, one is type-I fixed-charge $\tilde{\bar{f}}_j$ for jth TS, and second one is type-II fixed charge $\tilde{\bar{F}}_{ij}$ and $\tilde{\bar{F}}_{jk}$ which are also used as general charge for truck load constraints. Also δ^1_{ij} and δ^2_{jk} are defined the number of slots (*i.e.*, integer variable) for the case of truck load constraints and fractional variables for the case of type-II fixed-charge. The 1st and 2nd constraints are defined as source and demand constraints. Also 3rd and 4th constraints are product blending constraints. 5th constraints indicate the transfer constraints, *i.e.*, total supplied items are distributed to their destinations without any deterioration. Supply and demand parameters are considered in fuzzy-rough nature and the purity levels are in crisp nature. Non-negativity restrictions are defined by 6th constraints. We use the notation with abbreviation $\tilde{\overline{(:)}}$ for fuzzy-rough parameters in the model.

4.4. Equivalent deterministic model

We cannot directly extract the solution of MOFCTP due to the existence of fuzzy-rough variables. So we take the advantage of the expected-value operator E (defined in Def. 3.6) to transform Model 1 into crisp form.

Theorem 4.1 ([17]). *Let* $\tilde{\bar{c}}^1_{ij}$, $\tilde{\bar{c}}^2_{jk}$, $\tilde{\bar{F}}^1_{ij}$, $\tilde{\bar{F}}^2_{jk}$, $\tilde{\bar{f}}_j$, $\tilde{\bar{\alpha}}_j$, $\tilde{\bar{t}}^1_{ij}$, $\tilde{\bar{t}}^2_{jk}$, $\tilde{\bar{a}}_i$ *and* $\tilde{\bar{b}}_k$ *are independent fuzzy-rough variables corresponding to the regular fuzzy-rough distributions* $\phi_{\tilde{\bar{c}}^1_{ij}}$, $\phi_{\tilde{\bar{c}}^2_{jk}}$, $\phi_{\tilde{\bar{F}}^1_{ij}}$, $\phi_{\tilde{\bar{F}}^2_{jk}}$, $\phi_{\tilde{\bar{f}}_j}$, $\phi_{\tilde{\bar{\alpha}}_j}$, $\phi_{\tilde{\bar{t}}^1_{ij}}$, $\phi_{\tilde{\bar{t}}^2_{jk}}$, $\phi_{\tilde{\bar{a}}_i}$ *and* $\phi_{\tilde{\bar{b}}_k}$ *respectively.*

Therefore, the deterministic equivalents expected value model is presented by Model 2 as follows:

Model 2

$$\begin{aligned}
\text{minimize } \ E\left[\tilde{\bar{Z}}_1(x,y)\right] &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left[x_{ij}\int_0^1 \phi^{-1}_{\tilde{\bar{c}}^1_{ij}}(\beta)\mathrm{d}\beta + \delta^1_{ij}\int_0^1 \phi^{-1}_{\tilde{\bar{F}}^1_{ij}}(\beta)\mathrm{d}\beta\right] \\
&+ \sum_{j=1}^{n}\sum_{k=1}^{K}\left[y_{jk}\int_0^1 \phi^{-1}_{\tilde{\bar{c}}^2_{jk}}(\beta)\mathrm{d}\beta + \delta^2_{jk}\int_0^1 \phi^{-1}_{\tilde{\bar{F}}^2_{jk}}(\beta)\mathrm{d}\beta\right] \\
&+ \sum_{j=1}^{n}\left[\phi\left(\sum_{i=1}^{m} x_{ij}\right)\cdot\int_0^1 \phi^{-1}_{\tilde{\bar{f}}_j}(\beta)\mathrm{d}\beta\right] + \sum_{j=1}^{n}\left[\left(\sum_{i=1}^{m} x_{ij}\right)\cdot\int_0^1 \phi^{-1}_{\tilde{\bar{\alpha}}_j}(\beta)\mathrm{d}\beta\right] \\
\text{minimize } \ Z_2(x,y) &= \left(\sum_{i=1}^{m}\sum_{j=1}^{n} D^1_{ij}x_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{K} D^2_{jk}y_{jk}\right)e_{CO_2}
\end{aligned}$$

$$\text{minimize}\ \ E\left[\tilde{\tilde{Z}}_3(x,y)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\eta_{ij}^1 \int_0^1 \phi_{\tilde{\tilde{t}}_{ij}^1}^{-1}(\beta)\mathrm{d}\beta\right] + \sum_{j=1}^{n}\sum_{k=1}^{K}\left[\eta_{jk}^2 \int_0^1 \phi_{\tilde{\tilde{t}}_{jk}^2}^{-1}(\beta)\mathrm{d}\beta\right]$$

$$\text{subject to}\ \ \sum_{j=1}^{n} x_{ij} \le \left[\int_0^1 \phi_{\tilde{\tilde{a}}_i}^{-1}(\beta)\mathrm{d}\beta\right] \quad (i = 1, 2, \ldots, m), \tag{4.1}$$

$$\sum_{j=1}^{n} y_{jk} \ge \left[\int_0^1 \phi_{\tilde{\tilde{b}}_k}^{-1}(\beta)\mathrm{d}\beta\right] \quad (k = 1, 2, \ldots, K), \tag{4.2}$$

$$\sum_{i=1}^{m} \left[p_i - p_j^{\min}\right] x_{ij} \ge 0 \quad (j = 1, 2, \ldots, n), \tag{4.3}$$

$$\sum_{j=1}^{n} \left[p_j^{\min} - p_k^{\min}\right] y_{jk} \ge 0 \quad (k = 1, 2, \ldots, K), \tag{4.4}$$

$$\sum_{i=1}^{m} x_{ij} = \sum_{k=1}^{K} y_{jk} \quad (j = 1, 2, \ldots, n), \tag{4.5}$$

$$x_{ij} \ge 0, y_{jk} \ge 0, \forall\ i,\ j,\ k. \tag{4.6}$$

The feasibility condition of this TP is written as:

$$\sum_{i=1}^{m} \int_0^1 \phi_{\tilde{\tilde{a}}_i}^{-1}(\beta)\mathrm{d}\beta \ge \sum_{k=1}^{K} \int_0^1 \phi_{\tilde{\tilde{b}}_k}^{-1}(\beta)\mathrm{d}\beta.$$

Models 1 and 2 have $(mn + nK)$ number of variables in second and third objective function, and in the first objective function exists $(mn+nK+n)$ number of variables (for type-II fixed-charge case) and $(2mn+2nK+n)$ number of variables (for truck load constraints case). Total number of constraints in each model is equal to $(m + 2n + 2K + mn + nK)$.

Definition 4.2. Pareto-optimal solution of Model 2 is a feasible solution $(x^*,\ y^*) = \{(x_{ij}^*,\ y_{jk}^*) : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n; k = 1, 2, \ldots, K\}$ such that there exists no other feasible solution $(x,\ y) = \{(x_{ij},\ y_{jk}) : i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, K\}$ with $Z_l(x,\ y) \le Z_l(x^*,\ y^*),\ l = 1, 2, 3$ and $Z_l(x,\ y) < Z_l(x^*,\ y^*)$ for at least one l.

5. Solution methodology

In multi-objective optimization problem, there does not always subsist a solution which is the best for all the objective functions. Again objective functions are contradict to each other. That is the solution will be the best for one objective function and that may be worst for another objective function.

To achieve Pareto-optimal solution there exist various fuzzy and non-fuzzy techniques. Most commonly used non-fuzzy techniques are as: goal programming (GP), ϵ-constraint method [31], weighted sum method, global criteria method (GCM) [5, 31], etc. Again most general fuzzy techniques are fuzzy programming (FP) [5, 21, 25, 32], fuzzy goal programming (FGP), fuzzy TOPSIS approach [32], Intuitionistic fuzzy programming (IFP) [29], Intuitionistic fuzzy TOPSIS approach [29], neutrosophic linear programming (NLP) [4]. All these methods are applicable for finding Pareto-optimal solution of any multi-objective decision making problem. Among these methods we select two fuzzy methods namely **FP** and **NLP**, and one non-fuzzy method is **GCM** which are very simple with less computational time for deriving the optimal solution of the proposed model.

TABLE 2. Pay-off matrix.

	Z_1	Z_2	Z_3
${X_1}^*$	$Z_1({X_1}^*)$	$Z_2({X_1}^*)$	$Z_3({X_1}^*)$
${X_2}^*$	$Z_1({X_2}^*)$	$Z_2({X_2}^*)$	$Z_3({X_2}^*)$
${X_3}^*$	$Z_1({X_3}^*)$	$Z_2({X_3}^*)$	$Z_3({X_3}^*)$

5.1. Fuzzy programming (FP)

FP was initiated by Zimmerman [41] for solving multi-objective optimization problem and it is very easy for finding solution. Therefore to find overall Pareto-optimal solution, we take the advantage of FP which is applied to solve MOFCTP. Hence to solve the proposed Model 2 in FP, we depict the steps as:

Step 5.1.1. Transform the fuzzy-rough problem into crisp problem using expected value operator.
Step 5.1.2. Solve each objective function independently with subject to all constraints.
Step 5.1.3. Select the tolerance of each objective function.
Step 5.1.4. Determine the positive ideal solution (PIS) and negative ideal solution (NIS) obtained from pay-off matrix, displaying in Table 2 and defined as PIS $= {Z_l}^* = \min\,\{Z_l({X_1}^*),\ Z_l({X_2}^*),\ Z_l({X_3}^*)\}$ $(l = 1, 2, 3)$ and NIS $= {Z_l}' = \max\,\{Z_l({X_1}^*),\ Z_l({X_2}^*),\ Z_l({X_3}^*)\}$ $(l = 1, 2, 3)$ respectively.
Step 5.1.5. Formulate the membership function $\mu_l(Z_l(x, y))$ corresponding to each objective function $Z_l(x, y);\ (l = 1, 2, 3)$.

$$\mu_l(Z_l(x,y)) = \begin{cases} 1, & \text{if } \ Z_l(x,y) \leq L_l, \\ \frac{U_l - Z_l(x,y)}{U_l - L_l}, & \text{if } \ L_l \leq Z_l(x,y) \leq U_l, \\ 0, & \text{if } \ Z_l(x,y) \geq U_l, \end{cases}$$

Here $U_l =$ PIS for Z_l, and $L_l =$ NIS for Z_l.
Step 5.1.6. Maximize the formulated membership function, and then single objective linear programming model with θ as aspiration level, is defined as:

Model 3A

$$\begin{aligned} \text{maximize} \ \ & \theta \\ \text{subject to} \ \ & \mu_l(Z_l(x,y)) \geq \theta, (l = 1, 2, 3), \\ & \theta \in [0, 1], \\ & \text{Constraints (4.1)}-\text{(4.6)}. \end{aligned}$$

Model 3A is transformed into simplified form which is Model 3B as:

Model 3B

$$\begin{aligned} \text{maximize} \ \ & \theta \\ \text{subject to} \ \ & Z_l(x,y) + (U_l - L_l)\theta \leq U_l, (l = 1, 2, 3), \\ & \theta \in [0, 1], \\ & \text{Constraints (4.1)}-\text{(4.6)}. \end{aligned}$$

Step 5.1.7. Solve Model 3B by LINGO iterative scheme and GAMS software with parameter θ.

Theorem 5.1. *If $(x^*, y^*) = \{(x_{ij}, y_{jk}) :\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n; k = 1, 2, \ldots, K\}$ is an optimal solution of Model 3B then it is also Pareto-optimal (non-dominated) solution of Model 2.*

Proof. Let (x^*, y^*) is not a Pareto optimal (non-dominated) solution of Model 2. Therefore, from Def. 4.1, we consider that there exist at least one x and at least one y such that $Z_l(x,y) \leq Z_l(x^*,y^*)$ for $l = 1,2,3$ and $Z_l(x,y) < Z_l(x^*,y^*)$ for at least one l. Therefore membership function $\mu_l(Z_l(x,y))$ is strictly decreasing with respect to the corresponding objective function Z_l in [0, 1]. Hence $\mu_l(Z_l(x,y)) \geq \mu_l(Z_l(x^*,y^*))$ $\forall$ l and $\mu_l(Z_l(x,y)) > \mu_l(Z_l(x^*,y^*))$ for at least one l. Now $\theta = \min\{\mu_l(Z_l(x,y))\} \geq \min\{\mu_l(Z_l(x^*,y^*))\} = \theta^*$ which is a contradiction that (x^*,y^*) is an optimal solution of Model 3B. Here θ^* is the value of θ at (x^*,y^*). This completes the proof of the theorem. □

5.2. Neutrosophic linear programming (NLP)

Here, we improve NLP to derive the Pareto-optimal solution of multi-objective decision making problem. NLP is a modified and extended method that finds the Pareto-optimal solution of multi-objective problem. Neutrosophic set was defined by Smarandache [33] with incorporating truth membership function, indeterminacy membership function and falsity membership function for every objective function. In this programming we maximize the truth and indeterminacy membership functions whereas minimize falsity membership function. As the truth membership function is chosen for maximum and the falsity membership function is minimum, so the solution that extracted from NLP is very close to Pareto-optimal solution of the corresponding model. To solve the proposed model by NLP, we describe the following steps as:

Step 5.2.1. Transform the fuzzy-rough problem into crisp problem by expected value operator.

Step 5.2.2. Solve each objective function individually with subject to all constraints.

Step 5.2.3. Determine the upper bound and lower bound *i.e.*, PIS and NIS for each objective function from pay-off matrix defined in Step 5.1.4.

Step 5.2.4. Design the truth membership function and indeterminacy membership function with highest degree and falsity membership function with least degree.

Step 5.2.5. Setting the tolerance and constructing the membership functions according to the bounds as:

$$T_l(Z_l(x,y)) = \begin{cases} 1, & \text{if } Z_l(x,y) \leq L_l^T, \\ 1 - \frac{Z_l(x,y) - L_l^T}{U_l^T - L_l^T}, & \text{if } L_l^T \leq Z_l(x,y) \leq U_l^T, \\ 0, & \text{if } Z_l(x,y) \geq U_l^T, \end{cases}$$

$$I_l(Z_l(x,y)) = \begin{cases} 0, & \text{if } Z_l(x,y) \leq L_l^I, \\ 1 - \frac{Z_l(x,y) - L_l^I}{U_l^I - L_l^I}, & \text{if } L_l^I \leq Z_l(x,y) \leq U_l^I, \\ 0, & \text{if } Z_l(x,y) \geq U_l^I, \end{cases}$$

$$F_l(Z_l(x,y)) = \begin{cases} 0, & \text{if } Z_l(x,y) \leq L_l^F, \\ 1 - \frac{Z_l(x,y) - L_l^F}{U_l^F - L_l^F}, & \text{if } L_l^F \leq Z_l(x,y) \leq U_l^F, \\ 1, & \text{if } Z_l(x,y) \geq U_l^F, \end{cases}$$

Here $U_l^T = U_l =$ PIS for Z_l, and $L_l^T = L_l =$ NIS for Z_l; $U_l^F = U_l^T$, $L_l^F = L_l^T + t_l(U_l^T - L_l^T)$; $L_l^I = L_l^T$, $U_l^I = L_l^T + s_l(U_l^T - L_l^T)$; $t_l, s_l \in (0,1)$ are predetermined real numbers.

Step 5.2.6. Choose the values of θ, η and ζ in $[0,\ 1]$ for each neutrosophic number as the truth, indeterminacy and falsity degrees respectively.

Step 5.2.7. Constitute NLP that represents in Model 4A.

Model 4A

$$\begin{aligned} \text{maximize} \quad & T_l(Z_l(x,y)) \ (l = 1,2,3) \\ \text{maximize} \quad & I_l(Z_l(x,y)) \ (l = 1,2,3) \\ \text{minimize} \quad & F_l(Z_l(x,y)) \ (l = 1,2,3) \\ \text{subject to} \quad & \text{Constraints } (4.1)-(4.6). \end{aligned}$$

Model 4A can be reduced to Model 4B as:

Model 4B

$$\begin{aligned}
\text{maximize} \quad & \theta \\
\text{maximize} \quad & \eta \\
\text{minimize} \quad & \zeta \\
\text{subject to} \quad & T_l(Z_l(x,y)) \geq \theta,\ I_l(Z_l(x,y)) \geq \eta,\ F_l(Z_l(x,y)) \leq \zeta, \\
& \theta + \eta + \zeta \leq 3,\ \theta + \eta + \zeta \geq 0,\ \theta \geq \zeta,\ \theta \geq \eta, \\
& \theta, \eta, \zeta \in [0,1], (l = 1,2,3), \\
& \text{Constraints } (4.1)-(4.6).
\end{aligned}$$

Now the simplified model of NLP (Model 4B) that derives the Pareto-optimal solution of MOTP (*i.e.*, Model 4C) as follows:

Model 4C

$$\begin{aligned}
\text{maximize} \quad & \theta + \eta - \zeta \\
\text{subject to} \quad & Z_l(x,y) + (U_l^T - L_l^T)\theta \leq U_l^T, \\
& Z_l(x,y) + (U_l^I - L_l^I)\eta \leq U_l^I, \\
& Z_l(x,y) - (U_l^F - L_l^F)\zeta \leq U_l^F, \\
& \theta + \eta + \zeta \leq 3,\ \theta + \eta + \zeta \geq 0,\ \theta \geq \zeta,\ \theta \geq \eta, \\
& \theta, \eta, \zeta \in [0,1], (l = 1,2,3), \\
& \text{Constraints } (4.1)-(4.6).
\end{aligned}$$

Step 5.2.8. Solve Model 4C by LINGO iterative scheme and GAMS software, and obtain Pareto-optimal solution.

Theorem 5.2. *If* $(x^*, y^*) = \{(x_{ij}, y_{jk}) : \ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ k = 1,2,\ldots,K\}$ *is an optimal solution of Model 4C then it is also Pareto-optimal (non-dominated) solution of Model 2.*

Proof. Let (x^*, y^*) is not a Pareto optimal (non-dominated) solution of Model 2. Therefore, from Def. 4.1, we choose that there exist at least one x and one y such that $Z_l(x,y) \leq Z_l(x^*,y^*)$ for $l = 1,2,3$ and $Z_l(x,y) < Z_l(x^*,y^*)$ for at least one l. Therefore truth and indeterminacy membership functions $\mu_l(Z_l(x,y))$ and $\sigma_l(Z_l(x,y))$ are strictly decreasing with respect to the corresponding objective function Z_l in $[0,1]$ respectively. Again the falsity membership function $\nu_l(Z_l(x,y))$ strictly increases with respect to the objective function Z_l in $[0,1]$. Hence $\mu_l(Z_l(x,y)) \geq \mu_l(Z_l(x^*,y^*))\ \forall\ l$ and $\mu_l(Z_l(x,y)) > \mu_l(Z_l(x^*,y^*))$ for at least one l. Similarly $\sigma_l(Z_l(x,y)) \geq \sigma_l(Z_l(x^*,y^*))\ \forall\ l$ and $\sigma_l(Z_l(x,y)) > \sigma_l(Z_l(x^*,y^*))$ for at least one l. Also $\nu_l(Z_l(x,y)) \leq \nu_l(Z_l(x^*,y^*))\ \forall\ l$ and $\nu_l(Z_l(x,y)) < \nu_l(Z_l(x^*,y^*))$ for at least one l. Now $(\theta + \eta - \zeta) = \min\ \{\mu_l(Z_l(x,y)),\ \sigma_l(Z_l(x,y)),\ \nu_l(Z_l(x,y))\} \geq \min\ \{\mu_l(Z_l(x^*,y^*)),\ \sigma_l(Z_l(x^*,y^*)),\ \nu_l(Z_l(x^*,y^*))\} = (\theta^* + \eta^* - \zeta^*)$ which is a contradiction that (x^*, y^*) is an optimal solution of Model 4C. Here θ^*, η^* and ζ^* are the values of θ, η and ζ at (x^*, y^*) respectively. This ends the proof of the theorem. □

5.3. Global criteria method (GCM)

Here we recall another modified and advanced approach GCM that provides the Pareto-optimal solution of multi-objective problem by minimizing the distance between some reference points in the feasible objective region. Since Model 2 does not provide overall Pareto-optimal solution, therefore to find overall Pareto-optimal solution, GCM for Model 2 can be depicted by the following steps as:

Step 5.3.1. Transform the fuzzy-rough problem into crisp problem with help of expected value operator.
Step 5.3.2. Solve each objective function independently with subject to all constraints.
Step 5.3.3. Determine the PIS (U_l^T) and NIS (L_l^T) value of each objective function Z_l from pay-off table defined in Step 5.1.4 and formulate the crisp model is as:

Model 5

$$\begin{aligned} \text{minimize} \quad & F(x) = \left[\sum\nolimits_{l=1}^{3} \left(\frac{Z_l(x) - U_l^T}{L_l^T - U_l^T} \right)^q \right]^{\frac{1}{q}} \\ \text{subject to} \quad & \text{Constraints (4.1)–(4.6).} \end{aligned}$$

Here $q = 1$, used for Manhattan distance; $q = 2$, used for Euclidean distance; and $q = \infty$, used for Tchebycheff distance.
Step 5.3.4. Solve Model 5 by LINGO iterative scheme and GAMS software, and we achieve the Pareto-optimal solution.

Definition 5.3. The Pareto-optimal solution of GCM is defined as the minimum distance between the ideal solution and the desired solution in feasible region. If ${Z_l}^*$ is the ideal solution of the objective function Z_l then the Pareto-optimal solution of Model 2 is defined as ${Z_l}^* = \min ||{Z_l}' - Z_l||_\infty \ \forall\, l$.

5.4. Contributions and limitations of proposed approach

In this subsection, we define main contributions and limitations of our proposed model as:

– Our proposed model is solved with help of three methods such as FP, NLP and GCM in which NLP which is modified and the most preferable approach, as it is not only find the membership function value but also maximize truth and indeterminacy membership value and minimize falsity membership value. This method leads the Pareto-optimal solution which is very close to an exact solution.
– In competitive economic condition and industrial problem, where single uncertainty is not capable to tackle the situation, and hence this situation is tackled by considering fuzzy-rough environment which is known as two-fold uncertainty.
– TP always relates with vehicle capacity, and again that capacity may be greater or less than the compared transported amount of items. For the fact of low vehicle capacity we consider truck load constraints and the transportation is completed with repeated number of slots which are the integers defined as δ_{ij}^1 and δ_{jk}^2. For transportation cost, we consider two situations. One case is type-II fixed-charge that depends on total amount of items. Another case is the truck load constraints, where cost depends on a number of slots. These two cases lead to contradictory situation as the total transportation cost is different in two cases. Hence we verify two cases and also prefer a better case by observing the outcomes and investigating the results of numerical examples.
– TS is another step between source and destination that made the transportation system is to more flexible and easy. As it helps to reduce air pollution, reduce traffic, gives information about passing of average number of vehicles for a period of time and minimize maintenance cost by transferring the product item with small transfer cost and fixed-charge (type-I). Our proposed problem is handled by considering two decision variables, one from source to TS and another from TS to destination by keeping fixed with all other required conditions.
– For industrial planning problem where product blending is an important fact to provide the materials with minimum quality that to be required at destination. In our model, we include product blending constraints that may optimize the allocation of transportation by blending raw materials with different purity levels.
– The first limitation of our proposed model is that we cannot consider the vehicle capacity which is larger than the transported amount (*i.e.*, STP case). And on that case we do not analyze the transportation cost, fixed-charge, transfer cost, transported time and carbon emission, etc. Also here we choose only one type

TABLE 3. Purity level, weight capacity and CO_2 emission.

Purity level	$p_1 = 98\%,\ p_2 = 90\%,\ {p_1}^1 = 95\%,\ {p_2}^1 = 85\%,\ {p_1}^2 = 93\%,\ {p_2}^2 = 90\%,\ {p_3}^2 = 80\%.$
Weight capacity	$W = 1351.5$(litres)$=$ 8.5(barrel).
CO_2 emission	$e_{CO_2} = 0.00013\,\text{ton/km}.$

TABLE 4. Distance D^1_{ij} and D^2_{jk} in km.

D^1_{ij}	$D^1_{11} = 250,\ D^1_{12} = 300;$ $D^1_{21} = 275,\ D^1_{22} = 315$
D^2_{jk}	$D^2_{11} = 300,\ D^2_{12} = 270,$ $D^2_{13} = 250;\ D^2_{21} = 350,$ $D^2_{22} = 200,\ D^2_{23} = 215.$

TABLE 5. Supply and demand in fuzzy-rough.

Supply	$\tilde{\bar{a}}_1 = (a_1 - 5, a_1 - 4, a_1 + 4, a_1 + 5),$ $a_1 = ([42.5, 43.0][42.0, 43.5]);$ $\tilde{\bar{a}}_2 = (a_2 - 5, a_2 - 4, a_2 + 4, a_2 + 5),$ $a_2 = ([52.0, 52.5][51.5, 53.0]).$
Demand	$\tilde{\bar{b}}_1 = (b_1 - 6, b_1 - 5, b_1 + 5, b_1 + 6),$ $b_1 = ([21.2, 21.8][21.0, 22.0]);$ $\tilde{\bar{b}}_2 = (b_2 - 6, b_2 - 5, b_2 + 5, b_2 + 6),$ $b_2 = ([21.0, 21.5][20.8, 22.0]);$ $\tilde{\bar{b}}_3 = (b_3 - 6, b_3 - 5, b_3 + 5, b_3 + 6),$ $b_3 = ([20.0, 20.5][19.0, 21.0]).$

of vehicle (*i.e.*, same weight capacity), not different type of vehicles (*i.e.*, different weight capacities) which may be made the system more flexible.

6. Case study

To validate the formulated model, we consider two numerical examples. Example 1 is a simple problem with small size, but Example 2 is a case study with large size in realistic data which is obtained from three industrial plants. These two problems with the data are briefly defined subsequently as follows:

Example 6.1. Here we assume two sources ($i = 1, 2$), two TSs ($j = 1, 2$) and three destinations ($k = 1, 2, 3$) with maximum availability and minimum demand. Type-I fixed-charge for TS, transfer cost, weight capacity of vehicle, rate of carbon emission, normal purity of product available at source, required at TS and destination, supply from source, demand of destination, transportation cost, distance, transportation time, type-II fixed-charge or general charge are all given in Tables 3–12. Hence all the data except distance, carbon emission, weight capacity of vehicle and purity of product are all fuzzy-rough nature.

Utilizing the data from Tables 3–12 in Model 2, we offer Model 6 as below:

Table 6. Type-1 fixed-charge and transfer cost in fuzzy-rough.

Fixed-charge	$\tilde{\tilde{f}}_1 = (f_1 - 3, f_1 - 2, f_1 + 2, f_1 + 3),\ f_1 = ([5, 9][4, 11]);$ $\tilde{\tilde{f}}_2 = (f_2 - 3, f_2 - 2, f_2 + 2, f_2 + 3),\ f_2 = ([7, 10][5, 12]).$
Transfer cost	$\tilde{\tilde{\alpha}}_1 = (\alpha_1 - 4, \alpha_1 - 3, \alpha_1 + 3, \alpha_1 + 4),\ \alpha_1 = ([6, 8][5, 10]);$ $\tilde{\tilde{\alpha}}_2 = (\alpha_2 - 4, \alpha_2 - 3, \alpha_2 + 3, \alpha_2 + 4),\ \alpha_2 = ([7, 10][6, 12]).$

Table 7. Transportation cost $\tilde{\tilde{c}}^1_{ij}$ ($) in fuzzy-rough.

$\tilde{\tilde{c}}^1_{ij}$	
$\tilde{\tilde{c}}^1_{11}$	$(c^1_{11} - 2, c^1_{11} - 1, c^1_{11} + 1, c^1_{11} + 2),$ $c^1_{11} = ([6, 10][4, 13]);$
$\tilde{\tilde{c}}^1_{12}$	$(c^1_{12} - 2, c^1_{12} - 1, c^1_{12} + 1, c^1_{12} + 2),$ $c^1_{12} = ([4, 7][3, 12]);$
$\tilde{\tilde{c}}^1_{21}$	$(c^1_{21} - 2, c^1_{21} - 1, c^1_{21} + 1, c^1_{21} + 2),$ $c^1_{21} = ([5, 9][4, 15]);$
$\tilde{\tilde{c}}^1_{22}$	$(c^1_{22} - 2, c^1_{22} - 1, c^1_{22} + 1, c^1_{22} + 2),$ $c^1_{22} = ([8, 11][6, 17]).$

Table 8. Transportation cost $\tilde{\tilde{c}}^2_{jk}$ ($) in fuzzy-rough.

$\tilde{\tilde{c}}^2_{jk}$	
$\tilde{\tilde{c}}^2_{11}$	$(c^2_{11} - 2, c^2_{11} - 1, c^2_{11} + 1, c^2_{11} + 2),$ $c^2_{11} = ([4, 6][3, 8]);$
$\tilde{\tilde{c}}^2_{12}$	$(c^2_{12} - 2, c^2_{12} - 1, c^2_{12} + 1, c^2_{12} + 2),$ $c^2_{12} = ([5, 9][3, 13]);$
$\tilde{\tilde{c}}^2_{13}$	$(c^2_{13} - 2, c^2_{13} - 1, c^2_{13} + 1, c^2_{13} + 2),$ $c^2_{13} = ([6, 8][4, 10]);$
$\tilde{\tilde{c}}^2_{21}$	$(c^2_{21} - 2, c^2_{21} - 1, c^2_{21} + 1, c^2_{21} + 2),$ $c^2_{21} = ([6, 10][5, 12]);$
$\tilde{\tilde{c}}^2_{22}$	$(c^2_{22} - 2, c^2_{22} - 1, c^2_{22} + 1, c^2_{22} + 2),$ $c^2_{22} = ([5, 7][3, 10]);$
$\tilde{\tilde{c}}^2_{23}$	$(c^2_{23} - 2, c^2_{23} - 1, c^2_{23} + 1, c^2_{23} + 2),$ $c^2_{23} = ([7, 9][5, 14]).$

Model 6

$$
\begin{aligned}
\text{minimize} \quad Z_1 = {} & (8.25 + 71.25/8.5)x_{11} + (6.5 + 67.5/8.5)x_{12} + (8.25 + 75/8.5)x_{21} \\
& +(10.5 + 82.5/8.5)x_{22} + (5.25 + 78.75/8.5)y_{11} + (7.5 + 72.5/8.5)y_{12} \\
& +(7 + 83.75/8.5)y_{13} + (8.25 + 81.25/8.5)y_{21} + (6.25 + 85/8.5)y_{22} \\
& +(8.75 + 87.5/8.5)y_{23} + 7.25\phi(x_{11} + x_{21}) + 8.5\phi(x_{12} + x_{22}) \\
& +7.25(x_{11} + x_{21}) + 8.75(x_{12} + x_{22})
\end{aligned}
$$

TABLE 9. Transportation time $\tilde{\tilde{t}}_{ij}^{1}$ (hour) in fuzzy-rough.

$\tilde{\tilde{t}}_{ij}^{1}$	
$\tilde{\tilde{t}}_{11}^{1}$	$(t_{11}^1 - 4, t_{11}^1 - 3, t_{11}^1 + 3, t_{11}^1 + 4)$, $t_{11}^1 = ([8,9][6,12])$;
$\tilde{\tilde{t}}_{12}^{1}$	$(t_{12}^1 - 4, t_{12}^1 - 3, t_{12}^1 + 3, t_{12}^1 + 4)$, $t_{12}^1 = ([4,7][2,10])$;
$\tilde{\tilde{t}}_{21}^{1}$	$(t_{21}^1 - 4, t_{21}^1 - 3, t_{21}^1 + 3, t_{21}^1 + 4)$, $t_{21}^1 = ([8,11][7,16])$;
$\tilde{\tilde{t}}_{22}^{1}$	$(t_{22}^1 - 4, t_{22}^1 - 3, t_{22}^1 + 3, t_{22}^1 + 4)$, $t_{22}^1 = ([5,7][3,10])$.

TABLE 10. Transportation time $\tilde{\tilde{t}}_{jk}^{2}$ (hour) in fuzzy-rough.

$\tilde{\tilde{t}}_{jk}^{2}$	
$\tilde{\tilde{t}}_{11}^{2}$	$(t_{11}^2 - 4, t_{11}^2 - 3, t_{11}^2 + 3, t_{11}^2 + 4)$, $t_{11}^2 = ([4,8][2,10])$;
$\tilde{\tilde{t}}_{12}^{2}$	$(t_{12}^2 - 4, t_{12}^2 - 3, t_{12}^2 + 3, t_{12}^2 + 4)$, $t_{12}^2 = ([6,9][5,12])$;
$\tilde{\tilde{t}}_{13}^{2}$	$(t_{13}^2 - 4, t_{13}^2 - 3, t_{13}^2 + 3, t_{13}^2 + 4)$, $t_{13}^2 = ([7,10][5,14])$;
$\tilde{\tilde{t}}_{21}^{2}$	$(t_{21}^2 - 4, t_{21}^2 - 3, t_{21}^2 + 3, t_{21}^2 + 4)$, $t_{21}^2 = ([5,8][3,10])$;
$\tilde{\tilde{t}}_{22}^{2}$	$(t_{22}^2 - 4, t_{22}^2 - 3, t_{22}^2 + 3, t_{22}^2 + 4)$, $t_{22}^2 = ([7,10][5,14])$;
$\tilde{\tilde{t}}_{23}^{2}$	$(t_{23}^2 - 4, t_{23}^2 - 3, t_{23}^2 + 3, t_{23}^2 + 4)$, $t_{23}^2 = ([9,11][7,15])$;

TABLE 11. Type-II fixed-charge or general charge $\overline{\tilde{F}}_{ij}^{1}$ (\$) in fuzzy-rough.

$\overline{\tilde{F}}_{ij}^{1}$	
$\overline{\tilde{F}}_{11}^{1}$	$(F_{11}^1 - 3, F_{11}^1 - 2, F_{11}^1 + 2, F_{11}^1 + 3)$, $F_{11}^1 = ([60,80][55,90])$;
$\overline{\tilde{F}}_{12}^{1}$	$(F_{12}^1 - 3, F_{12}^1 - 2, F_{12}^1 + 2, F_{12}^1 + 3)$, $F_{12}^1 = ([55,80][50,85])$;
$\overline{\tilde{F}}_{21}^{1}$	$(F_{21}^1 - 3, F_{21}^1 - 2, F_{21}^1 + 2, F_{21}^1 + 3)$, $F_{21}^1 = ([70,80][65,85])$;
$\overline{\tilde{F}}_{22}^{1}$	$(F_{22}^1 - 3, F_{22}^1 - 2, F_{22}^1 + 2, F_{22}^1 + 3)$, $F_{22}^1 = ([80,85][75,90])$.

TABLE 12. Type-II fixed-charge or general charge $\tilde{\overline{F}}^2_{jk}$ (\$) in fuzzy-rough.

$\tilde{\overline{F}}^2_{jk}$	
$\tilde{\overline{F}}^2_{11}$	$(F^2_{11}-3, F^2_{11}-2, F^2_{11}+2, F^2_{11}+3)$, $F^2_{11}=([70,90][60,95])$;
$\tilde{\overline{F}}^2_{12}$	$(F^2_{12}-3, F^2_{12}-2, F^2_{12}+2, F^2_{12}+3)$, $F^2_{12}=([65,80][55,90])$;
$\tilde{\overline{F}}^2_{13}$	$(F^2_{13}-3, F^2_{13}-2, F^2_{13}+2, F^2_{13}+3)$, $F^2_{13}=([75,95][65,100])$;
$\tilde{\overline{F}}^2_{21}$	$(F^2_{21}-3, F^2_{21}-2, F^2_{21}+2, F^2_{21}+3)$, $F^2_{21}=([75,85][70,95])$;
$\tilde{\overline{F}}^2_{22}$	$(F^2_{22}-3, F^2_{22}-2, F^2_{22}+2, F^1_{22}+3)$, $F^2_{22}=([80,90][70,100])$;
$\tilde{\overline{F}}^2_{23}$	$(F^2_{23}-3, F^2_{23}-2, F^2_{23}+2, F^1_{23}+3)$, $F^2_{23}=([85,90][80,95])$.

$$\begin{aligned}
\text{minimize}\quad & Z_2 = 0.00013(250x_{11}+300x_{12}+275x_{21}+315x_{22}+300y_{11}+270y_{12}\\
&\quad +250y_{13}+350y_{21}+200y_{22}+215y_{23})\\
\text{minimize}\quad & Z_3 = 8.75\eta^1_{11}+5.75\eta^1_{12}+10.5\eta^1_{21}+6.25\eta^1_{22}+6\eta^2_{11}+8\eta^2_{12}+9\eta^2_{13}+6.5\eta^2_{21}\\
&\quad +9\eta^2_{22}+10.5\eta^2_{23}\\
\text{subject to}\quad & x_{11}+x_{12}\le 42.75, x_{21}+x_{22}\le 52.25,\\
& y_{11}+y_{21}\ge 21.5, y_{12}+y_{22}\ge 21.325, y_{13}+y_{23}\ge 20.125,\\
& 0.03x_{11}-0.05x_{21}\ge 0, 0.13x_{12}+0.05x_{22}\ge 0,\\
& 0.02y_{11}-0.08y_{21}\ge 0, 0.05y_{12}-0.05y_{22}\ge 0, 0.15y_{13}+0.05y_{23}\ge 0,\\
& x_{11}+x_{21}=y_{11}+y_{12}+y_{13}, x_{12}+x_{22}=y_{21}+y_{22}+y_{23},\\
& x_{ij}\ge 0, y_{jk}\ge 0, \forall\ i,\ j,\ k.
\end{aligned}$$

Example 6.2. In this example, we represent a case study with realistic data which are collected from three ($m = 3$) reputed petroleum refinery companies. They collect crude oil by outsourcing through vessels and keep into storage tanks. Since crude oils are collected from various sources, the average purity levels of each company might differ and these are blended in charging tanks prior to distribution. The characteristic of a particular gasoline blend is to prevent igniting too early and it is measured by its octane rating, which is produced in several classes. Gasoline is a blending product and the requirement of minimum RON (research octane number) in market is 91. Actually, an isomeric form of octane *i.e.*, isooctane (2,2,4 trimethyl pentane) has octane number 100. All refineries do not produce the same product. Different companies often blend different hydrocarbons aromatics and other ingredients in varying proportions with the crude oil as per their own choice and requirement for maintaining quality of the gasolines they produce. We connect with three companies namely Indian Oil Corporation Limited (IOCL) from Paradip Refinery (Odisha), IOCL from Haldia Refinery company (West Bengal), and Hindustan Petroleum Corporation Limited (HPCL) from Visakhapatnam Refinery (Andhra Pradesh) who supply different types of gasoline. In this problem we have survey that the report of one type of blended gasoline collected from these three companies with different purity levels for existence of different rating of octane number. The average octane ratings of gasoline 1, gasoline 2 and gasoline 3 obtained from

TABLE 13. Distance D^1_{ij} and D^2_{jk} in km.

D^1_{ij}	$D^1_{11}=410,\ D^1_{12}=365,\ D^1_{13}=280;$ $D^1_{21}=320,\ D^1_{22}=395,\ D^1_{23}=430;$ $D^1_{31}=340,\ D^1_{32}=385,\ D^1_{33}=420.$
D^2_{jk}	$D^2_{11}=325,\ D^2_{12}=390,\ D^2_{13}=440,$ $D^2_{14}=500;\ D^2_{21}=380,\ D^2_{22}=405,$ $D^2_{23}=460;\ D^2_{24}=305;\ D^2_{11}=290,$ $D^2_{12}=340,\ D^2_{13}=415,\ D^2_{14}=495.$

TABLE 14. Supply and demand in fuzzy-rough.

Supply
$\tilde{\tilde{a}}_1=(a_1-8,a_1,a_1+8),\ a_1=([54,58][50,62]);$ $\tilde{\tilde{a}}_2=(a_2-6,a_2,a_2+6),\ a_2=([48,50][46,52]);$ $\tilde{\tilde{a}}_3=(a_3-8,a_3,a_3+8),\ a_3=([56,60][52,64]).$
Demand
$\tilde{\tilde{b}}_1=(b_1-4,b_1,b_1+4),\ b_1=([32,34][30,36]);$ $\tilde{\tilde{b}}_2=(b_2-5,b_2,b_2+5),\ b_2=([40,45][35,50]);$ $\tilde{\tilde{b}}_3=(b_3-7,b_3,b_3+7),\ b_3=([42,44][40,46]);$ $\tilde{\tilde{b}}_4=(b_4-6,b_4,b_4+6),\ b_4=([39,41][37,43]);$

three sources are at least $p_1 = 90\%, p_2 = 95\%$ and $p_3 = 98\%$ respectively. The shifted gasolines are then transported to various distribution centers by tankers through TS. Here we select four distribution centers ($K = 4$) which are situated in the states of Jharkhand, Bihar, Madhya Pradesh and Chattisgarh of India and three TSs ($n = 3$) between distribution centers and supply centers. The required average octane rating of gasoline of TS and distribution centers are respectively defined as ($p^1_1 = 87\%, p^1_2 = 93\%, p^1_3 = 96\%$) and ($p^2_1 = 85\%, p^2_2 = 90\%, p^2_3 = 92\%, p^2_4 = 95\%$). Our problem is to transport these gasolines to their destinations by minimum transportation cost, transfer cost with type-I fixed-charge and type-II fixed-charge or general charge, minimum carbon emission and minimum time by considering product blending constraints without ignoring the source and demand. In this processing system most of the data (except distance, carbon emission, weight capacity of vehicle and purity of product) are considered as fuzzy-rough (triangular) due to various unexpected critical situations. All the data are represented in Tables 13–21. Here cost considering in \$ per unit, total carbon emission in ton and time in hour. Also weight capacity, $W = 1271.84/1272$(litres) $= 8$(barrel) and rate of CO_2 emission, $e_{CO_2} = 0.00020$ ton/km.

Here the tabulated data are transformed into crisp data by expected value operator defined by Definition 3.6 and the deterministic model is formulated as of Model 6/Model 2. To find overall Pareto-optimal solution of both examples, we use FP, referred to Subsection 5.1; NLP, mentioned to Subsection 5.2 and GCM, discussed to Subsection 5.3.

7. Computational results and discussion

In this section we compute the results of our proposed problem that obtained with the help of three methods. The following is the Pareto-optimal solution of each method.

FP: Following the Subsection 5.1 and using the LINGO iterative scheme and GAMS, we calculate the solutions for both examples as:

TABLE 15. Type-1 fixed-charge and transfer cost in fuzzy-rough.

Fixed-charge
$\tilde{\tilde{f}}_1 = (f_1 - 4, f_1, f_1 + 4),\ f_1 = ([7, 11][3, 15]);$
$\tilde{\tilde{f}}_2 = (f_2 - 2, f_2, f_2 + 2),\ f_2 = ([5, 6][4, 7]);$
$\tilde{\tilde{f}}_3 = (f_3 - 3, f_3, f_2 + 3),\ f_3 = ([7, 9][5, 11]).$
Transfer cost
$\tilde{\tilde{\alpha}}_1 = (\alpha_1 - 2, \alpha_1, \alpha_1 + 2),\ \alpha_1 = ([4, 5][3, 6]);$
$\tilde{\tilde{\alpha}}_2 = (\alpha_2 - 3, \alpha_2, \alpha_2 + 3),\ \alpha_2 = ([6, 8][4, 10]);$
$\tilde{\tilde{\alpha}}_3 = (\alpha_3 - 4, \alpha_3, \alpha_3 + 4),\ \alpha_2 = ([7, 9][5, 11]);$

TABLE 16. Transportation cost $\tilde{\tilde{c}}^1_{ij}$ (\$) in fuzzy-rough.

$\tilde{\tilde{c}}^1_{ij}$
$\tilde{\tilde{c}}^1_{11} = (c^1_{11} - 3, c^1_{11}, c^1_{11} + 3),\ c^1_{11} = ([5, 6][4, 7]);$
$\tilde{\tilde{c}}^1_{12} = (c^1_{12} - 2, c^1_{12}, c^1_{12} + 2),\ c^1_{12} = ([6, 7][5, 8]);$
$\tilde{\tilde{c}}^1_{13} = (c^1_{13} - 1, c^1_{13}, c^1_{13} + 1)\ c^1_{13} = ([8, 10][6, 12]);$
$\tilde{\tilde{c}}^1_{21} = (c^1_{21} - 2, c^1_{21}, c^1_{21} + 2),\ c^1_{21} = ([4, 5][3, 6]);$
$\tilde{\tilde{c}}^1_{22} = (c^1_{22} - 2, c^1_{22}, c^1_{22} + 2),\ c^1_{22} = ([7, 9][5, 11]);$
$\tilde{\tilde{c}}^1_{23} = (c^1_{23} - 2, c^1_{23}, c^1_{23} + 2)\ c^1_{23} = ([5, 7][3, 9]).$

TABLE 17. Transportation cost $\tilde{\tilde{c}}^2_{jk}$ (\$) in fuzzy-rough.

$\tilde{\tilde{c}}^2_{jk}$
$\tilde{\tilde{c}}^2_{11} = (c^2_{11} - 3, c^2_{11}, c^2_{11} + 3),\ c^2_{11} = ([6, 9][3, 12]);$
$\tilde{\tilde{c}}^2_{12} = (c^2_{12} - 3, c^2_{12}, c^2_{12} + 3),\ c^2_{12} = ([7, 10][4, 13]);$
$\tilde{\tilde{c}}^2_{13} = (c^2_{13} - 3, c^2_{13}, c^2_{13} + 3),\ c^2_{13} = ([6, 7][5, 8]);$
$\tilde{\tilde{c}}^2_{14} = (c^2_{14} - 3, c^2_{14}, c^2_{14} + 3),\ c^2_{14} = ([6, 8][4, 10]);$
$\tilde{\tilde{c}}^2_{21} = (c^2_{21} - 2, c^2_{21}, c^2_{21} + 2),\ c^2_{21} = ([5, 6][4, 7]);$
$\tilde{\tilde{c}}^2_{22} = (c^2_{22} - 2, c^2_{22}, c^2_{22} + 2),\ c^2_{22} = ([7, 8][6, 9]);$
$\tilde{\tilde{c}}^2_{23} = (c^2_{23} - 2, c^2_{23}, c^2_{23} + 2),\ c^2_{23} = ([8, 11][5, 14]);$
$\tilde{\tilde{c}}^2_{24} = (c^2_{24} - 2, c^2_{24}, c^2_{24} + 2),\ c^2_{24} = ([7, 9][5, 11]);$
$\tilde{\tilde{c}}^2_{31} = (c^2_{31} - 2, c^2_{31}, c^2_{31} + 2),\ c^2_{23} = ([5, 6][4, 7]);$
$\tilde{\tilde{c}}^2_{32} = (c^2_{22} - 2, c^2_{22}, c^2_{22} + 2),\ c^2_{22} = ([8, 10][6, 12]);$
$\tilde{\tilde{c}}^2_{33} = (c^2_{23} - 2, c^2_{23}, c^2_{23} + 2),\ c^2_{23} = ([8, 9][7, 10]);$
$\tilde{\tilde{c}}^2_{34} = (c^2_{24} - 2, c^2_{24}, c^2_{24} + 2),\ c^2_{24} = ([5, 7][3, 9]).$

TABLE 18. Transportation time $\tilde{\tilde{t}}^1_{ij}$ (hour) in fuzzy-rough.

$\tilde{\tilde{t}}^1_{ij}$
$\tilde{\tilde{t}}^1_{11} = (t^1_{11} - 4, t^1_{11}, t^1_{11} + 4)$, $t^1_{11} = ([11, 13][9, 15])$;
$\tilde{\tilde{t}}^1_{12} = (t^1_{12} - 4, t^1_{12}, t^1_{12} + 4)$, $t^1_{12} = ([10, 12][8, 14])$;
$\tilde{\tilde{t}}^1_{13} = (t^1_{13} - 4, t^1_{13} + 3, t^1_{13} + 4)$ $t^1_{13} = ([6, 8][4, 10])$;
$\tilde{\tilde{t}}^1_{21} = (t^1_{21} - 3, t^1_{21}, t^1_{21} + 3)$, $t^1_{21} = ([7, 9][5, 11])$;
$\tilde{\tilde{t}}^1_{22} = (t^1_{22} - 3, t^1_{22}, t^1_{22} + 3)$, $t^1_{22} = ([9, 10][8, 11])$;
$\tilde{\tilde{t}}^1_{23} = (t^1_{23} - 3, t^1_{23}, t^1_{23} + 3)$ $t^1_{23} = ([11, 12][10, 13])$;
$\tilde{\tilde{t}}^1_{31} = (t^1_{31} - 3, t^1_{31}, t^1_{31} + 3)$, $t^1_{31} = ([8, 10][6, 12])$;
$\tilde{\tilde{t}}^1_{32} = (t^1_{32} - 3, t^1_{32}, t^1_{32} + 3)$, $t^1_{32} = ([6, 8][4, 10])$;
$\tilde{\tilde{t}}^1_{33} = (t^1_{33} - 3, t^1_{33}, t^1_{33} + 3)$ $t^1_{33} = ([10, 11][9, 12])$.

TABLE 19. Transportation time $\tilde{\tilde{t}}^2_{jk}$ (hour) in fuzzy-rough.

$\tilde{\tilde{t}}^2_{jk}$
$\tilde{\tilde{t}}^2_{11} = (t^2_{11} - 4, t^2_{11}, t^2_{11} + 4)$, $t^2_{11} = ([7, 9][5, 11])$;
$\tilde{\tilde{t}}^2_{12} = (t^2_{12} - 4, t^2_{12}, t^2_{12} + 4)$, $t^2_{12} = ([8, 10][6, 12])$;
$\tilde{\tilde{t}}^2_{13} = (t^2_{13} - 4, t^2_{13}, t^2_{13} + 4)$, $t^2_{13} = ([10, 12][8, 14])$;
$\tilde{\tilde{t}}^2_{14} = (t^2_{14} - 4, t^2_{14}, t^2_{14} + 4)$, $t^2_{14} = ([11, 13][9, 15])$;
$\tilde{\tilde{t}}^2_{21} = (t^2_{21} - 3, t^2_{21}, t^2_{21} + 3)$, $t^2_{21} = ([9, 10][8, 11])$;
$\tilde{\tilde{t}}^2_{22} = (t^2_{22} - 3, t^2_{22}, t^2_{22} + 3)$, $t^2_{22} = ([9, 11][7, 13])$;
$\tilde{\tilde{t}}^2_{23} = (t^2_{23} - 3, t^2_{23}, t^2_{23} + 3)$, $t^2_{23} = ([11, 12][10, 13])$;
$\tilde{\tilde{t}}^2_{24} = (t^2_{24} - 3, t^2_{24}, t^2_{24} + 3)$, $t^2_{24} = ([7, 8][6, 9])$;
$\tilde{\tilde{t}}^2_{31} = (t^2_{31} - 2, t^2_{31}, t^2_{31} + 2)$, $t^2_{31} = ([6, 8][4, 10])$;
$\tilde{\tilde{t}}^2_{32} = (t^2_{32} - 2, t^2_{32}, t^2_{32} + 2)$, $t^2_{32} = ([8, 9][7, 10])$;
$\tilde{\tilde{t}}^2_{33} = (t^2_{33} - 2, t^2_{33}, t^2_{33} + 2)$, $t^2_{33} = ([10, 11][9, 12])$;
$\tilde{\tilde{t}}^2_{34} = (t^2_{34} - 2, t^2_{34}, t^2_{34} + 2)$, $t^2_{34} = ([12, 13][11, 14])$.

Ex 1 (LINGO): $\theta = 1$; $x_{11} = 33.667$, $x_{12} = 9.083$, $x_{21} = 20.2$, $y_{11} = 21.5$, $y_{12} = 21.325$, $y_{13} = 11.042$, other variables are zero.

Ex 1 (GAMS): $\theta = 1$; $x_{11} = 42.75$, $x_{21} = 20.2$, $y_{11} = 21.5$, $y_{12} = 21.325$, $y_{13} = 20.125$, other variables are zero.

Ex 2 (LINGO): $\theta = 1$; $x_{11} = 22.995$, $x_{12} = 16.49$, $x_{13} = 16.515$, $x_{21} = 40.9$, $x_{22} = 3.6, x_{32} = 8.454$, $x_{33} = 49.546$, $y_{11} = 23.534$, $y_{12} = 21.25$, $y_{13} = 19.111$, $y_{22} = 21.25$, $y_{24} = 7.294$, $y_{31} = 9.466$, $y_{33} = 23.889$, $y_{34} = 32.706$, other variables are zero.

Ex 2 (GAMS): $\theta = 1$; $x_{13} = 56$, $x_{21} = 49$, $x_{31} = 15.025$, $x_{32} = 36.795$, $x_{33} = 1.68$, $y_{12} = 28.33$, $y_{13} = 35.692$, $y_{21} = 16.153$, $y_{23} = 7.308$, $y_{24} = 13.333$, $y_{31} = 16.847$, $y_{32} = 14.167$, $y_{34} = 26.667$, other variables are zero.

TABLE 20. Type-II fixed-charge or general charge $\tilde{\bar{F}}^1_{ij}$ (\$) in fuzzy-rough.

$\tilde{\bar{F}}^1_{ij}$
$\tilde{\bar{F}}^1_{11} = (F^1_{11} - 3, F^1_{11}, F^1_{11} + 3),\ F^1_{11} = ([52, 54][50, 56]);$
$\tilde{\bar{F}}^1_{12} = (F^1_{12} - 3, F^1_{12}, F^1_{12} + 3),\ F^1_{12} = ([46, 47][45, 48]);$
$\tilde{\bar{F}}^1_{13} = (F^1_{13} - 3, F^1_{13}, F^1_{13} + 3)\ F^1_{13} = ([40, 42][38, 44])$
$\tilde{\bar{F}}^1_{21} = (F^1_{21} - 4, F^1_{21}, F^1_{21} + 4),\ F^1_{21} = ([56, 57][55, 58]);$
$\tilde{\bar{F}}^1_{22} = (F^1_{22} - 4, F^1_{22}, F^1_{22} + 3),\ F^1_{22} = ([54, 56][52, 58]).$
$\tilde{\bar{F}}^1_{23} = (F^1_{23} - 4, F^1_{23}, F^1_{23} + 3)\ F^1_{23} = ([50, 52][48, 54])$
$\tilde{\bar{F}}^1_{31} = (F^1_{31} - 2, F^1_{31}, F^1_{31} + 2),\ F^1_{31} = ([45, 50][40, 55]);$
$\tilde{\bar{F}}^1_{32} = (F^1_{32} - 2, F^1_{32}, F^1_{32} + 2),\ F^1_{32} = ([47, 49][45, 51]).$
$\tilde{\bar{F}}^1_{33} = (F^1_{33} - 2, F^1_{33}, F^1_{33} + 2)\ F^1_{23} = ([55, 60][50, 65])$

TABLE 21. Type-II fixed-charge or general charge $\tilde{\bar{F}}^2_{jk}$ (\$) in fuzzy-rough.

$\tilde{\bar{F}}^2_{jk}$
$\tilde{\bar{F}}^2_{11} = (F^2_{11} - 3, F^2_{11}, F^2_{11} + 3),\ F^2_{11} = ([40, 45][35, 50]);$
$\tilde{\bar{F}}^2_{12} = (F^2_{12} - 3, F^2_{12}, F^2_{12} + 3),\ F^2_{12} = ([48, 50][46, 52]);$
$\tilde{\bar{F}}^2_{13} = (F^2_{13} - 3, F^2_{13}, F^2_{13} + 3),\ F^2_{13} = ([40, 42][38, 44]);$
$\tilde{\bar{F}}^2_{14} = (F^2_{14} - 3, F^2_{14}, F^2_{14} + 3),\ F^2_{14} = ([45, 47][43, 49]);$
$\tilde{\bar{F}}^2_{21} = (F^2_{21} - 4, F^2_{21}, F^2_{21} + 4),\ F^2_{21} = ([42, 44][40, 46]);$
$\tilde{\bar{F}}^2_{22} = (F^2_{22} - 4, F^2_{22}, F^1_{22} + 4),\ F^2_{22} = ([50, 55][45, 60]);$
$\tilde{\bar{F}}^2_{23} = (F^2_{23} - 4, F^2_{23}, F^1_{23} + 4),\ F^2_{23} = ([45, 47][43, 49]);$
$\tilde{\bar{F}}^2_{24} = (F^2_{24} - 4, F^2_{24}, F^1_{24} + 4),\ F^2_{24} = ([41, 42][40, 43]);$
$\tilde{\bar{F}}^2_{21} = (F^2_{21} - 4, F^2_{21}, F^2_{21} + 4),\ F^2_{31} = ([53, 56][50, 59]);$
$\tilde{\bar{F}}^2_{32} = (F^2_{32} - 1, F^2_{32}, F^1_{32} + 1),\ F^2_{22} = ([60, 65][55, 70]);$
$\tilde{\bar{F}}^2_{23} = (F^2_{33} - 1, F^2_{33}, F^1_{33} + 1),\ F^2_{23} = ([62, 64][60, 66]);$
$\tilde{\bar{F}}^2_{24} = (F^2_{34} - 4, F^2_{34}, F^1_{34} + 1),\ F^2_{24} = ([50, 52][48, 54]).$

NLP: Pursuing the Subsection 5.2 and using the LINGO iterative scheme and GAMS, we obtain the solutions for both examples as:

Ex 1 (LINGO): $\theta = 1,\ \eta = 1,\ \zeta = 0;\ x_{11} = 42.75,\ x_{21} = 20.2,\ y_{11} = 21.5,\ y_{12} = 21.325,\ y_{13} = 20.125,$ other variables are zero.

Ex 1 (GAMS): $\theta = 1,\ \eta = 1,\ \zeta = 0;\ x_{11} = 40.802,\ x_{12} = 1.948,\ x_{22} = 20.2,\ y_{11} = 21.5,\ y_{12} = 19.377,$ $y_{13} = 20.125,\ y_{22} = 1.948,$ other variables are zero.

Ex 2 (LINGO): $\theta = 1,\ \eta = 1,\ \zeta = 0.10765 \times 10^{-6};\ x_{12} = 44.833,\ x_{13} = 6.667,\ x_{21} = 49,\ x_{32} = 38,$ $x_{33} = 20,\ y_{11} = 33,\ y_{12} = 10.042,\ y_{13} = 5.958,\ y_{22} = 32.457,\ y_{23} = 37.042,\ y_{24} = 13.333,\ y_{34} = 26.667,$ other variables are zero.

TABLE 22. Obtained solution from FP, NLP and GCM.

	Example 1(LINGO)				Example 1(GAMS)			
Method	Z_1^f	Z_1^t	Z_2	Z_3	Z_1^f	Z_1^t	Z_2	Z_3
FP	2534.739	2860.037	4.37	58.5	2512.615	2754.162	4.352	42.25
NLP	**2512.615**	2754.162	**4.352**	**42.25**	2512.955	2833.747	4.348	57
GCM	2576.016	3063.945	4.314	73.75	2576.087	3063.992	4.316	73.75

TABLE 23. Obtained solution from FP, NLP and GCM.

	Example 2(LINGO)				Example 2(GAMS)			
Method	Z_1^f	Z_1^t	Z_2	Z_3	Z_1^f	Z_1^t	Z_2	Z_3
FP	5301.793	5723.767	24.356	140.5	5465.16	5786.638	22.14	118
NLP	5313.663	5628.161	23.533	113	**5462.924**	5782.924	**21.828**	**108**
GCM	5291.54	5555.915	23.613	122.5	5255.89	5628.516	21.89	110.5

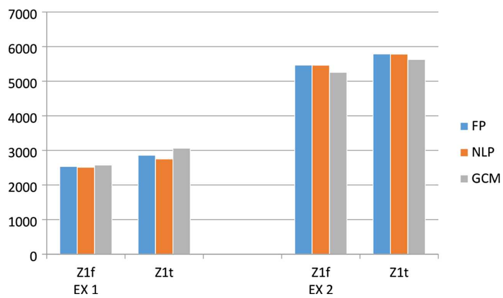

FIGURE 2. Objective function Z_1^f, Z_1^t in FP, NLP and GCM.

Ex 2 (GAMS): $\theta = 1,\ \eta = 1,\ \zeta = 0.002;\ x_{13} = 56,\ x_{21} = 44.5,\ x_{32} = 56.32,\ x_{33} = 1.68,\ y_{11} = 32.412,\ y_{13} = 12.088,\ y_{22} = 12.075,\ y_{23} = 30.912,\ y_{24} = 13.333,\ y_{31} = 0.588,\ y_{32} = 30.425,\ y_{34} = 26.667$, other variables are zero.

GCM: Imitating the Subsection 5.3 and using the LINGO iterative scheme and GAMS, we derive the solutions for both examples as:

(Here we consider $q = 2$, *i.e.*, for the Euclidean distance)

Ex 1 (LINGO): $x_{11} = 39.915,\ x_{12} = 2.835,\ x_{21} = 9.692,\ x_{22} = 10.508,\ y_{11} = 21.5,\ y_{12} = 10.662,\ y_{13} = 17.445,\ y_{22} = 10.662,\ y_{23} = 2.68$, other variables are zero.

Ex 1 (GAMS): $x_{11} = 39.915,\ x_{12} = 2.835,\ x_{21} = 9.692,\ x_{22} = 10.508,\ y_{11} = 21.5,\ y_{12} = 10.663,\ y_{13} = 17.445,\ y_{22} = 10.663,\ y_{23} = 2.68$, other variables are zero.

Ex 2 (LINGO): $x_{12} = 44.833,\ x_{13} = 6.667,\ x_{21} = 49,\ x_{32} = 38,\ x_{33} = 20,\ y_{11} = 20.583,\ y_{12} = 21.25,\ y_{13} = 7.167,\ y_{21} = 12.417,\ y_{22} = 21.25,\ y_{23} = 35.833,\ y_{24} = 13.333,\ y_{34} = 26.667$, other variables are zero.

Ex 2 (GAMS): $x_{13} = 56,\ x_{21} = 44.5,\ x_{32} = 56.32,\ x_{33} = 1.68,\ y_{11} = 2.391,\ y_{13} = 42.109,\ y_{22} = 42.5,\ y_{23} = 0.487,\ y_{24} = 13.333,\ y_{31} = 30.609,\ y_{33} = 0.405,\ y_{34} = 26.667$, other variables are zero.

Hence in our numerical examples we solve two types of problem, one is type-II fixed-charge and another is truck load constraints. We also calculate the value of three objective functions defined in Tables 22 and 23. Here Z_1^f represents the transportation cost for the case of type-II fixed-charge. Z_1^t expresses the transportation cost

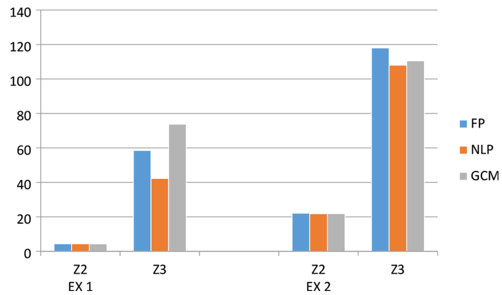

FIGURE 3. Objective function Z_2, Z_3 in FP, NLP and GCM.

for truck load constraints. Also Z_2 and Z_3 are the objective values of carbon emission and time of transportation respectively. The optimal values of these objective functions are calculated in three approaches, which are FP, NLP and GCM. Henceforth on comparing the results from Tables 22, 23 and Figures 2, 3, we always select the case of type-II fixed-charge for low vehicle capacity. The obtained optimal solutions of our proposed model from three methods are revealed that NLP provides a better result (indicated by bold face) than other two methods.

8. Managerial insights

Here, minimum transportation cost, transportation time and carbon emission are added in the formulation model, that can be applicable to any private organization relating to transportation sector. The proposed model provides a plan that helps to optimize the transportation policy by considering source, demand and product blending constraints. The following managerial insights are drawn out through this study as follows:

- The proposed two-stage TP becomes more flexible in the presence of fuzzy-rough environment and helps the decision makers to take the right decision easily in presence of any uncertainty.
- To justify the using of low vehicle capacity, there exist two different choices between truck load constraints and type-II fixed-charge.
- For more flexibility of minimum transportation cost, type-II fixed-charge addresses an improved result for multiple slots of low vehicle capacity and also provides less computational efforts than truck load constraints.
- From the discussion among the optimum values of the objectives, the organization can easily determine the impact of truck load constraints and type-II fixed-charge, and with this experiment he/she will be certainly select the best potential type of vehicles to pay minimum expense.
- Introduction of product blending as constraints in this formulated model plays positive role for customers' satisfaction. Organization can improve the way for obtaining an economical opportunity by blending raw materials. Also, both supply plants and demand plants are facilitated to realize considerable cost saving by blending raw materials in flexible way that we define in our formulated model.
- From Tables 22 and 23, it is analyzed that transportation cost with other objectives leads to considerable amount that can affect to design of any transportation network of any organizations or supply chain process with multiple slots.

9. Conclusion and outlook to future research scopes

In our formulated MOTP, we have considered two conflicting cases for low vehicle capacity such as truck load constraints and type-II fixed-charge. For truck load constraints, the cost depends only the number of slots whereas for type-II fixed-charge, the cost depends on the transported amount of items. In the presence of TS, we have incorporated the product blending constraints for transporting raw materials with different purity levels that meet the consumer with minimum purity level. TS supplies several facilities to the whole transportation sector. These sectors obtain the survey report from TS about the flow of average number of conveyances per day/per month/per year by connecting distinct supply plant to various demand plants. Also TS provides some urgent/basic essential maintenance of vehicles that reduce some major/minor risks and a smooth transportation obtain with optimum cost. By these services, TS yields transportation system to encourage fuel saving and helps to shrink the rate of air pollution by controlling traffic system in relaxed way. In a complex uncertain system, we have chosen fuzzy-rough data that adequate to tackle the situation. We have adopted three methods namely FP, NLP and GCM for solving our designed multi-objective model. Two numerical examples have been illustrated for showing the applicability and feasibility of the proposed model. A comparative study has been drawn among the derived Pareto-optimal solutions from the three methods, and we have concluded that a better result has been provided from NLP in compare to FP and GCM.

The content of the paper may be opened as a separate investigation for MOFCTP with truck load constraints and TS extended in various uncertain environments such as random rough uncertainty [31], type-2 fuzzy set [5], intuitionistic fuzzy set [26, 29, 30], etc. with linear or non-linear membership functions to accommodate the real-life scenario. As the formulated model has a very generic structure, it can be easily applied in different application based examples that face deep uncertainty and which involve various conflicting criteria/objectives, such as supply chain network design, environmental risk with climate change, decision support systems and portfolio management.

Conflict of interest. The authors would like to announce that there is no conflict of interest.

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